Lattice Studies of Baryon Resonances

- HADRON SPECTRUM COLLABORATION

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- Goals: Solve QCD
  - Theoretically determine the mass spectrum baryons, meson, hybrids, $\cdots$

- Means: Lattice QCD
  - Lots of interpolating field operators
  - Anisotropic lattices: $a_t = \frac{1}{3} a_s$
  - Variational method
  - Diagonalize matrices of correlations functions
  - Extract energies from eigenvalues of matrices.

I=$\frac{1}{2}$ BARYON SPECTRUM

- **USQCD Resources & Lattices**
- **Spin on the lattice**
- **A lattice view of the physical spectrum**
- **Variational method**
- **Pattern of low-lying states at two pion masses.**
- **Evidence for spin $\frac{5}{2}$**
- **Summary**

USQCD

Collaboration of collaborations

- $N_f = 2$ simulations for m_π = 416 MeV used
  - QCDOC Brookhaven National Laboratory
  -
  - BlueGene Teragrid Resource San Diego Supercomputer Center
- $N_f = 2$ simulations for m_π = 572 MeV used
  - Jaguar Cray XT3 National Center for Computational Science, Oak Ridge National Laboratory.
- and the Chroma software system (Edwards *et al.*)

Lattices for $N_f = 2$ SPECTRUM

- Szymanzik improved action
- u & d quarks with equal masses
- m_π = 416 MeV anisotropic lattice $24^3 \times 64$ (362 configs)
- a_s/a_t = 3 anisotropy
- a_s = 0.108(7) fm,
- m_π = 572 MeV anisotropic lattice $24^3 \times 64$ (430 configs)
- a_s/a_t = 3 anisotropy
- a_s = 0.113(7) fm,
- Smeared links, smeared quark fields

Quantum numbers of states

- Spins $\mathbf{J}^2$ and J_z are not diagonal.
- Irreducible representations (irreps) of double octahedral group are diagonal.
- Recover states of good $\mathbf{J}^2$ and J_z as patterns of octahedral irreps.

$$\sum_x \langle 0| \mathcal{T} B_k^{(\Lambda\lambda)}(\mathbf{x},t)\overline{B}_{k'}^{(\Lambda',\lambda')}(0)\,|0\rangle =$$

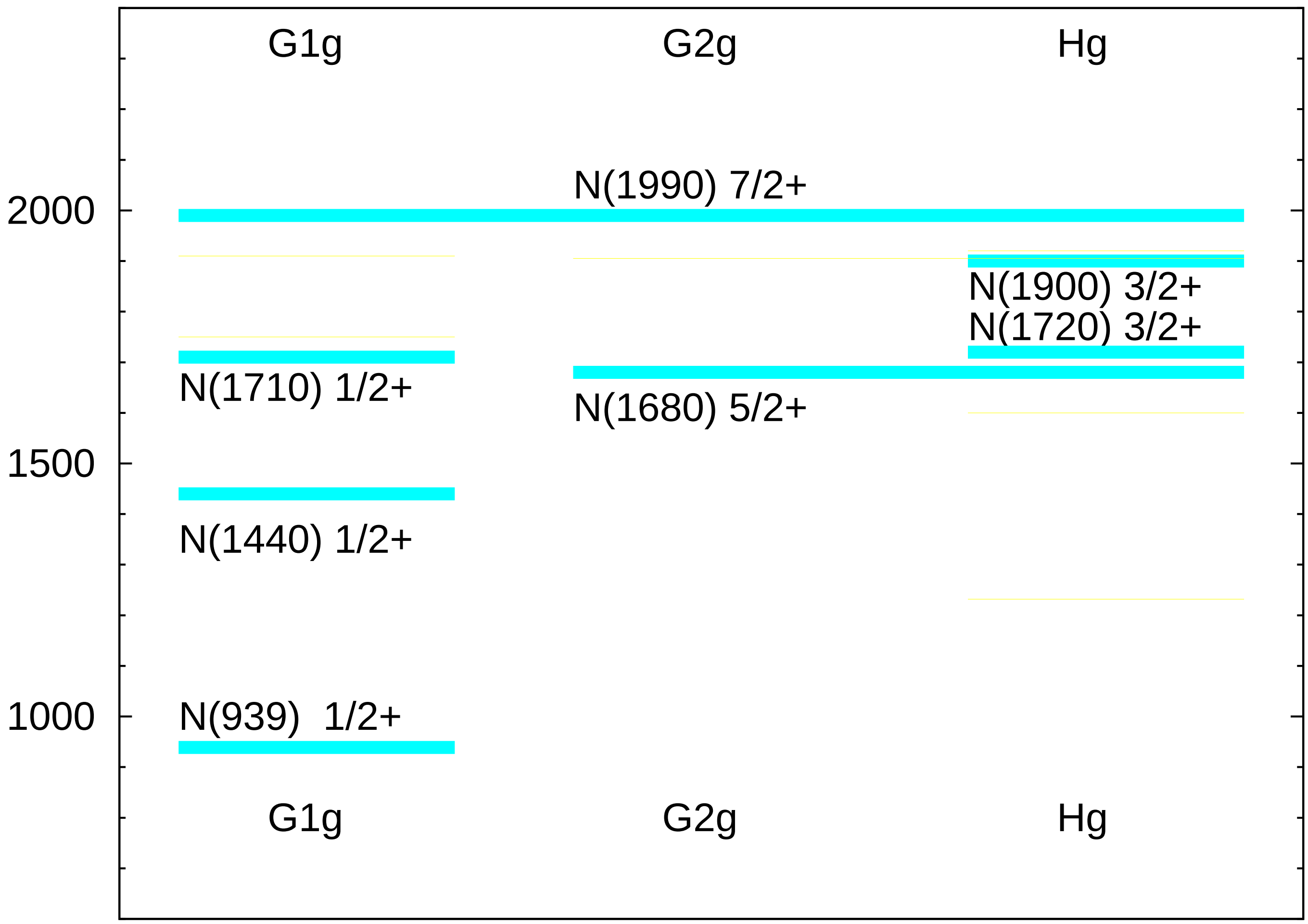
$$C_{kk'}^{(\Lambda,\lambda)}(t)\delta_{\mathbf{\Lambda\Lambda'}}\delta_{\lambda\lambda'}$$

Double octahedral group, $\mathcal{O}^D$ and subduction of J.

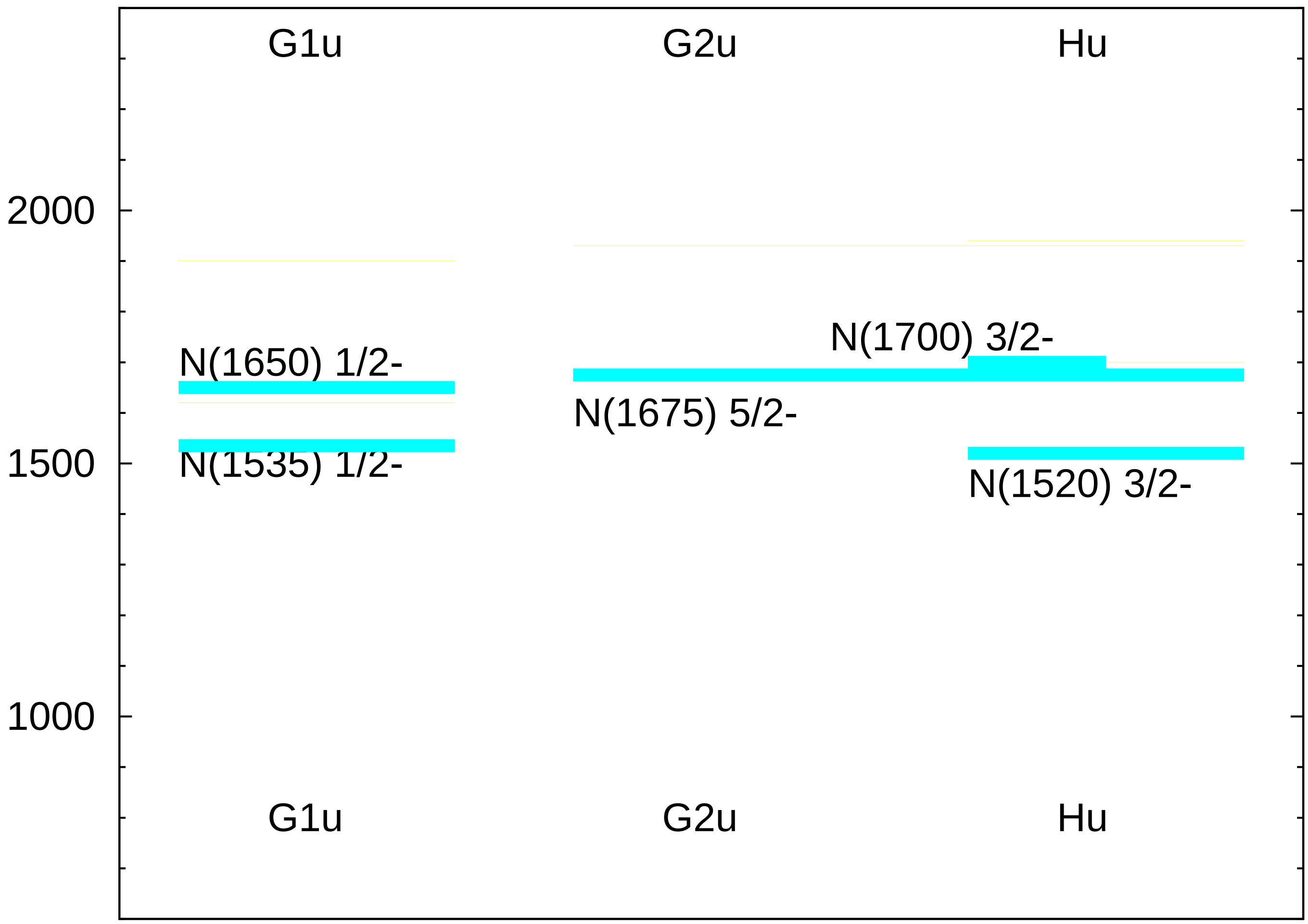
IR	Parity	IR Dimension	J $\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
G_{1g}	$+1$	2	1			1
H_g	$+1$	4		1	1	1
G_{2g}	$+1$	2			1	1
G_{1u}	-1	2	1			1
H_u	-1	4		1	1	1
G_{2u}	-1	2			1	1

- Spin $\frac{1}{2}$: Isolated G_1 state,
- Spin $\frac{3}{2}$: Isolated H state.
- Spin $\frac{5}{2}$: Degenerate G_2 and H states as $a \rightarrow 0$
- Spin $\frac{7}{2}$: Degenerate G_1, H and G_2 states as $a \rightarrow 0$

Physical positive-parity spectrum subduced



Physical negative-parity spectrum subduced



Need for nonlocal operators.

- **Three spin $\frac{1}{2}$ local operators can produce $S=\frac{1}{2}$ or $S=\frac{3}{2}$.**

- **Example:** $N_{121}=(u_1 d_2 - d_1 u_2)u_1$ **is $S=\frac{1}{2}$ Nucleon operator**

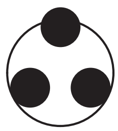
- **No higher spins can be produced.**

- **No G_2 irreps can be produced.**

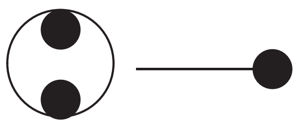
- **Nonlocal operators: provide $L=1$, $L=2$, ...**

- $L=1 \oplus S=\frac{3}{2} \rightarrow J=\frac{5}{2} \rightarrow G_2$

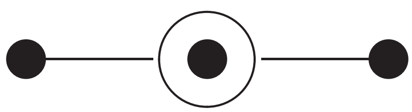
Types of nonlocal operators



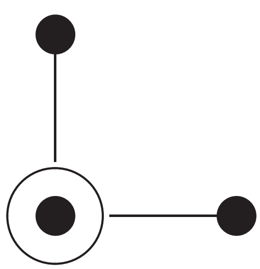
single-site



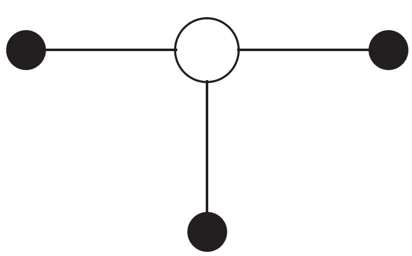
singly-displaced



doubly-displaced-I

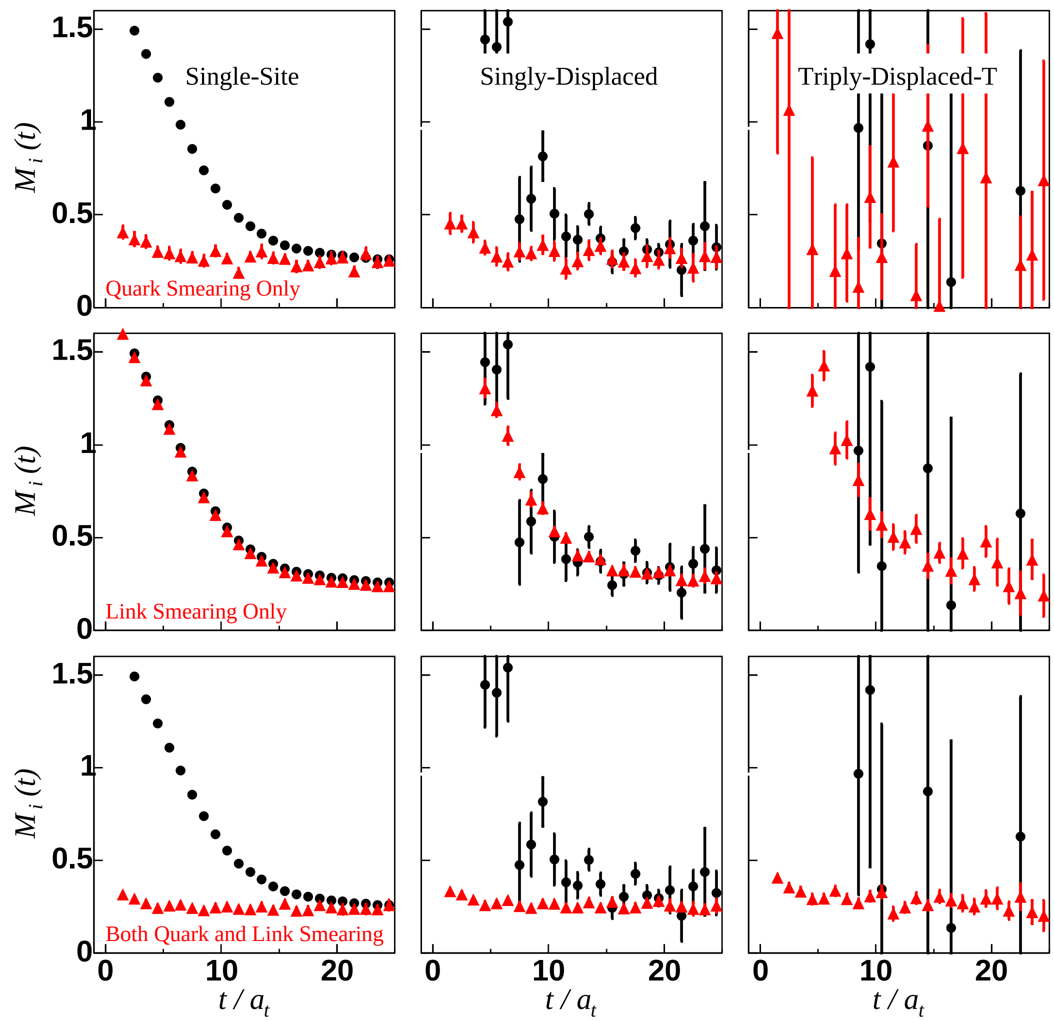


doubly-displaced-L



triply-displaced-T

Quark and link smearing



Effective masses $M(t)$ for unsmeared (circles) and smeared (triangles) operators: single site (left), singly displaced (center) , triply displaced T (right).

- **Top row: quark-field smearing only.**
- **Middle row: link-variable smearing only.**
- **Bottom row: both quark and link smearing.**

Pruning to sets of 16 operators

- 179 G_{1g} operators
- Delete ones with high intrinsic noise in self correlators
- Delete ones that are too parallel
- Delete ones with high condition numbers: $\lambda_{max}/\lambda_{min}$
- Keep 16 most promising operators

Lattice Charge-Conjugation Symmetry

$$C^{(\Lambda\lambda)}_{kk'}(t) = \mathcal{P}_{k'}\delta_{\mathbf{P},0}\Big[\sum_n \theta(t)\langle 0|B^{(\Lambda\lambda)}_k|n\rangle\langle n|\overline{B}^{(\Lambda\lambda)}_{k'}|0\rangle e^{-E_n t}$$
$$- \sum_{\bar{n}} \theta(-t)\langle 0|\overline{B}^{(\Lambda\lambda)}_{k'}|\bar{n}\rangle\langle \bar{n}|B^{(\Lambda\lambda)}_k|0\rangle e^{E_{\bar{n}} t}\Big], \tag{1}$$

Charge conjugation relations $|\bar{n}\rangle = \mathcal{C}\,|n\rangle\, e^{i\phi}$ produce a relation between correlation functions

$$C^{(\Lambda\lambda)}_{kk'}(t) = \delta_{\mathbf{P},0}\sum_n \Big[\theta(t)\mathcal{P}^{(\Lambda)}_{k'}\langle 0|B^{(\Lambda\lambda)}_k|n\rangle\langle n|\overline{B}^{(\Lambda\lambda)}_{k'}|0\rangle e^{-E_n t}$$
$$-\eta_t\theta(T-t)\mathcal{P}^{(\Lambda_c)}_{k'}\langle 0|B^{(\Lambda_c\lambda_c)}_k|n\rangle^*\langle n|\overline{B}^{(\Lambda_c\lambda_c)}_{k'}|0\rangle^* e^{-E_{\bar{n}}(T-t)}\Big]. \tag{2}$$

The forward propagating signal of a correlation function is equal to the backward propagating signal of the parity-reversed, complex-conjugated correlation function within the factor $-\eta_t$, i.e.,

$$C^{(\Lambda\lambda)}_{kk'}(t) = -\eta_t C^{(\Lambda_c\lambda_c)*}_{kk'}(T-t). \tag{3}$$

Improved statistics

- **For + parity operators use $\overline{B}^{(\Lambda)}$.**
- **For - parity operators use $\overline{B}^{(\Lambda_c)}$.**
- **Each provides a correlation function in $0 < t < T/2$ for BOTH parities.**
- **They are uncorrelated samples: $\approx$ 10 time slices apart.**
- **Roughly double the number of gauge configurations.**

Variational method

Diagonalize 16×16 matrices of correlation functions $C_{kk'}(t)$ to extract spectrum of energies.

1.) Calculate matrices of correlation functions

$$\sum_x \langle 0| \mathcal{T} B_k^{(\Lambda\lambda)}(\mathbf{x},t) \overline{B}_{k'}^{(\Lambda',\lambda')}(0) |0\rangle = C_{kk'}^{(\Lambda,\lambda)}(t)\delta_{\mathbf{\Lambda\Lambda'}}\delta_{\lambda\lambda'}$$

2.) Solve generalized eigenvalue eq.

$$\sum_{k'} \widetilde{C}_{kk'}^{(\Lambda)}(t) v_{k'}^{(n)} = \alpha^{(n)}(t,t_0) \sum_{k'} \widetilde{C}_{kk'}^{(\Lambda)}(t_0) v_{k'}^{(n)},$$

3.) Obtain principal eigenvalues

$$\alpha^{(n)}(t,t_0) \simeq e^{-E_n(t-t_0)}\Big(1+\mathcal{O}(e^{-|\delta E|t})\Big),$$

Principal correlators

Eigenvectors diagonalize the correlation matrix:

$$v_k^{(n)T}(t,t_0)\widetilde{C}_{kk'}^{(\Lambda)}(t)v_{k'}^{(n')}(t,t_0) = \alpha^{(n)}(t,t_0)\delta_{nn'}.$$

Eigenvalue is principal correlator:

$$\widetilde{C}_{nn}^{(\Lambda)}(t) = \alpha^{(n)}(t,t_0).$$

based on optimized operators:

$$\overline{\mathcal{O}}_n^{(\Lambda\lambda)} = \sum_k \widetilde{v}_k^{(n)}(t,t_0)\overline{B}_k^{(\Lambda\lambda)}.$$

Fit principal correlators

- **two exponential fits for all but G_{1u} channel**

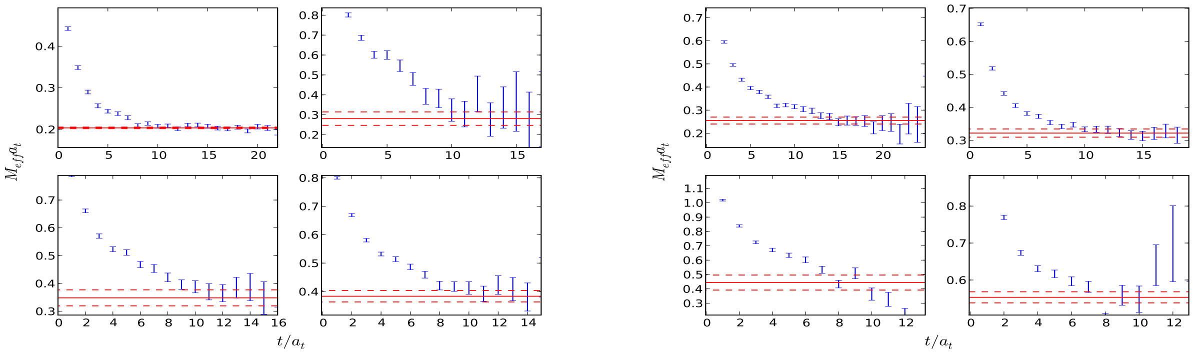
$$C_{fit}(t) = Ae^{-E'(t-t_0)} + (1-A)e^{-E(t-t_0)}$$

- **three-exponential fits in G_{1u} channel**

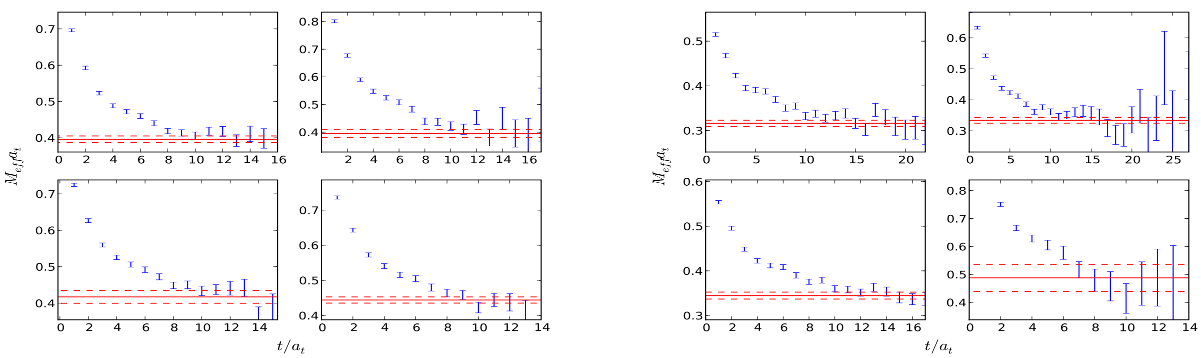
$$C_{fit}(t) = Ae^{-E'(t-t_0)} + (1-A-B)e^{-E(t-t_0)} + Be^{E_{G_{1g}}(t-t_0)}$$

The backward state in the G_{1u} channel is the nucleon state and it can contribute significantly because it is less massive and the lattice length is short (owing to anisotropy).

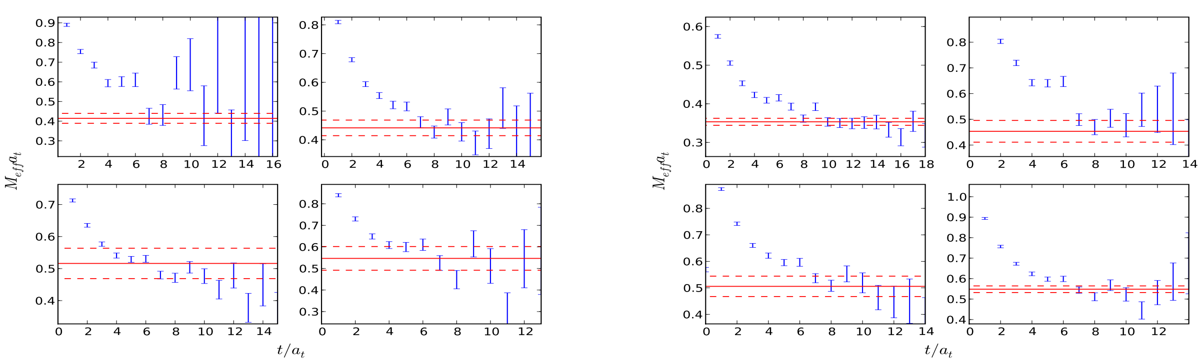
m_π = 400 MeV Effective masses



Left: G_{1g} **Right:** G_{1u}

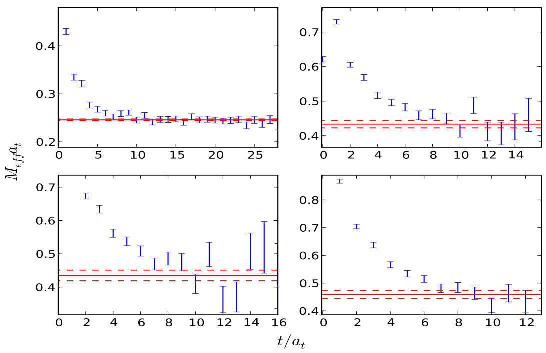


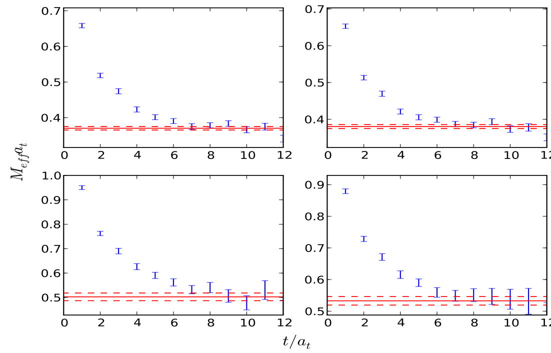
Left: H_g; **Right:** H_u



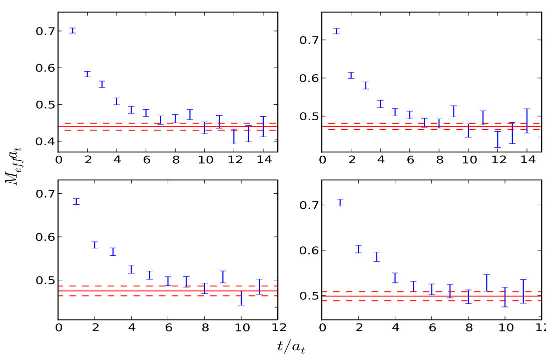
Left: G_{2g}; **Right:** G_{2u}

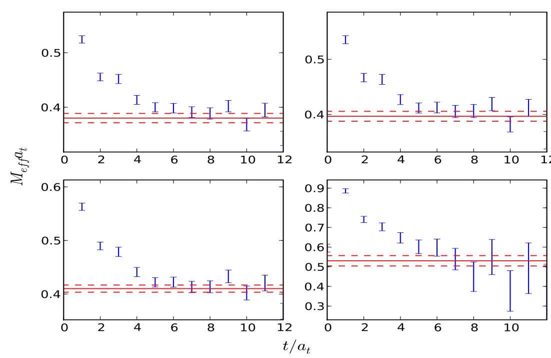
m_π = 572 MeV Effective masses



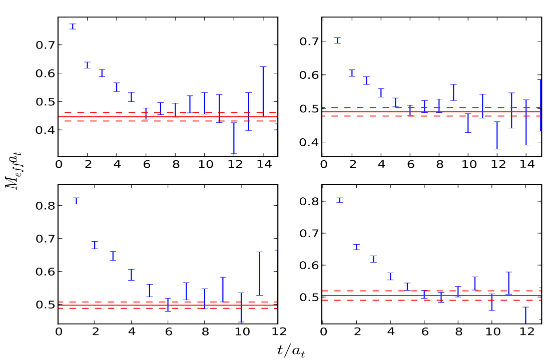


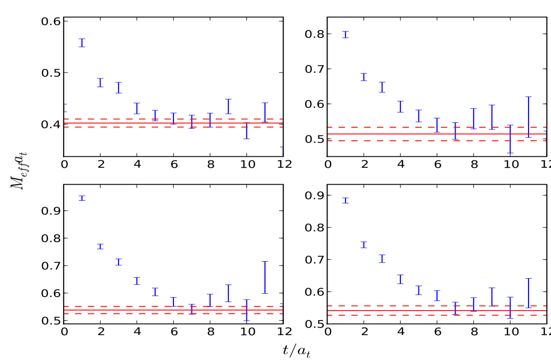
Left: G_{1g} **Right:** G_{1u}





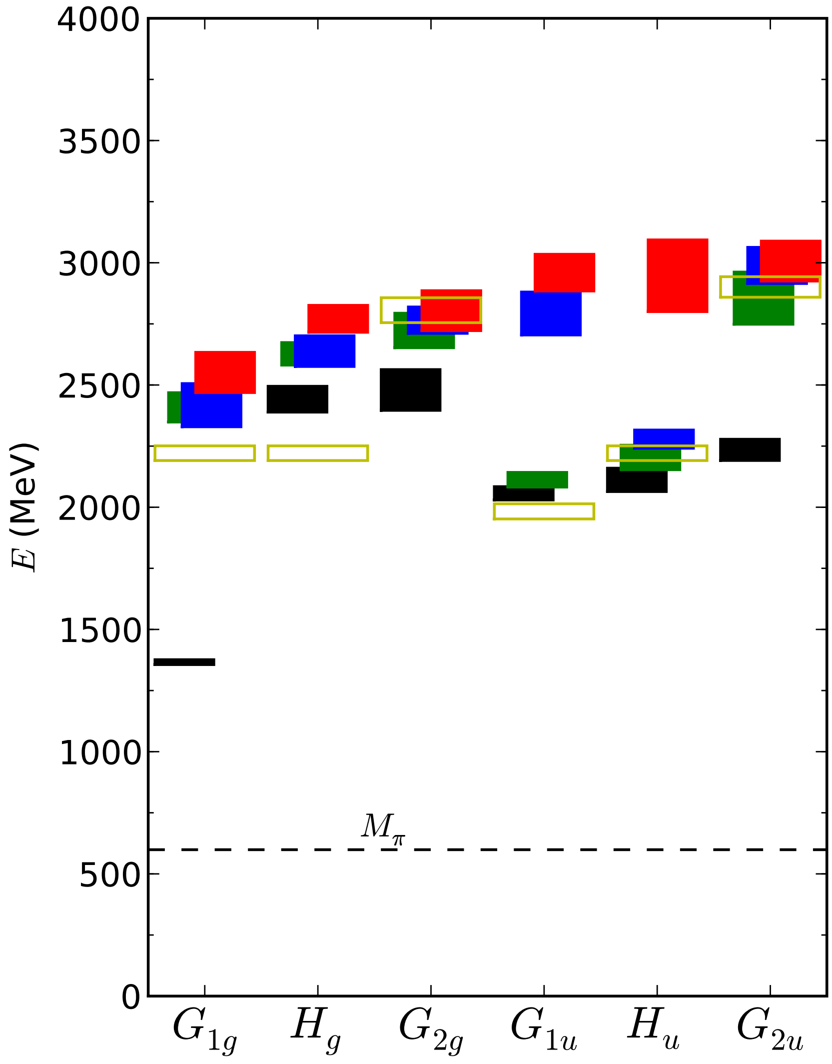
Left: H_g; **Right:** H_u



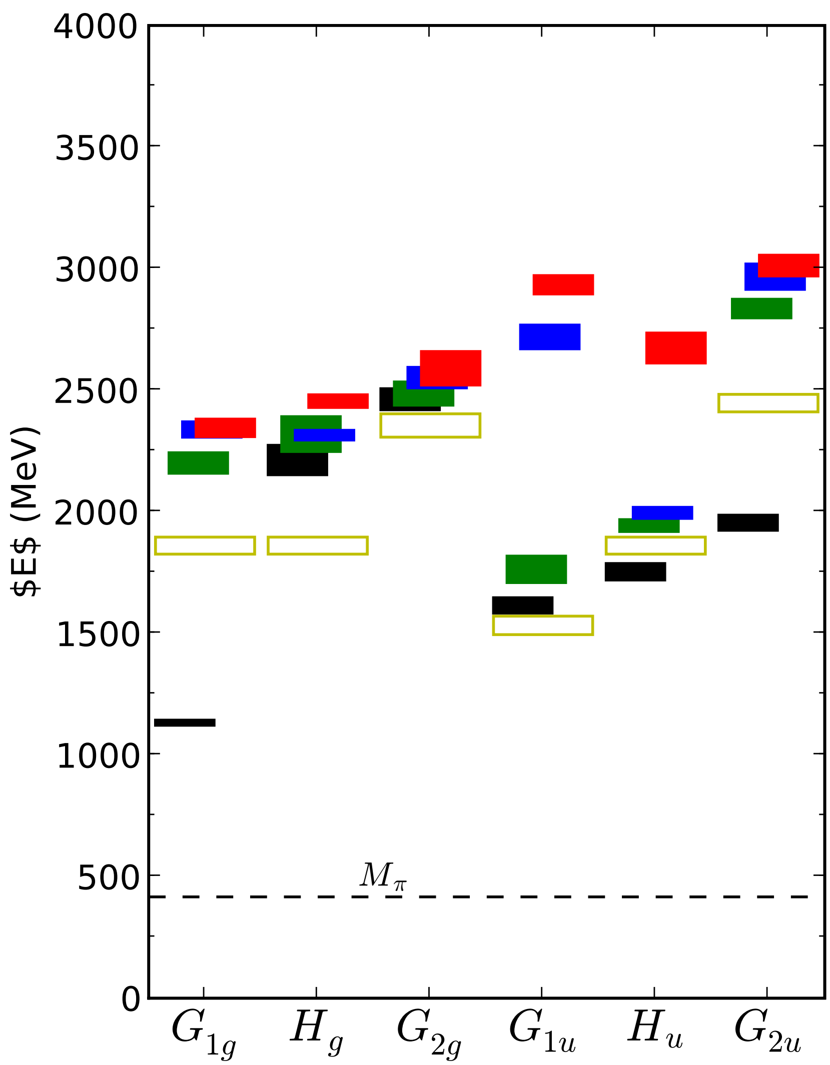


Left: G_{2g}; **Right:** G_{2u}

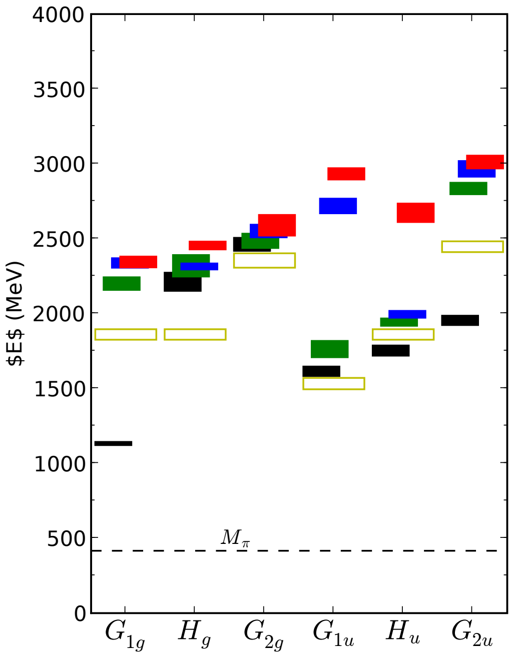
Pattern of lowest energies in each channel $m_\pi = 572$ MeV

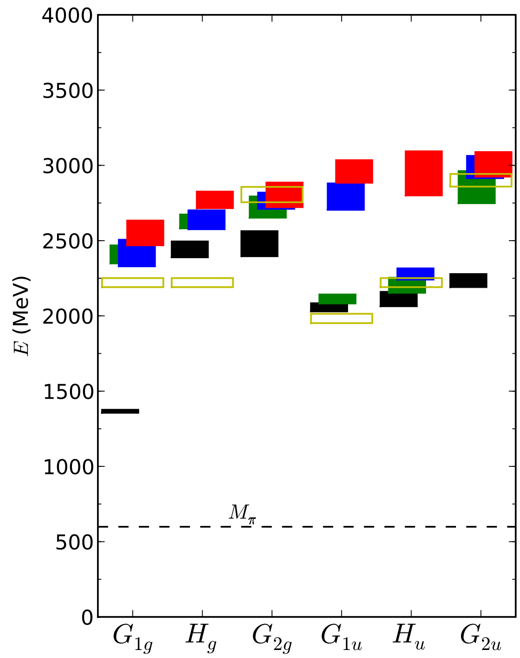


Pattern of lowest energies in each channel $m_\pi = 400$ MeV



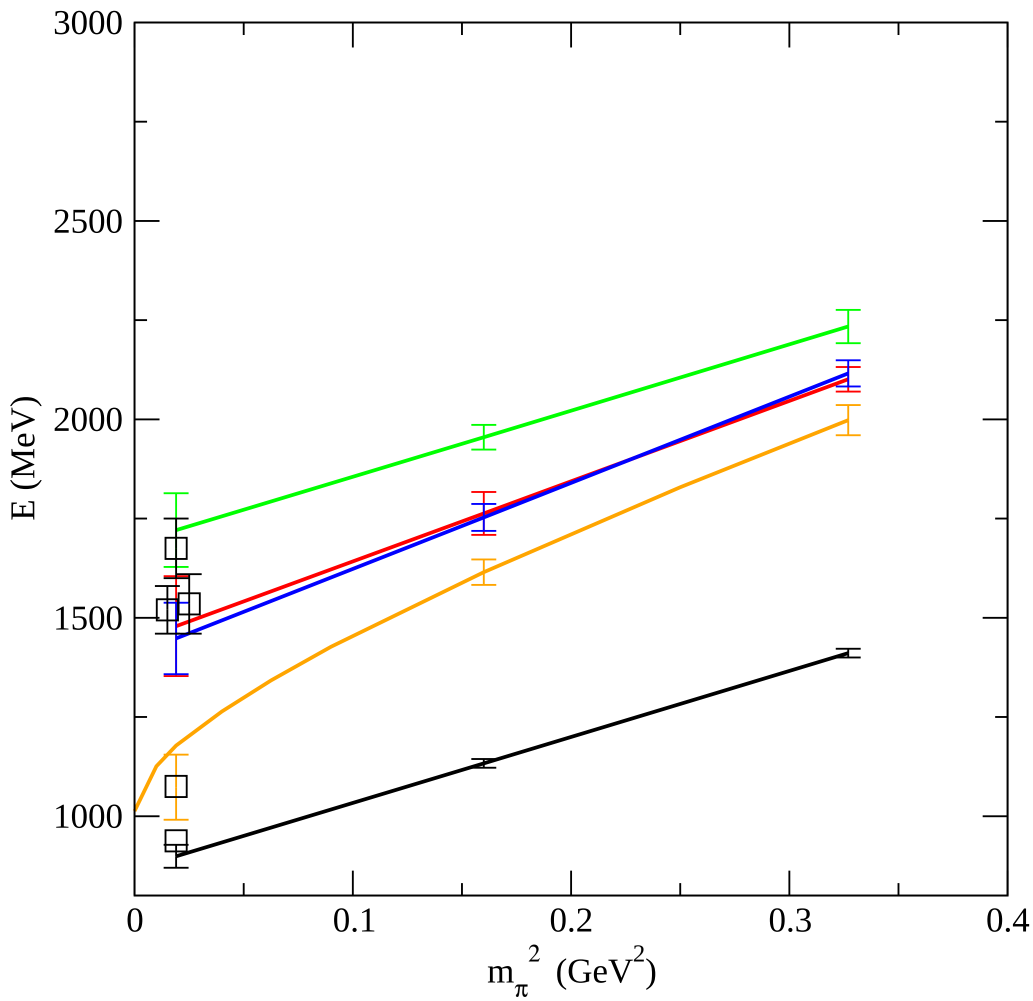
Comparison of $m_\pi = 400$ & 572 MeV





$m_\pi = 400$ MeV (left panel) and $m_\pi = 572$ MeV (right panel)

Extrapolations: $E = a + bm_\pi^2$



Lowest G_{1g}, **first excited** G_{1u}, **lowest** H_u , **lowest** G_{2u}. **Lowest** G_{1u} **state using** $E = a + m_\pi + bm_\pi^2$. **Degenerate** $G_{1u}(2)$ **and** $H_u(1)$ **energies matches** $\frac{3}{2}^-(1520) \approx \frac{1}{2}^-(1535)$

Signature of $\frac{5}{2}^-$ state

m_π	State	E
400	$H_u(2)$	1943(24)
400	$G_{2u}(1)$	1955(31)
572	$H_u(2)$	2236(36)
572	$G_{2u}(1)$	2234(42)

- Second H_u energy is degenerate with first G_{2u} energy at both pion masses
- Pattern $E(H_u(1)) < E(H_u(2)) = E(G_{2u}(1))$ matches $\frac{3}{2}^-(1520) < \frac{5}{2}^-(1675)$.

Progress

- $N_f = 2$ QCD at $m_\pi = 400$ and 572 MeV, $a_s \approx$0.1 fm.

- Lowest $I = \frac{1}{2}$ energies

- Many operators pruned to sets of 16.

- 24 energies at each m_π

- Pattern of lowest energies for negative parity is similar to the pattern of lowest physical resonance states.

- Evidence for $\frac{5}{2}^-$ state.
  - Partner G_{2u} and H_u states

- Energies above thresholds for multihadron states; Excited G_{1g} high.

Progress on related fronts

- **Form factor of G_{1g} first excited state at m_π =720 MeV.**
  - **arXiv:0803.3020 - First Lattice Study of the N-$P_{11}(1440)$ Transition Form Factors Huey-Wen Lin, Saul D. Cohen, Robert G. Edwards, David G. Richards**

- **E2/M1 ratio for $\Delta(1232)$**
  - **arXiv:0810.3976; arXiv:0710.4621: Delta-baryon electromagnetic form factors in lattice QCD C. Alexandrou, *et al.***

- **Charmonium**
  - **arXiv:0707.4162 Charmonium excited state spectrum in lattice QCD J.J. Dudek, R.G. Edwards, N. Mathur, D.G. Richards**

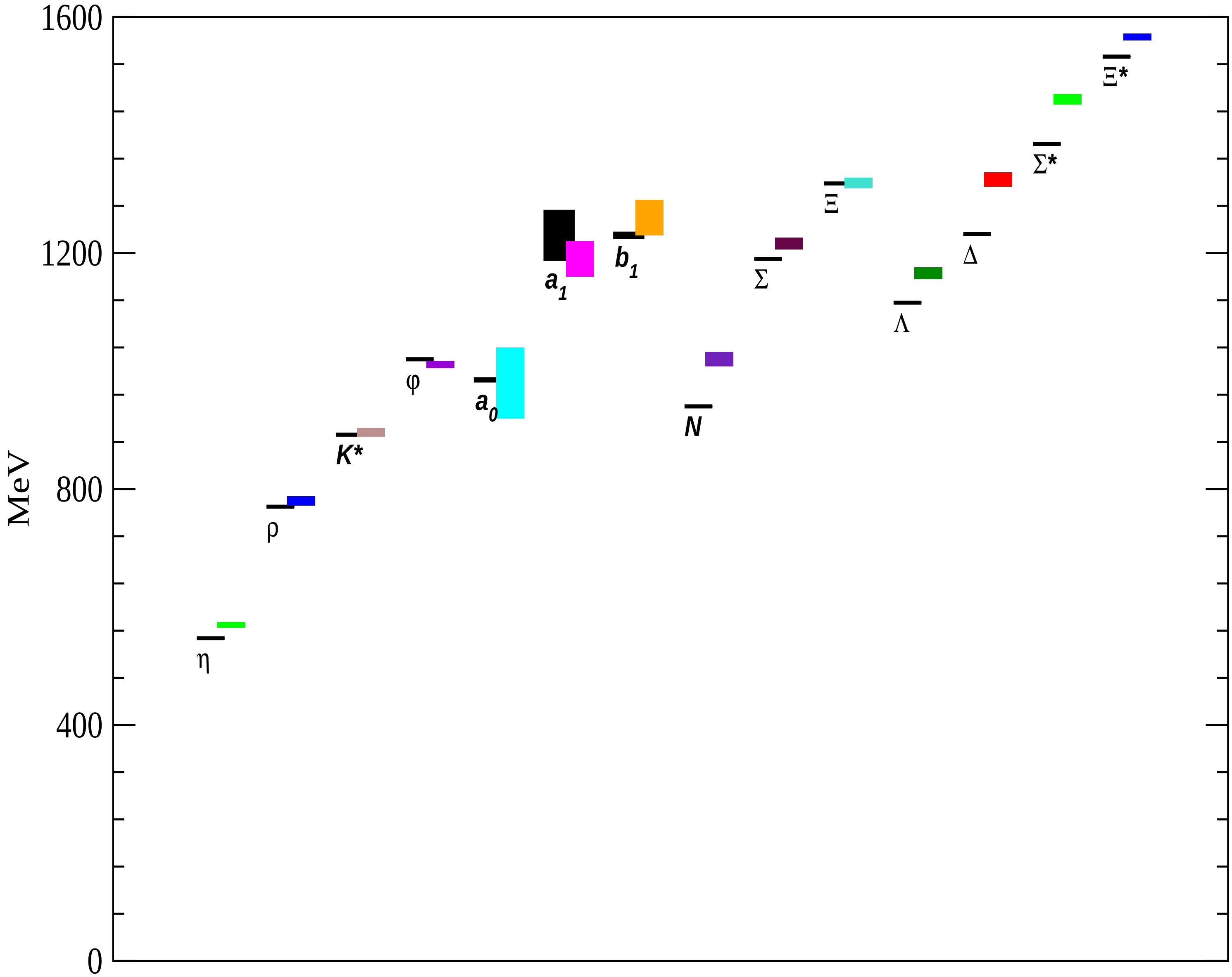
Prospectus

- $N_f = 2 + 1$ **QCD**

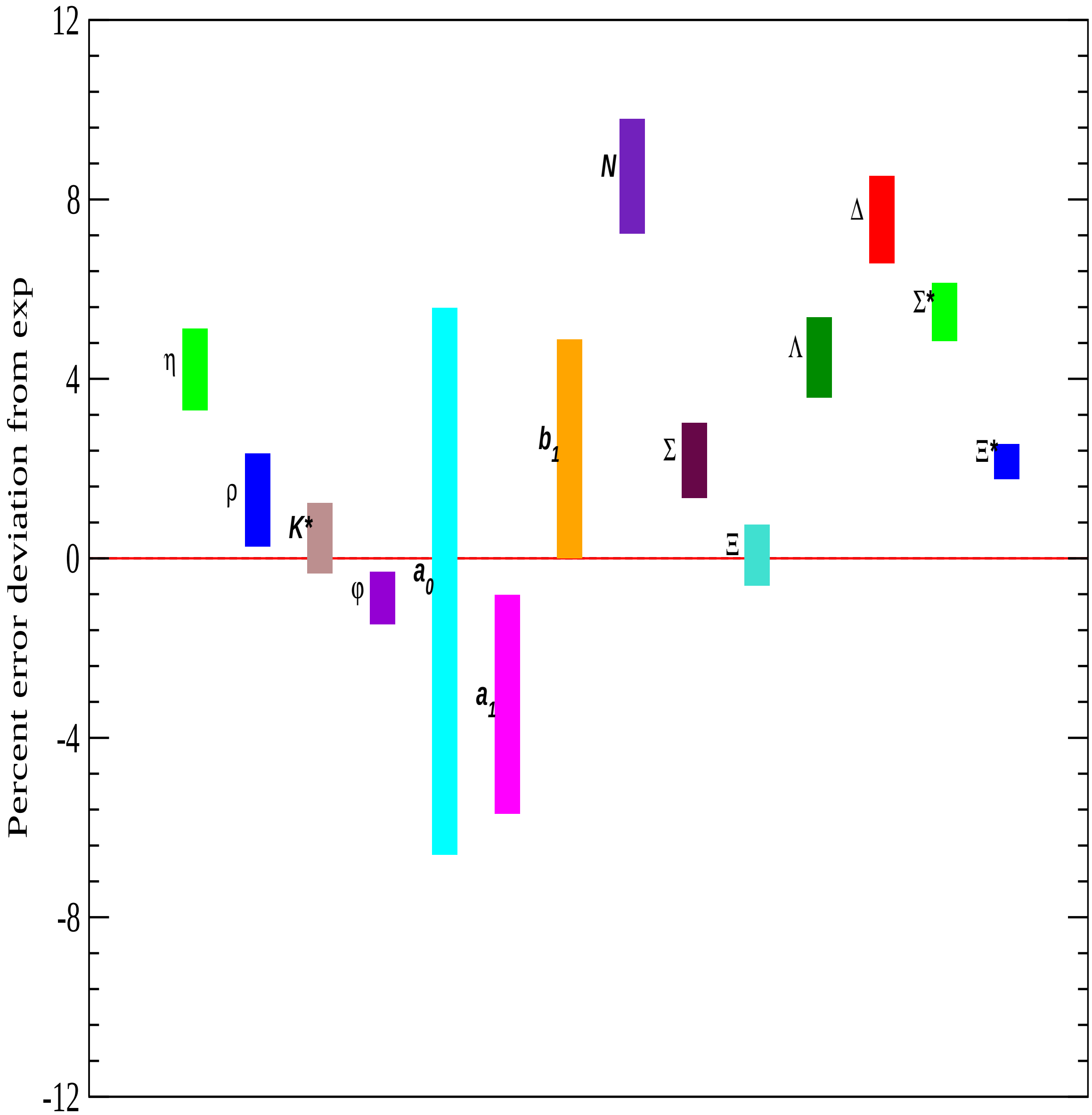
- **Multihadron operators**
  - **Include pion & baryon operators.**
  - **All-to-all propagators needed , e.g., for annihilation of** $\overline{q}$
  - **New methods: pattern-to-pattern propagators for many patterns**

- **Additional lattice volumes**

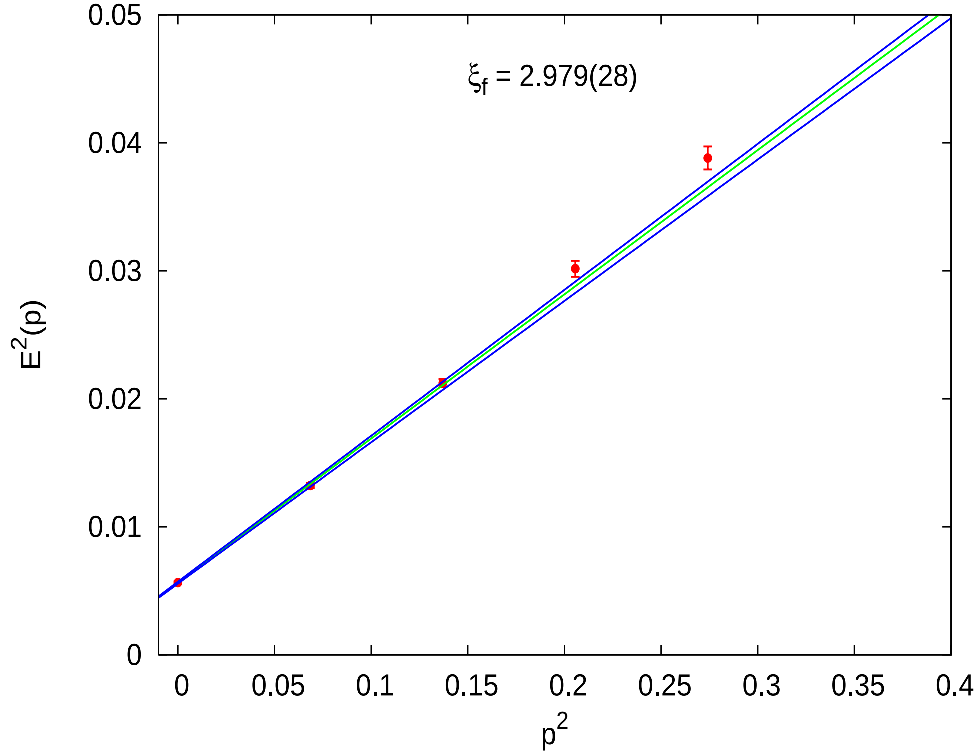
- **Lower pion masses: close to physical limit.**

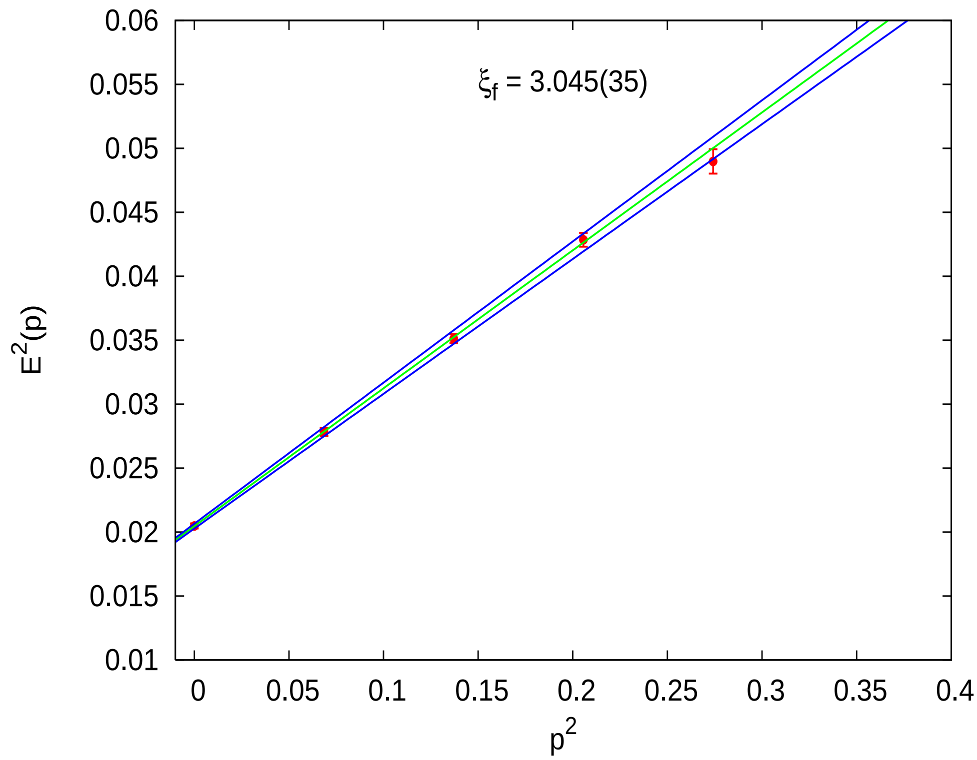


$N_f = 2 + 1$ **masses versus experiment.**



$N_f = 2 + 1$ **masses versus experiment.**





Dispersion relation: π and ρ