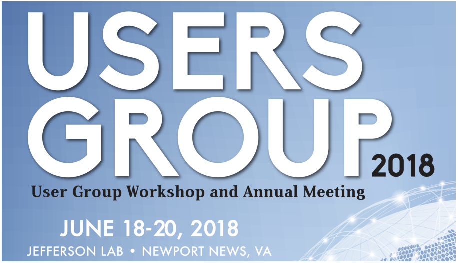
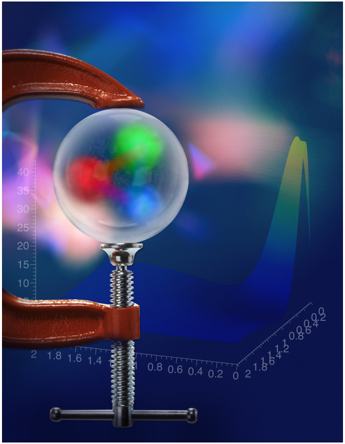
Proton Gravitational Form Factors

François-Xavier Girod

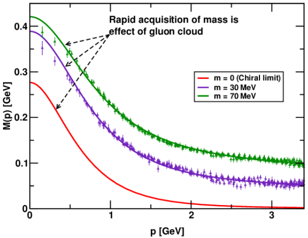


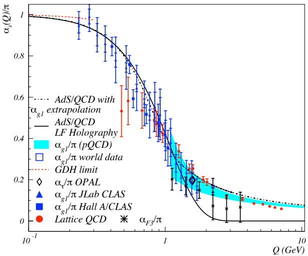


V. Burkert, L. Elouadrhiri, FXG
Nature **557**, 396–399 (2018)

Confinement Mechanism(s?)

Hadrons are singlets under SU(3)$_{color}$: No net color charge in asymptotic particle states





- **Linear growth of the static quark-antiquark pair**
  Area-law falloff for the Wilson loop
- **Gribov Confinement for light quarks**
  Analytical properties of the propagators in the infrared
  Instability of the vacuum above a supercritical charge

$$\alpha^{\text{crit}}_{\text{QED}} = 137 \text{ for a point-like nucleus}$$

$$\approx 180 \text{ for a finite size nucleus}$$

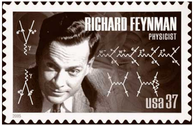
$$\frac{\alpha^{\text{crit}}_{\text{QCD}}}{\pi} = C_F^{-1}\left[1-\sqrt{\frac{2}{3}}\right] \approx 0.137$$

- **Light-Front AdS/QCD**
  quark and gluon chiral condensates confined!
  →condensates contribution to the cosmological constant already included in hadron mass
- Mass-Gap Millenium problem and Yang-Mills existence
  $1M from the Clay Mathematical Institute

Gravity and QCD

In some fundamental sense a *graviton* can be thought of as a *pair of vector bosons*: Gravity amplitudes appear as squared Yang-Mills amplitudes in the *Color-Kinematics Duality*

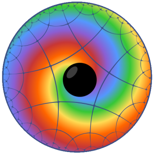
Z. Bern *et al.*
Gravity as the Square of Gauge Theory
*Phys. Rev. D*82 065003 (2010)



Understanding the deeper origin of these dualities is at the heart of string theory. Here a graviton (closed string) happens naturally as a pair of vector bosons (open strings). The **duality** between Gravity in the bulk and QCD on the boundary of AdS space, also called **holographic principle** is the currently the all time most cited high energy physics publication

J. Maldacena
The Large N limit of superconformal field theories and supergravity
*Int. J. Theor. Phys.*38 1113 (1999)
(13k citations as of June 2018)

Gravitational Form Factors from QCD bound states are observables of choice to test these dualities. Most promising avenue to understand the non-perturbative structure of gauge theories.

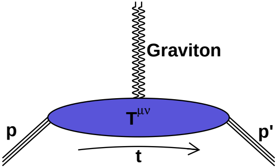


Energy Momentum Tensor

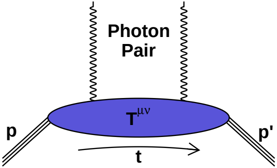
Gravitational Form Factors definition :

$$\langle p'|\hat{T}^q_{\mu\nu}|p\rangle = \bar{N}(p')\left[\; M_2^q(t)\,\frac{P_\mu P_\nu}{M} + J^q(t)\,\frac{\imath(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + d_1^q(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M}\;\right]N(p)$$

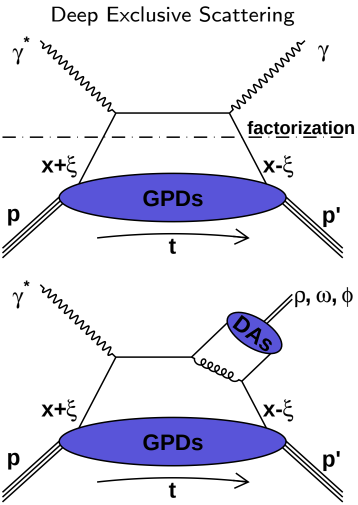
Confinement forces from space-space components of EMT
The graviton with spin 2 couples directly to EMT
But gravity is too weak to produce count rates in the detector



We can construct a spin 2 operator using two spin 1 operators
→ use a process with two photons to measure the EMT?
X. Ji *PRL* **78** 610 (1997) ; M. Polyakov *PLB* **555** 57 (2003)



Generalized Parton Distributions



$\gamma^* p \to \gamma p', \rho p', \omega p', \phi p'$

Bjorken regime :

$Q^2 \to \infty$, x_B fixed

t fixed $\ll Q^2$, $\xi \to \frac{x_B}{2-x_B}$

$$\frac{P^+}{2\pi} \int dy^- \, e^{ixP^+y^-} \langle p' | \bar{\psi}_q(0) \gamma^+ (1+\gamma^5) \psi(y) | p \rangle$$

$$= \bar{N}(p') \left[H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t) i\sigma^{+\nu} \frac{\Delta_\nu}{2M} + \tilde{H}^q(x,\xi,t)\gamma^+\gamma^5 + \tilde{E}^q(x,\xi,t)\gamma^5 \frac{\Delta^+}{2M} \right] N(p)$$

spin	N no flip	N flip
q no flip	H	E
q flip	$\tilde{H}$	$\tilde{E}$

3-D Imaging conjointly in transverse impact parameter and longitudinal momentum

Gravitational Form Factors and GPDs

Form Factors accessed *via* second x-moments :

$$\langle p'|\hat{T}^q_{\mu\nu}|p\rangle = \bar{N}(p')\left[\; M_2^q(t)\,\frac{P_\mu P_\nu}{M} + J^q(t)\,\frac{\imath(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + d_1^q(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M}\;\right] N(p)$$

Angular momentum distribution

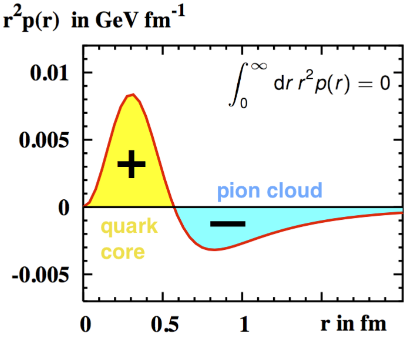
$$J^q(t) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x\,[H^q(x,\xi,t) + E^q(x,\xi,t)]$$

Mass and force/pressure distributions

$$M_2^q(t) + \frac{4}{5} d_1(t)\xi^2 = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x H^q(x,\xi,t)$$

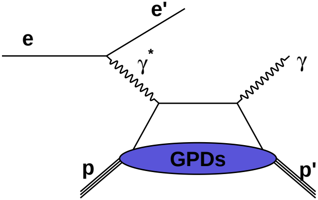
$$d_1(t) = 15M \int \mathrm{d}^3\vec{r}\, \frac{j_0(r\sqrt{-t})}{2t}\, p(r)$$

Distribution of pressure

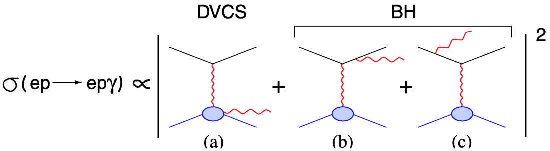


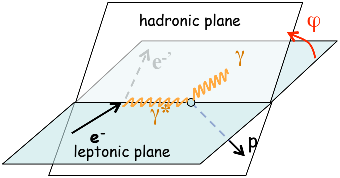
Deeply Virtual Compton Scattering

The cleanest GPD probe at low and medium energies



$$ep \to ep\gamma$$





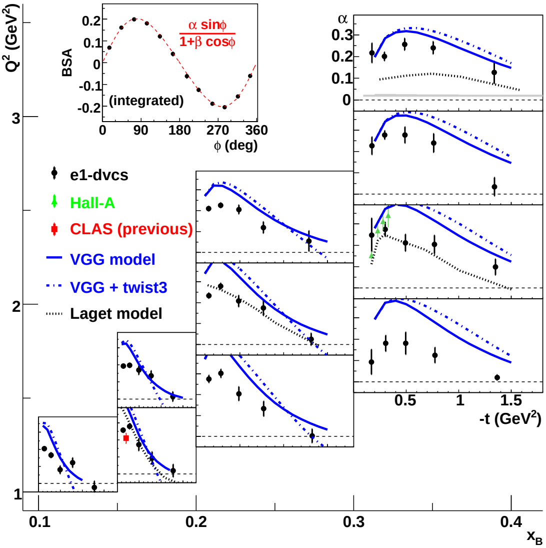
$$A_{LU} = \frac{d^4\sigma^{\rightarrow} - d^4\sigma^{\leftarrow}}{d^4\sigma^{\rightarrow} + d^4\sigma^{\leftarrow}} \overset{\text{twist-2}}{\approx} \frac{\alpha \sin\phi}{1 + \beta\cos\phi}$$

$$\alpha \propto \mathcal{I}\mathrm{m}\left(F_1\mathcal{H} + \xi G_M\tilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E}\right)$$

$$\mathcal{H}(\xi,t) = i\pi H(\xi,\xi,t) + \mathcal{P}\int_{-1}^{1} \mathrm{d}x\, \frac{H(x,\xi,t)}{x-\xi}$$

DVCS Beam Spin Asymmetry

$$F_1\mathcal{H} + \xi G_M\tilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E}$$



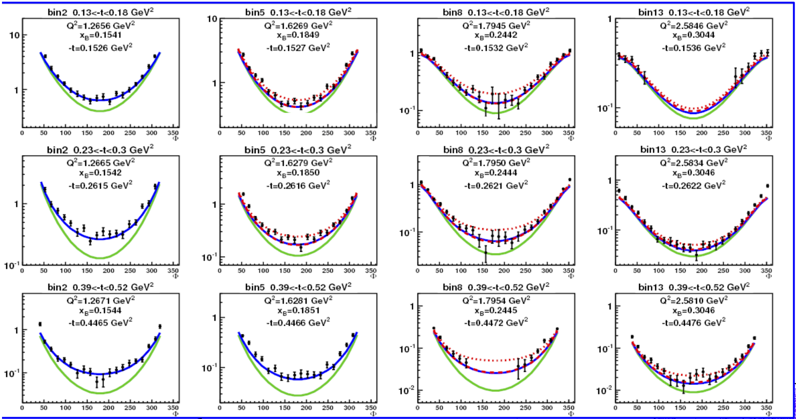
Precision in a large phase-space (x_B, Q^2, t)

Qualitative model agreement

quantitative constraints on parameters

F.-X. G. *et al.*, PRL **100** 162002 (2008)

DVCS Unpolarized Cross-Sections

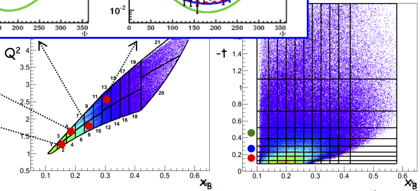


• $\frac{d^4\sigma_{ep\to ep\gamma}}{dQ^2 dx_B dt d\Phi}$ (nb/GeV^4)

BH — VGG (H only)

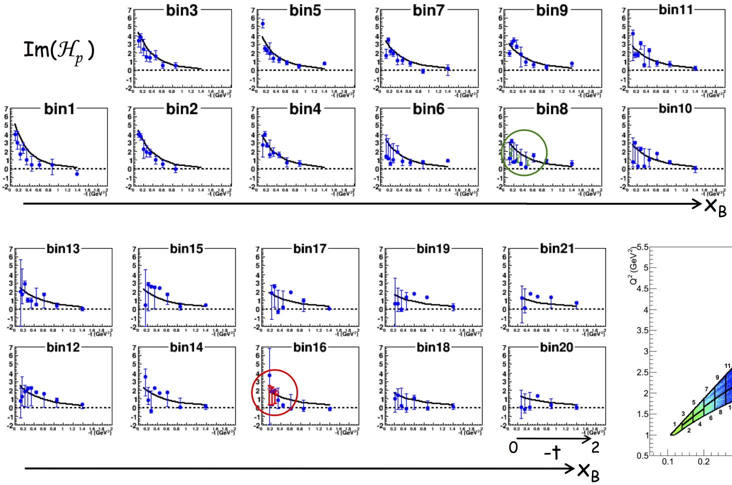
KM10 - - - KM10a

VGG : Vanderhaeghen, Guichon, Guidal *KM : Kumericki, Mueller*



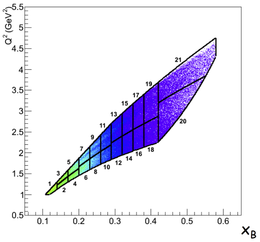
H.-S. Jo *et al.*, PRL **115** 212003 (2015)

Compton Form Factors



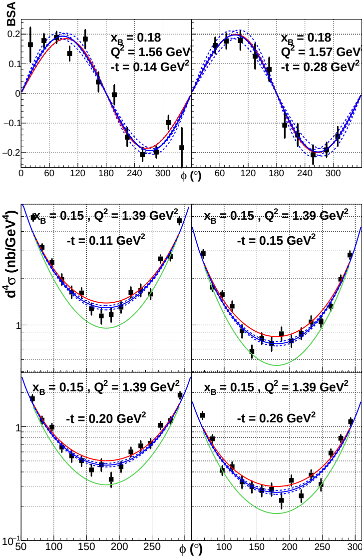
CFF fits (by M. Guidal) using:
the results of this work
Hall A σ and $\Delta\sigma$
CLAS A_{LU} and A_{UL} (published results)

— VGG predictions



The t-slope becomes flatter with increasing x_B:
valence quarks (higher x_B) at the center of the nucleon and sea quarks (small x_B) at its periphery

Global Fits to extract the D-term



Beam Spin Asymmetries

$$\mathrm{Im}\mathcal{H}(\xi,t) = \frac{r}{1+x}\left(\frac{2\xi}{1+\xi}\right)^{-\alpha(t)}\left(\frac{1-\xi}{1+\xi}\right)^{b}\left(\frac{1-\xi}{1+\xi}\,\frac{t}{M^2}\right)^{-1}$$

Unpolarized cross-sections

Use dispersion relation:

$$\mathrm{Re}\mathcal{H}(\xi,t) = D + \mathcal{P}\int dx\left(\frac{1}{\xi - x} - \frac{1}{\xi + x}\right)\mathrm{Im}\mathcal{H}(\xi,t)$$

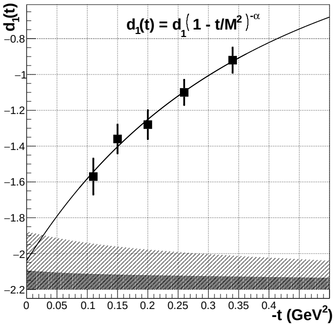
pure Bethe-Heitler

local fit + uncertainty range

resulting global fit

D-term Extraction

$$D^q\left(\frac{x}{\xi}, t\right) = \left(1 - \frac{x^2}{\xi^2}\right)\left[d_1^q(t)C_1^{3/2}\left(\frac{x}{\xi}\right) + d_3^q(t)C_3^{3/2}\left(\frac{x}{\xi}\right) + \cdots\right]$$



t-dependence of the D-term :

Dipole gives singular pressure at $r = 0$

Quadrupole implied by counting rules?

Exponential?

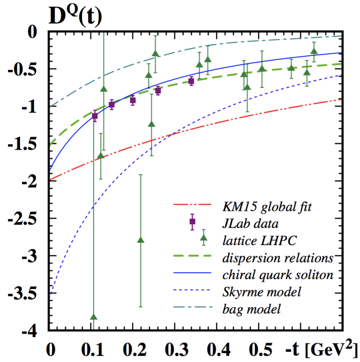
$\ldots$

$d_1(0) < 0$ dynamical stability of bound state

$d_1(0) = -2.04 \pm 0.14 \pm 0.33$

First Measurement of new fundamental quantity

D-term comparison with theory



Dispersion Relation Analysis

Chiral quark soliton model

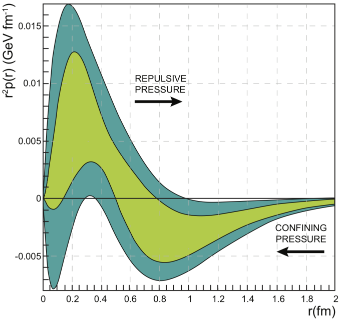
Lattice results LHPC

Global fit

M. V. Polyakov, P. Schweitzer arXiv:1805.06596 [hep-ph]

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N' \vert J^\mu_{\text{em}} \vert N \rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\text{C}$ $\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N' \vert J^\mu_{\text{weak}} \vert N \rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\text{grav}} \vert N \rangle$	$\longrightarrow$	$m = 938.272013(23)\,\text{MeV}/c^2$ $J = \frac{1}{2}$ $\boldsymbol{D} = \boldsymbol{?}$

Proton Pressure distribution results



The pressure at the core of the proton is $\sim 10^{35}$ Pa
About 10 times the pressure at the core of a neutron star

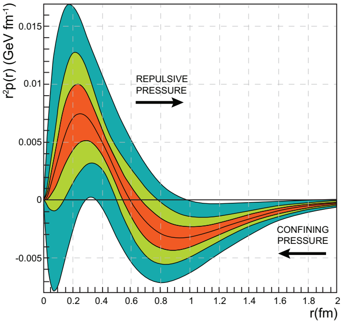
Positive pressure in the core (repulsive force)
Negative pressure at the periphery: pion cloud
Pressure node around $r \approx 0.6$ fm

Stability condition : $\int_0^\infty \mathrm{d}t\, r^2 p(r) = 0$

Rooted into Chiral Symmetry Breaking

World data fit
CLAS 6 GeV data

Proton Pressure distribution results



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Rooted into Chiral Symmetry Breaking

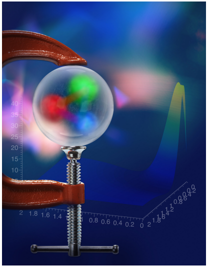
World data fit

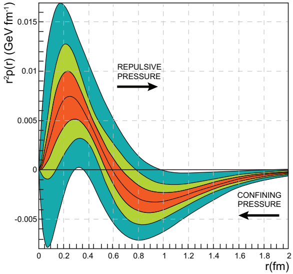
CLAS 6 GeV data

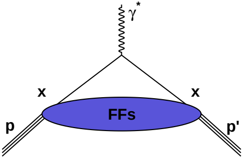
Projected CLAS12 data E12-16-010B

Summary and Outlook

- A new perspective on Exclusive Reactions Physics
- **First Ever Measurement of Gravitational Form Factors**
- Opens a new avenue to test confinement mechanism
- Partonic Energy Momentum Tensor
- Exciting times at the beginning of the 12 GeV high precision era!
- Will be an essential part of the EIC program as well





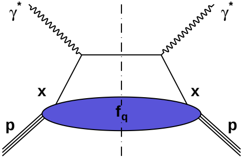


Form Factors - Fourier transform of transverse spatial distributions

$$\langle p'|\bar{\psi}_q(0)\gamma^+\psi(0)|p\rangle = \bar{N}(p')\left[F_1^q(t)\gamma^+ + F_2^q(t)i\sigma^{+\nu}\frac{\Delta_\nu}{2M}\right]N(p)$$

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t) \qquad \text{First x-moment}$$

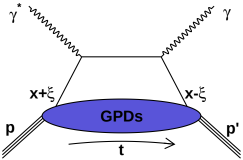
$$\int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t)$$



Parton longitudinal momentum fraction distributions

$$\frac{1}{4\pi}\int dy^-\, e^{ixp^+y^-}\langle p|\bar{\psi}_q(0)\gamma^+\psi(y)|p\rangle = f_q(x)$$

$$H^q(x,\xi=0,t=0) = f_q(x)$$



Generalized Parton Distributions

$$\frac{P^+}{2\pi}\int dy^-\, e^{ixP^+y^-}\langle p'|\bar{\psi}_q(0)\gamma^+(1+\gamma^5)\psi(y)|p\rangle$$

$$= \bar{N}(p')\left[H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t)i\sigma^{+\nu}\frac{\Delta_\nu}{2M}\right.$$

$$\left. + \tilde{H}^q(x,\xi,t)\gamma^+\gamma^5 + \tilde{E}^q(x,\xi,t)\gamma^5\frac{\Delta^+}{2M}\right]N(p)$$

CLAS12 GPD program

Number	Title	Contact	Days	Energy	Target
E12-06-108	Hard Exclusive Electroproduction of π^0 and η	Kubarovski	80	11	lH_2
E12-06-119	Deeply Virtual Compton Scattering	Sabatie	80	11	lH_2
E12-06-119	Deeply Virtual Compton Scattering	Sabatie	120	11	NH_3
E12-11-003	DVCS on Neutron Target	Niccolai	90	11	lD_2
E12-12-001	Timelike Compton Scat. & J/Ψ prod. in e^+e^-	Nadel-Turonski	120	11	lH_2
E12-12-007	Exclusive ϕ meson electroproduction	FXG	60	11	lH_2
C12-12-010	DVCS with a transverse target	Elouadrhiri	110	11	HD-ice
E12-16-010	DVCS with CLAS12 at 6.6 GeV and 8.8 GeV	Elouadrhiri	50+50	6.6 & 8.8	lH_2