Nucleon structure investigations from Lattice QCD

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JLab Users Group Meeting

June 18, 2018

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- ★ F. Steffens[5]
- ★ A. Vaquero [2]
- ★ Ch. Wiese [5]

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2. Cyprus Institute
3. Adam Mickiewicz University
4. Temple University
5. DESY Zeuthen

OUTLINE OF TALK

A. Motivation

B. Introduction to Lattice QCD

C. Selected nucleon structure studies

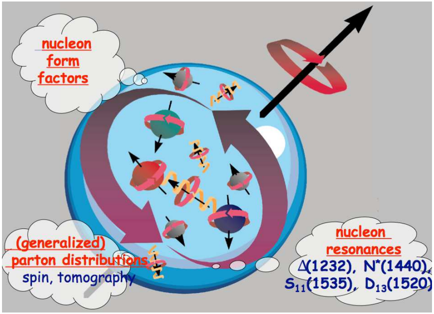
   1. E/M form factors
   2. Direct access to PDFs

D. Discussion

A

Motivation

Lattice QCD meets Nature



[M. Vanderhaeghen]

- ★ **4-D discretization, ab initio formulation of QCD**
- ★ **Make contact with well-known experimental data**
- ★ **Provide input for quantities not easily accessible in experiments**
- ★ **Guide New Physics searches**

Synergy with JLab program

★ Hall A:

E1207108 Precision Measurement of the Proton Elastic Cross Section at High Q^2
E1207109 Proton Form Factor Ratio Measurements at 13 and 15 $(GeV/c)^2$
E1209016 Measurement of Neutron EM Form Factor Ratio G_E^n/G_M^n at High Q^2
E1209019 Measurement of the Neutron Magnetic Form Factor up to $Q^2=18.0(GeV/c)^2$
E1210103 Measurement of the F_2^n/F_p^2, d/u Ratios and A=3 EMC Effect in DIS
E1214009 Ratio of the electric form factor in the mirror nuclei 3He and 3H
E1214011 Proton and Neutron Momentum Distributions in A=3 Asymmetric Nuclei

★ Hall B:

E1206109 The Longitudinal Spin Structure of the Nucleon
E1206112 Probing Proton's Quark Dynamics in Semi-Inclusive Pion Production at 12 GeV
E1206113 The Structure of the Free Neutron at Large x-Bjorken
E1211106 High Precision Measurement of the Proton Charge Radius

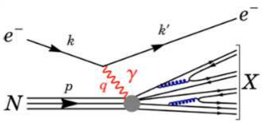
★ Hall C:

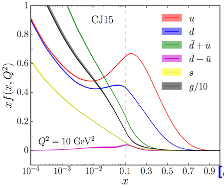
E1210002 Precision measurements of F_2 at large x in the resonance region and beyond
E1211009 G_E^n at Q^2 up to 7 $(GeV/c)^2$ from the Reaction $d(e,e'n)p$ via Recoil Polarimetry

★ Theory Group:

- Baryon and Meson Form Factors
- Proton radius and spin
- x-dependent PDFs (pseudo-PDFs, good Lattice Cross Sections)
- and many more

Probing Nucleon Structure





CJ15 PDFs

[A. Accardi et al., arXiv:1602.03154]

Parton Distribution Functions

- ★ Universal quantities for the description of the nucleon's structure (non-perturbative nature)
- ★ 1-dimensional picture of nucleon structure
- ★ Distribution functions are necessary for the analysis of Deep inelastic scattering data
- ★ Parametrized in terms of off-forward matrix of light-cone operators
- ★ Not directly accessible in a euclidean lattice

PDFs on the Lattice

★ **Moments of PDFs easily accessible in lattice QCD**

- **one relies on OPE to reconstruct the PDFs**
- **reconstruction difficult task:**

  $\Rightarrow$ **signal-to-noise is bad for higher moments**
  $\Rightarrow$ **n $>$ 3: operator mixing (unavoidable!)**

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★ **Alternative approaches to access PDFs**

- Hadronic Tensor [K.F. Liu, Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
- Compton amplitude and OPE [A. Chambers et al. (QCDSF), [arXiv:1703.01153]

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★ **Direct access to PDFs**

- quasi-PDFs [X. Ji, arXiv:1305.1539]
- pseudo-PDFs [A. Radyushkin, arXiv:1705.01488]
- good lattice cross-sections [Y-Q Ma&J. Qiu, arXiv:1709.03018]

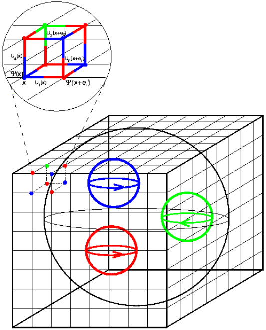
B

Introduction to Lattice QCD

Lattice formulation of QCD

★ **Space-time discretization on a finite-sized 4-D lattice**
- **Quark fields on lattice points**
- **Gluons on links**

★ **Serves as a regulator**
- **UV cut-off: inverse lat. spacing**
- **IR cut-off: inverse lattice size**



courtesy: USQCD

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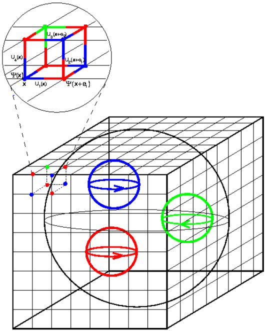


courtesy: USQCD

Technical Aspects

- ★ **Parameters (define cost of simulations):**
  - **quark masses (aim at physical values)**
  - **lattice spacing (ideally fine lattices)**
  - **lattice size (need large volumes)**
- ★ **Discretization not unique:**
  - **Wilson, Clover, Twisted Mass, Staggered, Overlap, Domain Wall**
  - **Mixed actions**

Nucleon on the Lattice

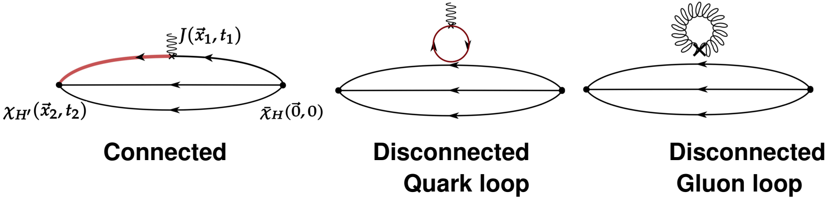
Multi-component task:

1. **Computation of 2pt- and 3pt-functions**

$$\text{2pt}: \quad G(\vec{q},t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\mathbf{\Gamma}^{\mathbf{0}}_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\overline{J}_\beta(0)\rangle$$

$$\text{3pt}: \quad G_{\mathcal{O}}(\mathbf{\Gamma}^{\kappa},\vec{q},t) =$$

$$\sum_{\vec{x}_f,\vec{x}} e^{i\vec{x}\cdot\vec{q}} e^{-i\vec{x}_f\cdot\vec{p}'}\,\mathbf{\Gamma}^{\kappa}_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\mathcal{O}(\vec{x},t)\overline{J}_\beta(0)\rangle$$



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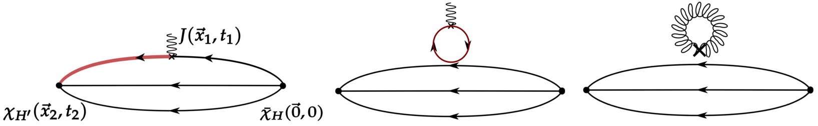


Connected | **Disconnected Quark loop** | **Disconnected Gluon loop**

2. Construction of optimized ratio

$$R^\mu_{\mathcal{O}}(\Gamma,\vec{q},t) = \frac{G_{\mathcal{O}}(\Gamma,\vec{q},t)}{G(\vec{0},t_f)} \times \sqrt{\frac{G(-\vec{q},t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(-\vec{q},t)G(-\vec{q},t_f)}}$$

Nucleon on the Lattice

3. Computation of 2pt- and 3pt-functions

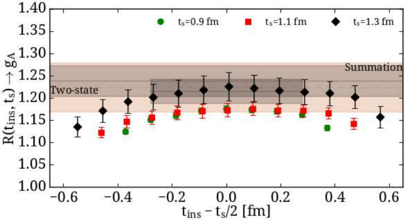
Plateau Method:

$$R_{\mathcal{O}}(\Gamma, \vec{q}, t) \underset{t-t_i \to \infty}{\overrightarrow{t_f - t \to \infty}} \Pi^{\mu}(\Gamma, \vec{q})$$

2-state fits:

Summation Method:

$$\sum_t R_{\mathcal{O}}(\Gamma, \vec{q}, t) \underset{t_f \to \infty}{\rightarrow} \mathcal{C} + \Pi^{\mu}(\Gamma, \vec{q}) \cdot t_f$$



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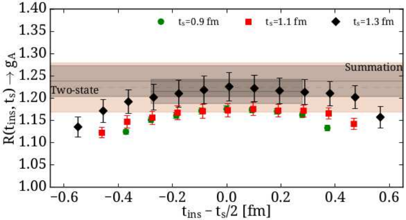

4. Renormalization:

connection to experiments

$$\Pi^{R}(\Gamma, \vec{q}) = Z_{\mathcal{O}}\, \Pi(\Gamma, \vec{q})$$

Simpler case!

Nucleon on the Lattice

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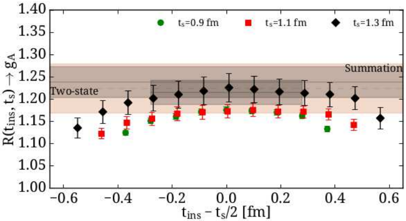

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$$\Pi^R(\Gamma,\vec{q}) = Z_{\mathcal{O}}\,\Pi(\Gamma,\vec{q})$$

Simpler case!

5. Extraction of form factors

e.g. Axial current:

$$A^3_\mu \equiv \bar{\psi}\,\gamma_\mu\,\gamma_5\,\frac{\tau^3}{2}\,\psi \Rightarrow \bar{u}_N(p')\left[\mathbf{G_A}(\mathbf{q^2})\,\gamma_\mu\,\gamma_5 + \mathbf{G_p}(\mathbf{q^2})\,\frac{q_\mu\,\gamma_5}{2\,m_N}\right]u_N(p)$$

Inherited Uncertainties

Laborious effort to eliminate uncertainties

Statistical errors significantly increase with:

- ★ decrease of pion mass
- ★ increase of momentum transfer Q^2 between source-sink
- ★ increase of source-sink separation ($T_{\rm sink}$)

Systematic

- ★ Cut-off Effects due to finite lattice spacing
- ★ Finite Volume Effects
- ★ Contamination from other hadron states
- ★ Chiral extrapolation for unphysical pion mass
- ★ Renormalization and mixing

C

Nucleon Structure

1

E/M Form Factors

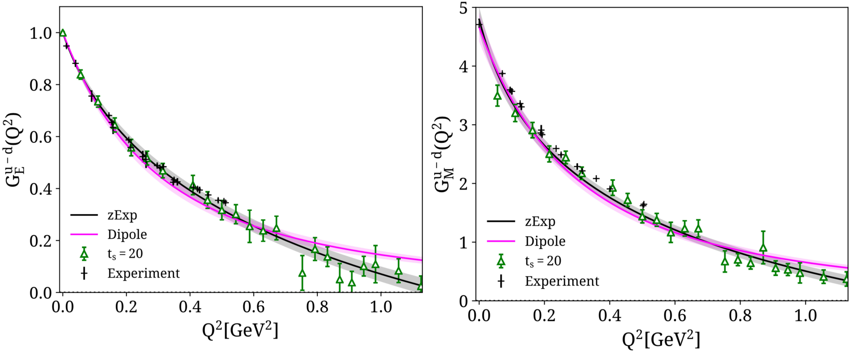
Nucleon EM Form Factors

New ensemble @ physical point:

★ N_f=2+1+1 **Twisted Mass fermions with clover term**

★ $64^3 \times 128$, a=0.08fm

★ **excited states investigations:** T_{sink}=1 − 1.6fm



★ G_E: **Very good agreement with experimental data**

★ G_M: T_{sink}=1.6fm **important for improvement in small Q^2 slope**

Nucleon EM Form Factors

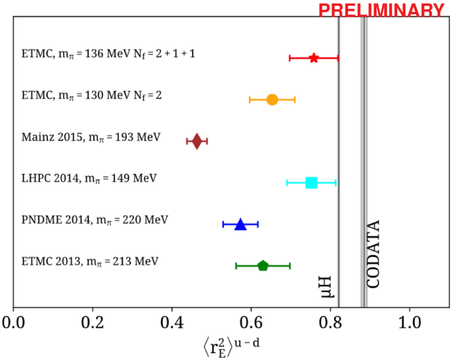
Study extended to:

- ★ Connected isoscalar flavor combination u+d
- ★ Disconnected contributions (very noisy, comput. costly)

⇩

allow flavor decomposition

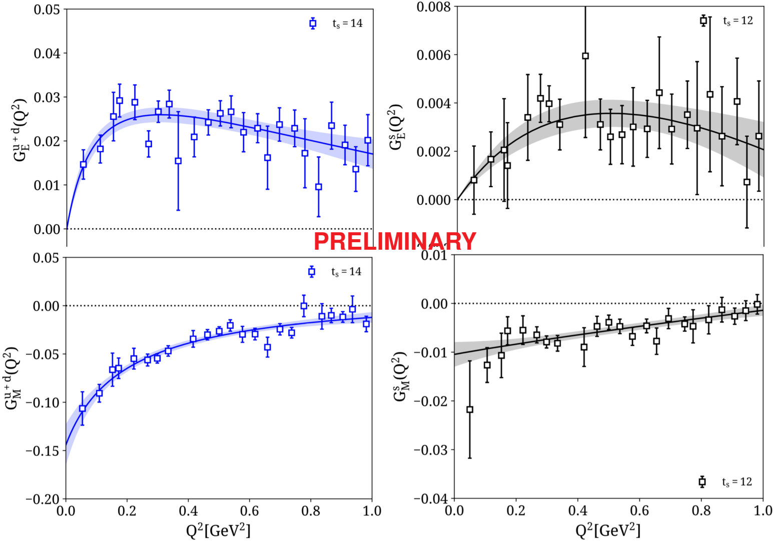
- ★ Magnetic moment
- ★ Charged radii



Target:

- ★ increase of statistics
- ★ refine fit methods

Nucleon EM Form Factors (disconnected)



★ **Clear signal using special techniques, e.g., hierarchical probing**

C

Nucleon Structure

2

Direct Access to PDFs

Access of PDFs on a Euclidean Lattice

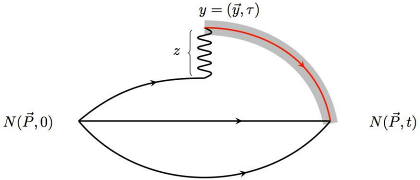
Novel direct approach:

[X.Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539]

★ Matrix elements of spatial operators with a Wilson line (length z)

★ Nucleon is boosted with momentum in spatial direction (e.g. z)

★ Renormalization more complicated than other nucleon quantities



Contact with light-cone PDFs:

★ Difference between quasi-PDFs and light-cone PDFs:
$\mathcal{O}\left(\frac{\Lambda^2_{QCD}}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$

★ Matching procedure (in LaMET) necessary
(provided that momenta are finite but feasibly large for lattice)

Parameters of Calculation

[C. Alexandrou et al. (ETMC), arXiv:1803.02685]

★ N_f=2 Twister Mass fermion action with clover term

★ Ensemble parameters:

β=2.10,	$c_{\rm SW}$=1.57751, a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$ $m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $m_\pi L = 2.98(1)$

★ Nucleon Momentum & Measurements

$P = \frac{6\pi}{L}$ (0.83 GeV)			$P = \frac{8\pi}{L}$ (1.11 GeV)			$P = \frac{10\pi}{L}$ (1.38 GeV)		
Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$
γ_3	100	9600	γ_3	425	38250	γ_3	655	58950
γ_0	50	4800	γ_0	425	38250	γ_0	655	58950
$\gamma_5\gamma_3$	65	6240	$\gamma_5\gamma_3$	425	38250	$\gamma_5\gamma_3$	655	58950

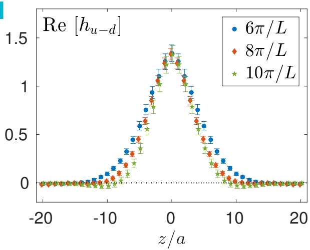
★ Excited states investigation:

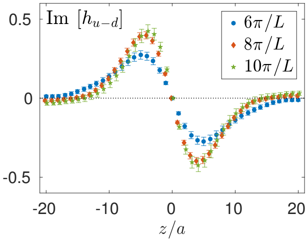
$T_{\rm sink}/a = 8,\ 10,\ 12\ (T_{\rm sink} = 0.75,\ 0.094,\ 1.13{\rm fm})$

Bare Nucleon Matrix Elements

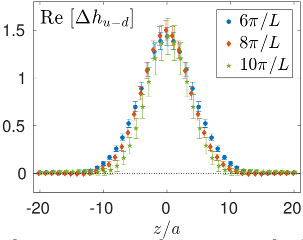
[C. Alexandrou et al. (ETMC), arXiv:1710.06408]   [C. Alexandrou et al. (ETMC), arXiv:1803.02685]

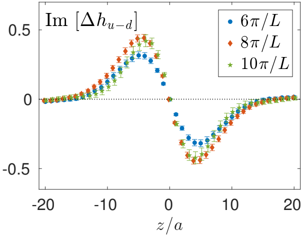
Unpolarized





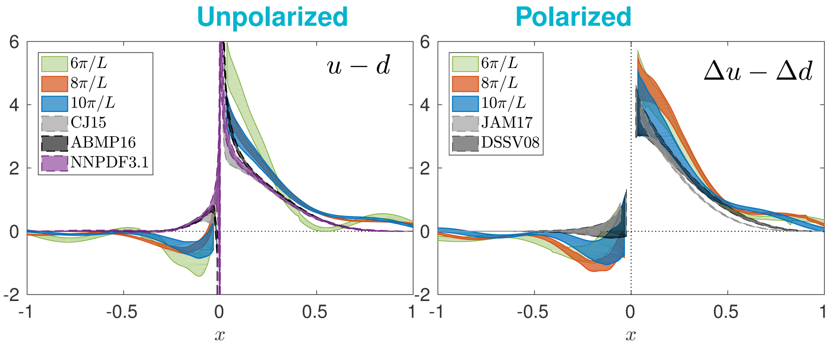
Polarized





★ **Addressing systematic uncertainties is imperative**

Towards light-cone PDFs



- ★ **Increasing momentum approaches the phenomenological fits**
- ★ **Saturation for** $p{=}8\pi/L$ **and** $p{=}10\pi/L$
- ★ $0{<}x{<}0.5$: **Lattice polarized PDF overlap with phenomenology**
- ★ **Negative** x **region: anti-quark contribution**

D

Discussion

Discussion

Lattice QCD has become a crucial part in understanding nucleon structure

- ★ **Simulations at the physical point**
- ★ **Addressing open questions (proton radius and spin**
- ★ **Investigations of more complicated quantities (quasi-PDfs, pseudo-PDFs, good LCSs)**

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Lattice QCD has become a crucial part in understanding nucleon structure

- ★ Simulations at the physical point
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Significant progress also in:

- ★ Spectroscopy
- ★ Hadron structure
- ★ Nuclear effects

THANK YOU

TMD Topical Collaboration

Grant No. PHY-1714407