Basics of Generalized Parton Distribution Theory and Phenomenology

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Recollections from 1996

Gauge invariant decomposition of nucleon spin and its spin - off.
Xiang-Dong Ji (MIT, LNS & Washington U., Seattle) .
MIT-CTP-2517, Mar 1996. 8pp.
Published in **Phys.Rev.Lett.78:610-613,1997**
e-Print Archive: **hep-ph/9603249**

Motivation: Understanding Spin Content of Proton

"The angular momentum operator in QCD ...

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk} \ ,$$

where M^{0ij} is the angular momentum density

$$M^{\alpha\mu\nu} = T^{\alpha\nu}x^\mu - T^{\alpha\mu}x^\nu \ .$$

... energy-momentum tensor $T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$

…form factors of quark and gluon energy-momentum tensors

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(\Delta^2)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M$$

$$+C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \bar{C}_{q,g}(\Delta^2)g^{\mu\nu}M\Big]U(P)$$

where $\bar{P}^\mu = (P^\mu + P^{\mu\prime})/2$, $\Delta^\mu = P^{\mu\prime} - P^\mu$

Spin content: $J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]$.

…one has to measure B-form factor, which is analogous to the Pauli form factor for the vector current

…no fundamental probe that couples to quark and gluon energy-momentum tensors ……

…measure off-forward matrix element of $TJ_\alpha(\xi)J_\beta(0)$ extrapolating the form factors to the forward limit.

The natural process to do this is Compton scattering

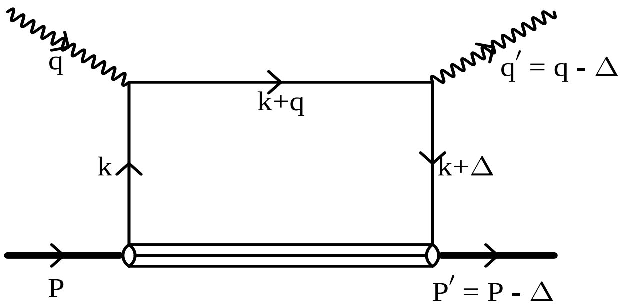
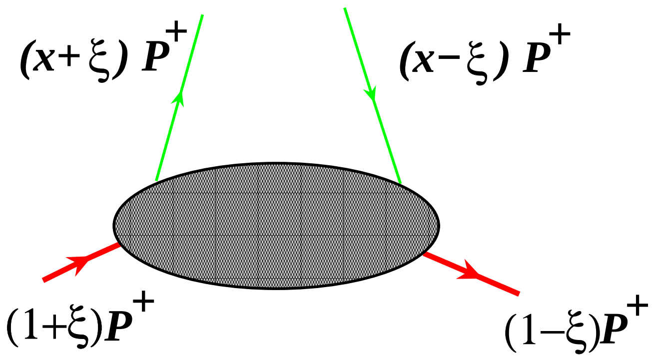


FIG. 1. Dominant scattering process in deeply-virtual Compton scattering

H, $\tilde{H}$, E and $\tilde{E}$ are new, off-forward, twist-two parton distributions

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}(-\lambda n/2) \gamma^\mu \psi(\lambda n/2) | P \rangle$$

$$= H(x, \Delta^2, \Delta\cdot n) \bar{U}(P') \gamma^\mu U(P) + E(x, \Delta^2, \Delta\cdot n) \bar{U}(P') \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M} U(P) + \ldots$$



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}(-\lambda n/2) \gamma^\mu \gamma_5 \psi(\lambda n/2) | P \rangle$$

$$= \tilde{H}(x, \Delta^2, \Delta\cdot n) \bar{U}(P') \gamma^\mu \gamma_5 U(P) + \tilde{E}(x, \Delta^2, \Delta\cdot n) \bar{U}(P') \frac{\gamma_5 \Delta^\mu}{2M} U(P) + \ldots$$

$$\Delta \cdot n = -2\xi P \cdot n$$

OFPDS ... have the characters of both ordinary parton distributions and nucleon form factors.

$H(x, 0, 0) = f_1(x), \quad \tilde{H}(x, 0, 0) = g_1(x)$

where $f_1(x)$ and $g_1(x)$ are quark and quark helicity distributions.

... first moment of the new distributions ... sum rules,

$$\int dx H(x, \Delta^2, \Delta \cdot n) = F_1(\Delta^2)$$

$$\int dx E(x, \Delta^2, \Delta \cdot n) = F_2(\Delta^2)$$

$$\int dx \tilde{H}(x, \Delta^2, \Delta \cdot n) = G_A(\Delta^2)$$

$$\int dx \tilde{E}(x, \Delta^2, \Delta \cdot n) = G_P(\Delta^2)$$

The most interesting sum rule relevant to the nucleon spin is

$$\int dx x[H(x, \Delta^2, \Delta \cdot n) + E(x, \Delta^2, \Delta \cdot n)] = A_q(\Delta^2) + B_q(\Delta^2)$$

$C_q(\Delta^2)$ contamination, drops out.

Extrapolating the sum rule to $\Delta^2 = 0$, the total quark (and hence quark orbital) contribution to the nucleon spin is obtained.

. . . practical aspects of the experiment . . . E and H can be measured either in unpolarized scattering, or in electron single-spin asymmetry through interference with the Bethe-Heitler amplitude, or in polarized electron scattering on a transversely polarized target.

. . . cross section is measurable, but statistics would be a challenging requirement."

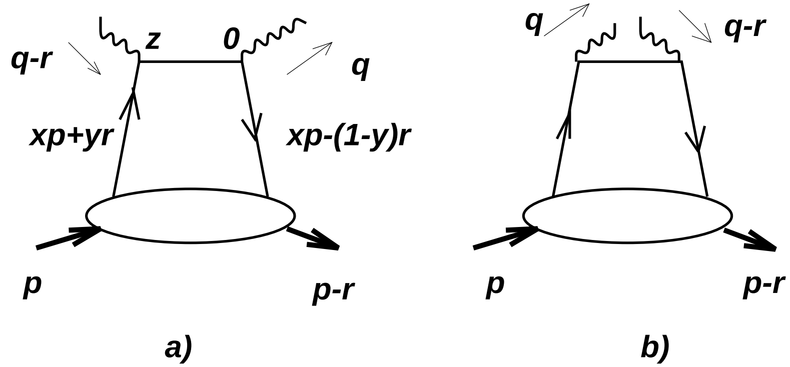
One month later:

Scaling limit of deeply virtual Compton scattering.
A.V. Radyushkin (Old Dominion U. & Jefferson Lab). CEBAF-TH-96-05, Apr 1996. 10pp.
Published in **Phys.Lett.B380:417-425,1996**
e-Print Archive: **hep-ph/9604317**

…as emphasized by Ji, DVCS amplitude has scaling behavior in the region of small t and fixed x_{Bj} which makes it a very interesting object on its own ground.

…alternative pQCD formalism for analysis of DVCS amplitude

…main point is that …one should treat
initial momentum p and momentum transfer r
on equal footing by introducing double distributions $F(x, y)$,
which specify the fractions of p and r, *resp.*,
carried by the constituents …



Handbag diagrams contributing into the DVCS amplitude.
The lower blob corresponds to double quark distributions.

These distributions have hybrid properties:
they look like distribution functions with respect to x
and like distribution amplitudes with respect to y.

. . . momenta p and r are proportional to each other $r = \zeta p$, . . .
(but) . . . specify momentum flow in two different channels

spectral constraints $x \geq 0$, $y \geq 0$, $x + y \leq 1$ . . . positive fractions
. . . $x + \zeta y$ for active quark and $\bar{x} - \zeta y$ for spectators . . . fraction
. . . by quark going out . . . $x - \bar{y}\zeta$ . . . both positive and negative . . .

reduction formulas for double distributions $F(x,y)$

$$\int_0^{1-x} F_a(x,y)\,dy = f_a(x)$$

DVCS amplitude:

$$T_V^a(\zeta) = \int_0^1 dx \int_0^{1-x} \left(\frac{1}{x - \zeta\bar{y} + i\epsilon} + \frac{1}{x + \zeta y} \right) F_a(x,y)\,dy$$

(bar convention: $\bar{x} \equiv 1-x, \bar{y} \equiv 1-y$, etc.) Imaginary part:

$$\frac{1}{\pi}\,\mathrm{Im}\,T_V^a(\zeta) = \frac{1}{\zeta}\int_0^{\zeta} F_a(x, 1-x/\zeta)dx = \int_0^1 F_a(\bar{y}\zeta, y)\,dy \equiv \Phi_a(\zeta)$$

y-integral ... is different from that in reduction formula

$$f_a(\zeta) = \int_0^{1-\zeta} F_a(\zeta, y)\,dy \neq \Phi_a(\zeta)$$

Evolution equation

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)F(x,y;\mu)=\int_0^1 d\xi\int_0^1 R(x,y;\xi,\eta;g)F(\xi,\eta;\mu)d\eta.$$

Since the integration over y converts $F(x,y)$ into the parton distribution function $f(x)$, whose evolution is governed by

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)f(x;\mu)=\int_x^1\frac{d\xi}{\xi}P(x/\xi;g)f(\xi;\mu)d\xi,$$

the kernel $R(x,y;\xi,\eta;g)$ must have the property

$$\int_0^{1-x}R(x,y;\xi,\eta;g)dy=\frac{1}{\xi}P(x/\xi).$$

For a similar reason, integrating $R(x,y;\xi,\eta;g)$ over x one should get the Brodsky-Lepage kernel:

$$\int_0^{1-y}R(x,y;\xi,\eta;g)dx=V(y,\eta;g).$$

To solve the evolution equation, I propose to combine the standard methods used to find solutions of the underlying DGLAP and Brodsky-Lepage evolution equations

nth x-moment of $F(x, y; \mu)$

$$F_n(y; \mu) = \int_0^1 x^n F(x, y; \mu) dx$$

the general solution of the evolution equation

$$F_n(y; \mu) = (y\bar{y})^{n+1} \sum_{k=0}^{\infty} A_{nk} C_k^{n+3/2}(y - \bar{y}) \left[\log(\mu/\Lambda)\right]^{-\gamma_k^{(n)}/\beta_0} ,$$

where $\beta_0 = 11 - \frac{2}{3} N_f$ is the lowest coefficient of the QCD β-function and the anomalous dimensions $\gamma_k^{(n)}$ ... coincide with the standard non-singlet anomalous dimensions γ_N: $\gamma_k^{(n)} = \gamma_{n+k+1}$

$\ldots$ in the formal $\mu \to \infty$ limit,

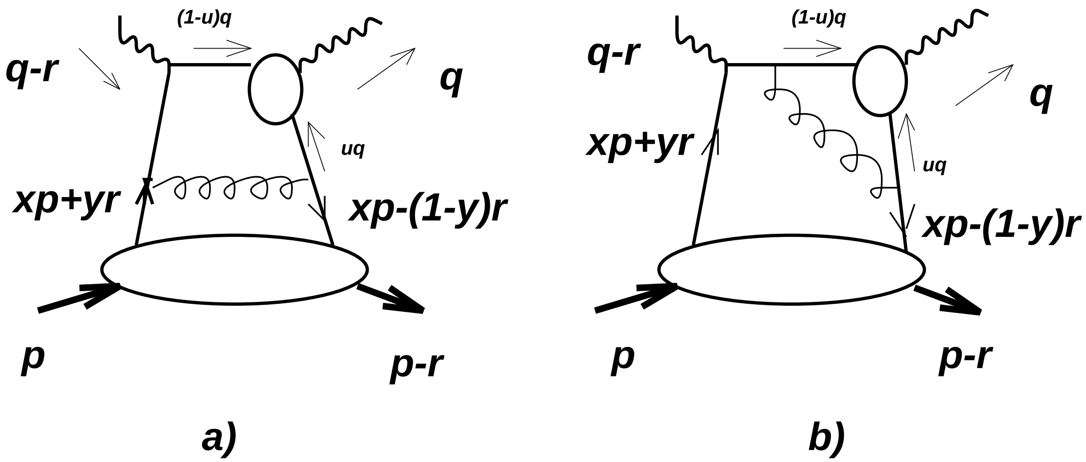
$$F(x, y; \mu \to \infty) \sim \delta(x) y \bar{y},$$

i.e., in each of its variables, the limiting function $F(x, y; \mu \to \infty)$ acquires the characteristic asymptotic form dictated by the nature of the variable:

$\delta(x)$ is specific for the distribution functions, while the $y\bar{y}$-form is the asymptotic shape for the lowest-twist two-body distribution amplitudes.

Infrared sensitivity and hard gluon exchange corrections.

…one of the photons in the DVCS amplitude is real …possibility of a long-distance propagation in the q-channel …means that …one should describe/parameterize it by introducing the distribution amplitude for the real photon



Simplest hard gluon exchange contributions to the DVCS amplitude. The upper blob corresponds to the photon distribution amplitude $\varphi_\gamma(u)$ and the lower one to double quark distributions $F(x,y), G(x,y)$.

By simple counting of propagators,

…contribution …behaves like α_s/Q^2 in the large-Q^2 limit.

Furthermore, due to the EM current conservation, …for a real photon, $q^2 = 0$ and $(q\epsilon) = 0$ …

the relevant contribution vanishes.

Hence, the hard gluon exchange corrections to the DVCS amplitude are rather strongly suppressed"

More detailed analysis was given by

Ji, Osborne (1998),

Collins, Freund (1998),

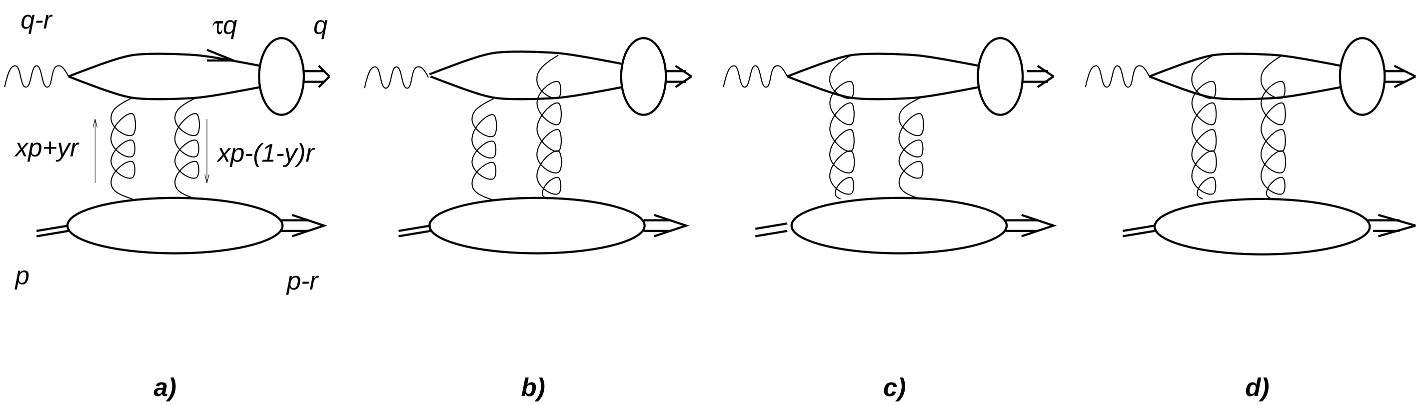
A.R. (1997)

. . . worth studying the possibility of observing the first signatures of the scaling behavior of DVCS at CEBAF, especially at upgraded energies

In a forthcoming paper, I discuss the gluonic double distributions $F_g(x, y)$ which play a crucial role in the perturbative QCD approach to hard diffractive electroproduction processes like $\gamma^* p \to p' \rho$.

One more month later:

Asymmetric Gluon Distributions and Hard Diffractive Electroproduction
A.V. Radyushkin (Old Dominion U. & Jefferson Lab). CEBAF-TH-96-06, May 1996. 11pp.
Published in **Phys.Lett.B385:333-342,1996**
e-Print Archive: **hep-ph/9605431**



Diagrams contributing to hard diffractive electroproduction of vector mesons.

"The usual gluon distribution function $xf_g(x)$ corresponds to the limit $r = 0$. Hence, . . . reduction formula:

$$\int_0^{1-x} F_g(x, y)\, dy = xf_g(x)$$

Asymmetric distribution functions[a]

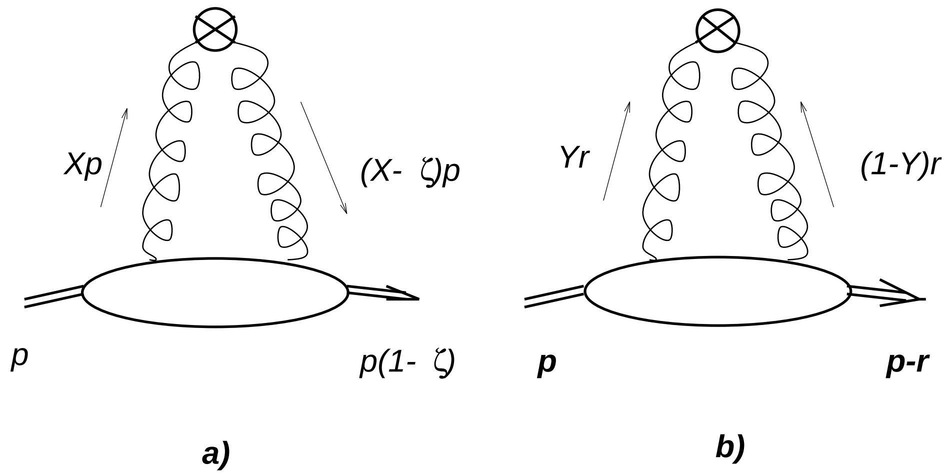
Since $r = \zeta p$, the variable y appears only in combinations $x + y\zeta \equiv X$ and $x - \bar{y}\zeta \equiv X - \zeta$, where X and $(X - \zeta)$ are the total fractions of the initial hadron momentum p carried by the gluons

integrate double distribution $F(X - y\zeta, y)$ over y to get

$$\mathcal{F}_\zeta(X) = \int_0^{\min\{X/\zeta, \bar{X}/\bar{\zeta}\}} F(X - y\zeta, y)\, dy,$$

variable X satisfies a natural constraint $0 \leq X \leq 1$

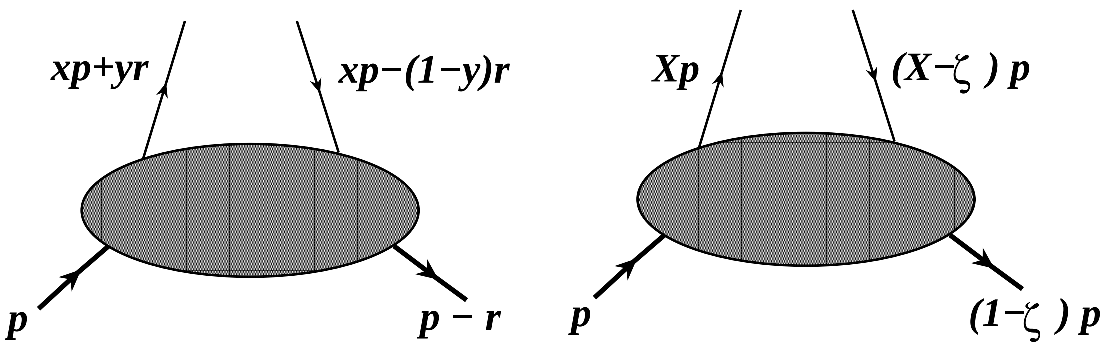
[a] Now called GPDs



In the region $X > \zeta$ ... function $\mathcal{F}_\zeta^g(X)$ ... describes a gluon with a positive fraction Xp ... coming back with a changed (but still positive) fraction $(X - \zeta)p$

Another region is $X < \zeta$), in which the "returning" gluon has negative fraction $(X - \zeta)$ ... more appropriate to treat it as a gluon going out of the hadron ... gluons carry now positive fractions $Y\zeta p \equiv Yr$ and $(1 - Y)r \equiv \bar{Y}r$ of the momentum transfer r ... like a distribution amplitude $\Psi_\zeta(Y)$ for a two-gluon state with the total momentum $r = \zeta p$

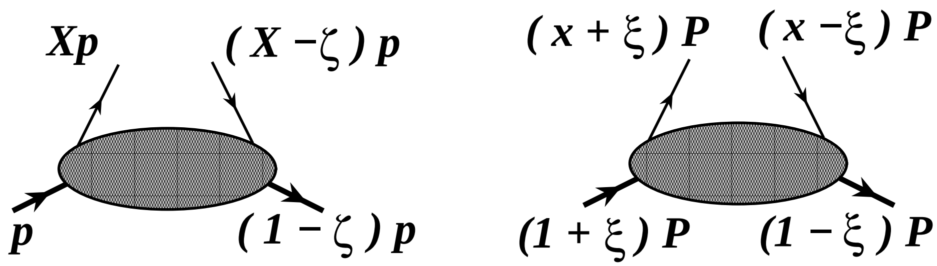
Conclusions. …one can describe asymmetric matrix element $\langle\bar{\zeta}p|G\ldots G|p\rangle$ either by the universal double distribution $F_g(x,y)$ or by the asymmetric distribution function[a] $\mathcal{F}_\zeta(X)$ which explicitly depends on the momentum asymmetry parameter[b] ζ and specifies the total fractions X and $X-\zeta$ of the original hadron momentum p carried by the gluons



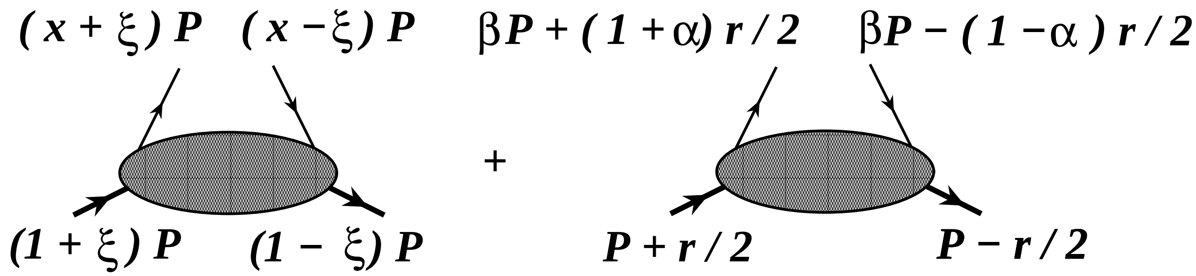
Comparison of DDs and NFPDs

[a] GPD

[b] skewness



Comparison of NFPDs and OFPDs



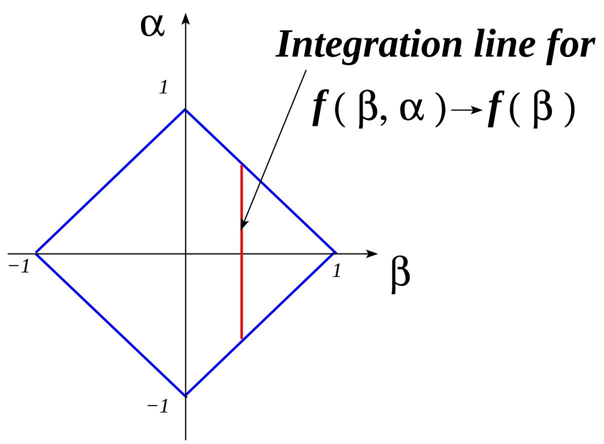
Comparison of OFPDs and symmetric DDs

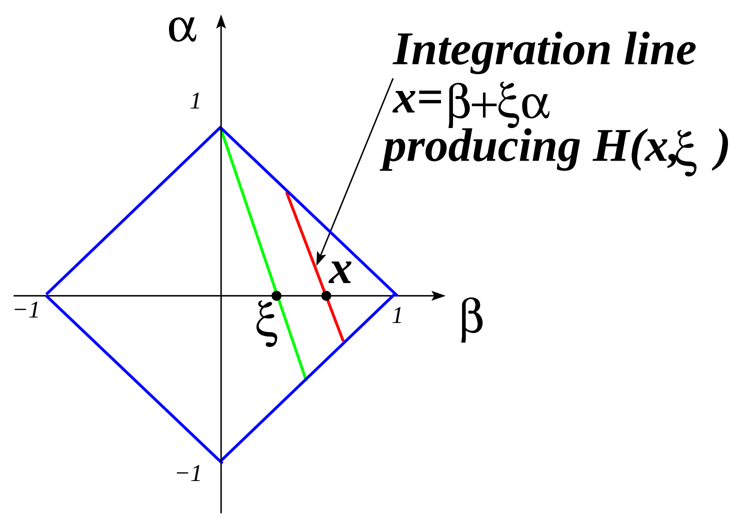
$$r = 2\xi p \Rightarrow x = \beta + \xi\alpha$$

forward limit $\xi = 0, t = 0$ corresponds to $x = \beta$

$\Rightarrow$ relation between DDs and usual parton densities

$$\int_{-1+|\beta|}^{1-|\beta|} f_a(\beta, \alpha; t = 0)\, d\alpha = f_a(\beta) \ .$$





Scanning pattern for DD $\rightarrow$ SPD conversion

"Munich" symmetry:

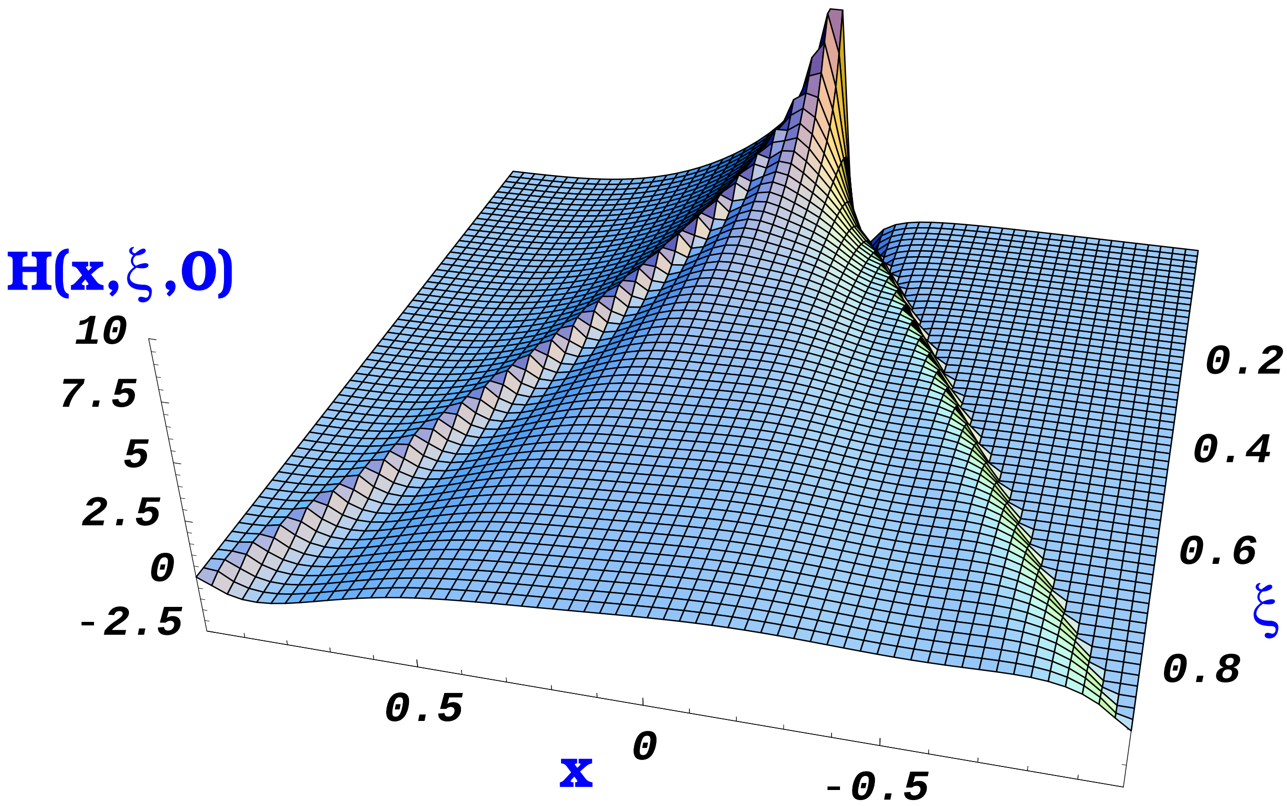
$$f_a(\beta, \alpha; t) = f_a(\beta, -\alpha; t)$$

Factorized model for DDs:

($\sim$ usual parton density in β-direction) $\otimes$
($\sim$ distribution amplitude in α-direction)

E.g.,

$$f_a(\beta, \alpha; t = 0) = f_a(\beta) \times \frac{3[(1 - |\beta|)^2 - \alpha^2]}{(1 - |\beta|)^3}$$



Factorized DD + D-term model (M. Vanderhaeghen).

Five years before 1996:

Wave Functions, Evolution Equations and Evolution Kernels from Light Ray Operators of QCD.
Dieter Mueller, D. Robaschik, B. Geyer (Leipzig U.), F.M. Dittes (Rossendorf, Forschungszentrum), J. Horejsi (Charles U.) NTZ-6-91, Dec 1998. 42pp.
Published in Fortsch.Phys.42:101,1994
e-Print Archive: hep-ph/9812448

Nonforward one-particle matrix elements ... lead to new distribution amplitudes ... depend ... on two scaling variables

They are applied for description of exclusive virtual Compton scattering in the Bjorken region near forward direction ...

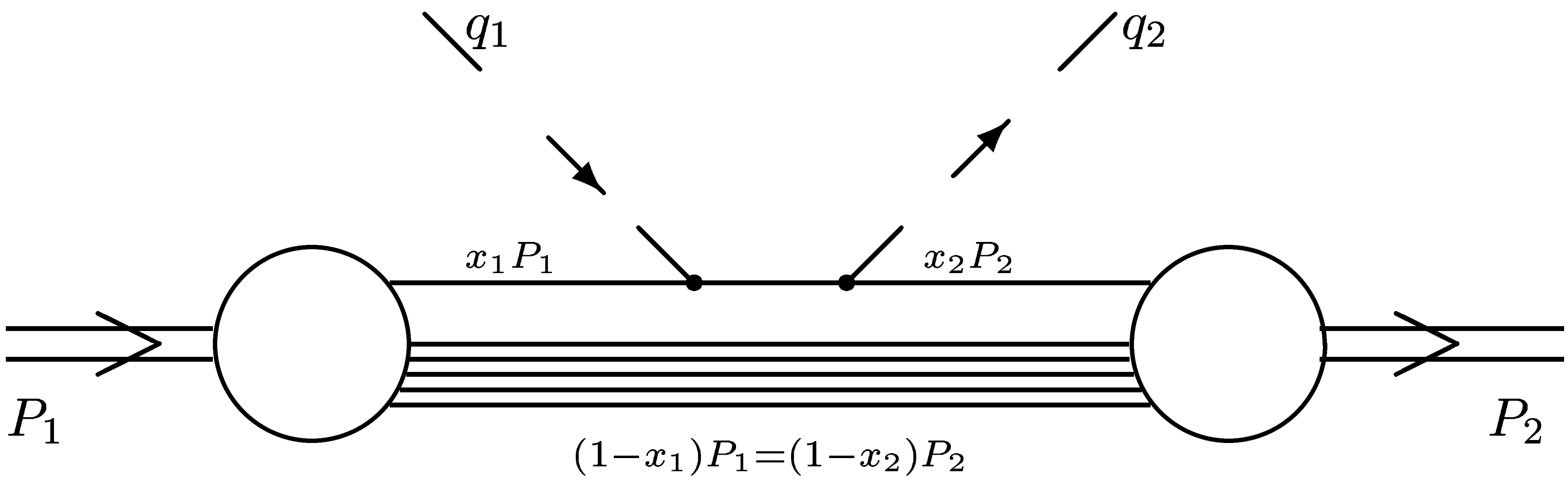
...evolution equations for these distribution amplitudes are derived ...evolution kernels follow from anomalous dimensions ...

Relations between different evolution kernels (especially the Altarelli-Parisi and the Brodsky-Lepage) kernels are derived ...

Paper discusses DDs, GPDs, spectral properties, etc.

Variables x_1, x_2 later appeared in discussing positivity constraints

$$x_1 = \frac{x+\xi}{1+\xi} \quad , \quad x_2 = \frac{x-\xi}{1-\xi}$$

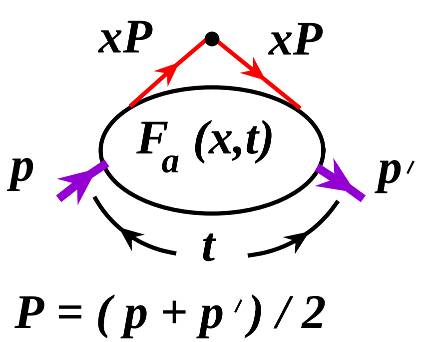


Leading contribution for the virtual Compton scattering in the Bjorken region

GPDs for large t

Nonforward parton densities and soft mechanism for form-factors and wide angle Compton scattering in QCD.
A.V. Radyushkin (Old Dominion U. & Jefferson Lab)
JLAB-THY-98-10, Mar 1998. 16pp.
Published in Phys.Rev.D58:114008,1998
e-Print Archive: hep-ph/9803316

Form factors in terms of $\xi = 0$ parton densities



$$F_{1a}(t) = \int_0^1 F_{1a}(x,t)\, dx$$

flavor components of form factors

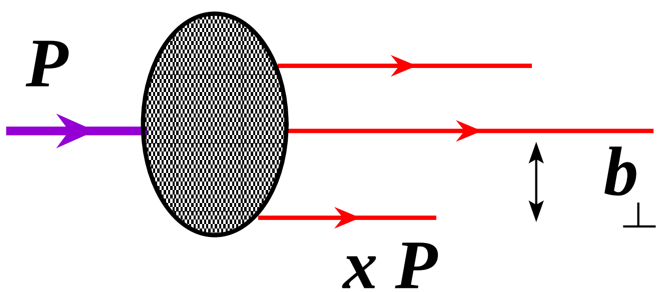
$$F_{1a}(x,t) \equiv e_a[\mathcal{F}_a(x,t) - \mathcal{F}_{\bar{a}}(x,t)]$$

Forward limit $t = 0$: $\mathcal{F}_{a(\bar{a})}(x, t = 0) = f_{a(\bar{a})}(x)$

Nonforward parton densitiess can be treated as Fourier transforms of the impact parameter $b_\perp$ distributions $f_1(x, b_\perp)$

$$\mathcal{F}(x,t) = \int f(x, b_\perp) e^{i(r_\perp b_\perp)} d^2 b_\perp$$

$(t = -r_\perp^2)$ describing the variation of parton densities in the transverse plane



Interplay between x and t dependences: simplest factorized ansatz

$$\mathcal{F}_a(x,t) = f_a(x) F_1(t)$$

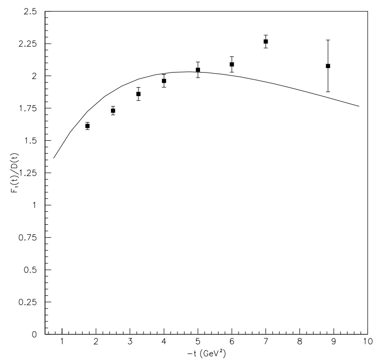
Reality may be more complicated: LC wave functions with Gaussian $k_\perp$ dependence

$$\Psi(x_i, k_{i\perp}) \sim \exp\left[-\frac{1}{\lambda^2}\sum_i k_{i\perp}^2/x_i\right]$$

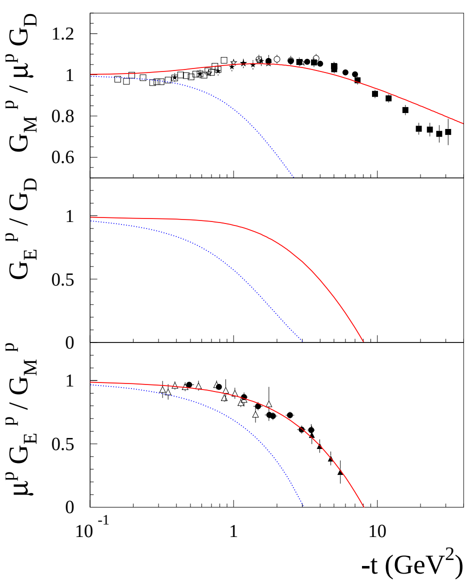
suggests

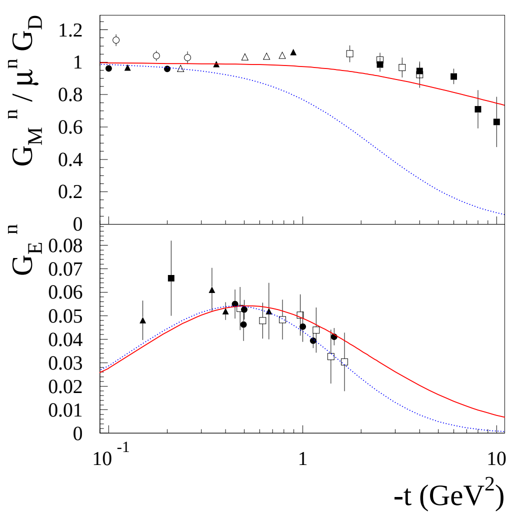
$$\mathcal{F}_a(x,t) = f_a(x) e^{\bar{x}t/2x\lambda^2}$$

λ^2 is adjusted to provide $\langle k_\perp^2 \rangle \approx (300\text{MeV})^2$



Proton $F_1(t)/F_{\text{dipole}}(t)$ form factor in Gaussian model





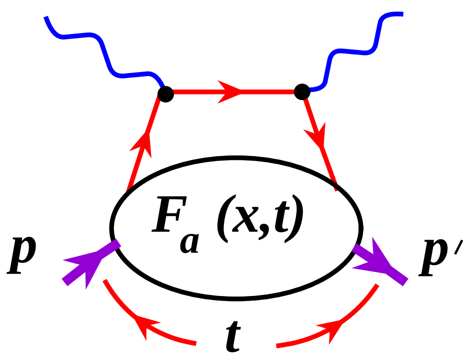
Proton & neutron form factors in Regge-type $\mathcal{F}_a(x,t) = f_a(x) x^{-t\alpha'}$ and $\mathcal{F}_a(x,t) = f_a(x) x^{-t\alpha'(1-x)}$ models

	M_2^q (MRST2002)	$2\,J^q$ (R2 model)	$2\,J^q$ (lattice)
u	0.40	0.63	0.734 ± 0.135
d	0.22	-0.06	-0.085 ± 0.088
s	0.03	0.03	
$u + d + s$	0.65	0.60	0.65 ± 0.16

Estimate of $2\,J^q$ (second column) for different quark flavors at scale $\mu^2 = 1$ GeV^2 in modified Regge model. Using MRST2002 for forward parton distributions, yields total quark momentum contributions M_2^q (first column). Third column shows the quenched lattice QCD results of [**?**], extrapolated to the physical pion mass, for $2\,J^u$ and $2\,J^d$

Nucleon form-factors from generalized parton distributions. M. Guidal (Orsay, IPN) , M.V. Polyakov (St. Petersburg, INP & Liege U.) , A.V. Radyushkin (Old Dominion U. & Jefferson Lab), M. Vanderhaeghen (Jefferson Lab & William-Mary Coll.) JLAB-THY-04-286, Oct 2004. 18pp. Published in Phys.Rev.D72:054013,2005 e-Print Archive: hep-ph/0410251

Application to wide-angle Compton scattering



Handbag term is ≈ given by

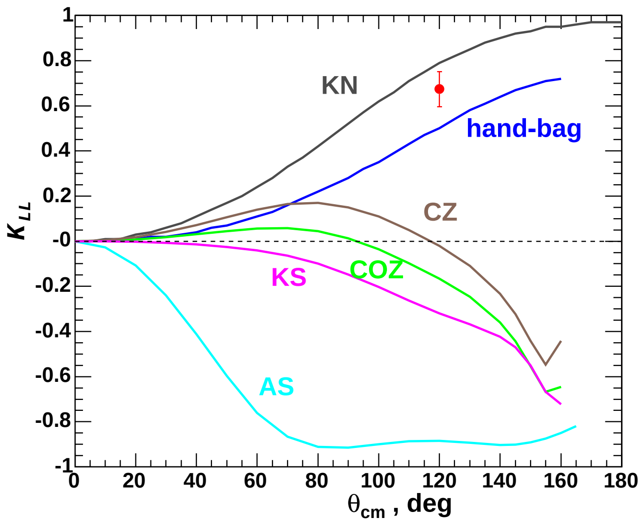
$$\left[\sum_a e_a^2 R_V^a(t)\right]^2 \left.\frac{d\sigma}{dt}\right|_{KN}$$

New form factors

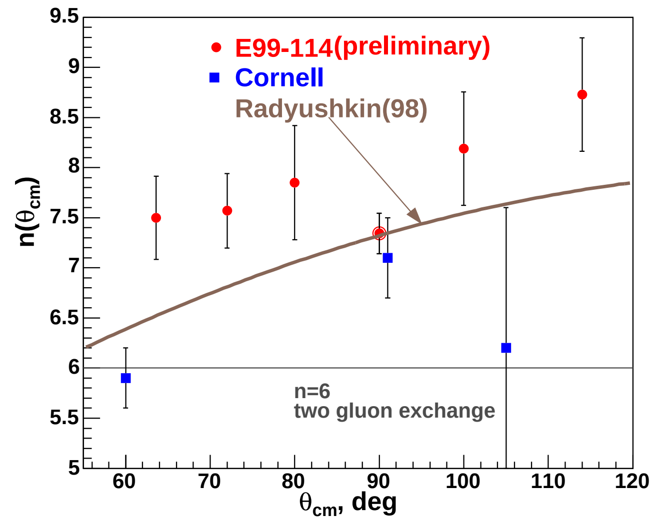
$$R_V^a(t) = \int_0^1 \frac{\mathcal{F}^a(x,t)}{x}\, dx$$

NB: $R_V^a(t)$ is obtained from the same NPD as for FF

Polarization transfer coefficient K_{LL}



Cross section scaling parameter



Comparison of JLab data with theoretical predictions

Polarization transfer in proton Compton scattering at high momentum transfer.
By Jefferson Lab Hall A Collaboration (D.J. Hamilton et al.) JLAB-PHY-04-280, Oct 2004. 4pp.
Published in Phys.Rev.Lett.94:242001,2005 e-Print Archive: nucl-ex/0410001

Major theoretical reviews on GPDs during 10 years since 1996

OFF FORWARD PARTON DISTRIBUTIONS.
Xiang-Dong Ji (Maryland U.) UMD-PP-98-092, DOE-ER-40762-144, Feb 1998. 29pp.
Published in J.Phys.G24:1181-1205,1998
e-Print Archive: hep-ph/9807358

HARD EXCLUSIVE REACTIONS AND THE STRUCTURE OF HADRONS.
K. Goeke (Ruhr U., Bochum), Maxim V. Polyakov (Ruhr U., Bochum & St. Petersburg, INP), M. Vanderhaeghen (Mainz U., Inst. Kernphys.) Jun 2001. 114pp.
Published in Prog.Part.Nucl.Phys.47:401-515,2001
e-Print Archive: hep-ph/0106012

GENERALIZED PARTON DISTRIBUTIONS.
M. Diehl (DESY) DESY-THESIS-2003-018, Jul 2003. 226pp.
Published in Phys.Rept.388:41-277,2003
e-Print Archive: hep-ph/0307382

UNRAVELING HADRON STRUCTURE WITH GENERALIZED PARTON DISTRIBUTIONS.
A.V. Belitsky (Arizona State U.), A.V. Radyushkin (Old Dominion U. & Jefferson Lab)
JLAB-THY-04-34, Apr 2005. 370pp.
Published in Phys.Rept.418:1-387,2005
e-Print Archive: hep-ph/0504030

Something has been done. But more work needed!