Nuclear axial currents and selected applications to few-nucleon systems

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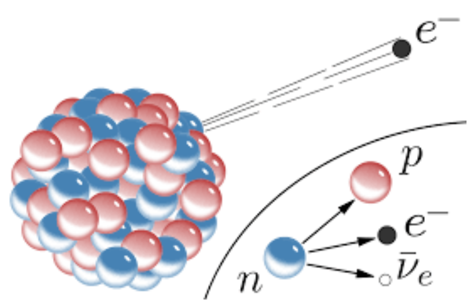
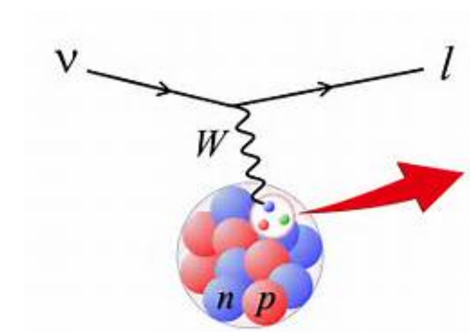
User Group Meeting
Jefferson Lab, Virginia
June 19, 2018

Nuclear weak processes



- β decays (single and double) important for
  - Precision tests of the Standard Model
  - g_A quenching (implications for $0\nu\beta\beta$)
  - Nuclear astrophysics (Sun chain reaction)

Well - known experimentally
excellent test for the theory

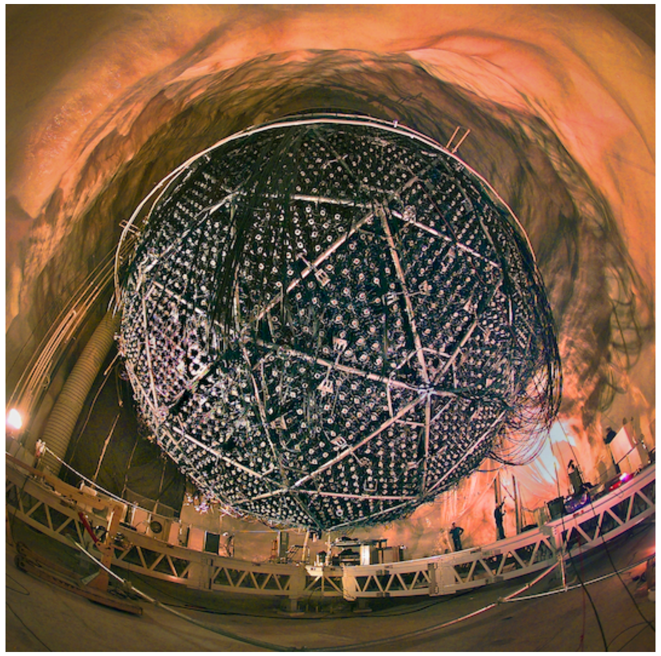


- $\nu-$nucleus scattering important for
  - Neutrino oscillations (SNO, …)
  - Leptonic CP violation
  - Nuclear astrophysics (Supernovae, ..)

Less - known experimentally
need of theoretical input

SNO

- Solar neutrino problem $\Phi_{^8\mathrm{B}}^{\mathrm{Expt.}} \sim \Phi_{^8\mathrm{B}}^{\mathrm{SSM}}$

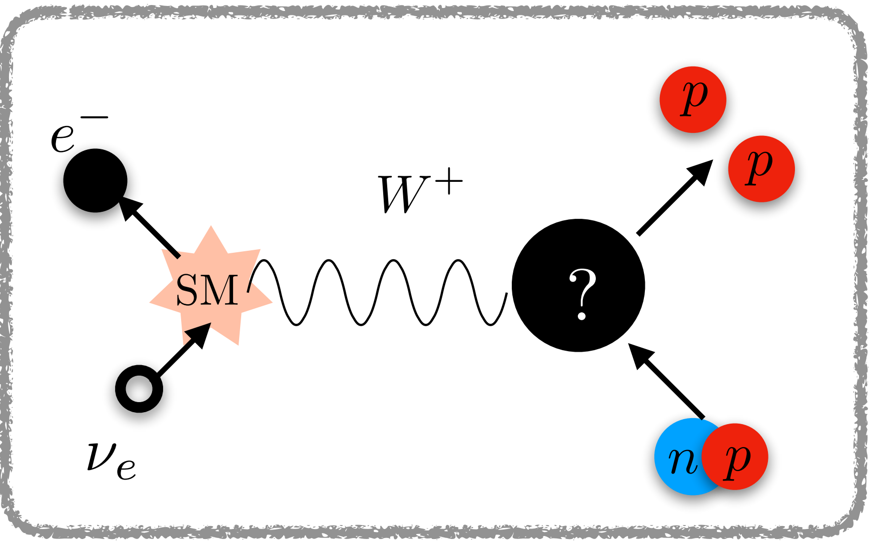


Heavy-water Cherenkov counter built to study neutrinos coming from $^8\mathrm{B}$ β- decay (5-15 MeV)

- CC : $\nu_e + d \to e^- + p + p$

  NC : $\nu_l + d \to \nu_l + n + p$

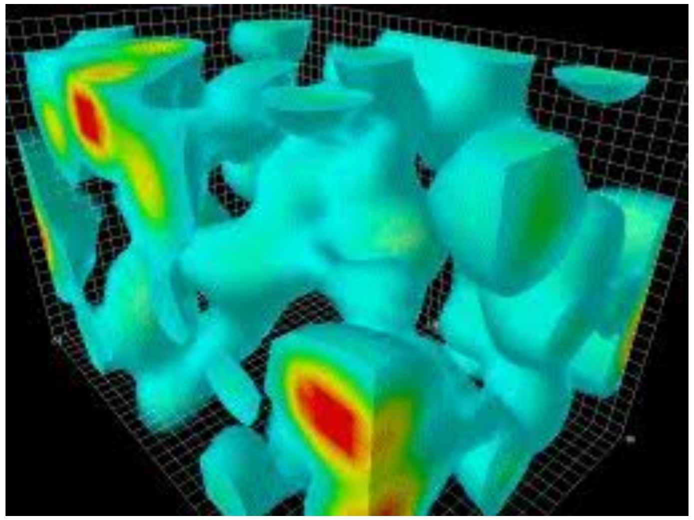
  ES : $\nu_l + e^- \to \nu_l + e^-$



Nuclear electroweak interactions?

Atomic nuclei are a complex quantum-many body systems of strongly interacting nucleons

Hadronic matrix elements difficult because of QCD



? ?

Nuclear Decay Half-Lives for Nuclei A = 3 - . . . .

Nuclide (β^- Decay)	Half-Life	Date	Nuclide (β^+ Decay)	Half-Life	Date
^{3}H	12.323 ± 0.020 yr	September, 2015	^{7}Be	53.22 ± 0.06 days	August, 2001
^{6}He	$806.89^{+0.25}_{-0.22}$ ms	August, 2015	^{8}B	770.3 ± 0.4 ms	July, 2015
^{8}He	119.0 ± 1.6 ms	September, 2015	^{9}C	126.5 ± 1.0 ms	July, 2015
^{8}Li	838.79 ± 0.36 ms	August, 2015	^{10}C	19.3015 ± 0.0017 sec	May, 2016
^{9}Li	177.7 ± 0.6 ms	August, 2015	^{11}C	20.3401 ± 0.0070 min	March, 2018
^{10}Be	$(1.51 \pm 0.06) \times 10^{6}$ yr	August, 2015	^{12}N	11.000 ± 0.016 ms	August, 2015
^{11}Li	8.74 ± 0.15 ms	August, 2015	^{13}N	9.967 ± 0.005 min	July, 2015

+ . . .

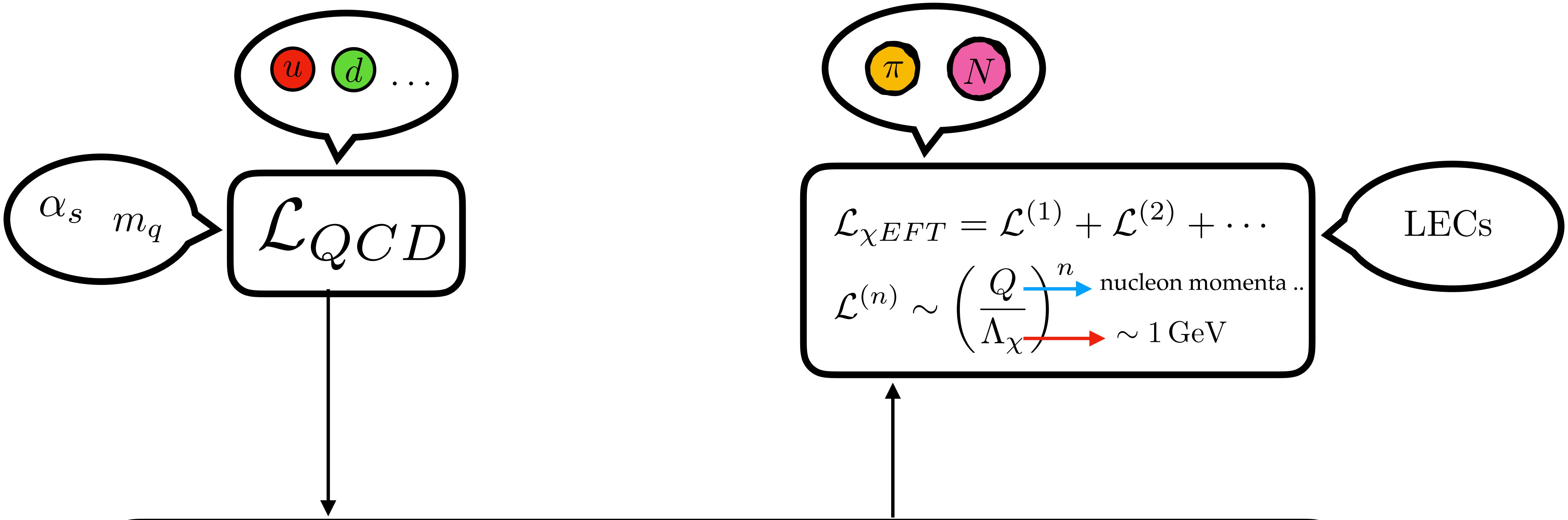
http://www.tunl.duke.edu/nucldata/HalfLife.shtml

Lattice QCD (non perturbative method)

Effective field theories (expansion in kinematic variables)

χ EFT

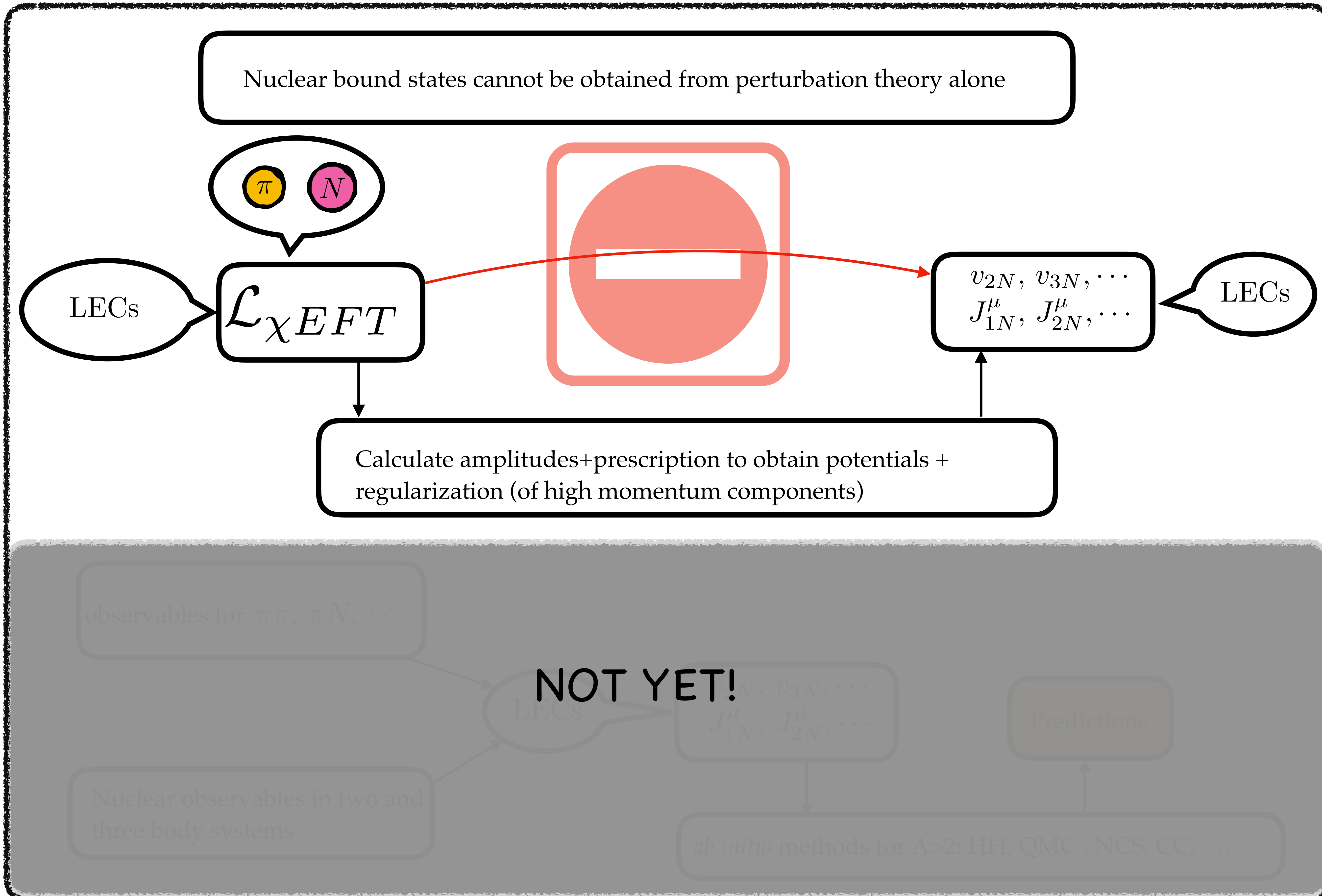
Build the most general Lagrangian with hadronic d.o.f. with the same exact symmetries and approximate symmetries of the underlying theory



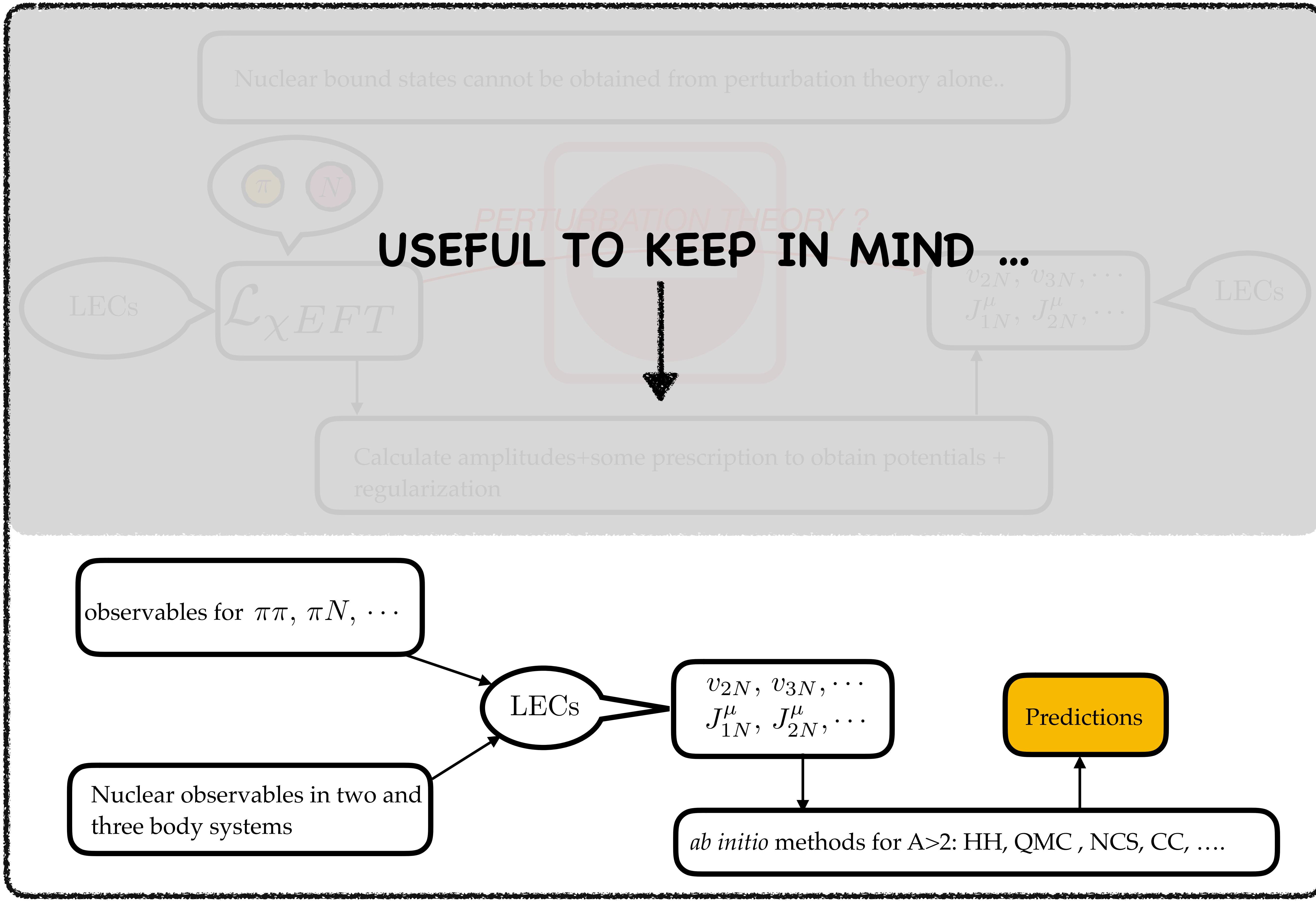
Approximate chiral symmetry requires the pion to couple to other pions and to nucleons by powers of its momentum

- S. Weinberg (1968-1979)

Nuclear χ EFT I



Nuclear χEFT II



Procedure

- Define a weak transition potential $v_5 = A^0_a \rho_{5,a} - \mathbf{A} \cdot j_{5,a}$ (similar to EM)
- We require the weak interaction potential to match the on shell scattering amplitude

$$T_5 = v_5 + v_5 \frac{1}{E_i - H_0 + i\epsilon} T_5$$

- Perturbative expansion in powers of the nucleon momenta

$$T_5 = T_5^{\rm LO} + T_5^{\rm NLO} + T_5^{\rm N2LO} + \cdots$$

$$v_5 = v_5^{\rm LO} + v_5^{\rm NLO} + v_5^{\rm N2LO} + \cdots$$

- Matching order by order

$$v_{5,a}^{\rm LO} = T_5^{\rm LO}$$

$$v_{5,a}^{\rm NLO} = T_5^{\rm NLO} - \left(v_{5,a}^{\rm LO} \frac{1}{E_i - E_I + i\epsilon} v^{\rm LO} + \text{permutations} \right)$$

$$\ldots$$

$\rightarrow$

$$\rho_{5,a} = \rho_{5,a}^{\rm LO} + \rho_{5,a}^{\rm NLO} + \rho_{5,a}^{\rm N2LO} + \cdots$$

$$\mathbf{j}_{5,a} = \mathbf{j}_{5,a}^{\rm LO} + \mathbf{j}_{5,a}^{\rm NLO} + \mathbf{j}_{5,a}^{\rm N2LO} + \cdots$$

Procedure

- A subtle point: operators derived are not unique!

$$v_{5,a}^{\rm LO} = T_5^{\rm LO}$$
$$v_{5,a}^{\rm NLO} = T_5^{\rm NLO} - \left(v_{5,a}^{\rm LO} \frac{1}{E_i - E_I + i\epsilon} v^{\rm LO} + \text{permutations} \right)$$
$$\ldots$$

Not uniquely defined

- Biblio

S. Weinberg (1990) TOPT; C. Ordonez and U. Van Kolck (1994-1996)

N. Kaiser et al. for nuclear potentials Feynman diagrams (1998)

S. Pastore et al. (2008-2011) for em currents, M. Piarulli et al. (2013) for em currents, TOPT

AB et al. (2016) for axial currents, TOPT

- Alternative approach using unitary transformations:

Epelbaum, Krebs, Meissner, et al. (1998-2017), for nuclear potentials, em and axial currents

Summary

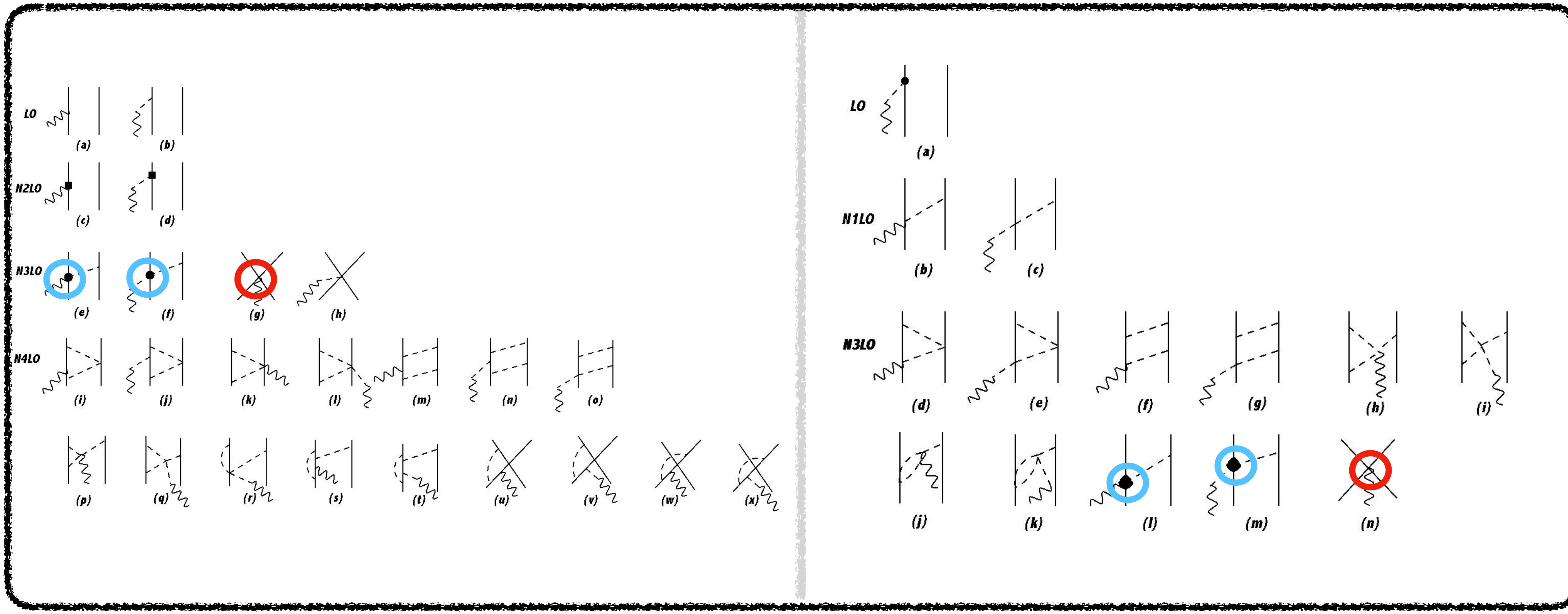
- Axial current and charge derived up to N4LO

- Self consistency checks:
  - ✓ Current conservation in the chiral limit
    $\mathbf{q} \cdot \mathbf{j}_{5,a} = [H, \rho_{5,a}] \longrightarrow \text{satisfied order by order}$
  - ✓ Renormalization of the axial charge (delicate cancellation of divergences)
  - ✓ Independence of the choice of the parametrization of the pion field

- Technical challenges:
  - "New" class of diagrams appear respect to EM currents, formalism had to be adjusted to include them
  - >1000 diagrams in TOPT (no software infrastructure available)

- Difference for some loop topologies with another recent derivation
  - H. Krebs, E. Epelbaum, and Meissner , Unitary transformation

Axial currents



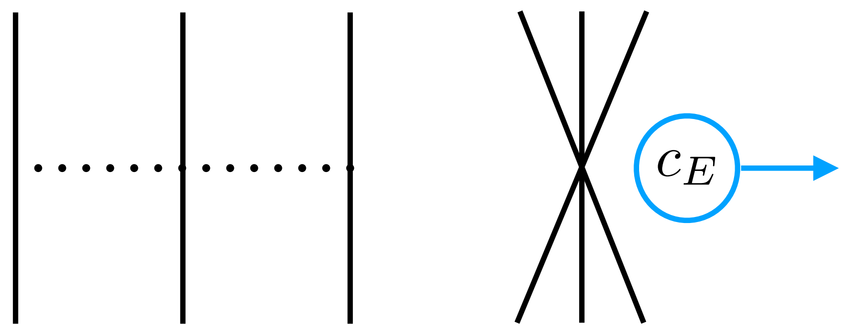
Strong and EM LECs partially known

1+4 "Weak" LECs ??

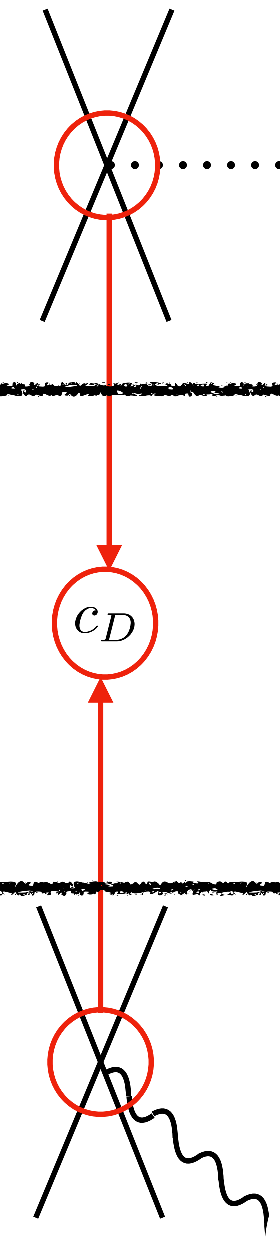
How do we fix them before ?

Actually…

LO chiral 3N force



Fixed to 3N
bindig energies



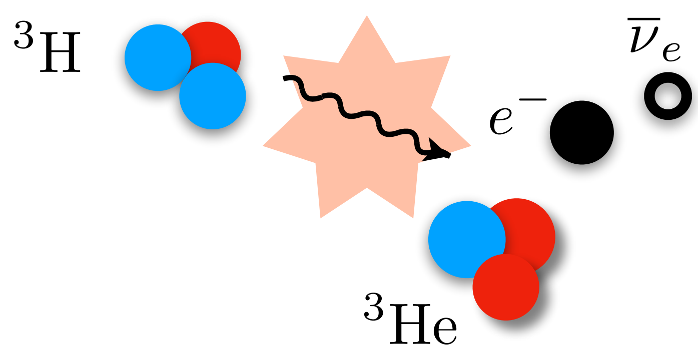
Gardestig and Phillips (2006)
Gazit et al. (2009), Marcucci et al. (2011), AB et al. (2017)
R. Schiavilla private communications (2018) (*correct relation*)

Axial current 2N contact term

Can be fixed with beta decays

Fix the LEC in axial current I

- We look at tritium beta decay rate (simplest beta decay), transition rate well known experimentally



$$(1+\delta_R)\, t f_V = \frac{K/G_V^2}{\langle \mathbf{F} \rangle^2 + f_A/f_V\, g_A^2 \langle \mathbf{GT} \rangle^2}$$

$\langle \mathbf{F} \rangle$: $\langle ^3\mathrm{He} || \rho^+ || ^3\mathrm{H} \rangle$

$\langle \mathbf{GT} \rangle$: $\langle ^3\mathrm{He} || \mathbf{j}_5^+ || ^3\mathrm{H} \rangle$

- Wave functions are obtained solving the 3-body Schrödinger equation
  (Pisa group specialty, Hyperspherical Harmonics, *ab initio* method)

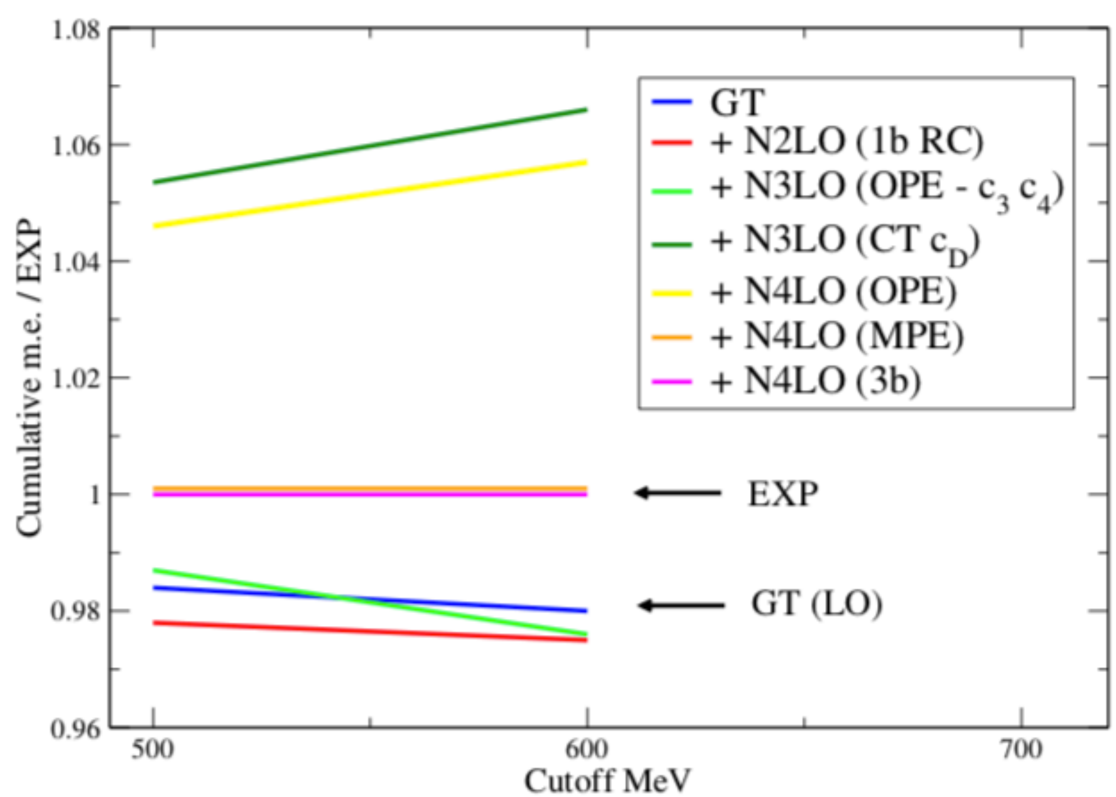
$$\hat{H}_{\chi\mathrm{EFT}}(c_D)|^3\mathrm{H}(c_D)\rangle = E_{^3\mathrm{H}}|^3\mathrm{H}(c_D)\rangle$$
$$\hat{H}_{\chi\mathrm{EFT}}(c_D)|^3\mathrm{He}(c_D)\rangle = E_{^3\mathrm{He}}|^3\mathrm{He}(c_D)\rangle$$

- Since the 3N potential depends on 2 unknown LECs we fix c_E to three-nucleon binding energies and we get a family of wave functions

Marcucci, Kievsky, Viviani, Rosati (1990-2018)

Fix the LEC in the axial current II

- Fitting of the triton GT matrix element using AV18+UIX and N4LO currents



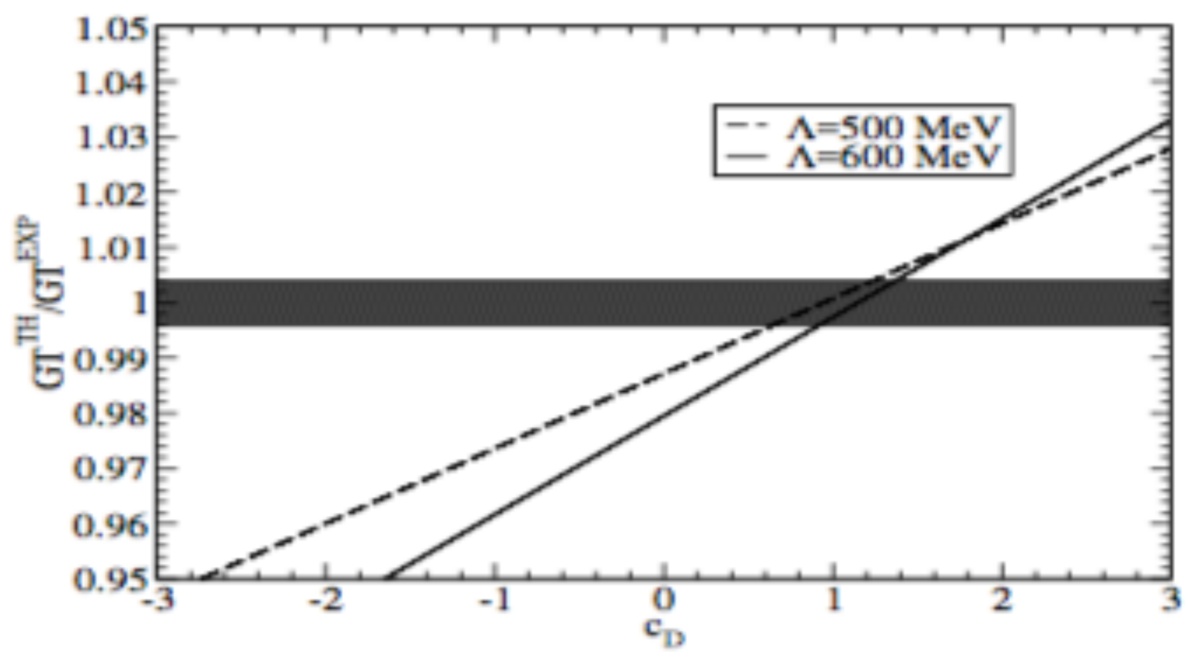
Courtesy of S. Pastore

AB, Schiavilla, Marcucci et al. (2016)

Fix the LEC in the axial current III

- Fitting of the triton GT matrix element $\langle ^3\mathrm{He}(c_D)|H_{\chi\mathrm{EFT}}(c_D)|^3\mathrm{H}(c_D)\rangle = f(c_D)$

Wave functions from
Entem-Machleidt
Chiral potential
N3LO currents



Courtesy of L. Marcucci

	Λ=500 MeV	Λ=600 MeV
c_D	$0.65 \div 1.24$	$0.92 \div 1.37$

AB, Schiavilla, Marcucci et al. (2017)
L. Marcucci et al. (2018)
AB, Schiavilla, Marcucci et al. (2018), in preparation

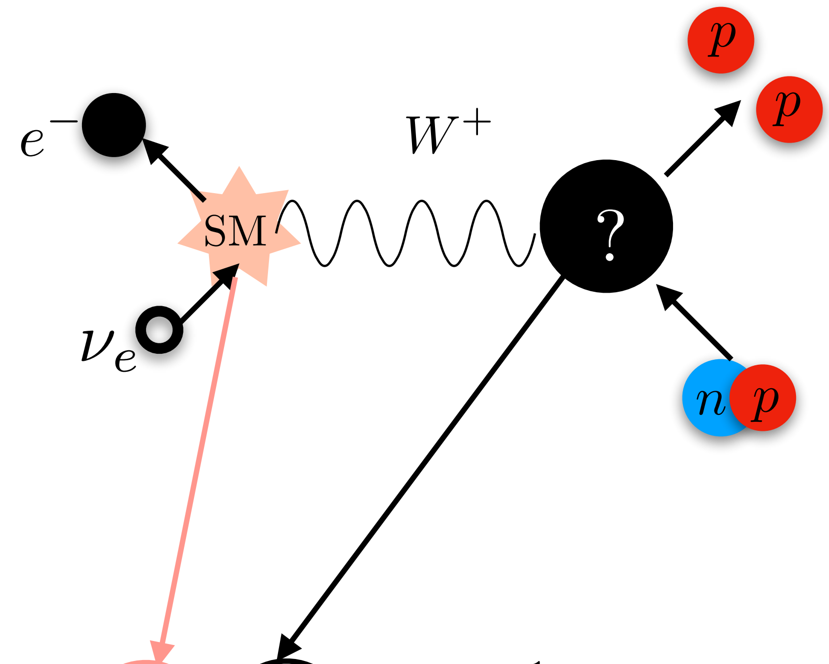
CONTRIBUTIONS

	500 MeV	600 MeV	
LO	0.9363	0.9322	→ Major contribution
N2LO	-0.569×10^{-2}	-0.457×10^{-2}	→ Relativistic correction to 1-body
N3LO(1π)	0.825×10^{-2}	0.043×10^{-2}	→ 2-body tree level, pion range
N4LO(loop)	-0.486×10^{-1}	-0.600×10^{-1}	→ Loop big effect
N4LO(3Bd)	-0.143×10^{-2}	-0.153×10^{-2}	→ 3-body currents, suppressed

Take home message

- LEC in the axial current determined using tritium beta decay
- Loop give important contribution
- Axial current acquires predictive power
- For axial charge as a first step we will assume LECs ~ 1
- Second not trivial application is low energy neutrino deuteron scattering

Neutrino deuterium



$E_\nu < 150\,\mathrm{MeV}$

chiral electroweak currents
(Vec. part fixed in Piarulli et al. 2013)

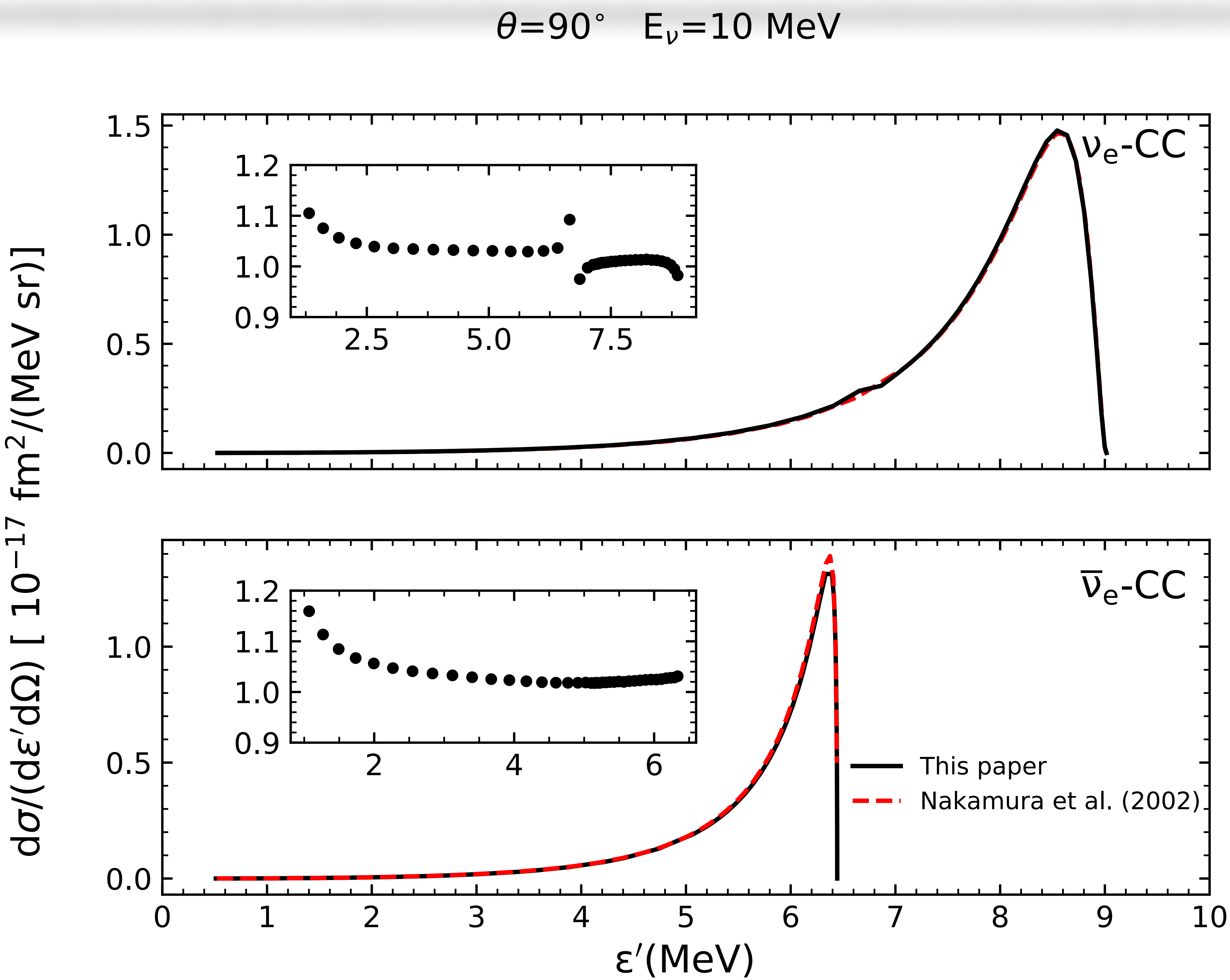
$$\left(\frac{d\sigma}{d\epsilon'\,d\Omega}\right)_{\nu\overline{\nu}} \propto G_F^2\, L_{\mu\nu}\, W^{\mu\nu}_{\mathrm{CC}} \rightarrow \frac{1}{3}\sum_{M,|f\rangle} \delta(E_i - E_f)\langle f|j^{\mu}_{\mathrm{CC}}|d,M\rangle\,\langle d,M|j^{\nu}_{\mathrm{CC}}|f\rangle^{\star}$$

Can be computed numerically

- Similar for neutral current process

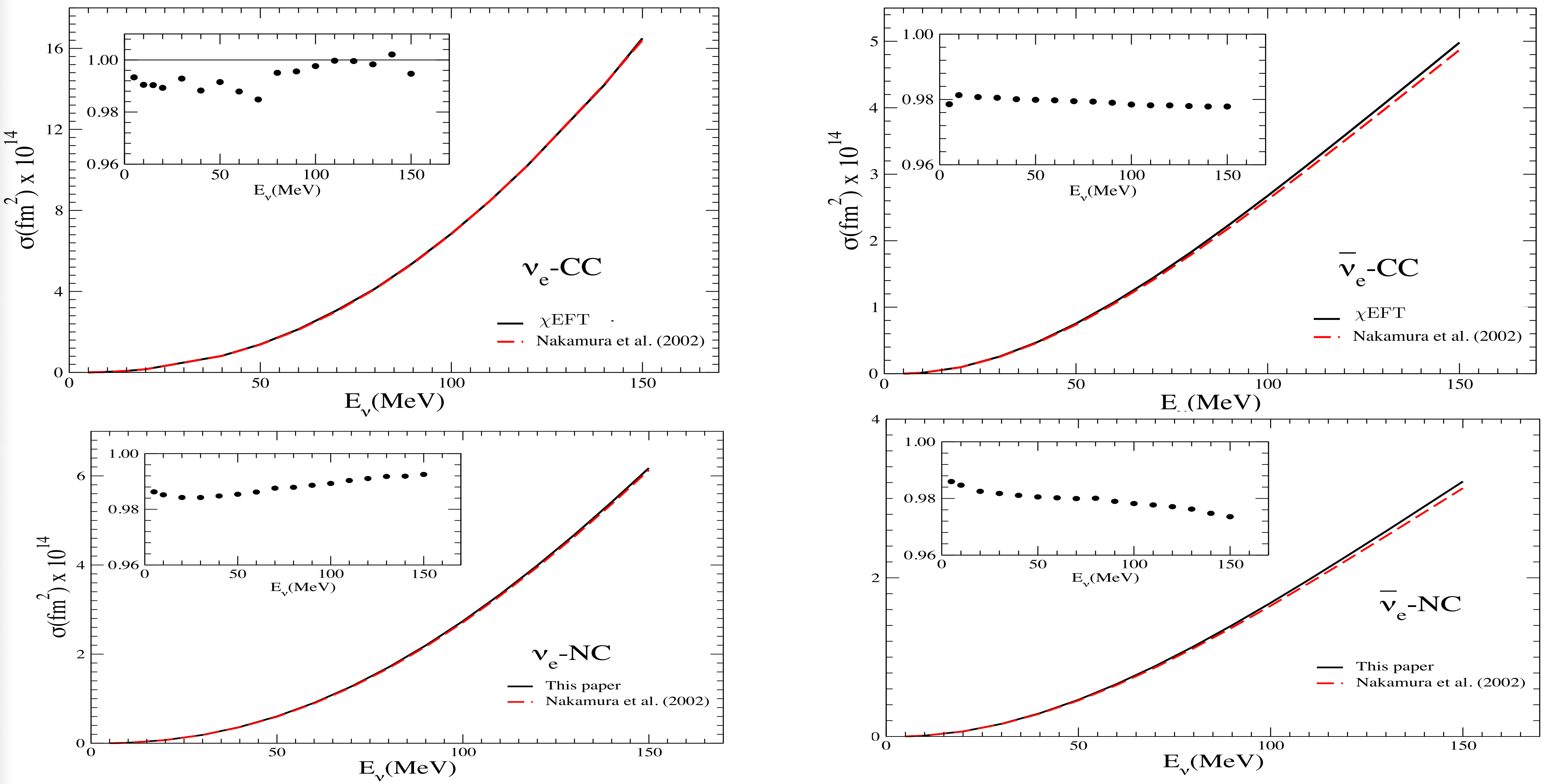
- Nakamura et al. (2002), Phenomelogical interactions
- Shen et al. (2011), Phenomenological interactions
- AB and Schiavilla 2017 (first chiral EFT calculation)

Results I: Differential cross sections



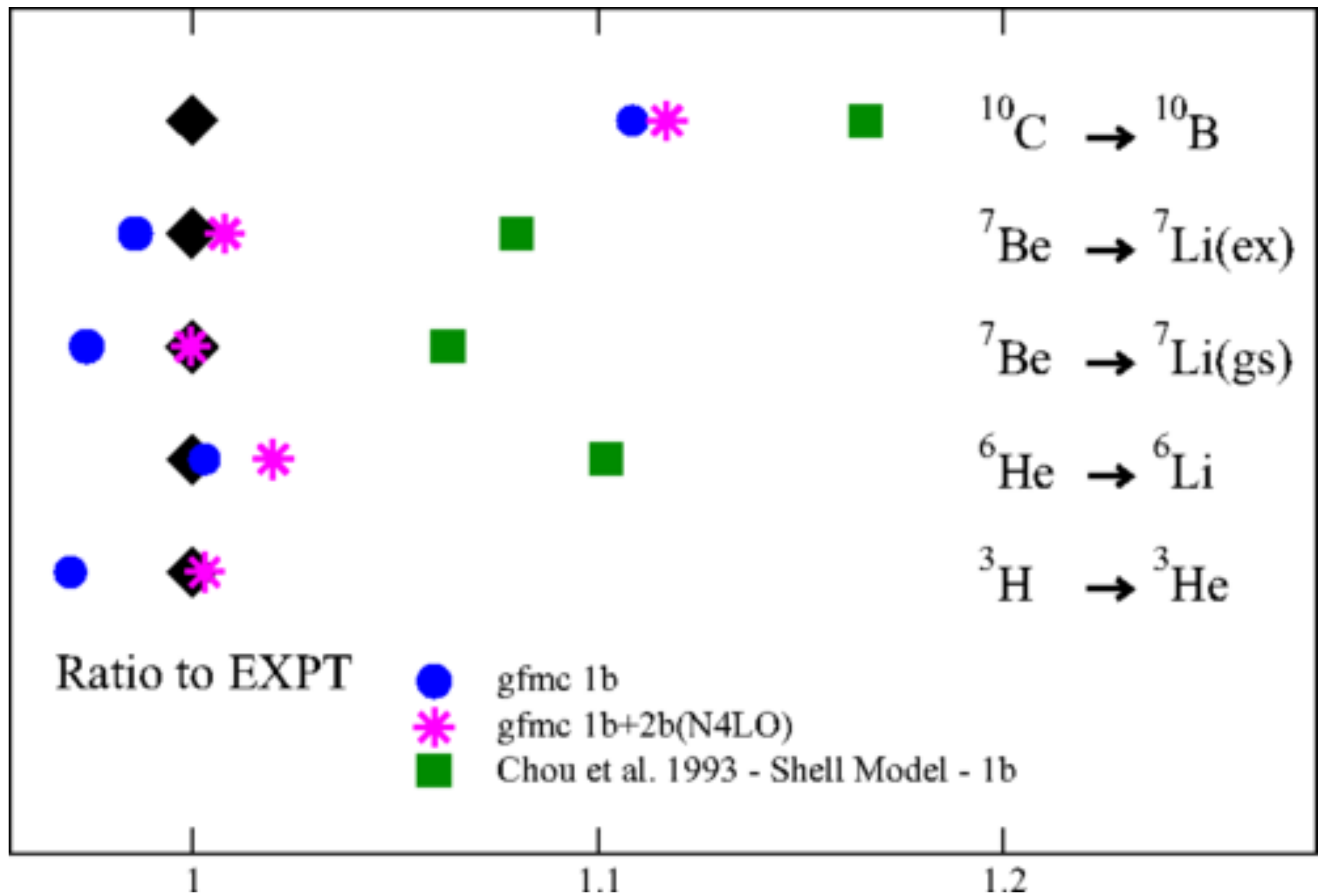
Results II: Total cross sections

$\Lambda = 500\,\text{MeV}$



for $\Lambda = 600\,\text{MeV}$ variation $\leq 1\%$

Currents used for beta decays



Electroweak currents, with phenomenological potentials, GFMC
2-body currents play a big role

Pastore et al. (2018)

Outlook

- ✓ Currents derived up to N4LO
- ✓ LEC in the axial current fixed with experimental GT matrix element
- ✓ Prediction for neutrino deuteron→confirm phenomenological approaches
- ✓ Hybrid calculations in beta decays denote big effect of two-body currents
- ○ Systematic study of theoretical uncertainty
- ○ LQCD to determine the LEC in the axial current (validation)
  (Savage et al. 2017)
- ○ Refine calculations for beta decays/include delta in the currents (?)
  (Goity et al. 2012)

Collaborators

L. Girlanda (INFN Lecce, Italy)

L. E. Marcucci (Univ. Pisa, Italy)

A. Kievsky (INFN Pisa, Italy)

S. Pastore (LANL, Washington University, USA)

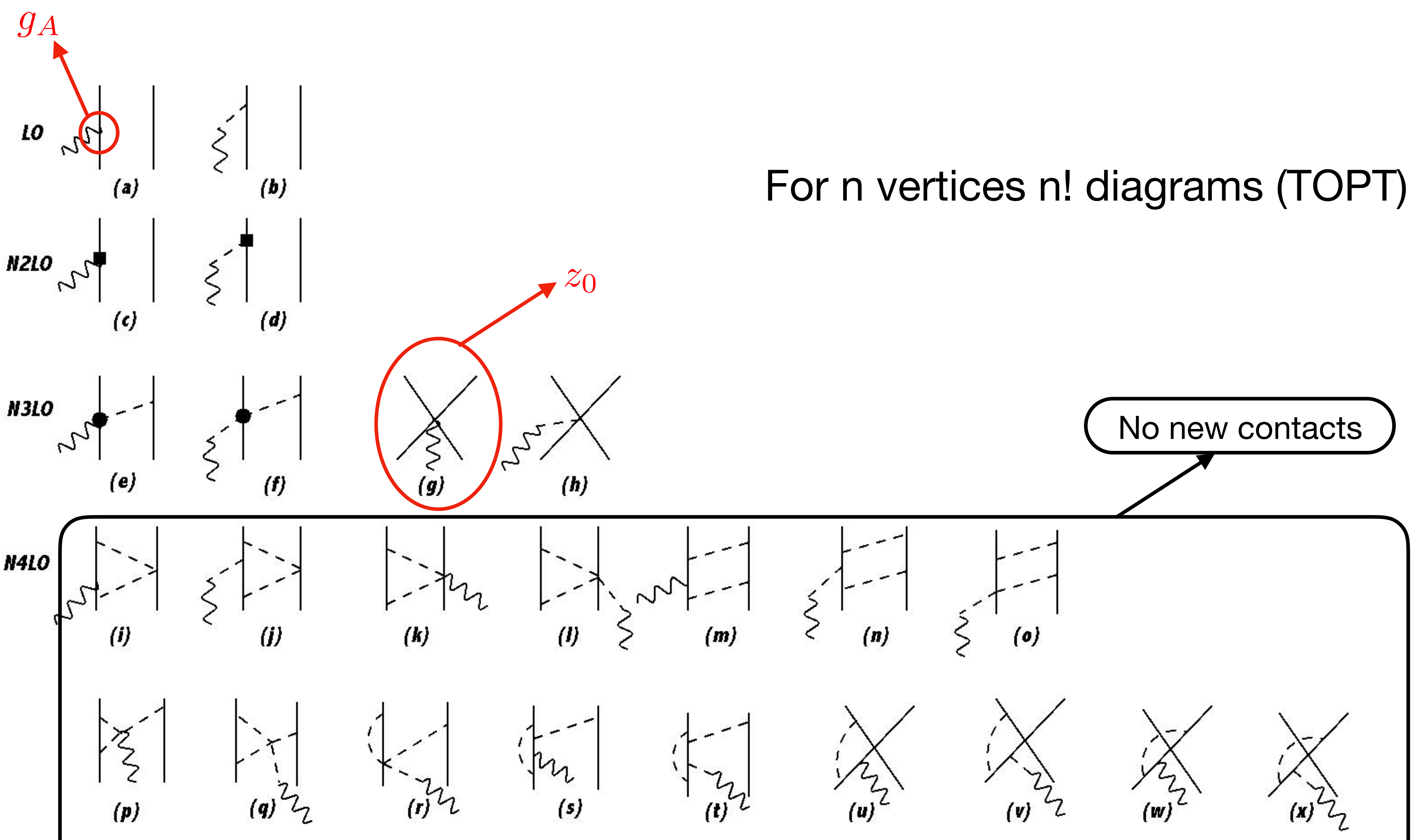
R. Schiavilla (JLab/ODU, USA) → Advisor

M. Viviani (INFN Pisa, Italy)

Thank you!

Collaborators

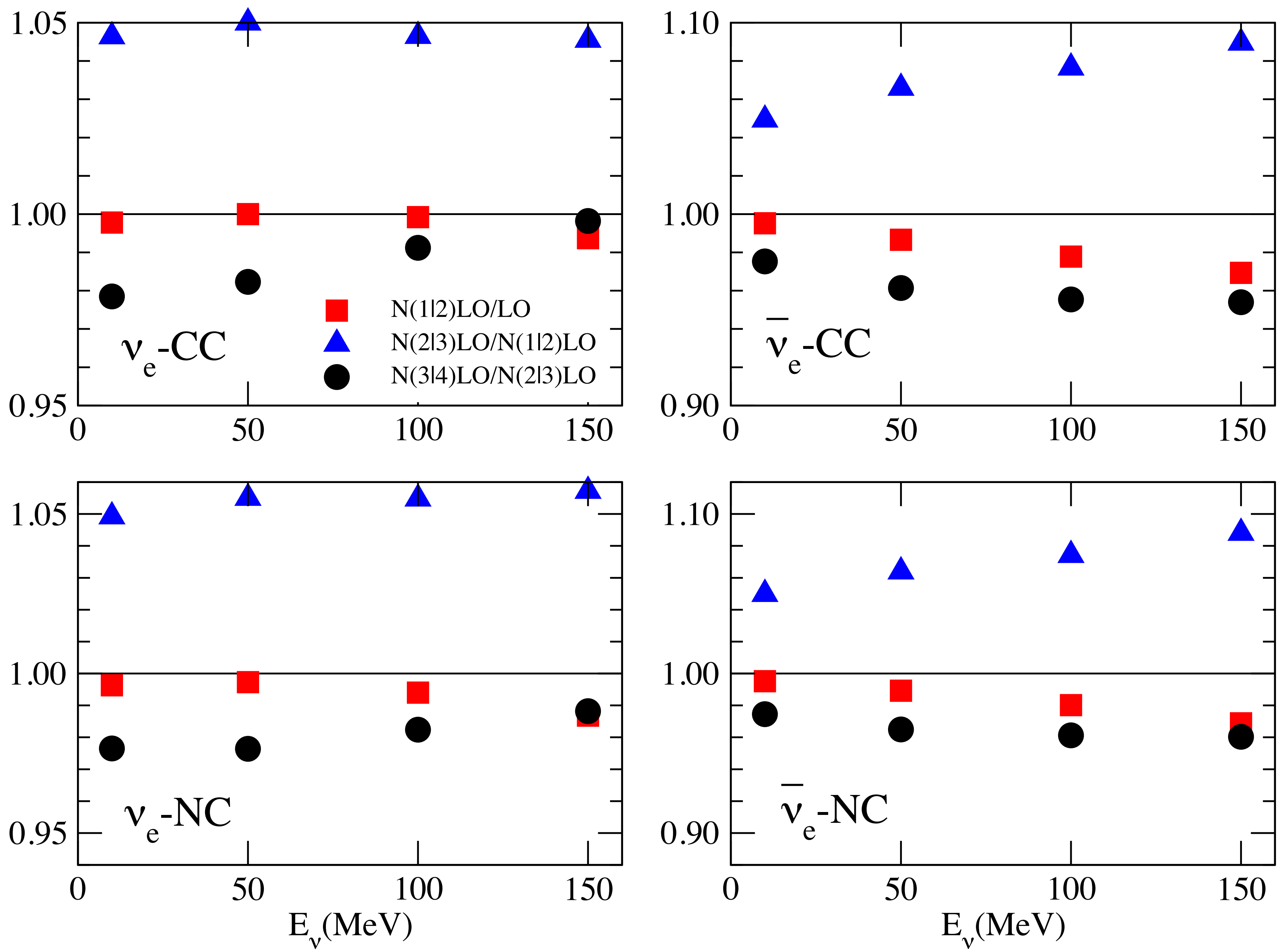
Axial current



For n vertices n! diagrams (TOPT)

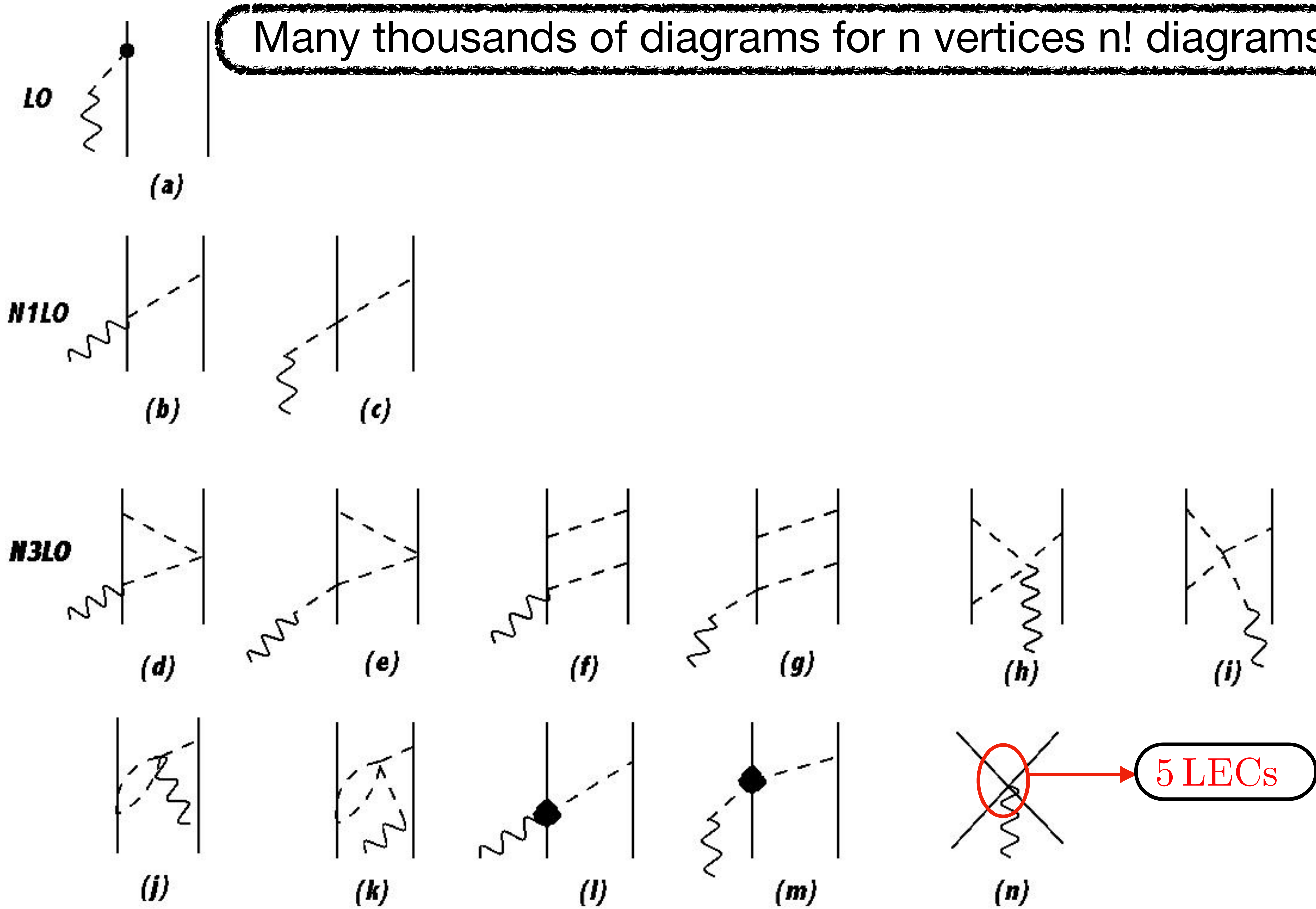
- Loop diagrams have been regularized (dim reg)

Convergence pattern



Axial charge

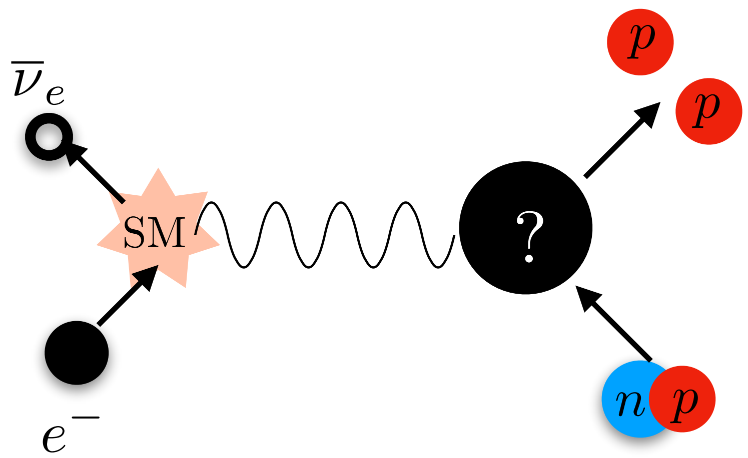
Many thousands of diagrams for n vertices n! diagrams (TOPT)

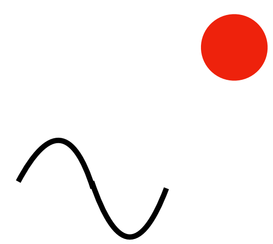


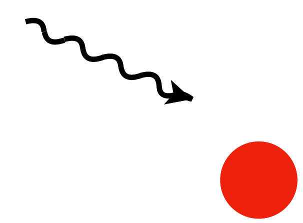
Loop diagrams have been regularized (dim reg) divergences are reabsorbed by contact terms and higher order πN couplings

Comparison with others ?

\beta decays Saori



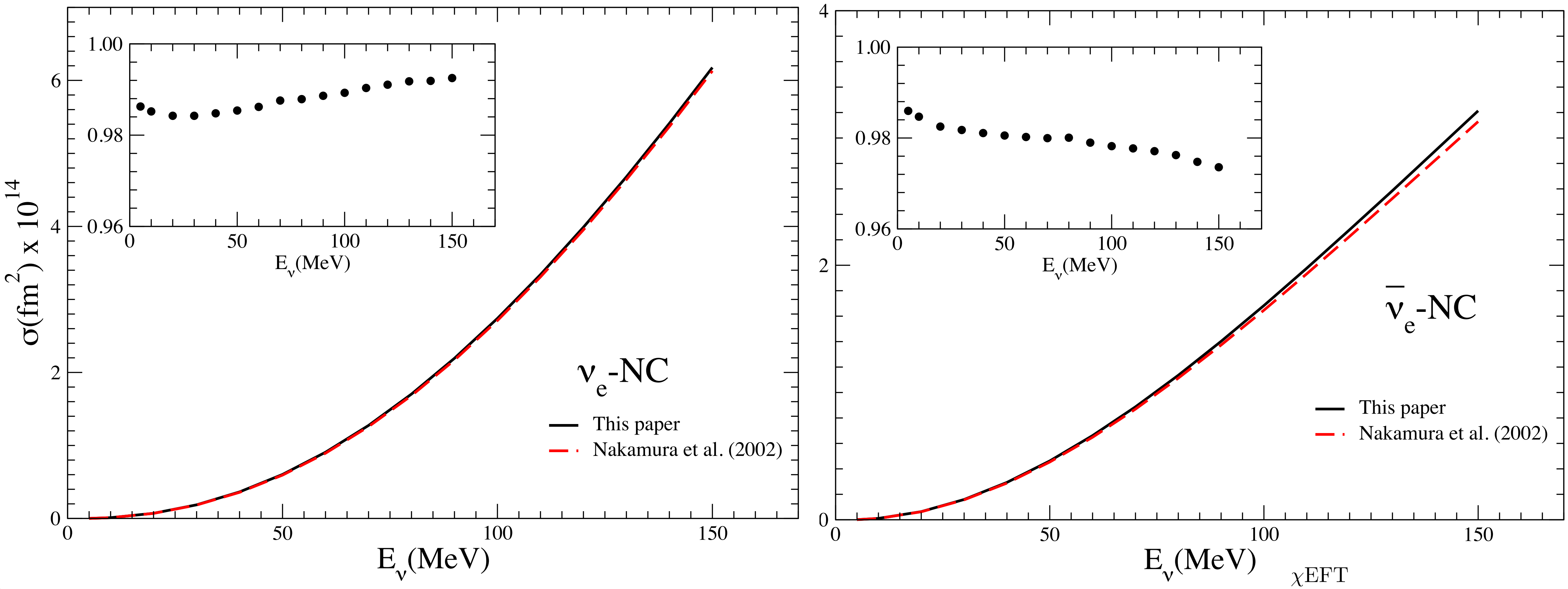




^{3}He

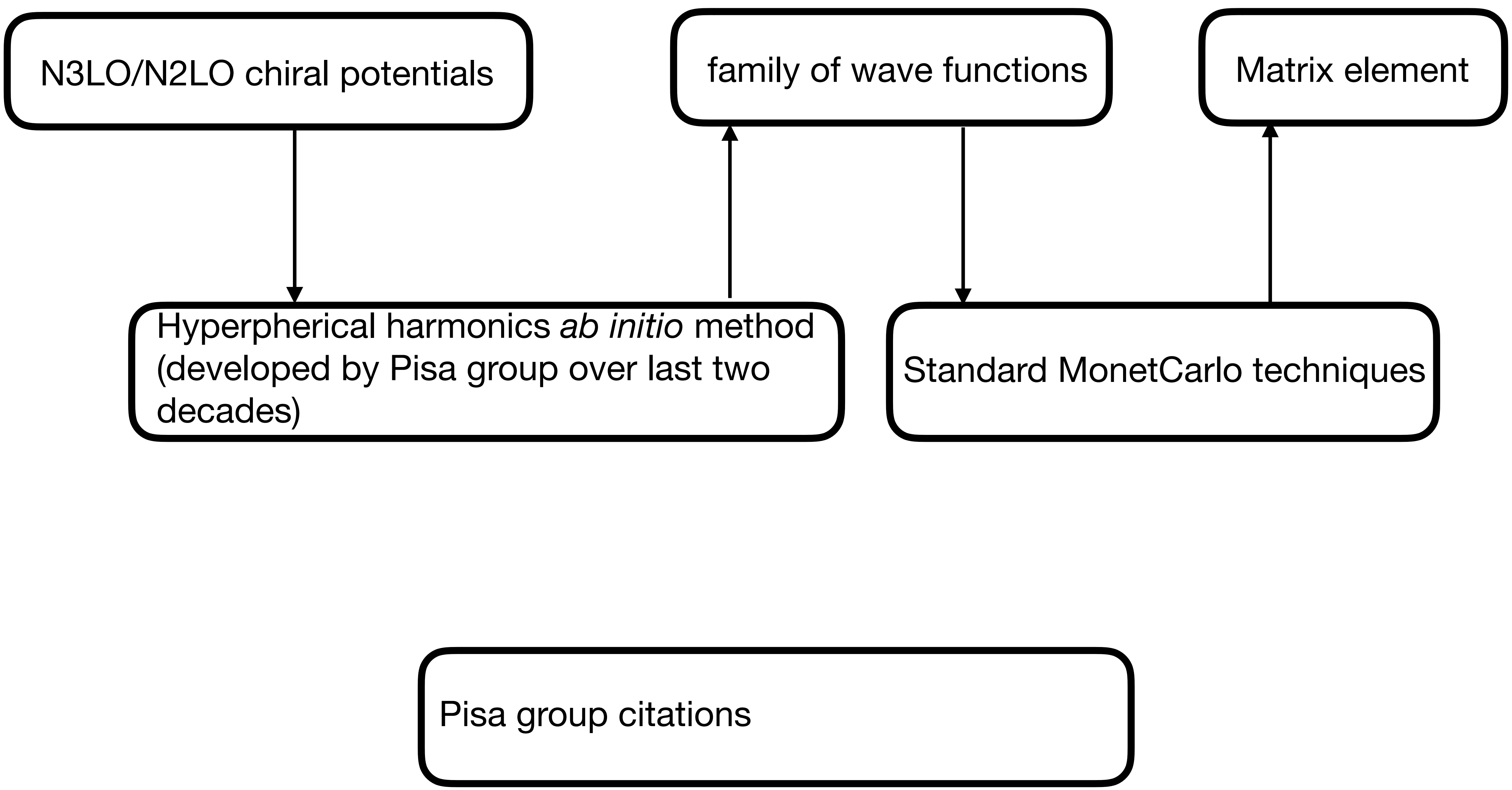
Results: Neutral currents

$\Lambda = 500\,\mathrm{MeV}$



for $\Lambda = 600\,\mathrm{MeV}$ variation $\leq 1\%$

Triton calculation



Λ	500 MeV	600 MeV
LO	0.9363 (0.9224)	0.9322 (0.9224)
N2LO	$-0.569(-0.844)\times10^{-2}$	$-0.457(-0.844)\times10^{-2}$
N3LO(OPE)	$0.825(1.304)\times10^{-2}$	$0.043(7.517)\times10^{-2}$
N4LO(Loop)	$-0.486(-0.650)\times10^{-1}$	$-0.600(-0.852)\times10^{-1}$
N4LO(3Bd)	$-0.143(-0.183)\times10^{-2}$	$-0.153(-0.205)\times10^{-2}$

- N3LO/N2LO full chiral
- AV18/UIX hybrid

- Loop give not negligible contribution
- Preliminary three-body currents seem negligible

500 MeV

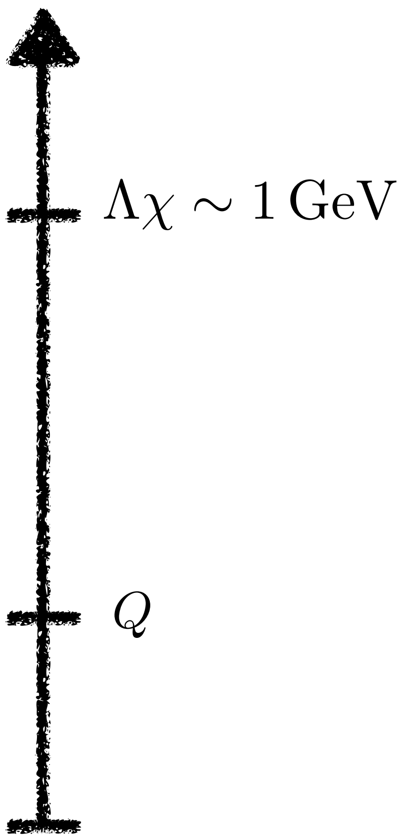
Axial currents: from amplitudes to nuclear operators II

- Matching holds for on shell scattering amplitude
- Matching is not unique →Nuclear operators are not unique
  - iterations of LS depend on the off-the-energy-shell extension of lower order currents and potentials
  - Not unique operators should be related by a unitary transformation (no general proof at the moment)

Outlook

Backup slides

χ Effective field theory



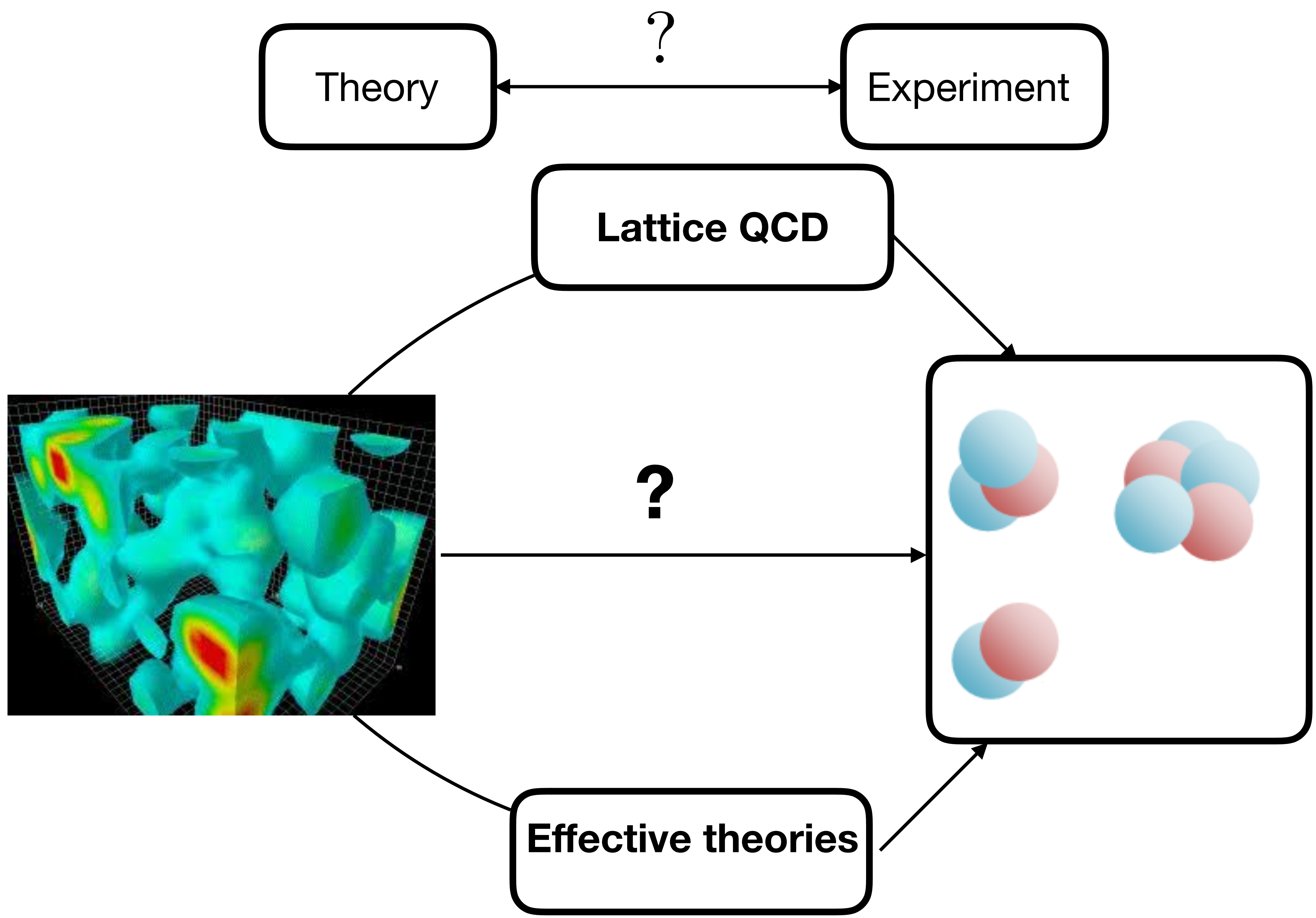
- Pion and nucleons degrees of freedom
- Exact
- Lagrangian is an expansion in powers of $Q/\Lambda\chi$

$$\mathcal{L}_{\chi\mathrm{EFT}} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$$

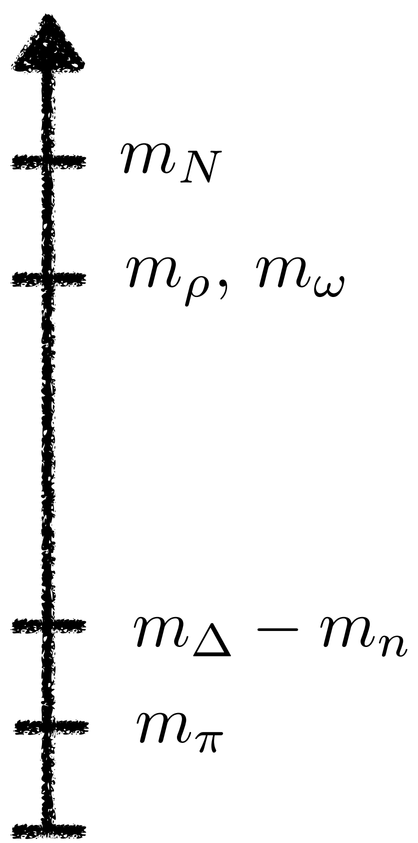
- Low energy constants (encode our ignorance)

Experiments (past and present) Lattice QCD (near future)

Theory approaches



Effective field theories

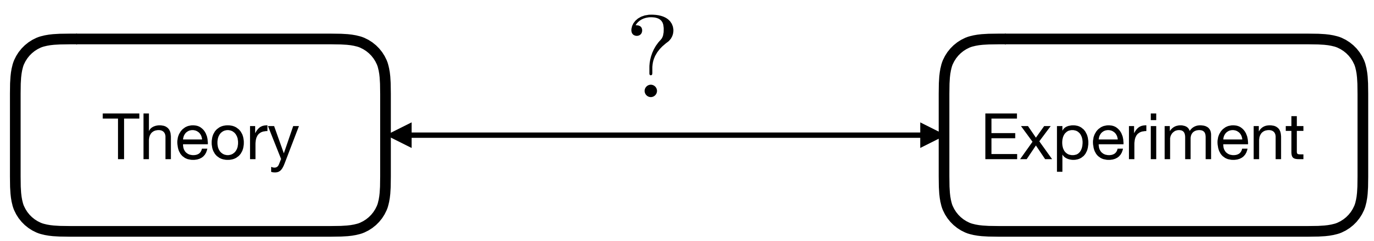


Low energy approximations of an underlying theory

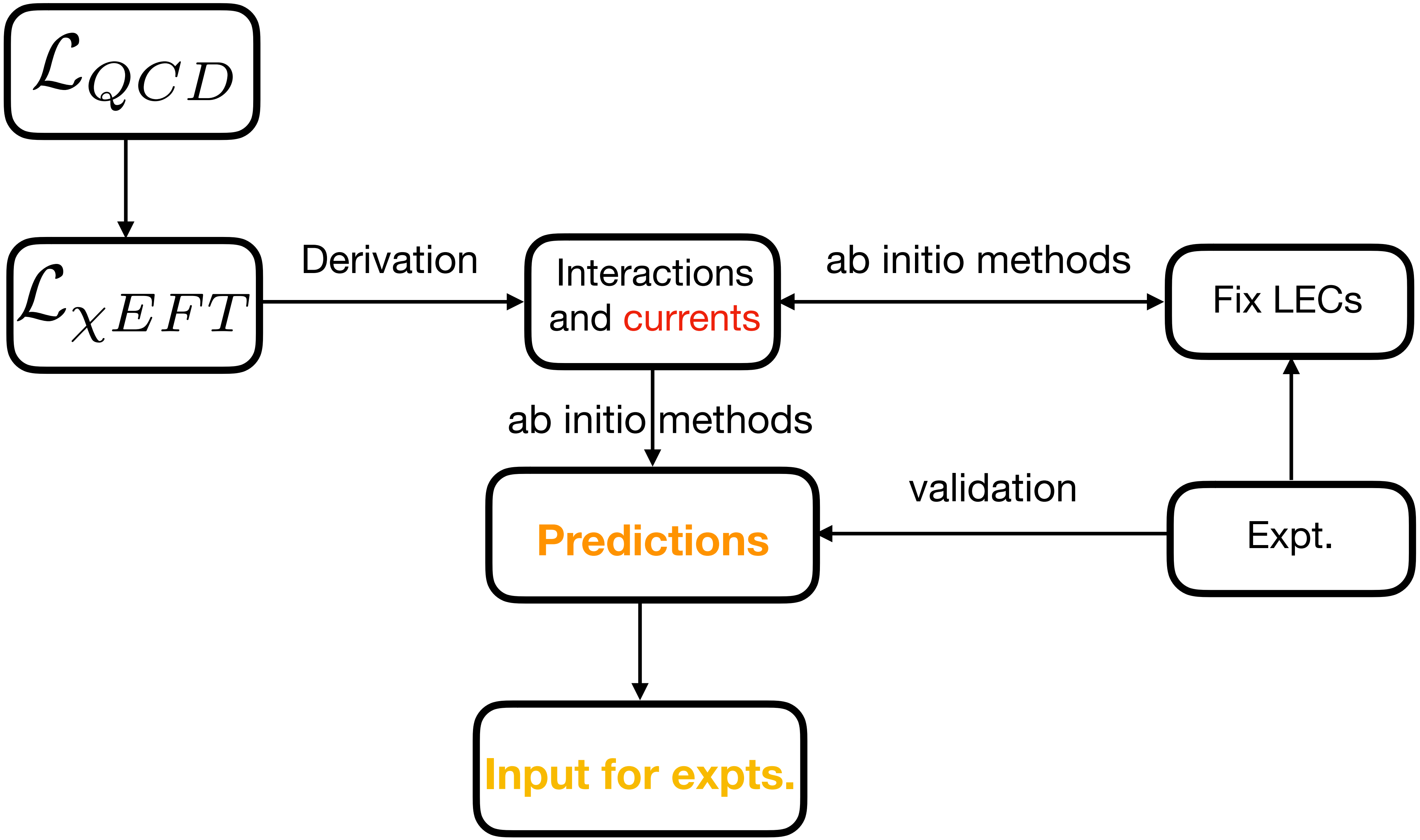
- Exploit separation of scales
- Build the most general Lagrangian consistent with the symmetries of the underlying theory

- Weinberg 1979

Strategy



Pipeline


$\mathcal{L}_{QCD}$
$\mathcal{L}_{\chi EFT}$
Derivation
Interactions and currents
ab initio methods
Fix LECs
ab initio methods
validation
Predictions
Expt.
Input for expts.