HEAVY QUARK CONTRIBUTION TO THE PROTON'S MAGNETIC MOMENT

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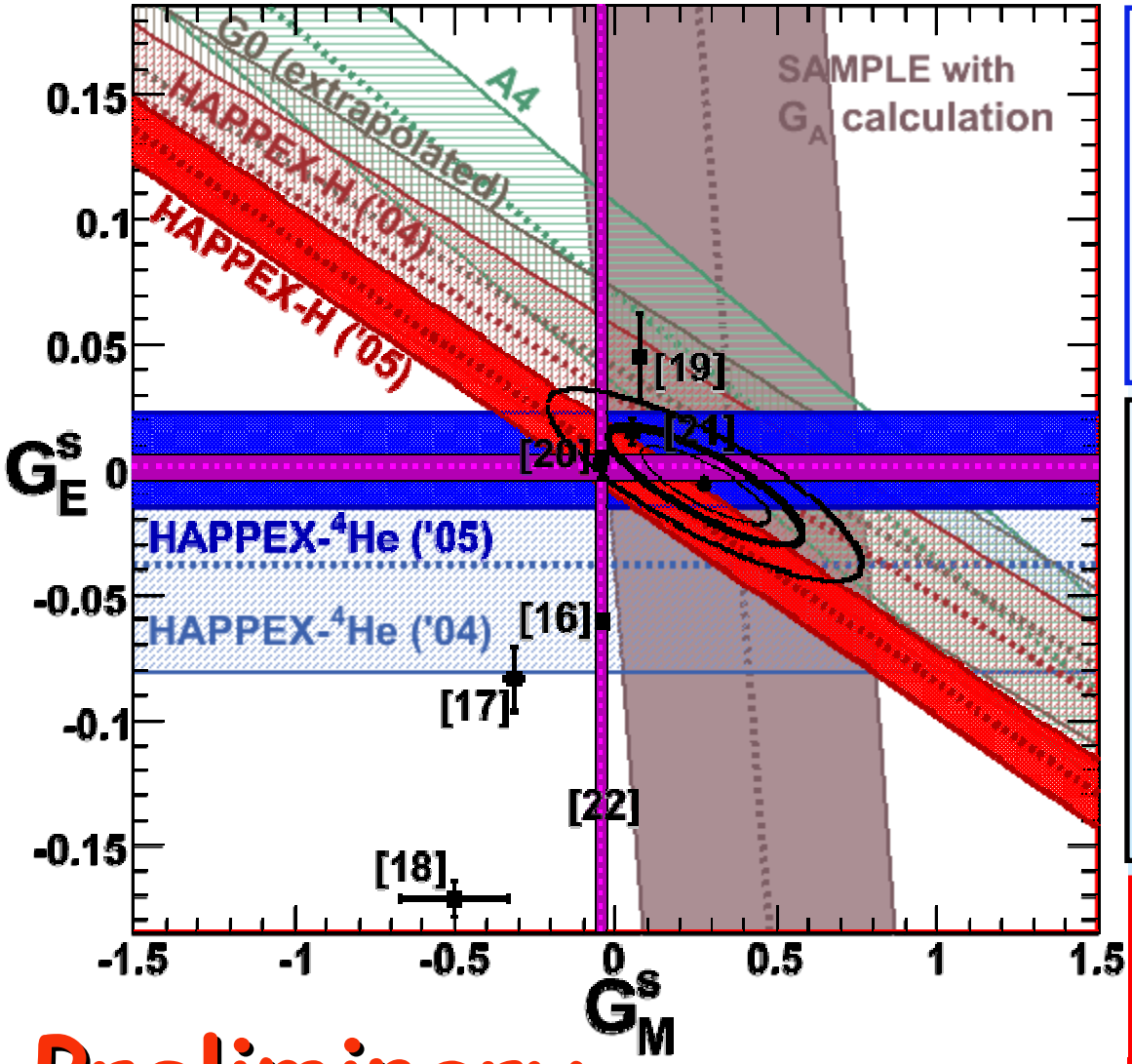
INTRODUCTION

- **Proton:**
  - ➢ **Naïve: 3 quarks bound by strong interaction**
  - ➢ **QCD: Sea of virtual gluons, quark-antiq. pairs**
    - ⇨ **What are its consequences for the macroscopic properties of the proton?**
- **Proton's magnetic moment:**
  - ➢ **Strange quark contribution?**
    - ▪ **Exciting experimental results**
    - ▪ **Theory: Difficult because $m_s \sim \Lambda_{\mathrm{QCD}}$**
    - ▪ **Learn from light and heavy quark limits**
- **Heavy sea-quark contribution?**

OUTLINE

- Experimental results vs theory
- Sea-quark contribution to the proton's magnetic moment
  - Light and heavy quark limits
- Warm-up: Muon contribution to the electron's magnetic moment
- Heavy sea-quark contribution to the proton's magnetic moment
- Implications for the physical strange quark

WORLD DATA vs THEORY



Experiments

$$G_M^s = 0.28 \pm 0.20$$

$$G_E^s = -0.006 \pm 0.016$$

Theory

16. Skyrme Model
17. Dispersion Relation
18. Dispersion Relation
19. Chiral Quark Soliton Model
20. Perturbative Chiral Quark Model
21. Lattice
22. Lattice + charge symmetry

Contradiction between theory and experiment?

Preliminary

K. Paschke, TJNAF 06

$(Q^2 \sim 0.1 \mathrm{GeV}^2)$

MAGNETIC MOMENT

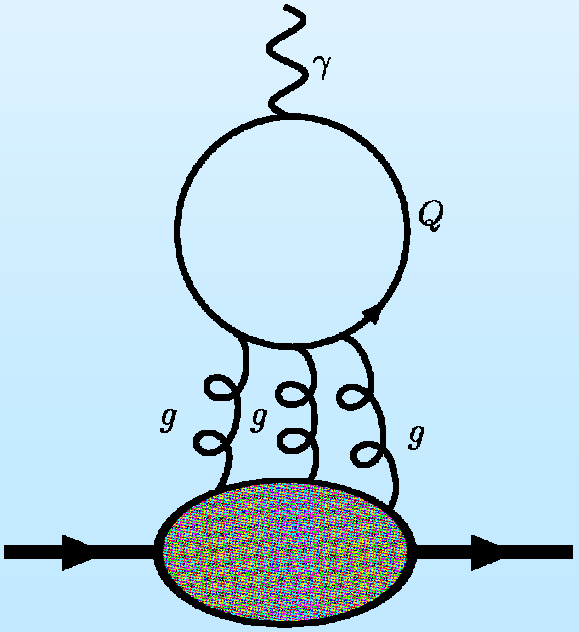
- **Proton polarized in z-direction**

$$\mu_p = \langle p\uparrow | \frac{1}{2} \int d^3r \left(\vec{r} \times \vec{j}^{\,\mathrm{em}} \right)_z | p\Uparrow \rangle / \langle p\uparrow | p\Uparrow \rangle$$

- **Light sea-quark, meson-cloud model:** $\delta\mu_p^{\mathrm{sea}} < 0$

Musolf and Burkardt, Leinweber et al.

- **Heavy sea-quark at lowest order**

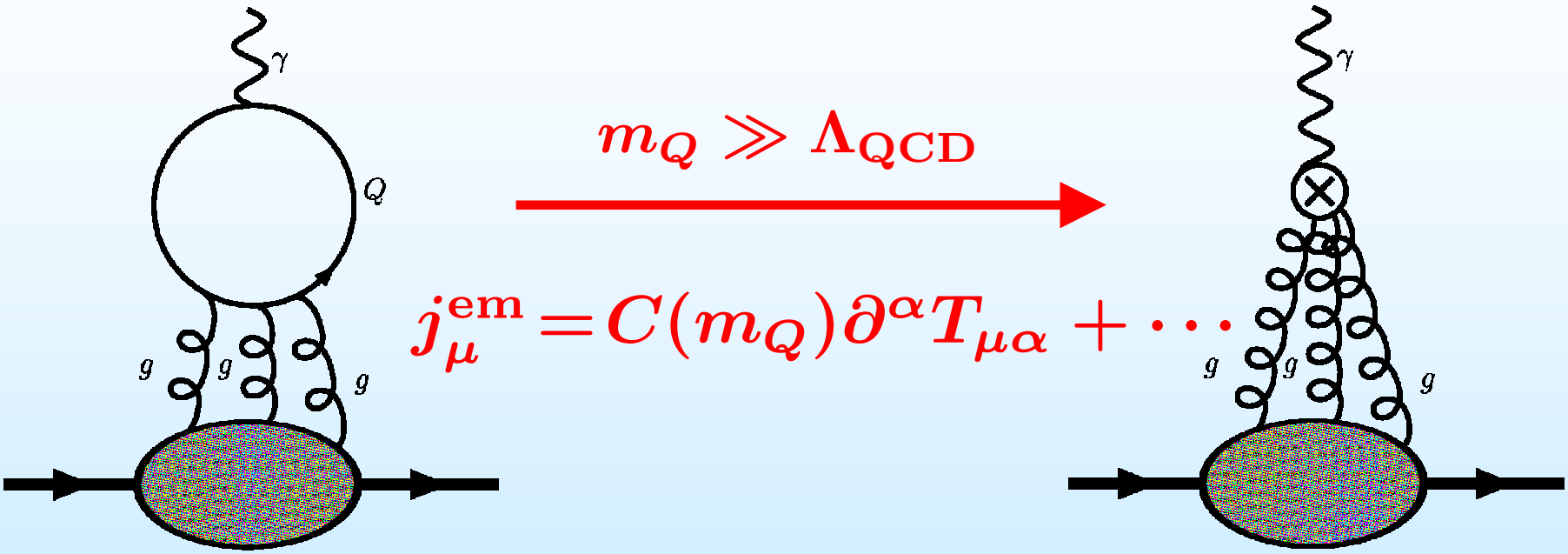


$$j_\mu^{\mathrm{em}} = \bar{Q}\gamma_\mu Q$$

Heavy sea-quark contribution to proton magnetic moment?

HEAVY STRANGE QUARK

- **Heavy quark limit $\Longrightarrow$ effective operator**



- $C(m_Q) = \kappa\, g^3(m_Q)/m_Q^4$
- $T_{\mu\alpha} = 14\, G_{\mu\sigma}\{G^{\sigma\tau}, G_{\tau\alpha}\} - 5\, G_{\sigma\tau}\{G^{\sigma\tau}, G_{\alpha\mu}\}$

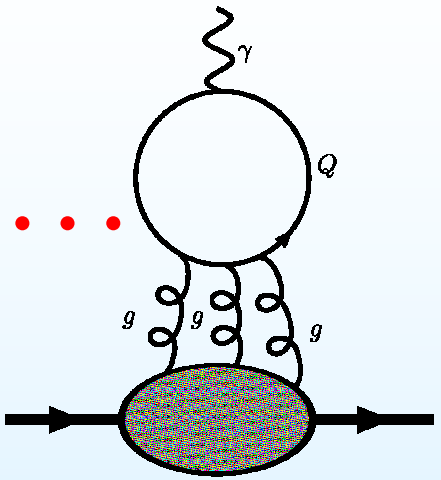
Kaplan and Manohar

- **Use effective $j_\mu^{\rm em}$ in def. of μ_p**

CONTRIBUTION TO μ_p

- **Heavy quark limit:**

$$\delta\mu_p^Q = C(m_Q)\ \langle p\uparrow|\ T_{yx}\ |p\Uparrow\rangle + \cdots$$

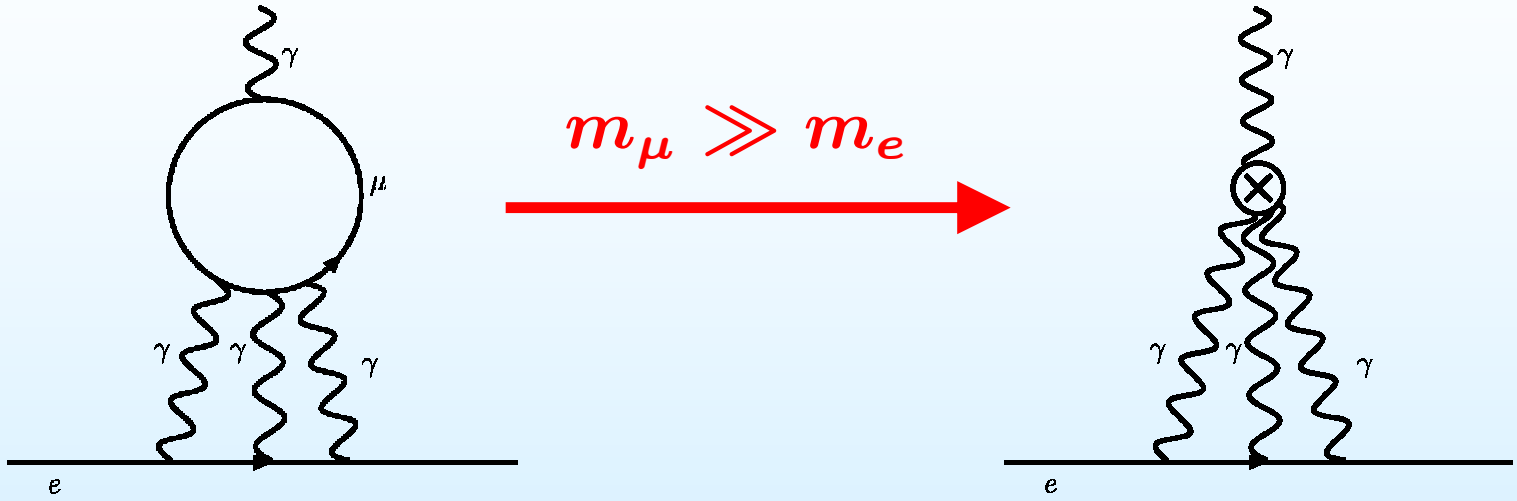


  - $C(m_Q) = \kappa\ g^3(m_Q)/m_Q^4$
  - $T_{yx} = \sum d^{abc}\Big[7(\vec{E}^a \cdot \vec{B}^b)\ E^{cz} - 2(\vec{E}^a \cdot \vec{E}^b - \vec{B}^a \cdot \vec{B}^b)\ B^{cz}\Big]$
  - **Light-by-light scattering at LO in QED** Euler and Kockel

- **Need to calculate** $\langle p\uparrow|\ T_{yx}\ |p\Uparrow\rangle$
- **Similar problem: muon contribution to** μ_e

MUON CONTRIBUTION TO μ_e

- **Heavy muon limit**



  - $\delta\mu_e^\mu = (C_e/m_\mu^4)\ \langle e|\ T_{yx}^\gamma\ |e\rangle + \cdots$

- **Exact calculation:**

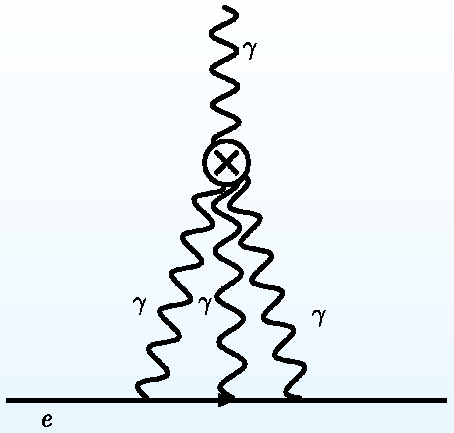
  - $\delta\mu_e^\mu/\mu_e = K_e\ \alpha_{\rm em}^3\ m_e^2/m_\mu^2 + \ldots\ > 0$

    Laporta and Remiddi

  - **Therefore:** $\langle e|\ T_{yx}^\gamma\ |e\rangle \sim m_\mu^2\ m_e$

MUON CONTRIBUTION TO μ_e

- **How can we understand this?**
  - **Power counting: k^3 divergence**
  - **Two scales: Mom. flow (m_μ), m_e**
  - **Factorization, dim. 3:**



$$\langle e|\, T^{\gamma}_{yx}\, |e\rangle = a\, m_\mu^3 + b\, m_\mu^2 m_e + c\, m_\mu m_e^2 + d\, m_e^3$$

MUON CONTRIBUTION TO μ_e

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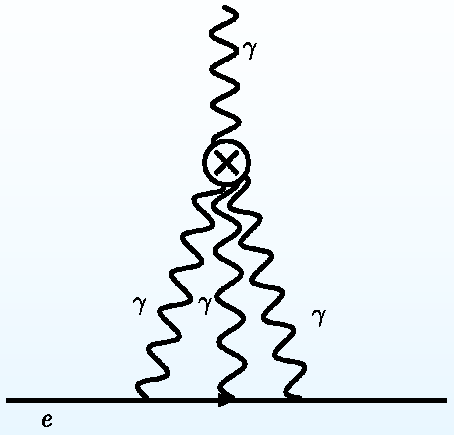

$$\langle e|\, T^{\gamma}_{yx}\, |e\rangle = a\, \cancel{m_\mu^3} + b\, m_\mu^2 m_e + c\, \cancel{m_\mu m_e^2} + d\, m_e^3$$

→ Lorentz invariance

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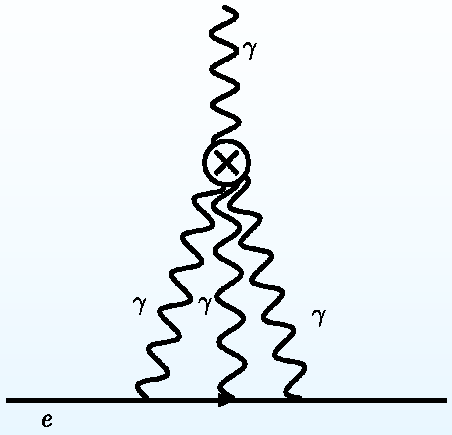

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→ **Lorentz invariance**

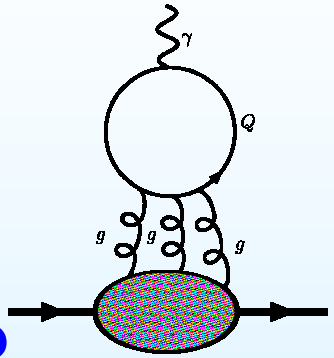
- **Symmetry** $\Rightarrow T^{\gamma}_{\mu\nu} = m_\mu^2 \left(\kappa\, m_e\, \bar{\psi}\sigma_{\mu\nu}\psi\right) + \cdots$

$$\Rightarrow \boxed{\delta\mu_e^{\mu} = (C_e/m_\mu^4)(\kappa\, m_\mu^2 m_e \langle e|\bar{\psi}\sigma_{yx}\psi|e\rangle) > 0}$$

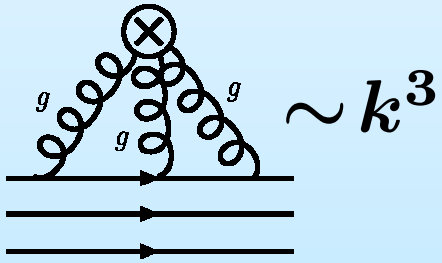
  - **Same result as exact calculation at LO**

CONTRIBUTION TO μ_p

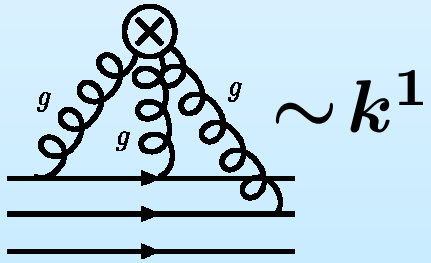
- **Heavy quark limit:** $\delta\mu_p^Q = C(m_Q)\ \langle p\uparrow|\ T_{yx}\ |p\Uparrow\rangle$
  - $C(m_Q) = \kappa\ g^3(m_Q)/m_Q^4$
  - $\langle p\uparrow|\ T_{yx}\ |p\Uparrow\rangle = \cancel{a\ m_Q^3} + b\ m_Q^2 \Lambda_{\rm QCD} + \cancel{c\ m_Q \Lambda_{\rm QCD}^2} + d\ \Lambda_{\rm QCD}^3$



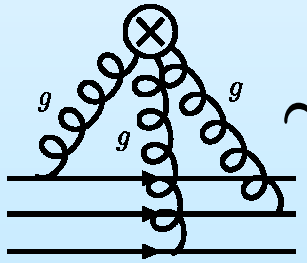
- **Contributions from 1, 2, and 3 quarks in proton:**



$\sim k^3$



$\sim k^1$



$\sim k^0$

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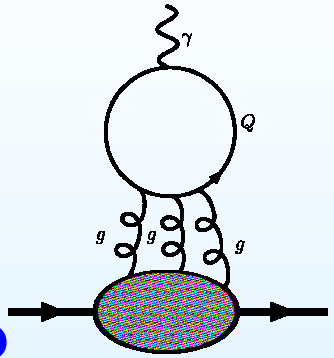

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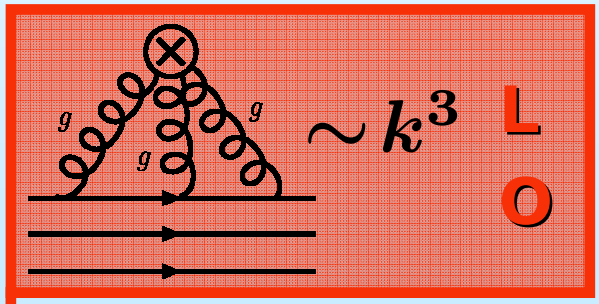

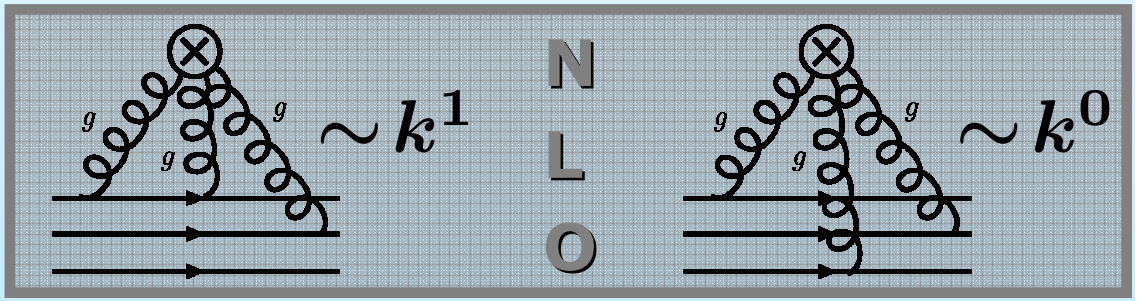

→ **Similar to muon contribution to μ_e**

$$\langle p\uparrow|\ T_{yx}\ |p\Uparrow\rangle_{\rm single\ quark} \sim m_Q^2 \Lambda_{\rm QCD} + \cdots$$

CONTRIBUTION TO μ_p

- **Heavy quark limit:** $\delta\mu_p^Q = C(m_Q)\ \langle p\uparrow|\ T_{yx}\ |p\Uparrow\rangle$

  - $\langle p\uparrow|T_{yx}|p\Uparrow\rangle = K m_Q^2\ \sum_f m_f\ \langle p\uparrow|\bar{\psi}_f \sigma_{yx} \psi_f|p\Uparrow\rangle$

CONTRIBUTION TO μ_p

- **Heavy quark limit:** $\delta\mu_p^Q = C(m_Q)\ \langle p\uparrow|\ T_{yx}\ |p\Uparrow\rangle$

  ➢ $\langle p\uparrow|T_{yx}|p\Uparrow\rangle = K m_Q^2 \underbrace{\sum_f m_f\ \langle p\uparrow|\bar{\psi}_f \sigma_{yx} \psi_f|p\Uparrow\rangle}$

  Proton tens. charge ⟶ $m_u\ \delta_u + m_d\ \delta_d\ > 0$

$\Rightarrow$ $\boxed{\delta\mu_p^Q = A(m_u\ \delta_u + m_d\ \delta_d)/m_Q^2 > 0}$

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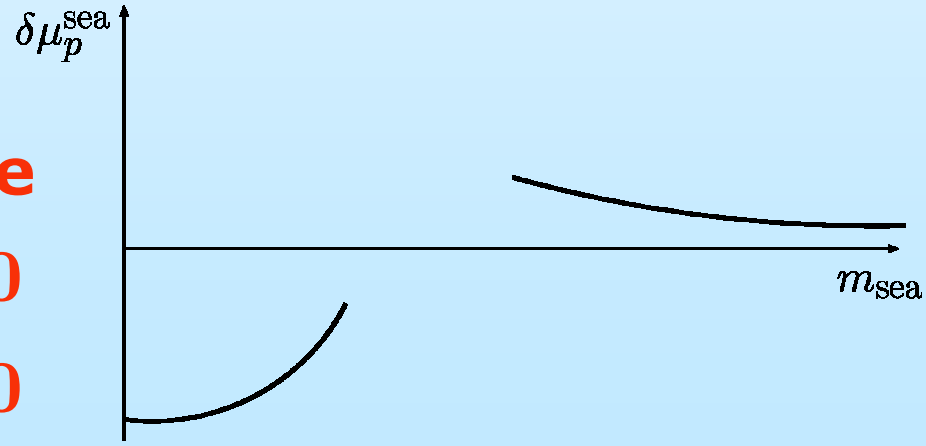
$$\Rightarrow \boxed{\delta\mu_p^Q = A(m_u\ \delta_u + m_d\ \delta_d)/m_Q^2 > 0}$$

- **Therefore:**

  Non-trivial dependence

  ➢ **Light sea:** $\delta\mu_p^{\rm sea} < 0$

  ➢ **Heavy sea:** $\delta\mu_p^{\rm sea} > 0$



QUARK MODELS OF THE PROTON

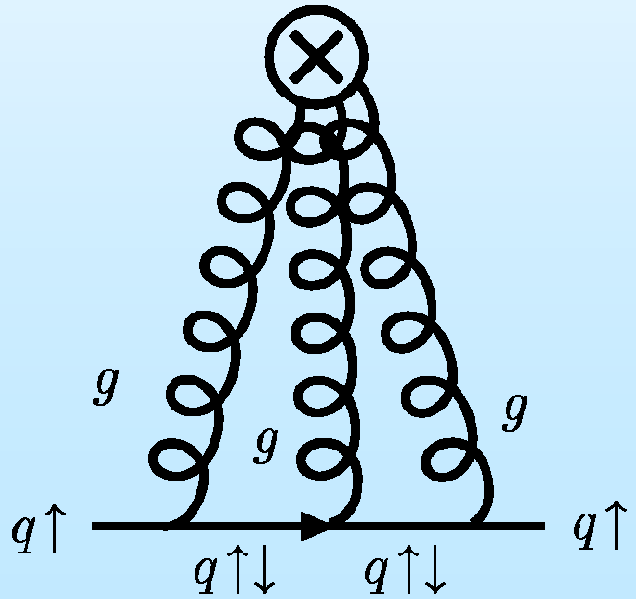
- Quark models: gluon fields generated by quarks
- Solve color Maxwell's equation in linear approx.

$$D_\mu G^{\mu\nu} = j^\nu$$

- Of course, other gluons also hold the proton together, generating potential between quarks, bag confinement, ...
- Assumption: correlation btwn polarized gluons and quarks are generated correctly by models
- Ignore non-linear effects
  - Same as 8 copies of e.m. fields

QUARK MODELS: RESULTS

- **Spin-dependent color magnetic field:**
  - ➢ **NRQ model: constit. quark magnetic dipole**
  - ➢ **MIT bag: quasi-massless quark orbit. motion**
- **Contribution from single polarized quark**
  - ➢ **Ground state = largest contribution**



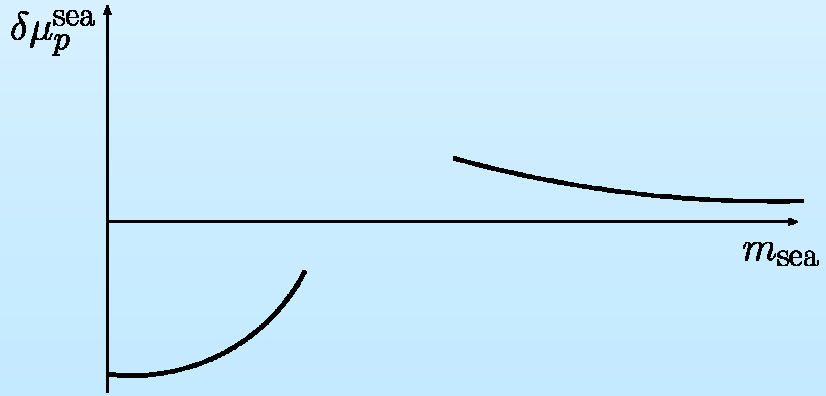
$$\delta\mu_p^Q / C\mu_p \simeq \begin{cases} 2.2\ \mathrm{fm}^{-4} & \textbf{NRQ} \\ 1.3\ \mathrm{fm}^{-4} & \textbf{Bag} \end{cases}$$

Extrapolation

Strange quark: $G_M^s \simeq 0.1$

CONCLUSIONS AND OUTLOOK

- Quark sea and the proton's macroscop. prop.?
- Strange quark and proton's magnetic moment
  - ➢Exciting experimental results
  - ➢Contradiction with theory?
- Sea quark contribution to μ_p
  - ➢Changes sign for light and heavy sea quark



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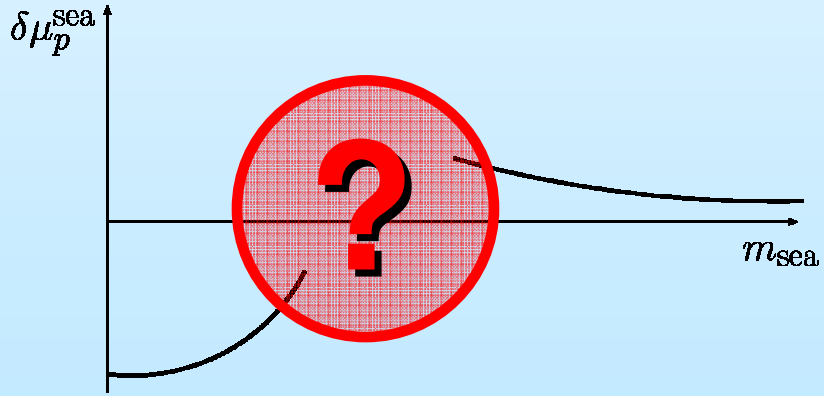

- Strange quark contribut.: Negative or positive?

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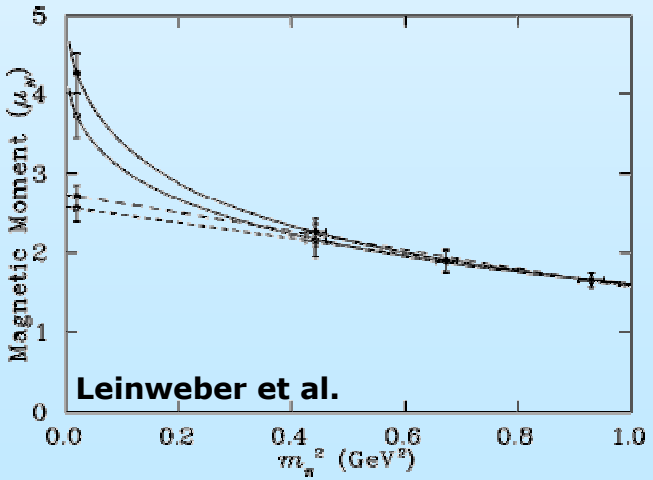

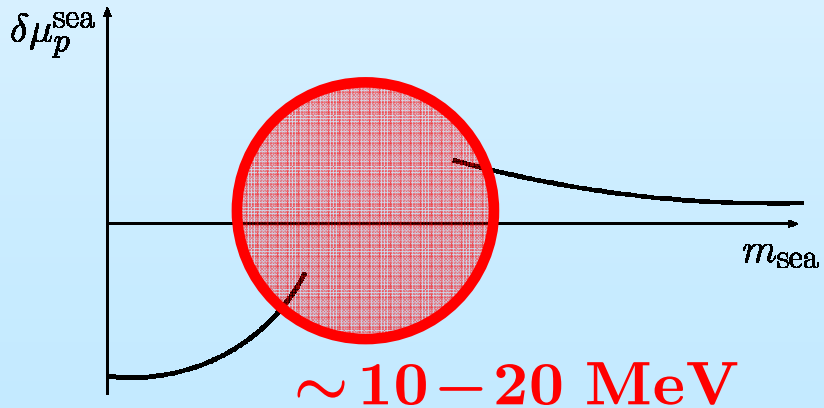

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