Resonances in coupled-channel scattering from Lattice QCD

David Wilson

based on work with the Hadron Spectrum Collaboration

had/spec

hadspec.org

JLab Users Group
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Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

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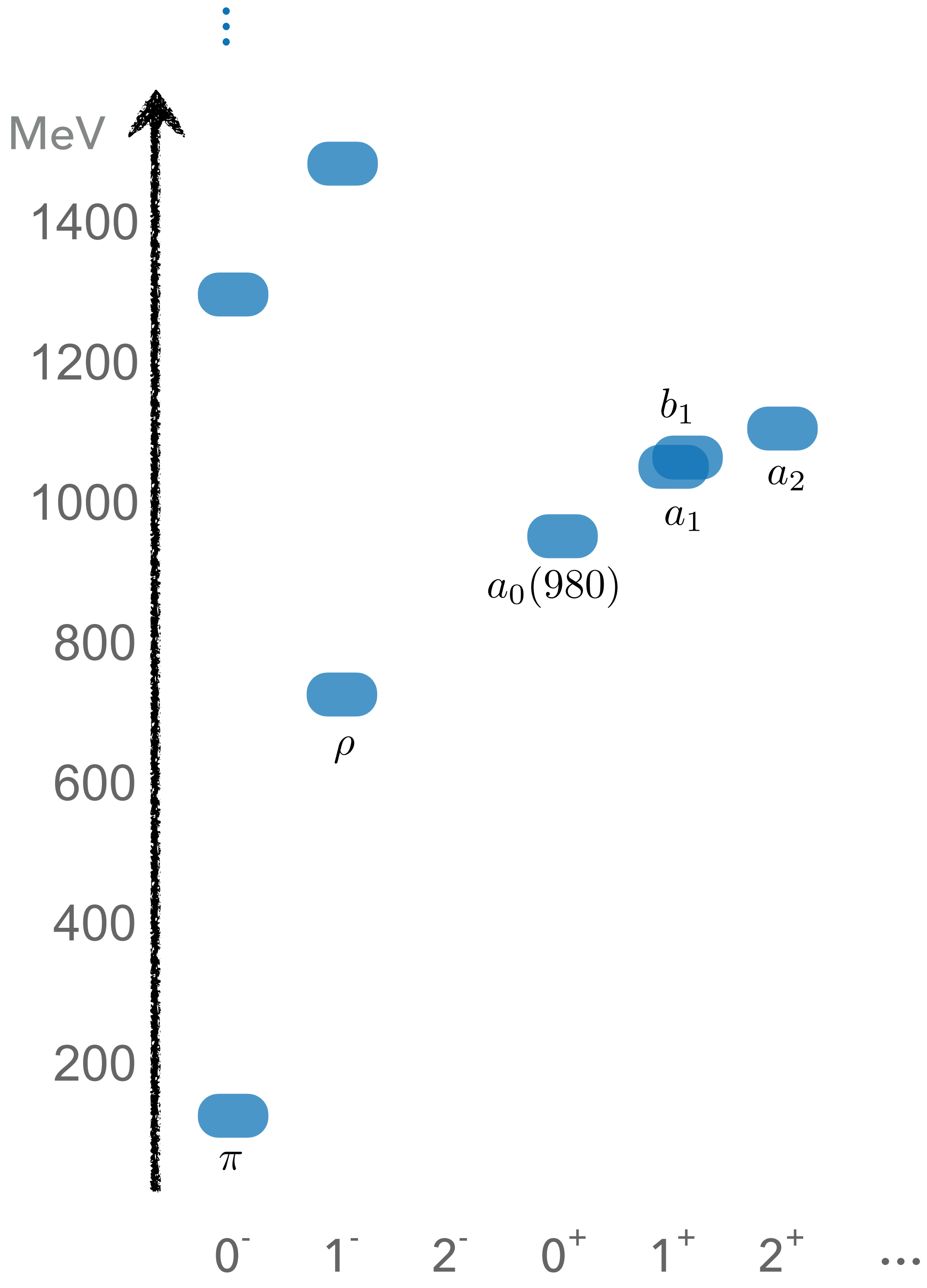
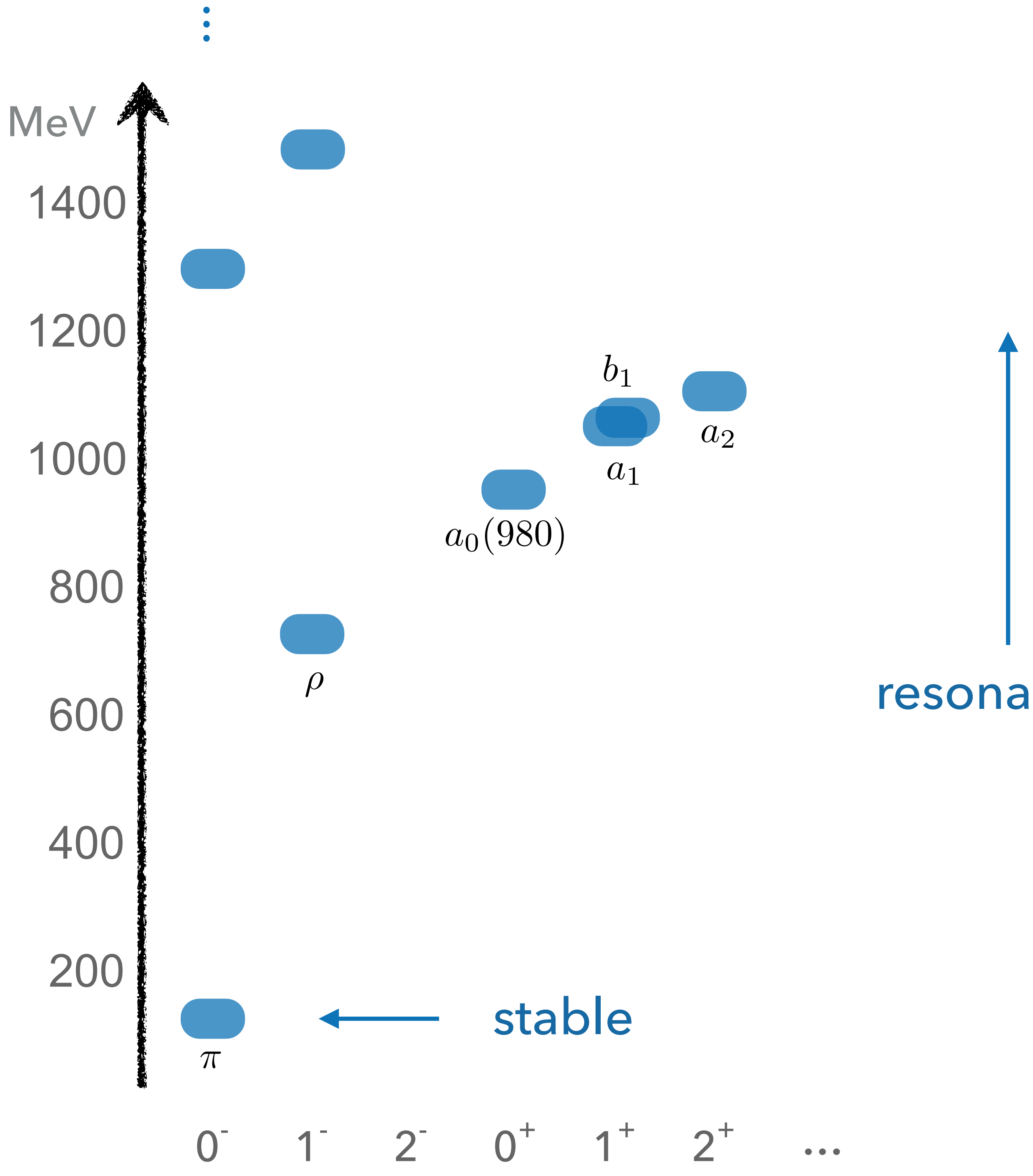


Light mesons with Isospin=1

Scattering and resonances

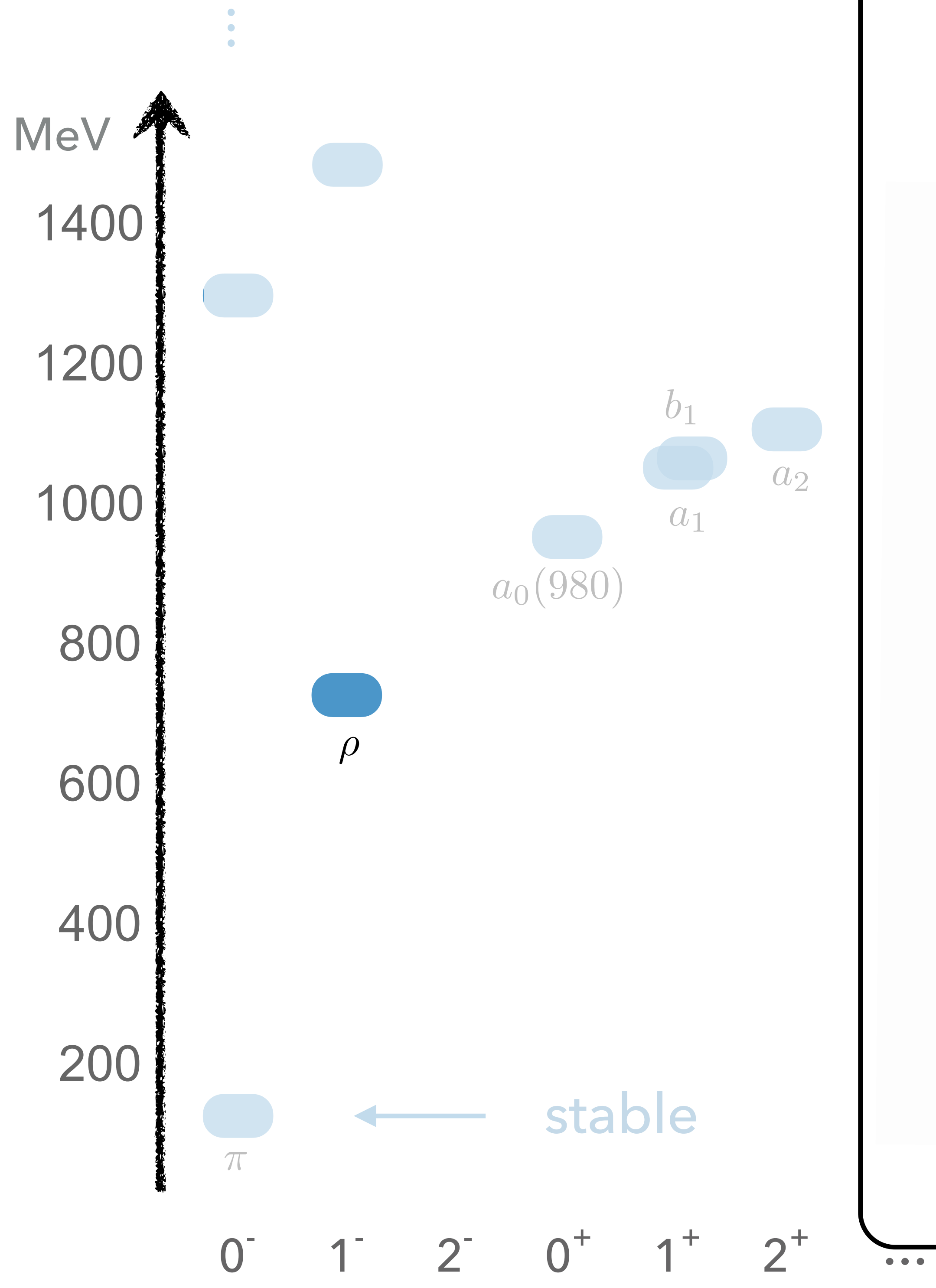
Light mesons with Isospin=1

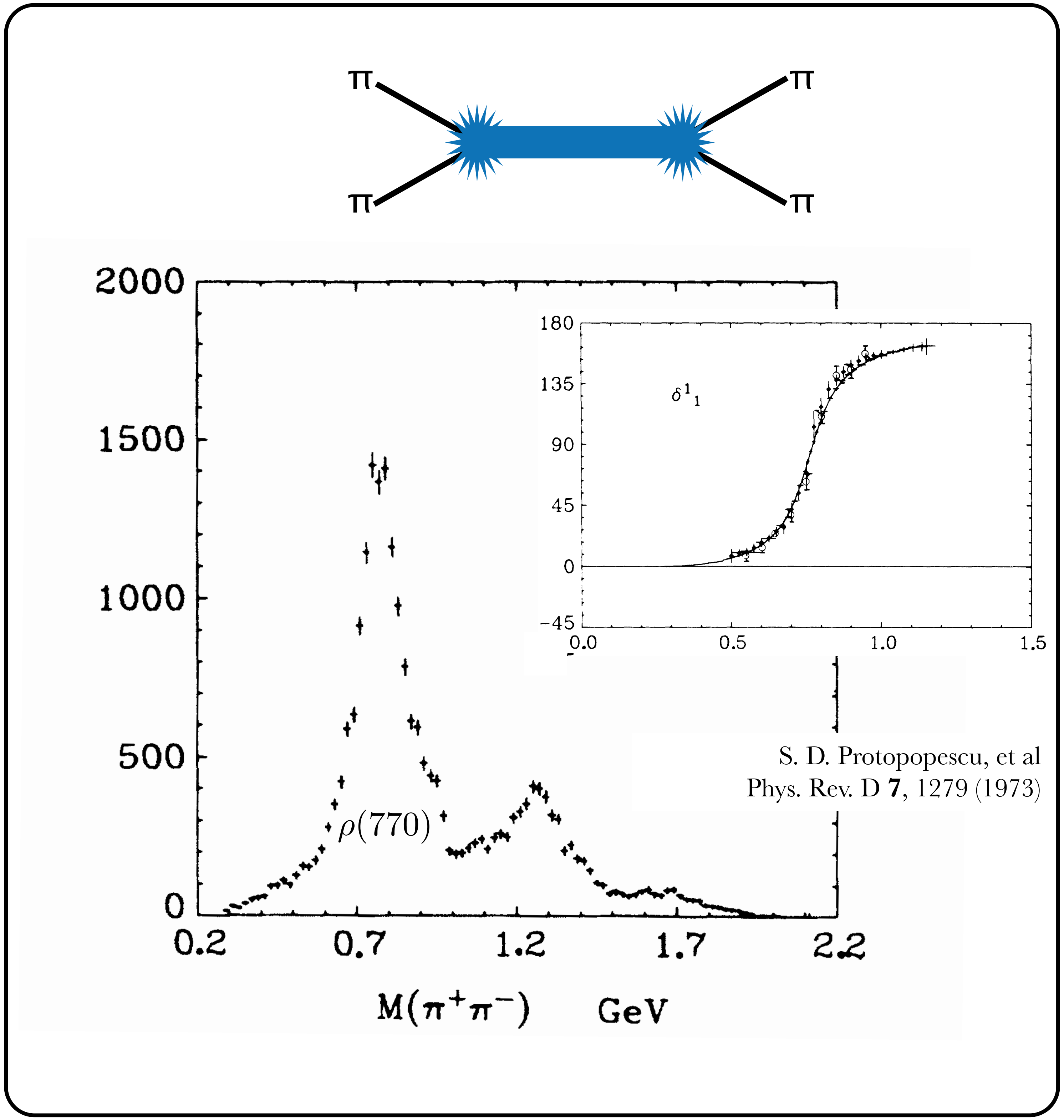


resonances

stable

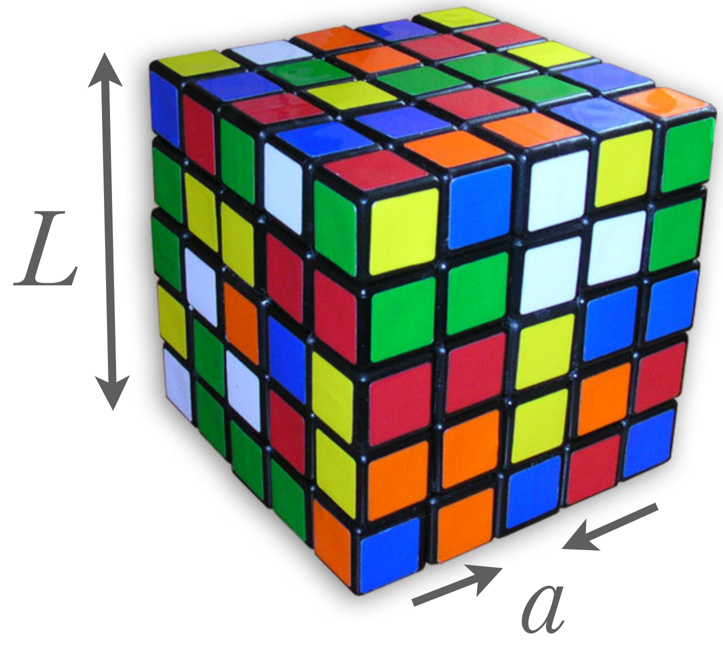
Scattering and resonances



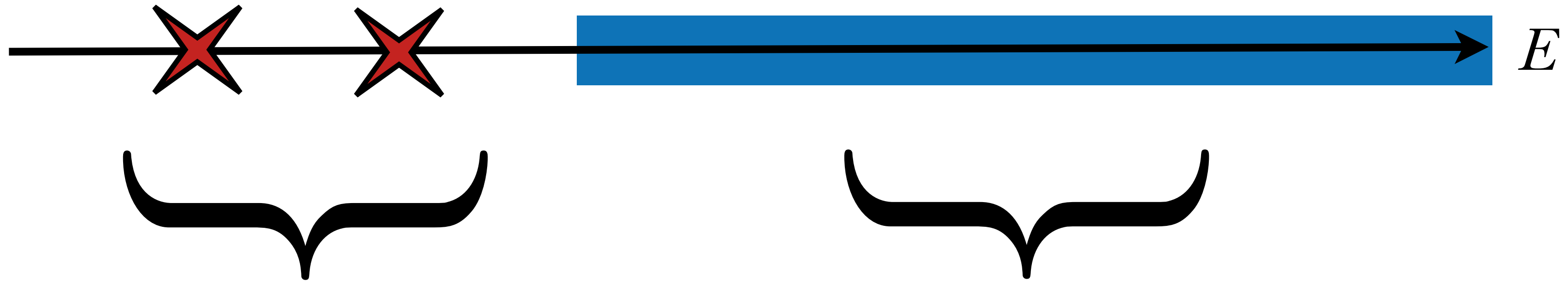


S. D. Protopopescu, et al
Phys. Rev. D **7**, 1279 (1973)

Scattering in a finite volume?



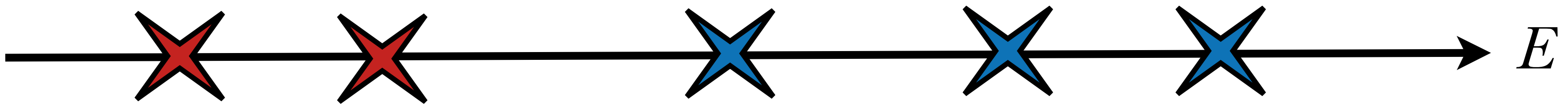
Infinite volume



Bound states

Meson-meson continuum

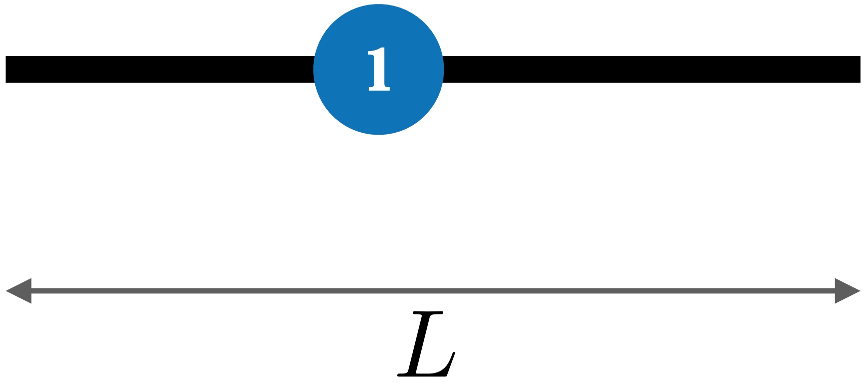
Finite volume

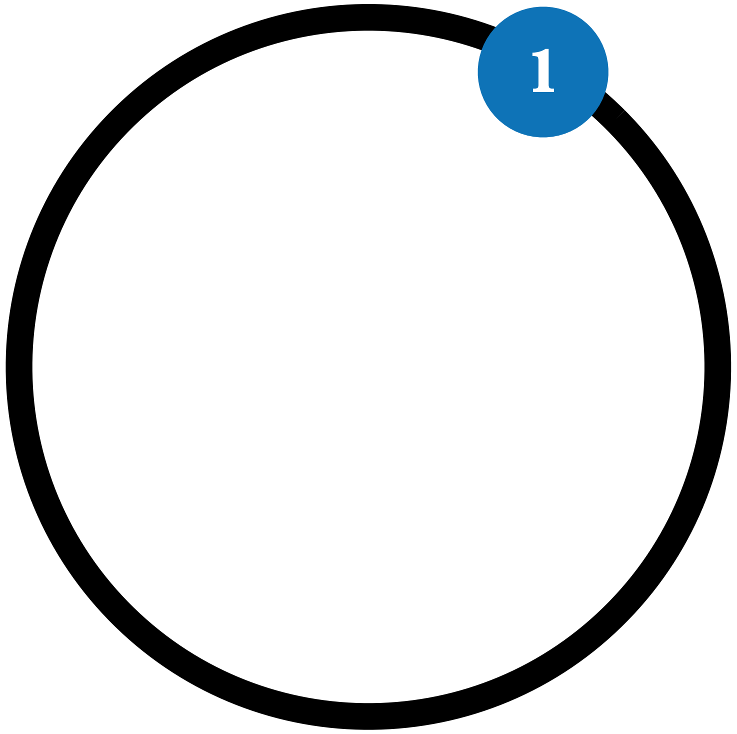


Momentum is quantised - no continuum $\vec{p} = \frac{2\pi}{L}\vec{n}$

Lüscher et al

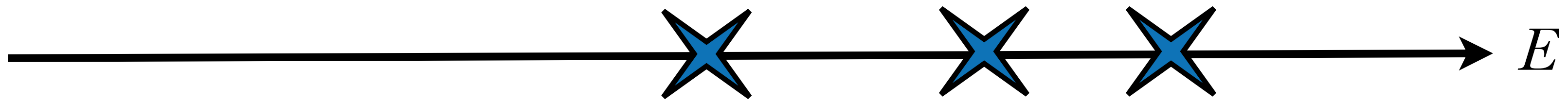
1-dimensional QM, periodic BC, single particle:



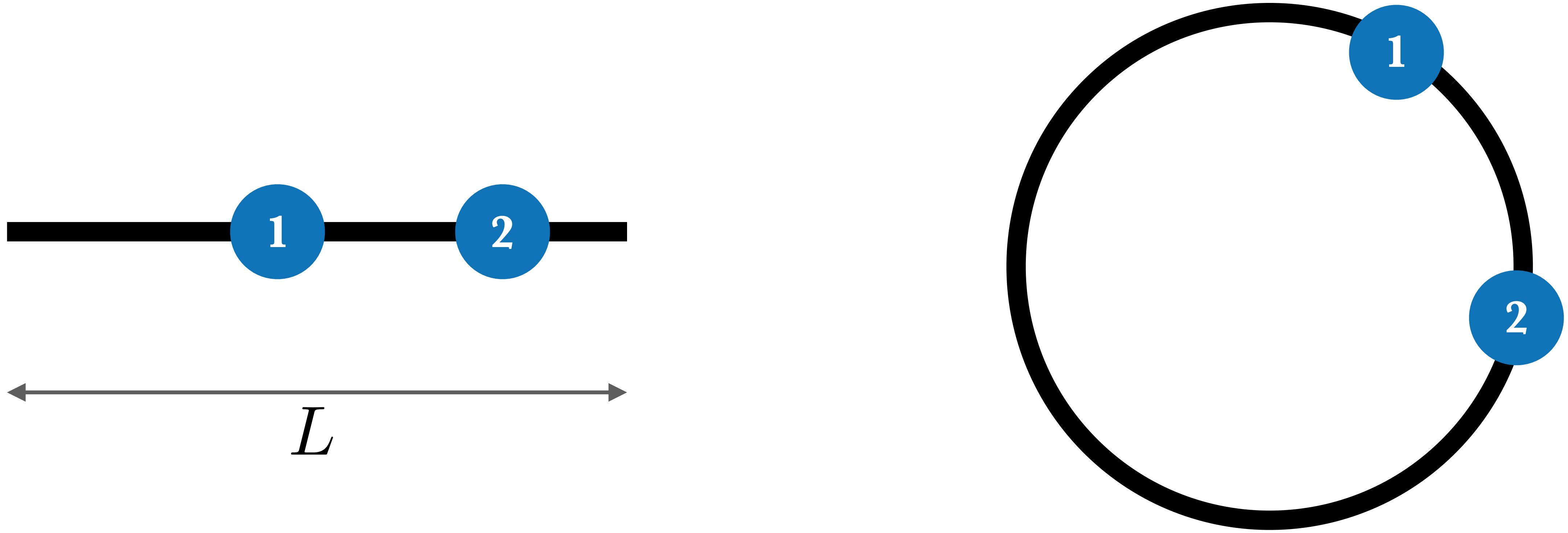


momentum is quantised: $p = \frac{2\pi n}{L}$

Lüscher et al



1-dimensional QM, periodic BC, two particles, no interactions

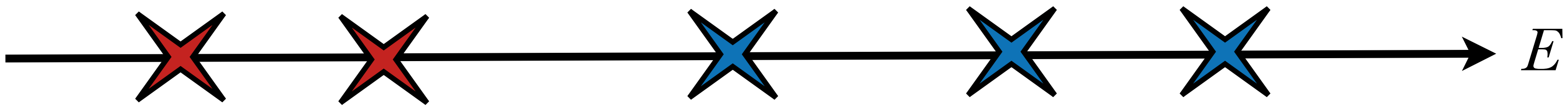


momentum is quantised: $p_i = \frac{2\pi n_i}{L}$

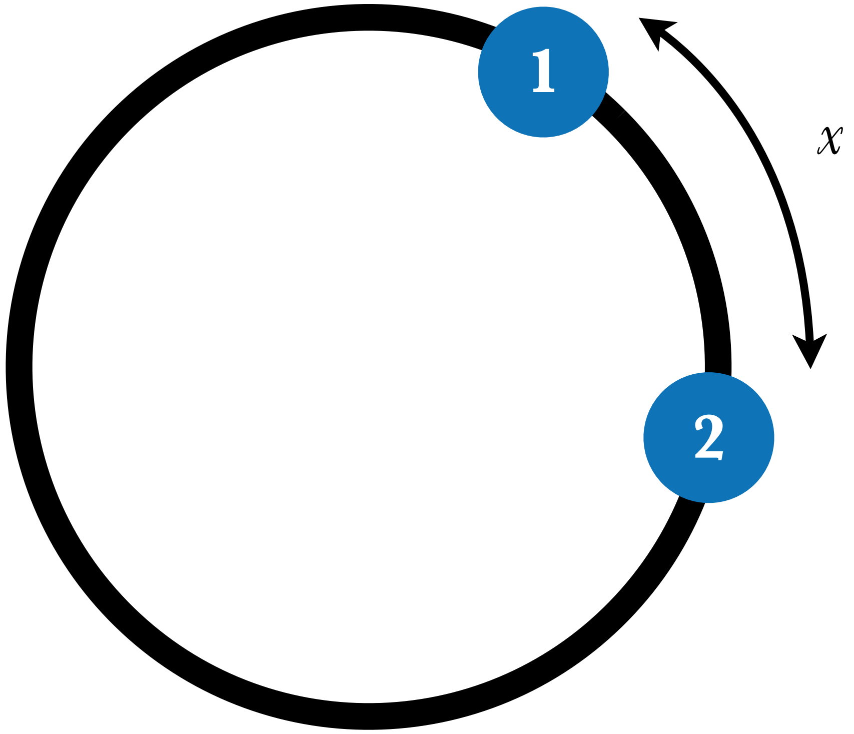
two particle energies are discrete:

$$E = (p_1^2 + m_1^2)^{\frac{1}{2}} + (p_2^2 + m_2^2)^{\frac{1}{2}}$$

Lüscher et al



1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$



$$\psi(0) = \psi(L), \quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$

Phase shifts via Lüscher's method:

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}$$

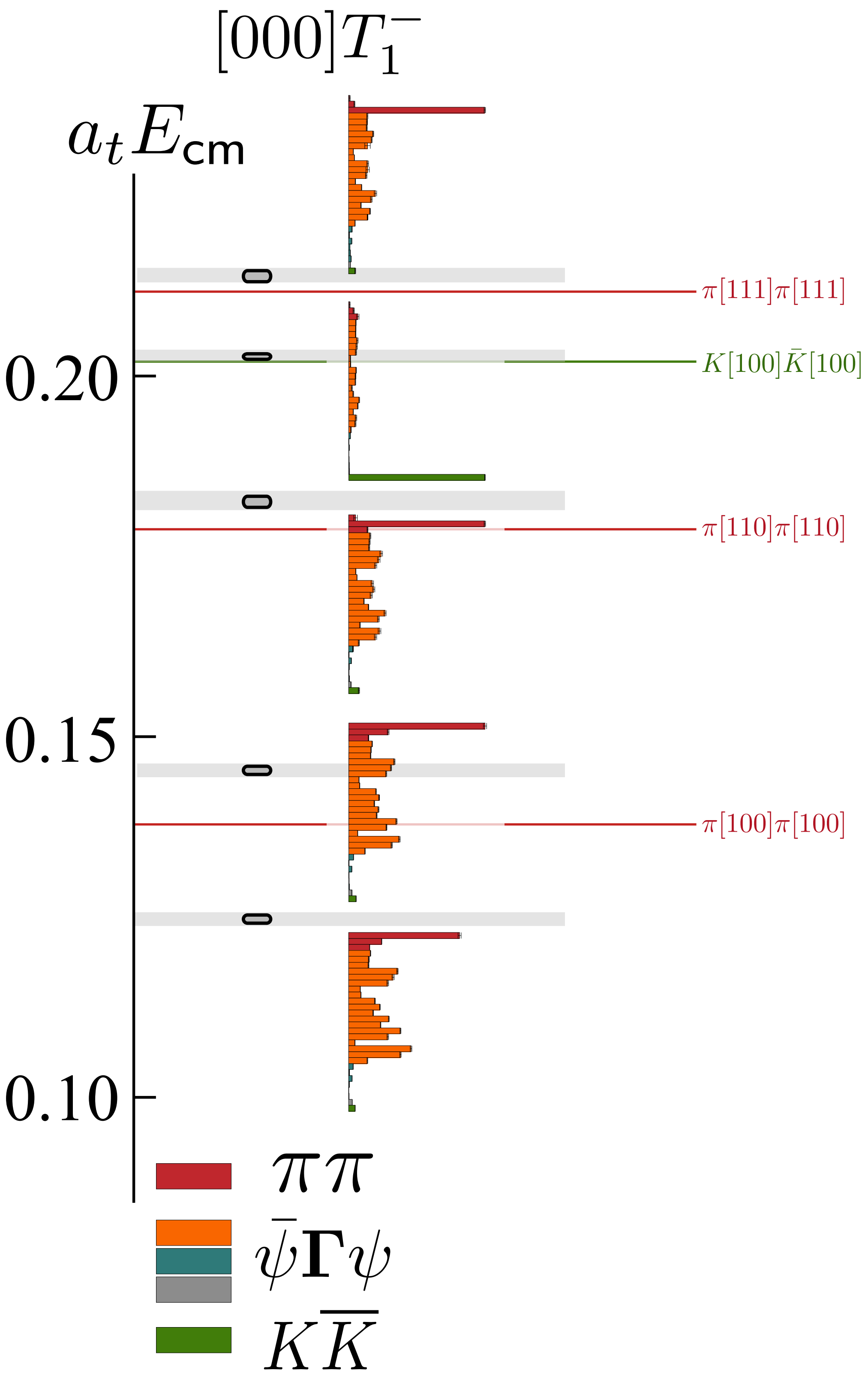
$$\mathcal{Z}_{00}(1; q^2) = \sum_{n \in \mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

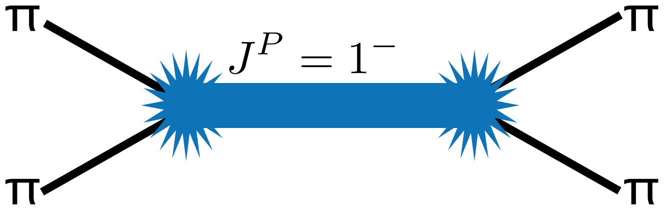
Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

rho resonance at m_π=236 MeV





excited state spectra from a variational method

$$C_{ij}(t) = \langle 0|\, \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\, |0\rangle$$

$$C_{ij}(t)v_j^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t,t_0)C_{ij}(t_0)v_j^{\mathfrak{n}}$$

operators used:

$$\bar{\psi}\Gamma\overleftrightarrow{D}\ldots\overleftrightarrow{D}\;\psi$$

local qq-like constructions

$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\;\Omega_\pi(\vec{p}_2)$$

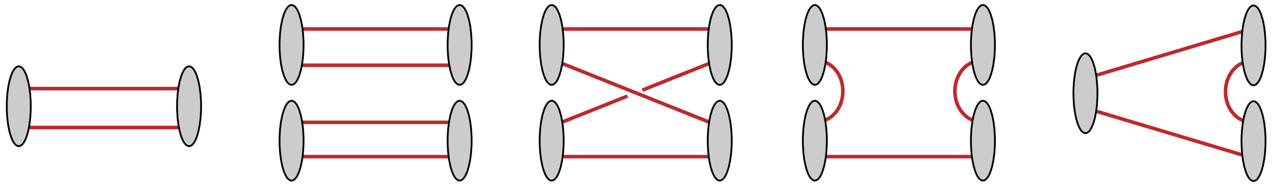
two-hadron constructions

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

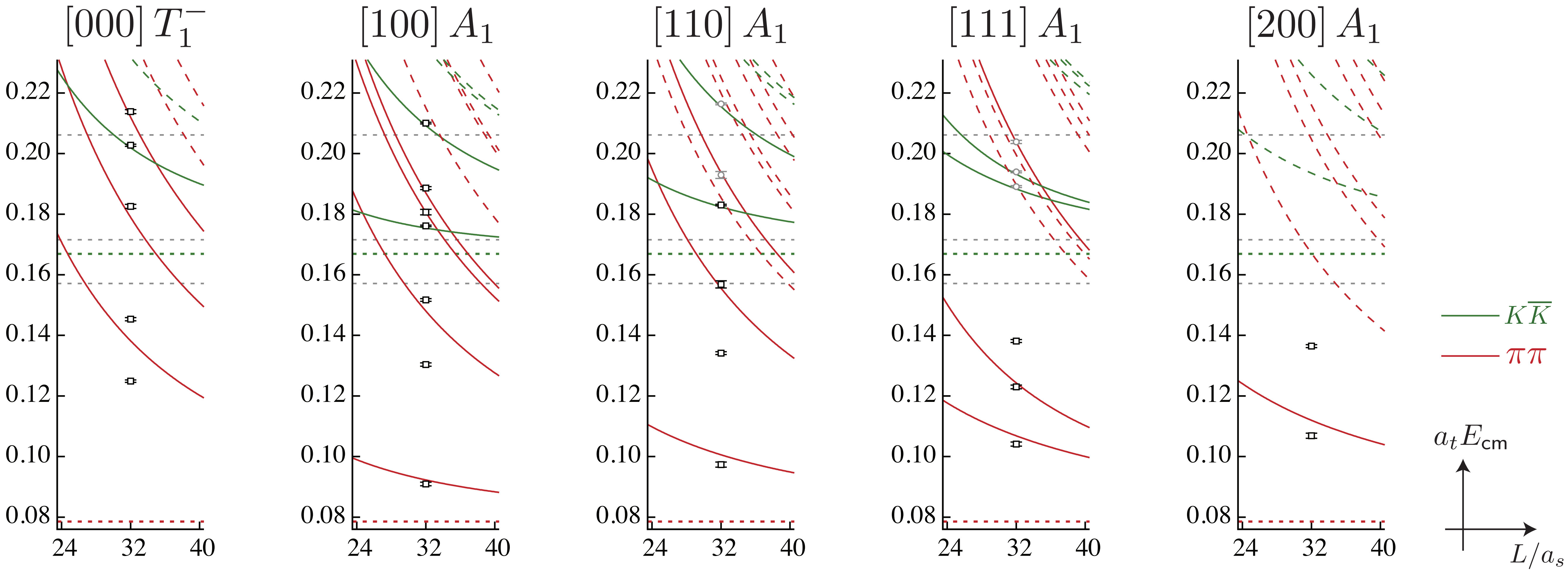
uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)

many wick contractions:



rho resonance at mpi=236 MeV



at-rest

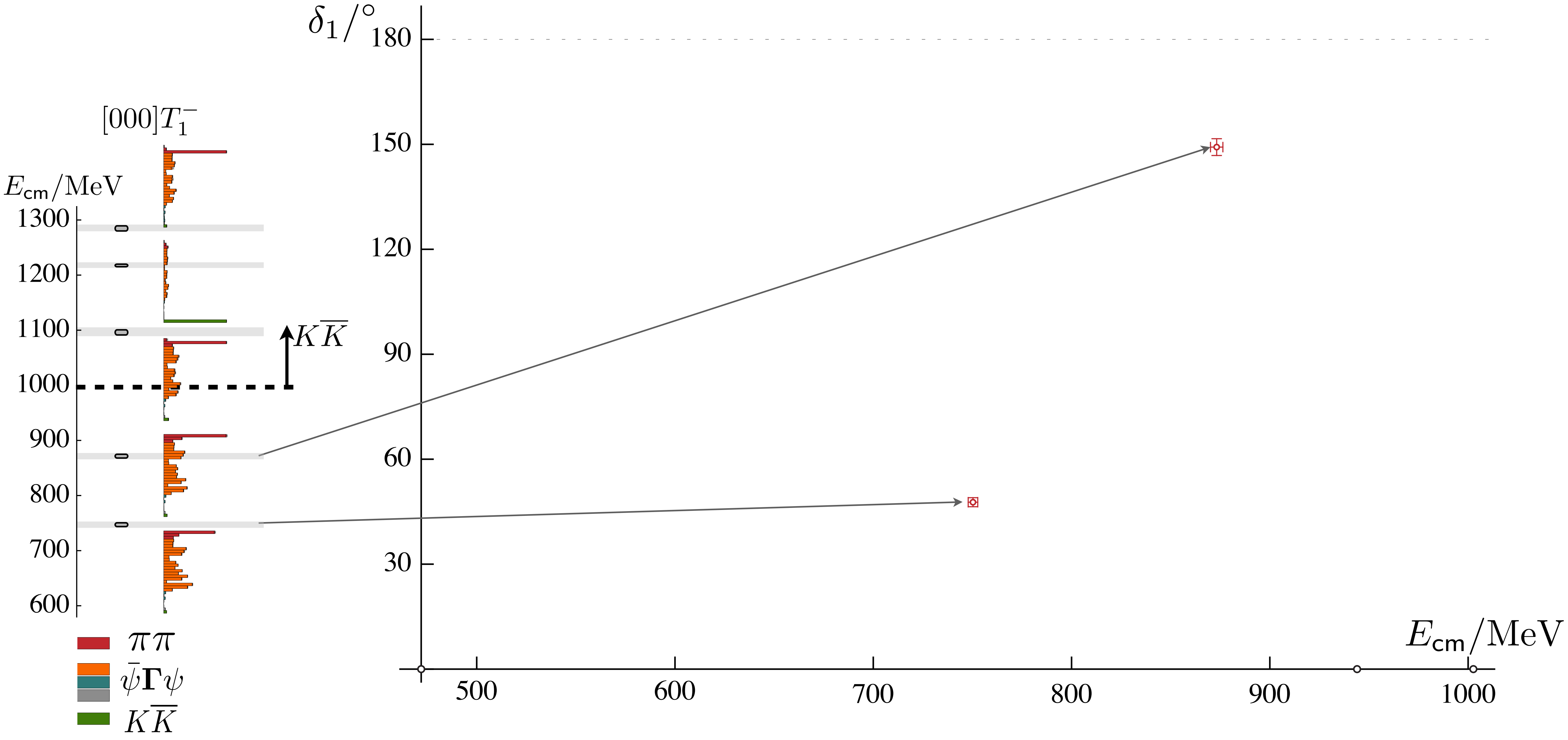
non-zero overall momentum

$m_\pi = 236$ MeV

rho resonance at mpi=236 MeV

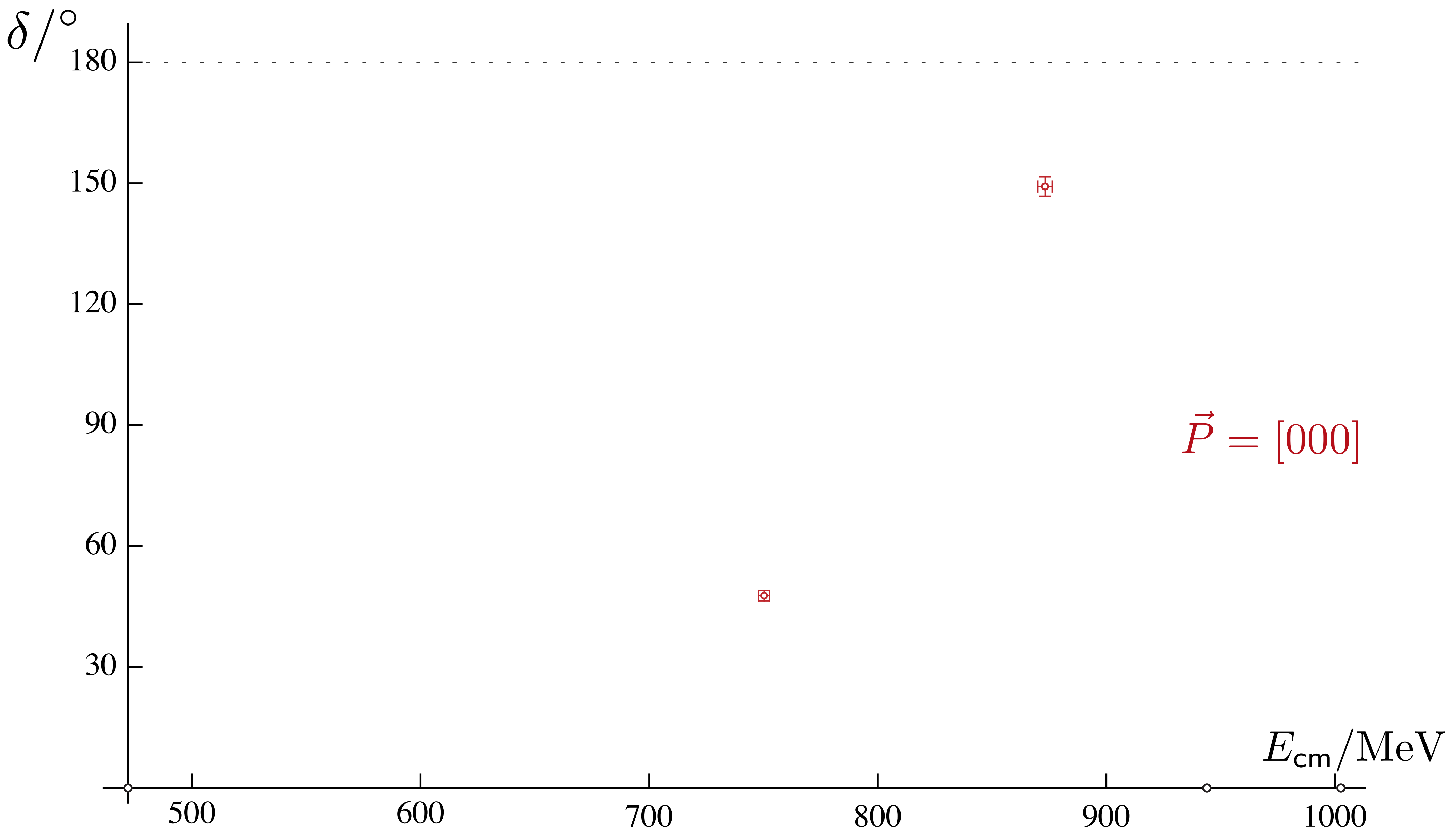
$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$



$$m_\pi = 236\ \mathrm{MeV}$$

rho resonance at mpi=236 MeV



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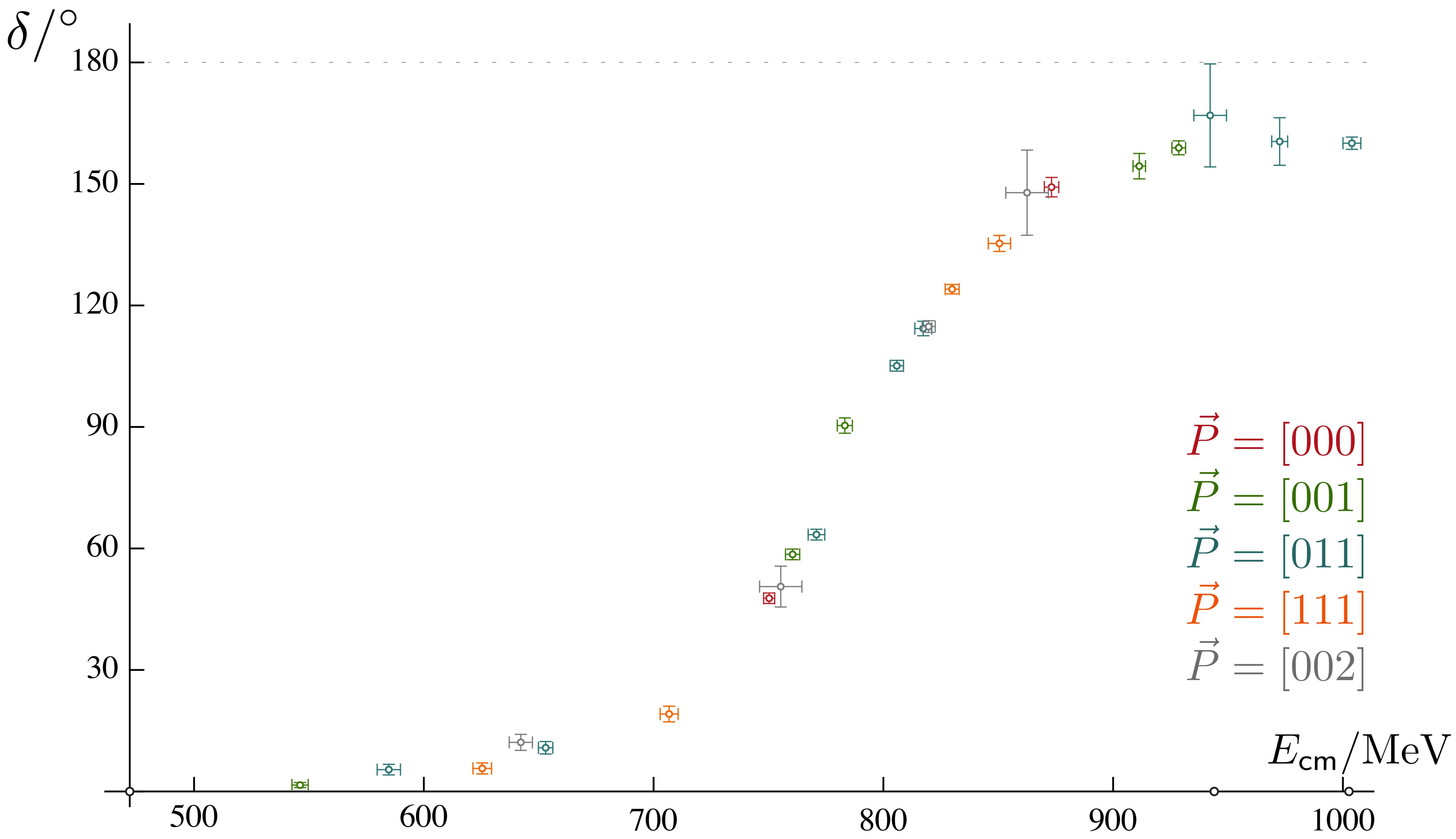

$m_\pi = 236$ MeV

rho resonance

S. D. Protopopescu, et al
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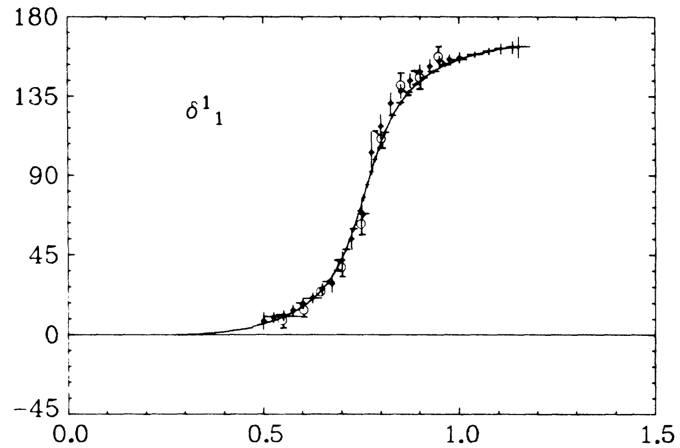
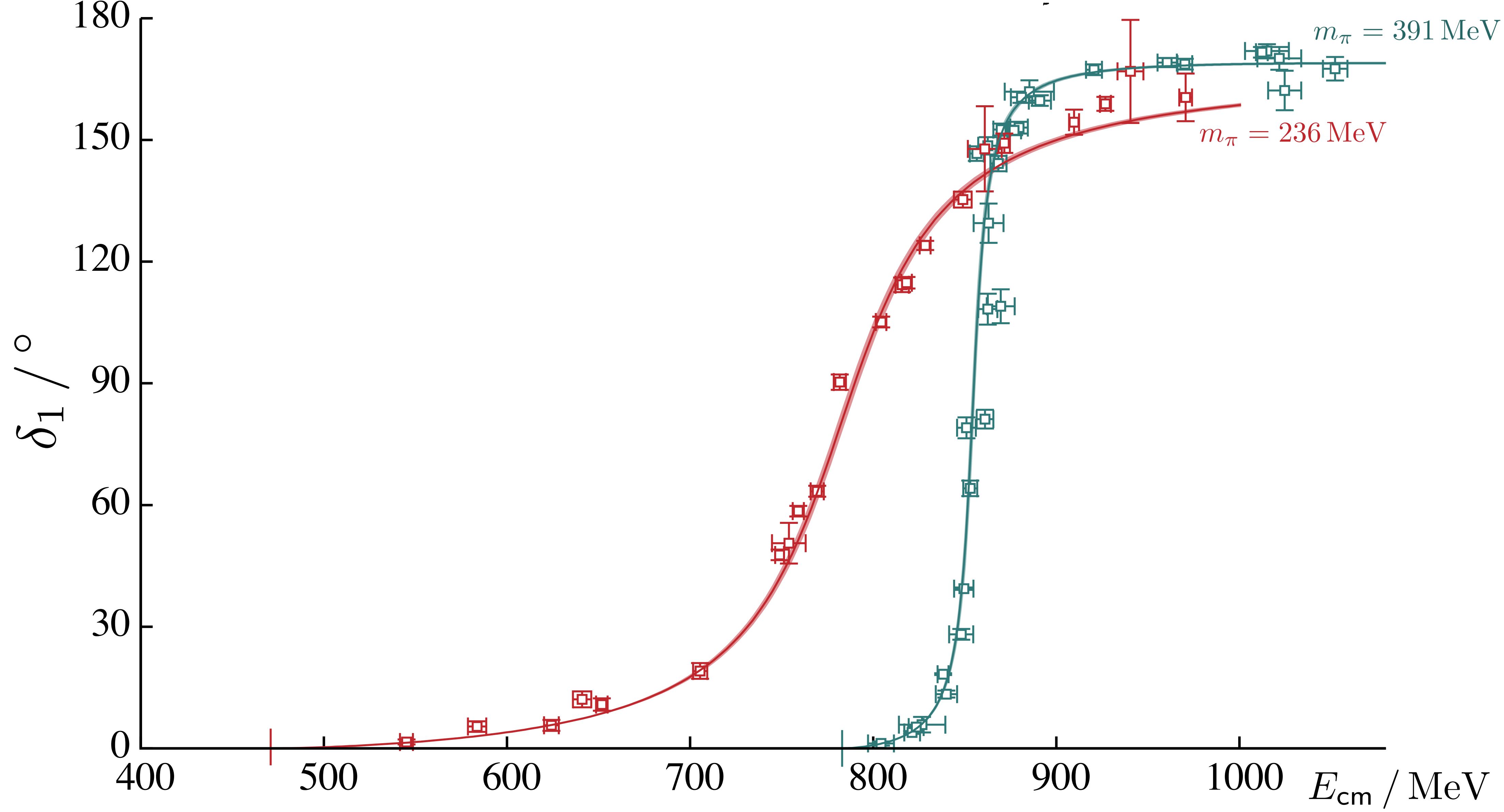
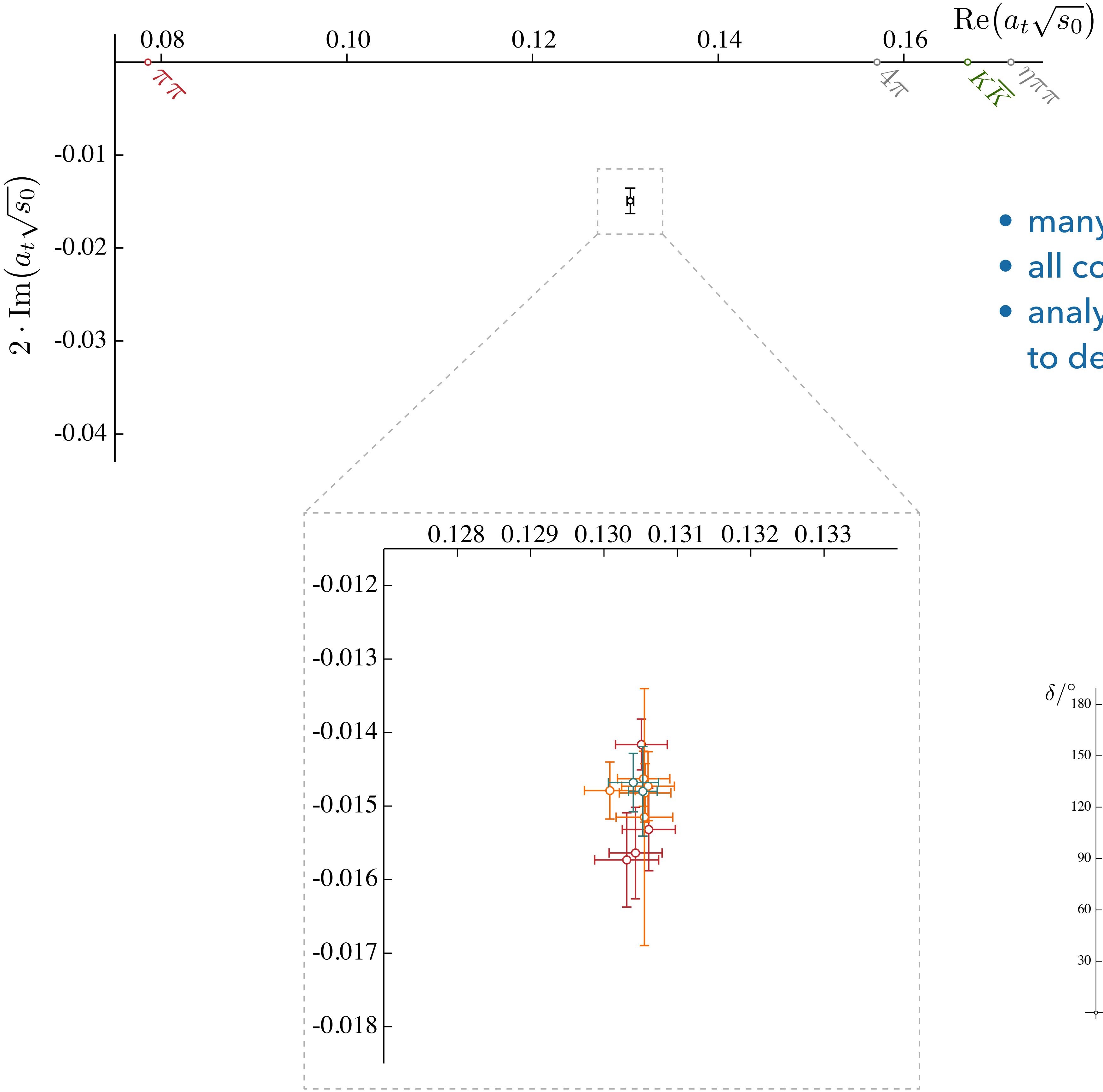




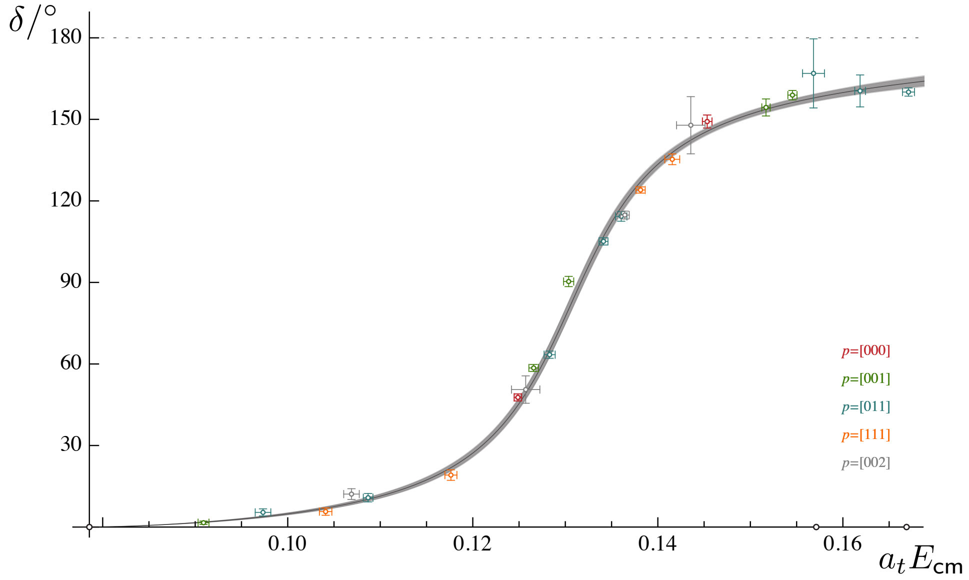
rho resonance pole



near a pole:

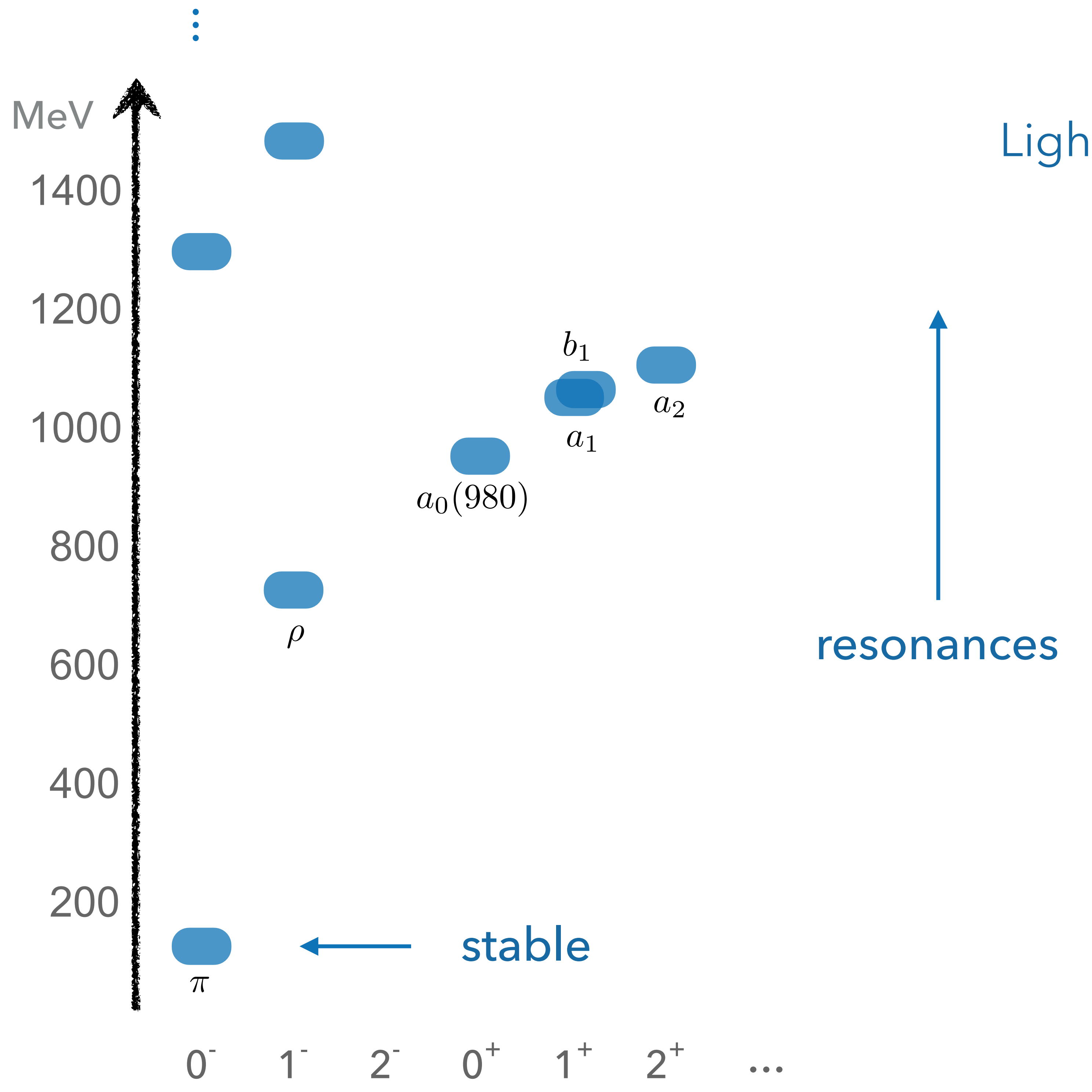
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

- many parameterisations of *t*
- all consistent are for real energies
- analytically continue to complex energies to determine t-matrix poles

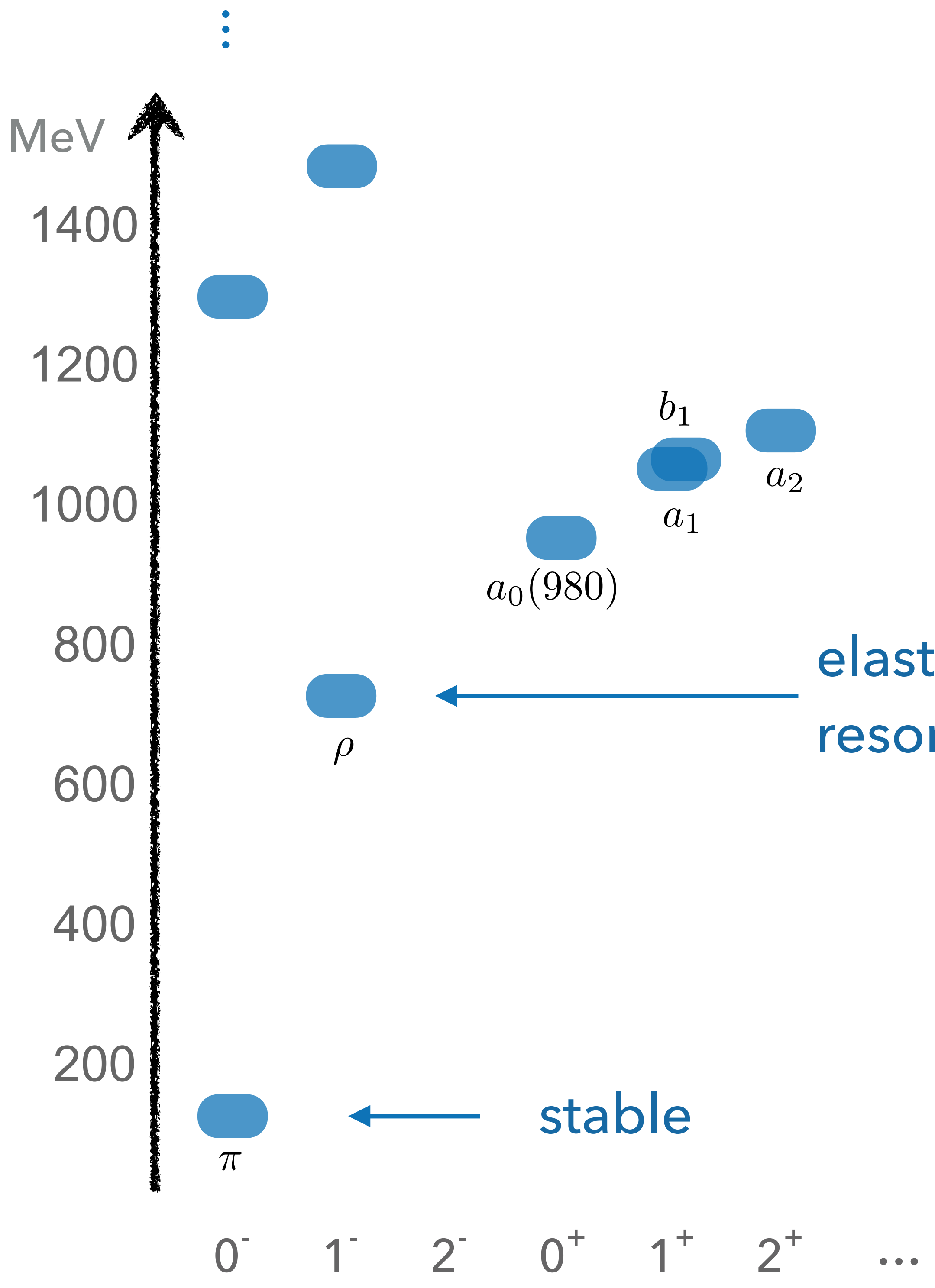


Scattering and resonances

Light mesons with Isospin=1



Coupled-channel scattering



stable

elastic resonances

resonances in multiple channels

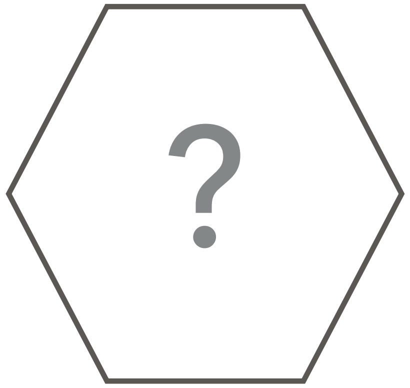
e.g. $\pi\eta \to \pi\eta$

$\to K\bar{K}$

$\pi\pi \to \pi\pi$

$\to K\bar{K}$

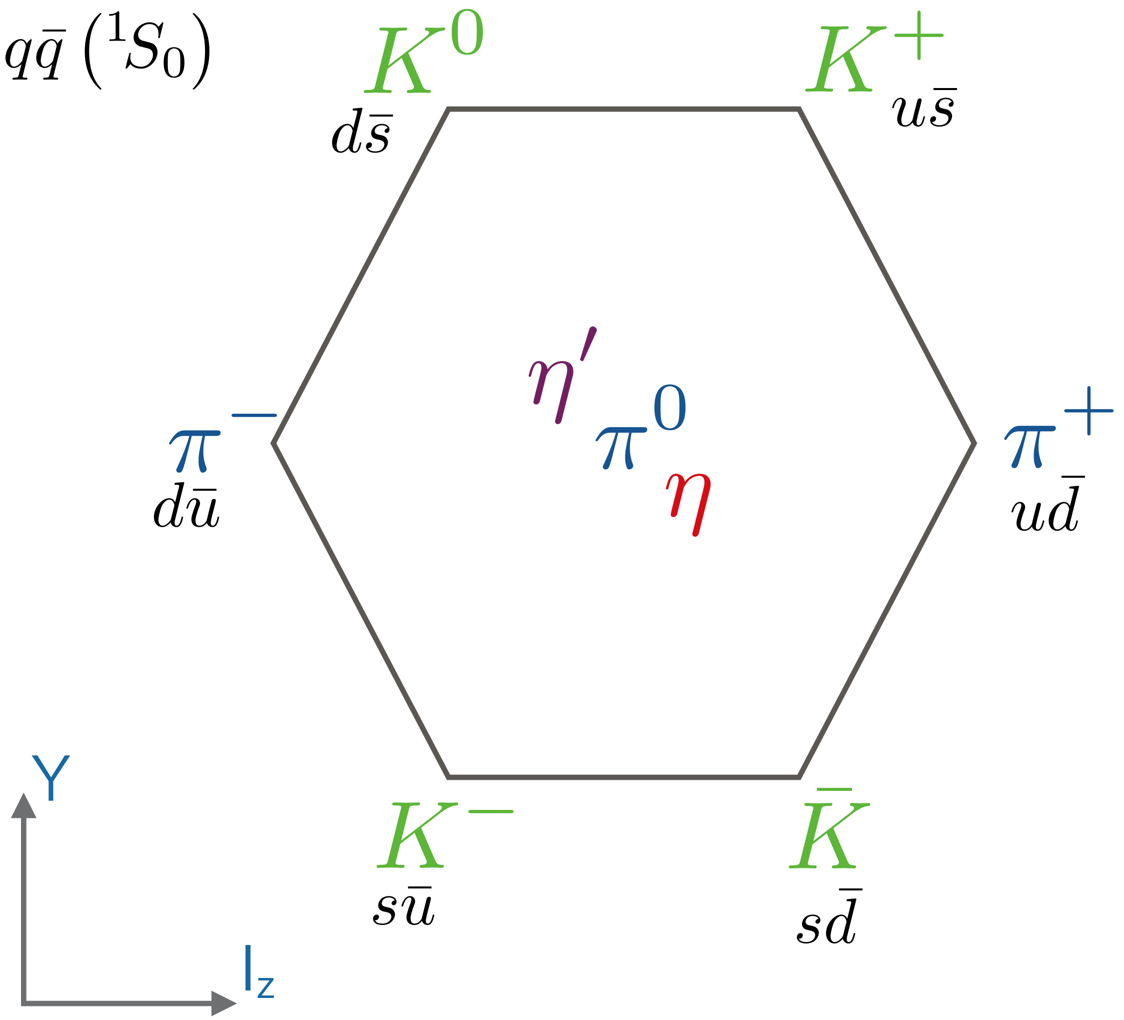
How do we understand the light scalars?



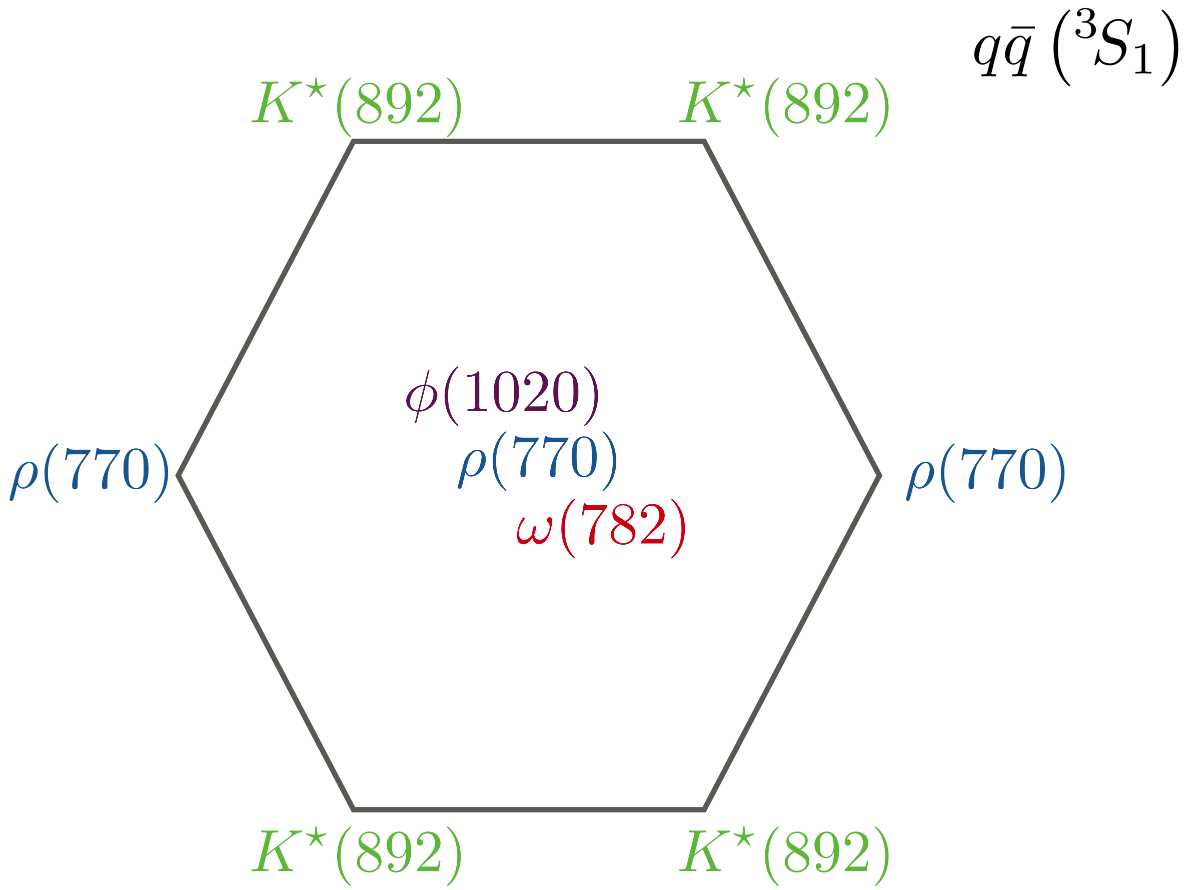
Quark model

Considering *u,d,s* quarks: $q\bar{q}\,({}^{2S+1}L_J)$

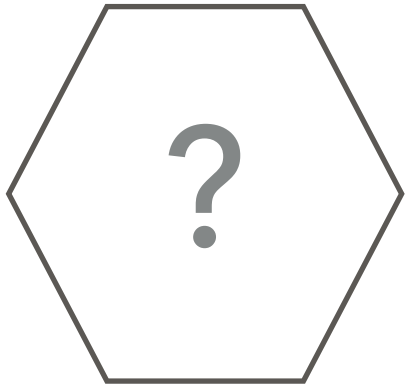
pseudoscalar nonet $J^P=0^-$

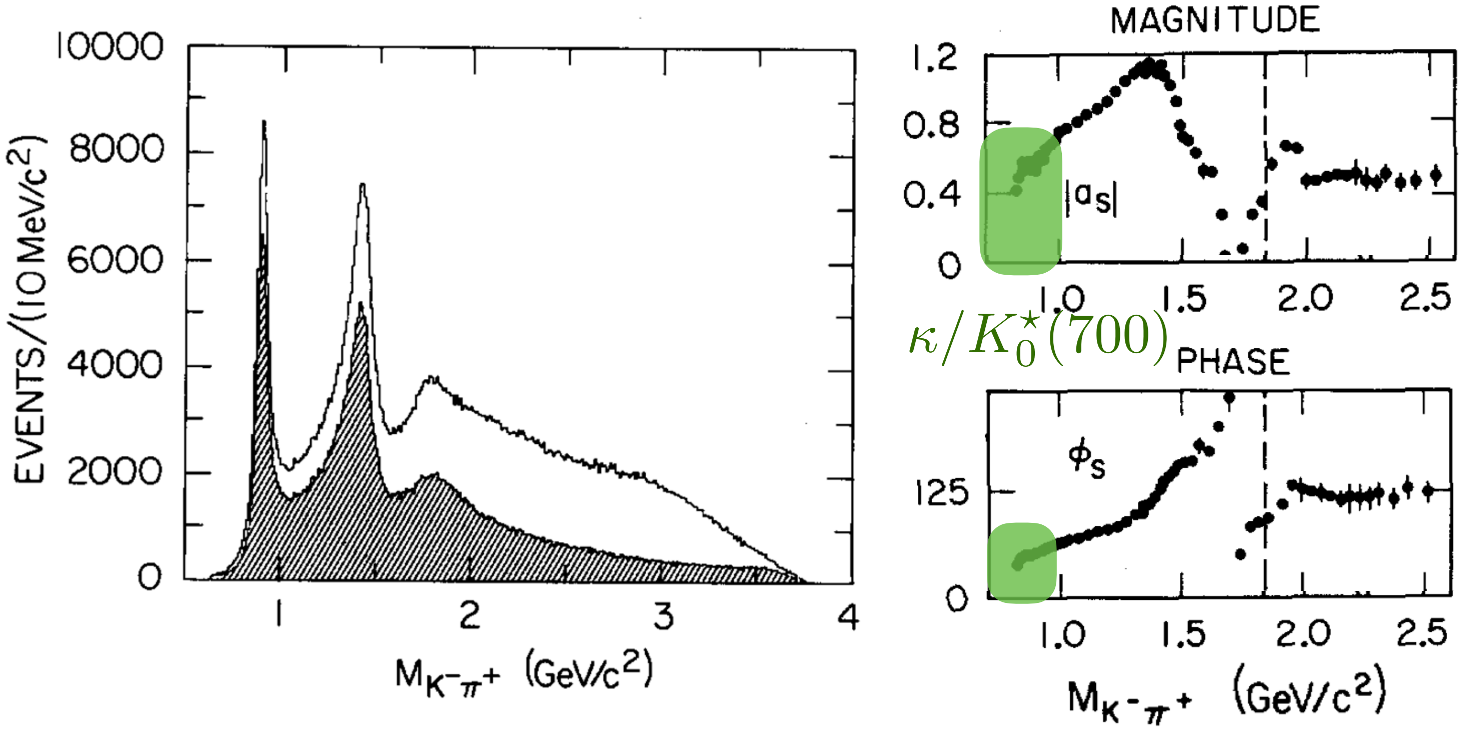


vector nonet $J^P=1^-$



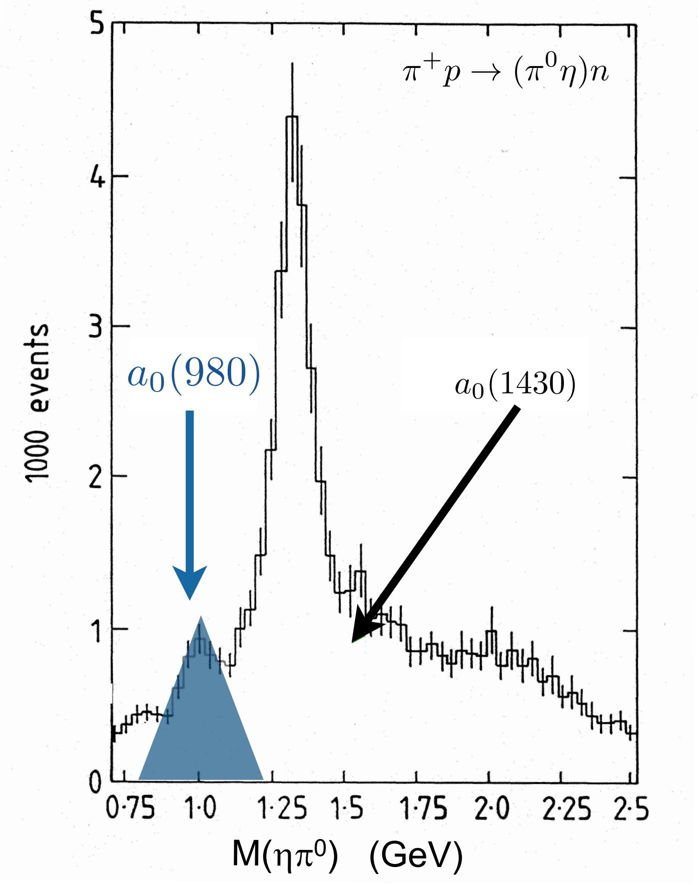
How do we understand the scalars?

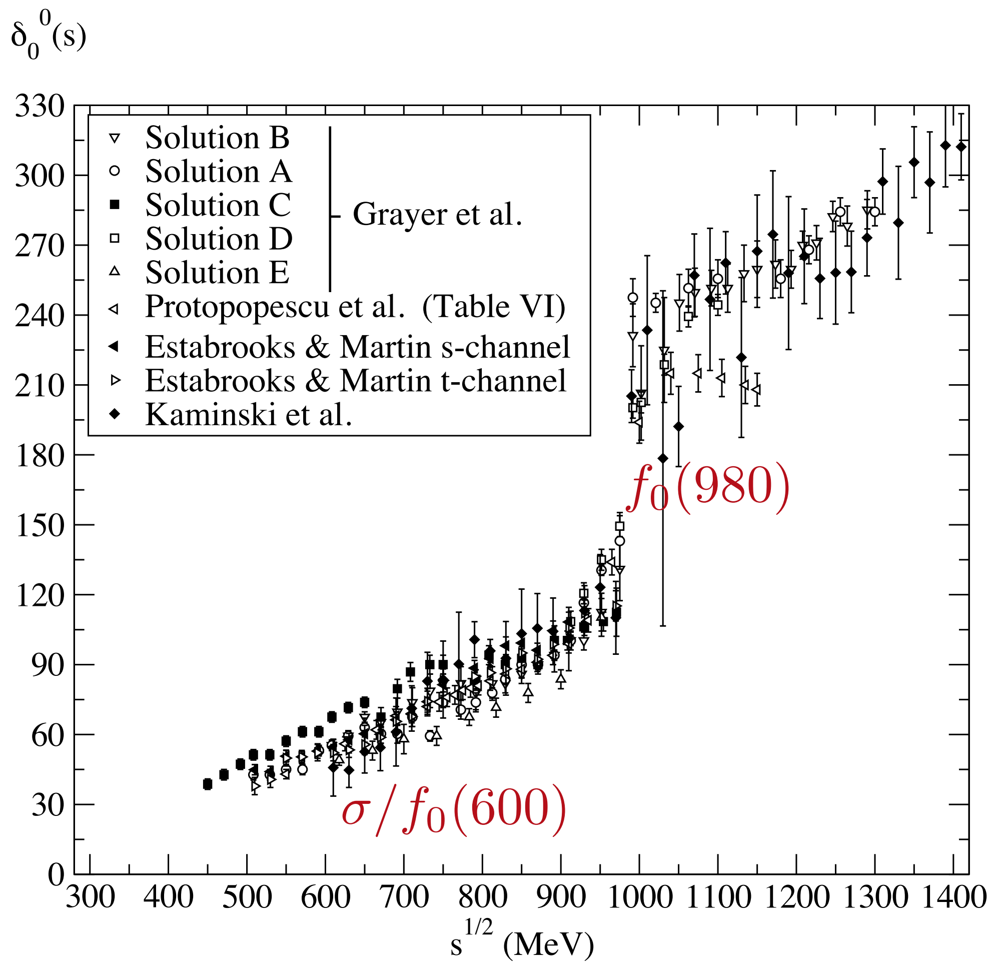




LASS experiment at SLAC $E_K = 11$ GeV

GAMS, Alde *et al* PLB 203 397, 1988.





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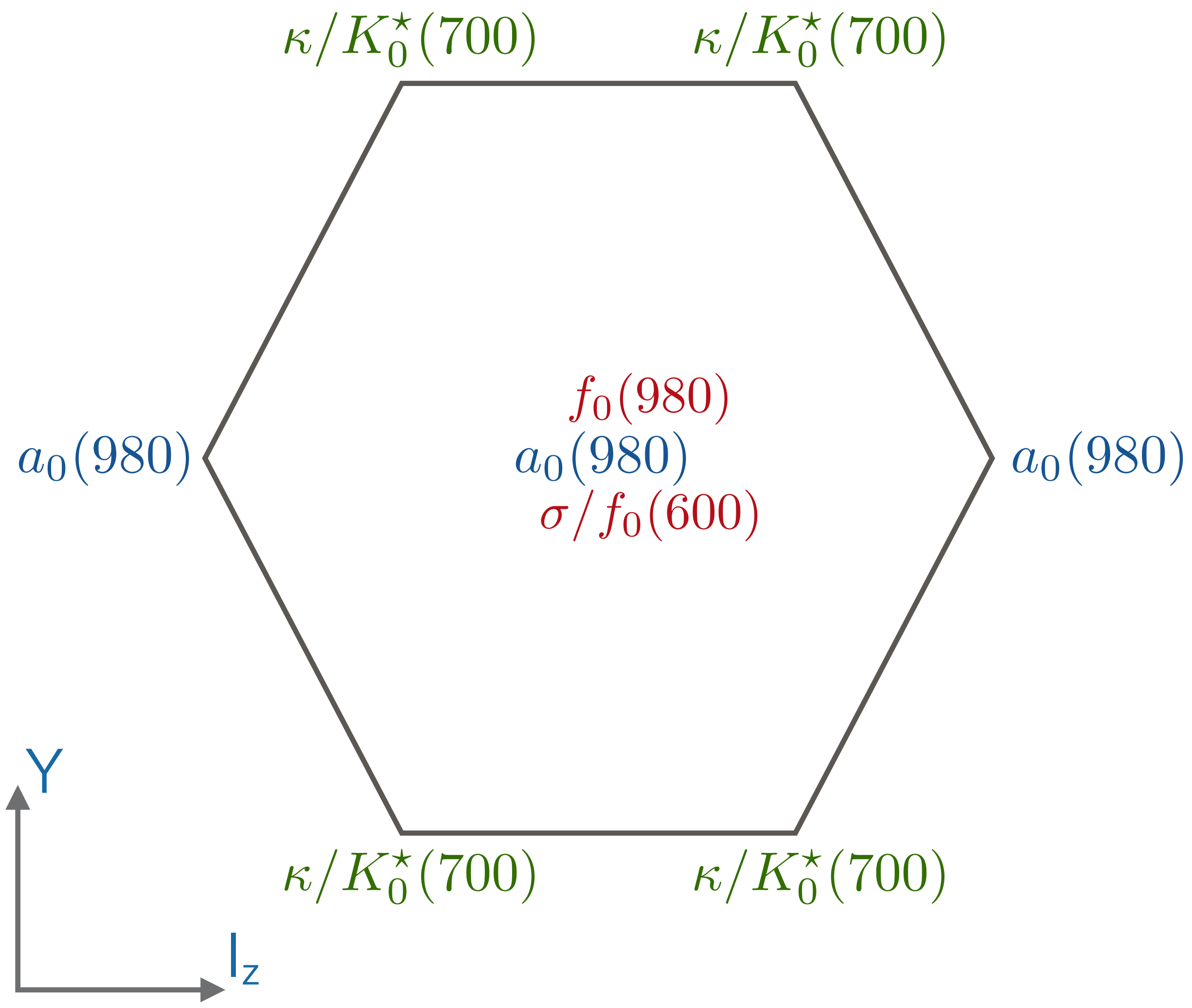
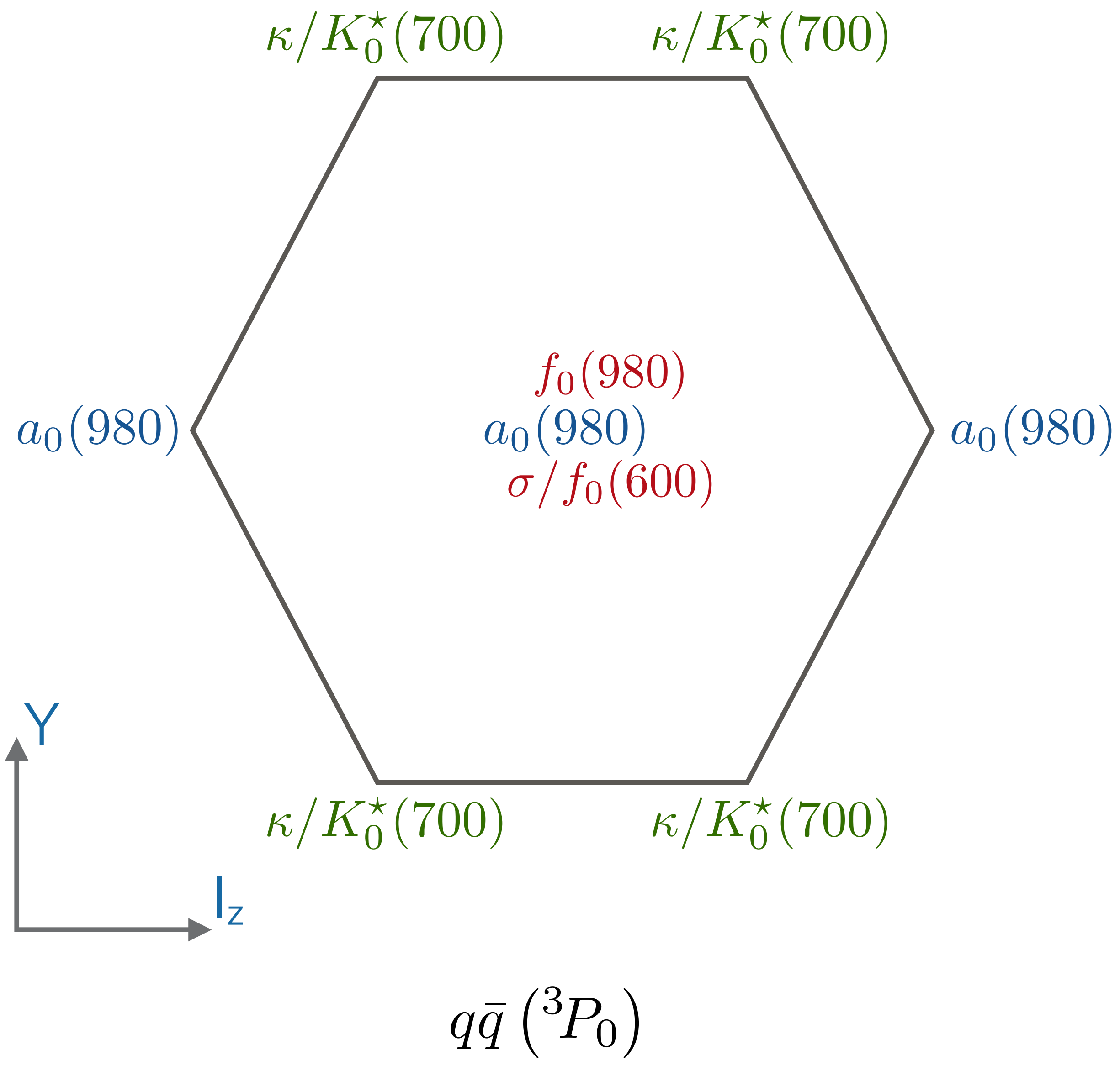
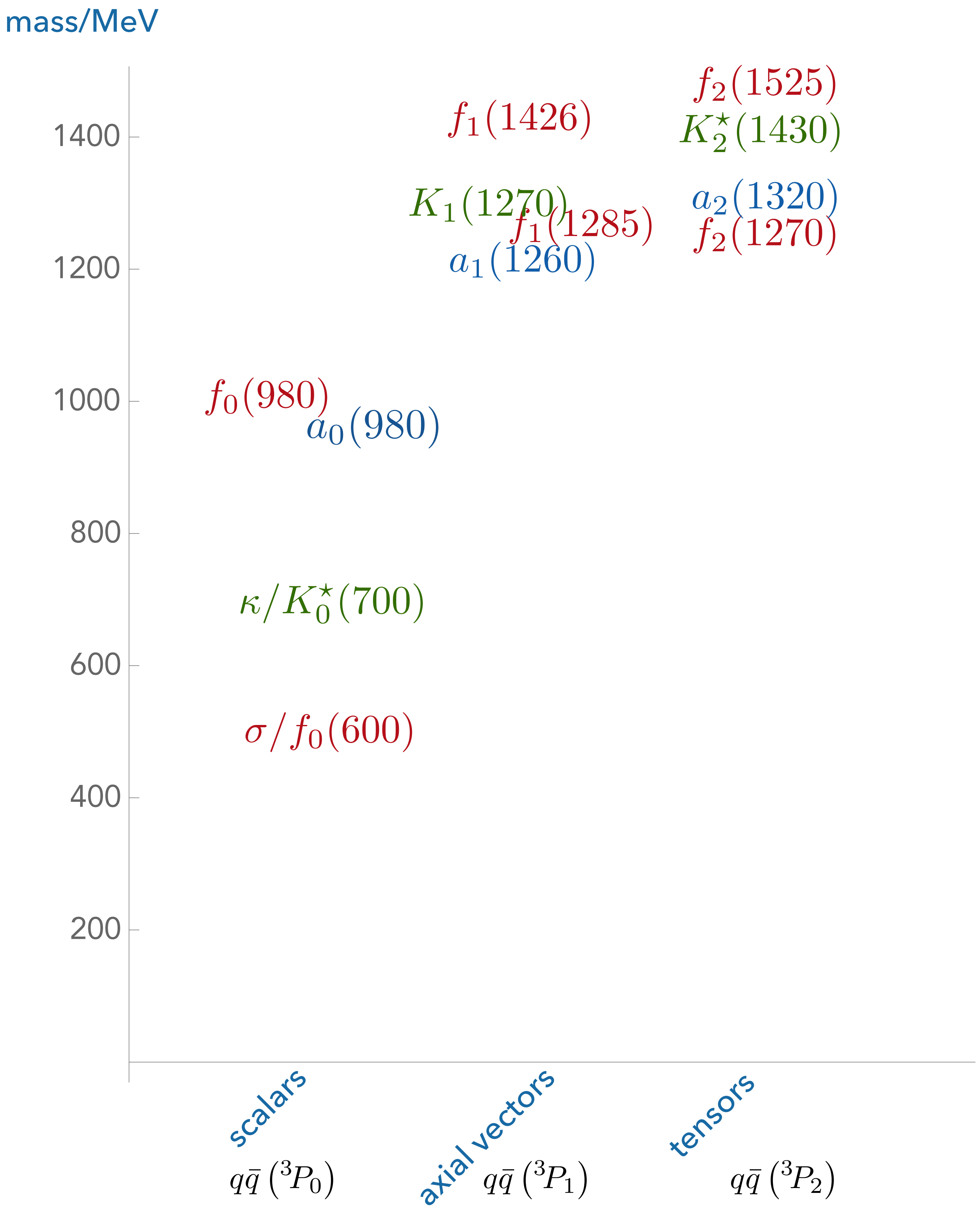


$q\bar{q}\left({}^3P_0\right)$

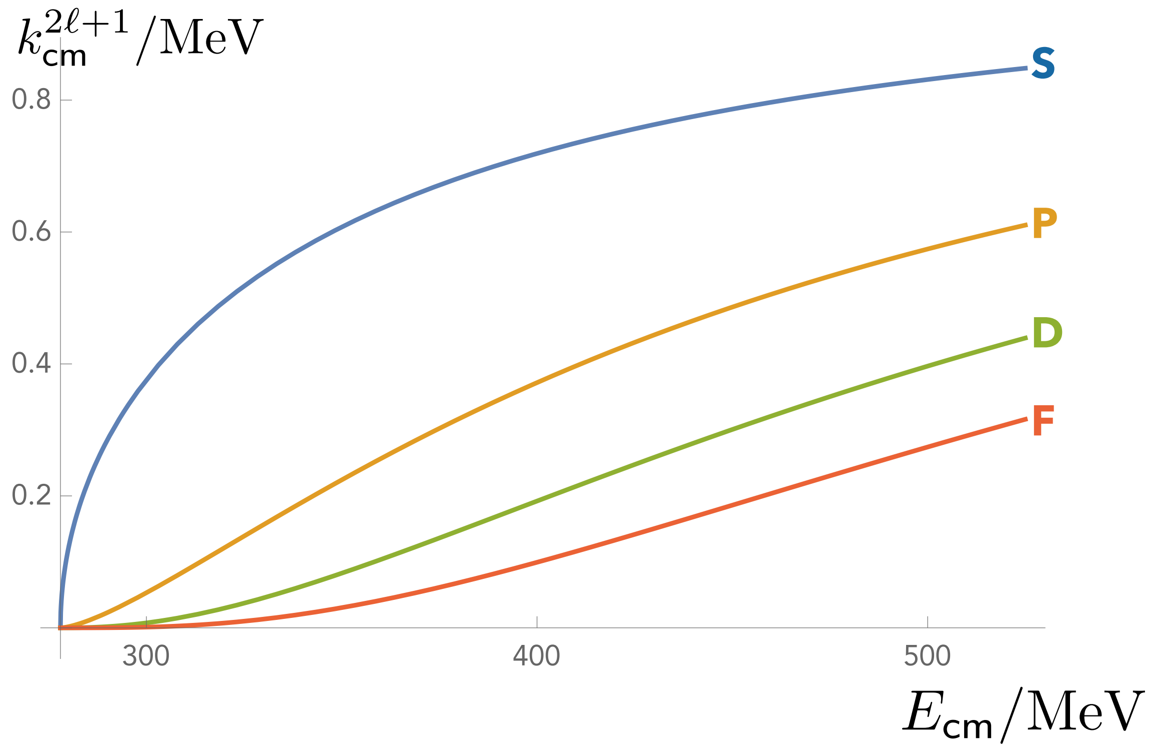
What about the masses?





Resonance physics is important

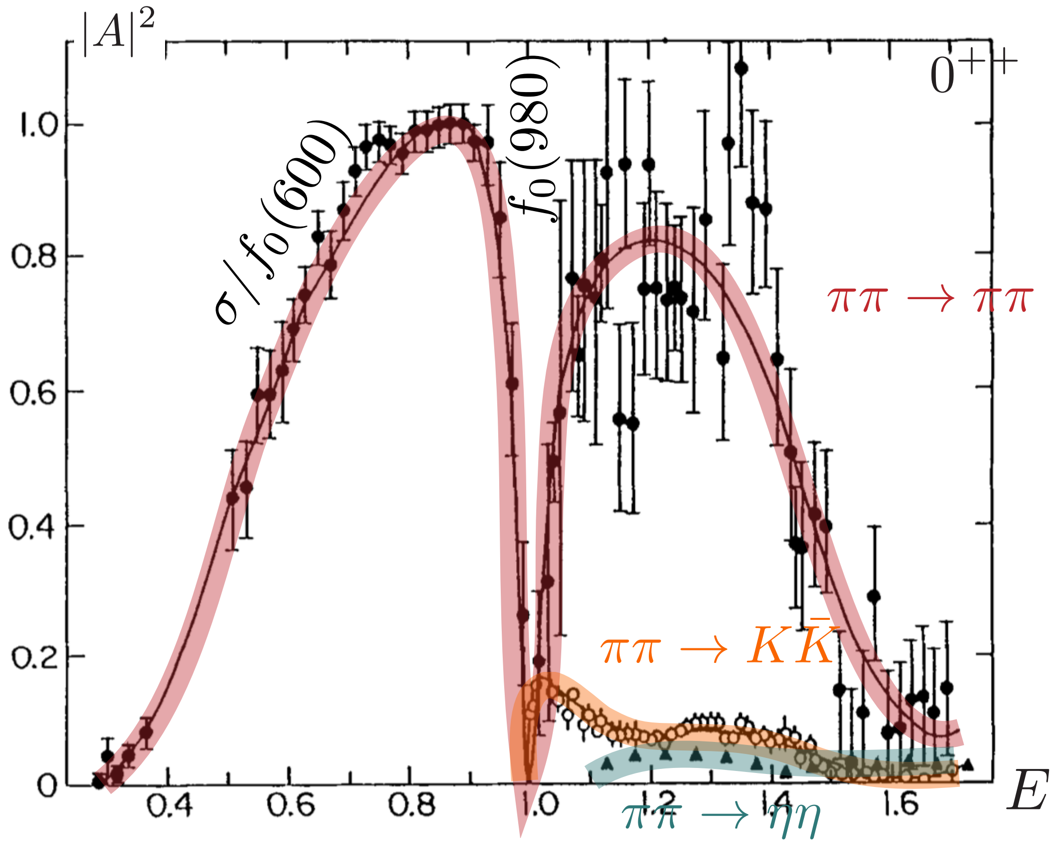
In the scalar sector, amplitudes grow rapidly from threshold:



σ and κ are broad (width ~ mass)

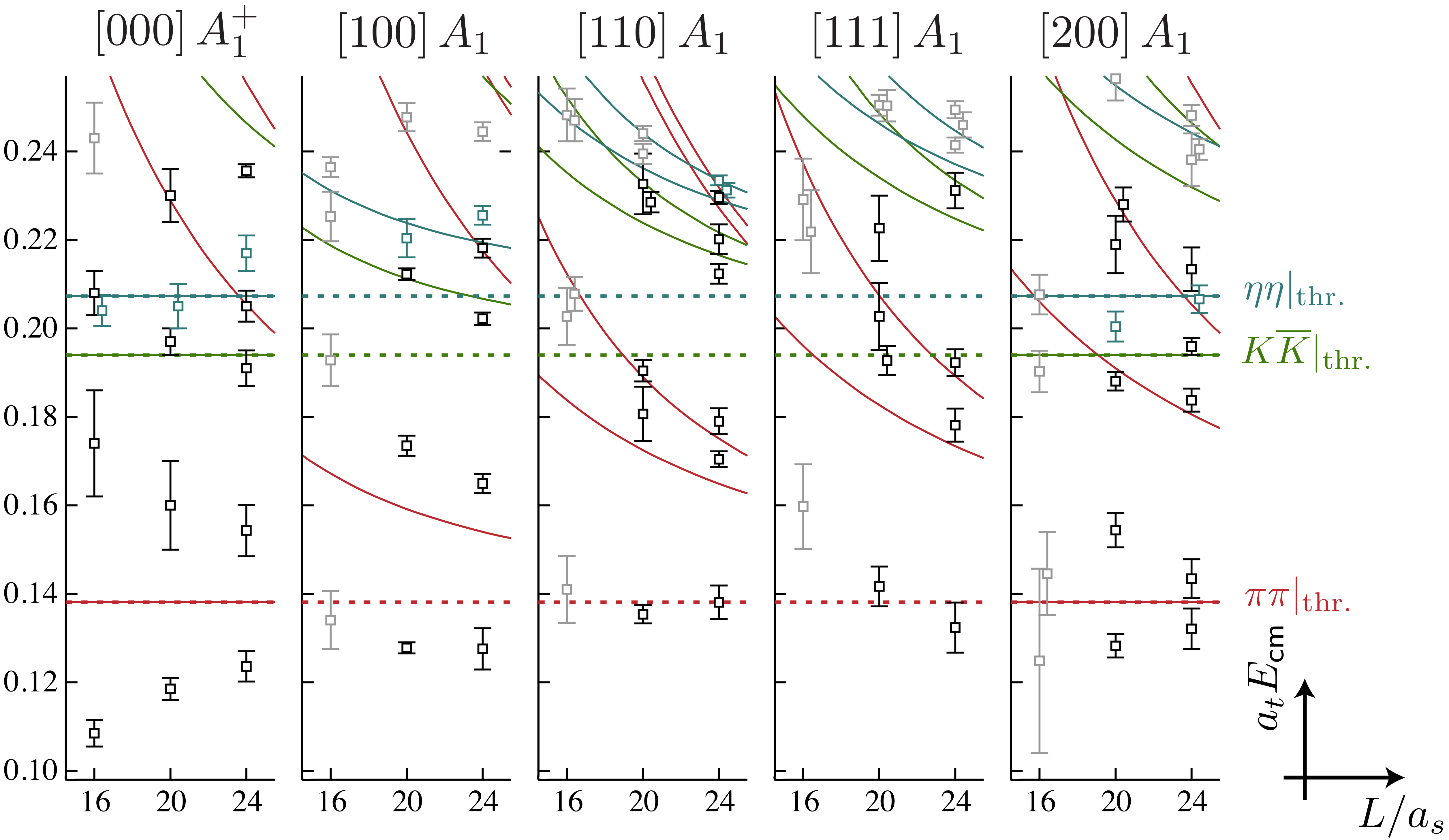
$f_0(980)$ and $a_0(980)$ lie very close to KK threshold

CERN-Munich, ANL, BNL



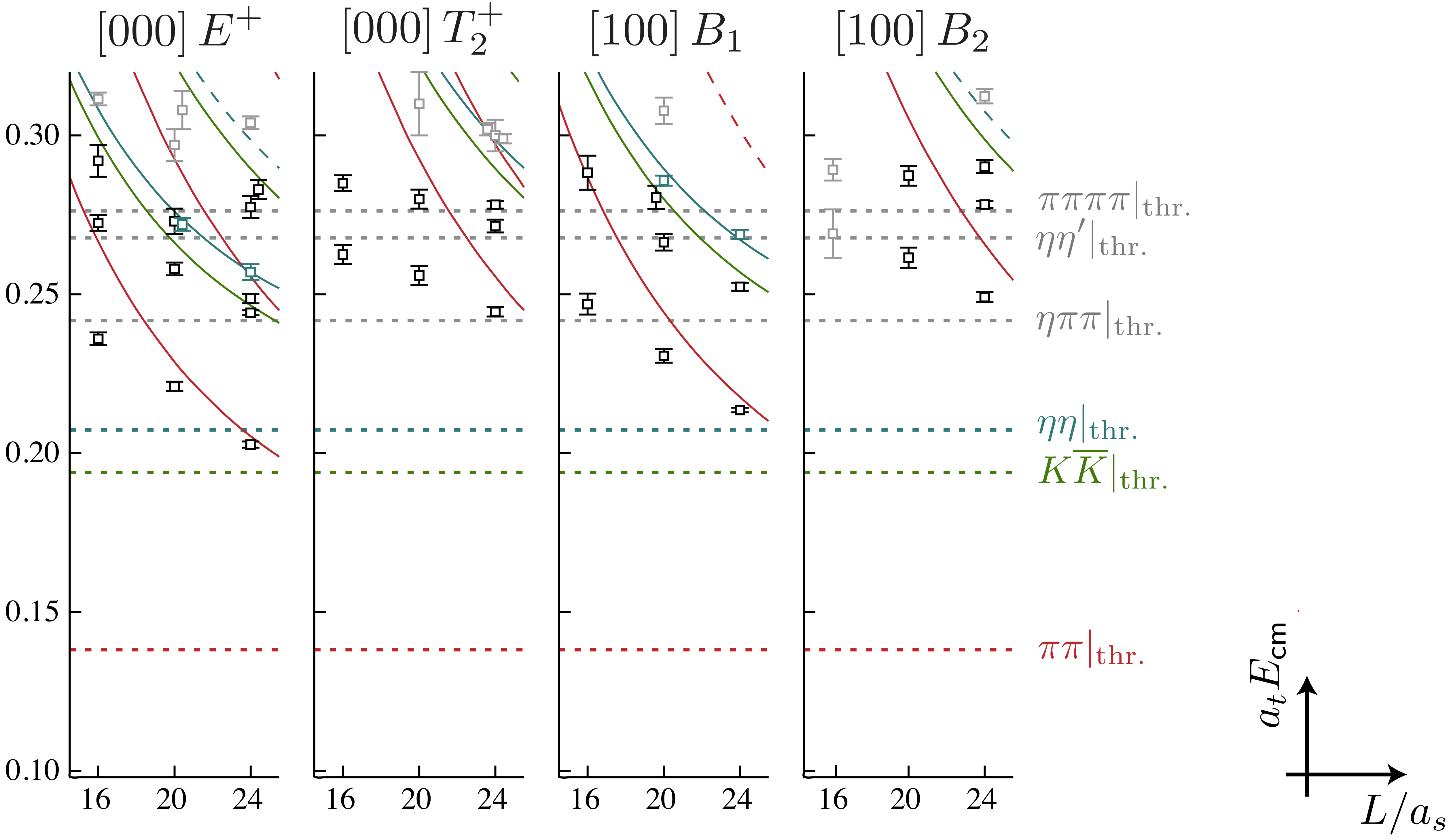
Isoscalar scalar spectra

Briceno et al, Phys. Rev. D 97, 054513 [Editors' Suggestion]



similar construction of operators: local $q\bar{q}$ & 2-hadron
conservatively 57 energy levels
dominated by S-wave interactions

Isoscalar tensor spectra



conservatively 34 energy levels
dominated by D-wave interactions

Extracting the t-matrix

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1}+i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi\to\pi\pi & \pi\pi\to K\bar{K} \\ K\bar{K}\to\pi\pi & K\bar{K}\to K\bar{K} \end{pmatrix}$$

determinant condition:
- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations
- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad\rightarrow\quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Chew-Mandelstam phase space:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$

use a dispersion relation to generate a real part from iρ

- any form real for real energies is valid
- we use a broad selection of K-matrices
- neglects left-hand cut

S-wave amplitudes

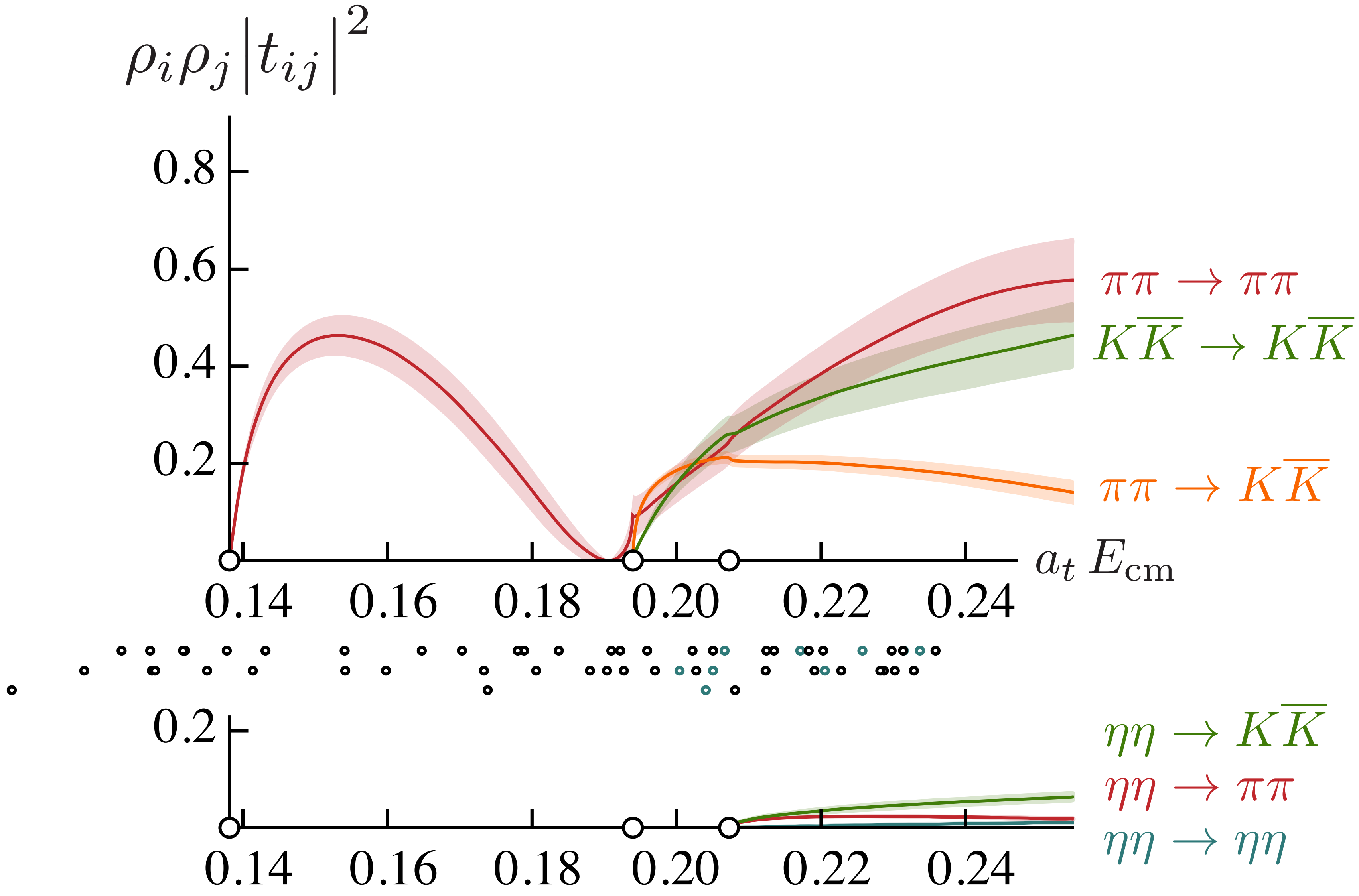
An example S-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels



S-wave amplitudes

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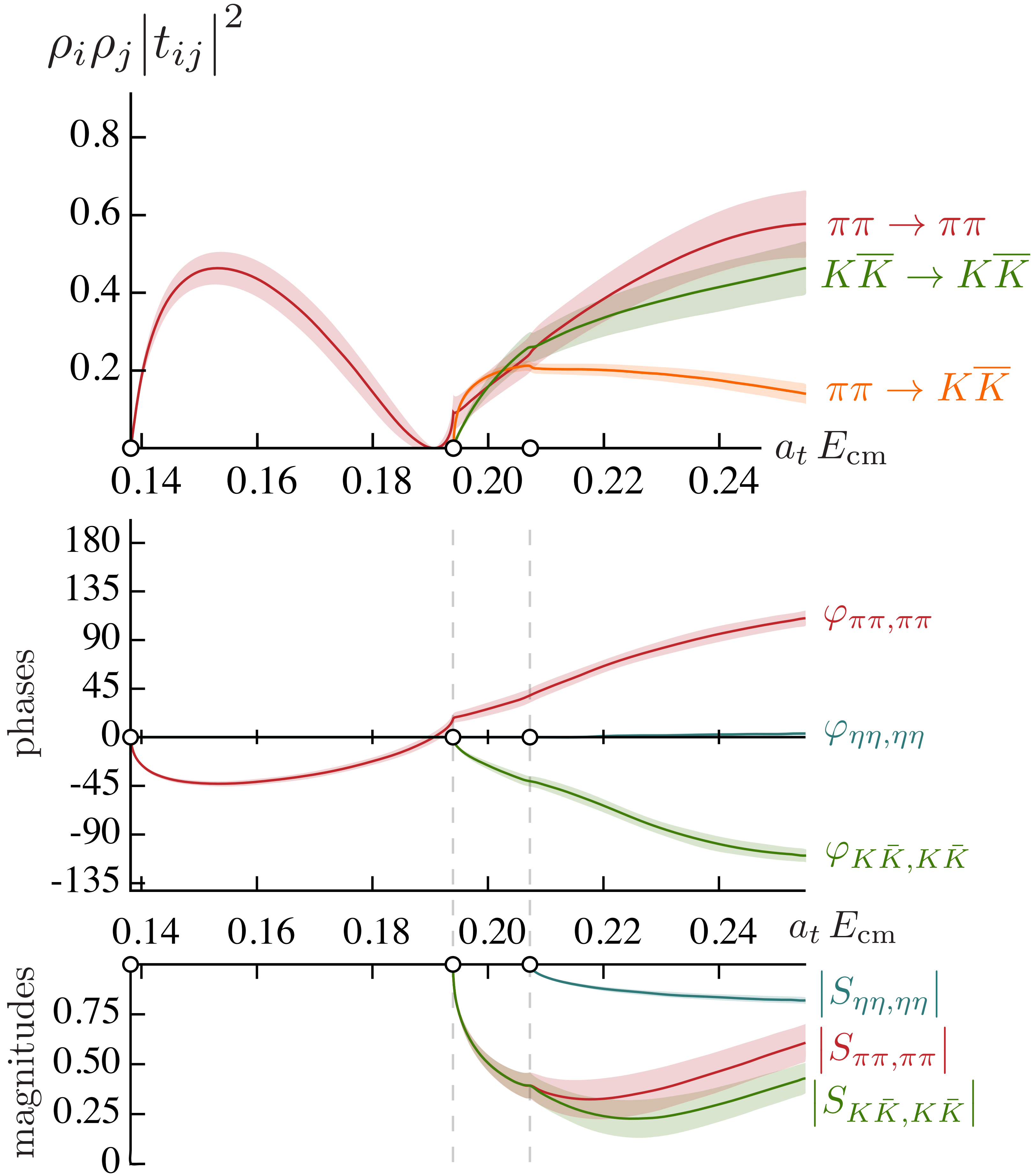
57 energy levels

$$S_{ii}(E_{\mathsf{cm}}) = |S_{ii}(E_{\mathsf{cm}})|\, e^{2i\phi_{ii}(E_{\mathsf{cm}})}$$



D-wave amplitudes

An example D-wave spectrum fit

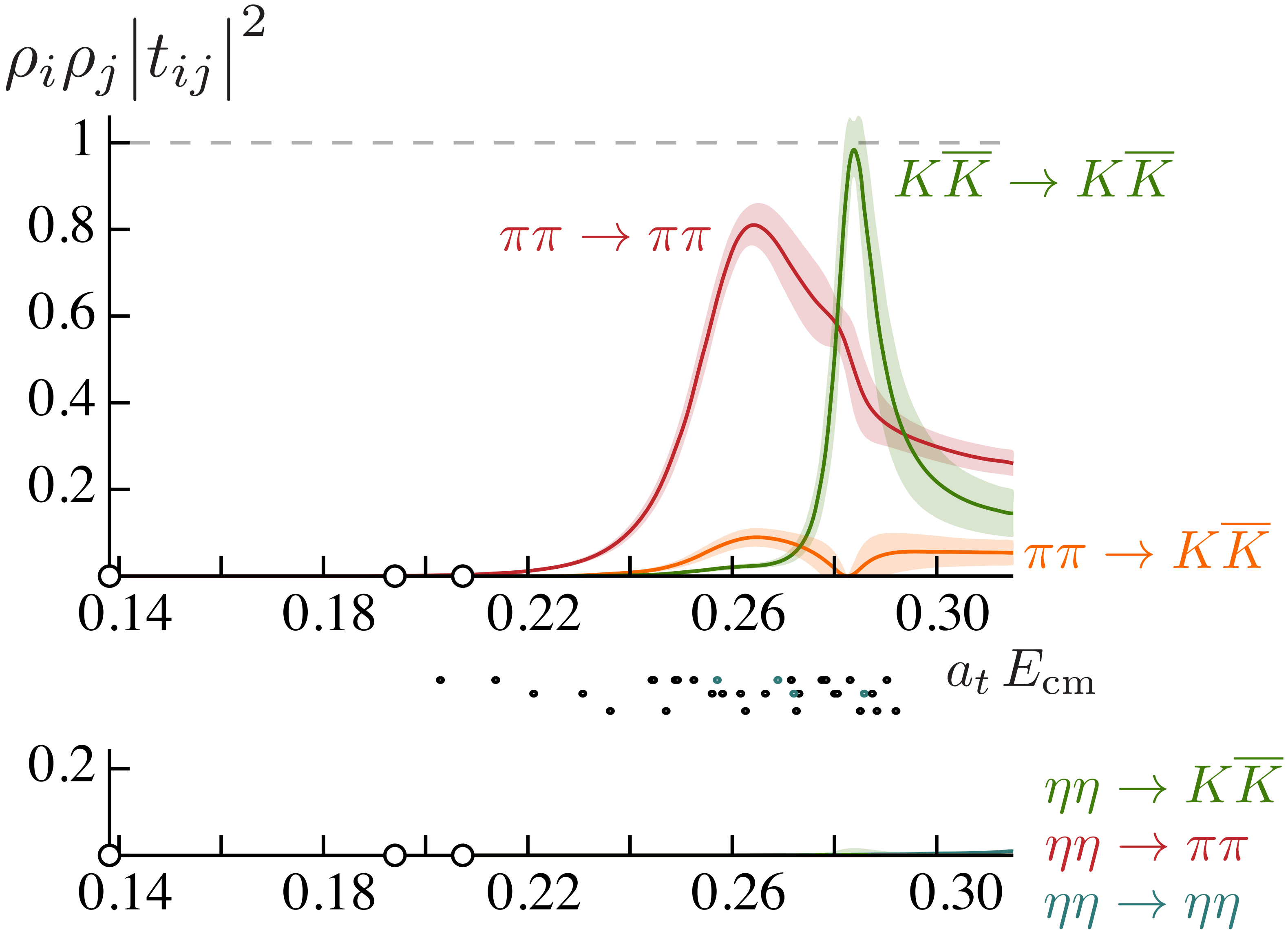
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$

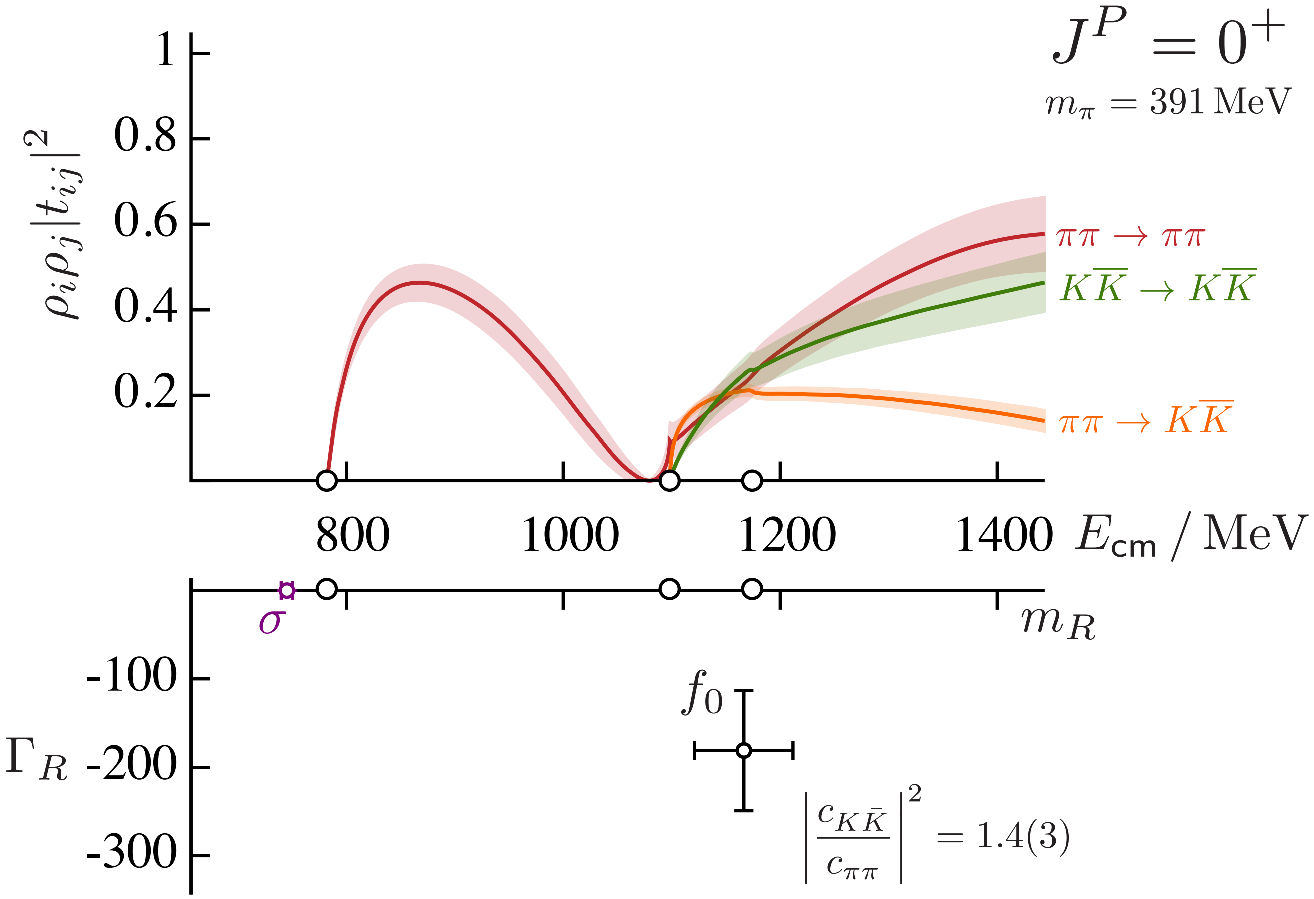
$$\gamma_{\eta\eta} \neq 0$$
$$\gamma_{ij} = 0 \quad \text{otherwise}$$

$$\chi^2/N_{\text{dof}} = \frac{28.9}{34-9} = 1.15$$

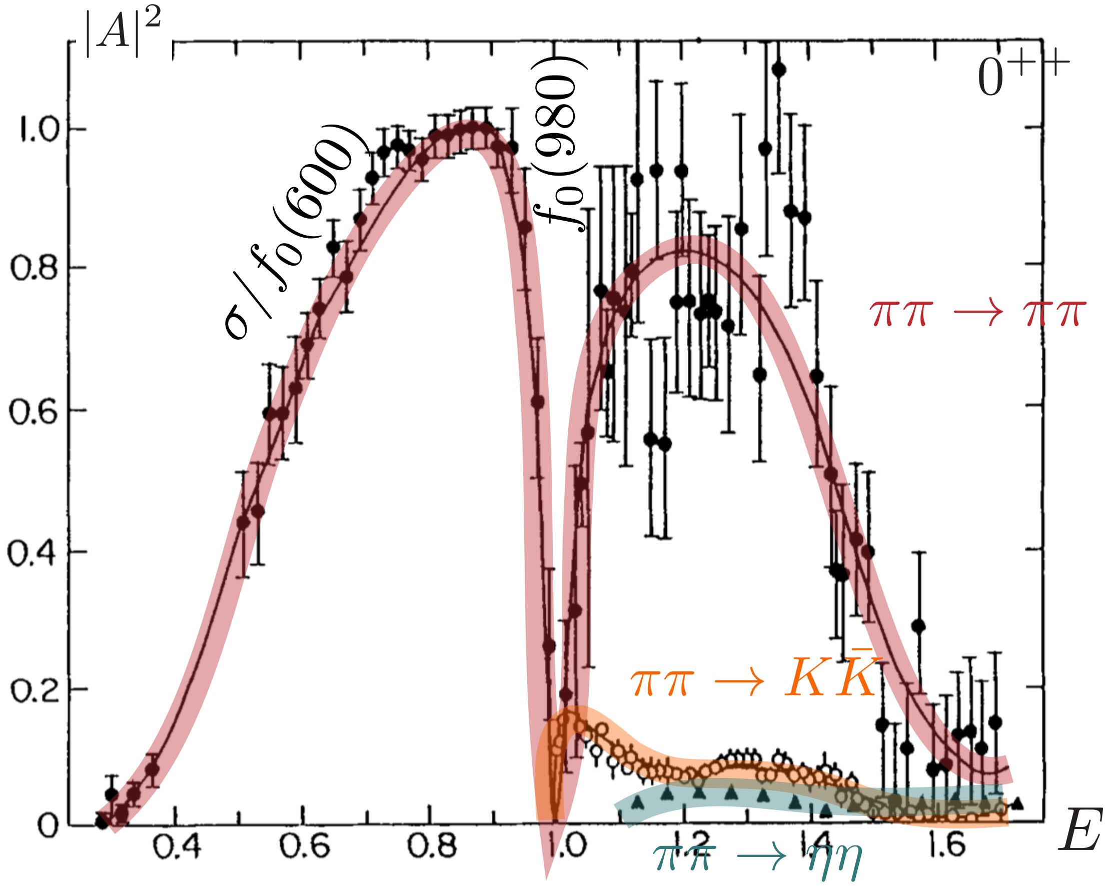
34 energy levels



Poles and analysis



CERN-Munich, ANL, BNL



The amplitudes

An example S-wave spectrum fit

$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

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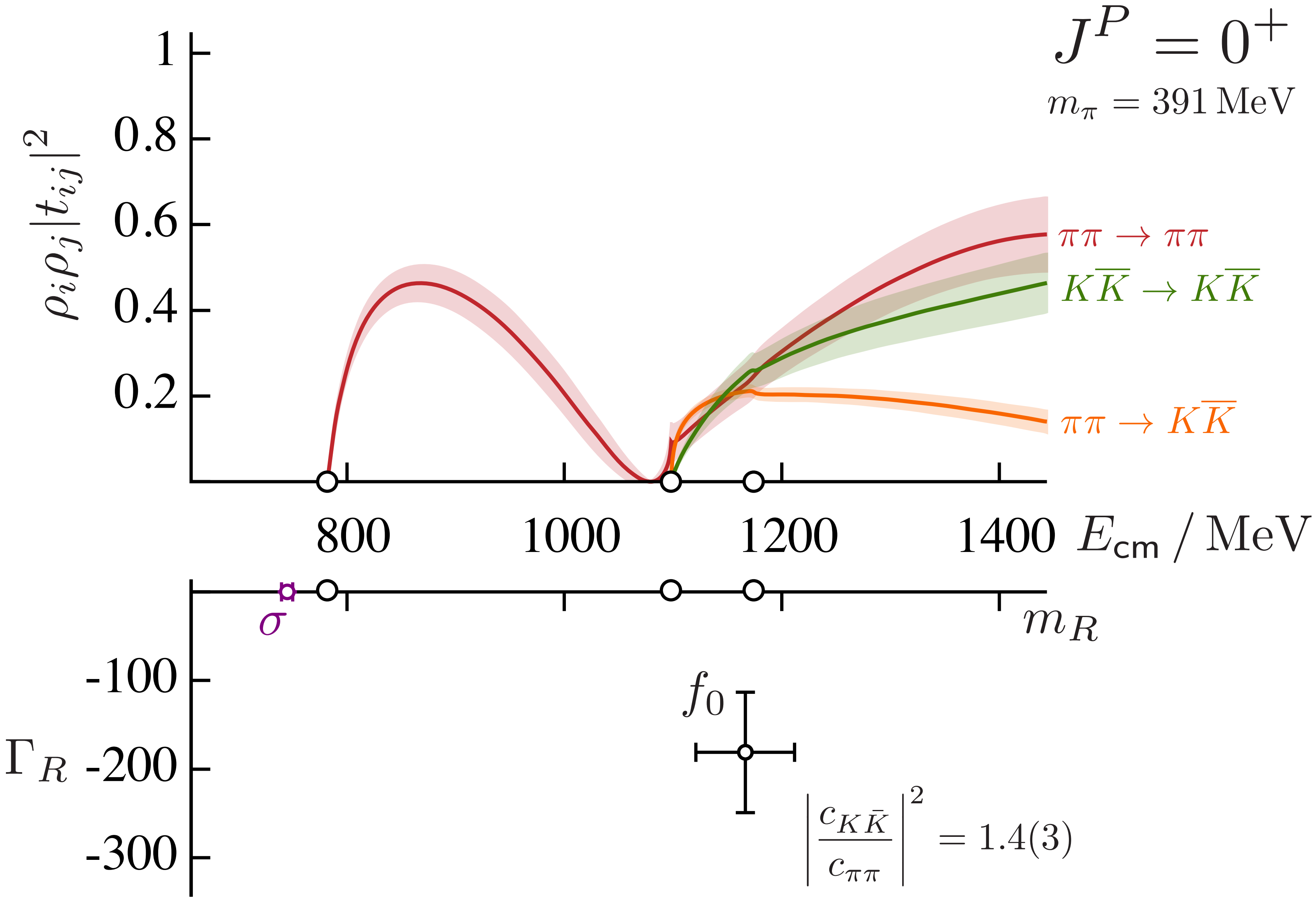
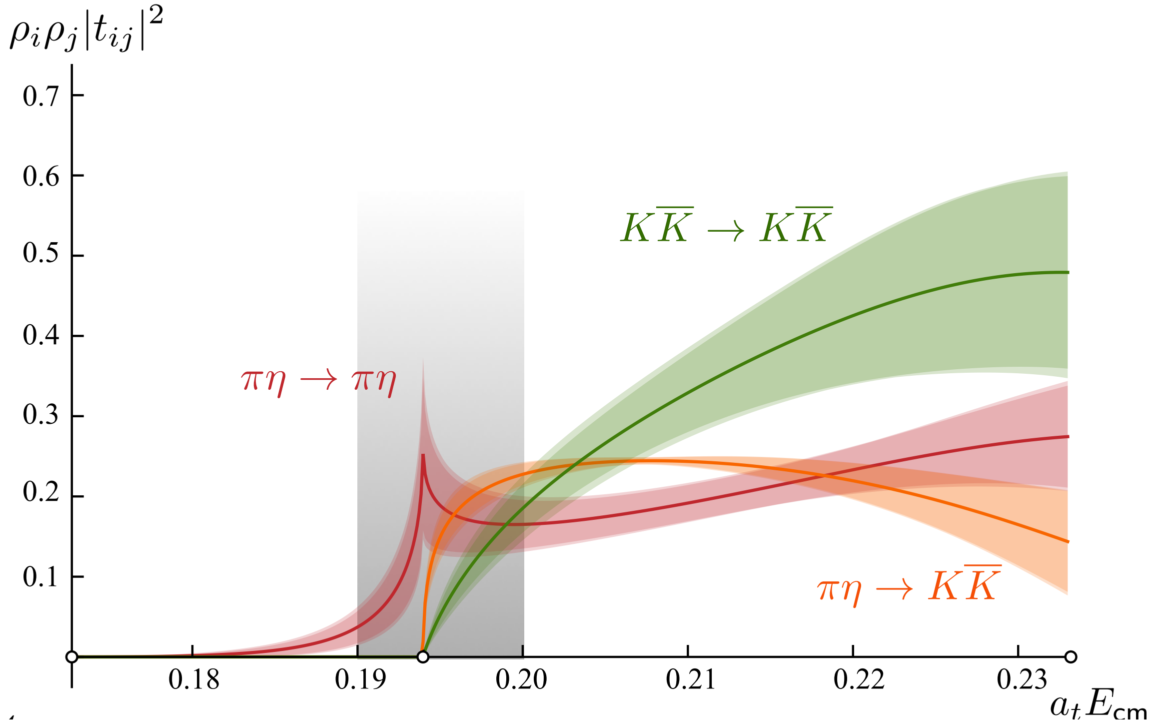
57 energy levels

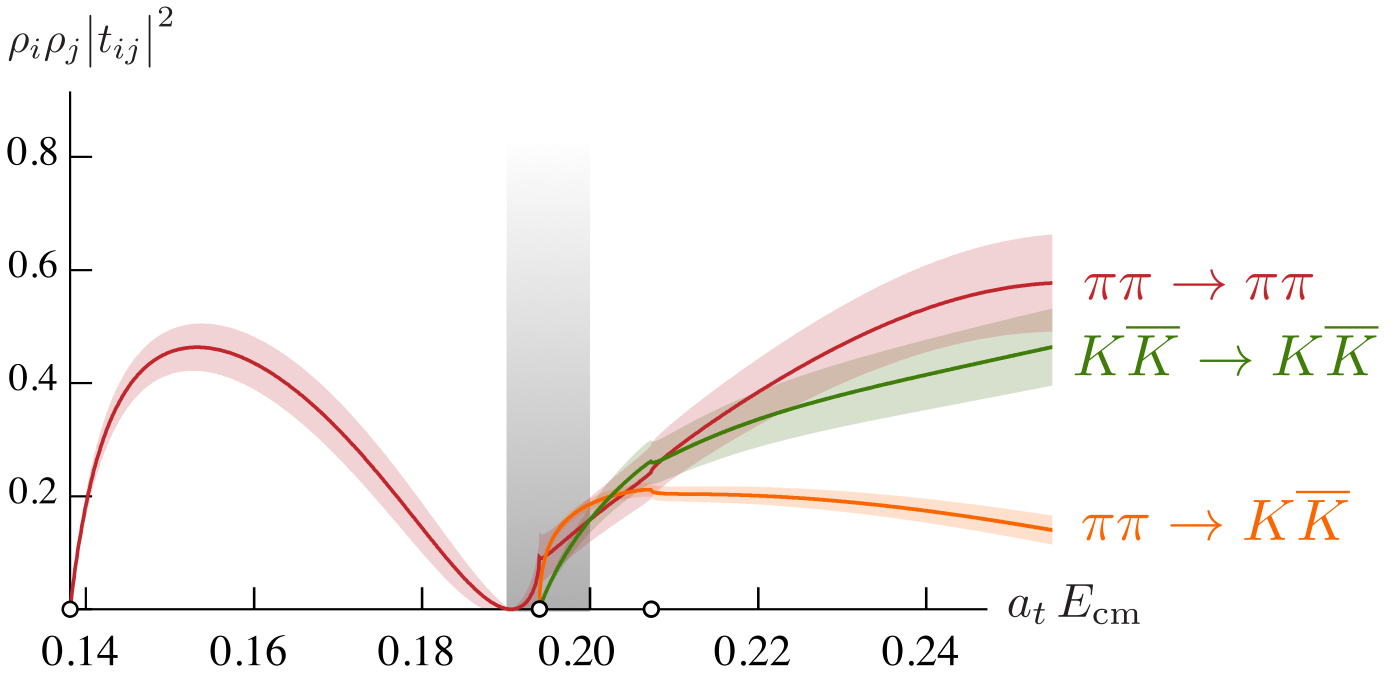


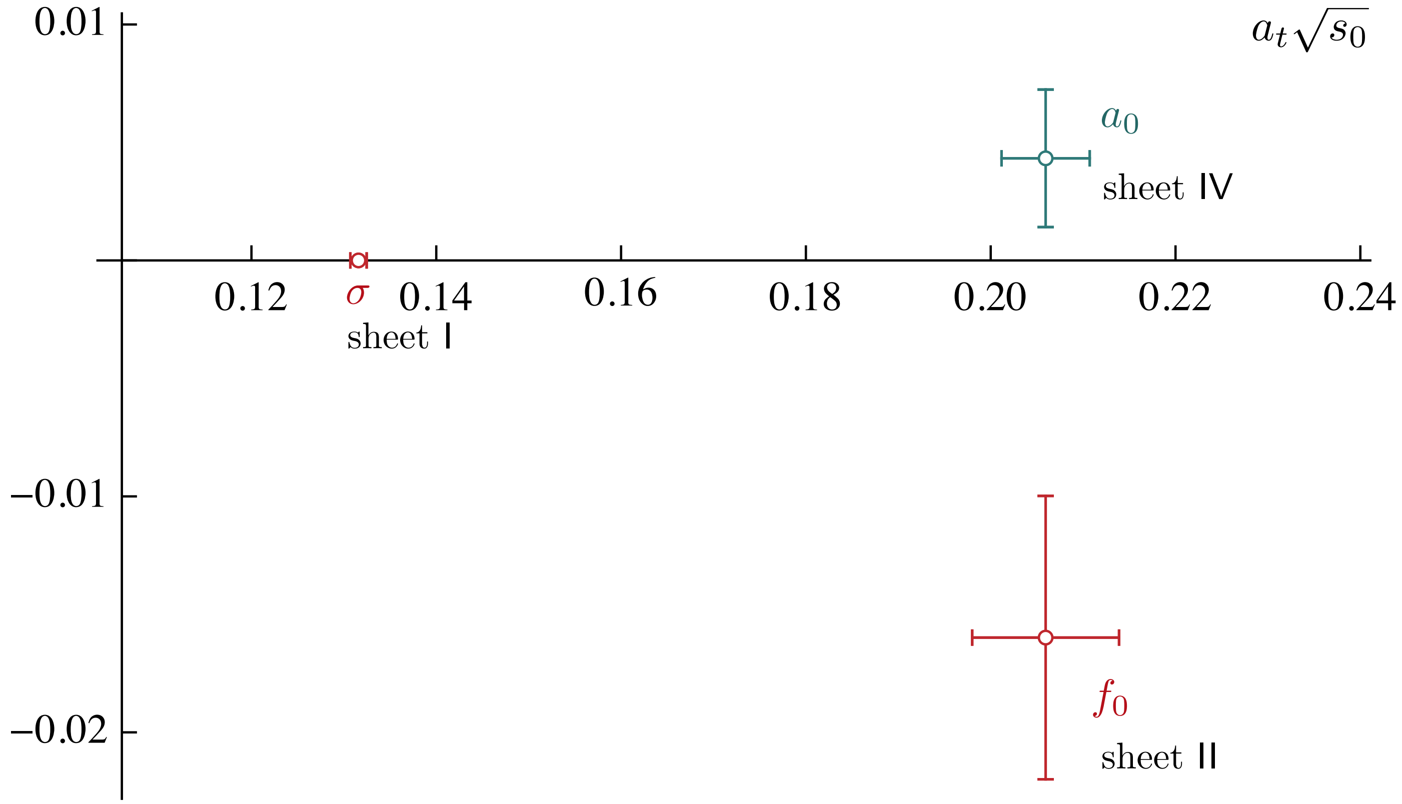
Near a t-matrix pole

$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

Comparing a_0 and f_0 resonance poles

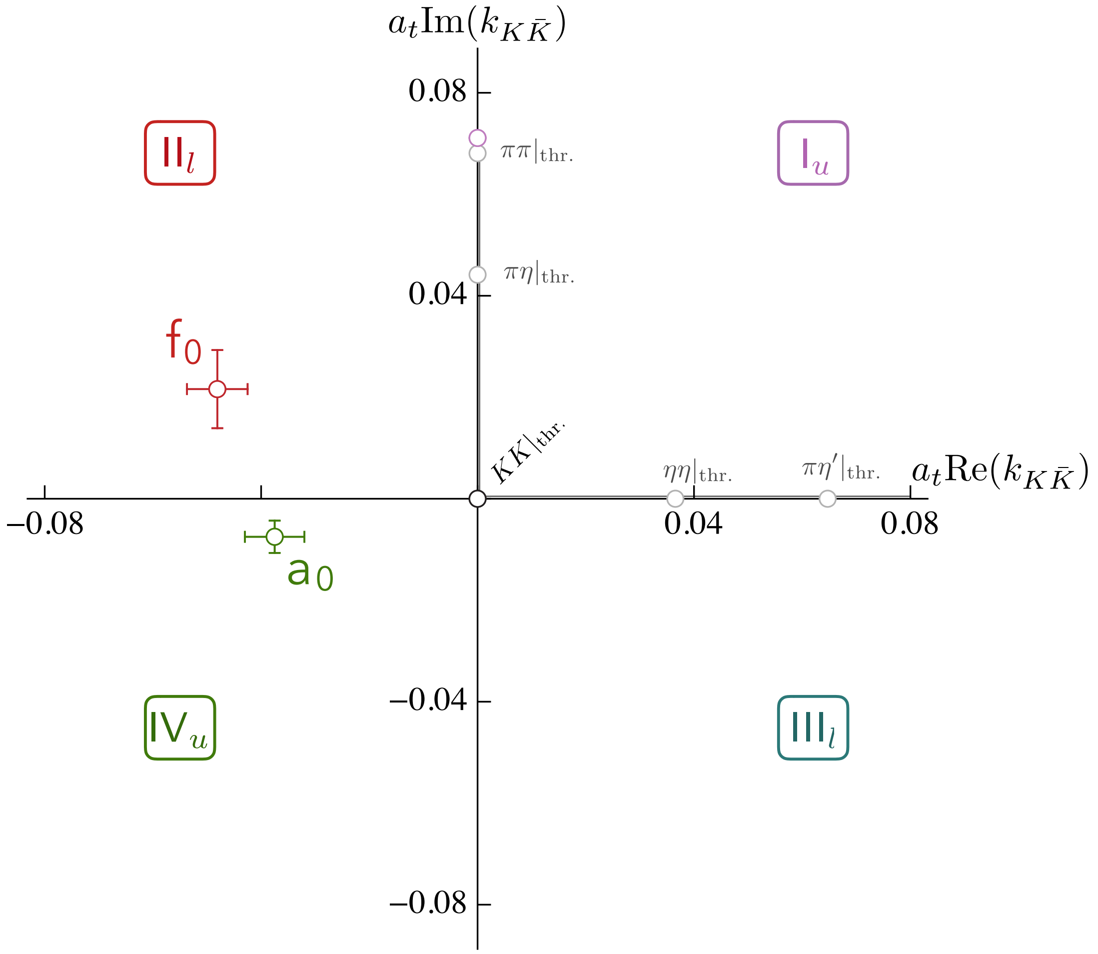


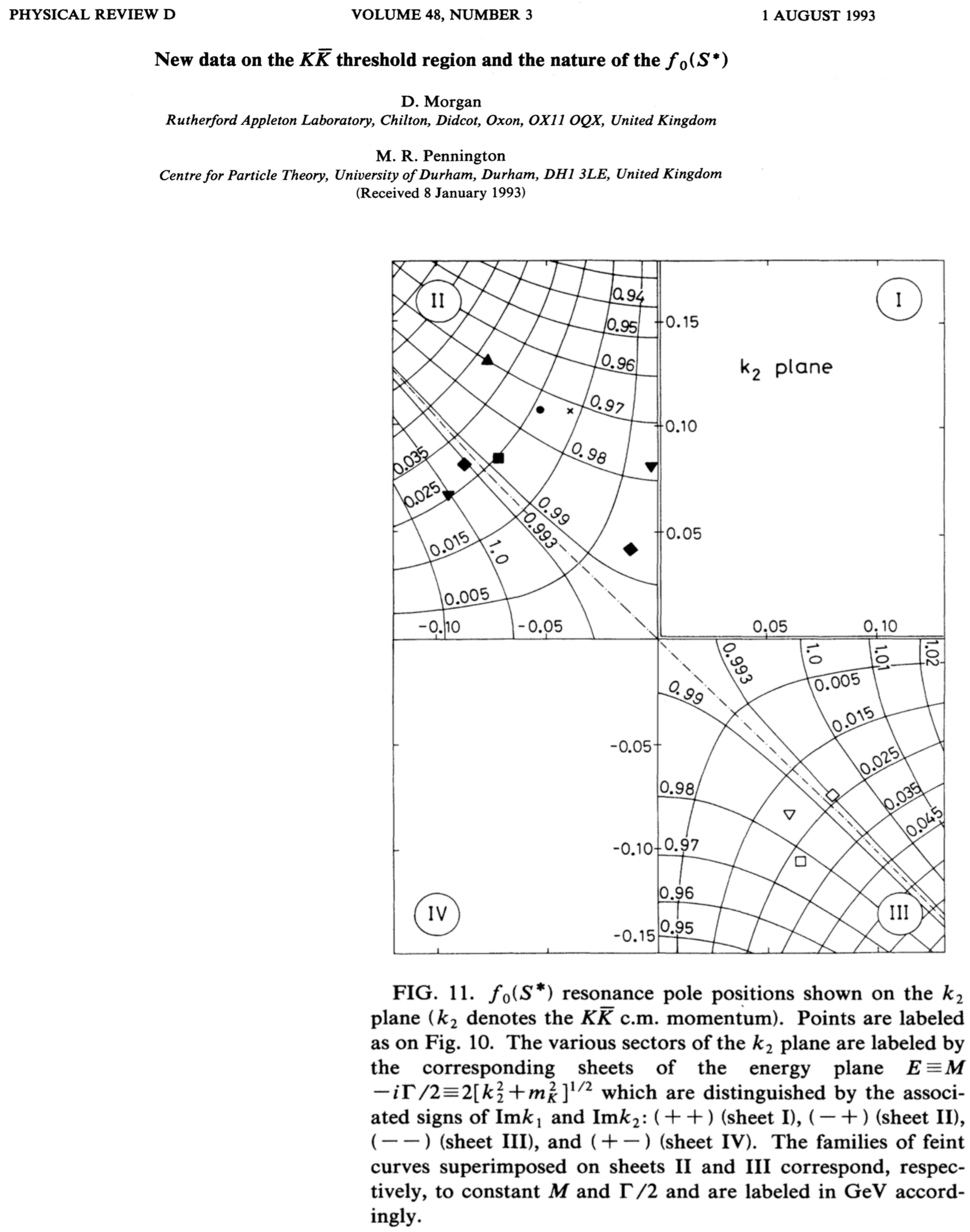




- rapid variation in both amplitudes at $K\bar{K}$ threshold
- both contain a nearby pole with a similar real part

Comparing a_0 and f_0 resonance poles





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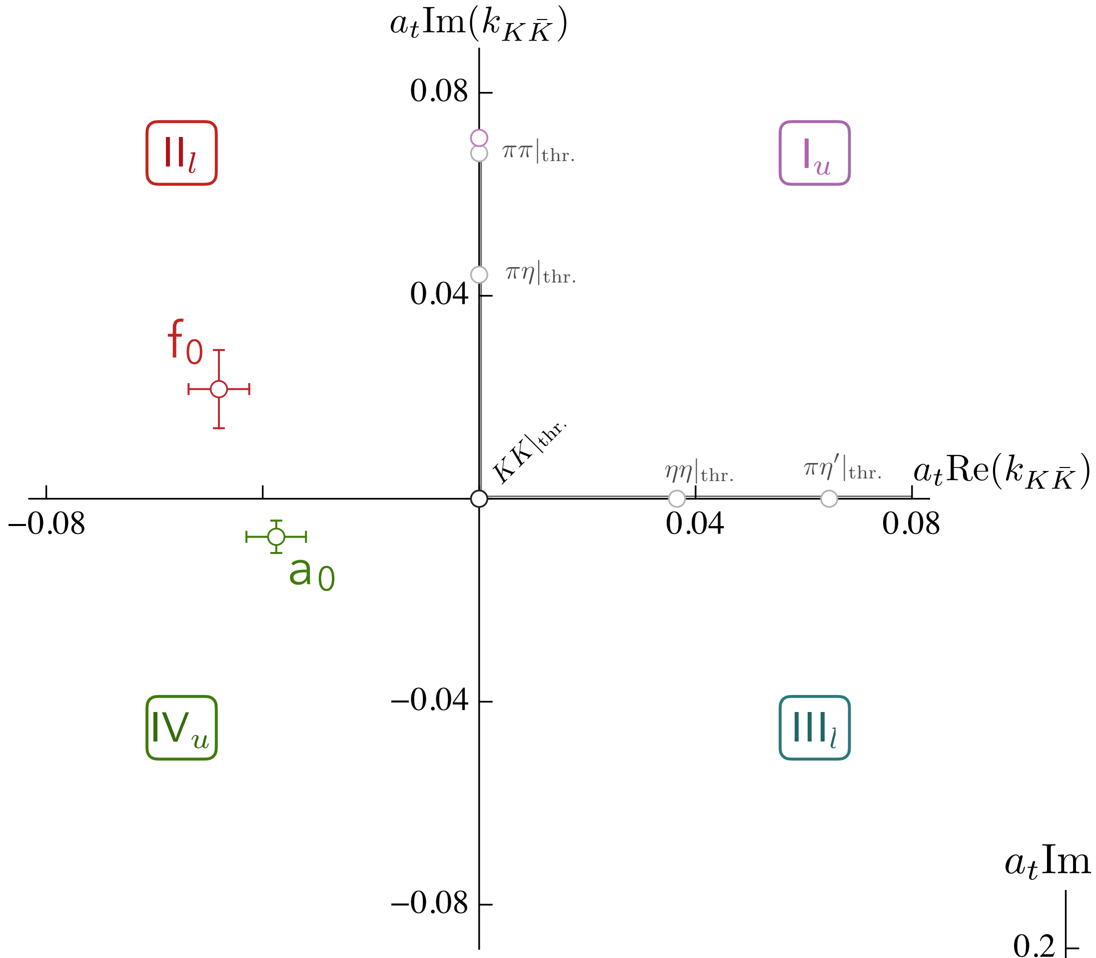
New data on the $K\bar{K}$ threshold region and the nature of the $f_0(S^*)$

D. Morgan
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom

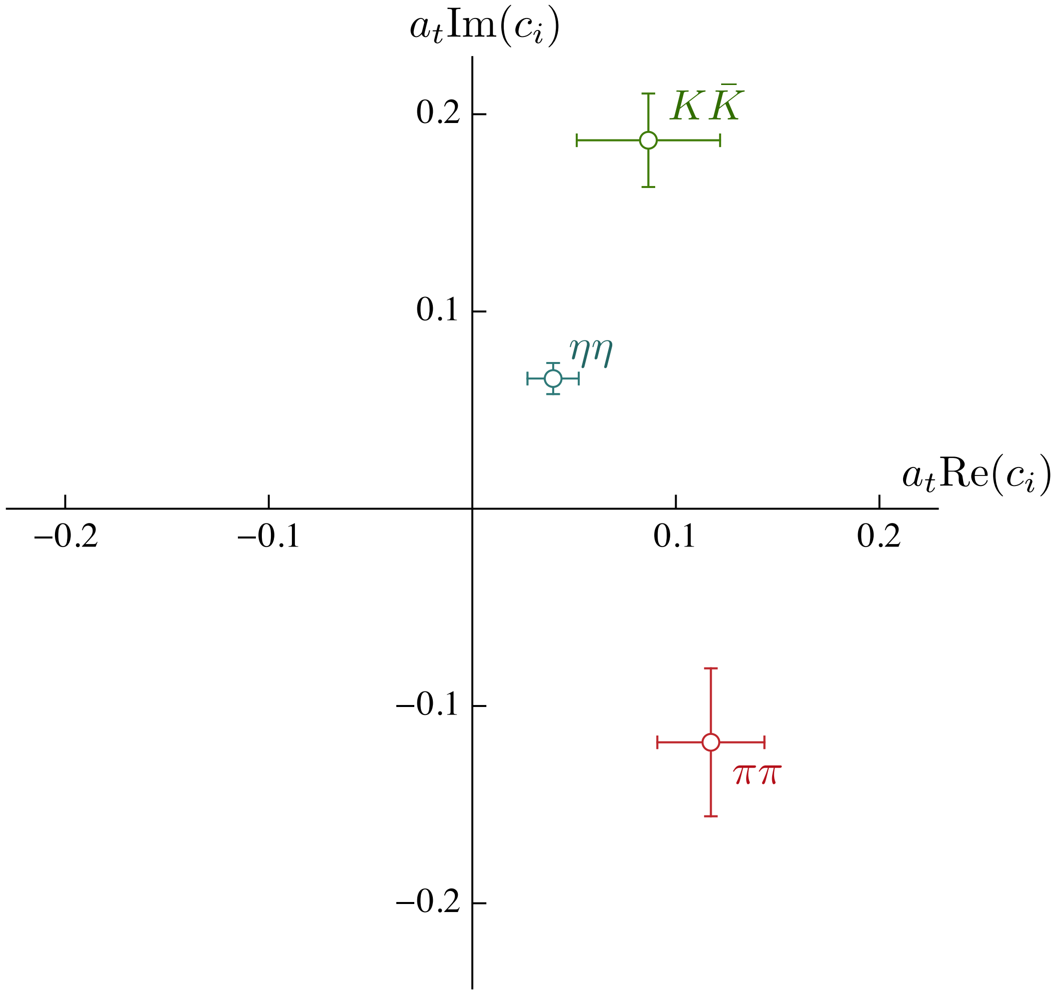
M. R. Pennington
Centre for Particle Theory, University of Durham, Durham, DH1 3LE, United Kingdom
(Received 8 January 1993)

FIG. 11. $f_0(S^*)$ resonance pole positions shown on the k_2 plane (k_2 denotes the $K\bar{K}$ c.m. momentum). Points are labeled as on Fig. 10. The various sectors of the k_2 plane are labeled by the corresponding sheets of the energy plane $E \equiv M - i\Gamma/2 \equiv 2[k_2^2 + m_K^2]^{1/2}$ which are distinguished by the associated signs of Imk_1 and Imk_2: $(++)$ (sheet I), $(-+)$ (sheet II), $(--)$ (sheet III), and $(+-)$ (sheet IV). The families of feint curves superimposed on sheets II and III correspond, respectively, to constant M and $\Gamma/2$ and are labeled in GeV accordingly.

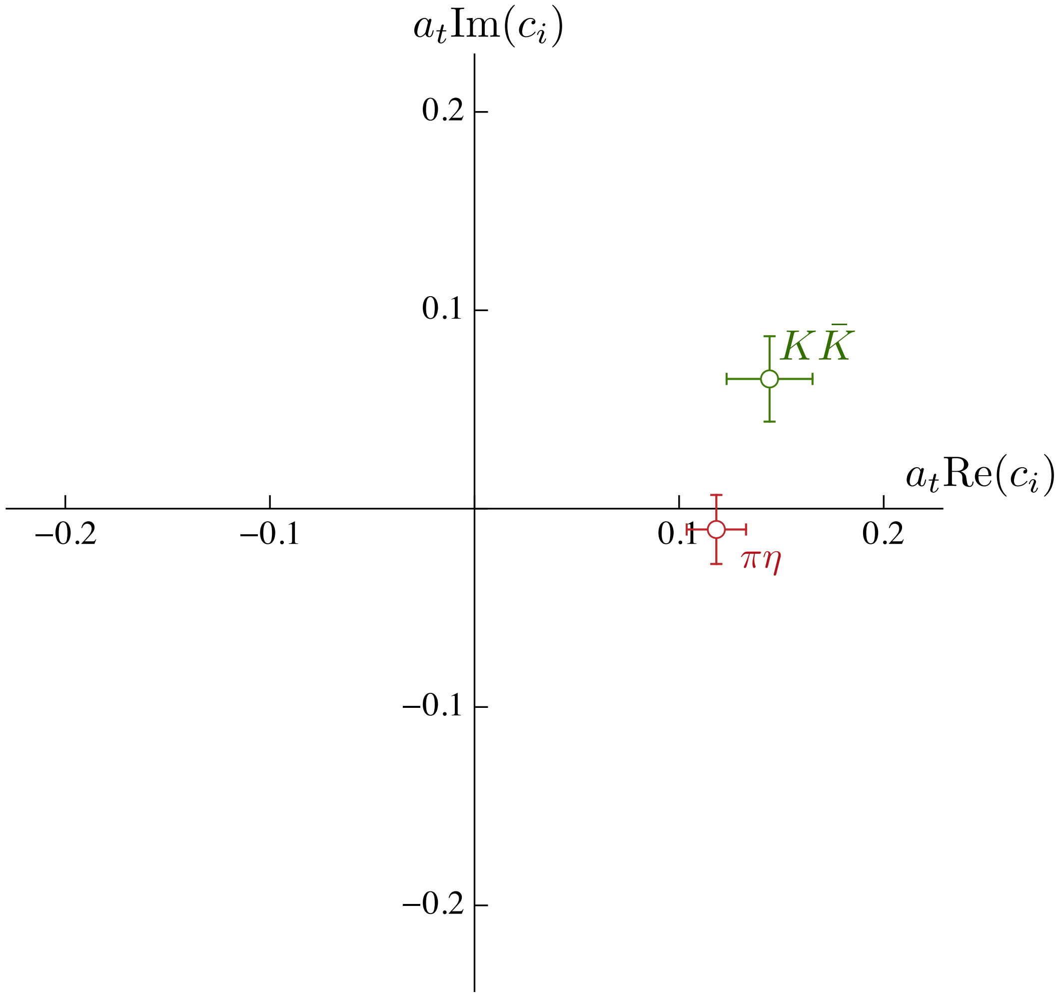
Comparing a_0 and f_0 resonance poles



f_0 couplings



a_0 couplings



D-wave amplitudes

An example D-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

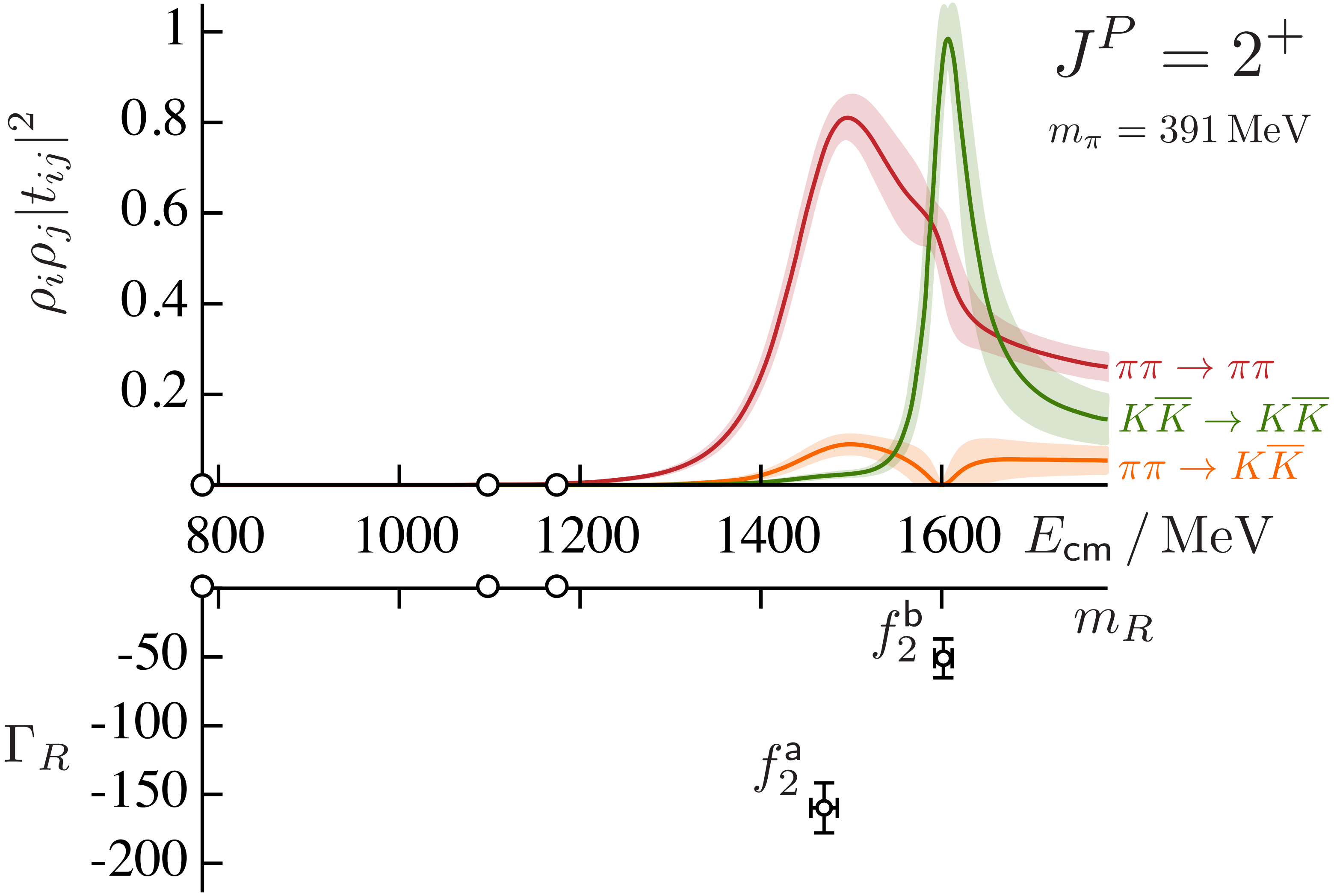
$$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$

$\gamma_{\eta\eta} \neq 0$

$\gamma_{ij} = 0$ otherwise

$$\chi^2/N_{\mathrm{dof}} = \frac{28.9}{34-9} = 1.15$$

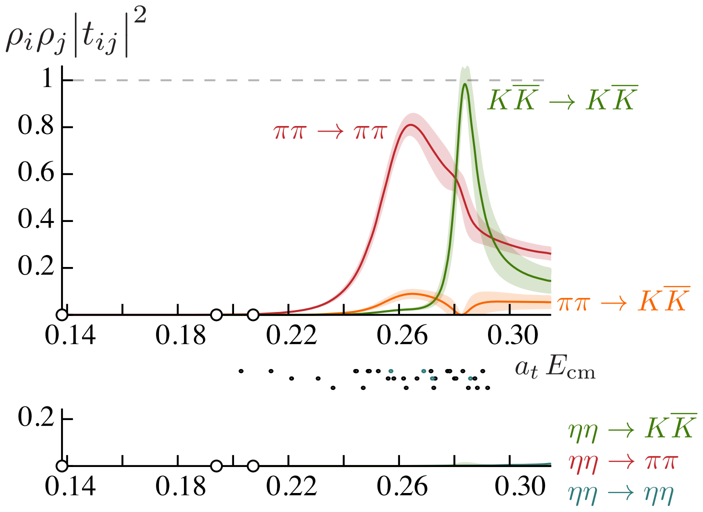
34 energy levels

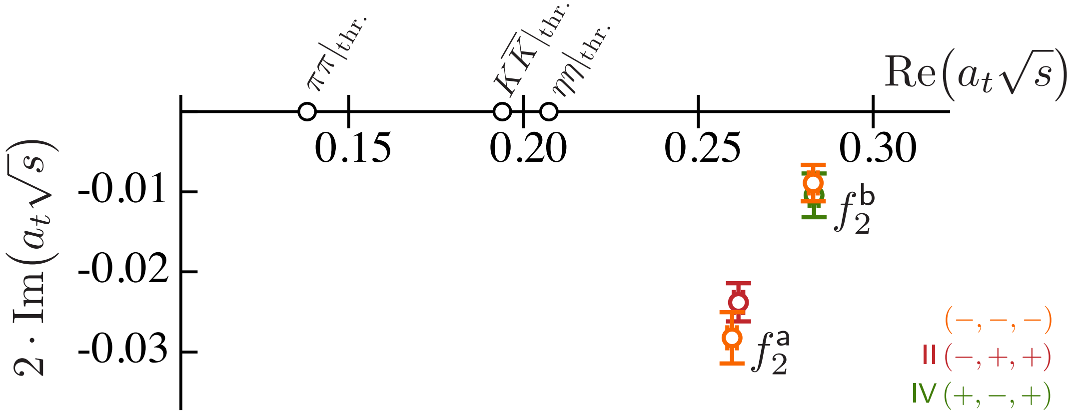


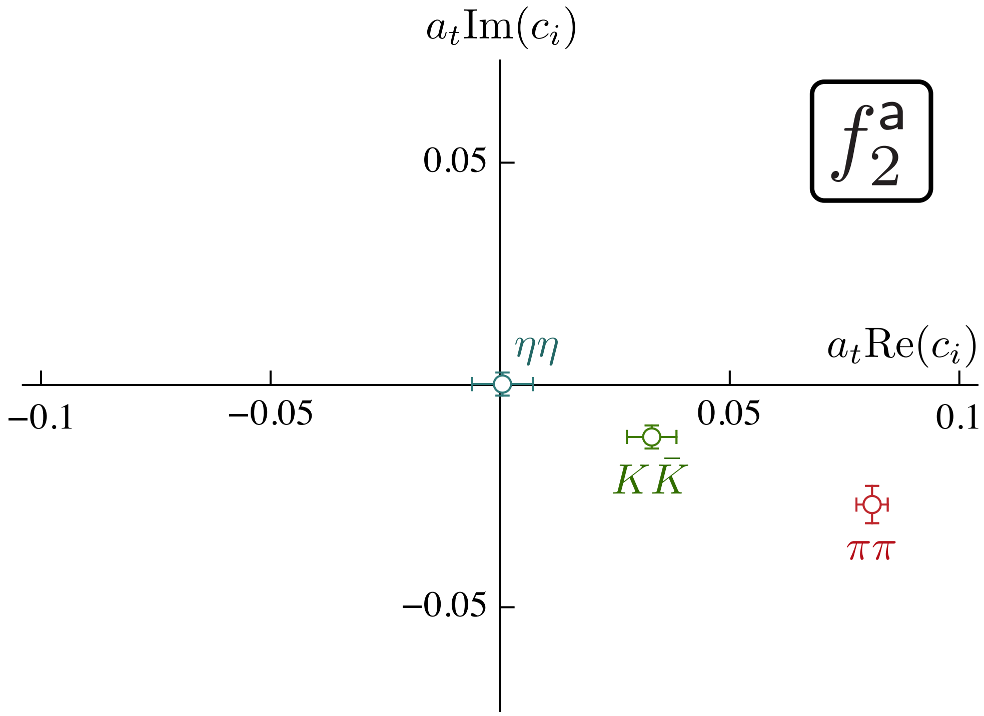
Near a t-matrix pole

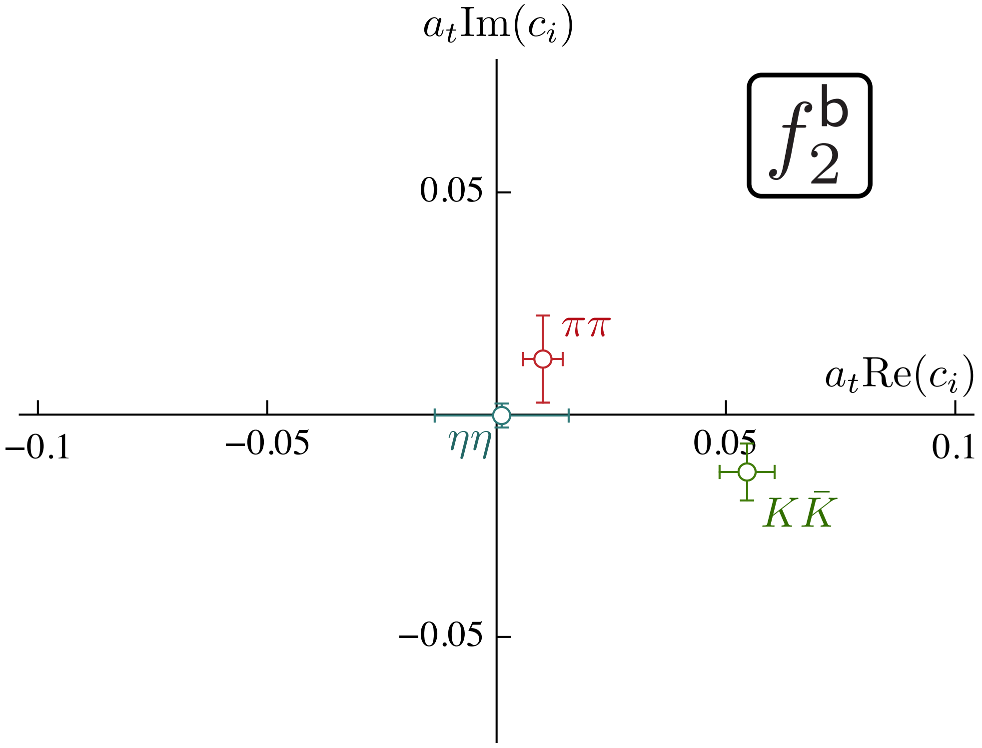
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

Tensor resonance poles









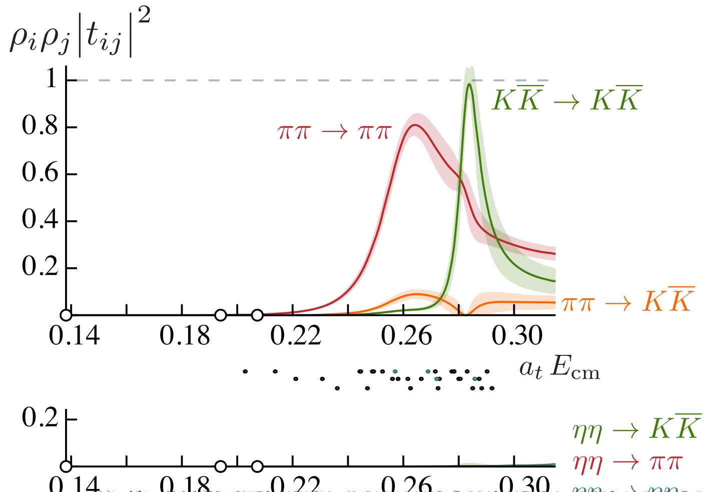
$f_2^{\mathsf{a}}: \quad \sqrt{s_0} = 1470(15) - \frac{i}{2}\,160(18)$ MeV

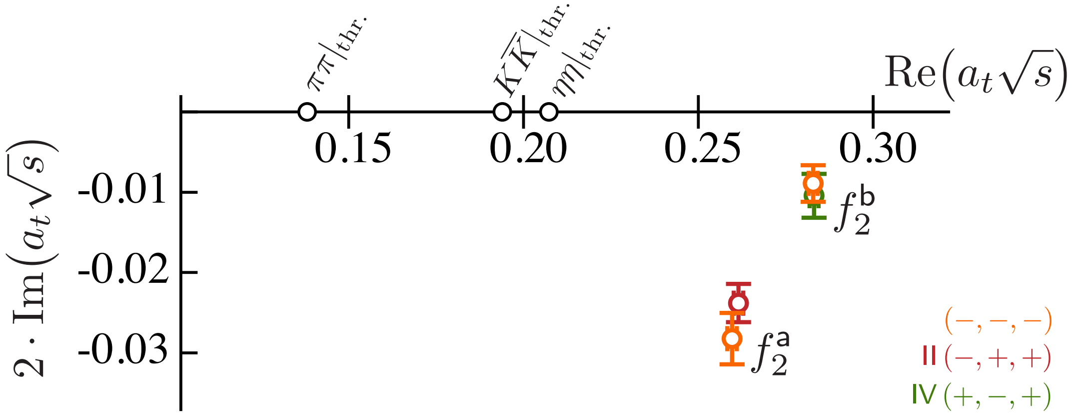
$\mathrm{Br}(f_2^{\mathsf{a}} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\mathsf{a}} \to K\overline{K}) \sim 12\%$

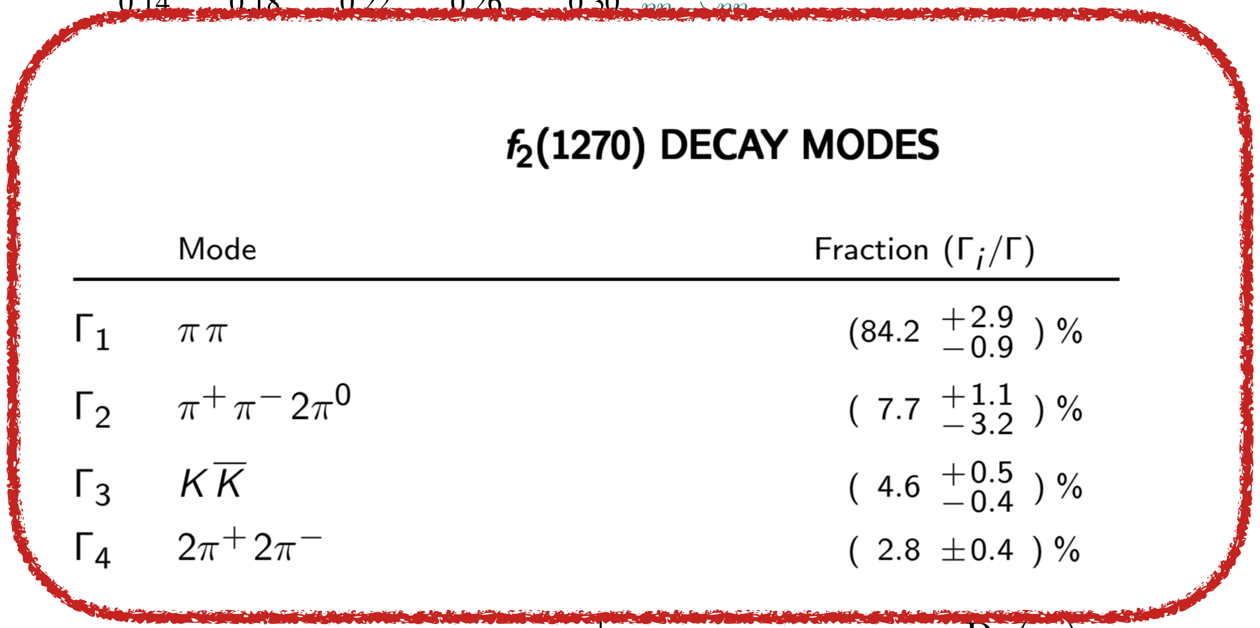
$f_2^{\mathsf{b}}: \quad \sqrt{s_0} = 1602(10) - \frac{i}{2}\,54(14)$ MeV

$\mathrm{Br}(f_2^{\mathsf{b}} \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^{\mathsf{b}} \to K\overline{K}) \sim 92\%$

Just for fun...

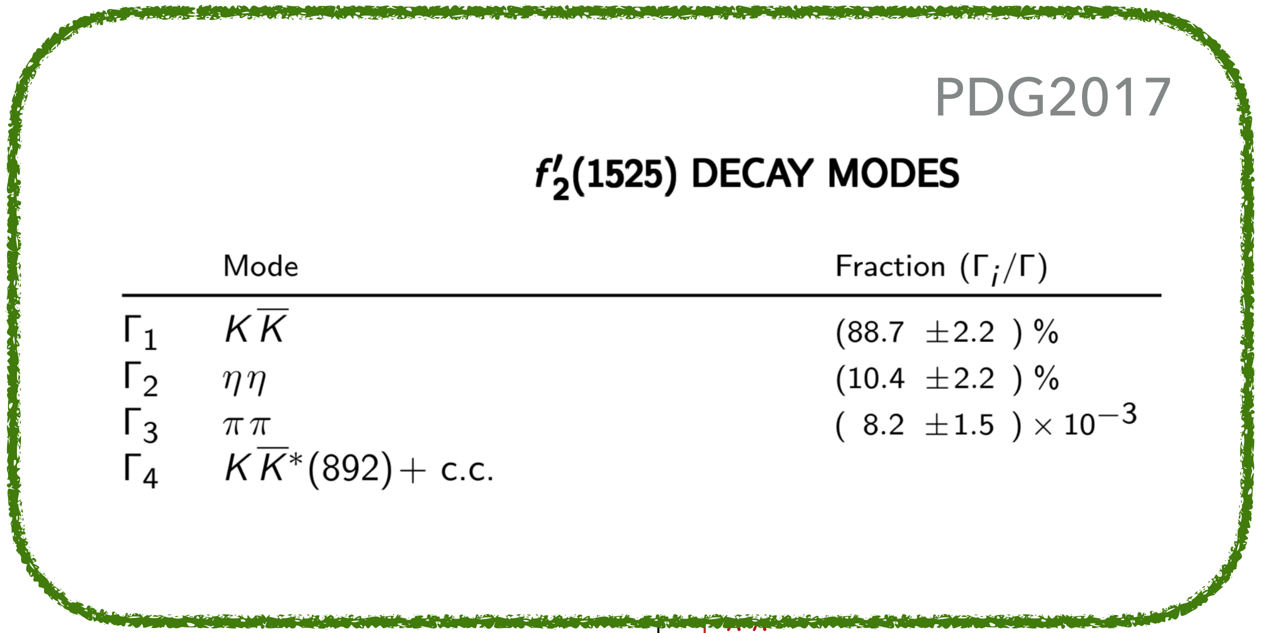






$f_2(1270)$ DECAY MODES

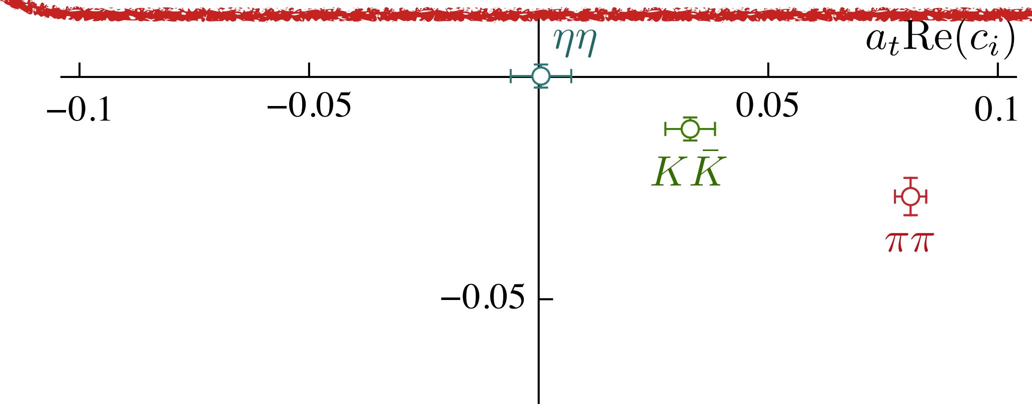
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	$(84.2\ ^{+2.9}_{-0.9})$ %
Γ_2	$\pi^+\pi^-2\pi^0$	$(\ 7.7\ ^{+1.1}_{-3.2})$ %
Γ_3	$K\overline{K}$	$(\ 4.6\ ^{+0.5}_{-0.4})$ %
Γ_4	$2\pi^+2\pi^-$	$(\ 2.8\ \pm0.4)$ %

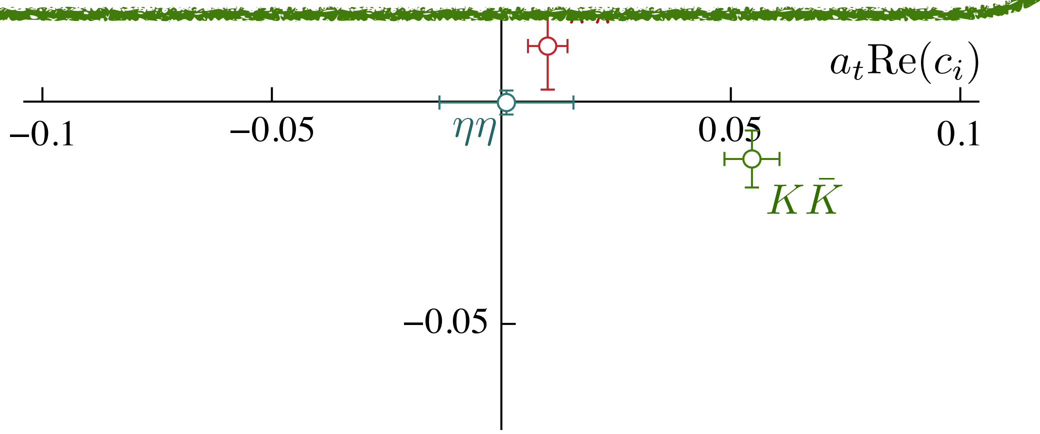


PDG2017

$f_2'(1525)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}$	$(88.7\ \pm2.2)$ %
Γ_2	$\eta\eta$	$(10.4\ \pm2.2)$ %
Γ_3	$\pi\pi$	$(\ 8.2\ \pm1.5) \times 10^{-3}$
Γ_4	$K\overline{K}^*(892)+$ c.c.	





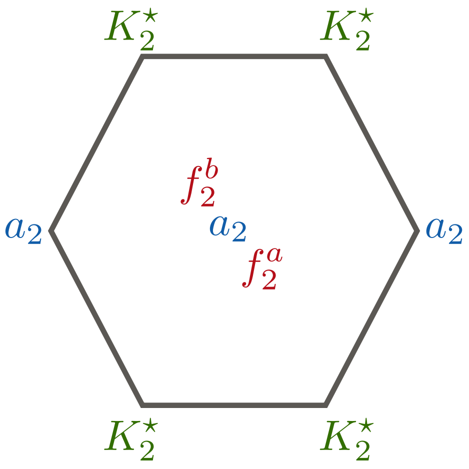
$$f_2^{\rm a}: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)\ \mathrm{MeV}$$

$$\mathrm{Br}(f_2^{\rm a}\to\pi\pi)\sim 85\%, \quad \mathrm{Br}(f_2^{\rm a}\to K\overline{K})\sim 12\%$$

$$f_2^{\rm b}: \quad \sqrt{s_0} = 1602(10) - \tfrac{i}{2}\,54(14)\ \mathrm{MeV}$$

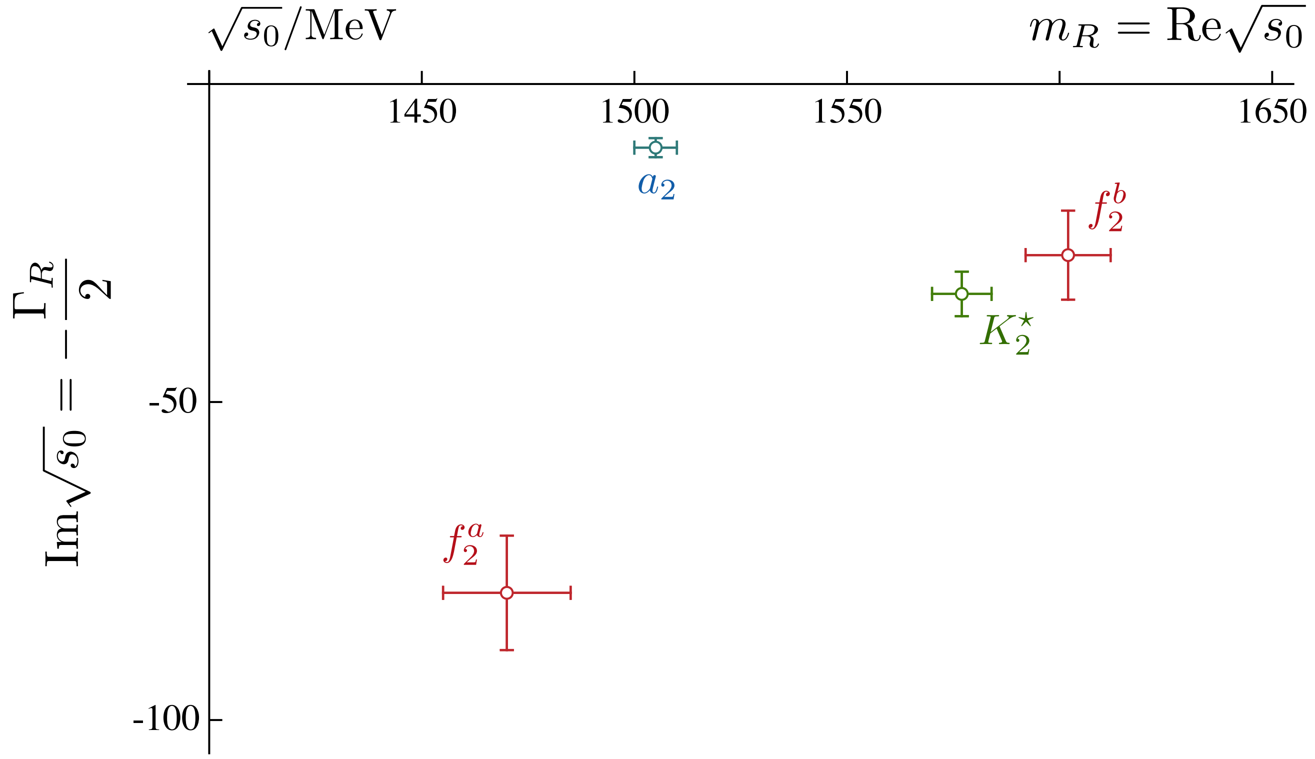
$$\mathrm{Br}(f_2^{\rm b}\to\pi\pi)\sim 8\%, \quad \mathrm{Br}(f_2^{\rm b}\to K\overline{K})\sim 92\%$$

The lightest tensors of QCD at mπ=391 MeV

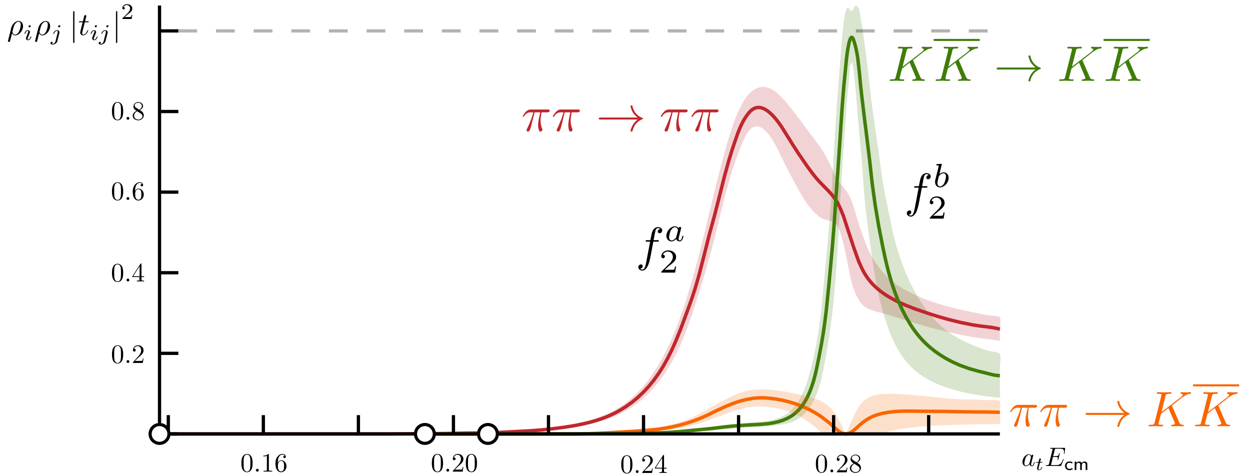


$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

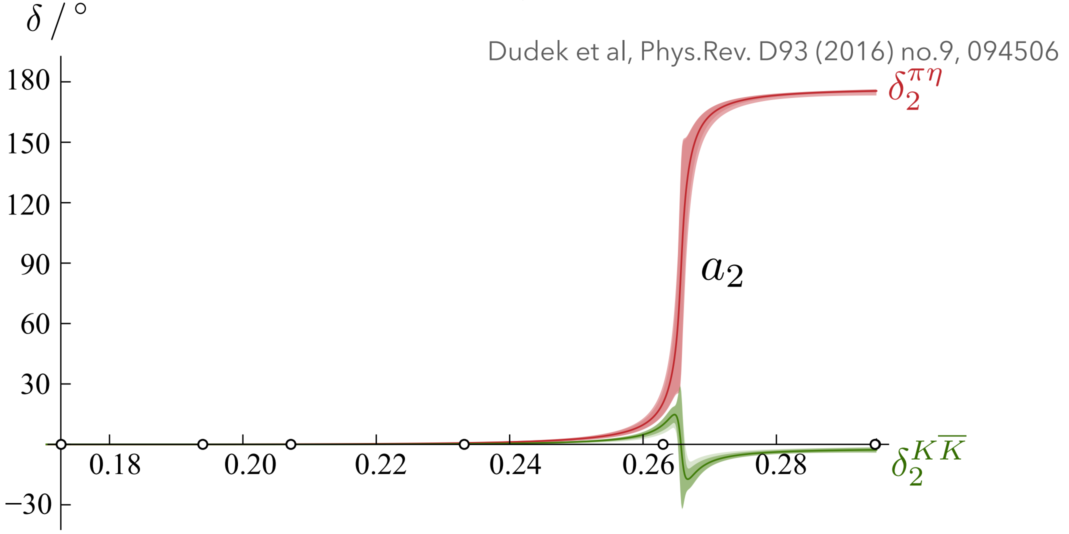
$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$



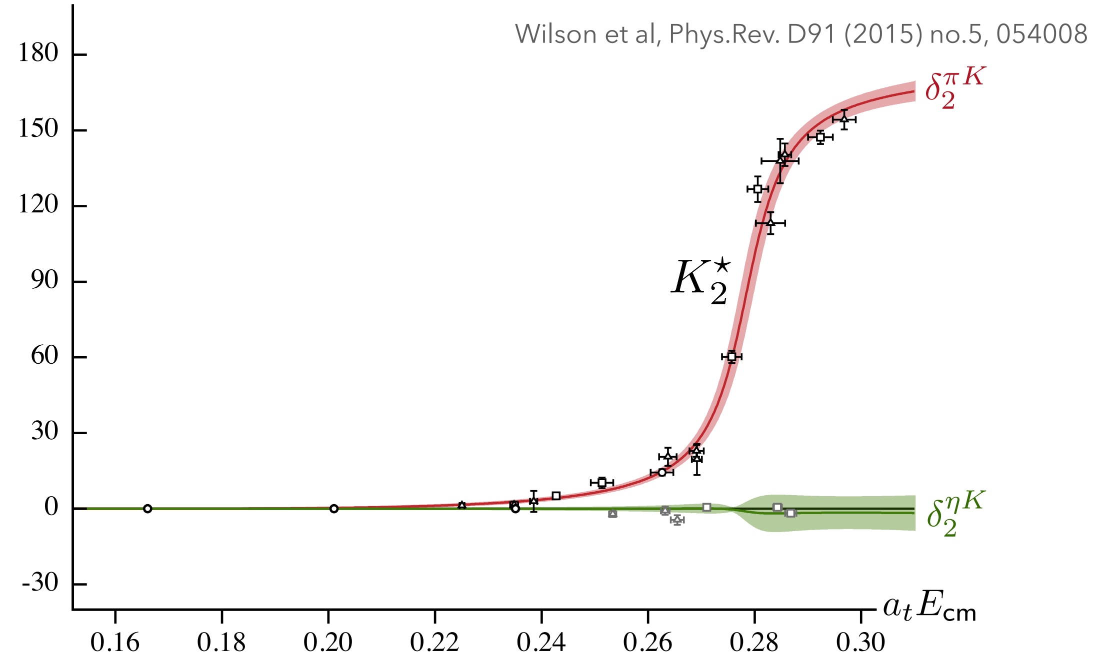
Briceno et al, arXiv:1708.06667



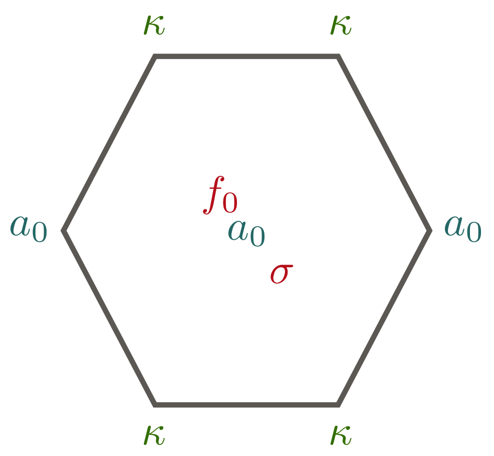
Dudek et al, Phys.Rev. D93 (2016) no.9, 094506



Wilson et al, Phys.Rev. D91 (2015) no.5, 054008

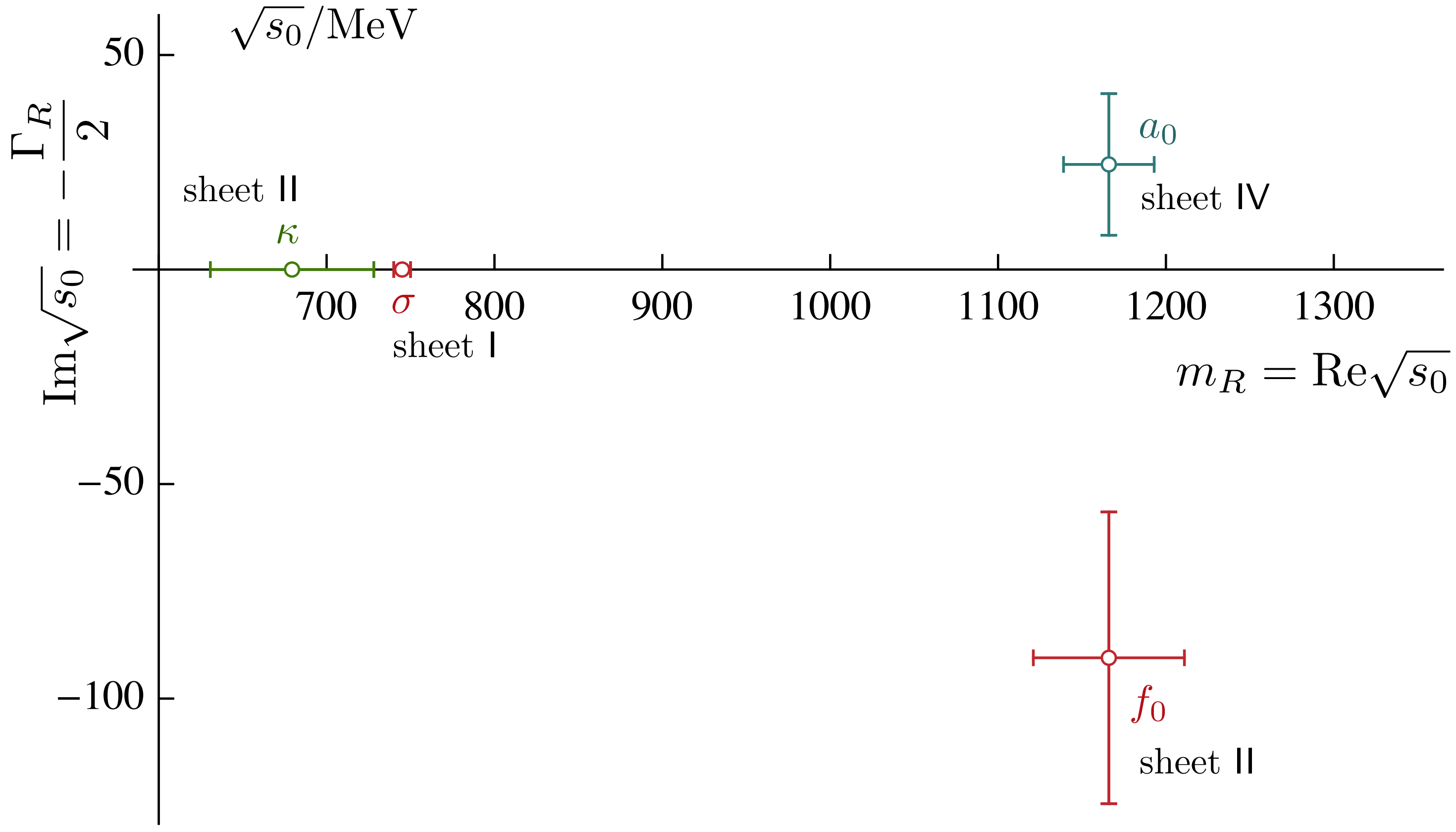


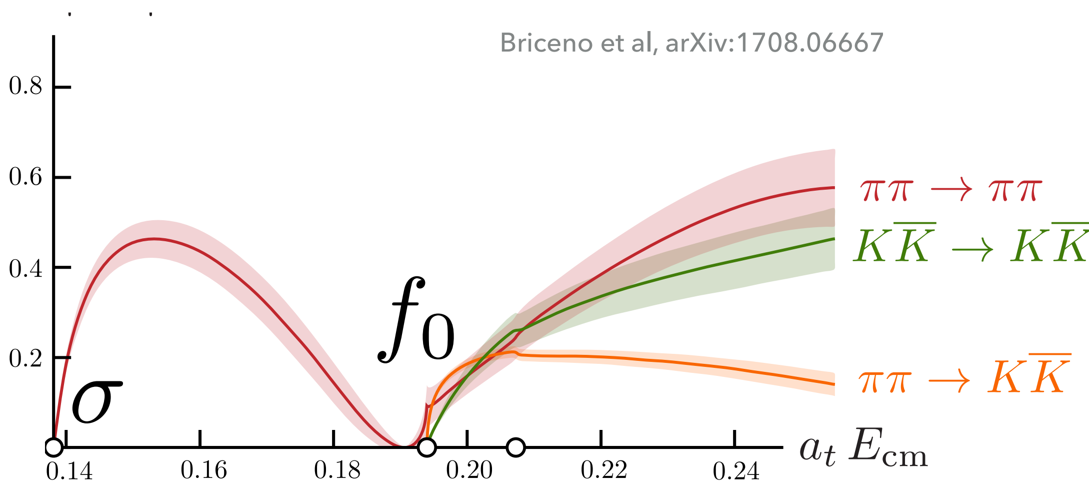
The lightest scalars of QCD at mπ=391 MeV



$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$





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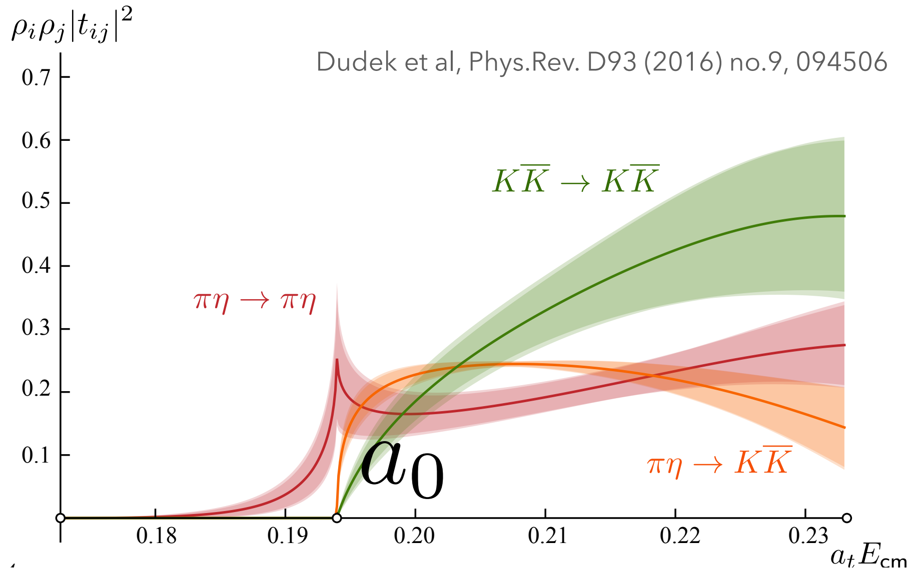

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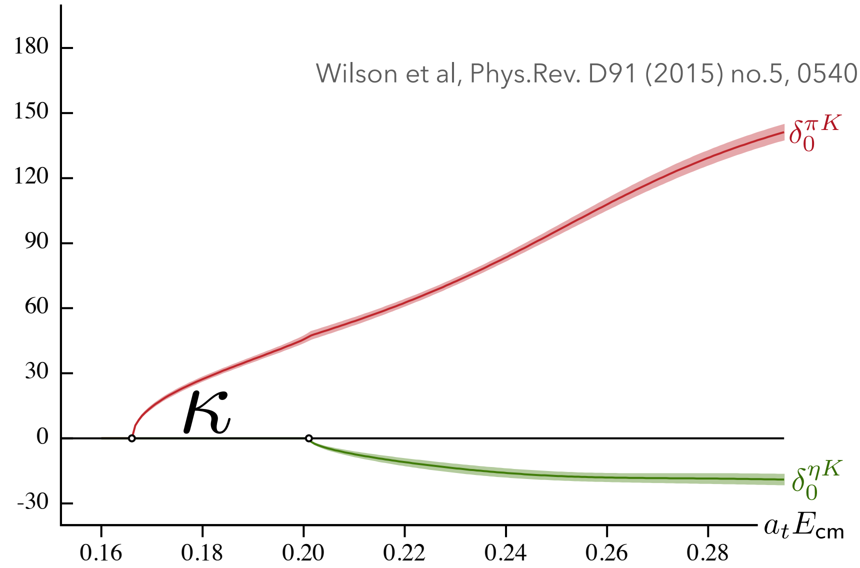


Wilson et al, Phys.Rev. D91 (2015) no.5, 054008

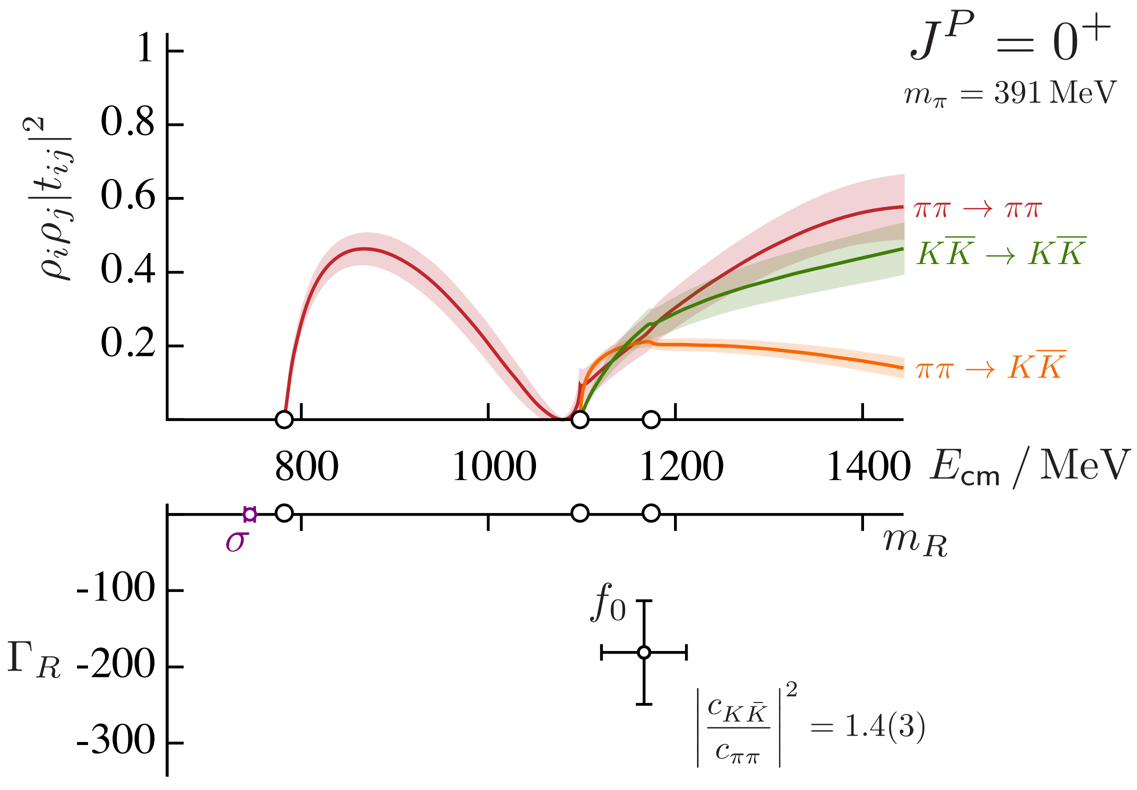
Summary

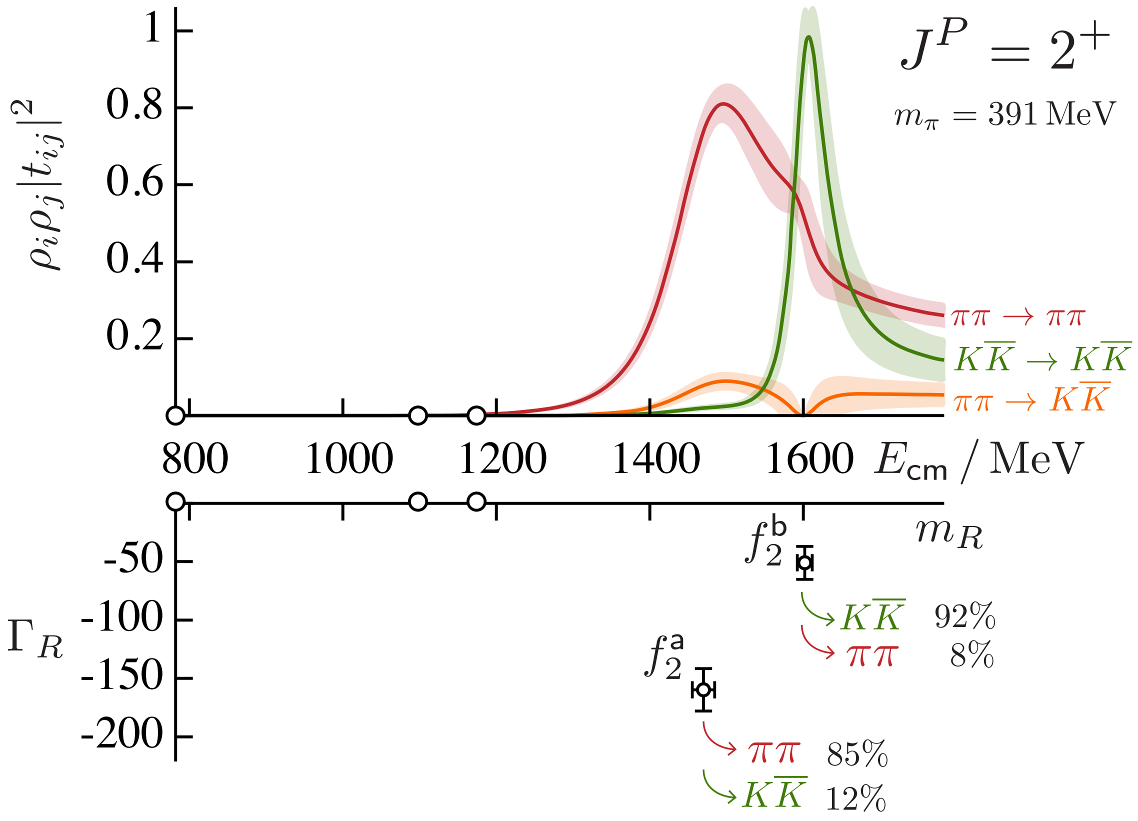
Scattering amplitudes of pairs of pseudoscalar hadrons can be computed from lattice QCD

Several channels with scalar, vector and tensor resonances have been computed

Control of 3+ body effects needed for
- lighter pion masses
- higher mass resonances

Scattering of particles with spin

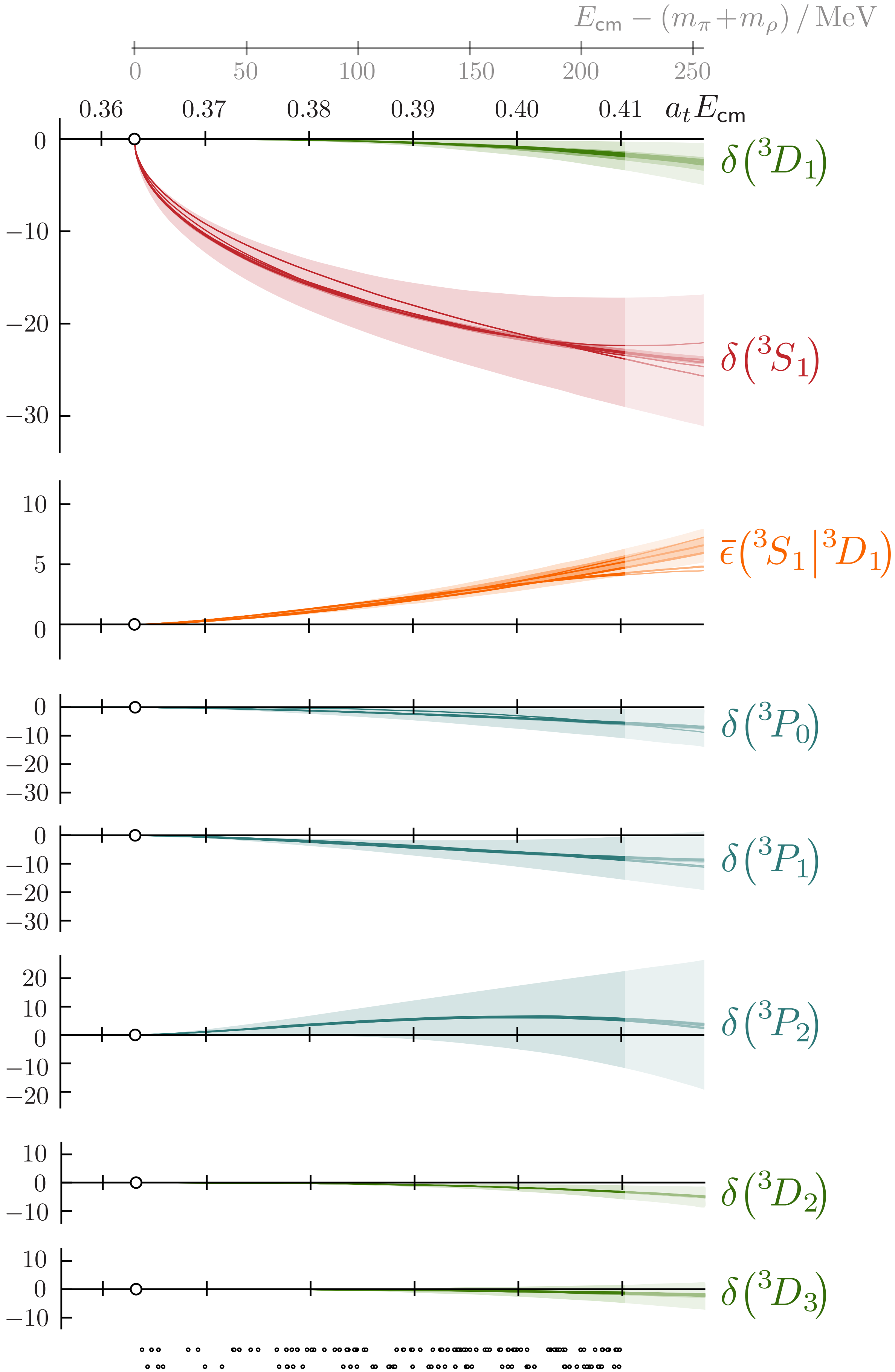




spinning particles

I = 2 $\rho\pi$ scattering, $m_\pi = 700$ MeV

Woss et al, arXiv:1802.05580



Many thanks to my collaborators:

Raul Briceño, Bipasha Chakraborty, **Jozef Dudek, Robert Edwards**, Balint, Joo, David Richards (Jefferson Lab & around)

Mike Peardon, Sinéad Ryan, Cian O'Hara, David Tims (Trinity College Dublin)

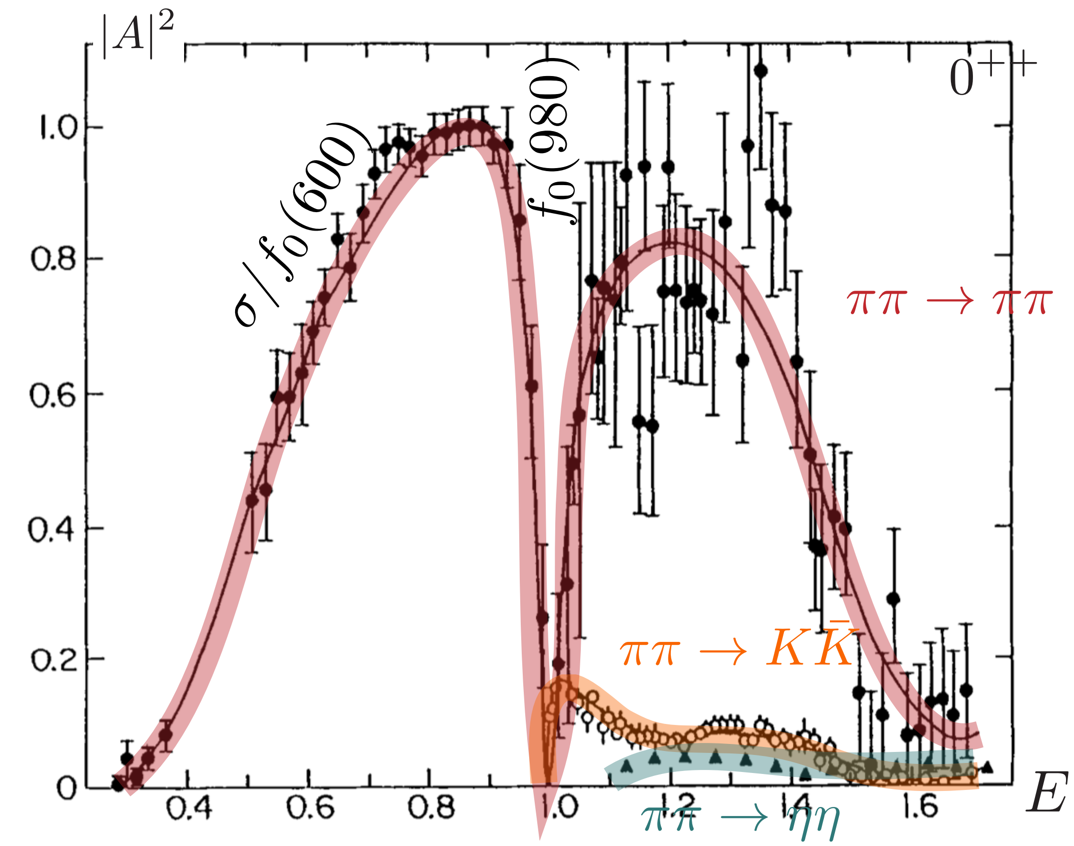
Christopher Thomas, Graham Moir, Gavin Cheung, **Antoni Woss** (University of Cambridge)

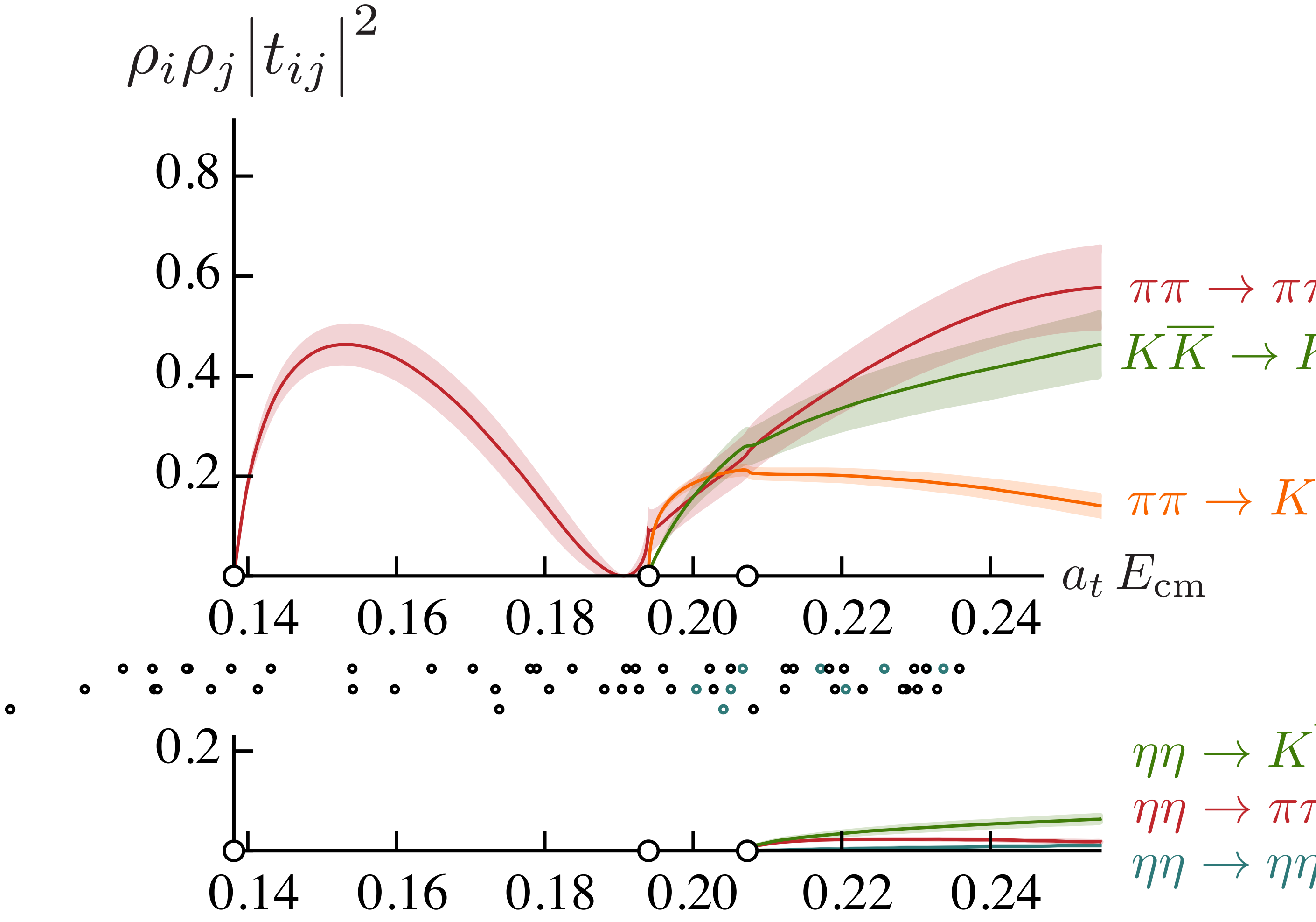
Nilmani Mathur (Tata Institute)

(**Bold** - authors of one or more of the papers mentioned)

Backup

CERN-Munich, ANL, BNL





Isoscalar scattering

3 volumes
L=16, 20, 24
anisotropic (3.5 finer spacing in time)
Wilson-Clover

m_π=391MeV

m_K=549MeV

$$\begin{bmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} & \pi\pi \to \eta\eta \\ & K\bar{K} \to K\bar{K} & K\bar{K} \to \eta\eta \\ & & \eta\eta \to \eta\eta \end{bmatrix}$$

operators used:

$\bar{\psi}\Gamma\overleftrightarrow{D}\ldots\overleftrightarrow{D}\,\psi$ local qq-like constructions

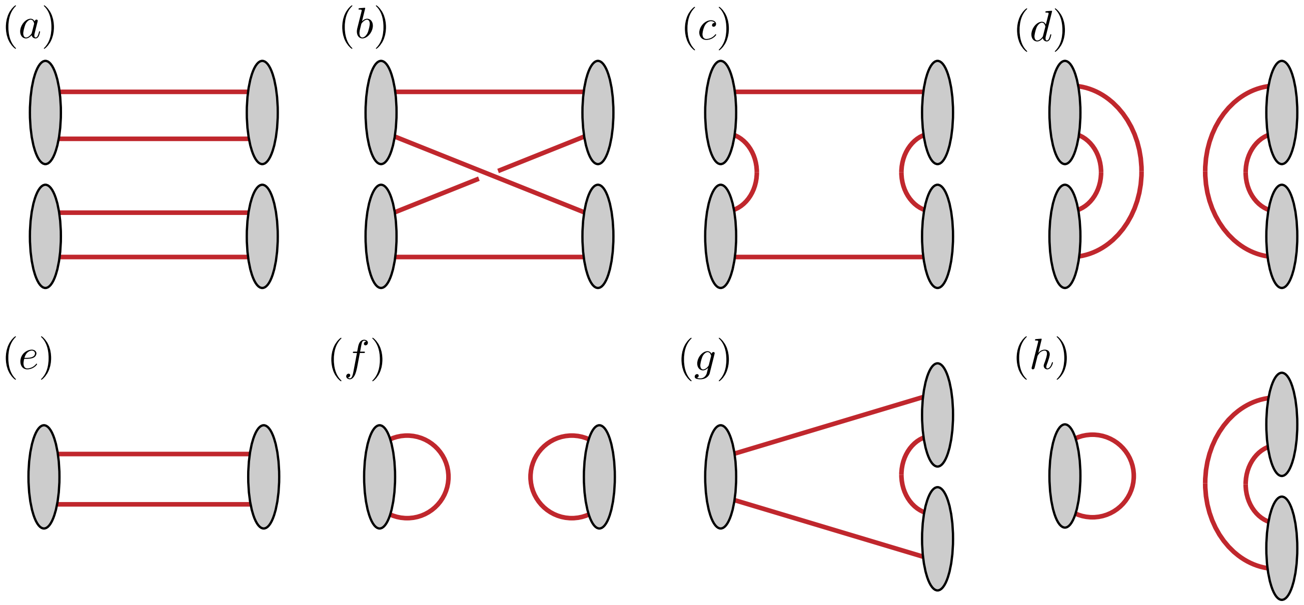
$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$

two-hadron constructions

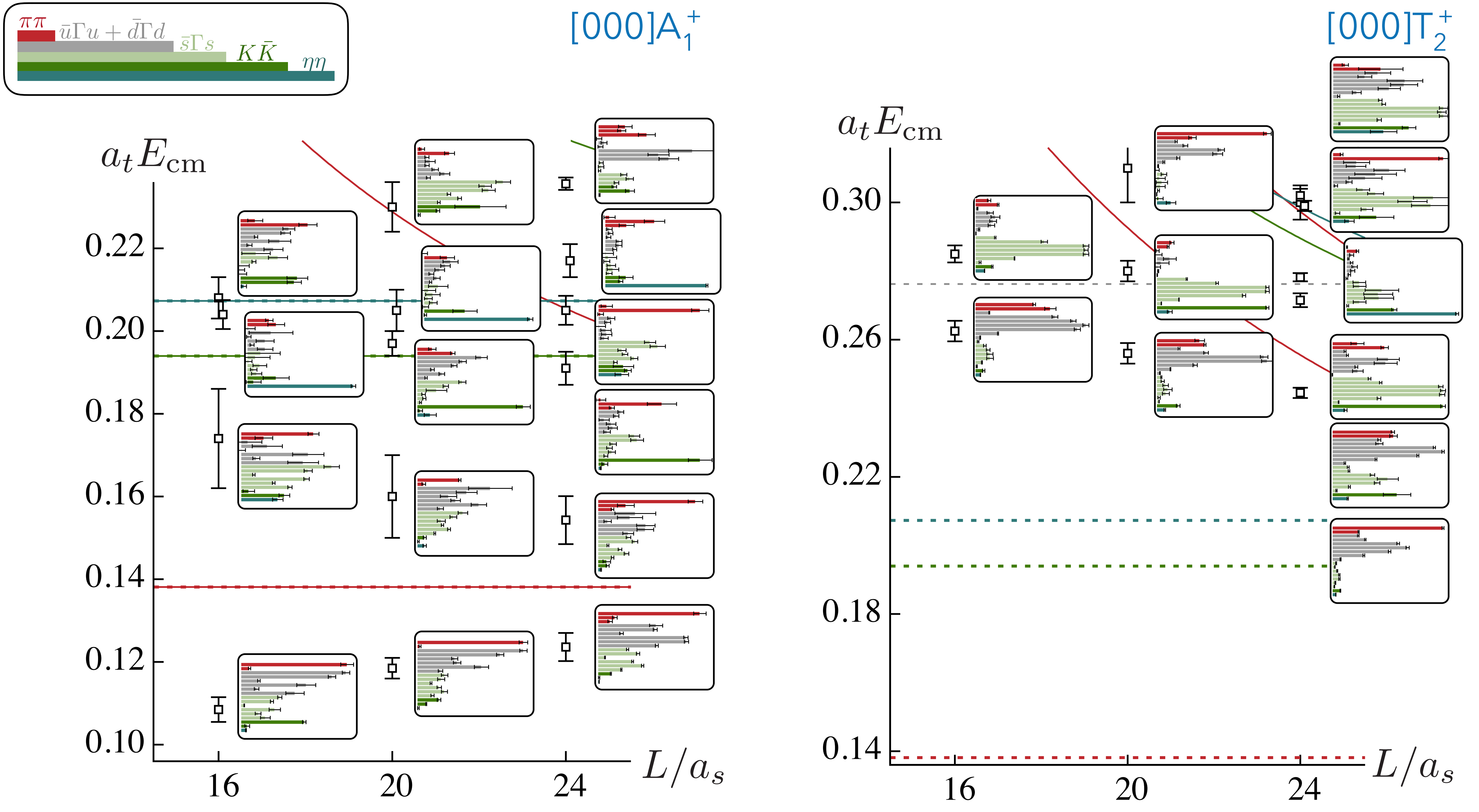
$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)
many wick contractions, eg just pi-pi & qq operators:



Spectra and overlaps



operator overlaps give some intuition

lots of mixing in the scalar sector

- essential to have meson-meson ops even below threshold
- can't always 'read-off' resonance content

recent review by Briceno, Dudek, Young:
arXiv:1706.06223

Extracting the t-matrix

Direct extension of the elastic quantization condition

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space

infinite volume scattering
t-matrix

known finite-volume
functions

Elastic scattering: Lüscher 1986,1991
Generalised to moving frames: Gottlieb, Rummukainen 1995
Unequal masses: Prelovsek, Leskovec 2012

Many derivations of the coupled-channel extension, **all in agreement**:
He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-½.

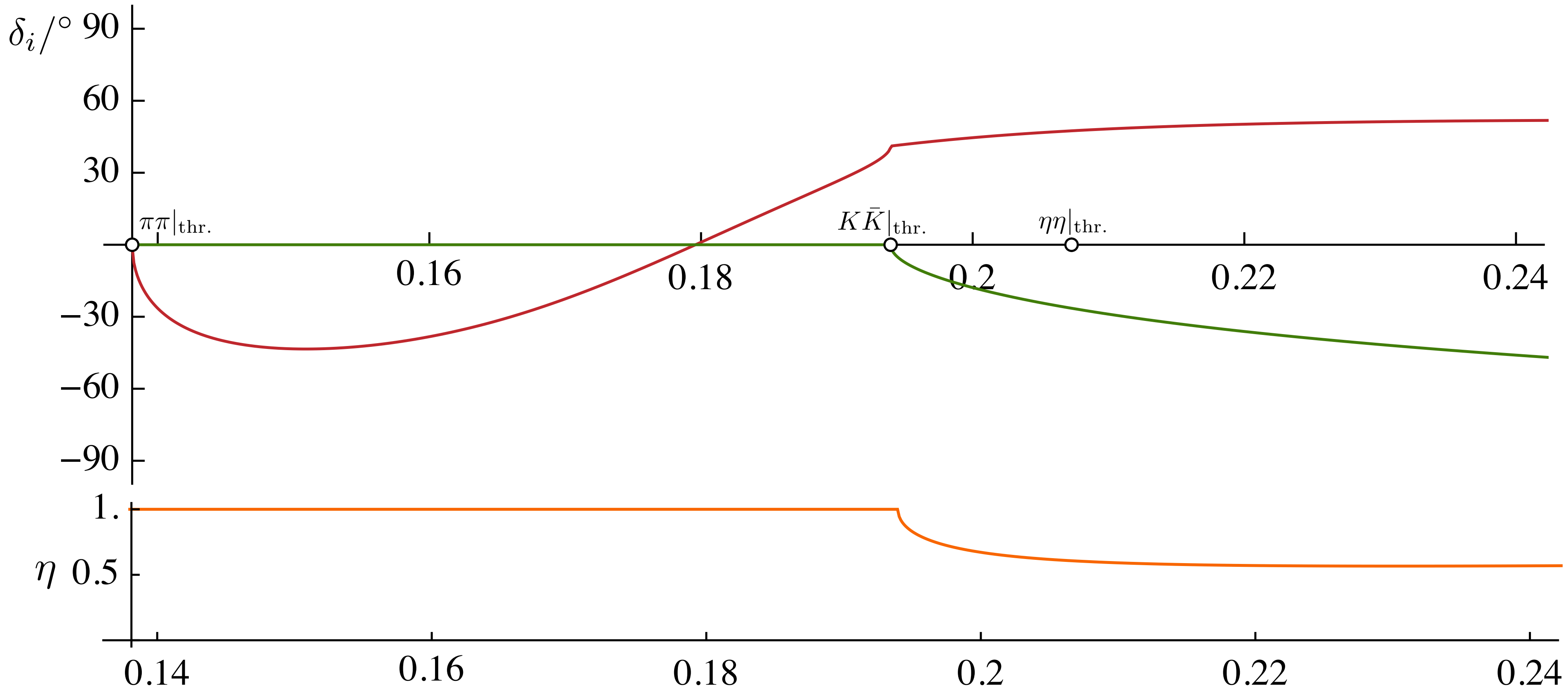
For progress on a general 3-body quantization condition - see other talks

Given some t-matrix...

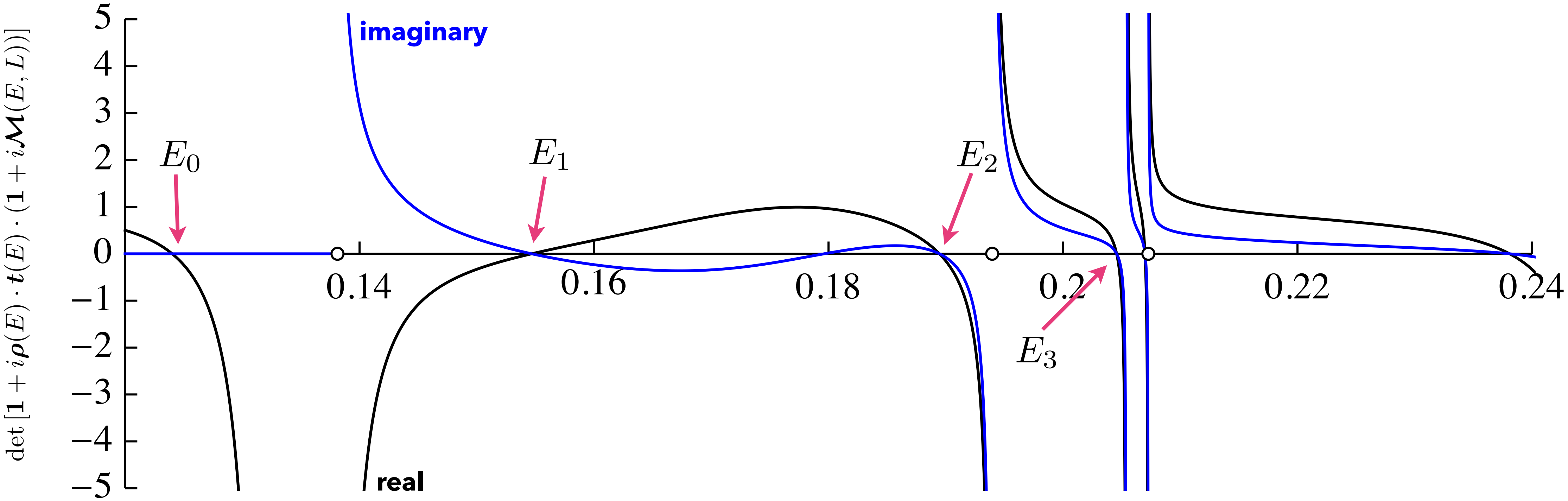
$$t_{11} = \frac{1}{2i\rho_1}\left(\eta e^{2i\delta_1} - 1\right)$$

$$t_{12} = \frac{1}{2\sqrt{\rho_1\rho_2}}(1-\eta^2)^{\frac{1}{2}} e^{i\delta_1+i\delta_2}$$

$$S_{ii} = \eta e^{2i\delta_i}$$



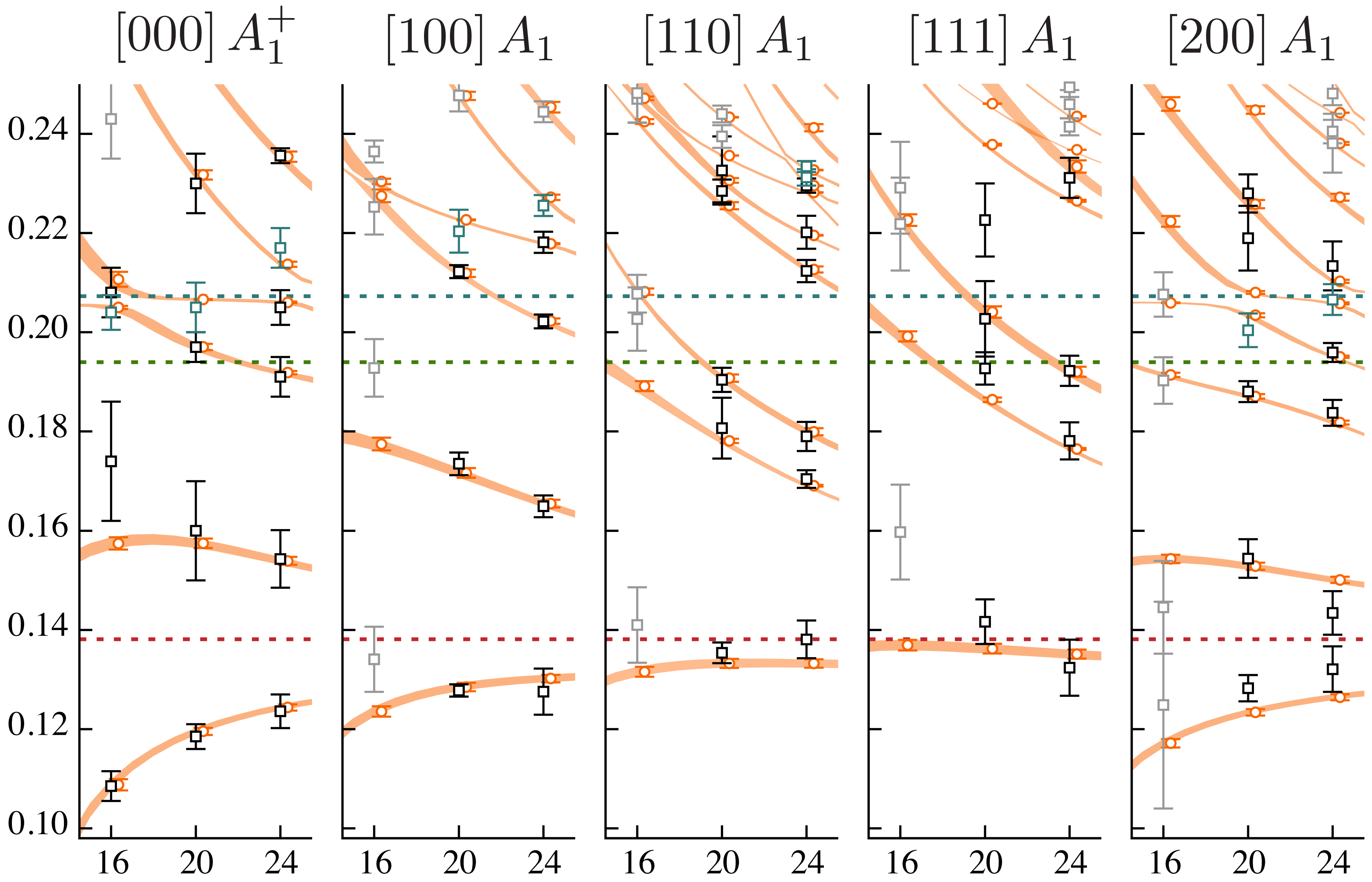
we can identify the zeros



Reproducing the spectrum

An example S-wave spectrum fit

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \boldsymbol{t}(E)\cdot \left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$



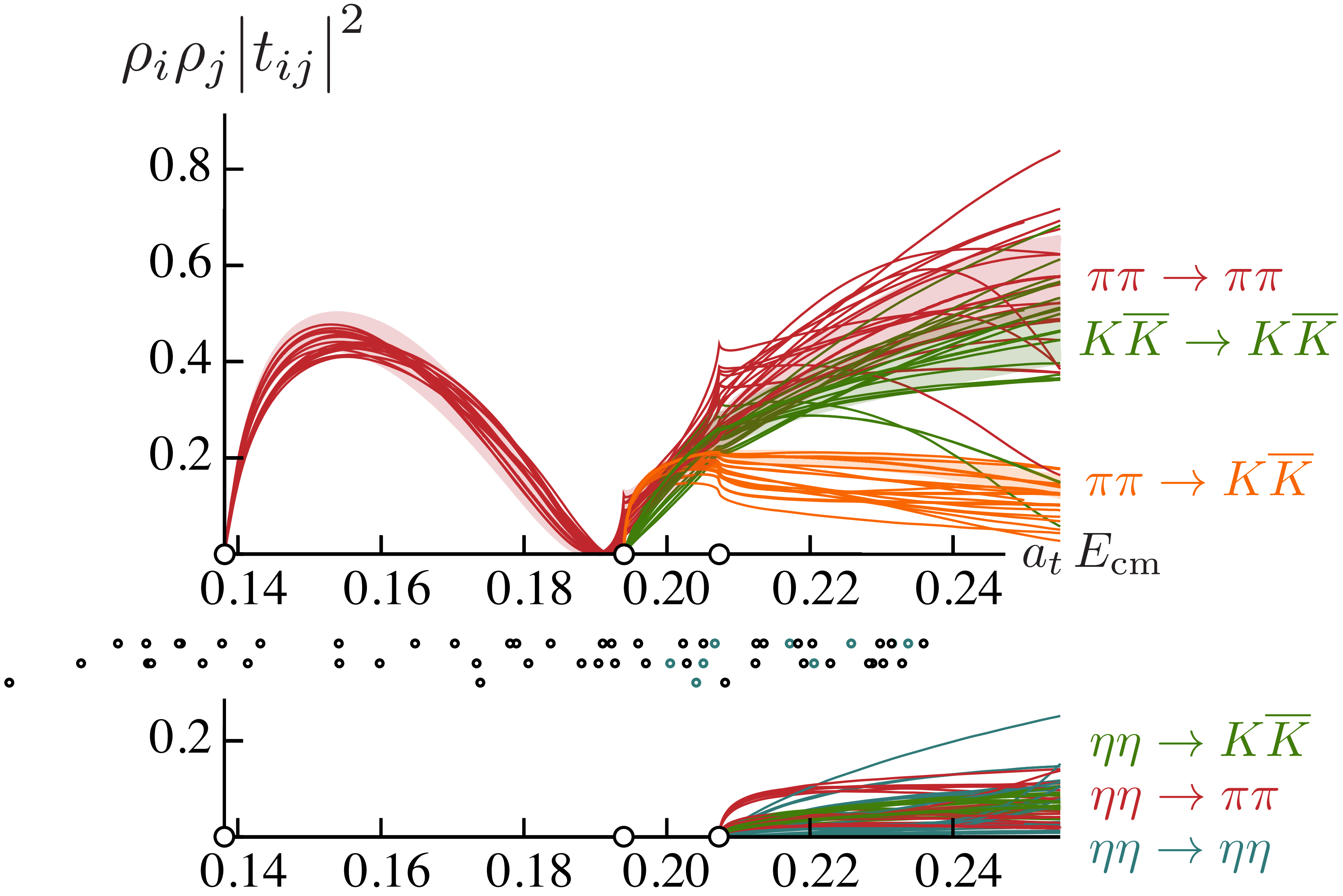
$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

S-wave amplitude variation

20 amplitudes

$\chi^2/N_{\mathrm{dof}} < 1.05$

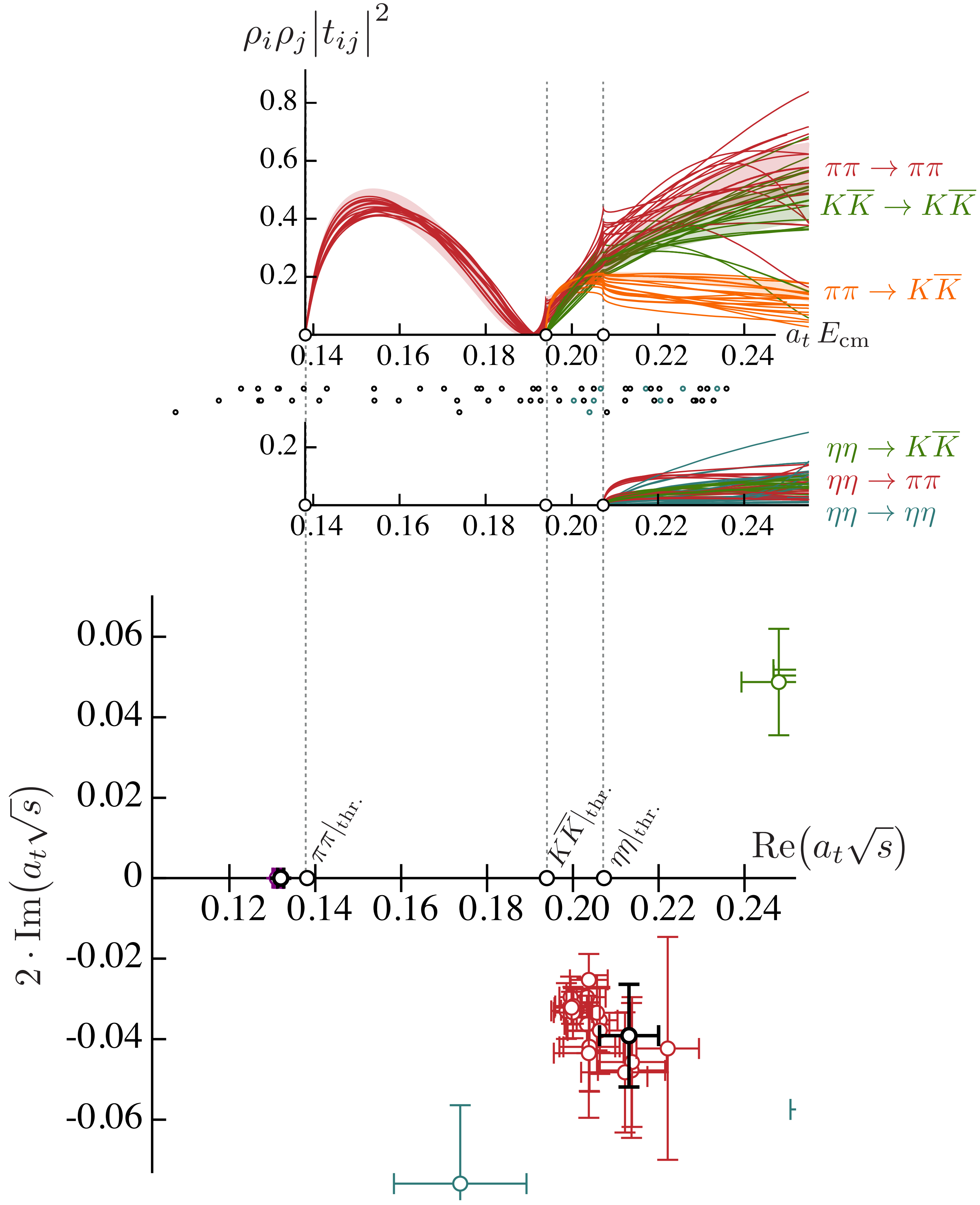
57 energy levels



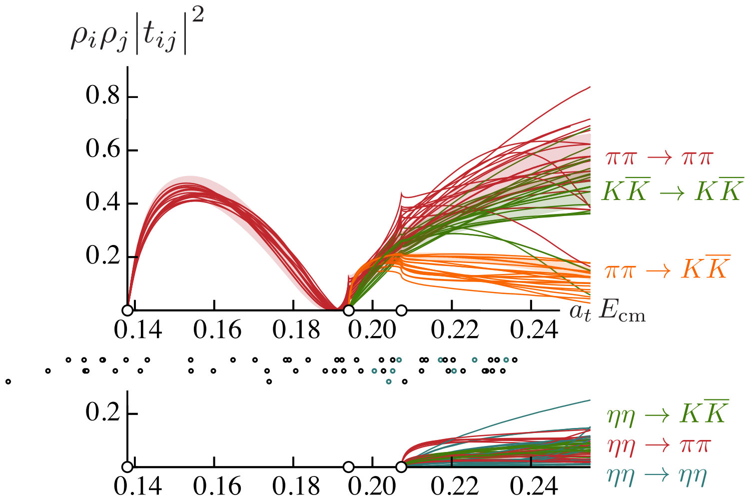
Scalar f0 poles

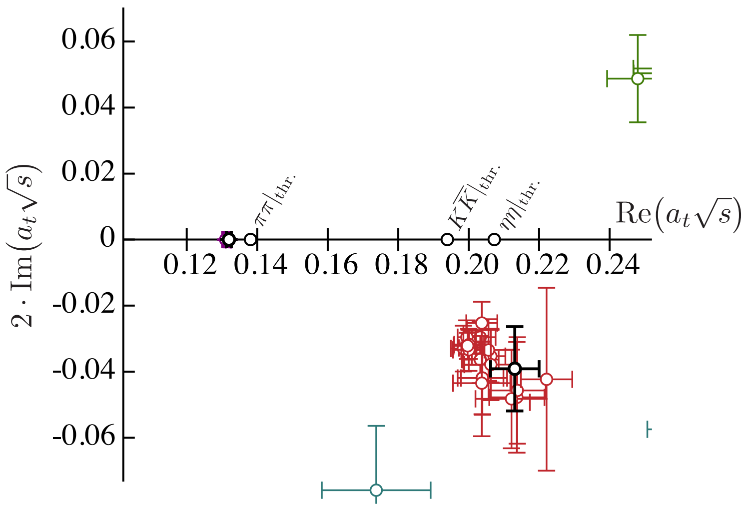
Near a t-matrix pole

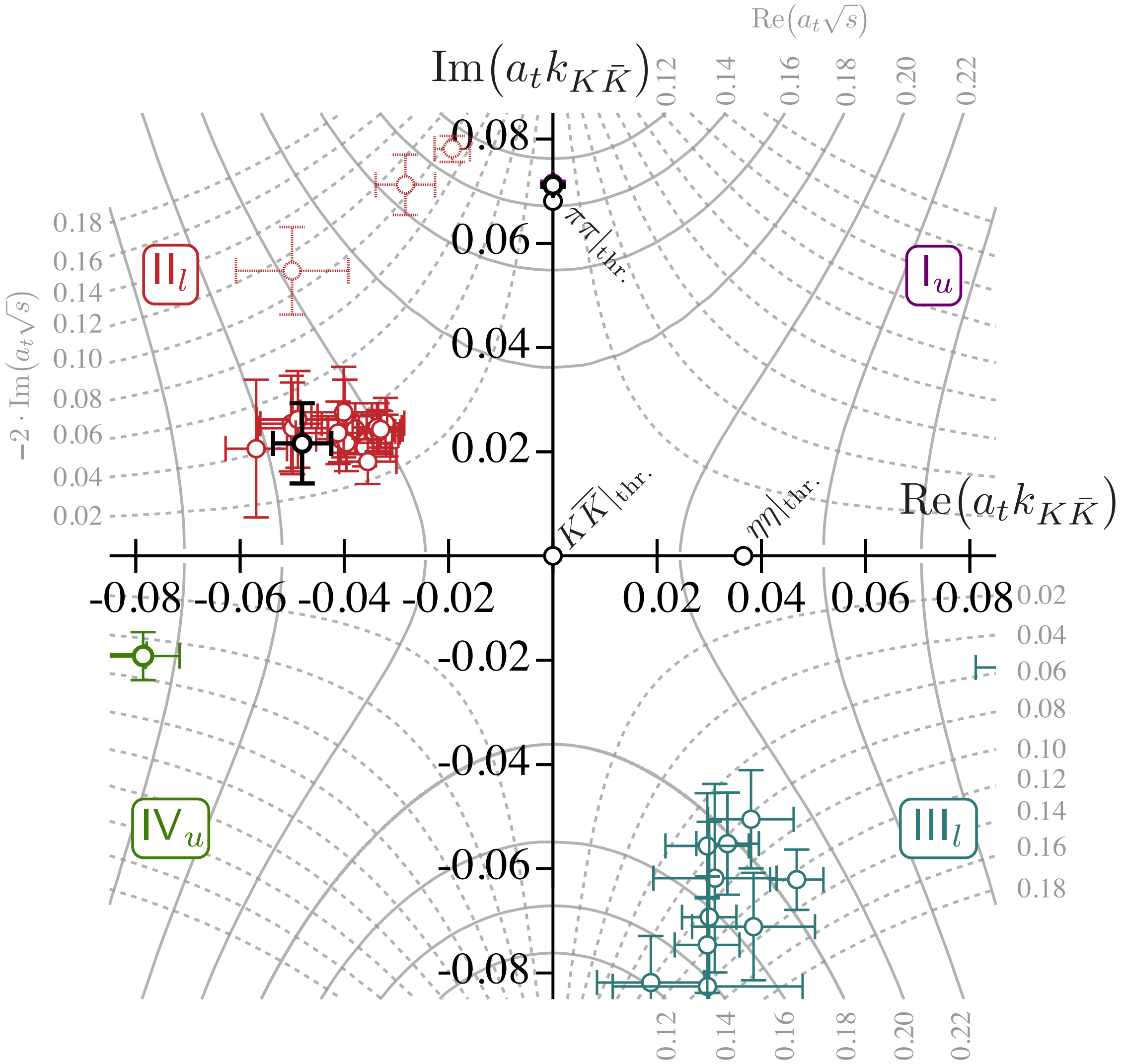
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

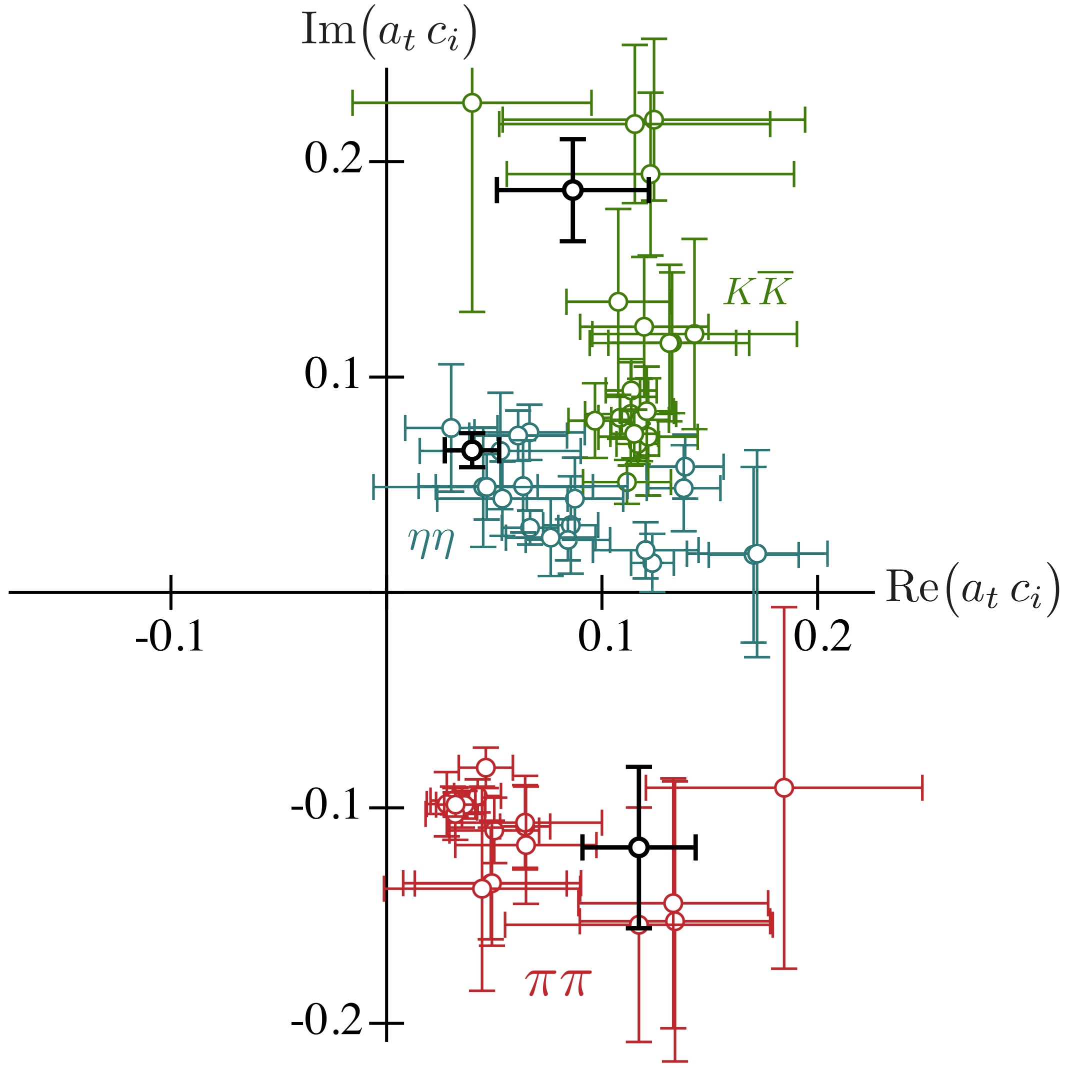


Scalar f0 resonance poles





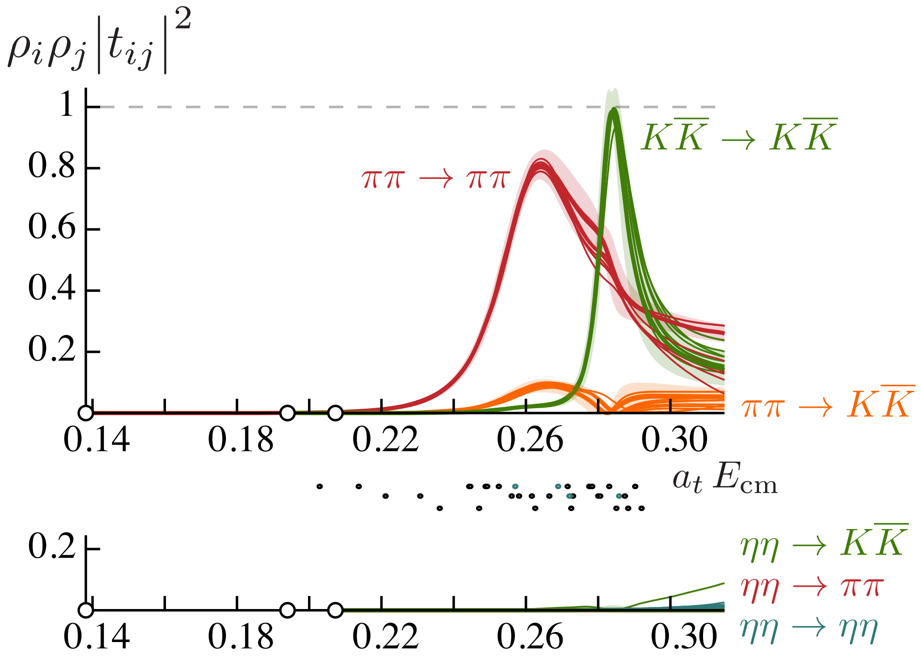


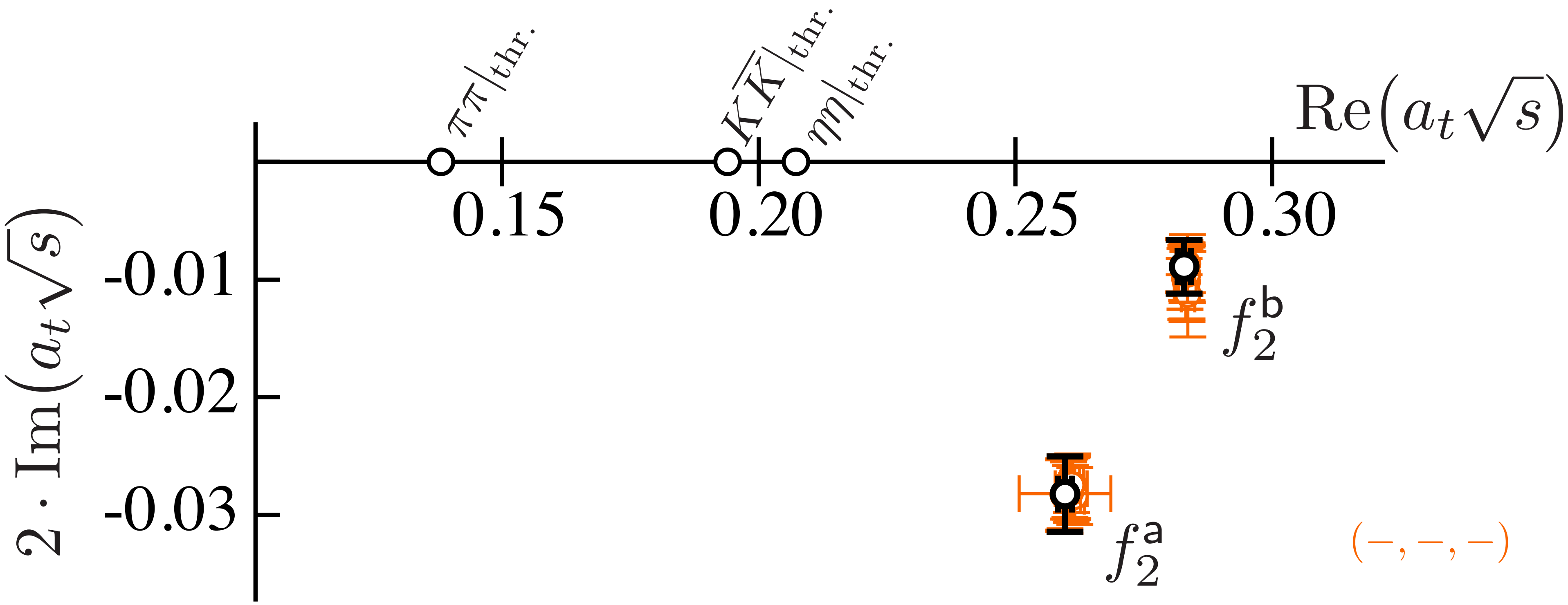


Tensor f2 resonance poles

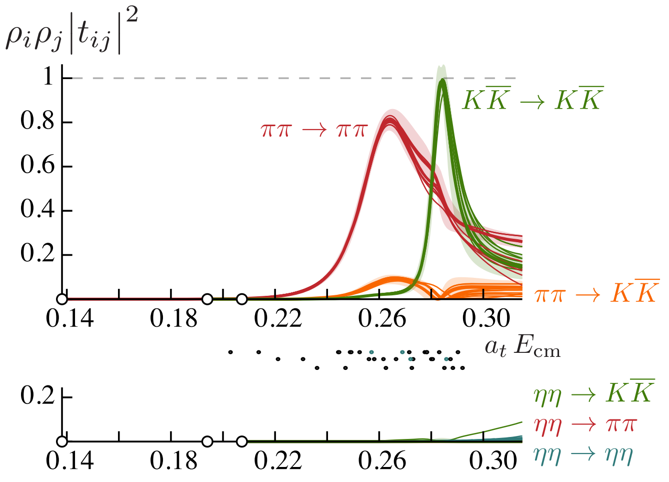
Near a t-matrix pole

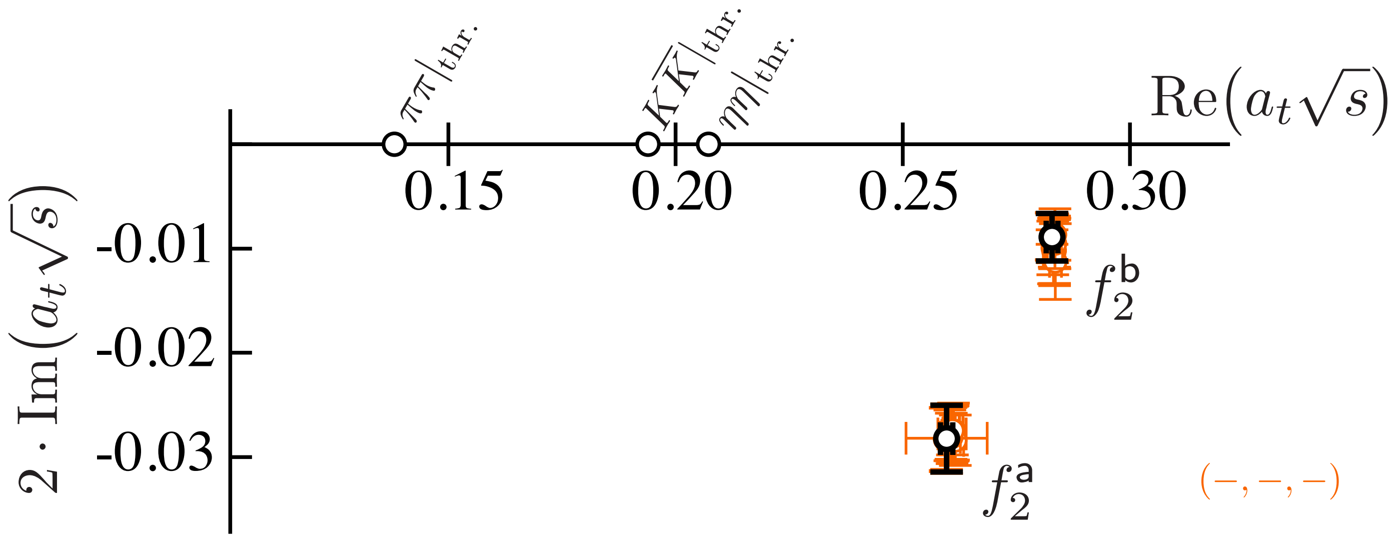
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

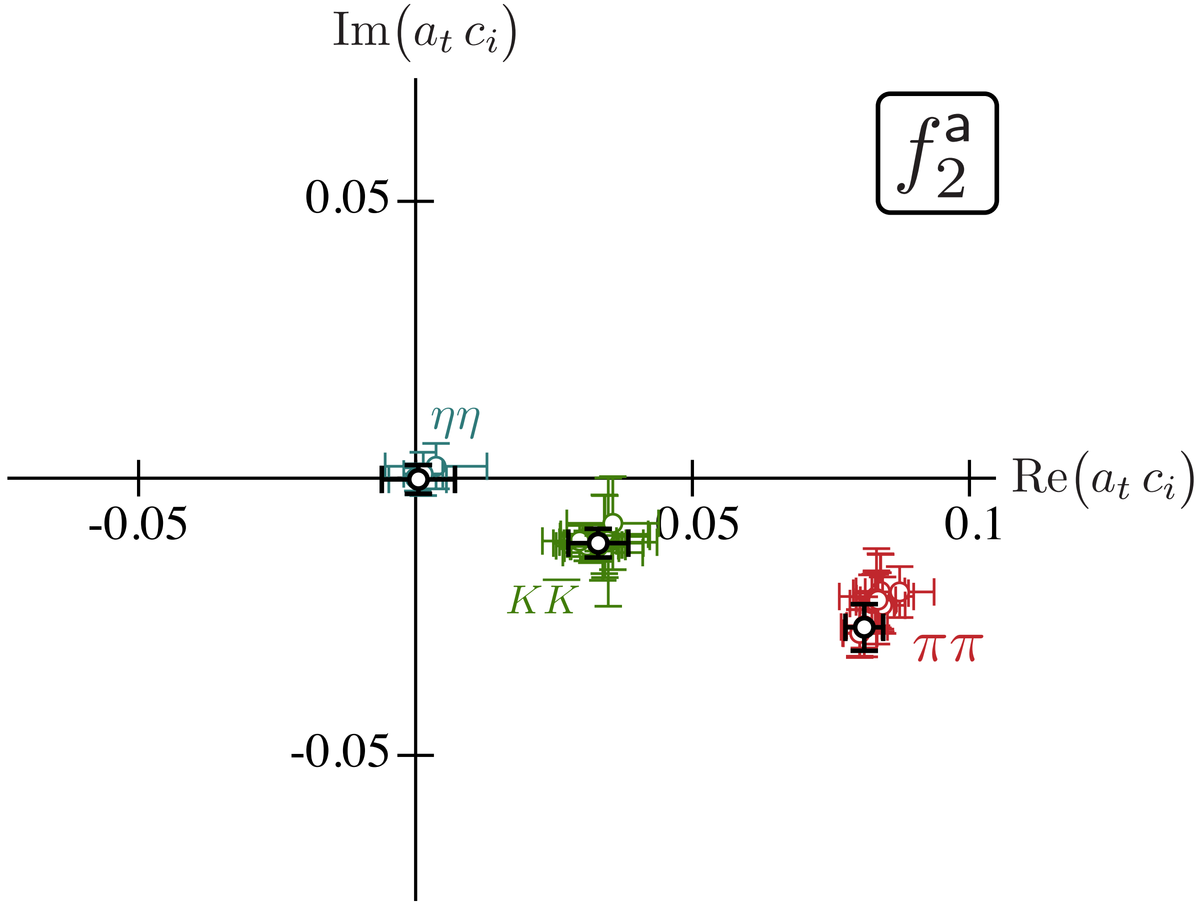


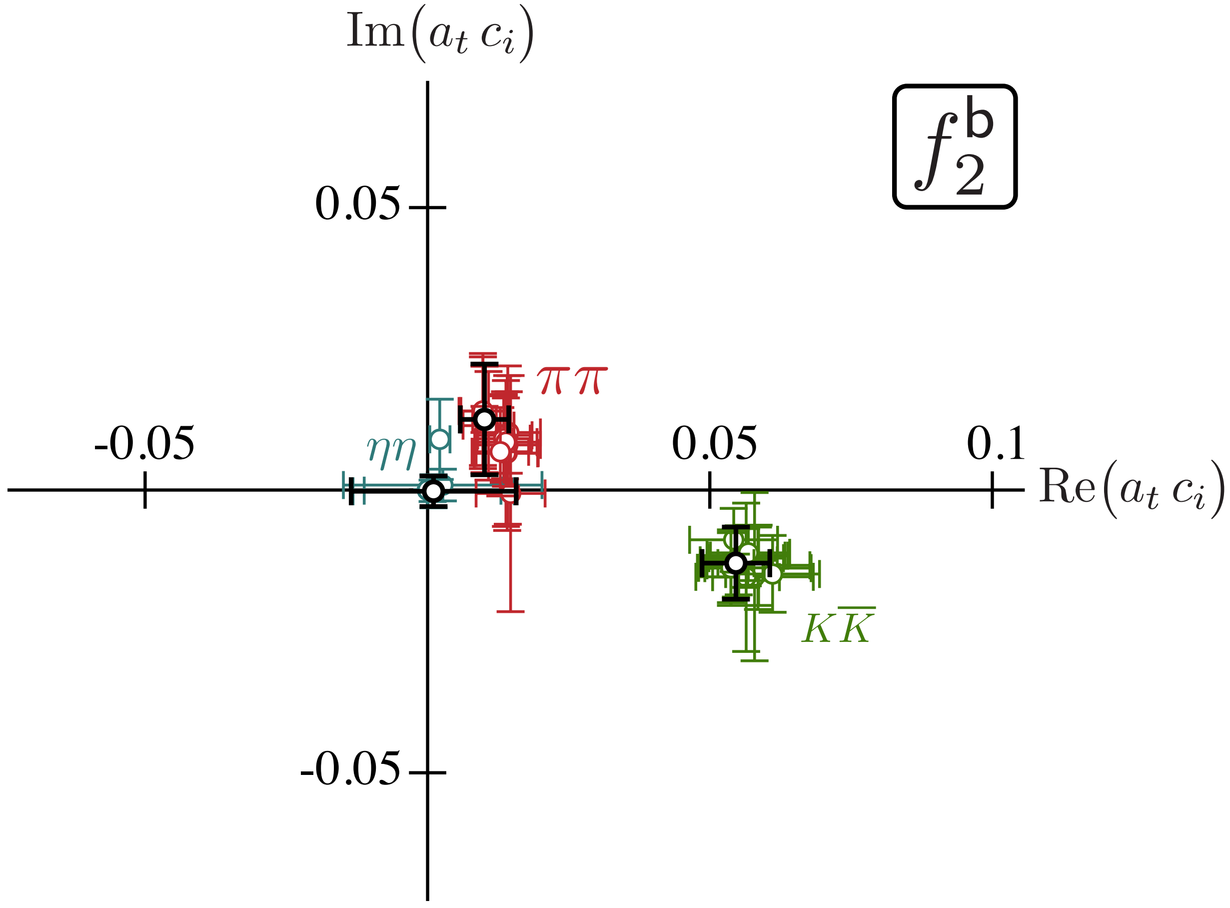


Tensor f2 resonance poles









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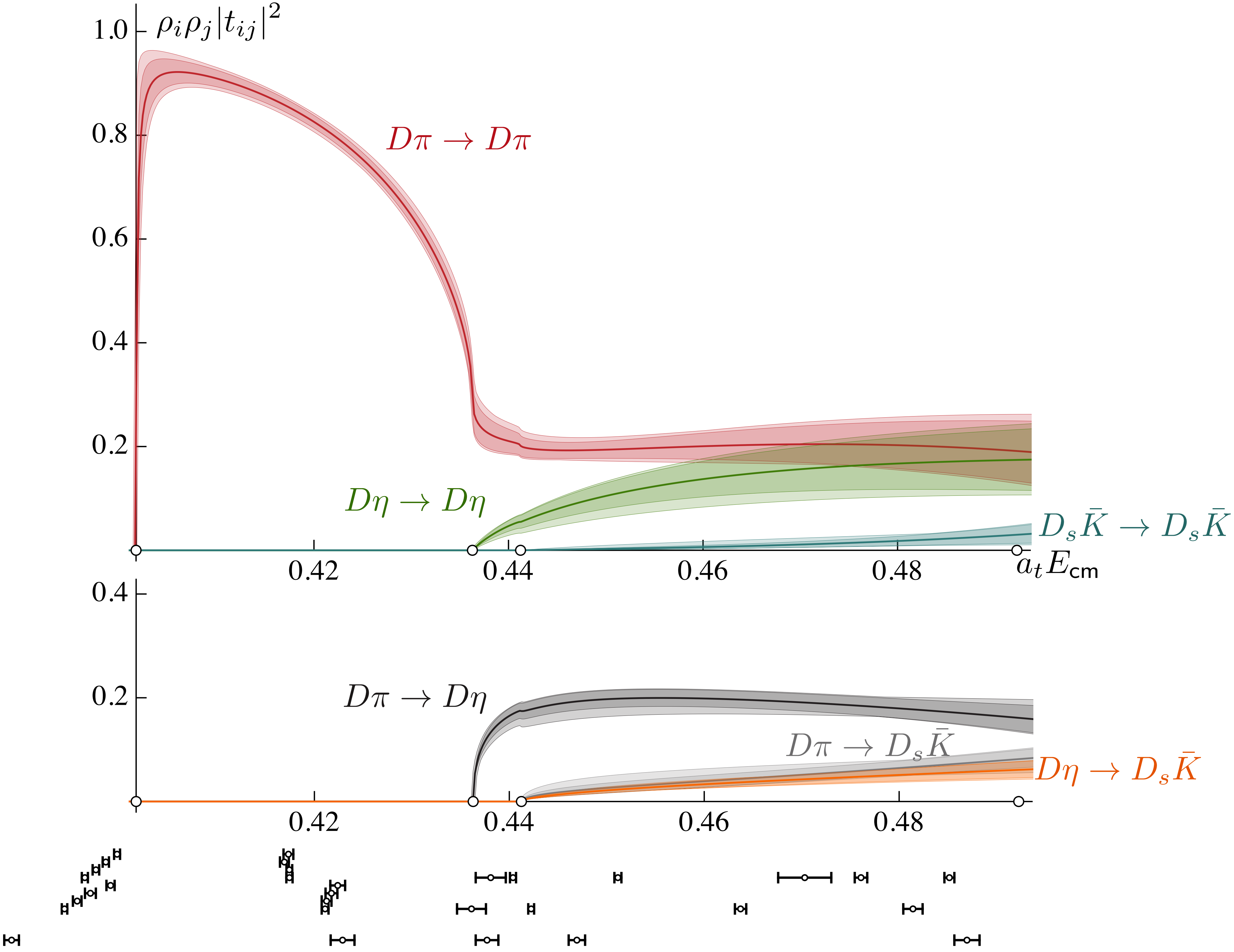
Dπ

Moir et al, JHEP 1610 (2016) 011



State composition

Several methods - many challenges similar to experimental analyses

- Weinberg method, uses renormalisation factor Z

  Useful for bound states - distinguishes composite meson-meson vs compact

- Morgan pole counting

  Generalisation of Weinberg - one pole ~ molecular, vs two poles ~ compact

- Photocouplings - determine radial extent
- Decay constants
- N_c dependence

  qq states become stable, meson-meson sink into continuum - Pelaez *et al*

- m_π dependence

  How a near-threshold state reacts to changes in the masses can give clues

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- m_π dependence

  How a near-threshold state reacts to changes in the masses can give clues

requires significant extra computation

State composition

- Weinberg method

Useful for bound states - distinguishes composite meson-meson vs compact

$$a = -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{m_\pi \epsilon}}$$

$$r = -\frac{Z}{1-Z}\frac{1}{\sqrt{m_\pi \epsilon}}$$

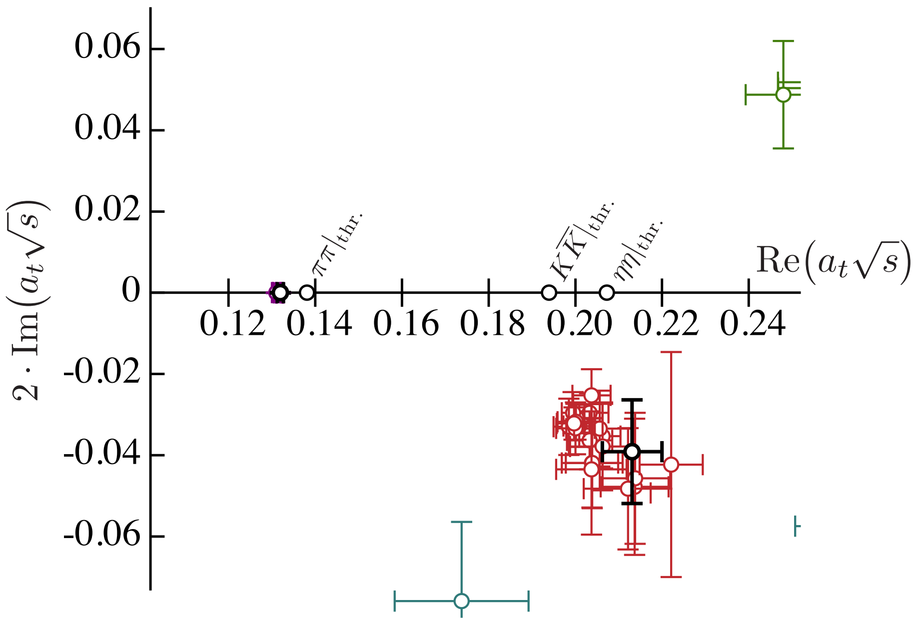
$$Z = 1 \sim \text{compact}$$

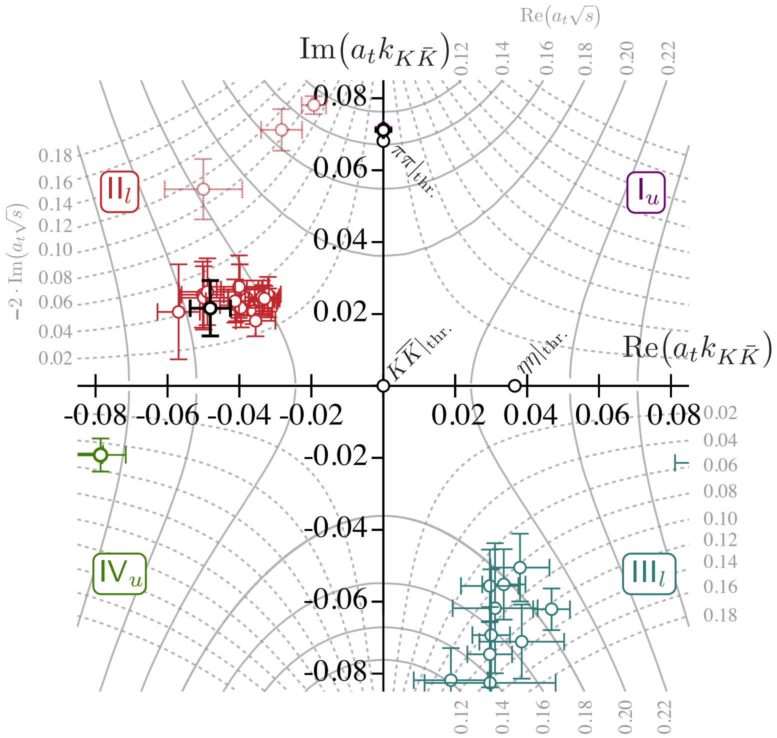
$$Z = 0 \sim \text{molecule}$$

$$Z \sim 0.3(1)$$

- Morgan pole counting

Generalisation of Weinberg - one pole ~ molecular, vs two poles ~ compact

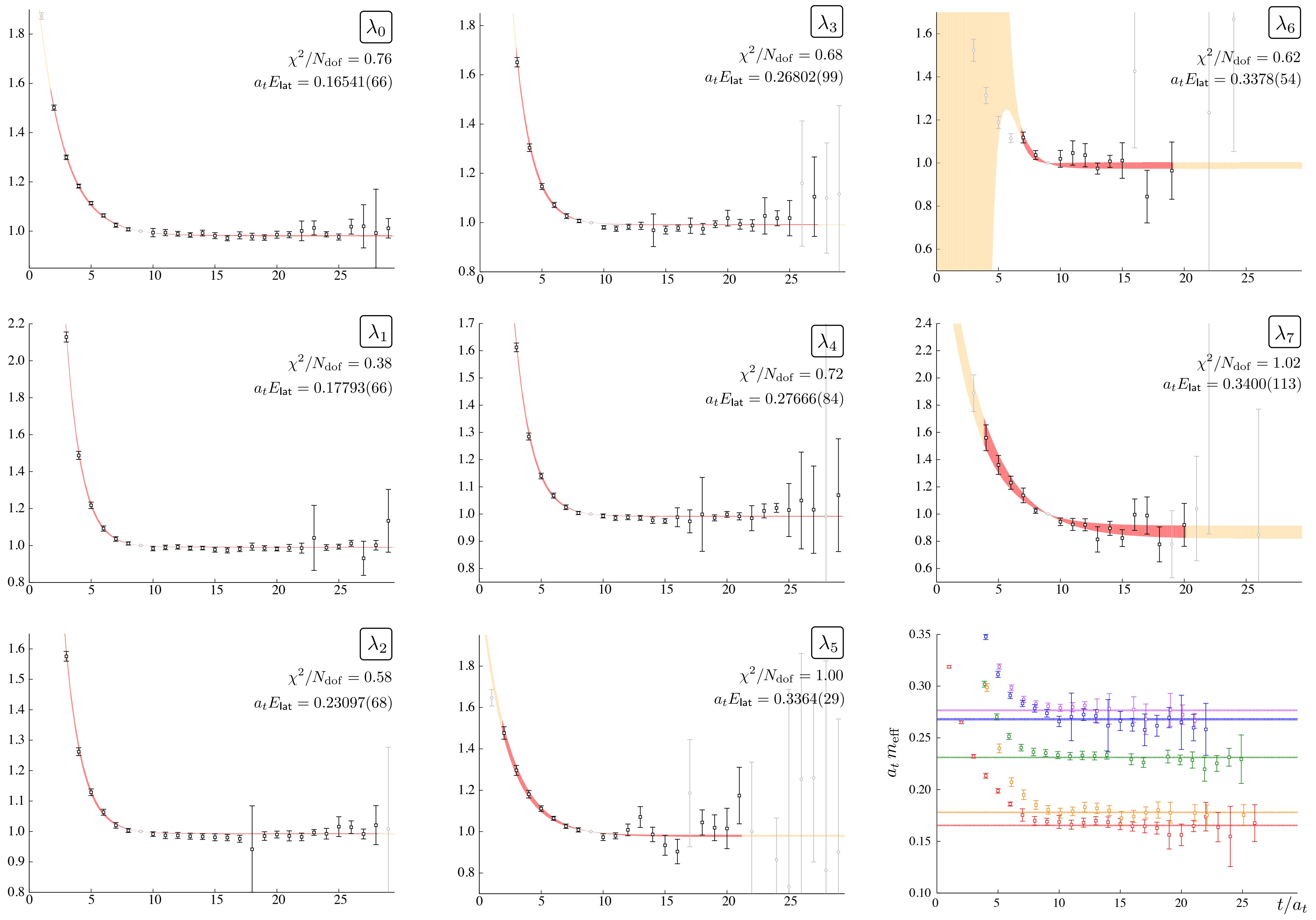




Principal correlators

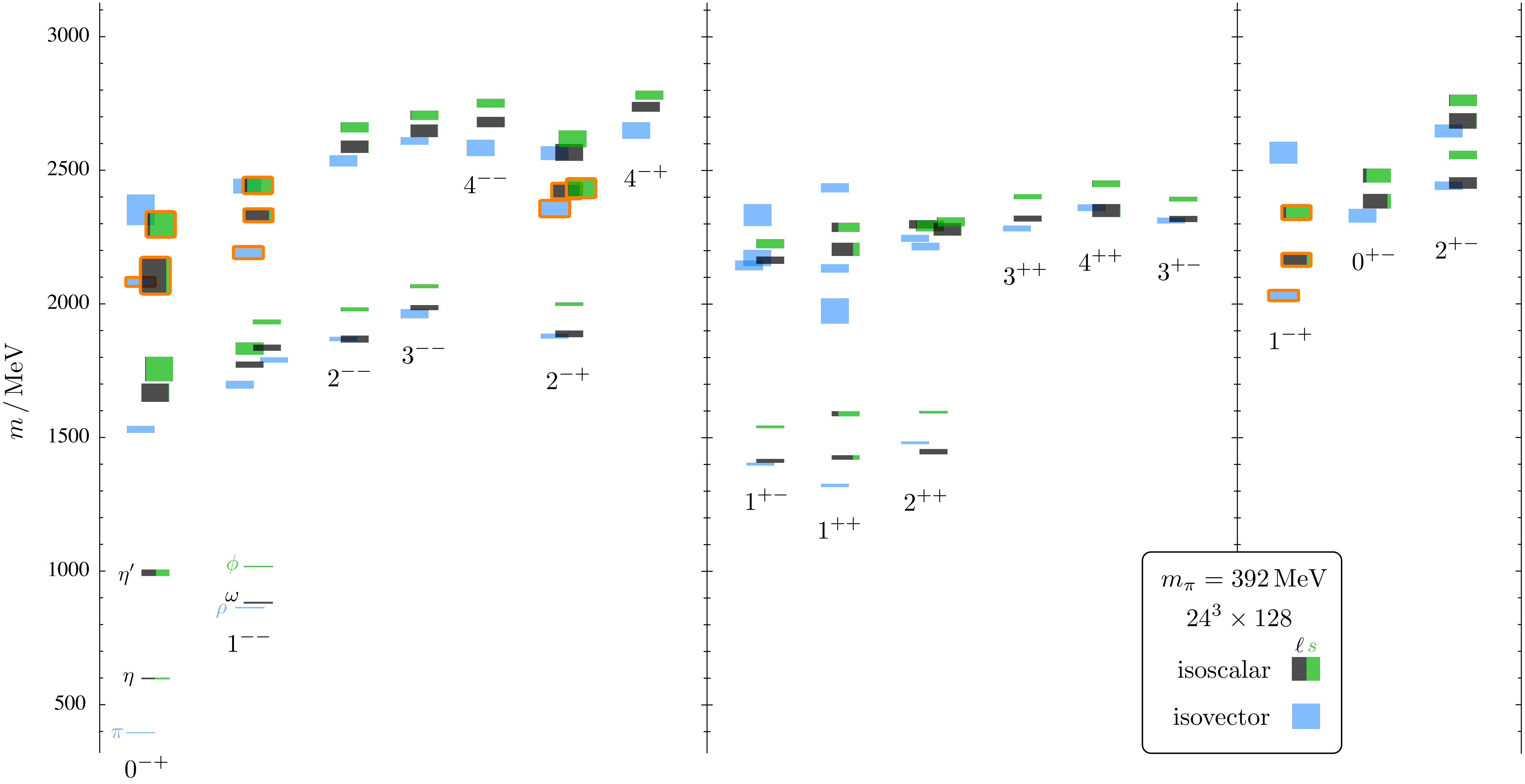
Obtain the finite volume spectrum using a variational method

$$C_{ij}(t)v_j^{\mathfrak{n}} = \lambda(t)_{\mathfrak{n}} C_{ij}(t_0)v_j^{\mathfrak{n}} \quad \rightarrow \quad \lambda_{\mathfrak{n}} \sim \exp(-E_{\mathfrak{n}} t)$$



J. J. Dudek, R. G. Edwards, P. Guo and C. E. Thomas, Phys.Rev. D88 (2013) 9, 094505.

light quark spectrum from qq̄ operators



J. J. Dudek, R. G. Edwards, P. Guo and C. E. Thomas, Phys.Rev. D88 (2013) 9, 094505.

K-matrix details

- *K*-matrix contains everything not constrained by unitarity

$$t^{-1}_{ij}(s) = K^{-1}_{ij}(s) - i\delta_{ij}\rho_i(s)$$

$$K = \frac{1}{m^2 - s}\begin{bmatrix} g^2_{\pi K} & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g^2_{\eta K} \end{bmatrix} + \begin{bmatrix} \gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K} \end{bmatrix}$$

- Chew-Mandelstam phase space -- include also *s*-channel cut along with imaginary part.

$$t^{-1}_{ij}(s) = K^{-1}_{ij}(s) + \delta_{ij}\; I_i(s)$$

$$I_i(s) = I_i(s_{\mathrm{thr}_i}) - \frac{s - s_{\mathrm{thr}_i}}{\pi}\int_{s_{\mathrm{thr}_i}}^{\infty} ds'\, \frac{\rho_i(s')}{(s' - s)(s' - s_{\mathrm{thr}_i})}$$

(Subtract at pole so that Re $I(s = m^2) = 0$)

- Threshold factors for *l*>0

$$t^{-1}_{ij}(s) = \frac{1}{(2k_i)^{\ell}} K^{-1}_{ij}(s) \frac{1}{(2k_j)^{\ell}} + \delta_{ij}\; I_i(s)$$

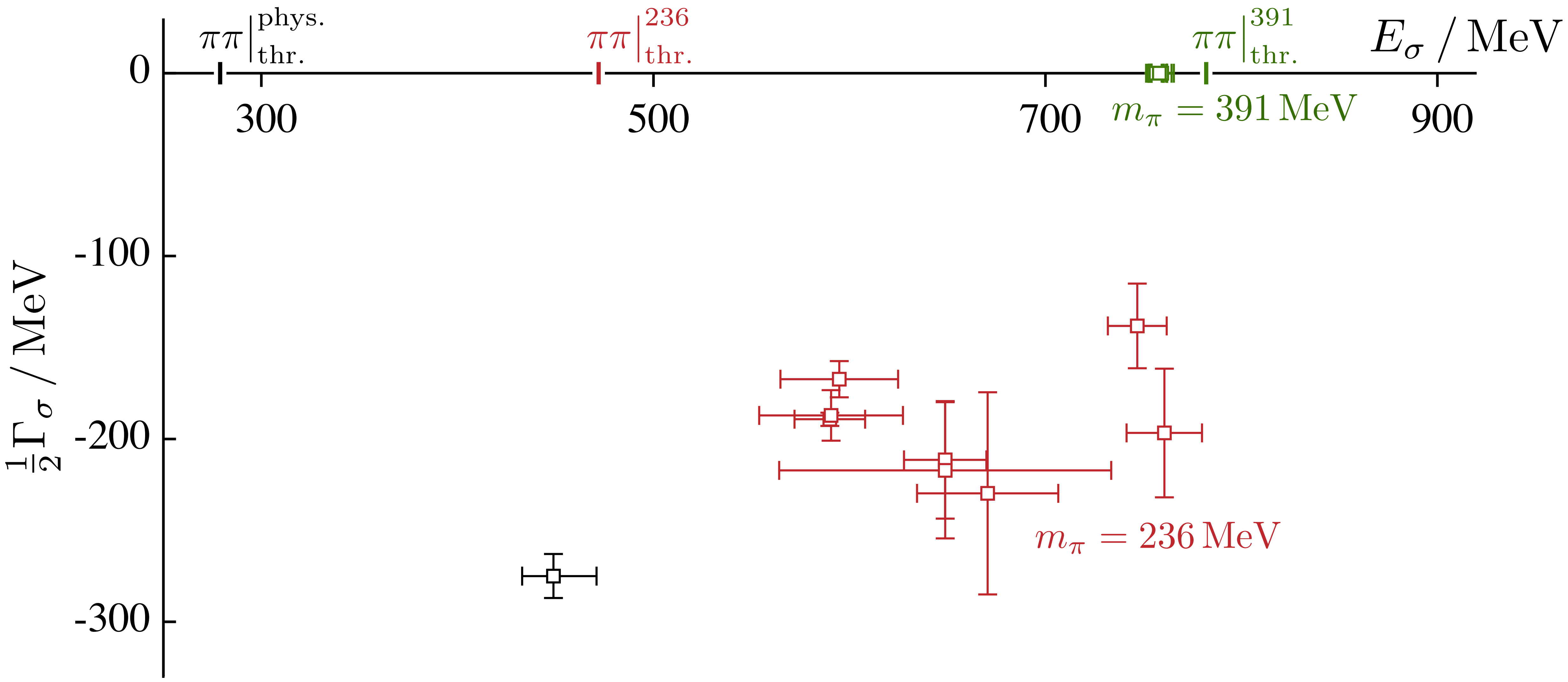
As used in Guo, Mitchell and Szczepaniak Phys.Rev. D82 (2010) 094002

No modifications were used in I(s) for higher waves.

Also tested phase space factors instead of k_i for thresholds.

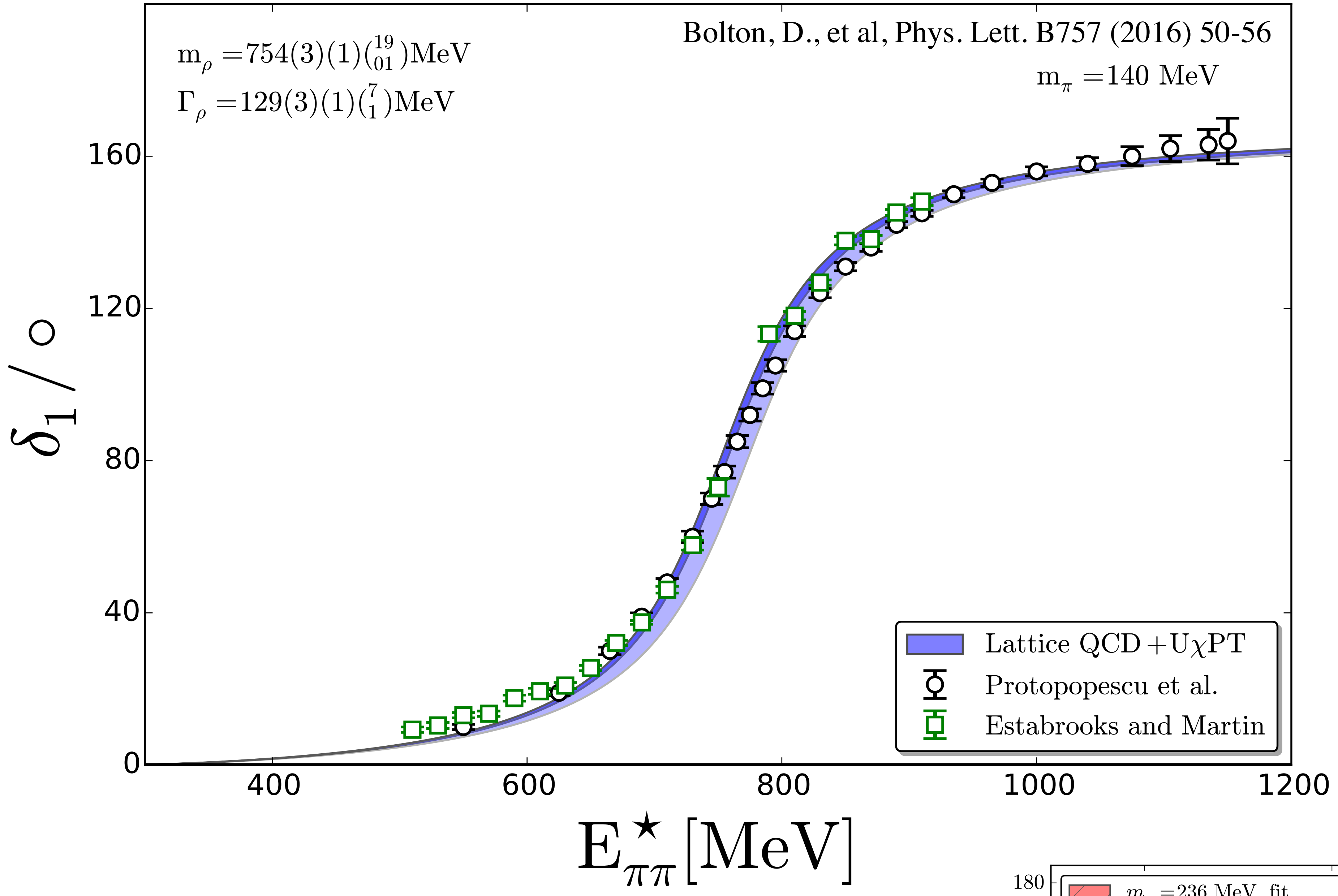
Sigma pole

arXiv:1607.05900,
PRL 118, 022002 (2017)

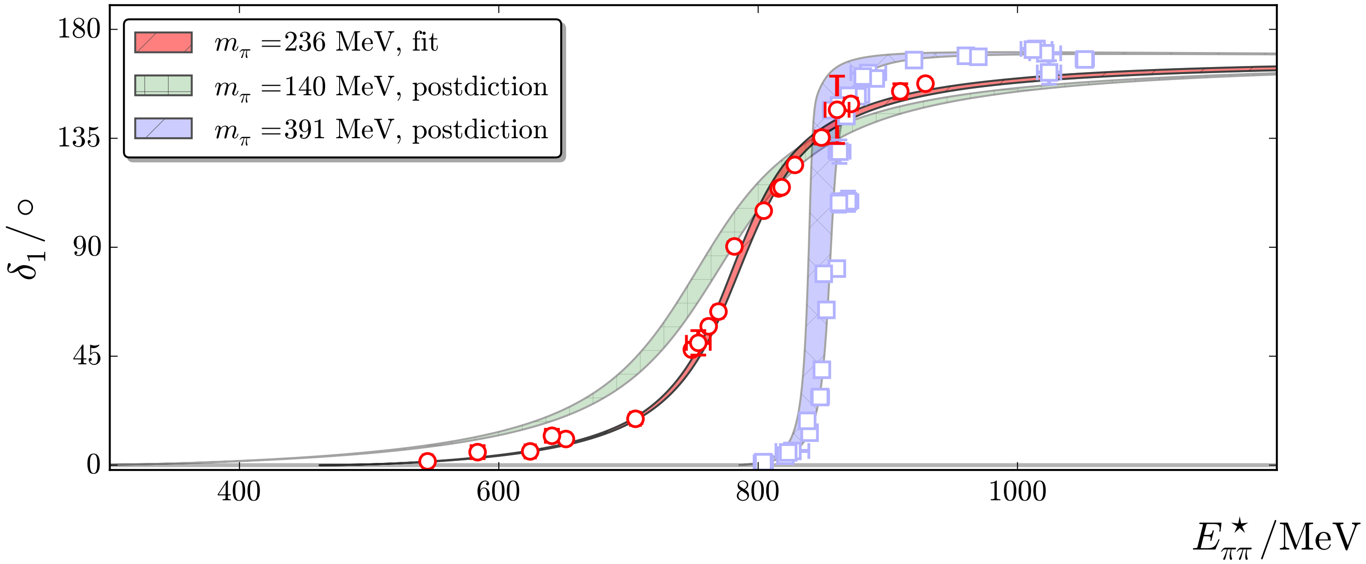


Black point: dispersive + exp.
J.R. Pelaez, Phys. Rep. 658, 1 (2016).

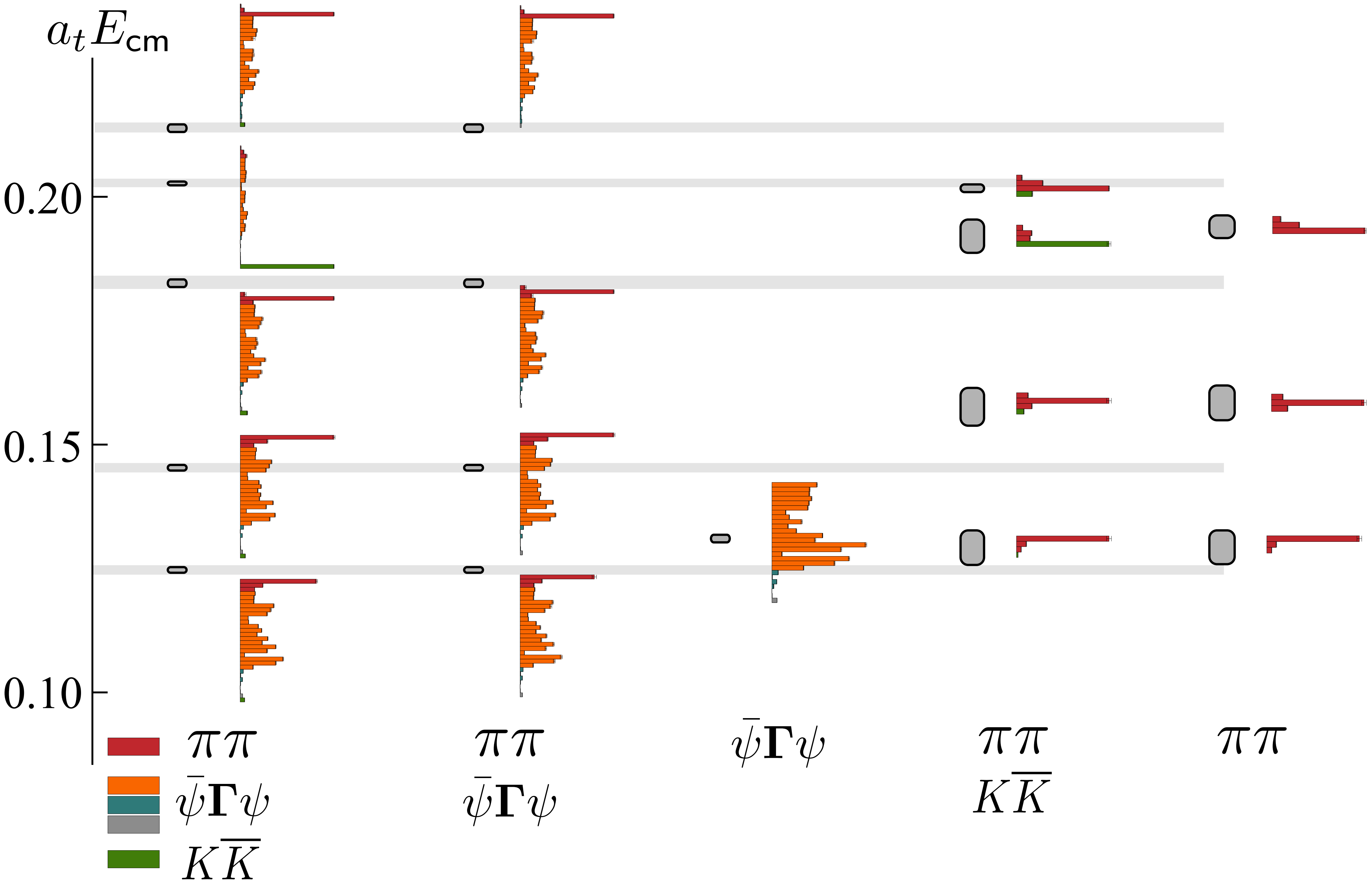
rho resonance extrapolation



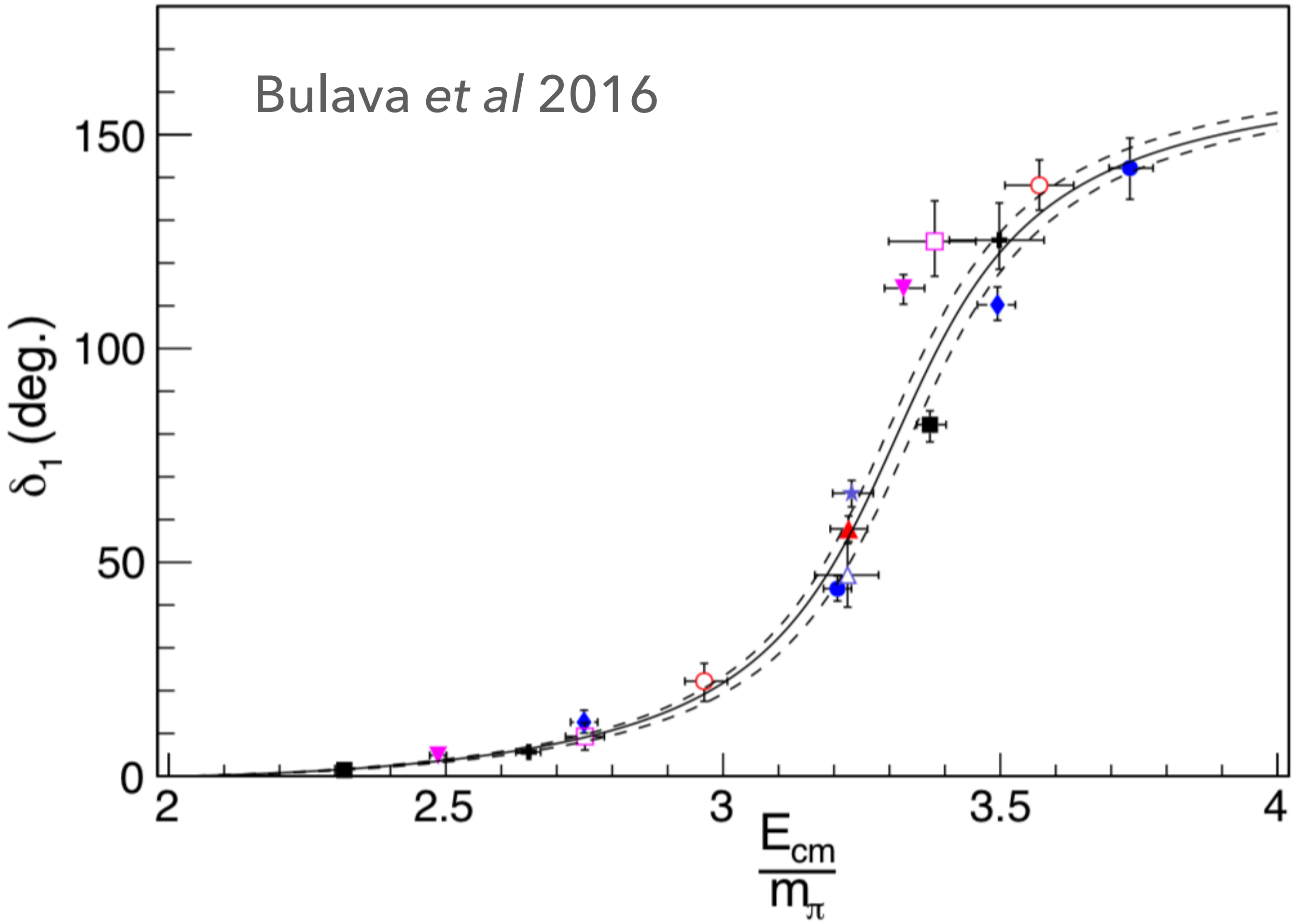
- fitted UChiPT LECS at 236 MeV
- Extrapolated up to 391 MeV
- Extrapolated down to 140 MeV



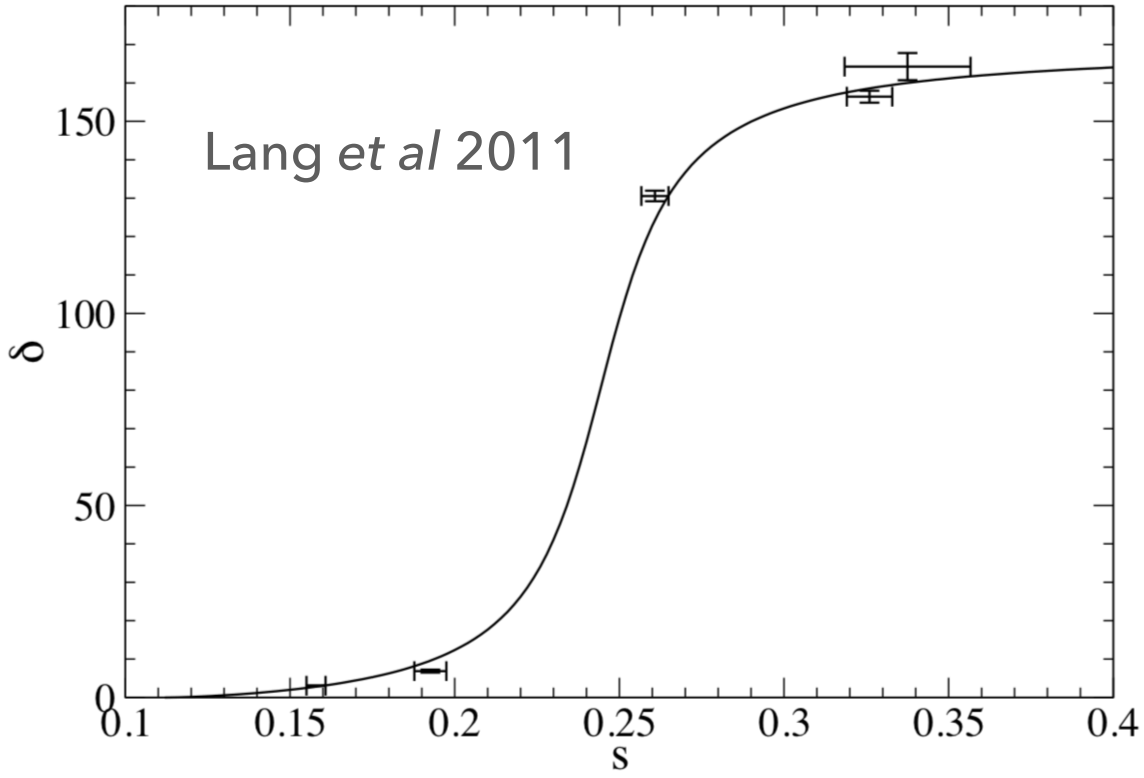
basis variation

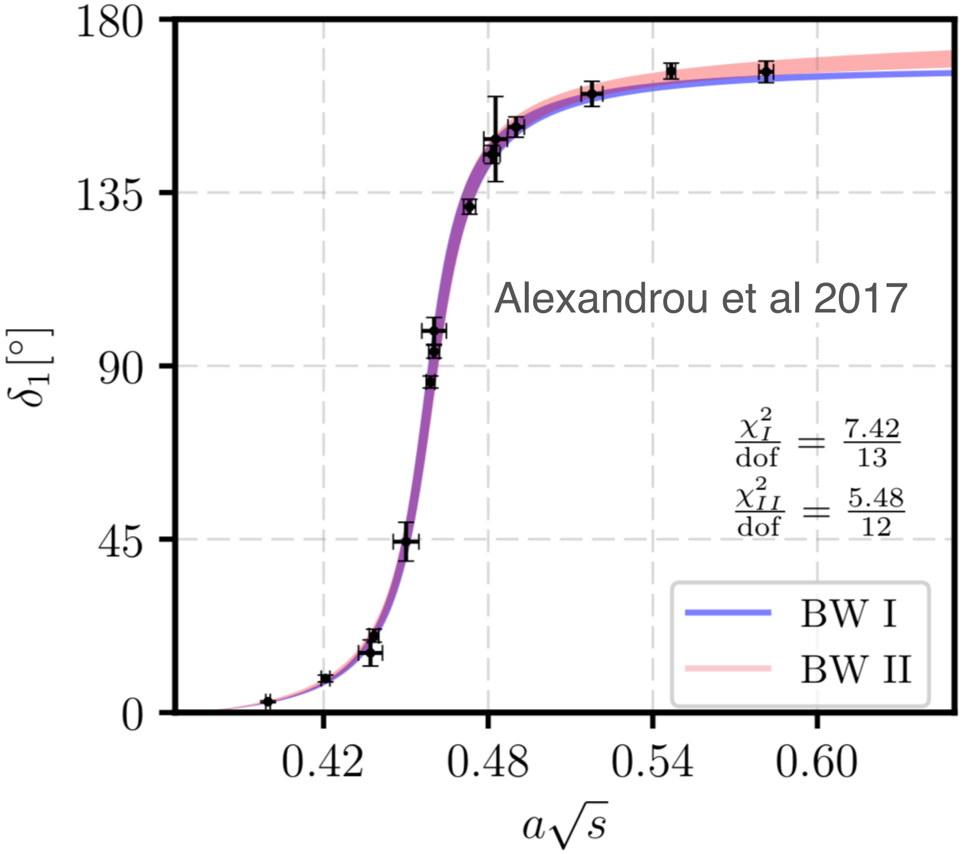


many other elastic resonance calculations

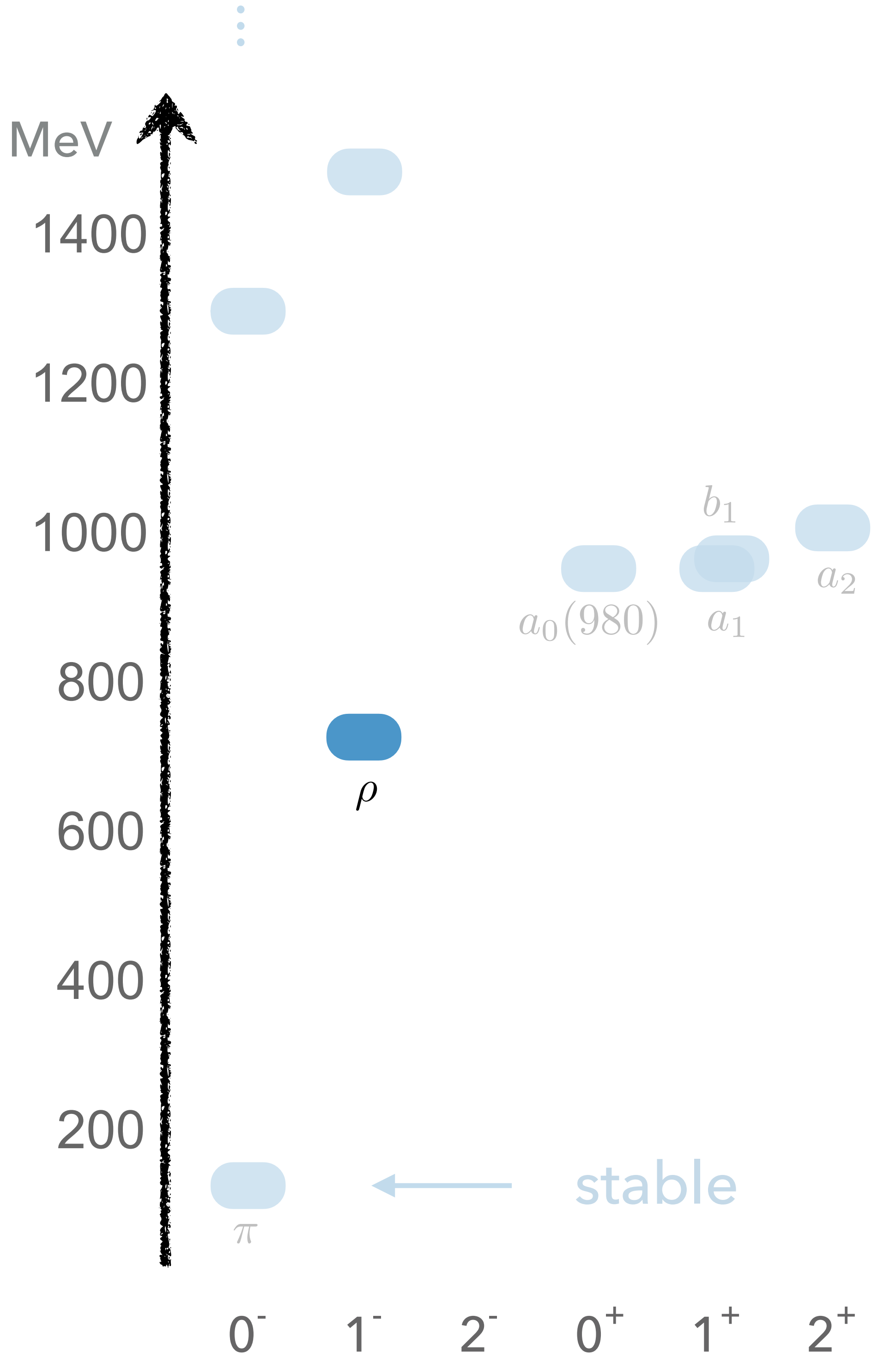


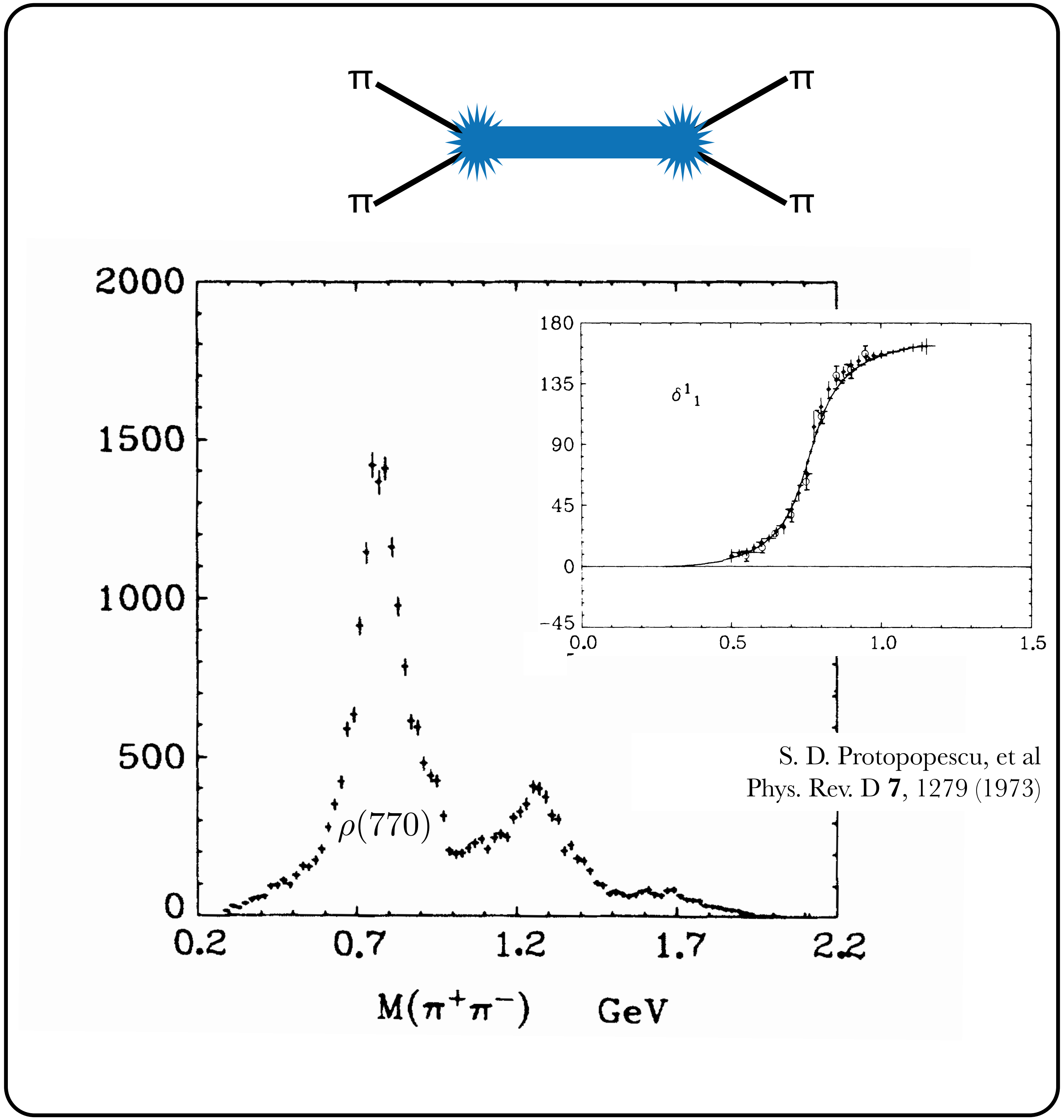
- just a selection for the ρ
- … many more in the literature
- nucleons, K, D, B, charmonium





Scattering and resonances

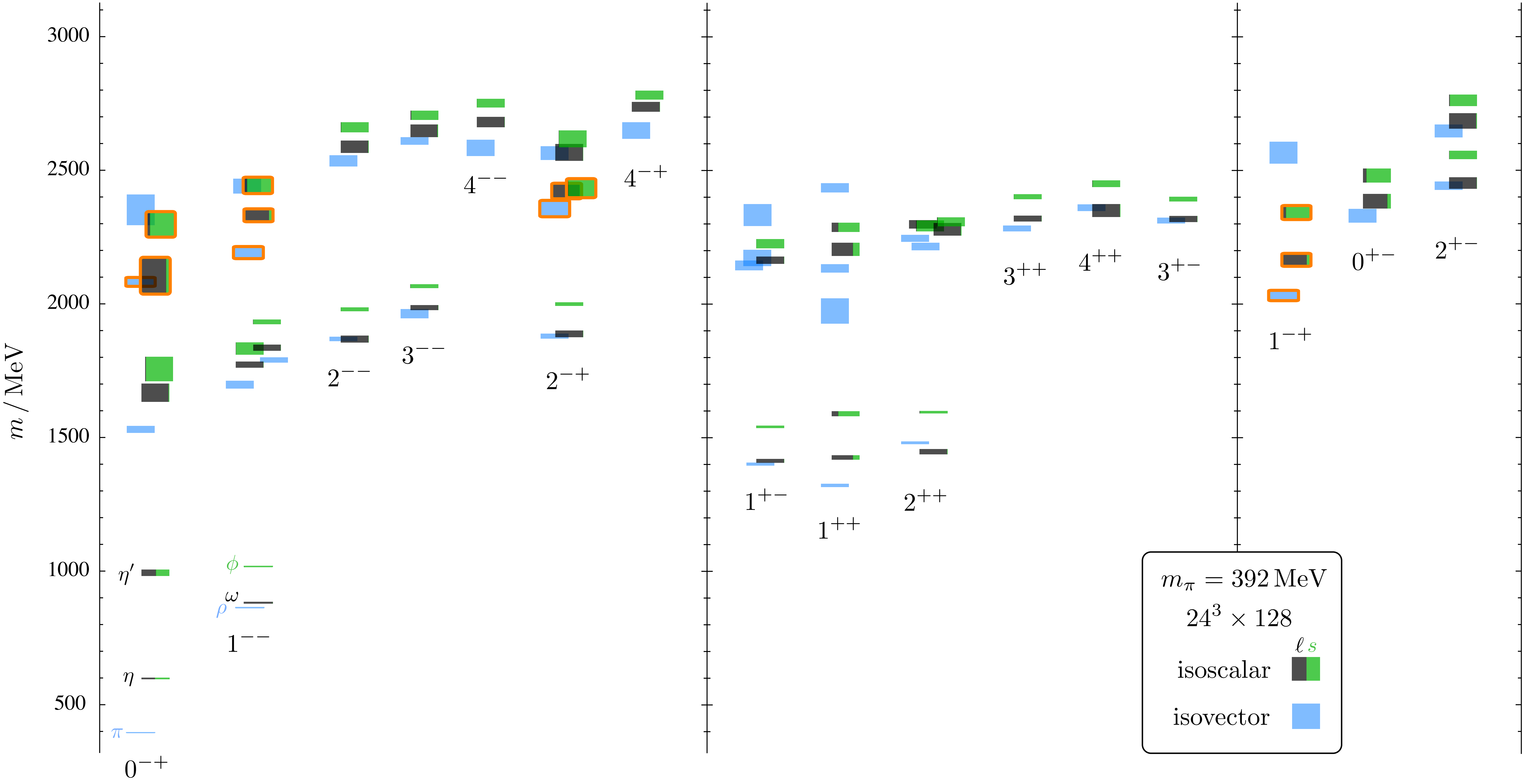




S. D. Protopopescu, et al
Phys. Rev. D **7**, 1279 (1973)

light quark spectrum from qq̄ operators

from a large basis of local qq-like operators



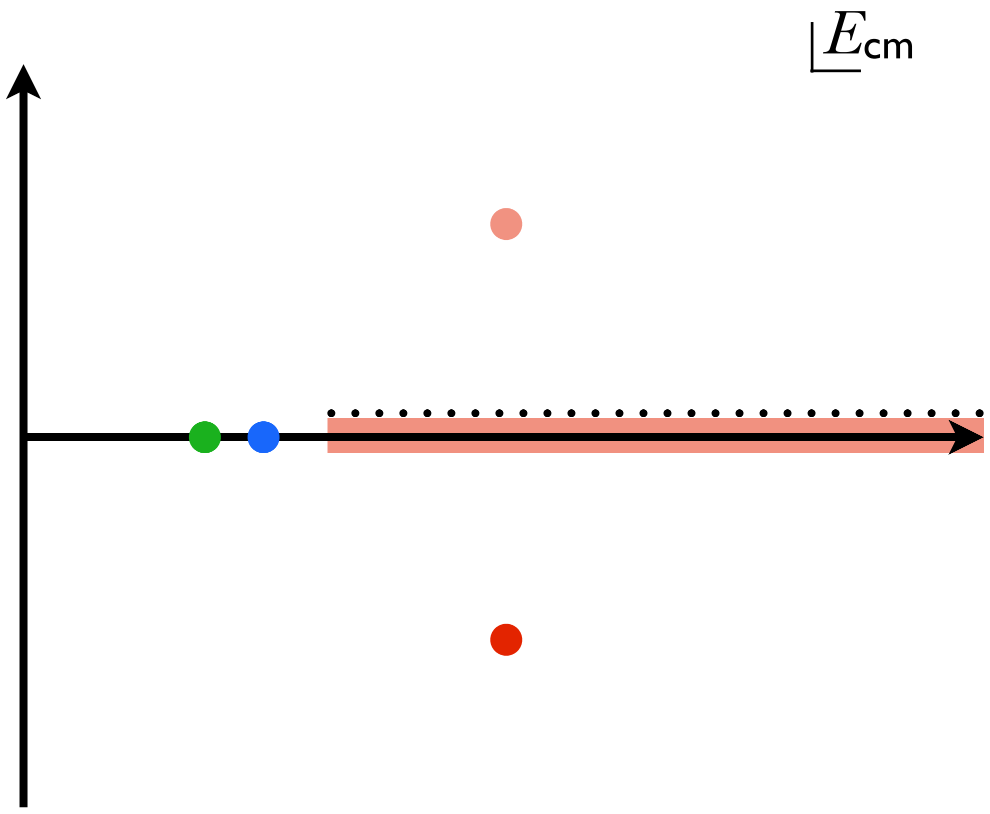
J. J. Dudek, R. G. Edwards, P. Guo and C. E. Thomas, Phys.Rev. D88 (2013) 9, 094505.

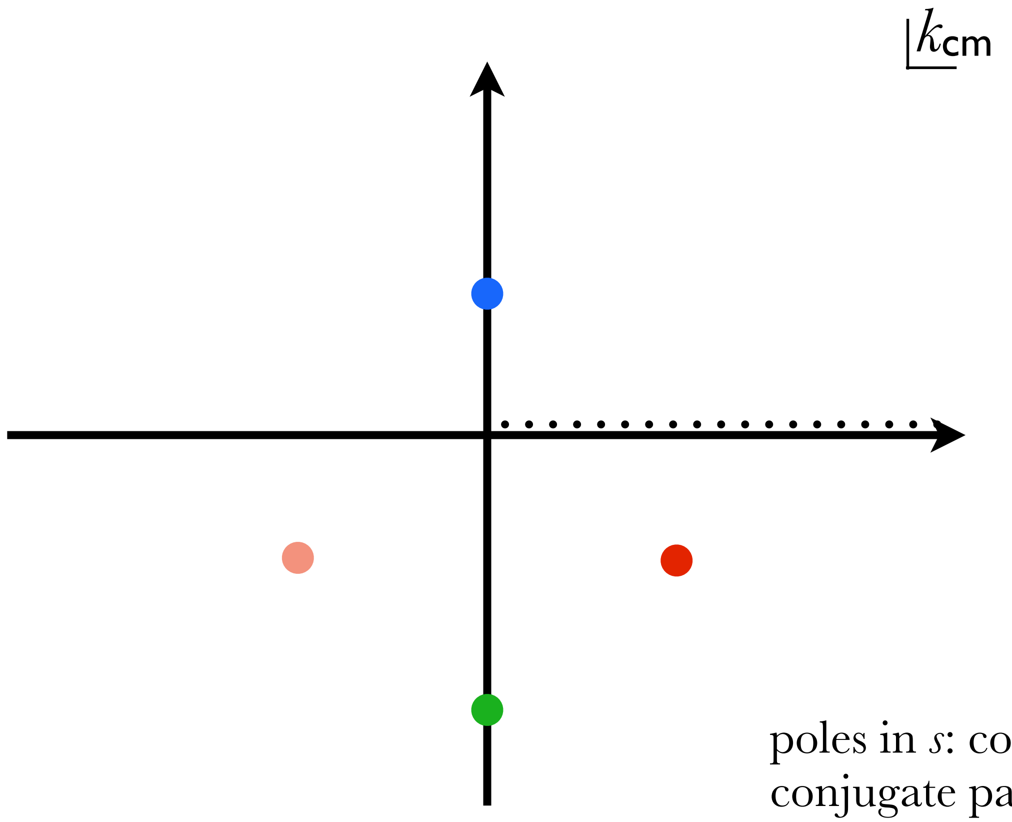
Poles in amplitudes

Multi-sheeted complex plane due to square-root branch cuts at each threshold, in single channel case for now:

$$t_{ij}(s \sim s_0) \sim \frac{c_i c_j}{s_0 - s}$$

$$k_{\mathrm{cm}} = \pm\frac{1}{2}\left(E_{\mathrm{cm}}^2 - 4m^2\right)^{\frac{1}{2}}$$





poles in s: complex conjugate pairs in k

Bound state

Resonance

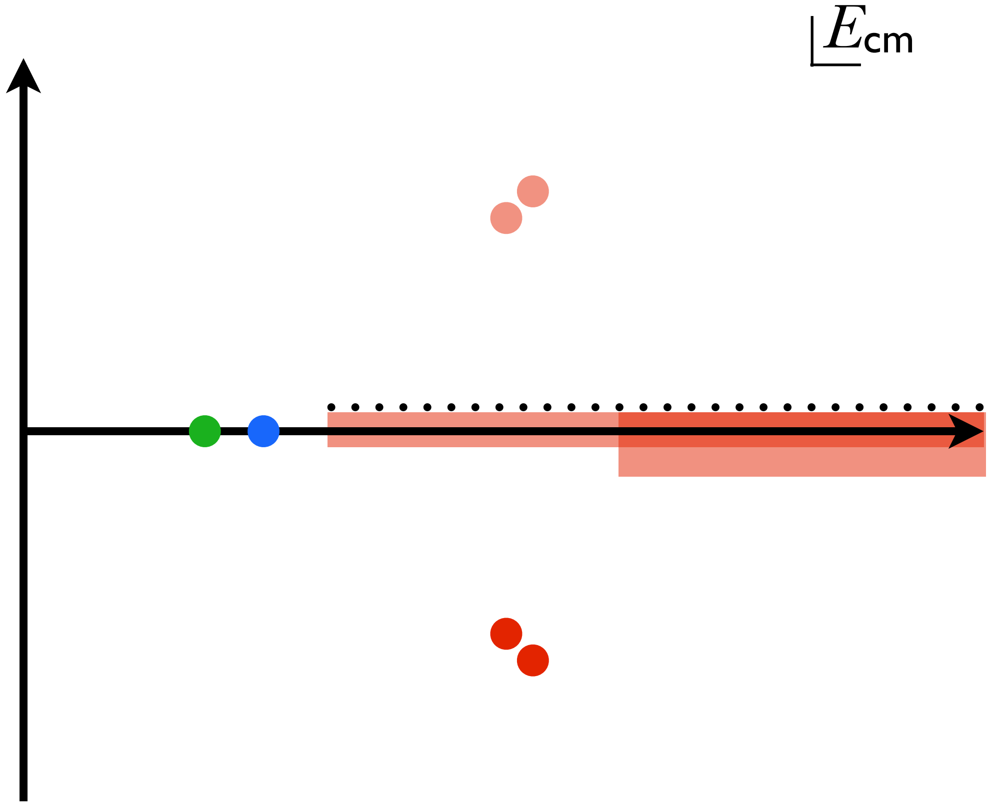
Virtual Bound state

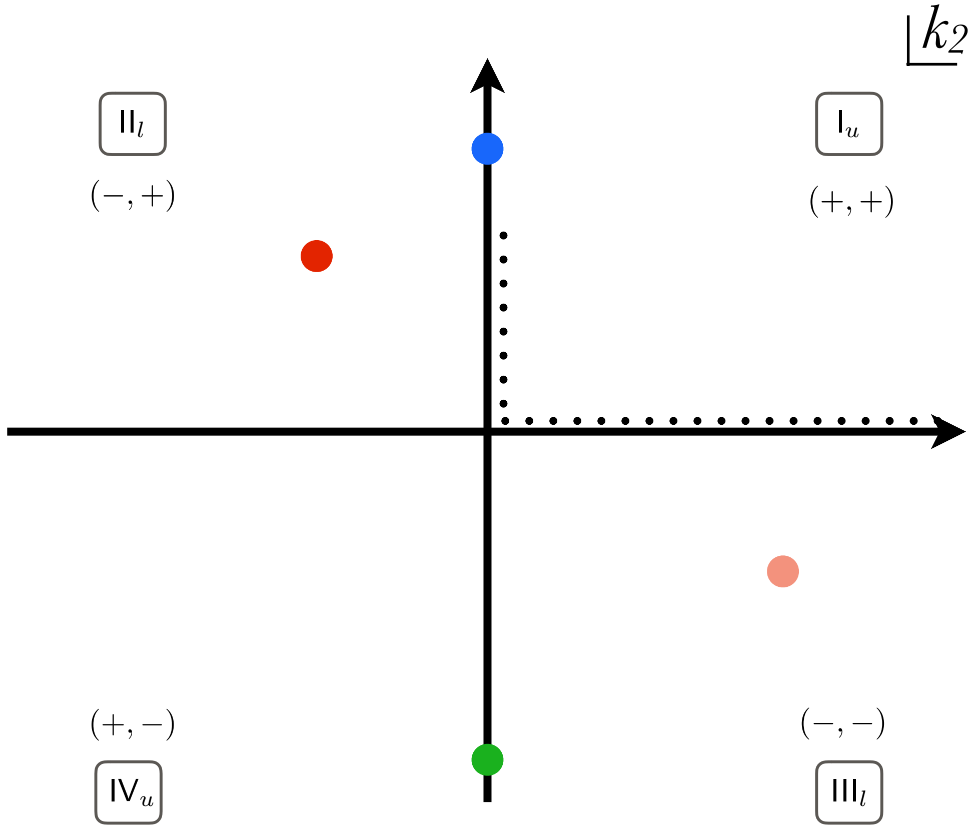
Poles in amplitudes

for n-channels, there are 2^n sheets

$$k_{\mathsf{cm}} = \pm\frac{1}{2}\left(E_{\mathsf{cm}}^2 - 4m^2\right)^{\frac{1}{2}}$$

$$t_{ij}(s \sim s_0) \sim \frac{c_i c_j}{s_0 - s}$$





Bound state

Resonances

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Poles in amplitudes

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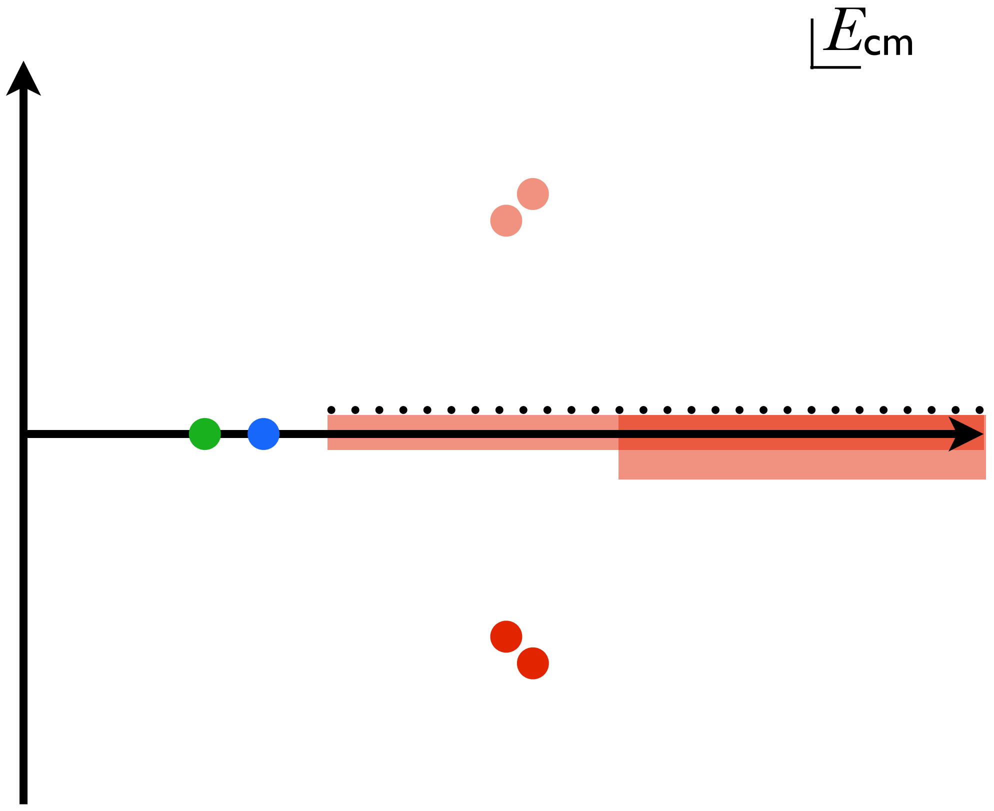
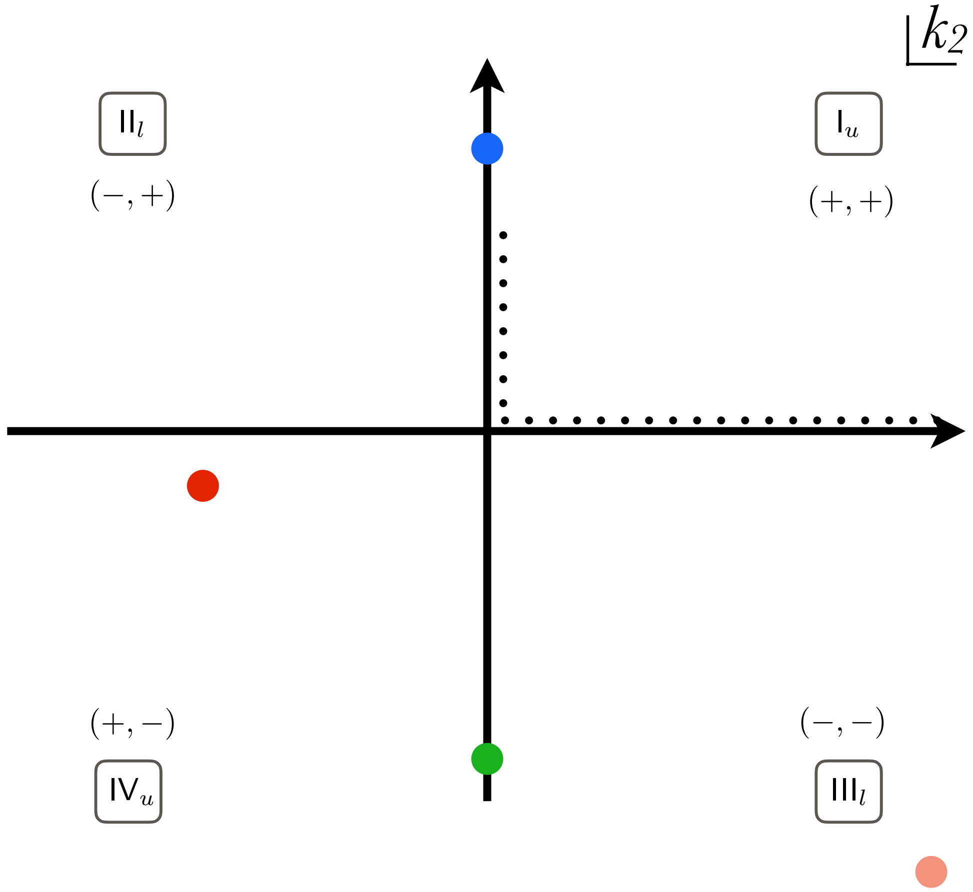




label sheets by signs of Im(k)

many distributions of pole positions possible

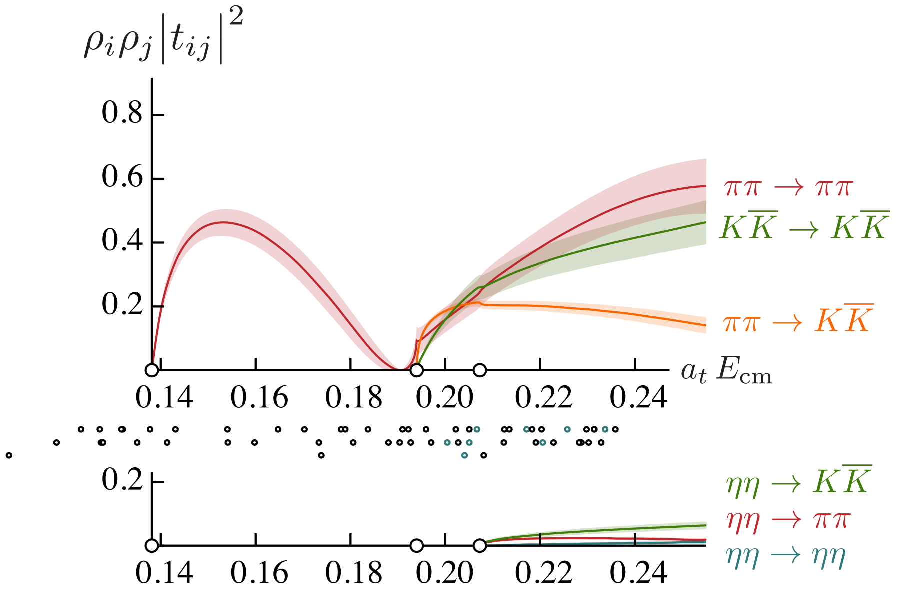
in some cases they can tell us about the composition the state

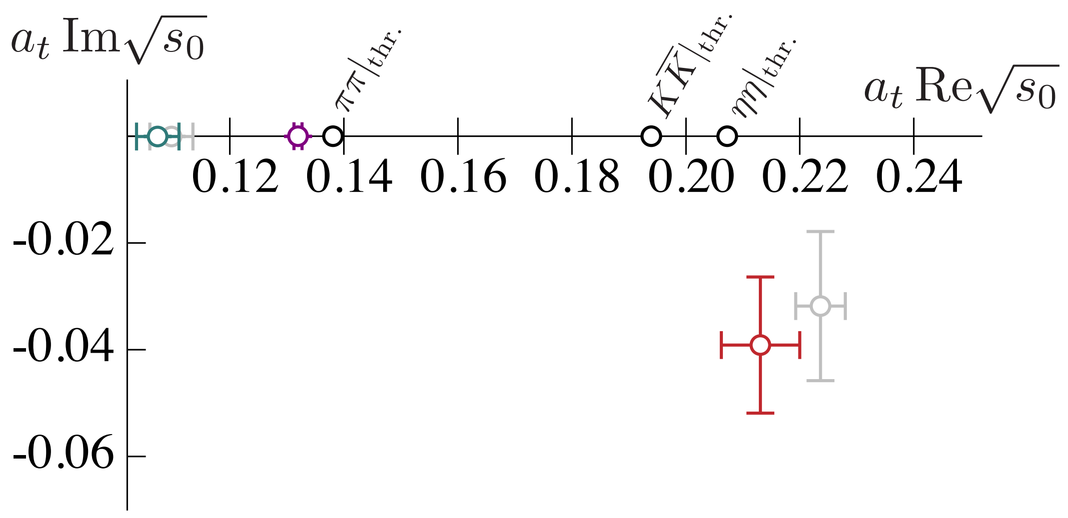
Bound state

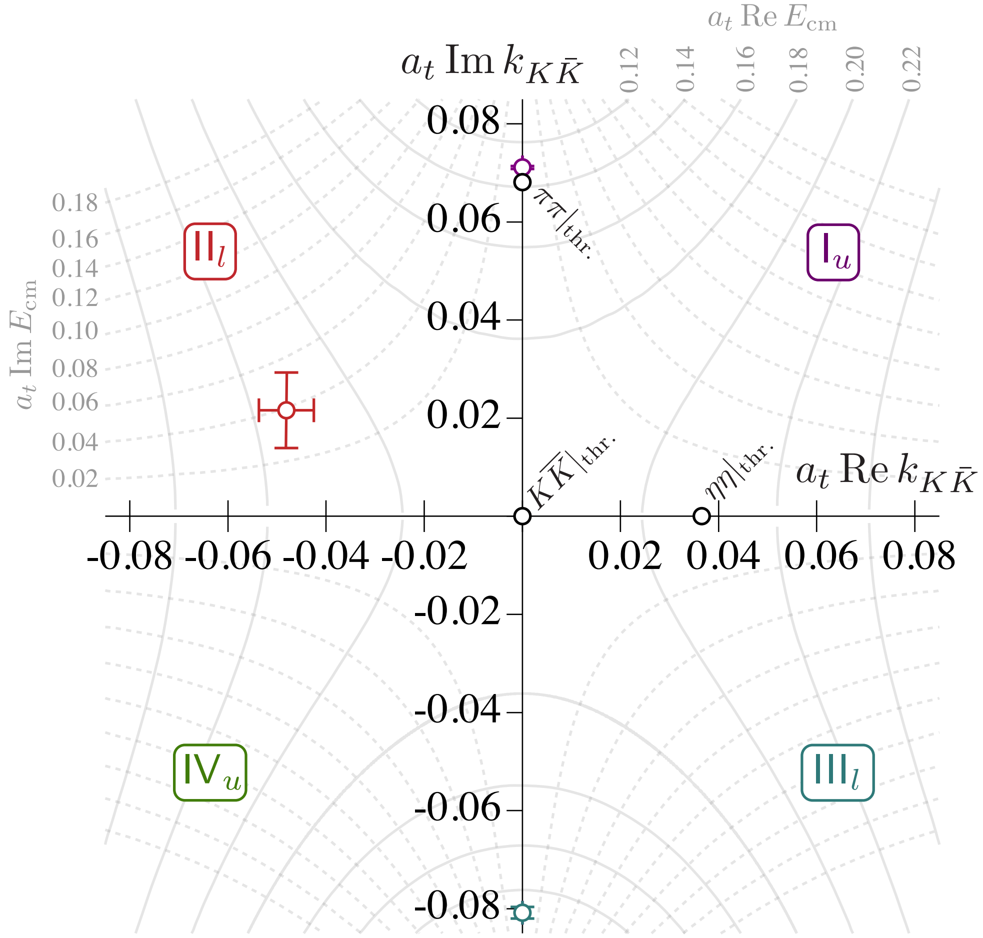
Resonances

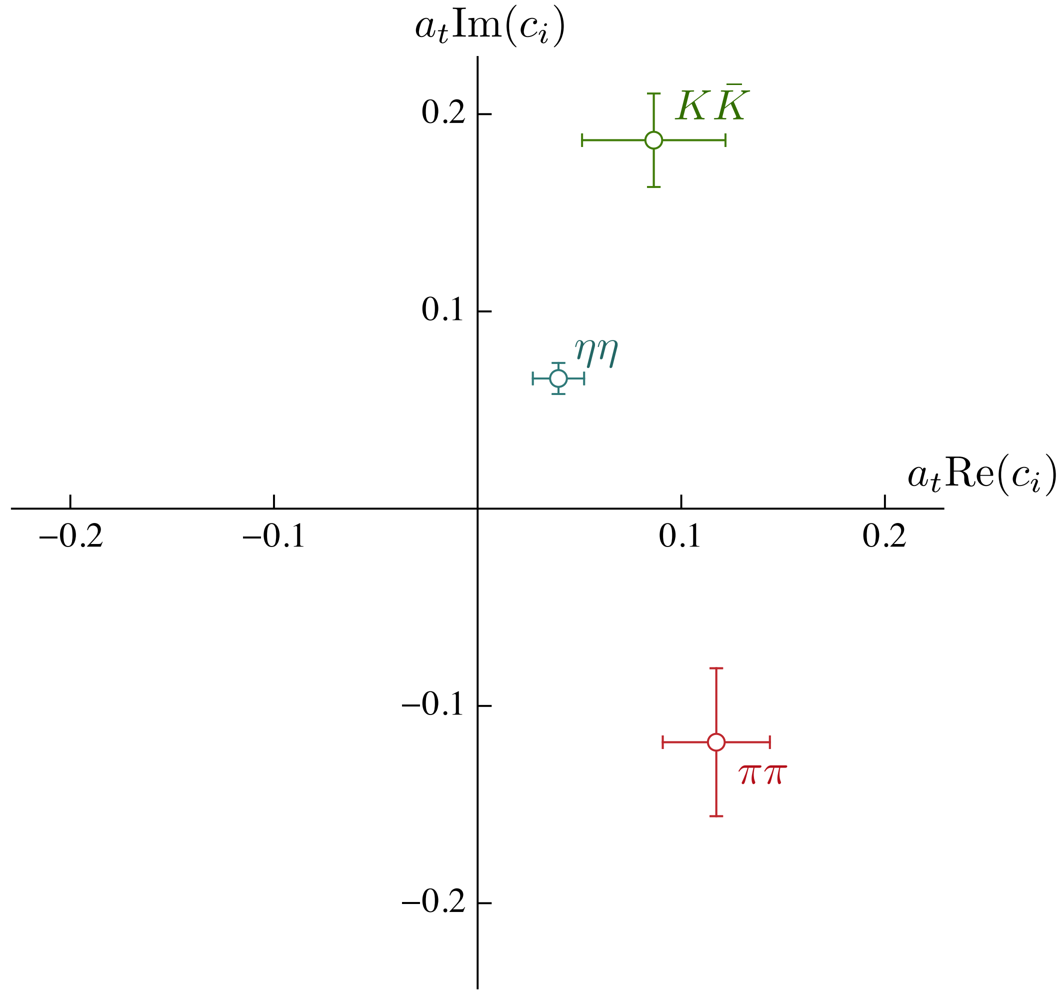
Virtual Bound state

Scalar resonance poles



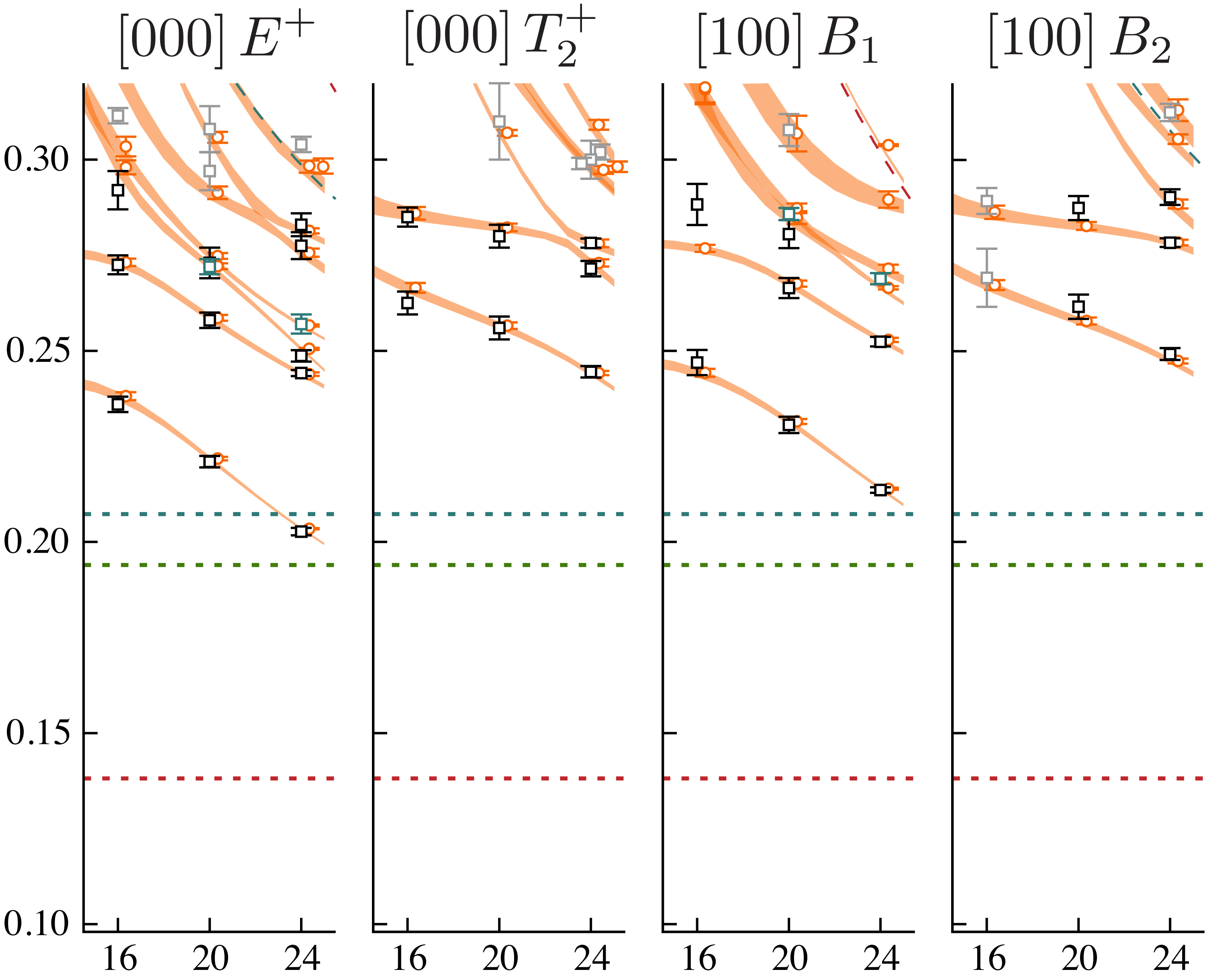






Reproducing the spectrum

An example D-wave spectrum fit



$$\chi^2/N_{\text{dof}} = \frac{28.9}{34-9} = 1.15$$