Lattice Calculations of Generalized Parton Distributions

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http://talks.drubryantrenner.org/jlab_gpds_6-13-06.pdf

Generalized Form Factors

- for example, unpolarized parton distributions

$$q(x) = < P,S| \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \overline{q}(-y^-/2)\gamma^+ q(y^-/2)|P,S>$$

- light-cone expansion generates unpolarized twist two operators

$$O_q^{\mu_1\cdots\mu_n} = \overline{q} iD^{(\mu_1} \ldots iD^{\mu_{n-1}}\gamma^{\mu_n)} q$$

- off-forward matrix elements of the twist two operators[1]

$$\langle P',S'|O_q^{\mu_1\cdots\mu_n}|P,S\rangle = \overline{U}(P',S')[\sum_{\substack{i=0\\ \text{even}}}^{n-1} A^q_{ni}(t) K^A_{ni}(P',P)$$

$$+ \sum_{\substack{i=0\\ \text{even}}}^{n-1} B^q_{ni}(t) K^B_{ni}(P',P) + \delta^n_{\text{even}} C^q_n(t) K^C_n(P',P)]U(P,S)$$

- similar expression for the polarized observables: $\tilde{A}^q_{ni}(t)$ and $\tilde{B}^q_{ni}(t)$

[1] X. D. Ji hep-ph/9807358

Moments of Generalized and Transverse Parton Distributions

- moments of generalized parton distributions (unpolarized example)

$$\int_{-1}^{1} dx\ x^{n-1} H_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} A^q_{n,2i}(t)(-2\xi)^{2i} + \mathrm{mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

$$\int_{-1}^{1} dx\ x^{n-1} E_q(x,\xi,t) = \sum_{i=0}^{[(n-1)/2]} B^q_{n,2i}(t)(-2\xi)^{2i} - \mathrm{mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

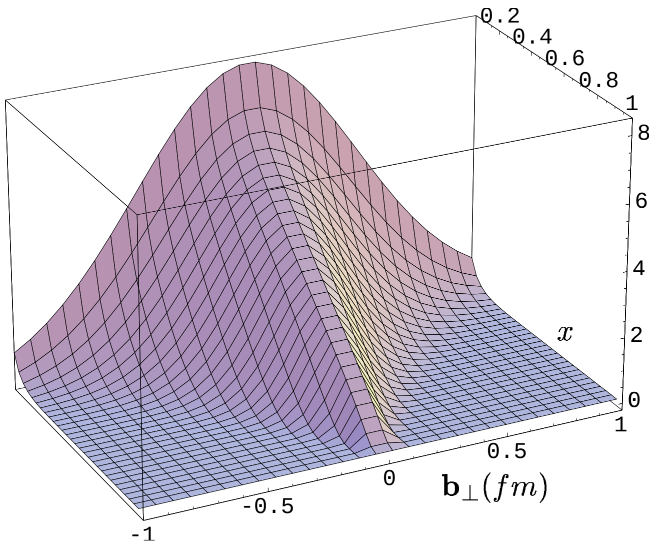
- transverse quark distributions, $\xi \to 0$ [1]

$$\int_{-1}^{1} dx\ x^{n-1}\, q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A^q_{n0}(-\vec{\Delta}^2_\perp)$$

$$\int_{-1}^{1} dx\ x^{n-1}\, \Delta q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \tilde{A}^q_{n0}(-\vec{\Delta}^2_\perp)$$

[1] M. Burkardt hep-ph/0005108

Transverse Distributions



$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2b_\perp\; e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int_{-1}^{1} dx\; x^{n-1}\, q(x,\vec{b}_\perp)$$

$$\langle b^2_\perp\rangle^q_{(n)} = -4\,\frac{A^{q\,\prime}_{n0}(0)}{A^q_{n0}(0)}$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x\to 1} q(x,\vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

- higher moments A^q_{n0} weight $x \sim 1$ more heavily

$$\lim_{n\to\infty} A^q_{n0}(t) \propto \int d^2b_\perp\; e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\delta^2(\vec{b}_\perp) = \text{constant}$$

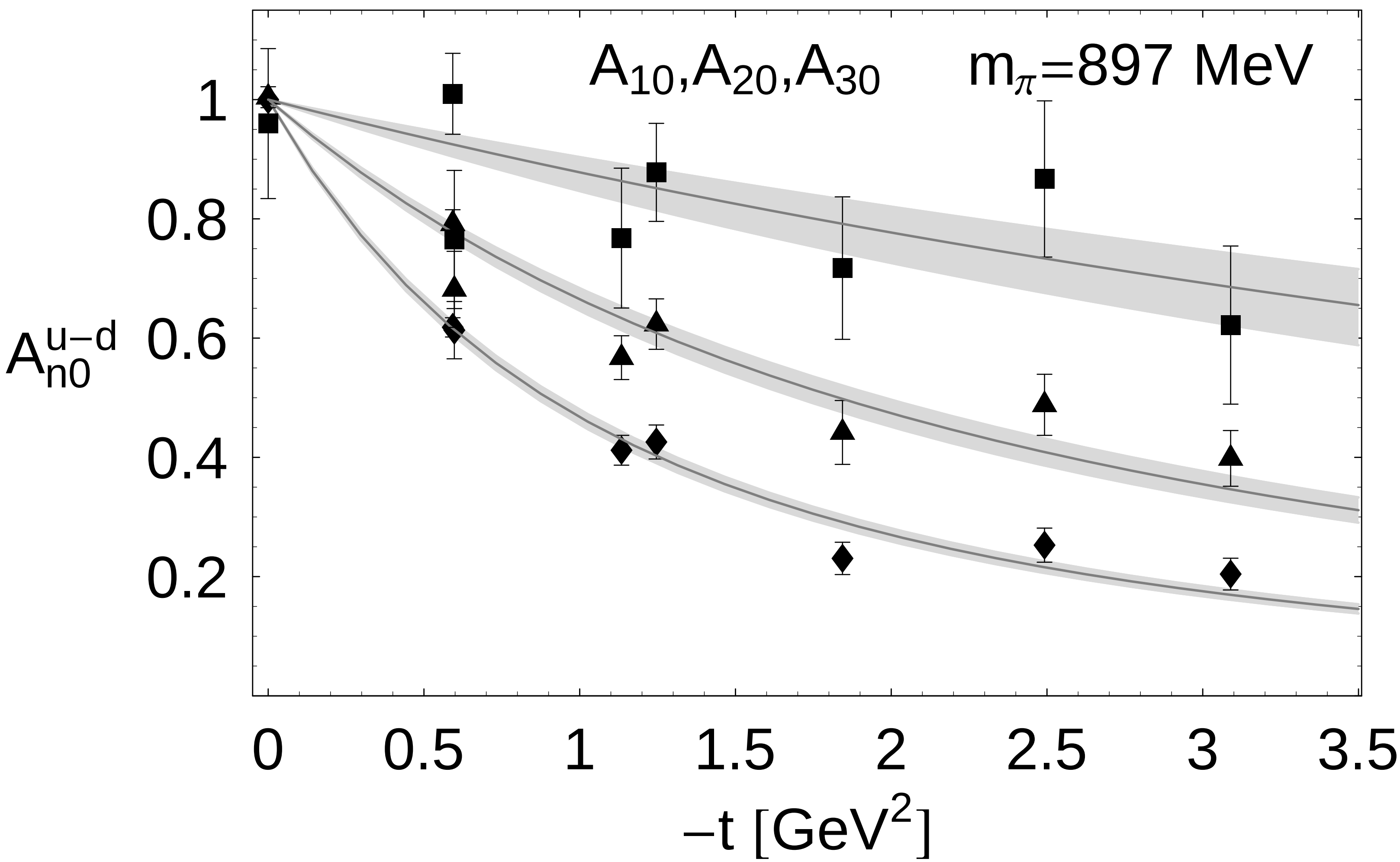
- slopes of A^q_{n0} should decrease as n increases

- A_{10}, A_{30}, $\tilde{A}_{20}$ measure $q-\overline{q}$ & $\tilde{A}_{10}$, $\tilde{A}_{30}$, A_{20} measure $q+\overline{q}$

graph from M. Burkardt hep-ph/0207047

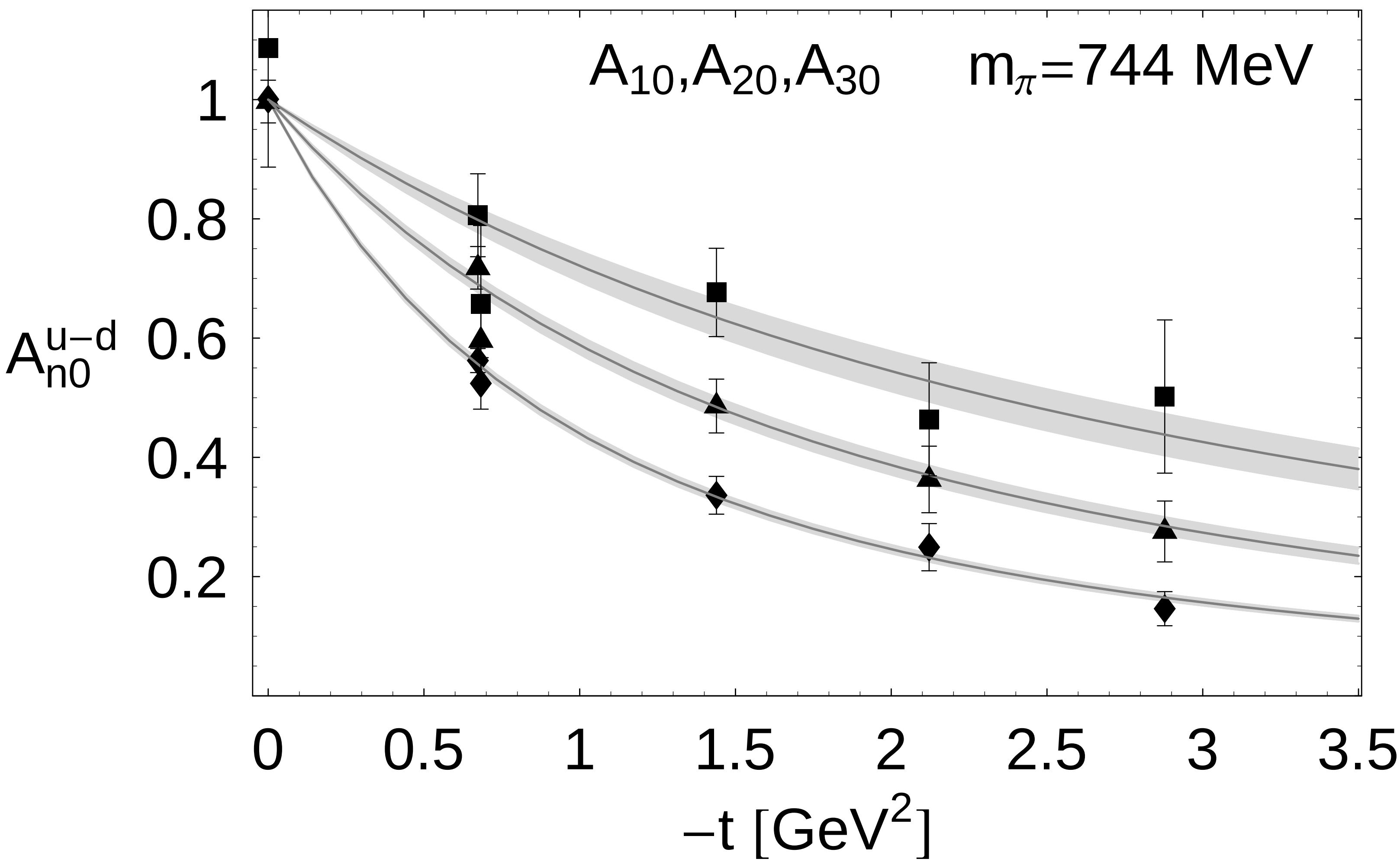
Transverse Distributions: $m_\pi = 897$ MeV

- slope of $A_{10}^{u-d} = -0.93 \pm 0.04\ (\text{GeV})^{-2}$
- slope of $A_{30}^{u-d} = -0.13 \pm 0.03\ (\text{GeV})^{-2}$ (factor of 7)



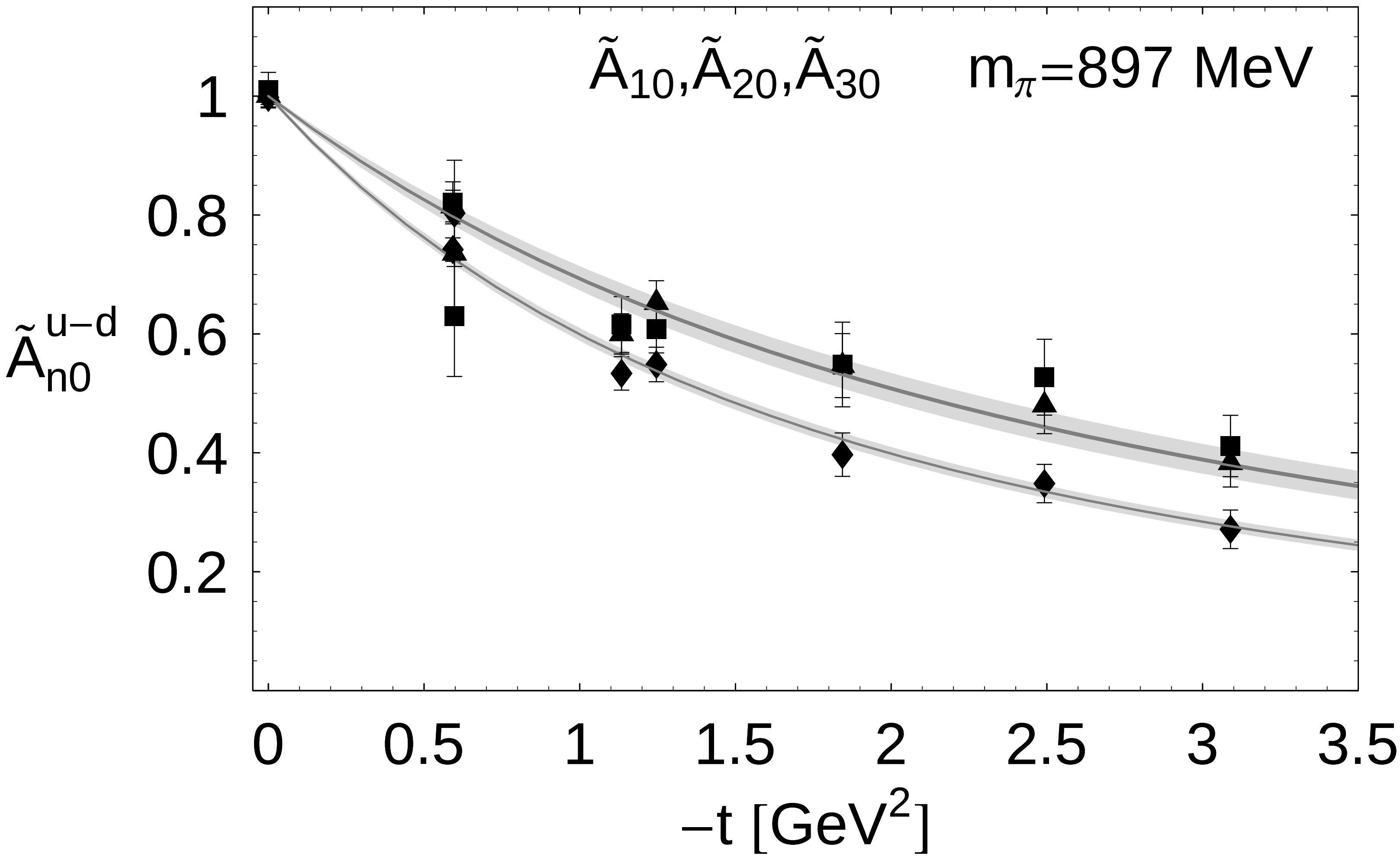
Transverse Distributions: Mass Dependence

- slope of $A_{10}^{u-d} = -1.02 \pm 0.03~(\mathrm{GeV})^{-2}$
- slope of $A_{30}^{u-d} = -0.36 \pm 0.04~(\mathrm{GeV})^{-2}$ (factor of 3)



Transverse Distributions: Spin Dependence

- slope of $\tilde{A}_{10}^{u-d} = -0.58 \pm 0.02\ (\mathrm{GeV})^{-2}$
- slope of $\tilde{A}_{30}^{u-d} = -0.40 \pm 0.03\ (\mathrm{GeV})^{-2}$ (factor of 1.5)



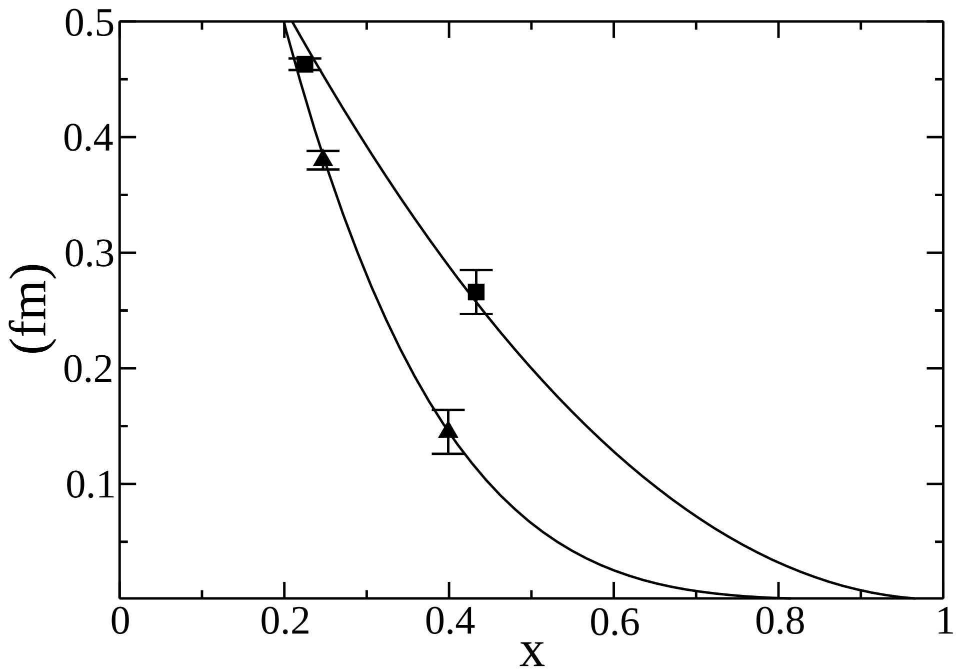
Transverse Distributions: x Dependence

- transverse rms radius (momentum space)

$$\left\langle b_\perp^2 \right\rangle_x = \frac{\int d^2 b_\perp \, b_\perp^2 \, q(x, \vec{b}_\perp)}{\int d^2 b_\perp \, q(x, \vec{b}_\perp)}$$

- transverse rms *moment* radius (Mellin space)

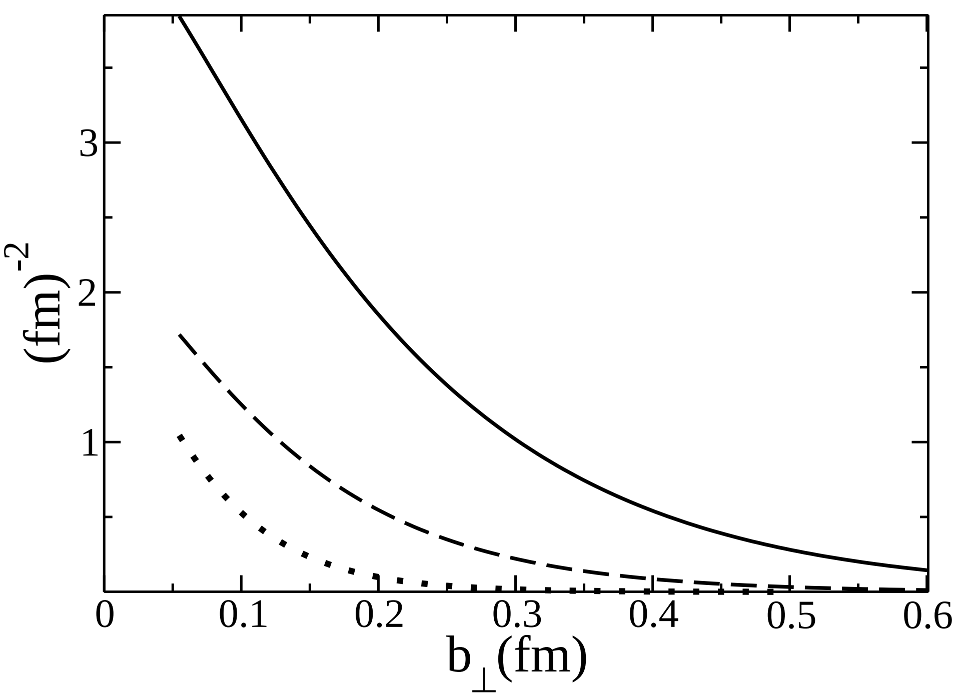
$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4} \left\langle b_\perp^2 \right\rangle_{(n)} \qquad \left\langle b_\perp^2 \right\rangle_{(n)} = \frac{\int d^2 b_\perp \, b_\perp^2 \; \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)}{\int d^2 b_\perp \; \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)}$$



Transverse Distributions: $\vec{b}_\perp$ Dependence

$$q_1(\vec{b}_\perp) = \int_{-1}^{1} dx\; q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{10}^q(-\vec{\Delta}_\perp^2)$$

$$q_2(\vec{b}_\perp) = \int_{-1}^{1} dx\; x\, q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{20}^q(-\vec{\Delta}_\perp^2)$$



Hybrid Lattice Calculation

- asqtad staggered sea quarks (MILC) with $a = 0.124$ fm

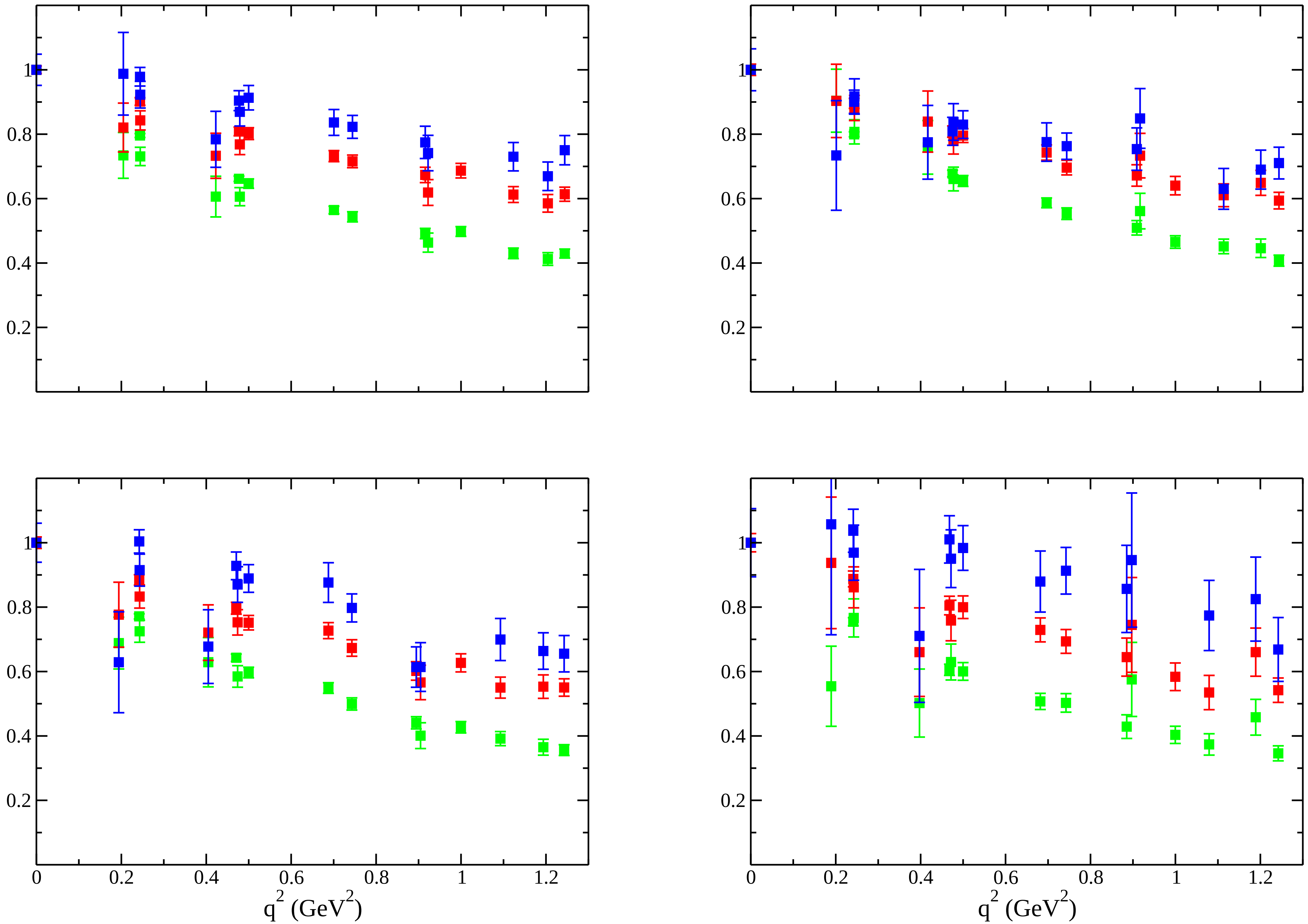
$am_{u/d}^{\text{asqtad}}$	L/a	L	m_π^{asqtad}	#
		fm	MeV	
0.05	20	2.52	770	425
0.04	"	"	692	350
0.03	"	"	601	564
0.02	"	"	495	486
0.01	"	"	357	656
0.01	28	3.53	357	270

- domain wall valence quarks with HYP smearing

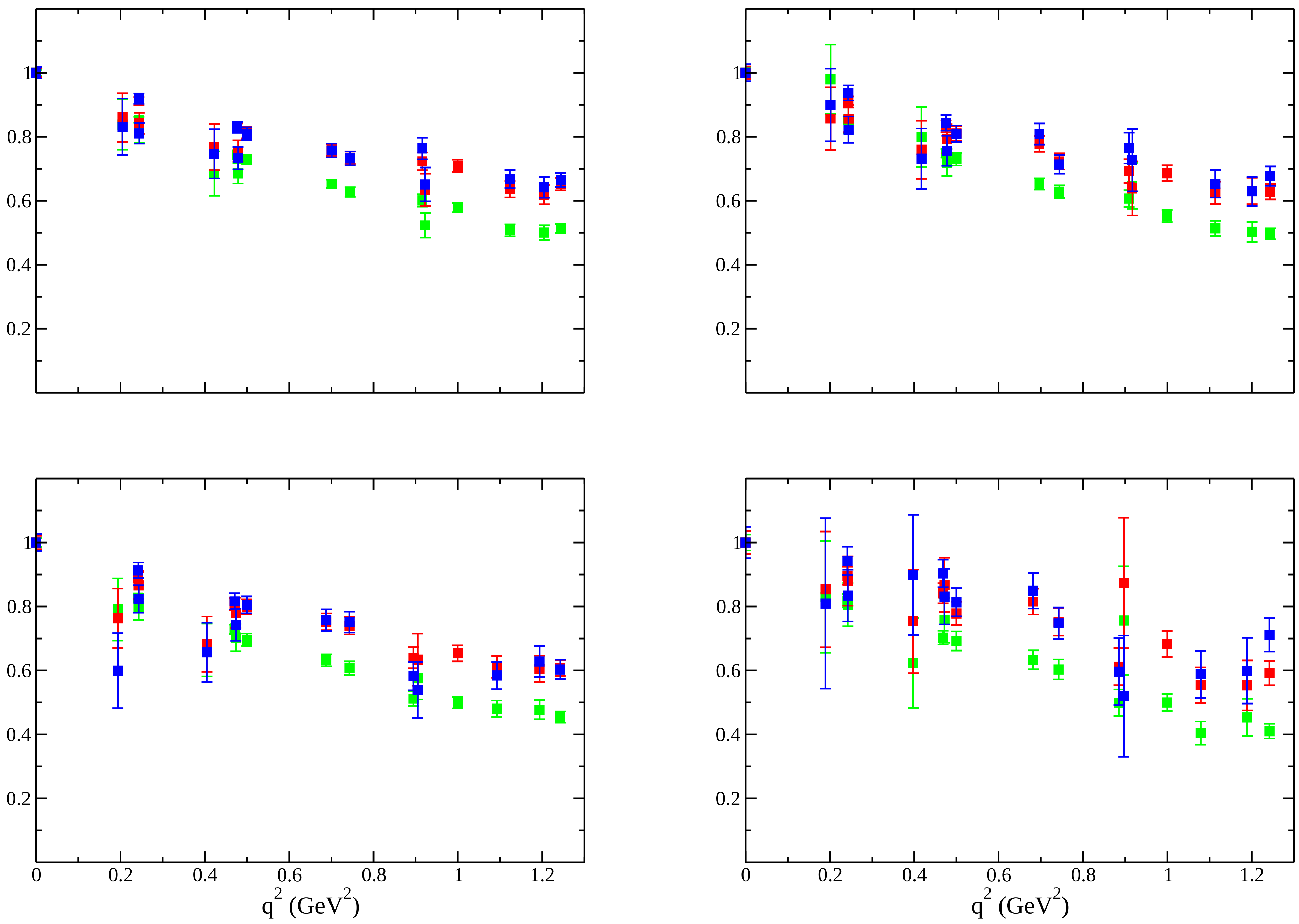
- one loop perturbative renormalization at $\mu = 2$ GeV

- please see hep-lat/0409130 for more details

Unpolarized Transverse Distributions: $A_{n0}(q^2)$



Polarized Transverse Distributions: $\tilde{A}_{n0}(q^2)$



Conclusions

- we can examine the transverse structure of the nucleon by way of the generalized parton distributions

- we can determine the low moments of the generalized parton distributions by calculating the generalized form factors in lattice QCD

- in this way we calculate directly the transverse, $\vec{b}_\perp$, dependence of the low moments

- we observe the transverse size of the nucleon, $\sqrt{\langle b_\perp^2 \rangle}$, shows a significant dependence on the longitudinal momentum fraction $\langle x \rangle$