

# Global analysis of parton densities and fragmentation functions

Carlota Andrés

Jefferson Lab

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# Why JAM?

## JAM: Jefferson Lab Angular Momentum Collaboration

- To study the quark and gluon structure of the nucleon by performing global fits of both spin-dependent ( $\Delta$ PDFs) and unpolarized parton distribution functions (PDFs)

## How?

- Analyzing the impact of JLab in a rigorous way
- JLab DIS data: large  $x_b$ , low-intermediate  $Q^2$  and  $W^2$
- Framework: (NLO) collinear factorization  
Higher twist (HT) and Target Mass Corrections (TMC) needed at large  $x_b$

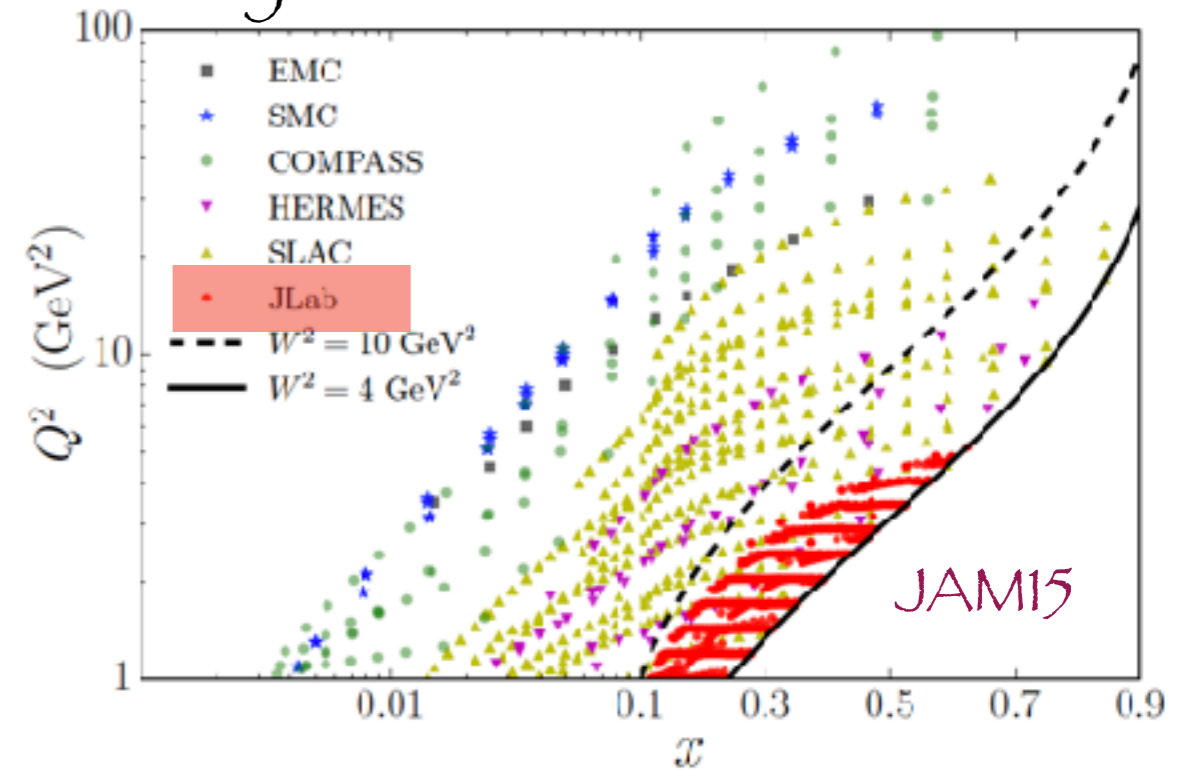
# Evolution of JAM

Iterative MC fitting technique

		JAM15	JAM16	JAM17	JAM18
Process	DIS	✓	✗	✓	✓
	SIA	✗	✓	✓	✓
	SIDIS	✗	✗	✓	✓
	DY	✗	✗	✗	✓
Function	$f$	✗	✗	✗	✓
	$\Delta f$	✓	✗	✓	✓
	$D_f^h$	✗	✓	✓	✓

# JAM15

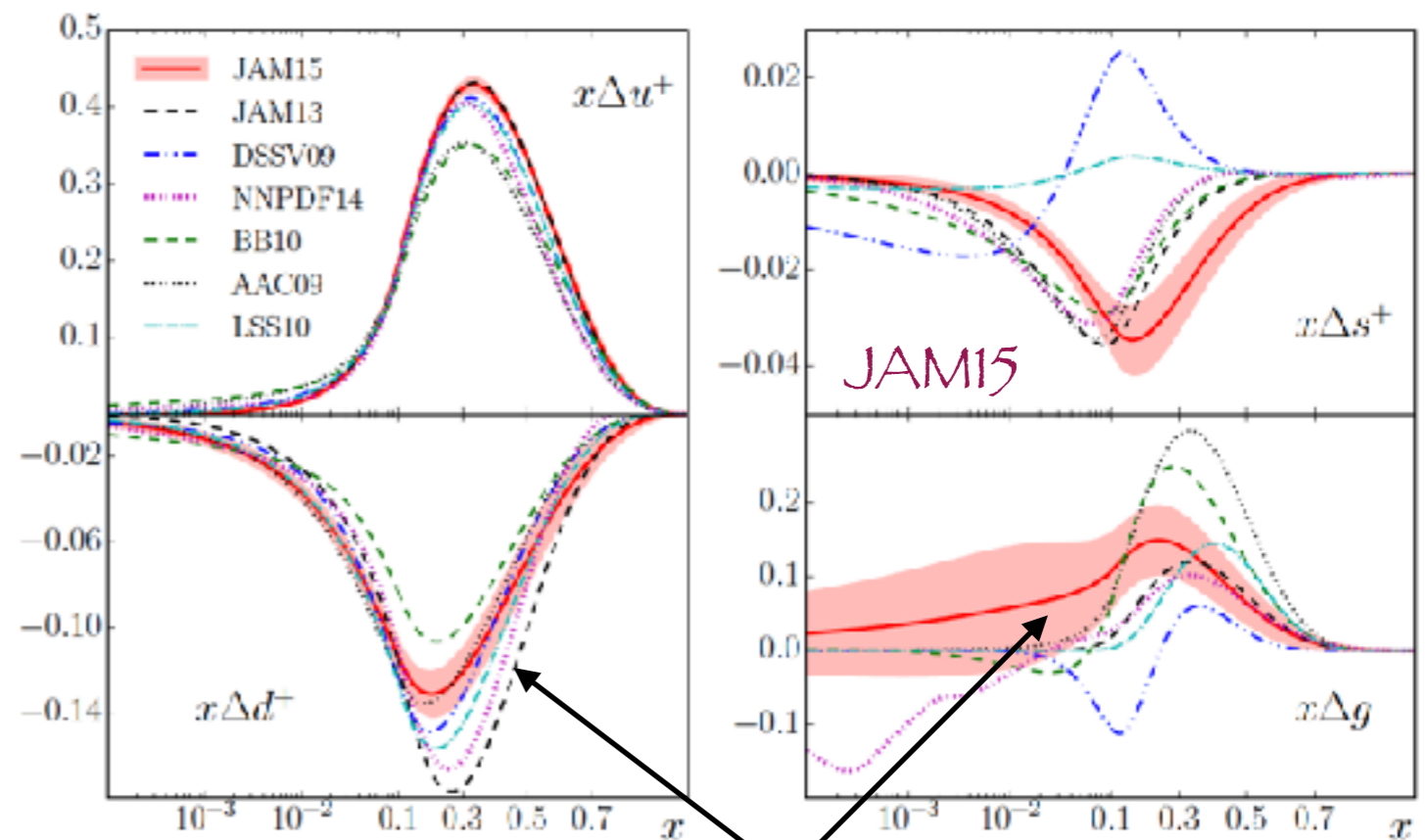
IMC analysis + all available JLab data



Uses CJ12 NLO unpolarized PDFs

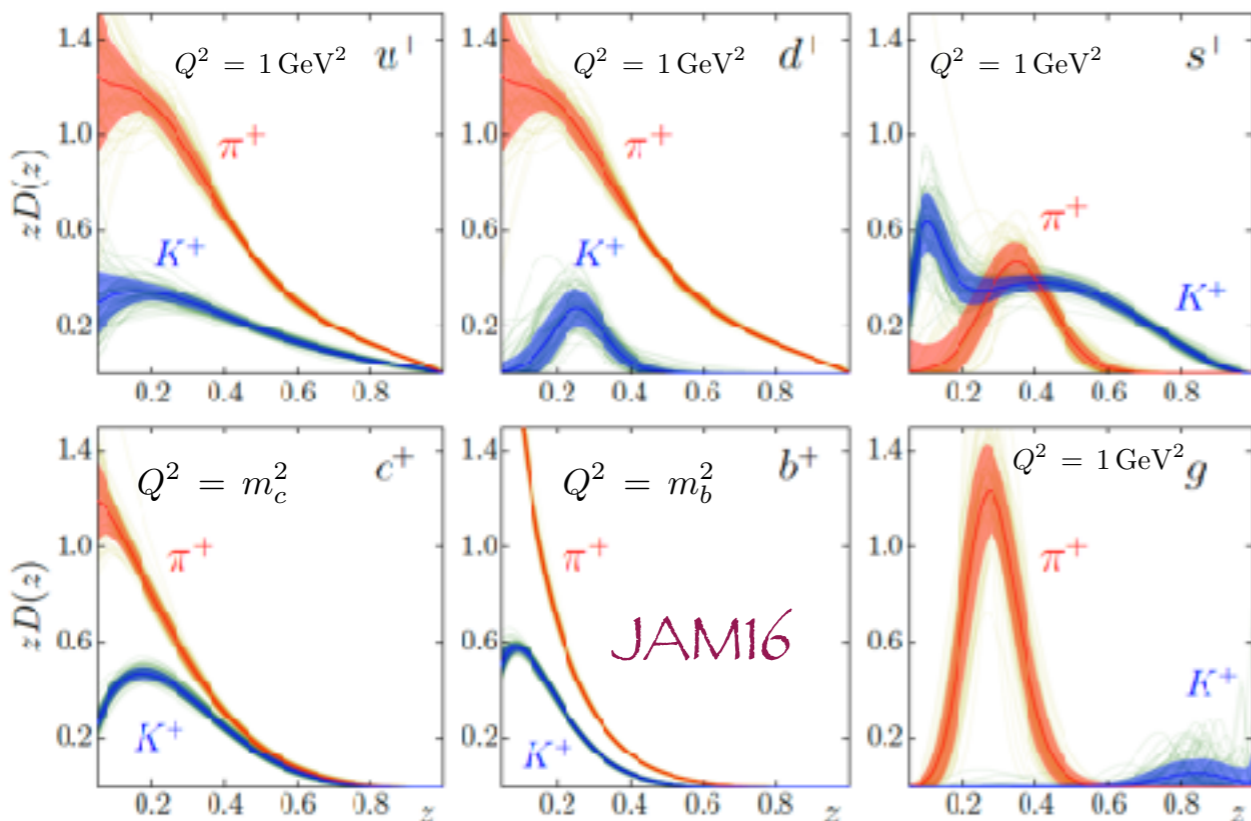
- $\Delta u^+$  and  $\Delta d^+$  consistent with previous analysis
- $\Delta s^+$  slightly harder

Sato, Melnitchouk, Kuhn, Ethier, Accardi  
Phys. Rev. D 93, 074005 (2016)



# JAM16

- First IMC analysis of FFs
- Only SIA included

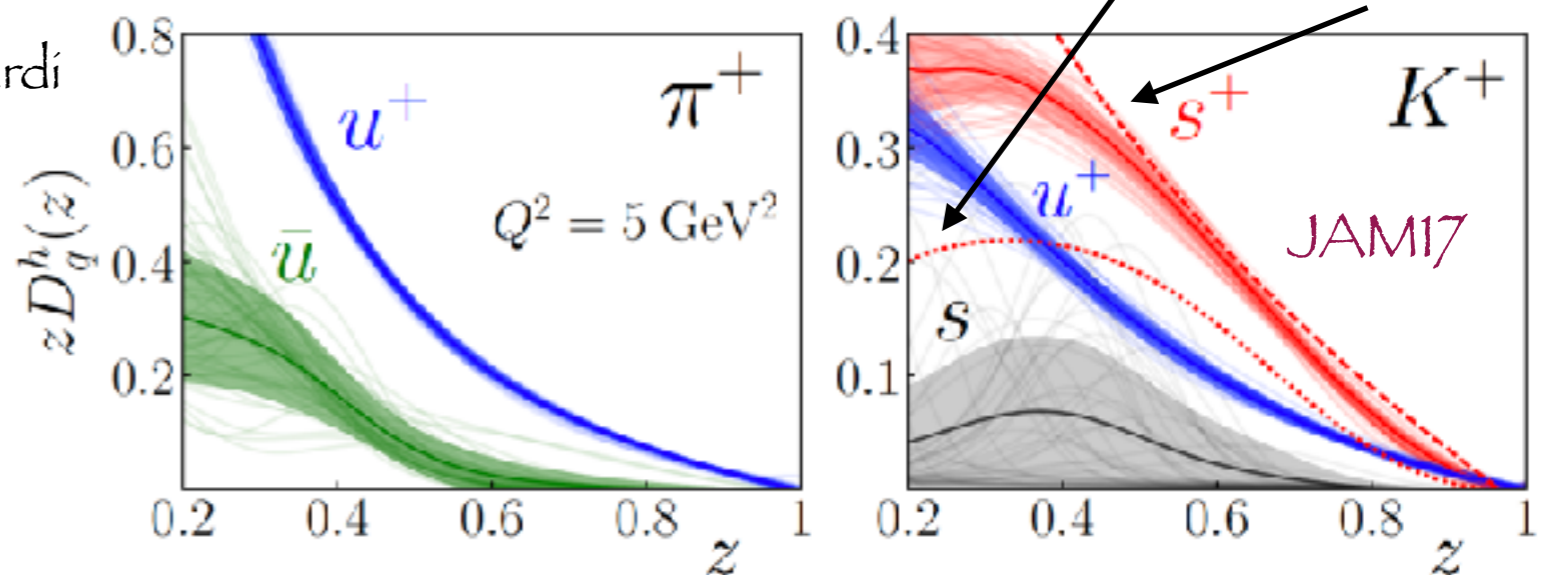
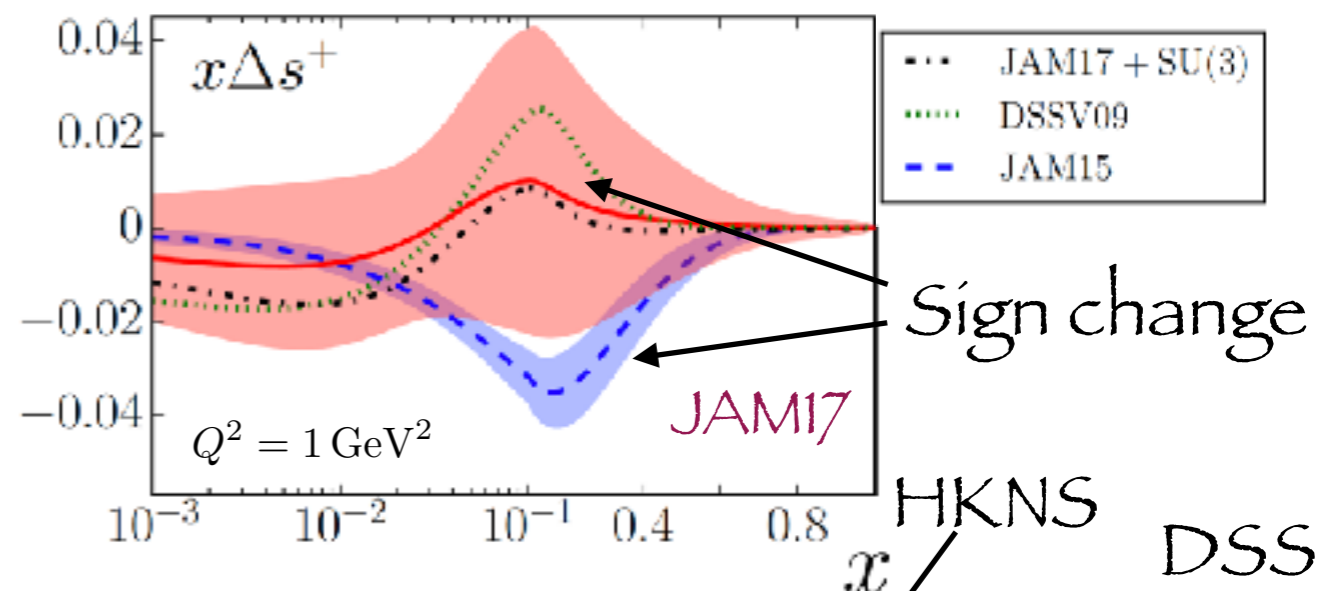


Sato, Ethier, Melnitchouk, Hirai, Kumano and Accardi  
 Phys. Rev. D 94, 114004 (2016)

JAM17 FFs better agreement  
 with other analysis

# JAM17

- First (simultaneous) MC analysis of polarized PDFs and FFs
- Polarized SIDIS, polarized DIS and SIA included



4 Ethier, Sato, Melnitchouk: Phys. Rev. Lett. 119, 132001 (2017)

JAM18

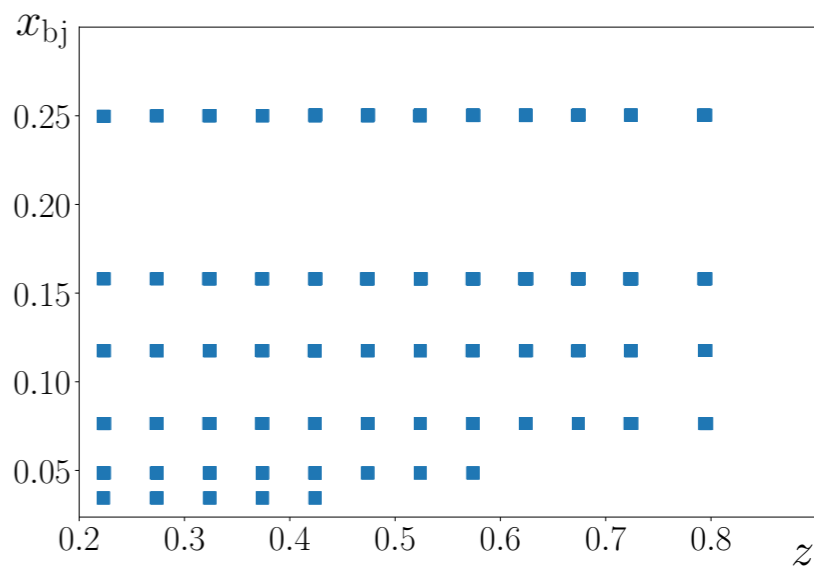
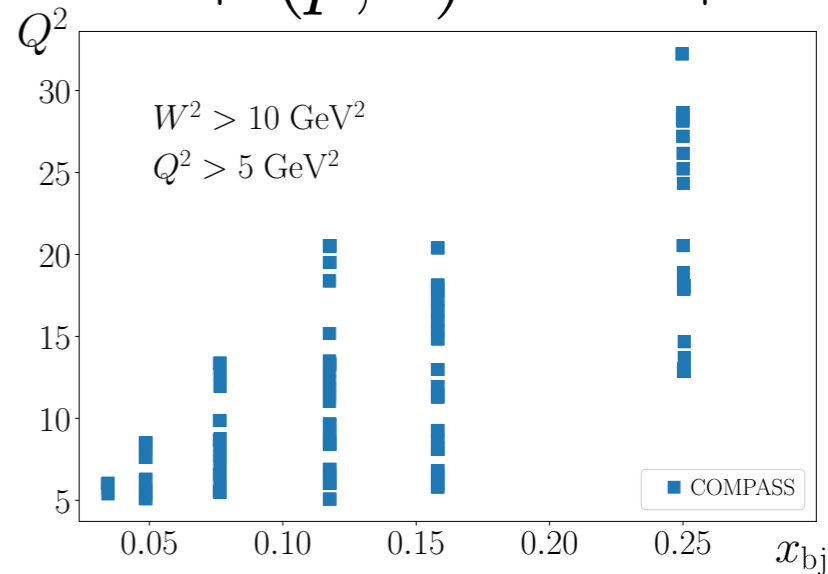
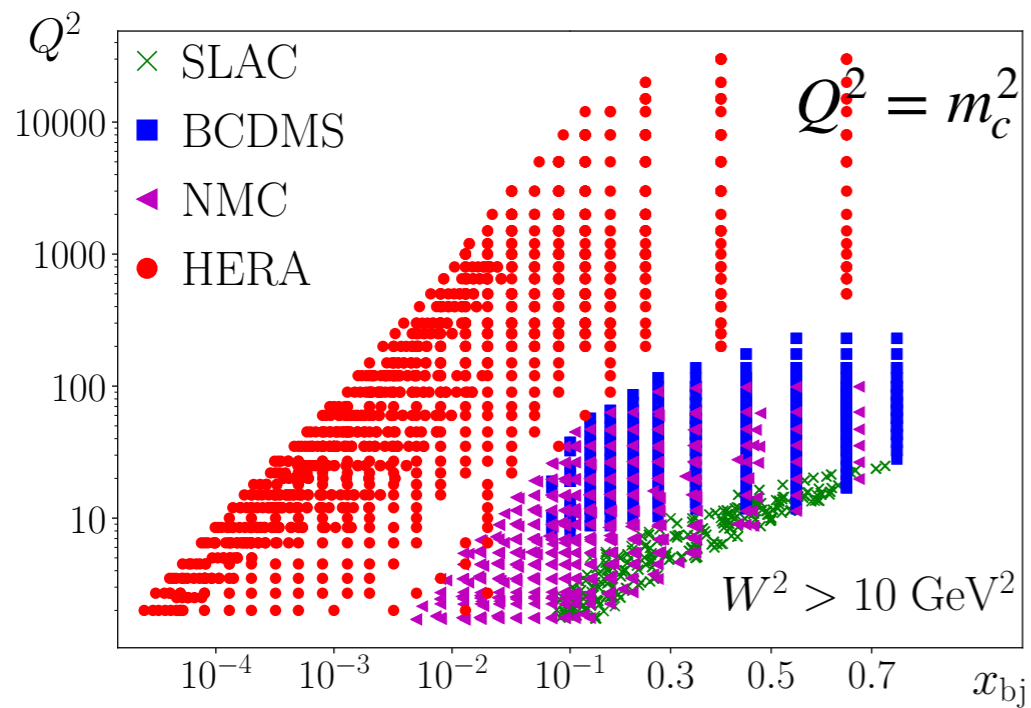
# Motivation

- Knowing the limits in  $x$  and  $Q^2$  of collinear factorization
- Testing the universality of PDFs ,FFs...
- All the data must be studied using the same theoretical framework
- First step: (first) combined analysis of unpolarized PDFs and FFs

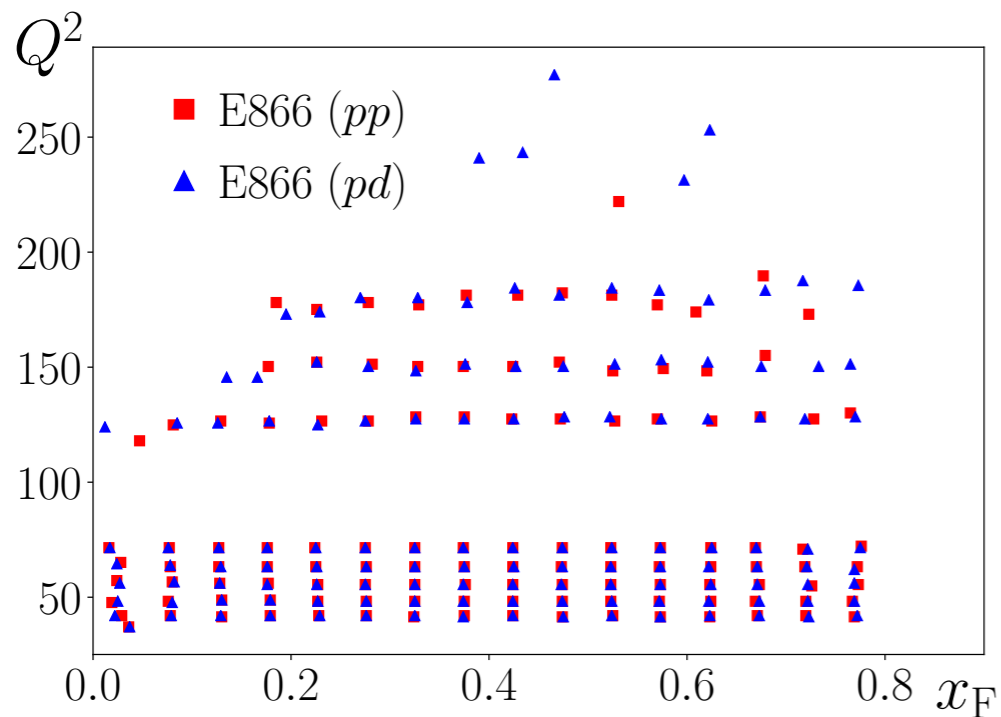
# Setup: JAM18 data

$$\text{SIDIS} : l + (p, d) \rightarrow l' + h + X$$

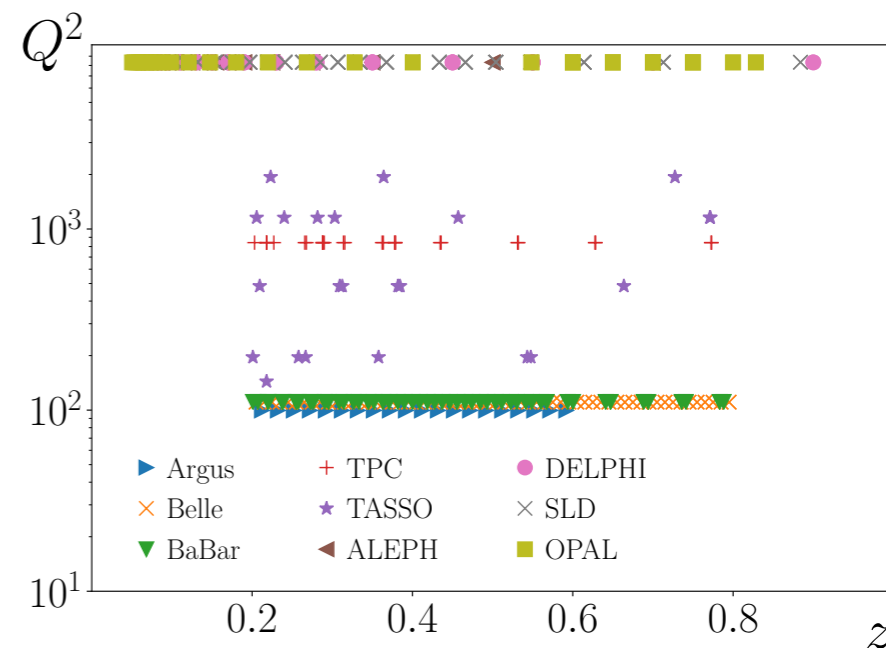
$$\text{DIS} : l + (p, d) \rightarrow l' + X$$



$$\text{DY} : p + (p, d) \rightarrow l\bar{l} + X$$



$$\text{SIA} : e^+ + e^- \rightarrow h + X$$



# Setup: theory

- All observables computed at NLO in pQCD
- DGLAP truncated evolution at order  $\alpha_s$  in Mellin space
- DIS cross sections computed at leading twist
- Nuclear smearing for deuterium DIS
- Heavy quark treatment : ZM-VFN
- Fitting methodology:
  - IMC based on Bayesian statistics
  - Future: Nested sampling



# Why IMC?

$$\chi^2 = \sum_e^{N_{exp}} \sum_i^{N_{data}} \frac{(D_i^e - T_i)^2}{(\sigma_i^e)^2}$$

- Typical PDF parametrization:

$$x\Delta f(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx) \longrightarrow \text{Multiple local minima!}$$

- Perform single  $\chi^2$  fit:

Parameters difficult to constrain

Hessian method for uncertainties  $\longrightarrow$  Introduces tolerance criteria

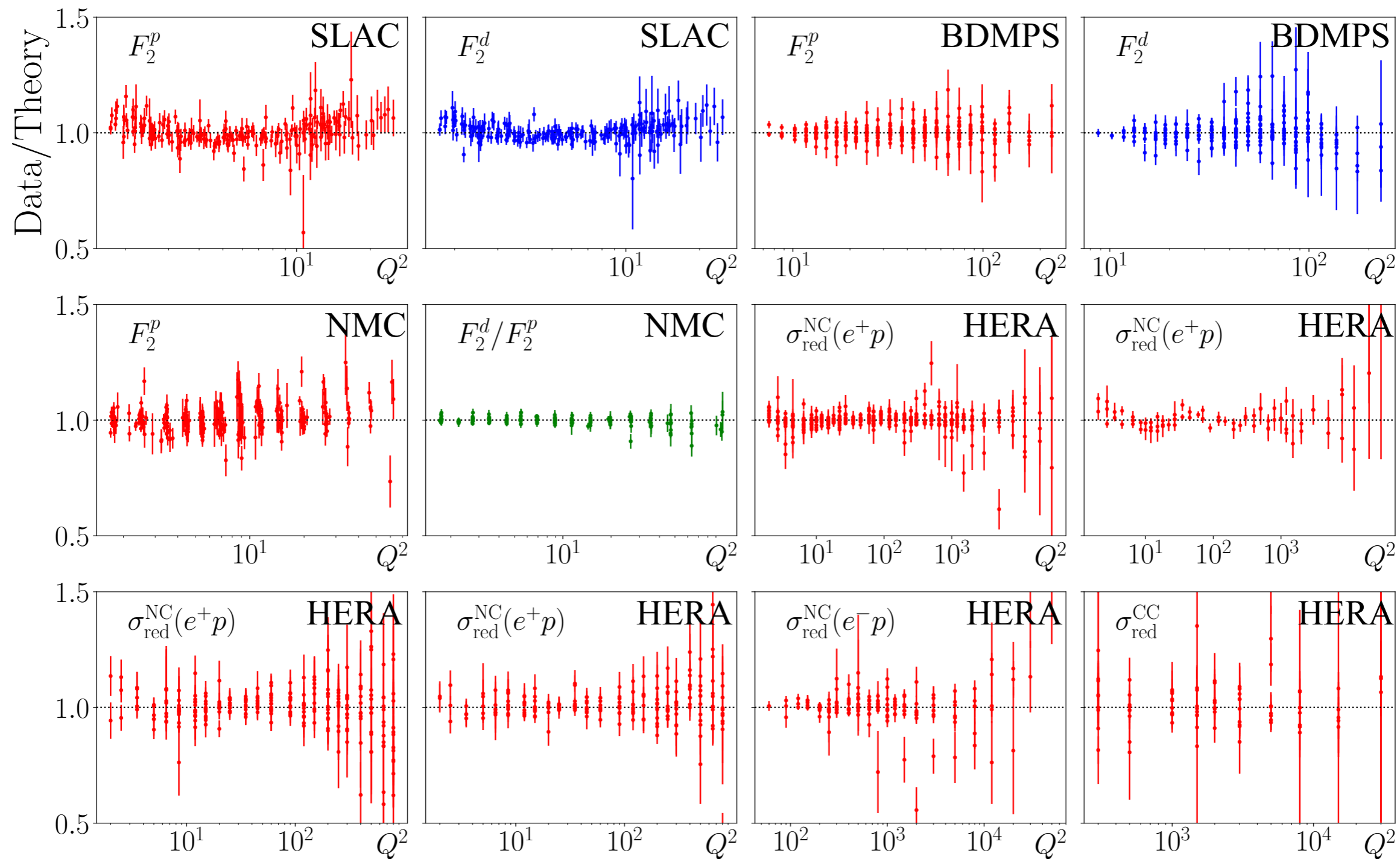
Unsuitable for simultaneous analysis of collinear distributions

- Monte Carlo methods:
  - Allows efficient exploration of the parameter space
  - Uncertainties directly obtained from MC replicas

JAM18 currently uses an IMC based on a Bayesian approach

# Data vs. theory: DIS

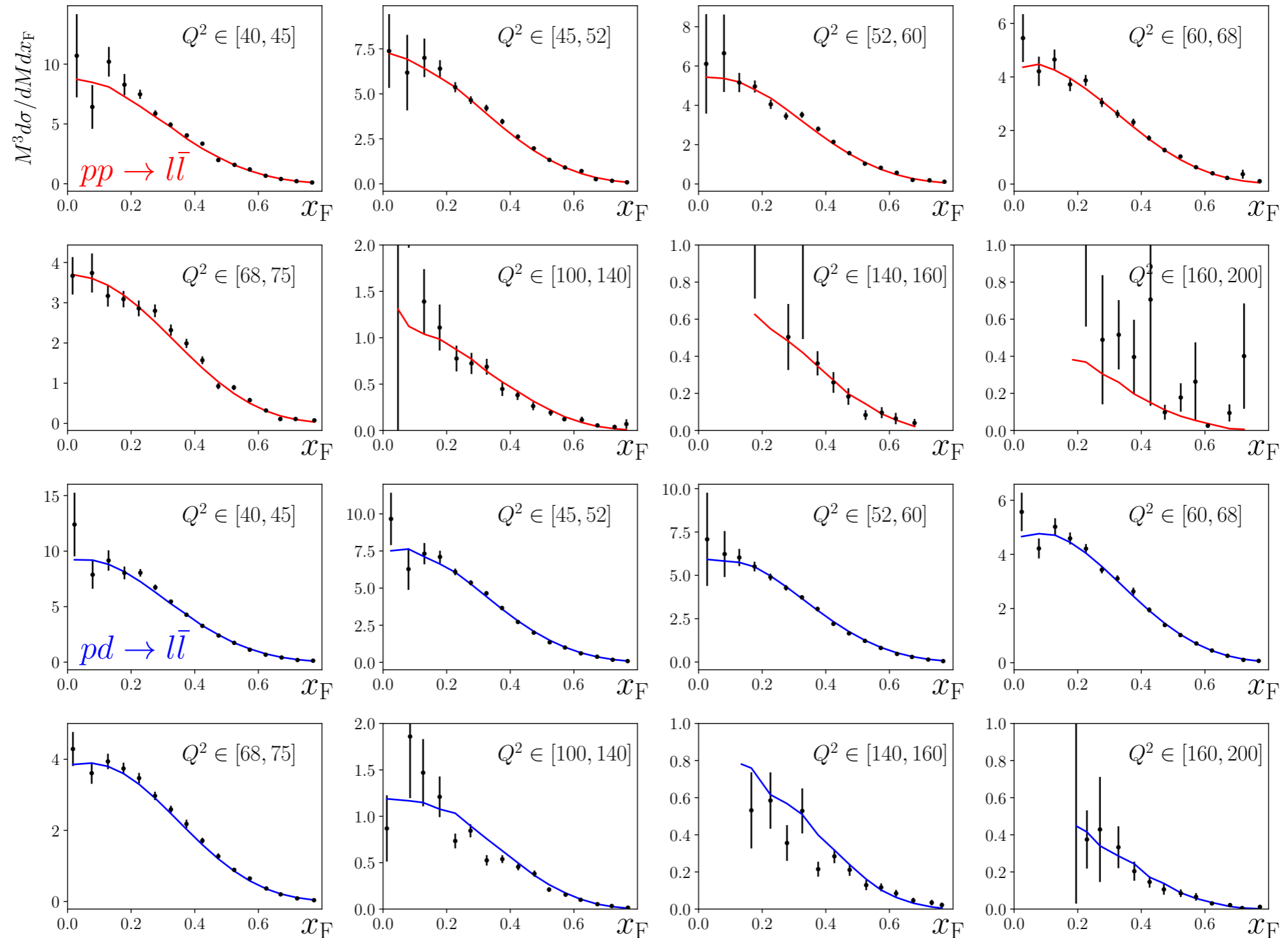
$$\text{DIS} : l + (p, d) \rightarrow l' + X$$



# Data vs. theory: DY

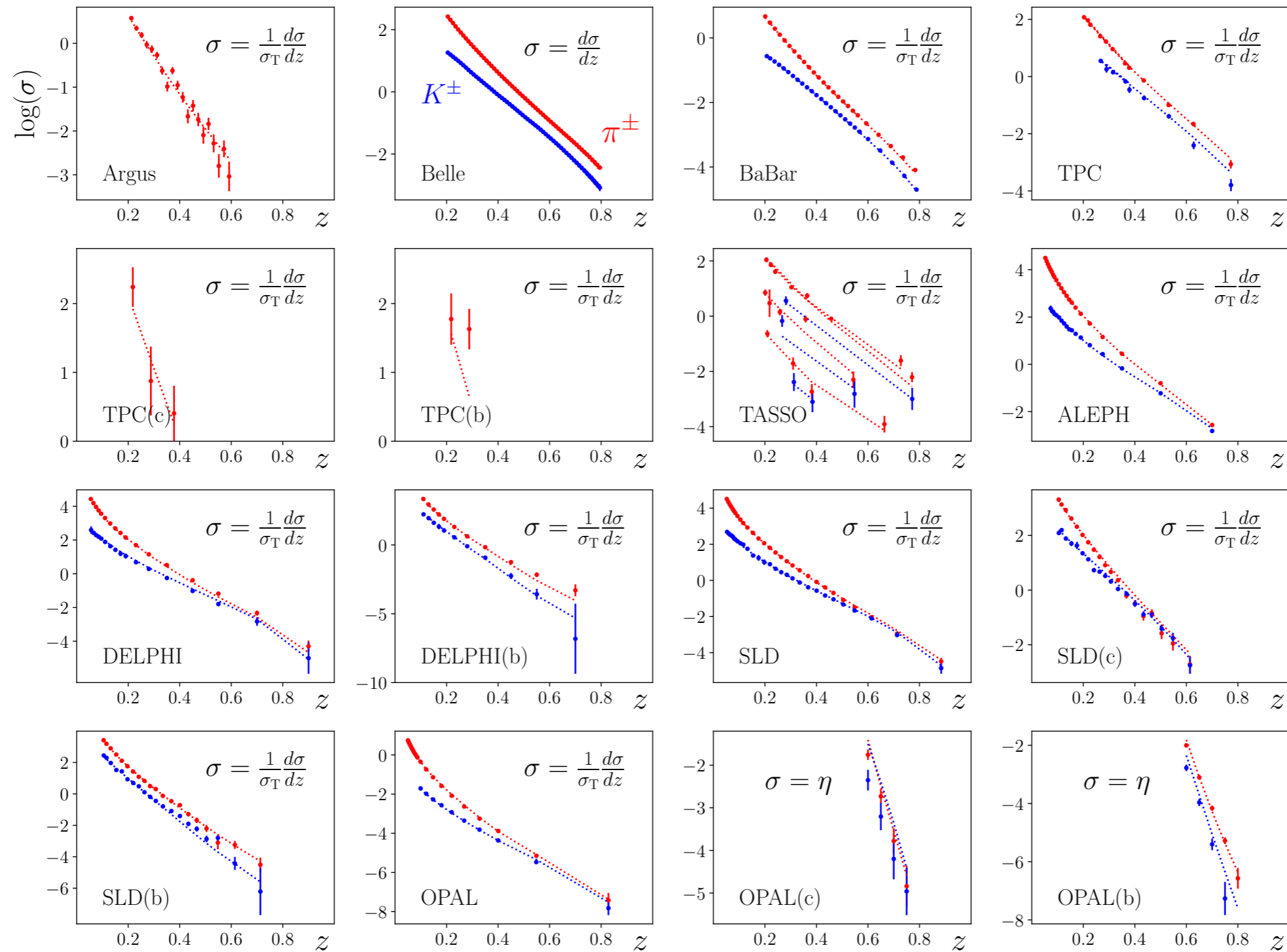
E866

DY :  $p + (p, d) \rightarrow l\bar{l} + X$



# Data vs. theory: SIA

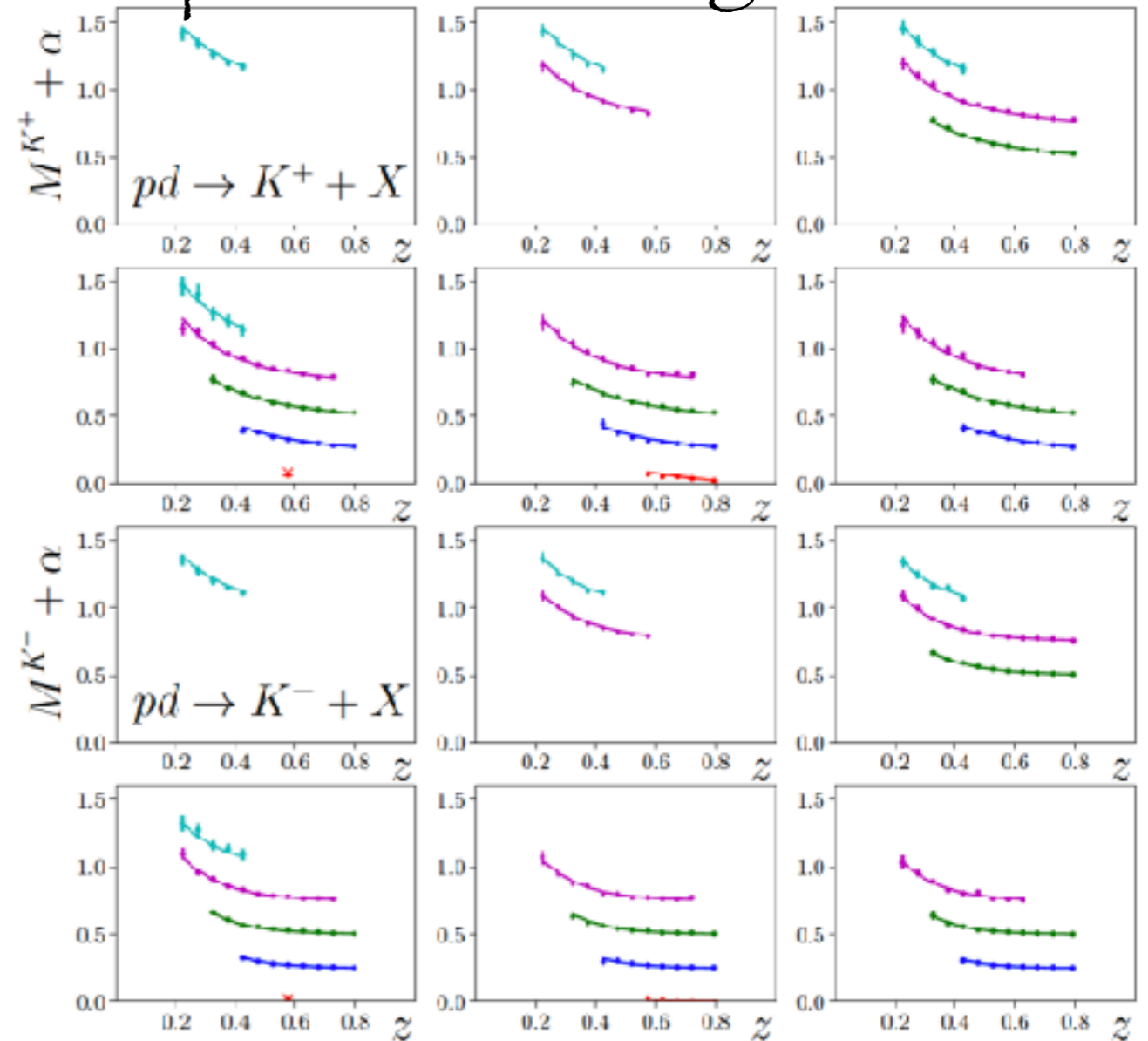
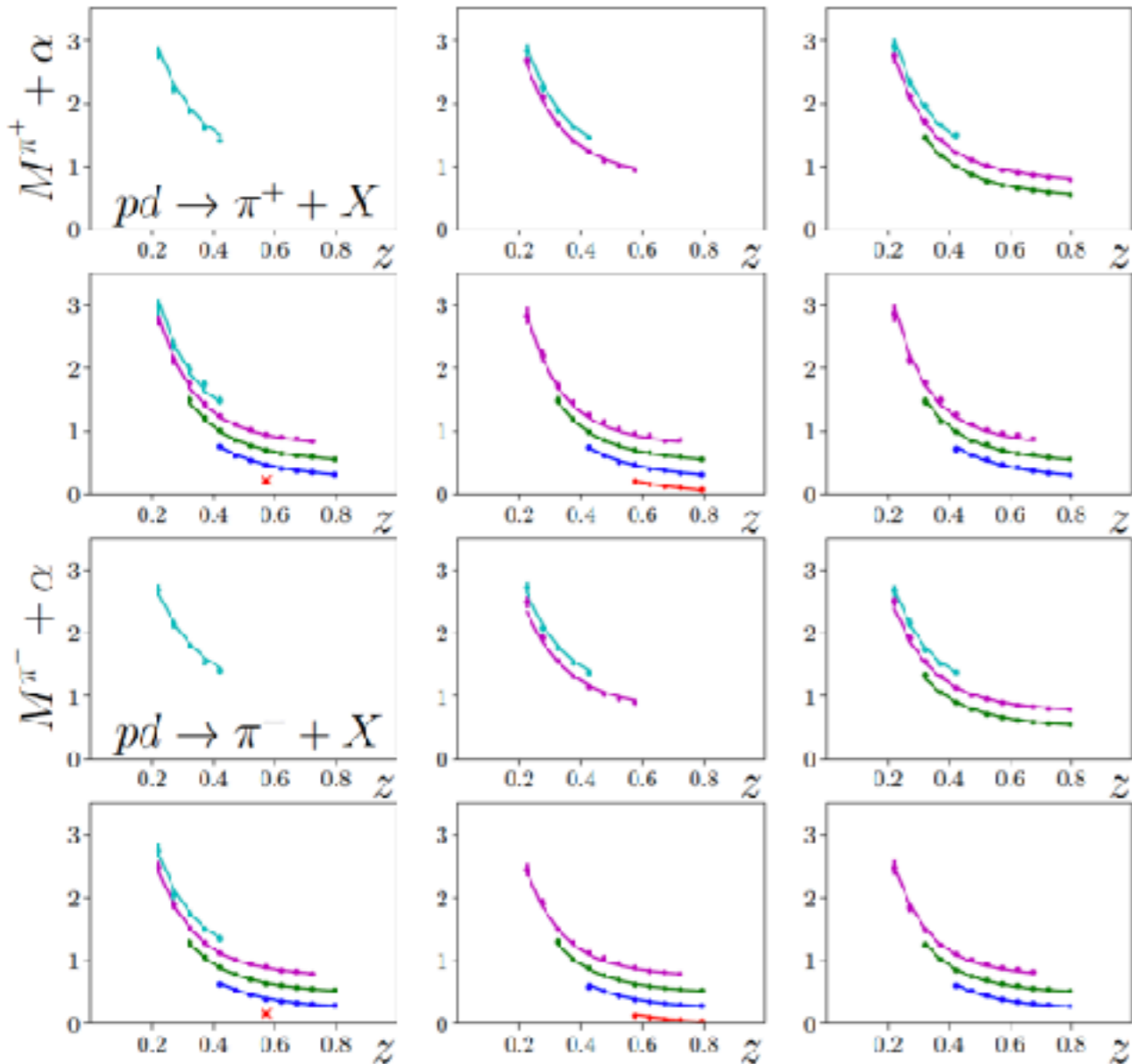
$$\text{SIA} : e^+ + e^- \rightarrow h + X$$



# Data vs. theory: SIDIS

First time SIDIS data are included in unpolarized PDFs global fit!

$$\text{SIDIS} : l + p/d \rightarrow l' + h + X$$



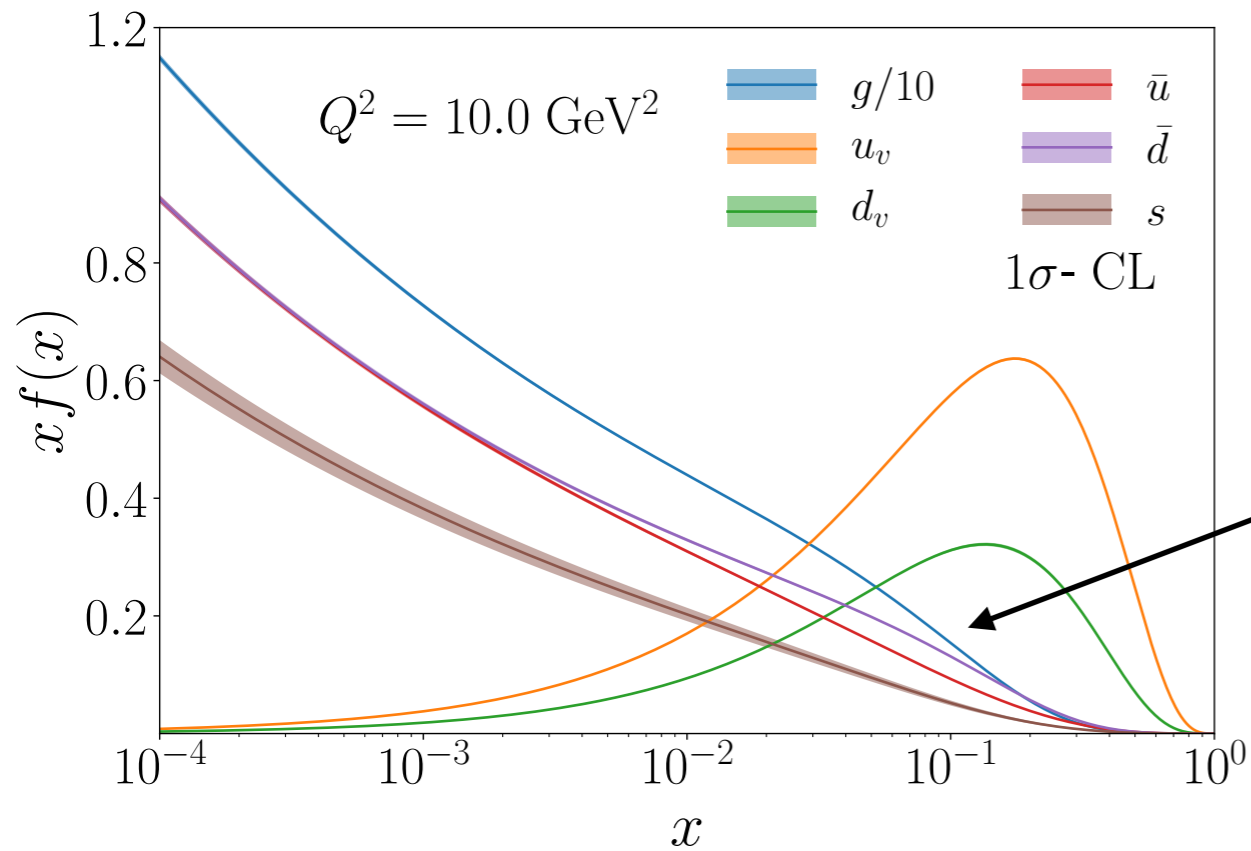
—  $y \in [0.10, 0.15], \alpha = 0.00$     —  $y \in [0.20, 0.30], \alpha = 0.50$   
—  $y \in [0.15, 0.20], \alpha = 0.25$     —  $y \in [0.30, 0.50], \alpha = 0.75$

COMPASS

—  $y \in [0.10, 0.15], \alpha = 0.00$     —  $y \in [0.20, 0.30], \alpha = 0.50$   
—  $y \in [0.15, 0.20], \alpha = 0.25$     —  $y \in [0.30, 0.50], \alpha = 0.75$

Difficult to fit low  $Q^2$  data  $\rightarrow$  only  $Q^2 > 5 \text{ GeV}^2$  data included

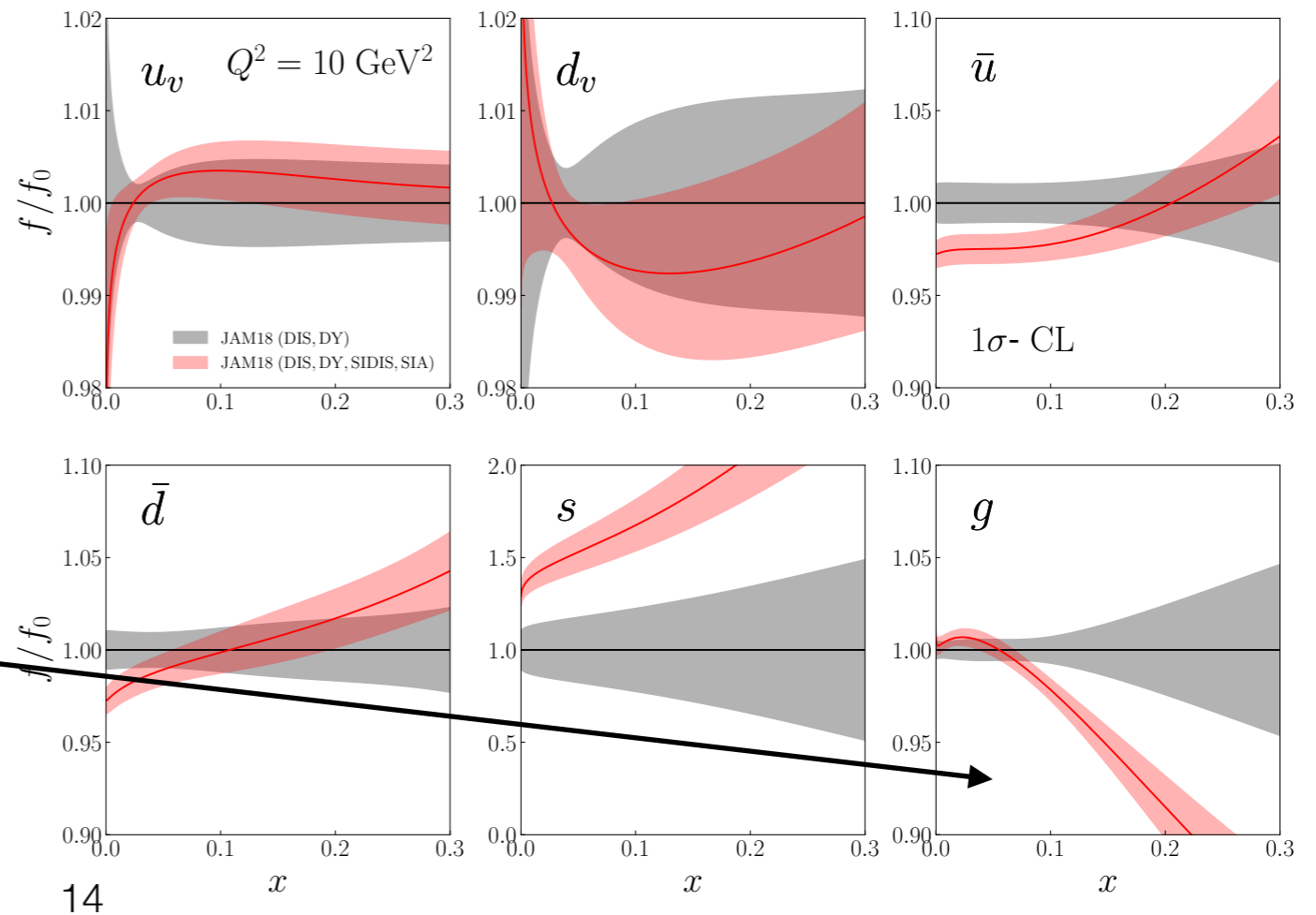
# Unpolarized PDFs (preliminary)



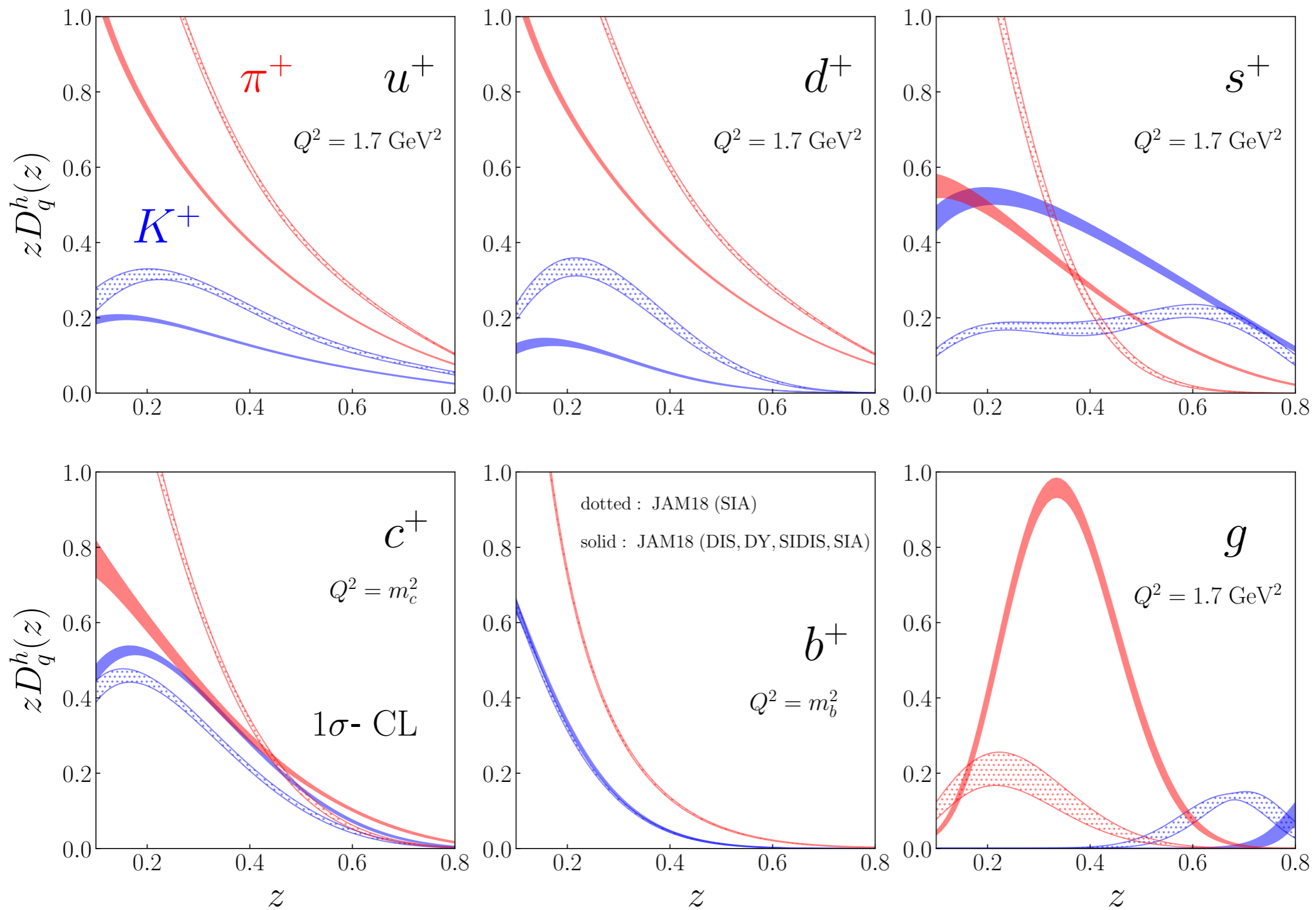
SIDIS supports the strange suppression

$\bar{u} - \bar{d}$  asymmetry at large  $x$

The gluon distribution changes significantly with the inclusion of SIDIS



# FFs (preliminary)



SIDIS data has large effect on flavor decomposition of FFs

# Summary

- MC statistical methods are important for a robust extraction of non-perturbative collinear distributions
  - Crucial for future Global TMDs, GPDs analysis
- First (preliminary) MC fit of PDFs and FFs using DIS, SIDIS and SIA data
- Strange PDF constrained by SIDIS data
- Significant effect of SIDIS data on flavor decomposition of FFs
- Difficulties in incorporating low  $Q^2 < 5 \text{ GeV}^2$  SIDIS data



# Outlook

- Impact of SIDIS data on  $s$  vs.  $\bar{s}$
- Introduce the HQ treatment: ACOT (GM-VFNS) ✓
- Use  $F_2^c$  and  $F_2^b$  HERA data GM-VFNS required!
- Likelihood sampling methods: Nested sampling ✓
- Inclusion of polarized DIS and SIDIS and extract  
PDFs, FFs, and  $\Delta$ PDFs
  - Simultaneous extraction of all non-perturbative input
  - Strict test of universality

Backup

# Iterative Monte Carlo (IMC)

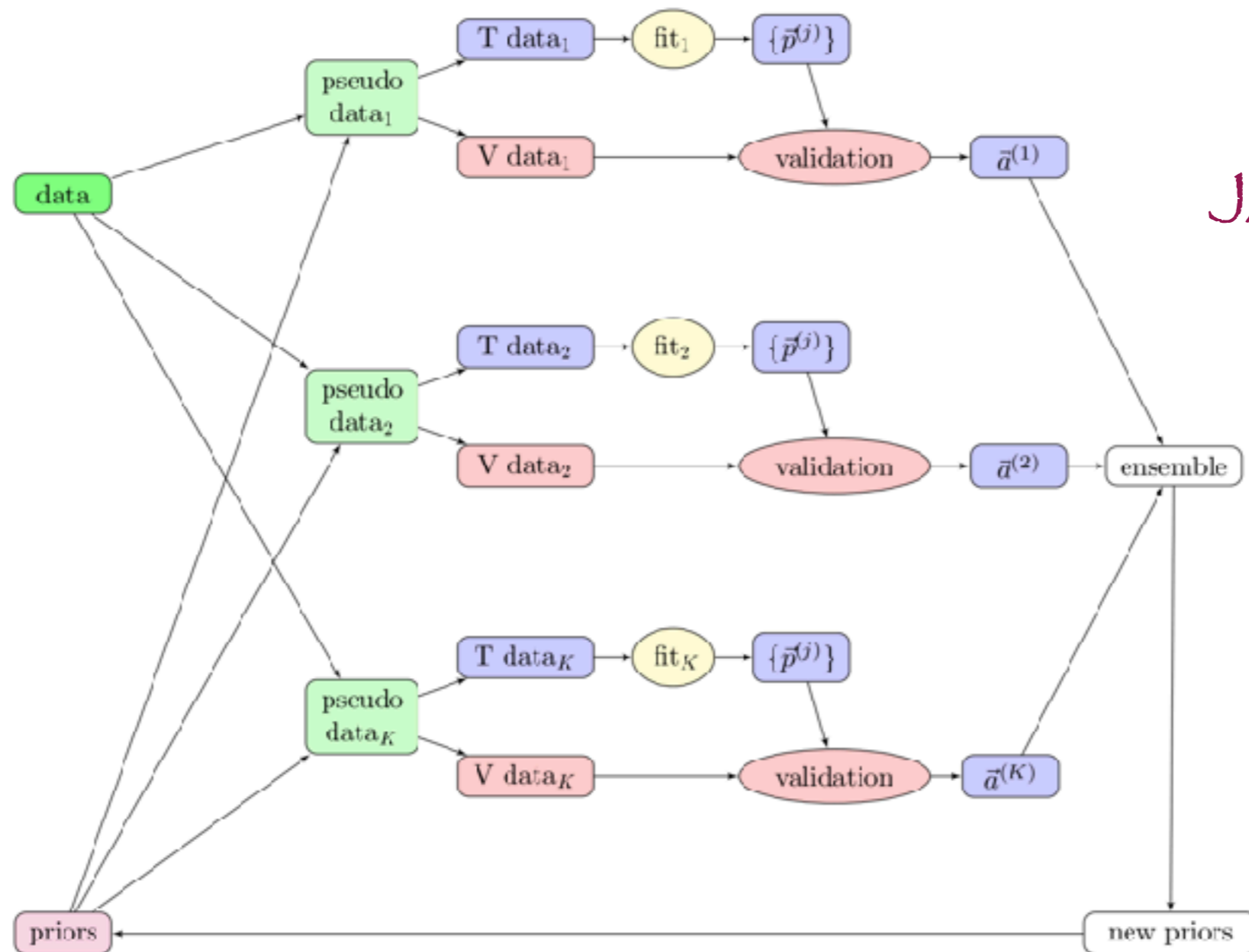
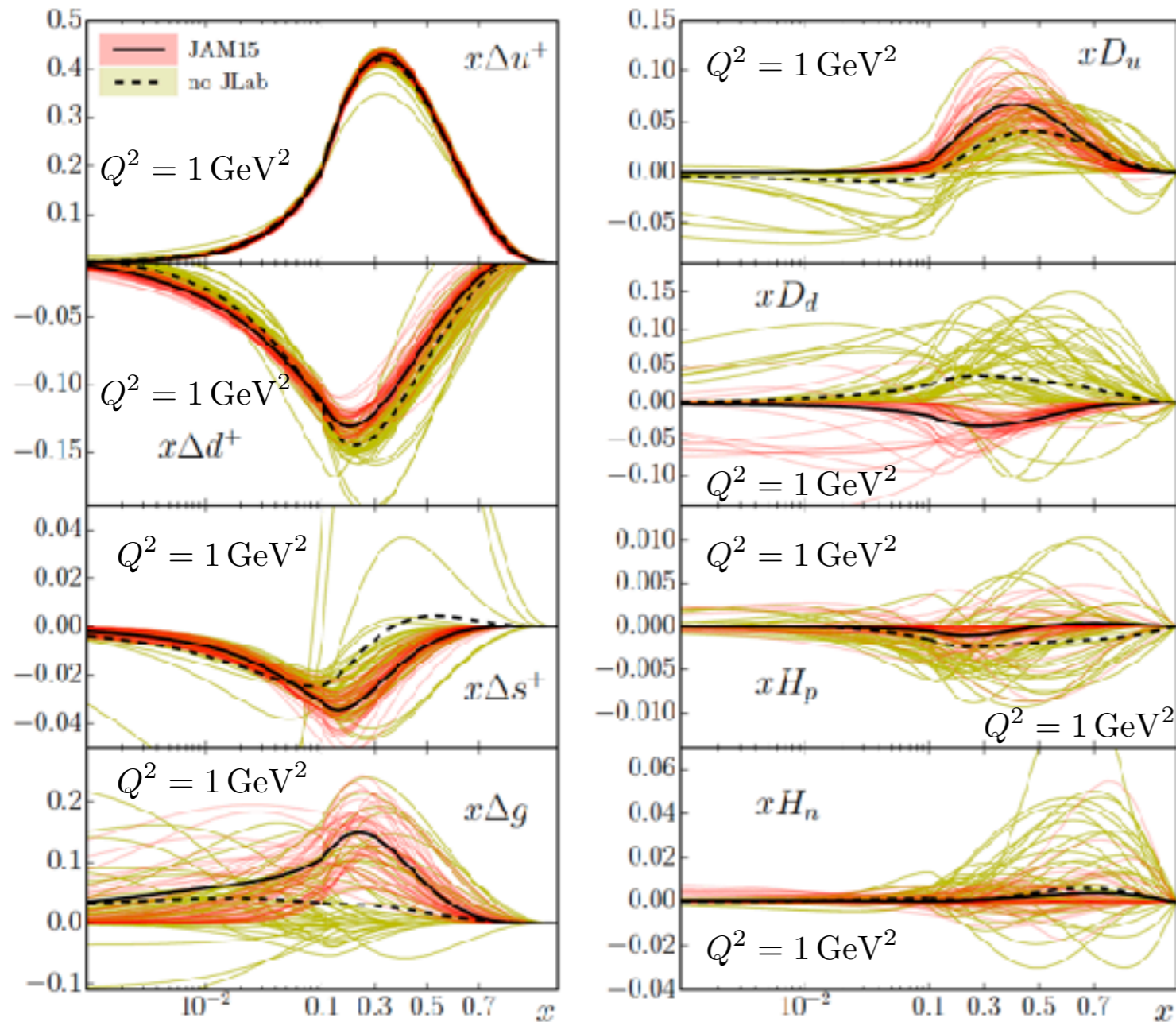


FIG. 1. Schematic illustration of the workflow for the iterative Monte Carlo fitting method. In the first stage,  $K$  pseudodata sets are generated, each of which is partitioned into training (T) and validation (V) subsets. For each pseudodata set, the training set is fitted and the parameters  $\{\vec{p}^{(j)}\}$  across all the minimization stages  $j$  are stored. The cross-validation procedure selects a single set of best-fit parameters  $\vec{a}^{(l)}$  from  $\{\vec{p}^{(j)}\}$  for each pseudodata set  $l$ , and the collection of  $\{\vec{a}^{(l)}; l = 1, \dots, K\}$  is then used as the priors for the next iteration.

# JAM15

## Impact of JLab data



$\Delta u^+$   $\Delta d^+$  : reduction of the uncertainties

$\Delta d^+$  less negative due to Jlab data



Harder  $\Delta s^+$

$\Delta s^+$  positive

$\Delta g$  big uncertainty

$\Delta g$  positive

Flavor decomposition of twist-3 distributions

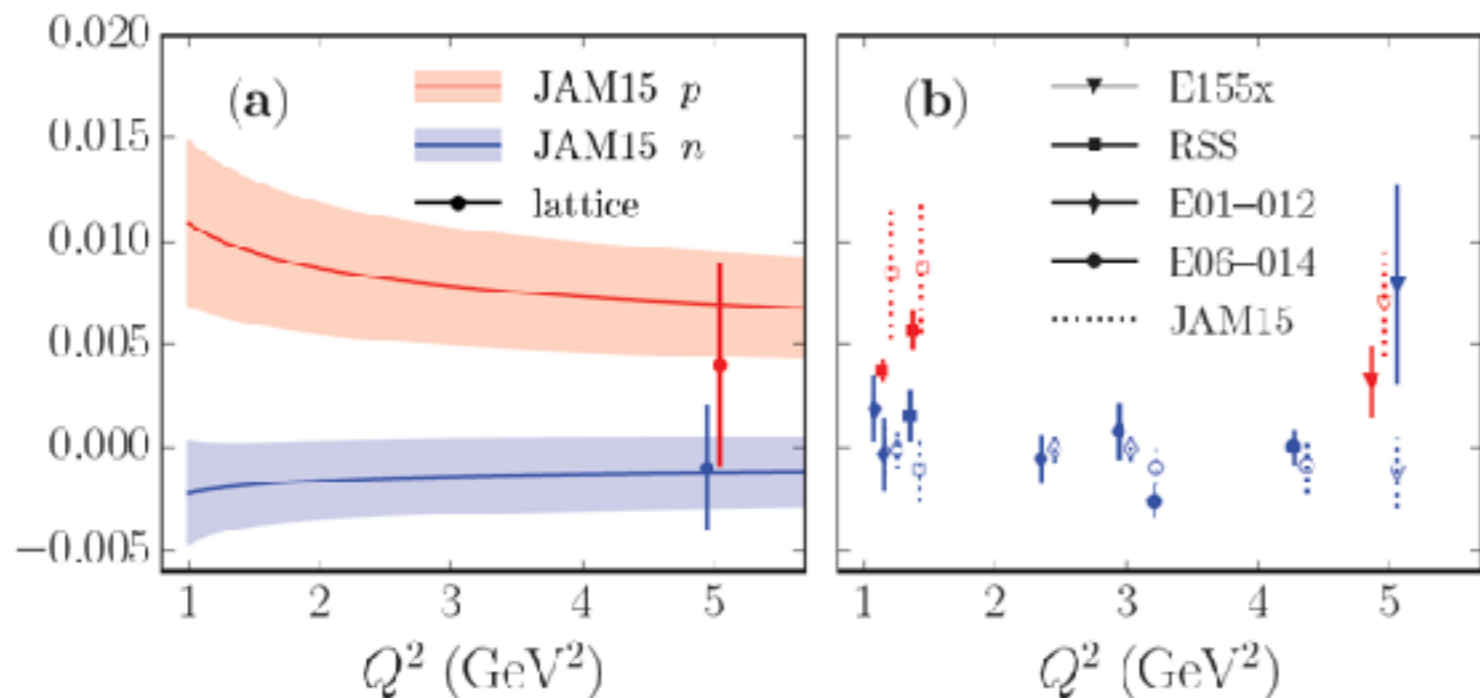
Twist-4 functions compatible with 0

$$g_1(x, Q^2) = g_1^{\text{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \dots) + g_1^{\text{T3+TMC}}(D_u, D_d) + g_1^{\text{T4}}(H_{p,n})$$

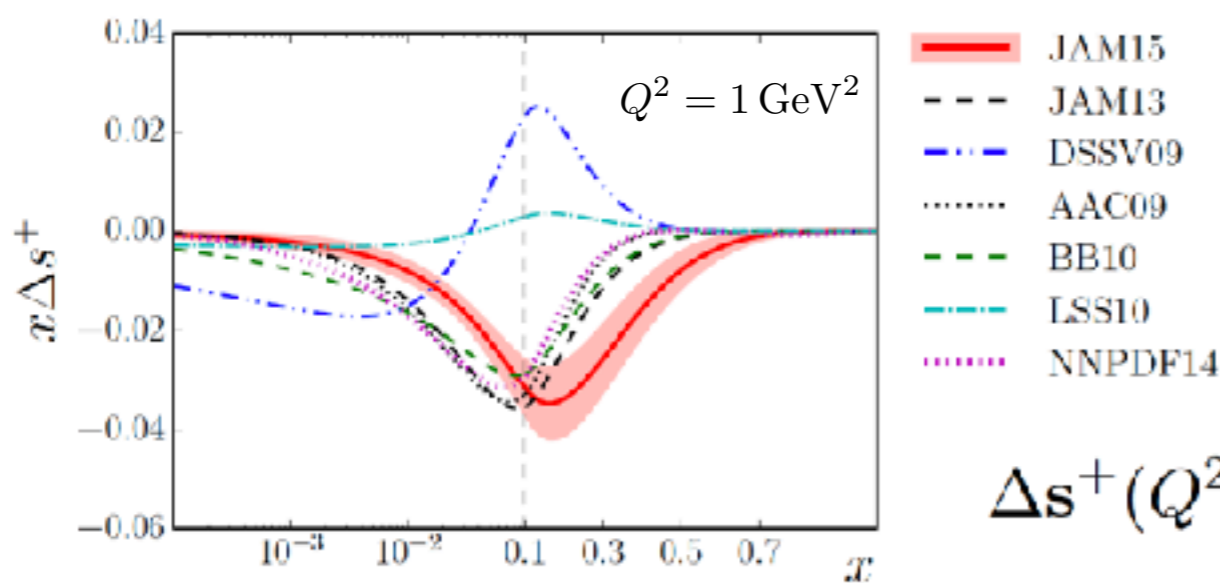
$$g_2(x, Q^2) = g_2^{\text{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \dots) + g_2^{\text{T3+TMC}}(D_u, D_d)$$

# JAM15

$$d_2(Q^2) = \int_0^1 dx x^2 (2g_1(x, Q^2) + 3g_2(x, Q^2))$$



$d_2$ -moment agrees with experimental data



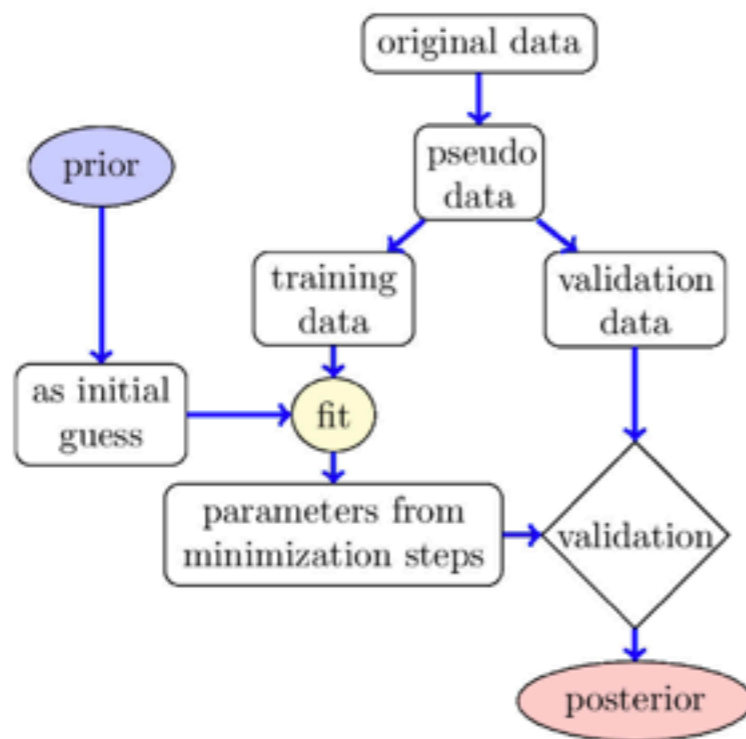
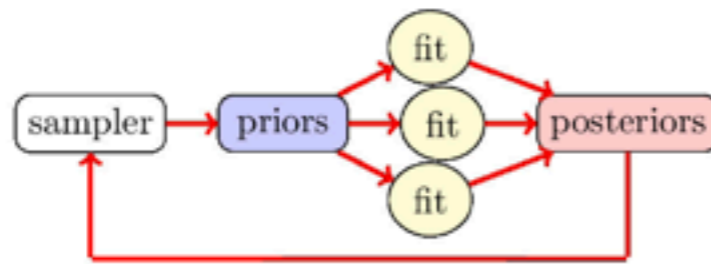
$$\Delta s^+(Q^2) = \int_0^1 dx \Delta s^+(x, Q^2)$$

JAM15:  $\Delta s^+ = -0.1 \pm 0.01$

DSSV09:  $\Delta s^+ = -0.11$   $Q^2 = 1 \text{ GeV}^2$

# Iterative Monte Carlo (IMC)

JAM16

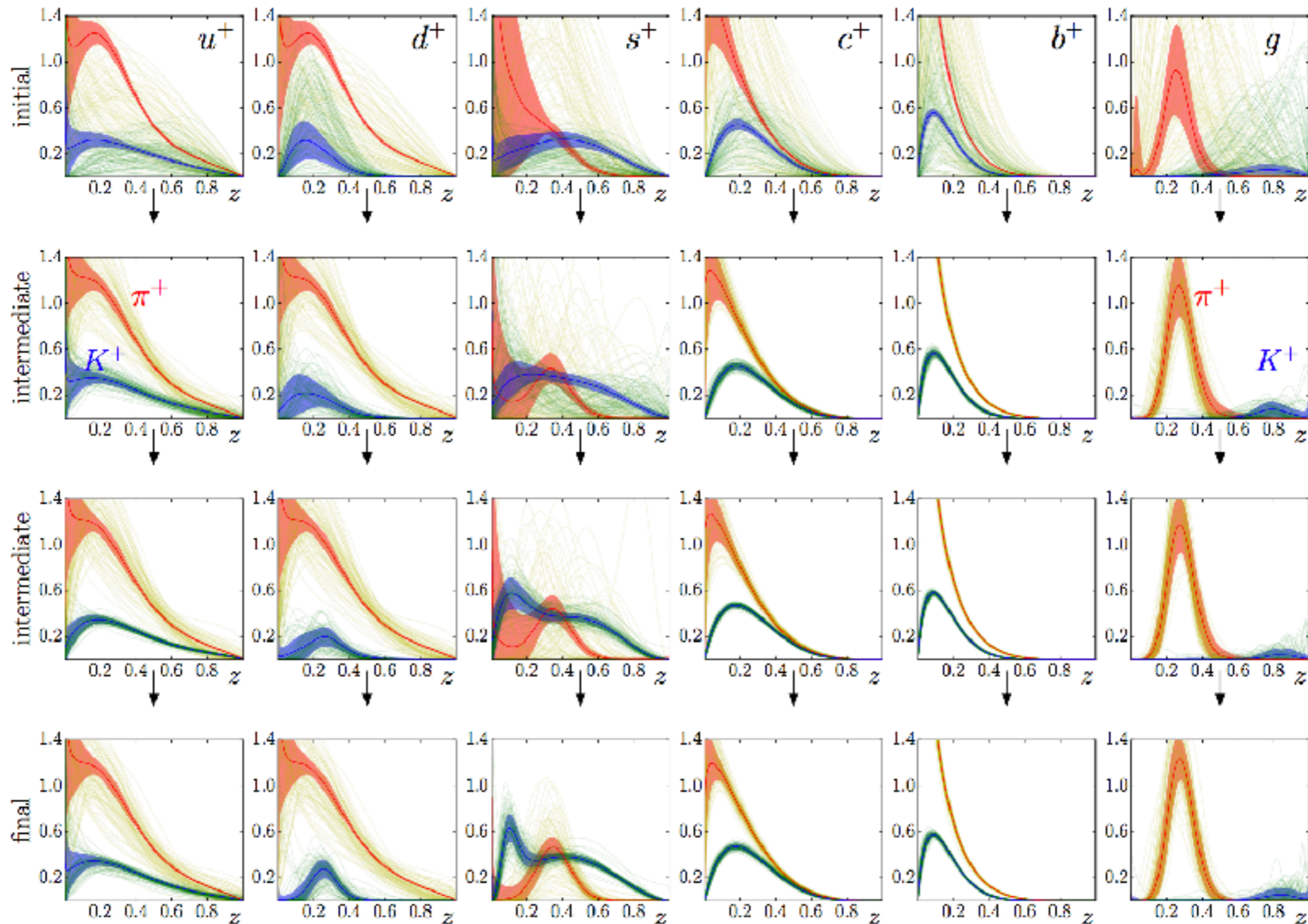


- Samples wide region of parameter space
- Data is partitioned for cross-validation – training set is fitted via chi-square minimization
- Posteriors used to construct sampler (multi-dimensional Gaussian, kernel density estimation, etc) – where parameters are chosen for the next iteration
- Procedure iterated until converged

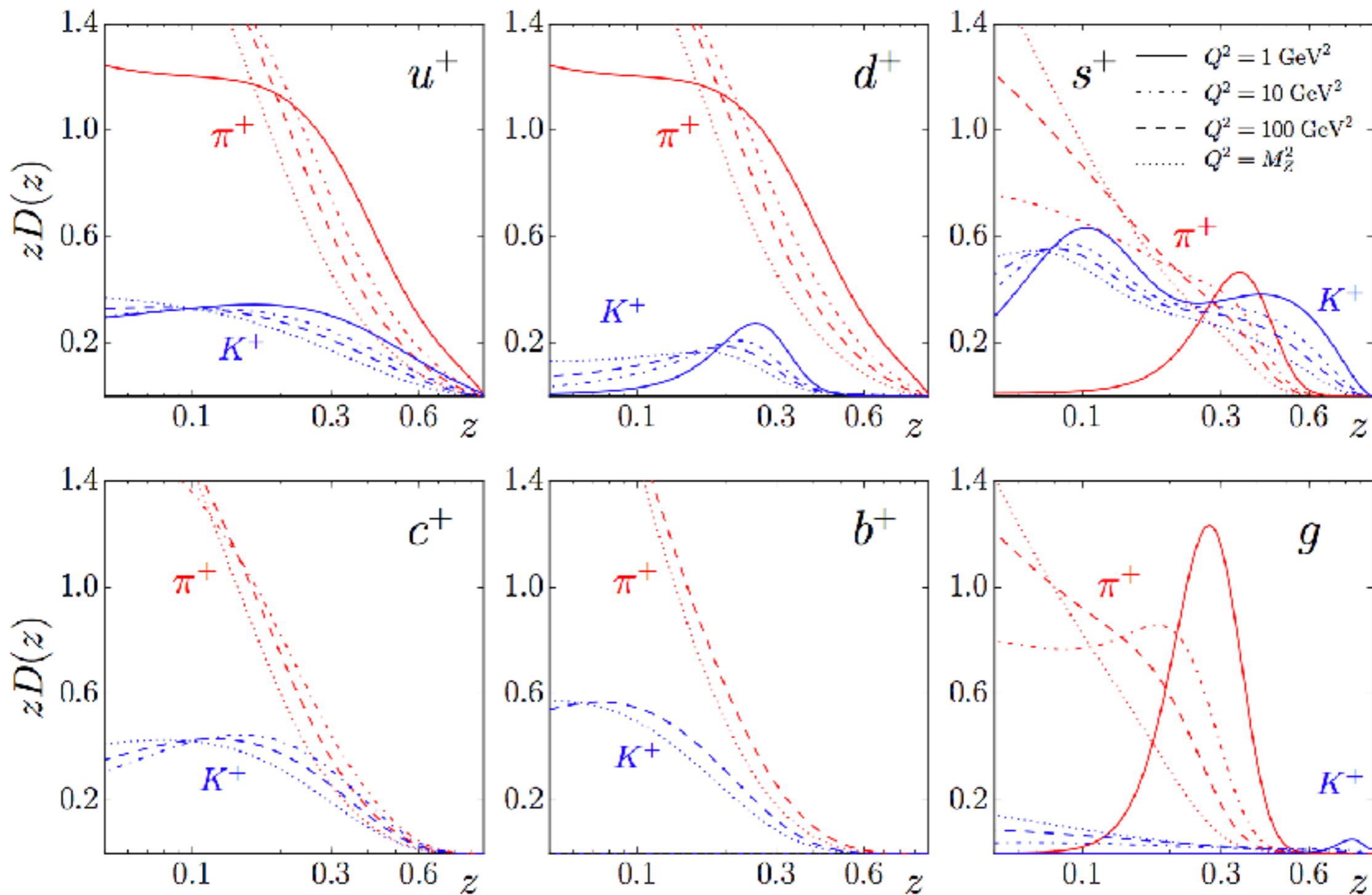
$$E[\mathcal{O}] = \frac{1}{n} \sum_{k=1}^n \mathcal{O}(\mathbf{a}_k)$$

$$V[\mathcal{O}] = \frac{1}{n} \sum_{k=1}^n (\mathcal{O}(\mathbf{a}_k) - E[\mathcal{O}])^2$$

# JAM16: iterative convergence



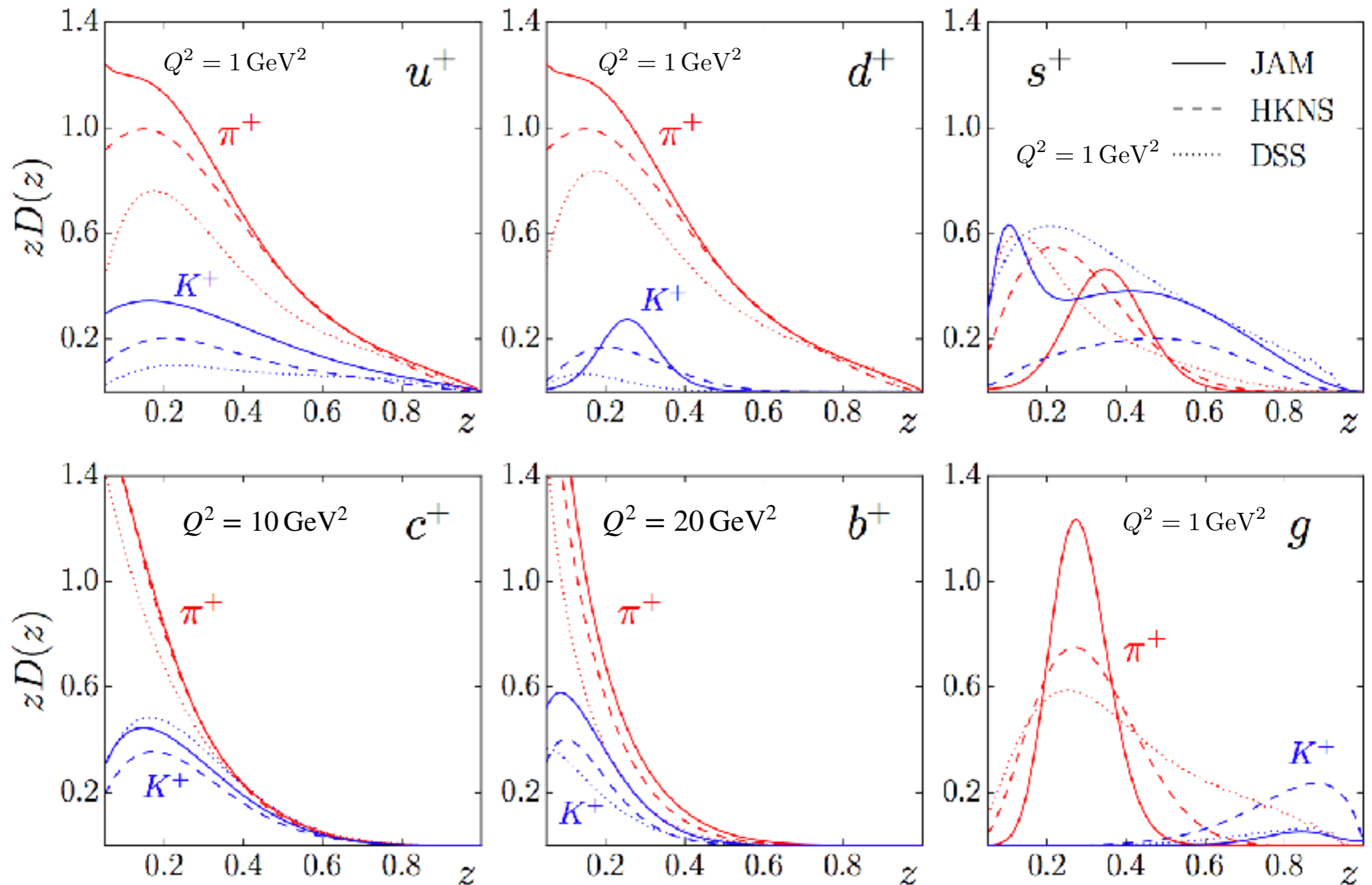
# JAM16: FFs evolution



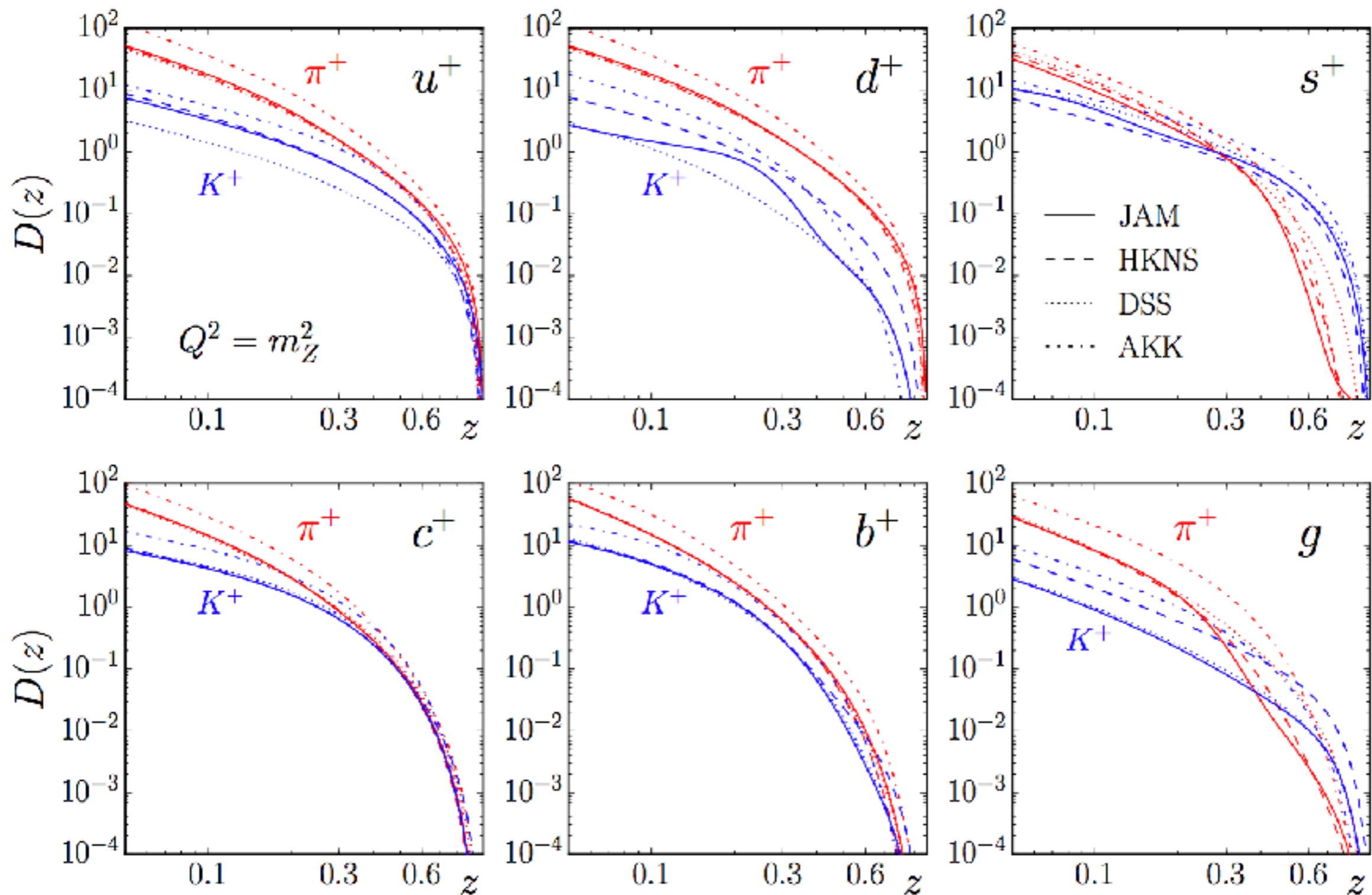
The gluon peak disappears



# JAM16: comparison



# JAM16: comparison II



# JAM17: motivation

Spin sum rule

$$\Delta\Sigma(Q^2) = \int_0^1 dx (\Delta u^+(x, Q^2) + \Delta d^+(x, Q^2) + \Delta s^+(x, Q^2))$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta G(Q^2) = \int_0^1 dx \Delta g(x, Q^2)$$

$$\Delta\Sigma(Q_{\text{EMC}}^2) \sim 0.1$$

- First moment of polarized structure function  $g_1$ :

$$\int_0^1 dx g_1^p(x, Q^2) = \frac{1}{36} [8\Delta\Sigma + 3g_A + a_8] \left(1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

→ DIS requires assumptions about triplet and octet axial charges to extract  $\Delta\Sigma$

- Assuming exact  $SU(2)_f$  and  $SU(3)_f$  values from weak baryon decays

$$\int dx (\Delta u^+ - \Delta d^+) = g_A \sim 1.269 \quad \int dx (\Delta u^+ + \Delta d^+ - 2\Delta s^+) = a_8 \sim 0.586$$

$$\Delta\Sigma_{[10^{-3}, 0.8]} \sim 0.3$$

Released in JAM17

# Parameterizations and Chi-square

Euler Beta function

Template function:  $T(x; \mathbf{a}) = \frac{M x^a (1-x)^b (1+c\sqrt{x})}{B(n+a, 1+b) + cB(n+\frac{1}{2}+a, 1+b)}$

- PDFs:  $n = 1$   $\Delta q^+, \Delta \bar{q}, \Delta g = T(x; \mathbf{a})$
- FFs:  $n = 2, c = 0$  Favored:  $D_{q^+}^h = T(z; \mathbf{a}) + T(z; \mathbf{a}')$   
Unfavored:  $D_{q^+,g}^h = T(z; \mathbf{a})$

Isospin → 
Pions:  
 $D_{\bar{u}}^{\pi^+} = D_d^{\pi^+} = T(z; \mathbf{a})$   
 $D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = \frac{1}{2} D_{s^+}^{\pi^+}$

Kaons:  
 $D_{\bar{u}}^{K^+} = D_d^{K^+} = \frac{1}{2} D_{d^+}^{K^+}$   
 $D_s^{K^+} = T(z; \mathbf{a})$

- Chi-squared definition:

$$\chi^2(\mathbf{a}) = \sum_e \left[ \sum_i \left( \frac{\mathcal{D}_i^{(e)} N_i^{(e)} - T_i^{(e)}(\mathbf{a})}{\alpha_i^{(e)} N_i^{(e)}} \right)^2 + \sum_k (r_k^{(e)})^2 \right] + \sum_\ell \left( \frac{a^{(\ell)} - \mu^{(\ell)}}{\sigma^{(\ell)}} \right)^2$$

syst + stat (quad.)

Normalization  
(Correlated uncertainties)

Penalty term

Nuisance parameters

# Polarized SIDIS

## Proton Spin Structure from SIDIS

- Measured via longitudinal double spin asymmetries

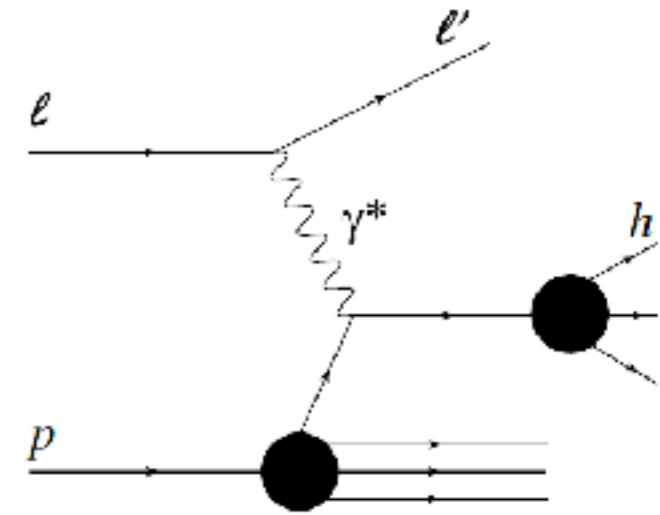
$$A_1^h(x, z, Q^2) = \frac{g_1^h(x, z, Q^2)}{F_1^h(x, z, Q^2)}$$

- Polarized structure function at NLO defined in terms of 2-D convolution

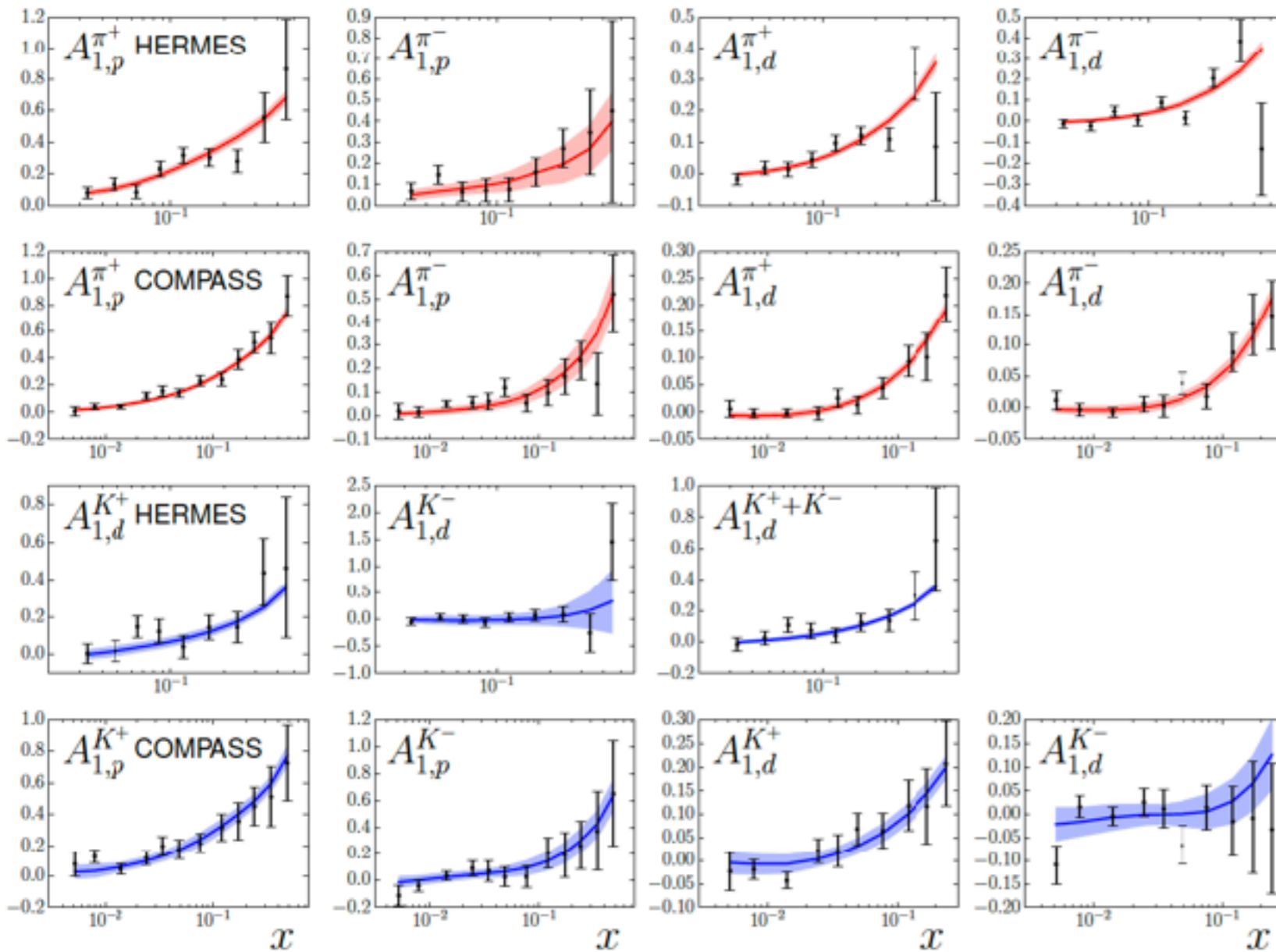
$$g_1^h(x, z, Q^2) = \frac{1}{2} \sum_q e_q^2 \left\{ \Delta q(x, Q^2) D_q^h(z, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \times \left( \Delta q \otimes \Delta C_{qq} \otimes D_q^h + \Delta q \otimes \Delta C_{gq} \otimes D_g^h + \Delta g \otimes \Delta C_{qg} \otimes D_q^h \right) \right\}$$

- To include SIDIS observables in the JAM global analyses, fragmentation functions (FFs) must be known
  - Choice of FF parameterizations available (HKNS & DSS) differed significantly in kaon sector – strongly impacts  $\Delta s^+$  extraction

JAM decided to use Monte Carlo procedure to extract FFs from SIA



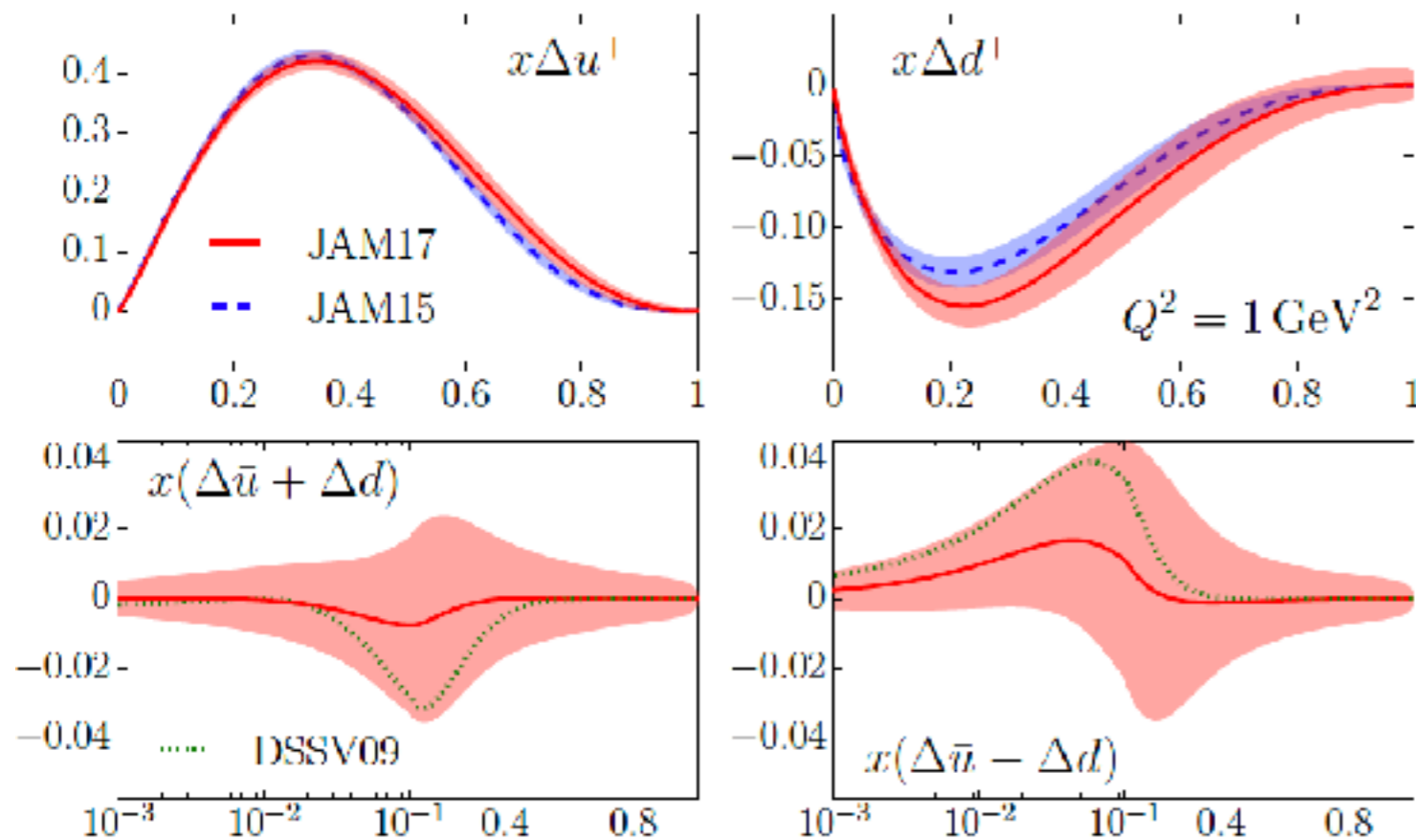
# JAM17: Data vs. Theory



$$A_1^h = \frac{g_1^h}{F_1^h}$$

process	target	$N_{\text{dat}}$	2
DIS	$p, d, {}^3\text{He}$	854	854.8
SIA ( $\pi^\pm, K^\pm$ )		850	997.1
SIDIS ( $\pi^\pm$ )			
HERMES	$d$	18	28.1
HERMES	$p$	18	14.2
COMPASS	$d$	20	8.0
COMPASS	$p$	24	18.2
SIDIS ( $K^\pm$ )			
HERMES	$d$	27	18.3
COMPASS	$d$	20	18.7
COMPASS	$p$	24	12.3
<b>Total:</b>		<b>1855</b>	<b>1969.7</b>

# JAM17: Polarized PDFs

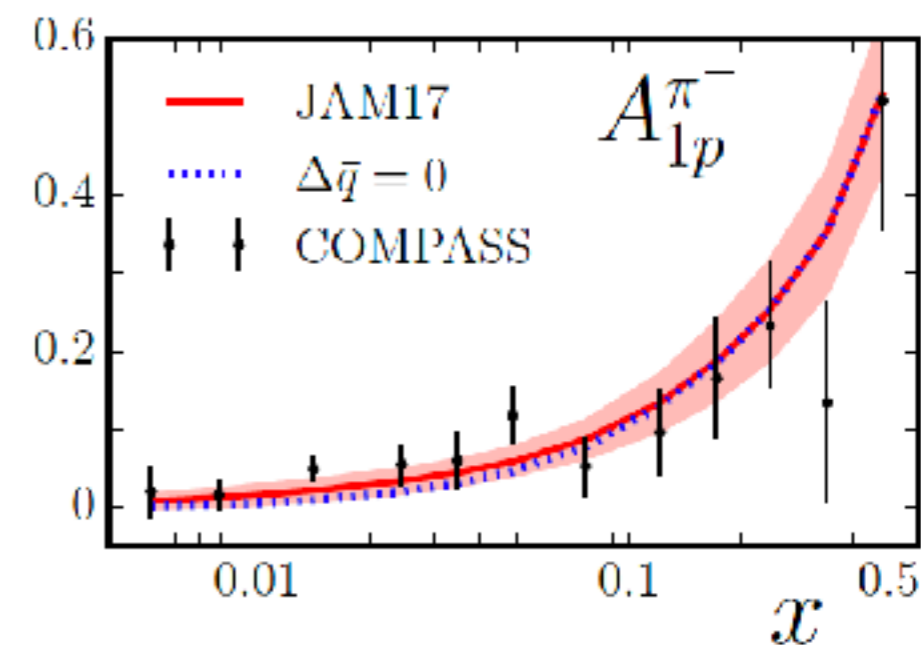


- Isoscalar sea distribution consistent with zero
- Isovector sea slightly prefers positive shape at low  $x$ 
  - Non-zero asymmetry given by small contributions from SIDIS asymmetries

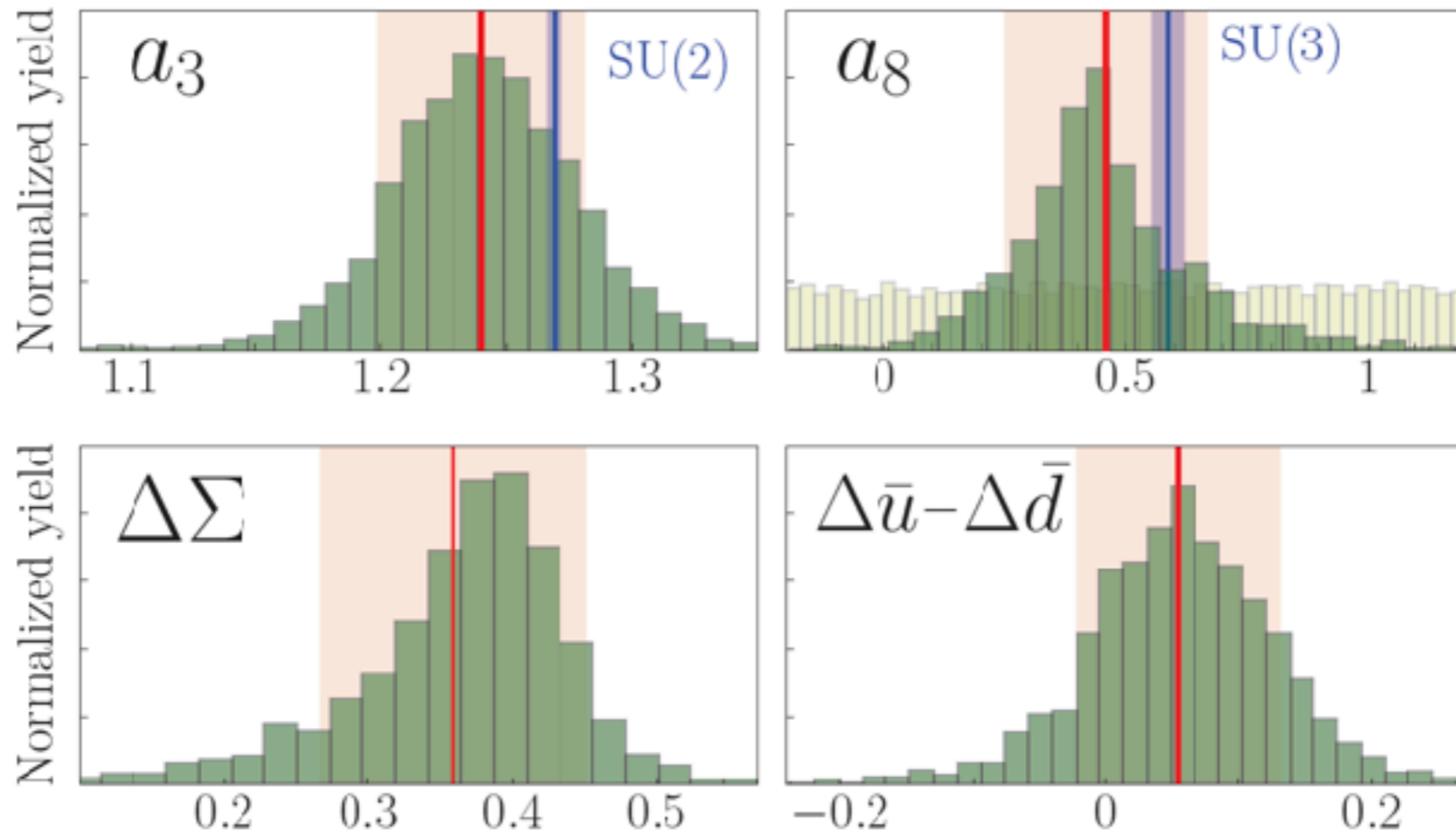
- $\Delta u^+$  consistent with previous analysis

- $\Delta d^+$  slightly larger in magnitude

→ Anti-correlation with  $\Lambda s^+$ , which is less negative than JAM15 at  $x \sim 0.2$



# JAM17: Lowest moments



$$g_A = 1.24 \pm 0.04 \quad \text{Confirmation of SU(2) symmetry to } \sim 2\%$$

$$a_8 = 0.46 \pm 0.21 \quad \sim 20\% \text{ SU(3) breaking } \pm \sim 20\%; \text{ large uncertainty}$$

- Need better determination of  $\Delta s^+$  moment to reduce  $a_8$  uncertainty!

$$\Delta s^+ = -0.03 \pm 0.09$$

$$\Delta\Sigma = 0.36 \pm 0.09$$



# JAM18: Parametrization

Parametrization  $\rightarrow$  generic template functions

$$T(\xi; \mathbf{a}) = N \frac{\xi^a (1 - \xi)^b (1 + c\sqrt{\xi} + d\xi)}{B(2 + a, b + 1) + cB(5/2 + a, b + 1) + dB(3 + a, b + 1)}$$

For PDFs 13 parameters

- +  $g, u_v, d_v, \bar{u}, \bar{d}, s = \bar{s}$
- + momentum sum rules and quark number sum rule

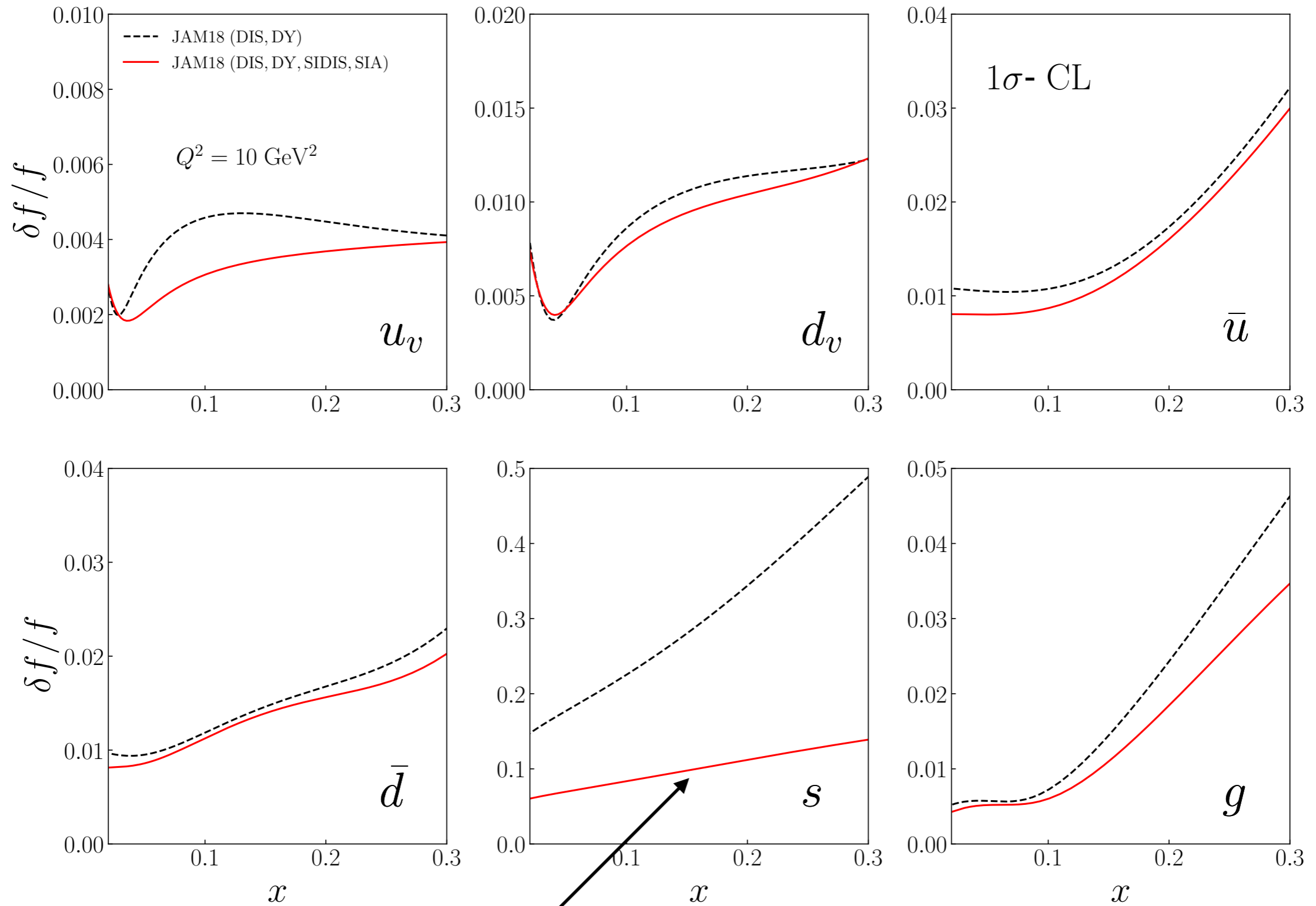
For FFs  $\pi^+$  16 parameters

- +  $g, u(\text{fav.}), d(\text{unfav.}), c, b$
- +  $u = \bar{d}, d = \bar{u} = s = \bar{s}$

For FFs  $K^+$  15 parameters

- +  $g, u(\text{fav.}), d(\text{unfav.}), \bar{s}(\text{fav.}), c, b$
- +  $d = \bar{d} = \bar{u} = s$

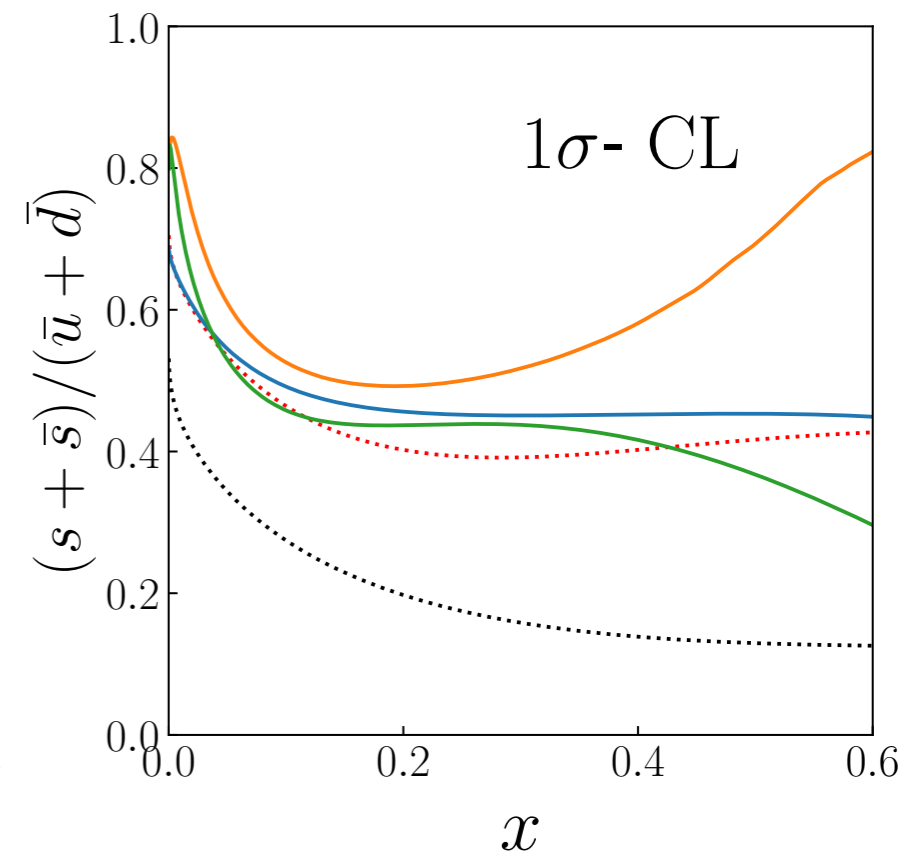
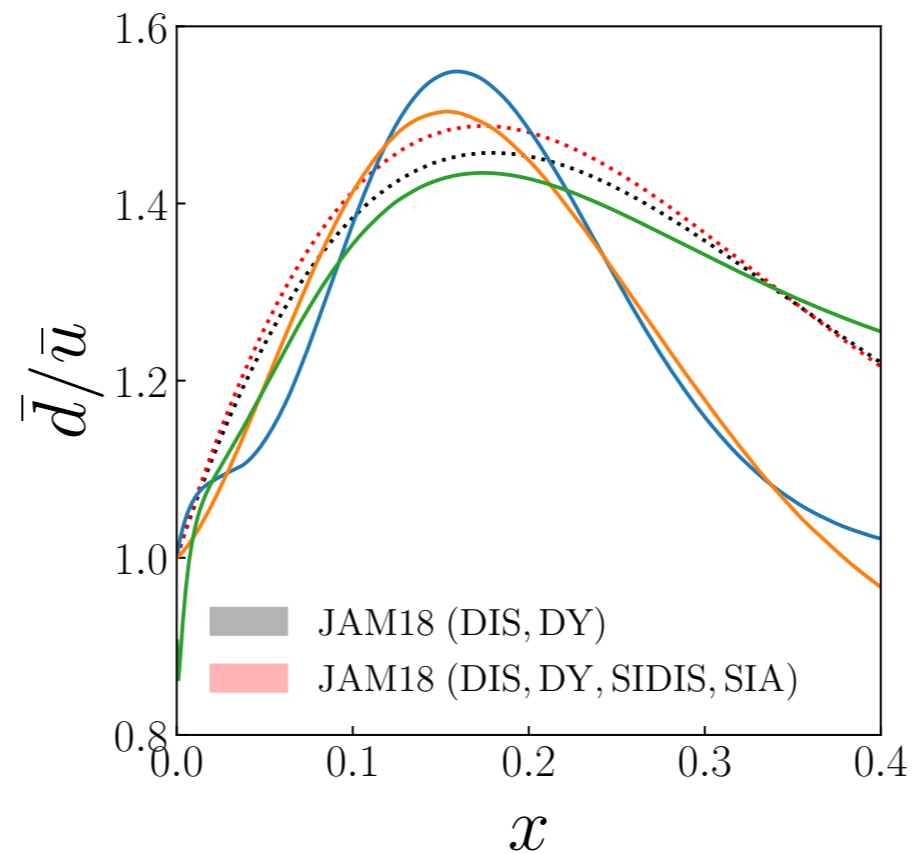
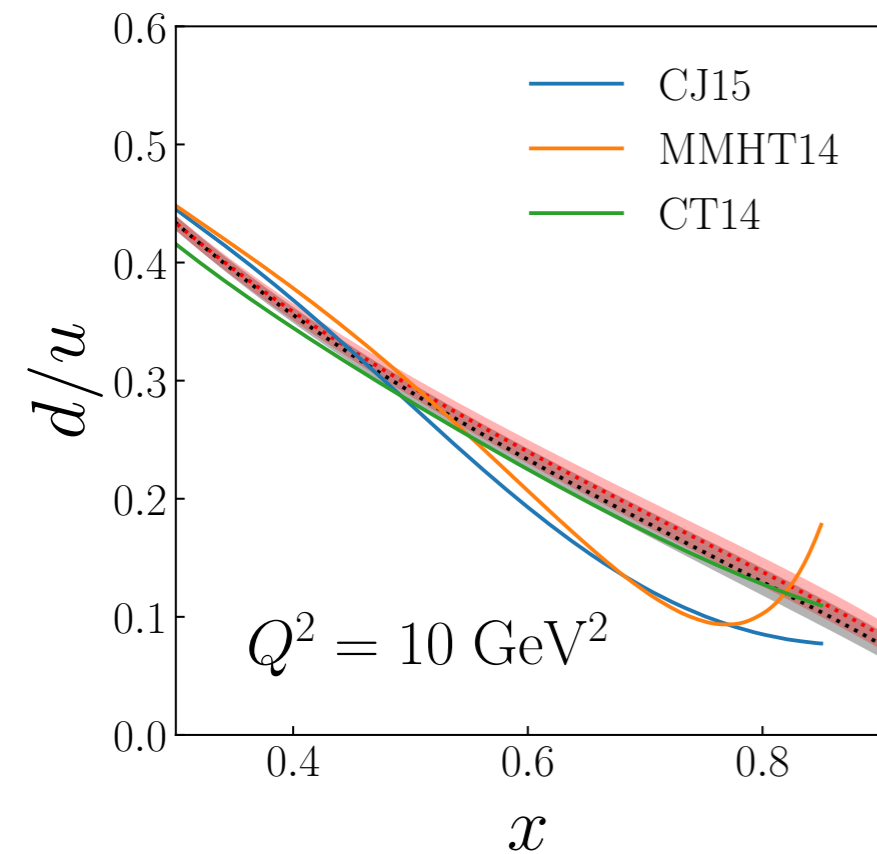
# Unpolarized PDFs (JAM18)



Decrease of strange uncertainties due to  $K$  SIDIS data

# Unpolarized PDFs (JAM18)

## Sea distributions



# Next steps

- Using  $F_2^c$  and  $F_2^b$  data? GM-VFNS required!

HERA data: [arXiv:1804.01019](https://arxiv.org/abs/1804.01019) [hep-ex]

52 points  $\sigma_{red}^{c\bar{c}}$ :  $3 \cdot 10^{-5} < x < 0.05$  and  $2.5 \text{ GeV}^2 < Q^2 < 2000 \text{ GeV}^2$

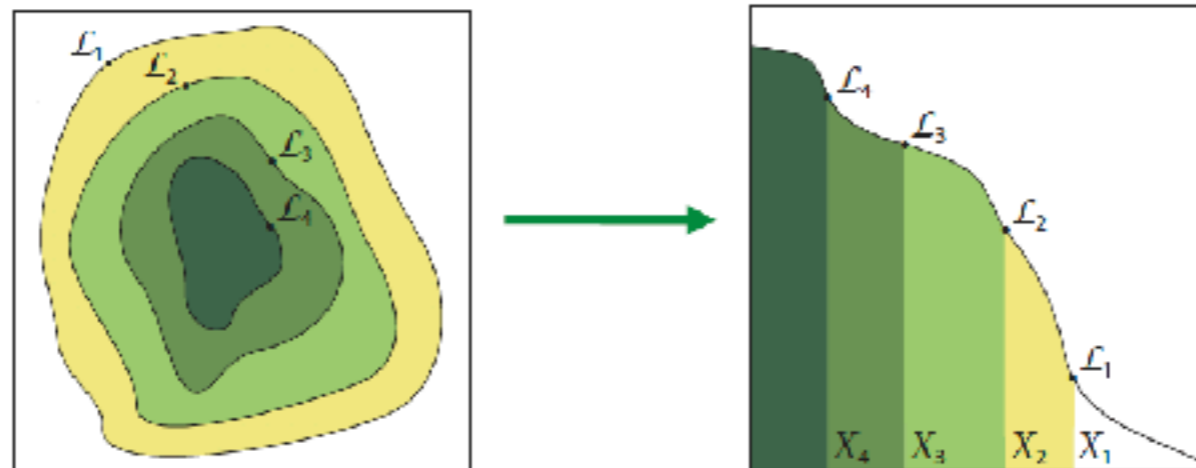
27 points  $\sigma_{red}^{b\bar{b}}$ :  $1.3 \cdot 10^{-4} < x < 0.05$  and  $2.5 \text{ GeV}^2 < Q^2 < 2000 \text{ GeV}^2$

## Methodology Shift: Nested Sampling

- Statistical mapping of multidimensional integral to 1-D

$$Z = \int d^n a \mathcal{L}(\text{data}|\vec{a})\pi(\vec{a}) = \int_0^1 dX \mathcal{L}(X)$$

where the *prior volume*  $dX = \pi(\vec{a})d^n a$



$$Z_i \sim \sum_i \mathcal{L}_i w_i$$

where  $w_i = \frac{1}{2} (X_{i-1} - X_i)$

Feroz et al. arXiv:1306.2144  
[astro-ph]

- Algorithm:**
  - Initialize  $X_0 = 1, L = 0$  and choose  $N$  active points  $X_1, X_2, \dots, X_N$  from prior
  - For each iteration, sample new point and remove lowest  $L_i$ , replacing with point such that  $L$  is monotonically increasing
  - Repeat until entire parameter space has been explored