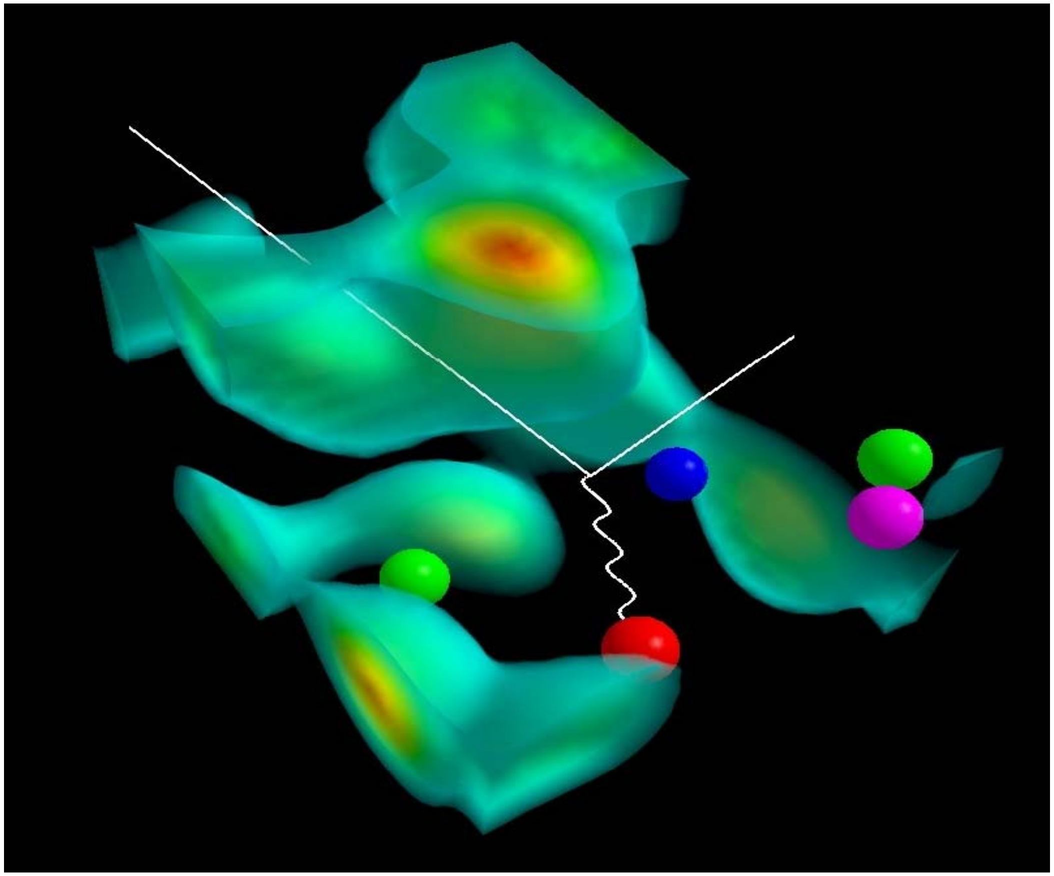
Experiment and Lattice : Building a Picture of Hadron Structure



Anthony W. Thomas

Synergy at JLab : November 22nd 2008

The Open Questions

- **Does lattice QCD precisely reproduce the best experimental data**
  - spectroscopy, form factors, DIS, GPDs?

- **Are some observables more likely to yield interesting constraints than others?**

- **What insight can LQCD yield into how QCD works?**

- **Can it give us physical insight?**

- **Are we able to take the lessons learnt in hadron structure and use them to understand nuclear structure better?**

Open questions (cont.)

- **LQCD opens a new axis in QCD, namely varying quark masses: what does this teach us?**
- **QQCD is typically within 10% of data, why?**
- **Chiral corrections grow with quark mass (e..g. LNA for $M_N \sim -5.6\ m_{GB}^3$) yet QQCD is limit of infinite sea quark mass?**
- **The nucleon electromagnetic form factors behave like $1/(1 + Q^2/\Lambda^2)$ with $\Lambda \sim 0.7$ GeV $\Rightarrow$ radius of convergence well below 1 GeV. What replaces the traditional power series expansion in Q^2 ?**

Formal Chiral Expansion

<u>Formal</u> expansion of Hadron mass:

$$M_N = c_0 + c_2\, m_\pi^2 + c_{LNA}\, m_\pi^3 + c_4\, m_\pi^4 + c_{NLNA}\, m_\pi^4 \ln m_\pi + c_6\, m_\pi^6 + ..$$

Mass in chiral limit

No term linear in m_π (in FULL QCD…… there is in QQCD)

First (hence "leading") non-analytic term $\sim m_q^{3/2}$ (LNA)
Source: $N \to N\pi \to N$

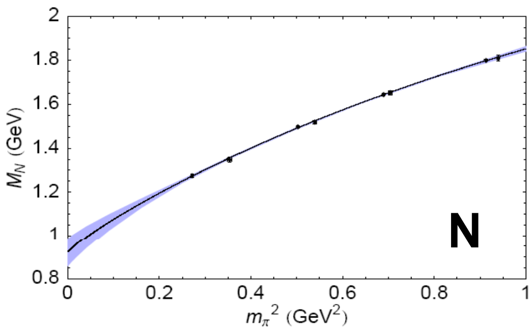
c_{LNA} MODEL INDEPENDENT

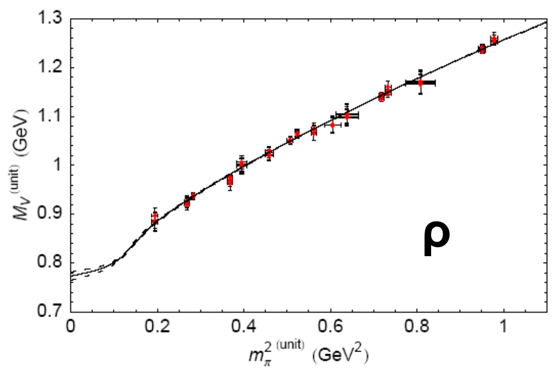
Another branch cut from $N \to \Delta\,\pi \to N$
- higher order in m_π
- hence "next-to-leading" non-analytic (NLNA)

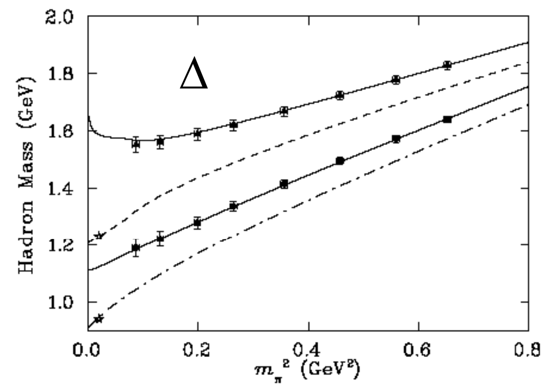
c_{NLNA} MODEL INEPENDENT

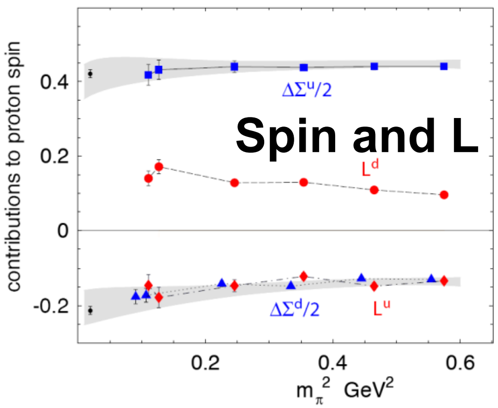
Convergence?

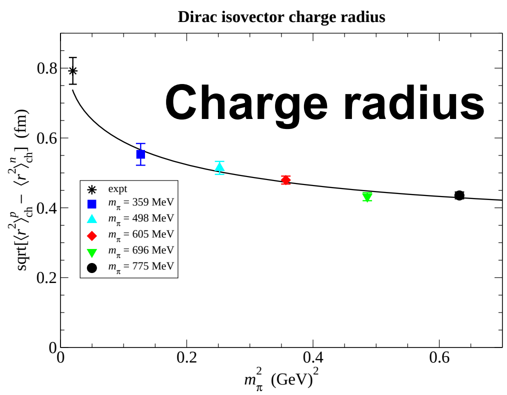
The “big picture”

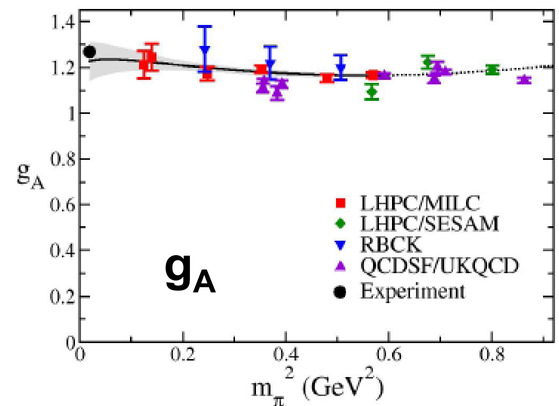


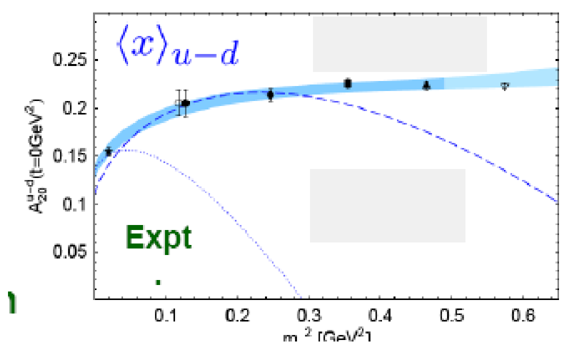








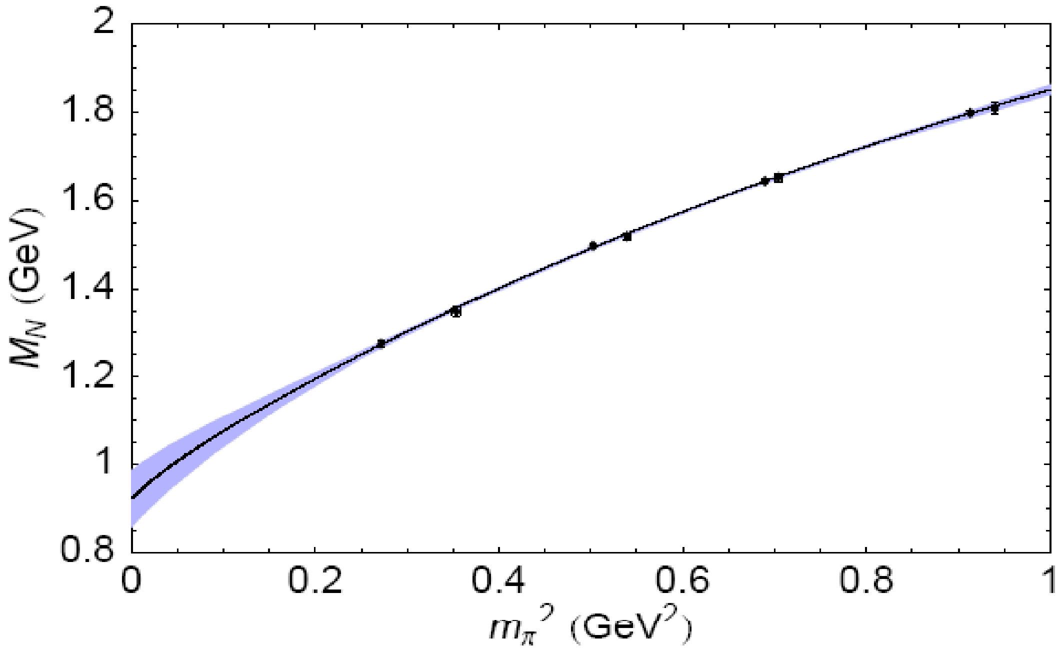




Is it believable that smooth behavior for m_π above 400 MeV is a result of a <u>different</u> accidental cancellation in every case??

$$a + b\, m_\pi^2 + c\, m_\pi^3 + d\, m_\pi^4 \ln m_\pi + m_\pi^5 + \ldots.$$

χ'al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon



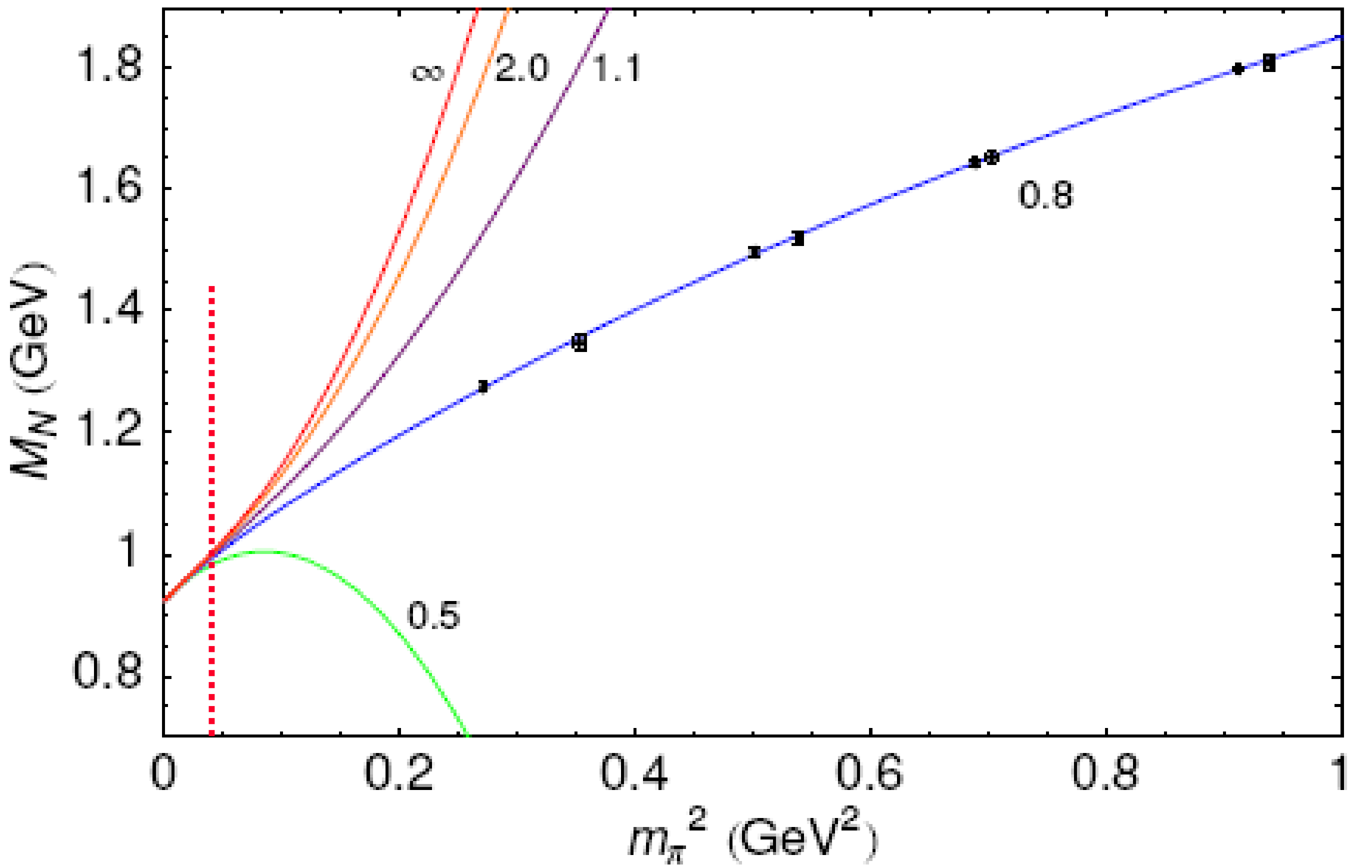
FRR give same answer to <<1% systematic error!

	Bare Coefficients				Renormalized Coefficients			
Regulator	a_0^Λ	a_2^Λ	a_4^Λ	Λ	c_0	c_2	c_4	m_N
Monopole	1.74	1.64	−0.49	0.5	0.923(65)	2.45(33)	20.5(15)	0.960(58)
Dipole	1.30	1.54	−0.49	0.8	0.922(65)	2.49(33)	18.9(15)	0.959(58)
Gaussian	1.17	1.48	−0.50	0.6	0.923(65)	2.48(33)	18.3(15)	0.960(58)
Sharp cutoff	1.06	1.47	−0.55	0.4	0.923(65)	2.61(33)	15.3(8)	0.961(58)
Dim. Reg. (BP)	0.79	4.15	+8.92	–	0.875(56)	3.14(25)	7.2(8)	0.923(51)

Leinweber et al., PRL 92 (2004) 242002

Power Counting Region?

Ensure coefficients c_0 , c_2 , c_4 all identical to 0.8 GeV fit

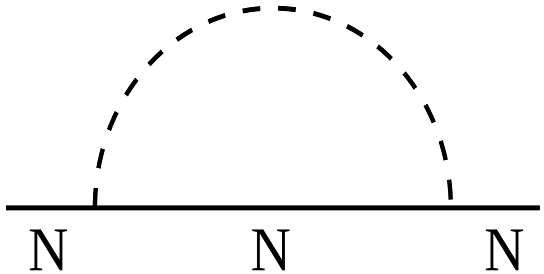


Leinweber, Thomas & Young, hep-lat/0501028

FRR works because…

- It preserves model independent LNA and NLNA behavior
- Form factor naturally yields GT discrepancy of right sign and magnitude – and therefore correct m_π^5 term!
  - i.e. correct NNLNA behavior
- N.B. Usual EFT yields this term only at two loops
- For <u>sound physical reasons</u>, FRR suppresses meson loops once m_π exceeds about 0.4 GeV
- Yields convergent series expansion over mass region covered by lattice data

Some details



$= c_{LNA}\, I_\pi \quad ; \; c_{LNA} = -3\, g_A^2/(32\, \pi\, f_\pi^2)$

$$I_\pi = \frac{2}{\pi} \int_0^\infty dk \, \frac{k^4\, u^2(k)}{k^2 + m_\pi^2}$$

$$I_\pi^{\mathrm{DIP}} = \frac{\Lambda^5 (m_\pi^2 + 4 m_\pi \Lambda + \Lambda^2)}{16 (m_\pi + \Lambda)^4} \sim \frac{\Lambda^3}{16} - \frac{5\Lambda}{16} m_\pi^2 + m_\pi^3 - \frac{35}{16\Lambda} m_\pi^4 + \ldots$$

(with dipole regulator; /// closed forms for other regulators)

Convergence from LNA to NLNA is Rapid – Using Finite Range Regularization

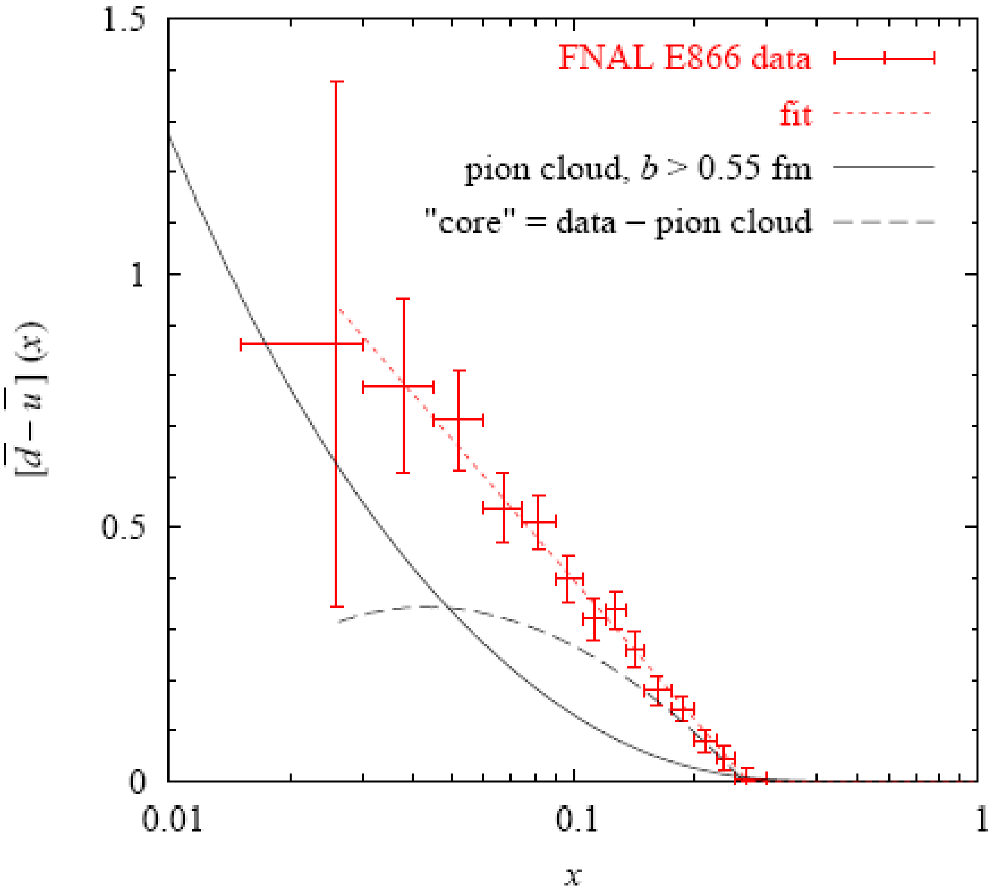
Regulator	LNA	NLNA
Sharp	968	961
Monopole	964	960
Dipole	963	959
Gaussian	960	960
Dim Reg	784	884

M_N in MeV

Pion cloud and sea quark flavor asymmetry in the impact parameter representation

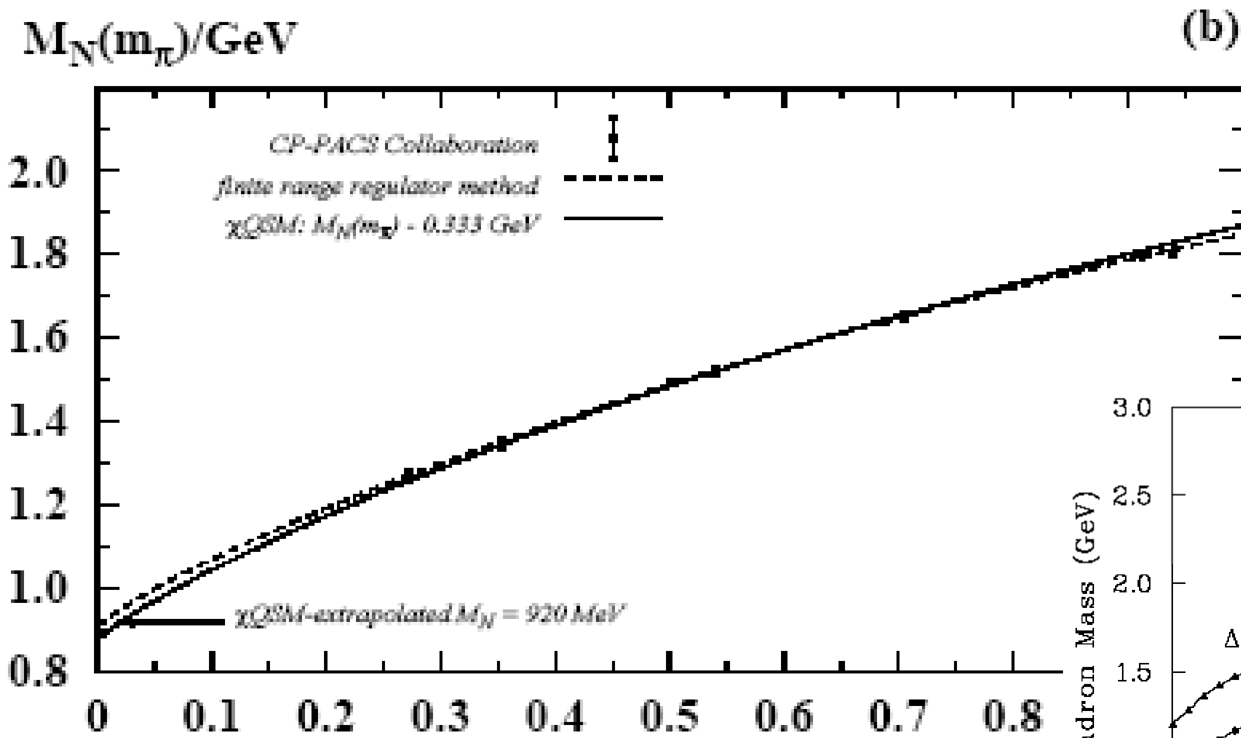
M. Strikman
Department of Physics, Pennsylvania State University, University Park, PA 16802, USA
E-mail: strikman@phys.psu.edu

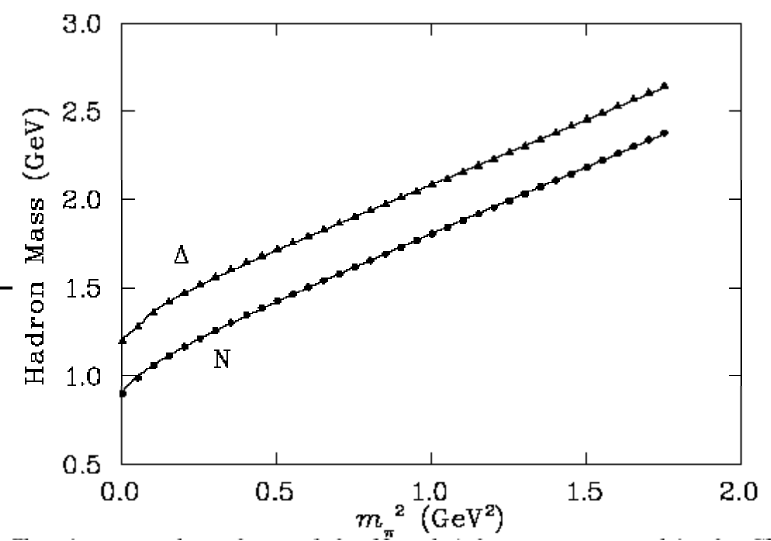
C. Weiss*
Theory Center, Jefferson Lab, Newport News, VA 23606, USA
E-mail: weiss@jlab.org



- Prediction of $\bar{d} - \bar{u} > 0$ from pion cloud 1983 **(AWT, Phys. Lett. B126, 97)**
- Here analysis establishes model independent piece, for b>0.55fm
- Inside is "non-chiral" core
- m_π > 400 MeV : pion cannot be distinguished from "core"
- chiral behavior disappears

Comparison with models:
e.g. χQSM and CBM





Goeke *et al.*, hep-lat/0505010

CBM: Leinweber et al.,
Phys.Rev.D61:074502,2000

Octet-baryon masses

SU(3) expansions plus FRR loops (π, η and K)

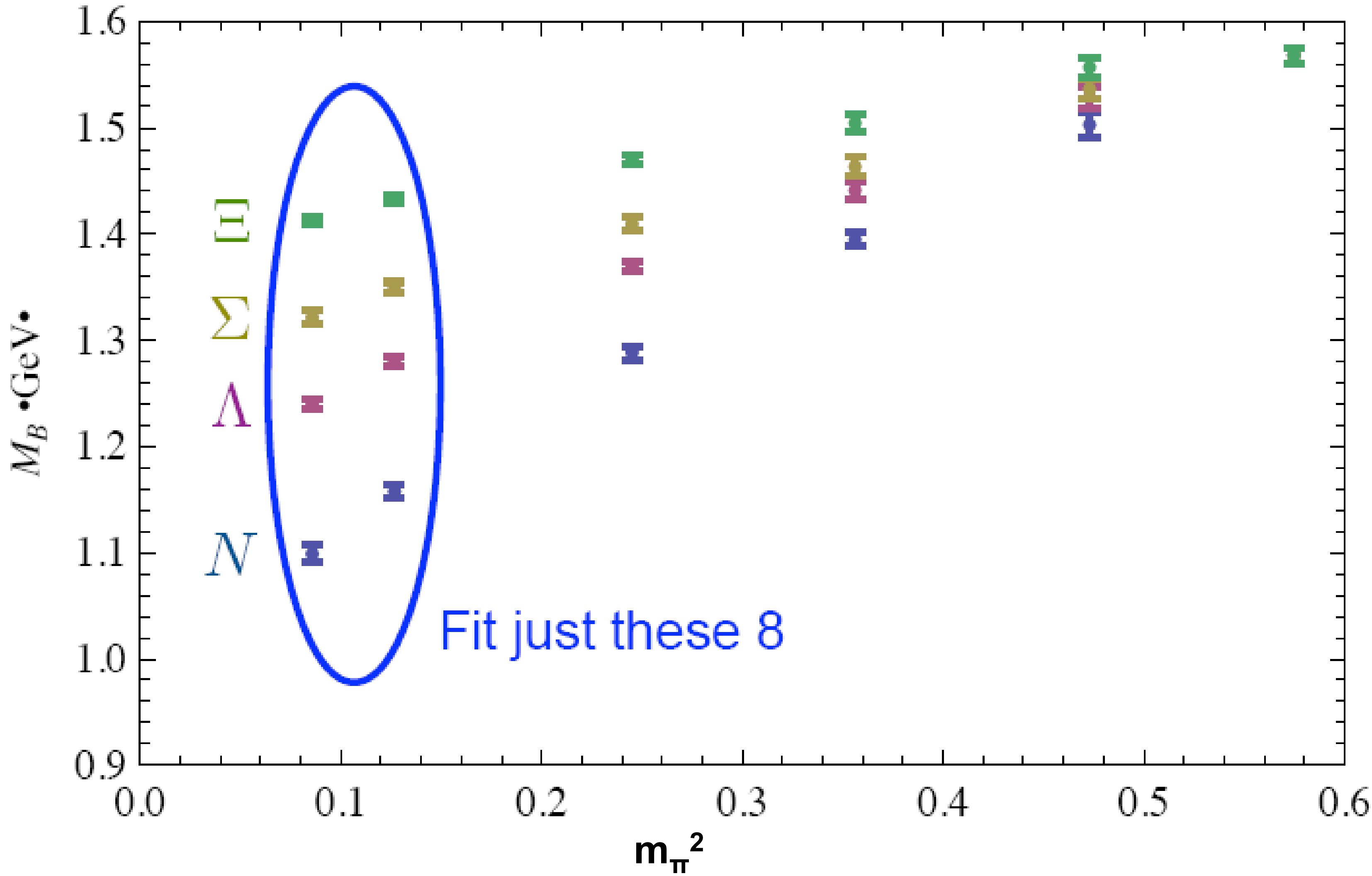
- Leading-order expansion *O*(1)

$$
\begin{aligned}
M_N &= M_0 + 2(\alpha_M + \beta_M)m_q + 2\sigma_M(2m_q + m_s) \\
M_\Lambda &= M_0 + (\alpha_M + 2\beta_M)m_q + \alpha_M m_s + 2\sigma_M(2m_q + m_s) \\
M_\Sigma &= M_0 + \frac{1}{3}(5\alpha_M + 2\beta_M)m_q + \frac{1}{3}(\alpha_M + 4\beta_M)m_s + 2\sigma_M(2m_q + m_s) \\
M_\Xi &= M_0 + \frac{1}{3}(\alpha_M + 4\beta_M)m_q + \frac{1}{3}(5\alpha_M + 2\beta_M)m_s + 2\sigma_M(2m_q + m_s)
\end{aligned}
$$

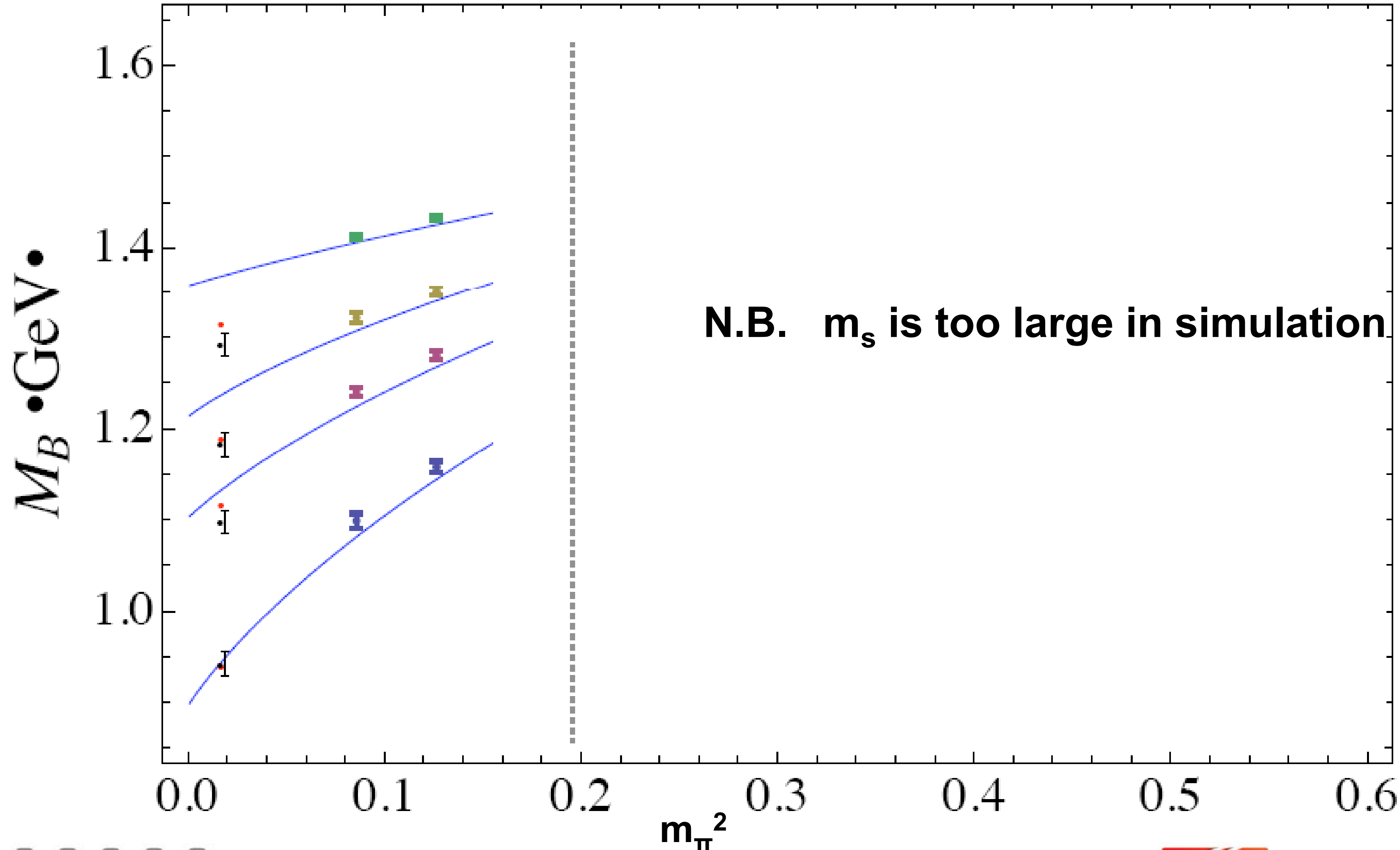
$$m_\pi^2 = 2Bm_q \quad m_K^2 = B(m_q + m_s)$$

$$m_q \to \frac{m_\pi^2}{2B}, \quad m_s \to \frac{2m_K^2 - m_\pi^2}{2B} \qquad \{\alpha, \beta, \sigma\} \to B\{\alpha', \beta', \sigma'\}$$

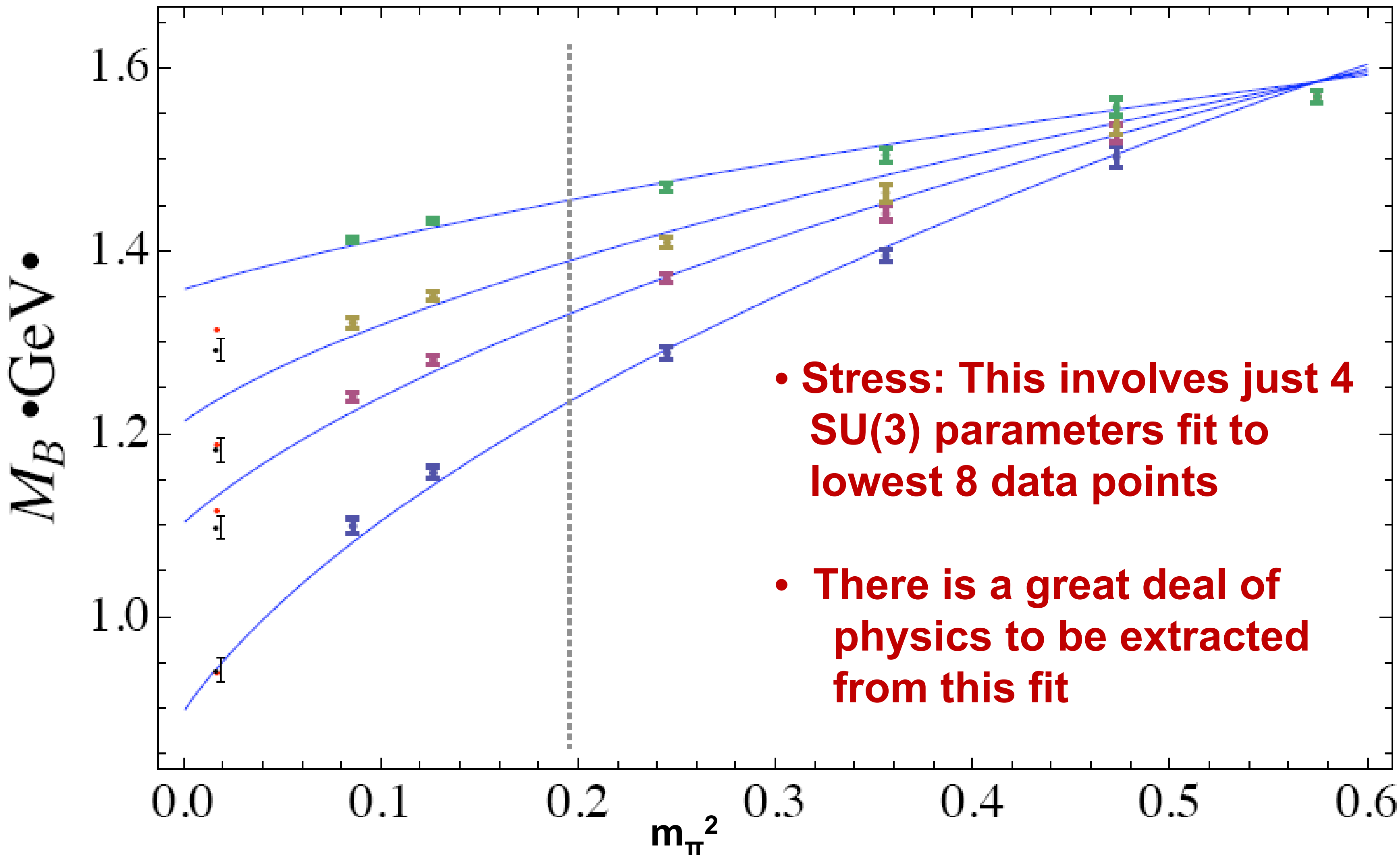
Lattice Simulation Results: LHPC



Fits to 2 lightest LHPC points



Fits to 2 lightest LHPC points

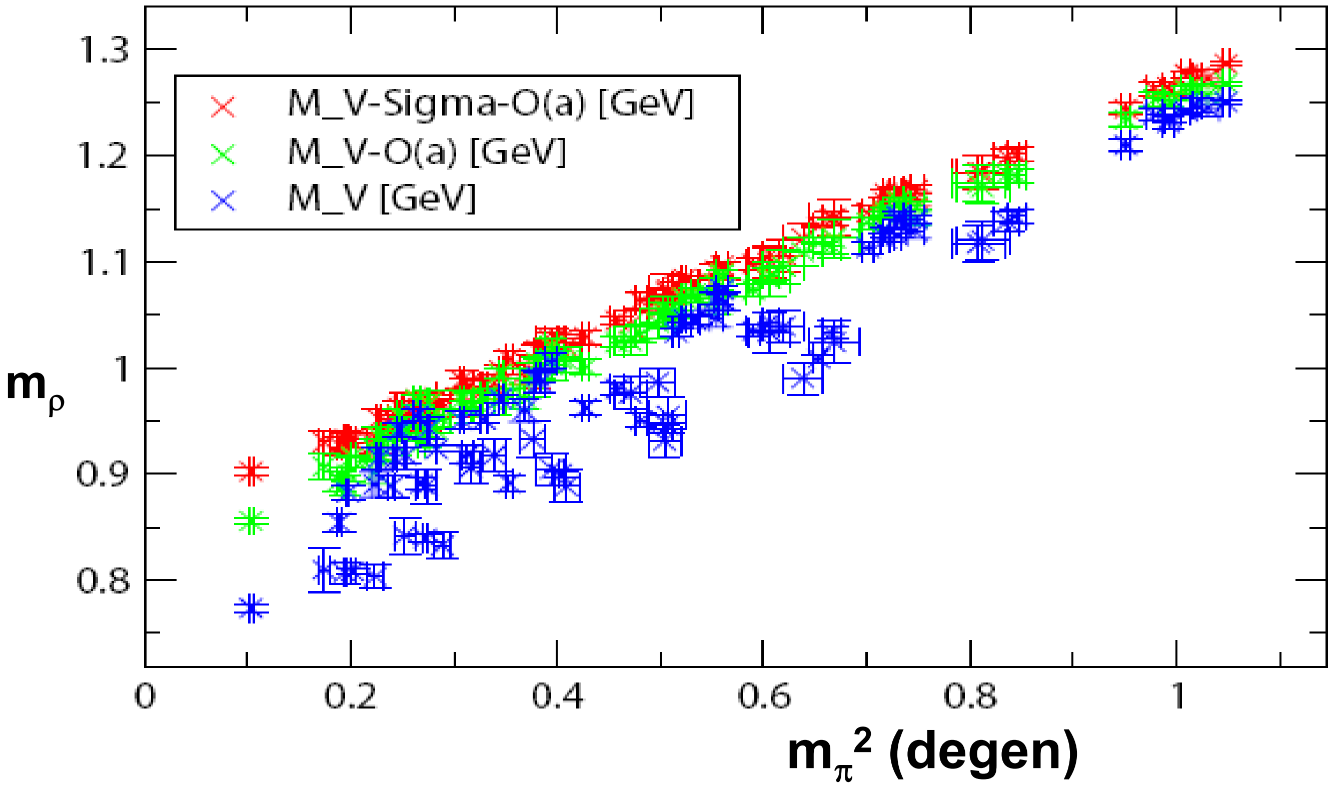


- Stress: This involves just 4 SU(3) parameters fit to lowest 8 data points
- There is a great deal of physics to be extracted from this fit

Young & Thomas, in preparation

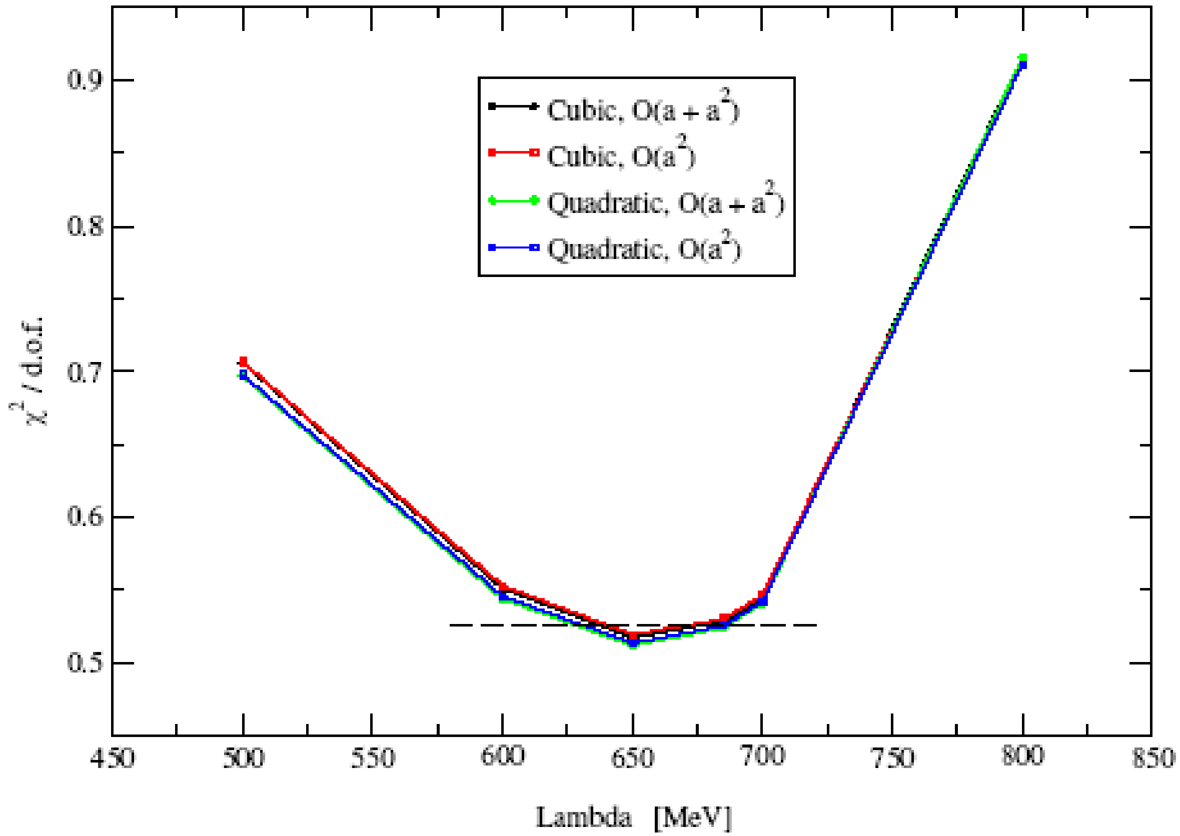
Analysis of pQQCD ρ data from CP PACS

i.e. $m_{val} \neq m_{sea}$



Fit with: $\sqrt{(M_V^{deg})^2 - \Sigma_{TOT}} = (a_0^{cont} + X_1 a + X_2 a^2) + a_2 (M_{PS}^{deg})^2 + a_4 (M_{PS}^{deg})^4 + a_6 (M_{PS}^{deg})^6$

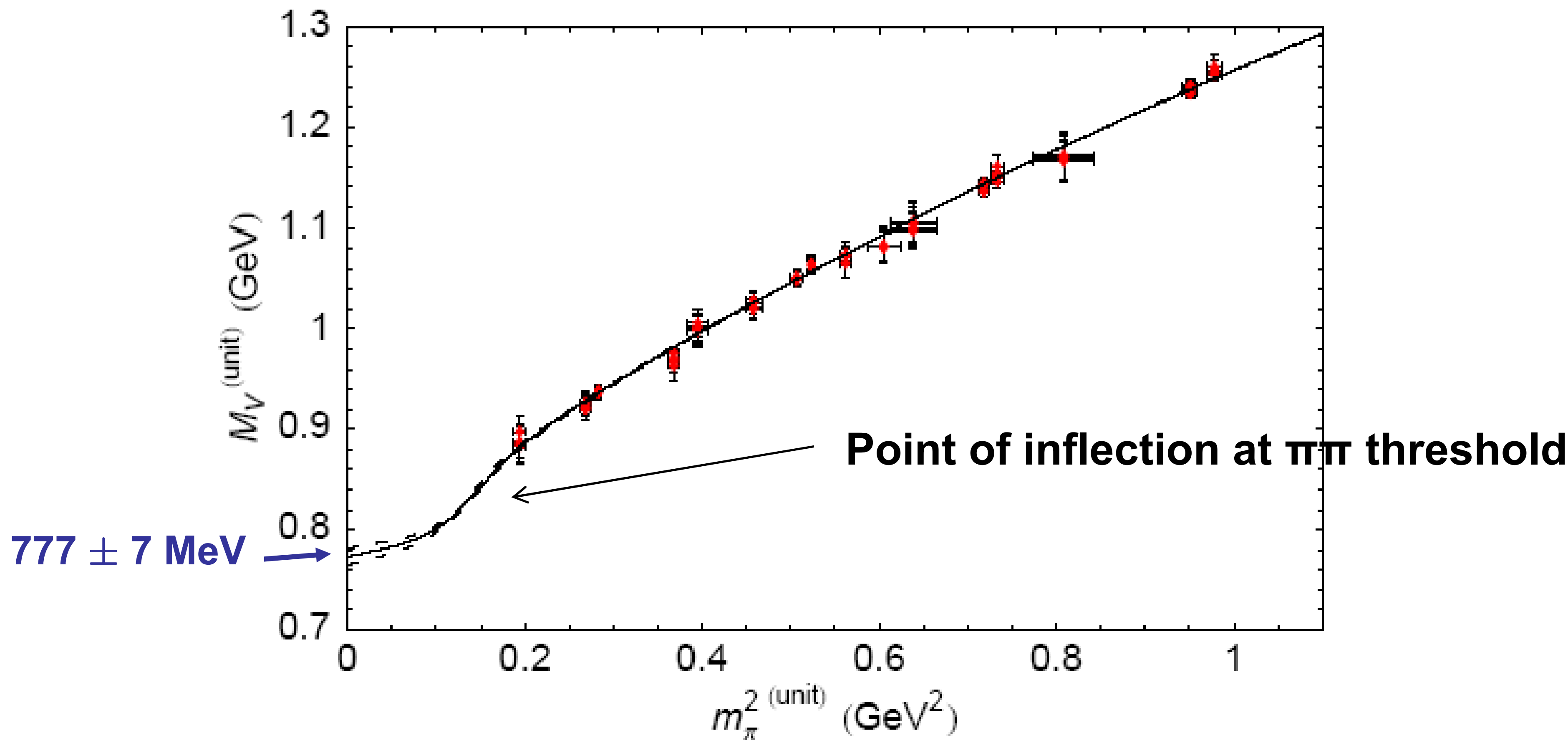
Mass (in Σ_{TOT}) well determined



$$\sqrt{(M_V^{deg})^2 - \Sigma_{TOT}} = (a_0^{cont} + X_1 a + X_2 a^2) + a_2(M_{PS}^{deg})^2 + a_4(M_{PS}^{deg})^4 + a_6(M_{PS}^{deg})^6$$

Infinite Volume Unitary Results

All 80 data points drop onto single, well defined curve !



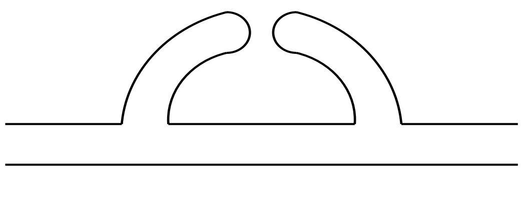
Allton, Young *et al*., hep-lat/0504022

Baryon Masses in Quenched QCD

Chiral behaviour in QQCD quite different from full QCD

η' is an additional Goldstone Boson , so that:

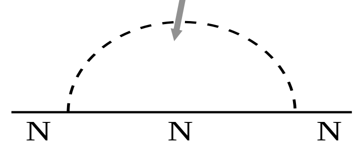
$$m_N = m_0 + c_1 m_\pi + c_2 m_\pi^2 + c_3 m_\pi^3 + c_4 m_\pi^4 + m_\pi^4 \ln m_\pi + \ldots$$

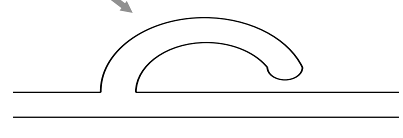


LNA term now ~ $m_q^{1/2}$

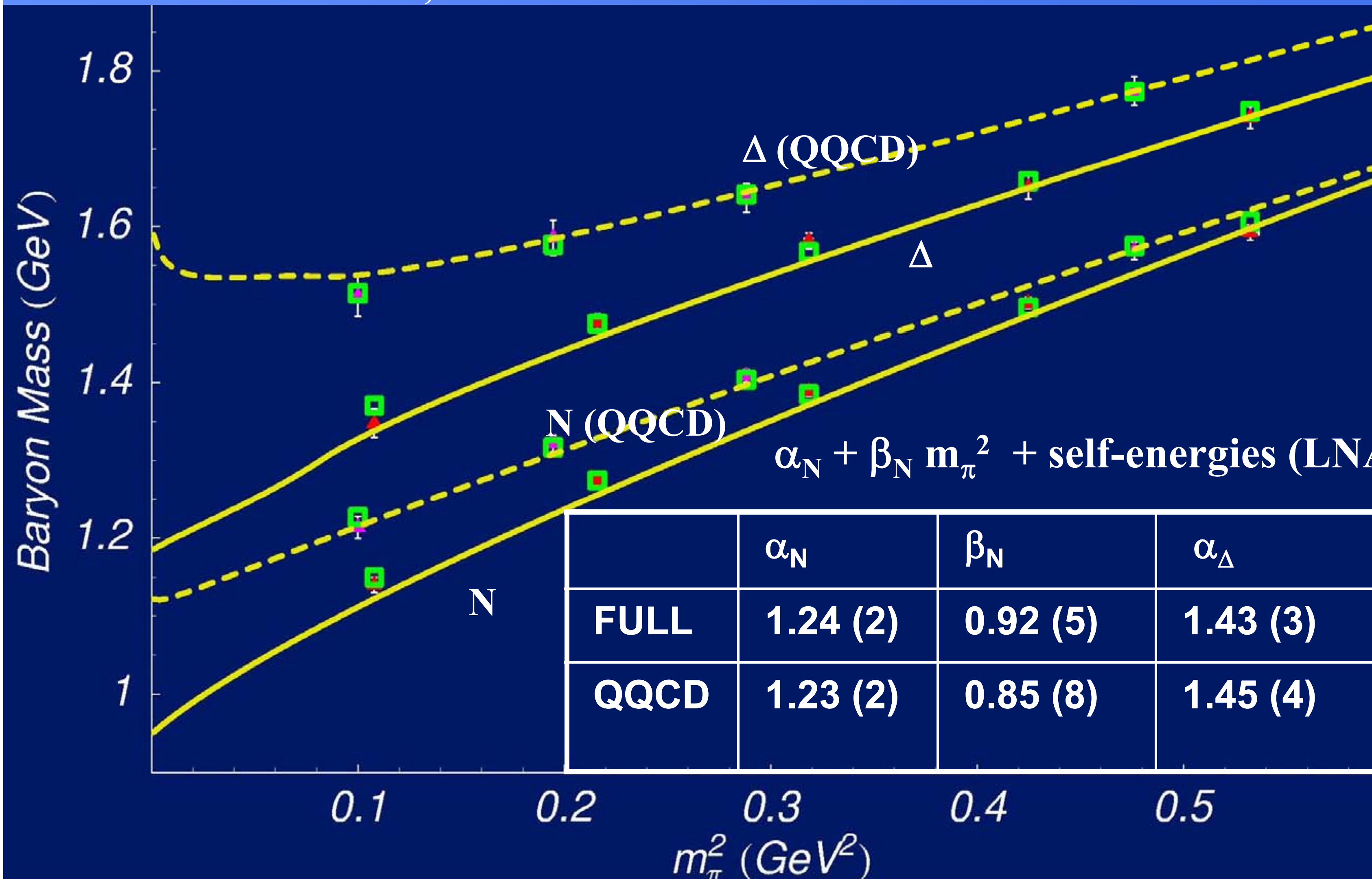
origin is η' double pole

Contribution from η' and π





- Lattice data (from **MILC Collaboration**) : red triangles
- Green boxes: fit evaluating σ's on <u>same finite grid as lattice</u>
- Lines are exact, continuum results



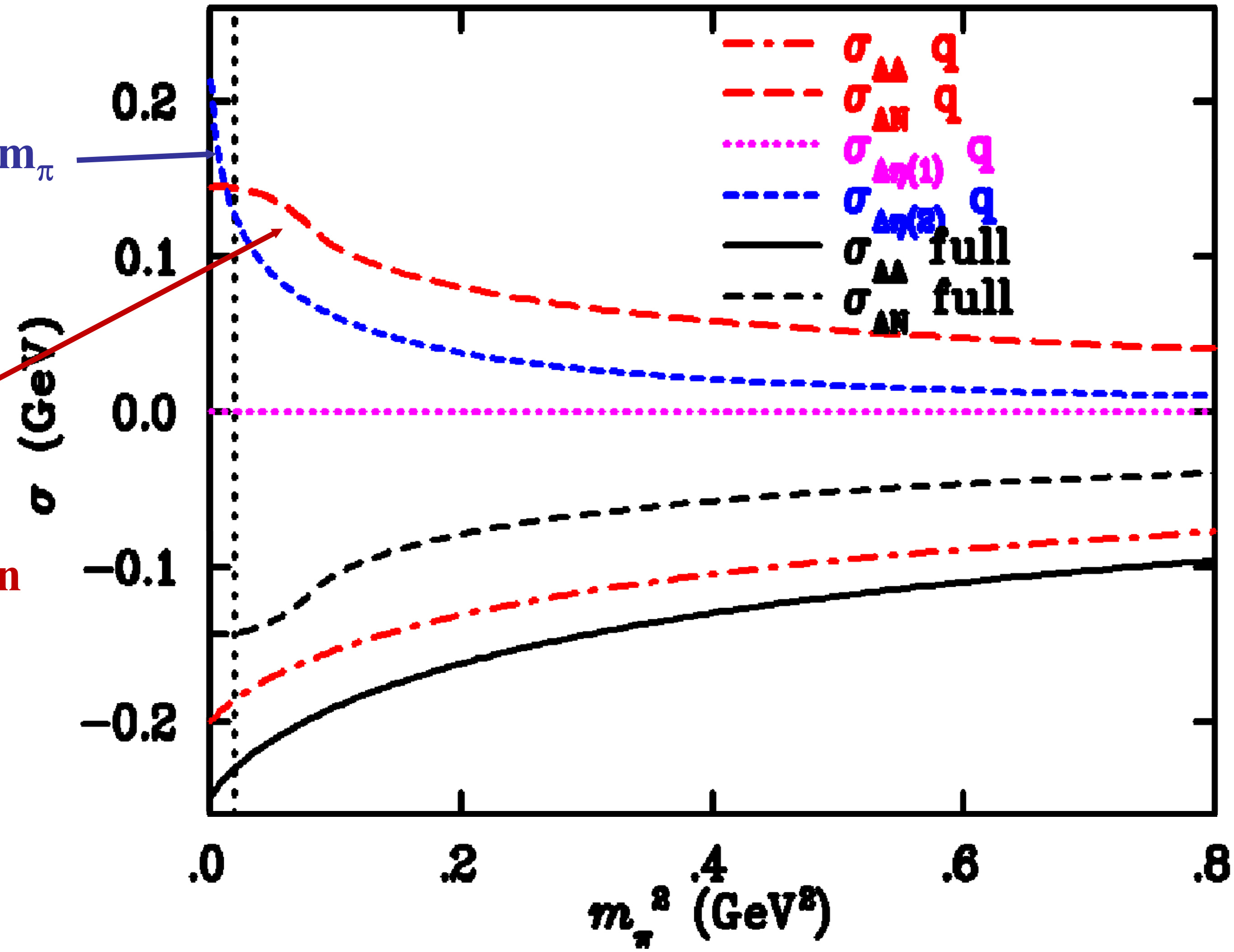
$\alpha_N + \beta_N m_\pi^2$ + self-energies (LNA+NLNA)

	α_N	β_N	α_Δ	β_Δ
FULL	1.24 (2)	0.92 (5)	1.43 (3)	0.75 (8)
QQCD	1.23 (2)	0.85 (8)	1.45 (4)	0.71 (11)

Young *et al.*, hep-lat/0111041; Phys. Rev. D66 (2002) 094507

Δ in QQCD

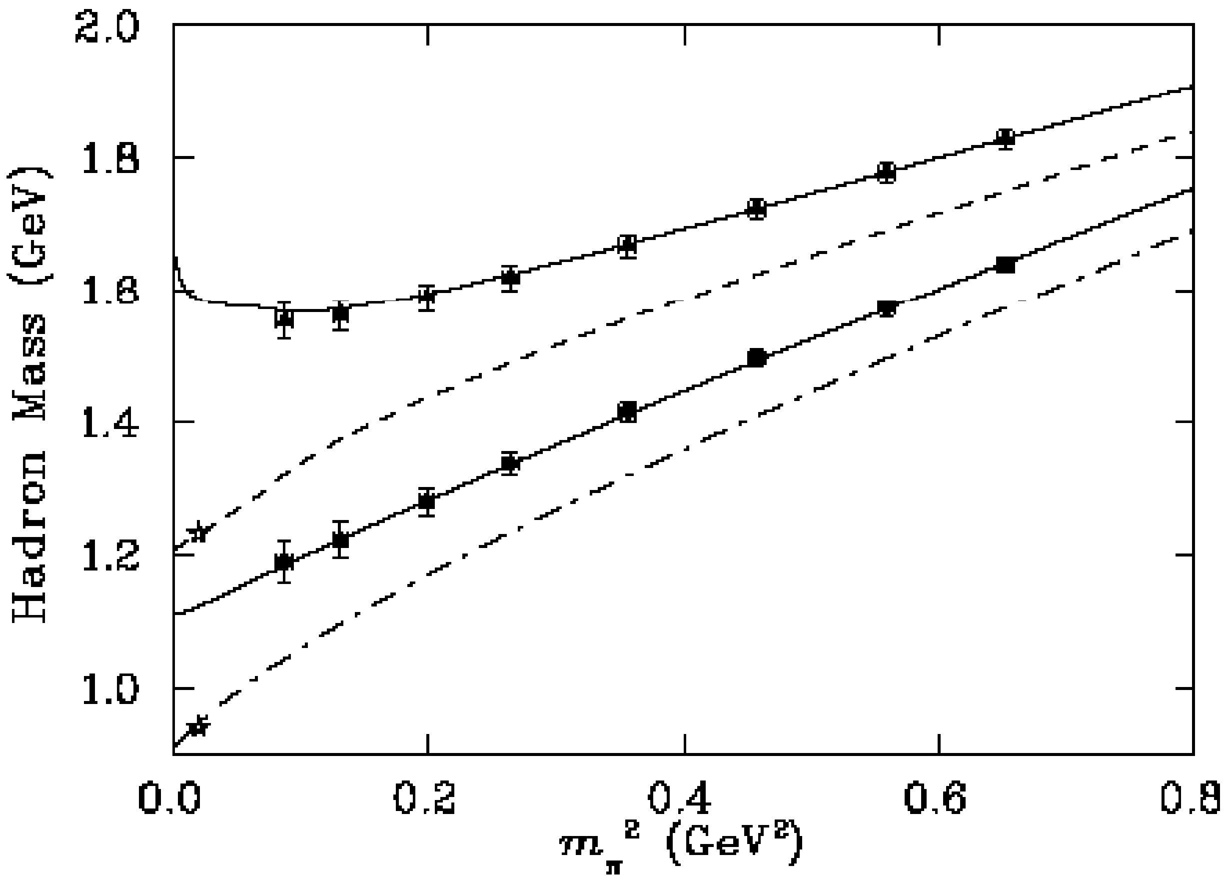
LNA term linear in m_π



Δ → N π contribution has opposite sign in QQCD (repulsive)

Overall σ_{QQCD} is repulsive !

Confirmation of Predicted Behavior of Δ



Zanotti et al., hep-lat/0407039

These results suggest following conjecture :

IF lattice scale is set using static quark potential (e.g. Sommer scale)
(insensitive to chiral physics)

Suppression of Goldstone loops for $m_\pi > \Lambda$ implies:
Analytic terms (e.g. $\alpha + \beta\, m_\pi^2 + \gamma\, m_\pi^4$)
representing "hadronic core" are the same in QQCD & QCD

Can then correct QQCD results by replacing LNA & NLNA behaviour in QQCD by corresponding terms in full QCD

Quenched QCD is then no longer an "uncontrolled approximation" !

Strangeness Widely Believed to Play a Major Role – Does

- **As much as 100 to 300 MeV of proton mass:**

$$M_N = \langle N(P)| - \frac{9\,\alpha_s}{4\,\pi}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) + m_u\bar{\psi}_u\psi_u + m_d\bar{\psi}_d\psi_d + m_s\bar{\psi}_s\psi_s|N(P)\rangle$$

$$\Delta M_N^{s-\mathrm{quarks}} = \frac{y m_s}{m_u + m_d}\,\sigma_N$$

y=0.2 ± 0.2 ?

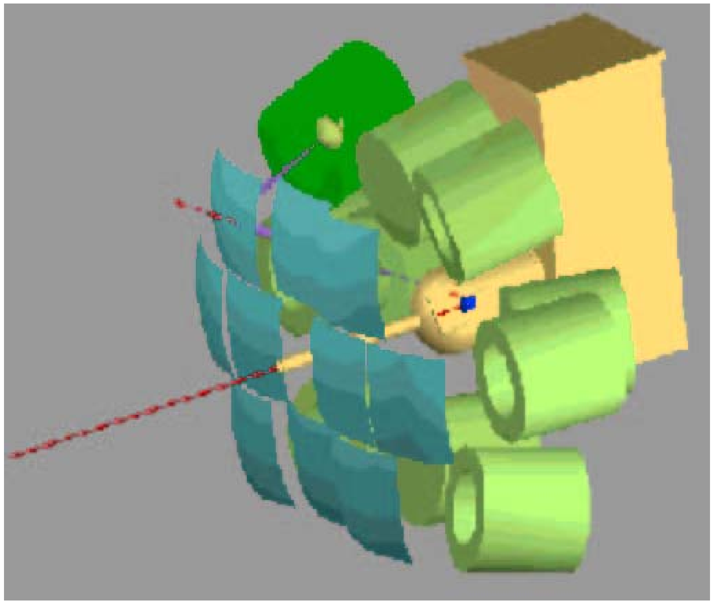
45 ± 8 MeV (or 70?)

Hence 110 ± 110 MeV (increasing to 180 for higher σ_N)

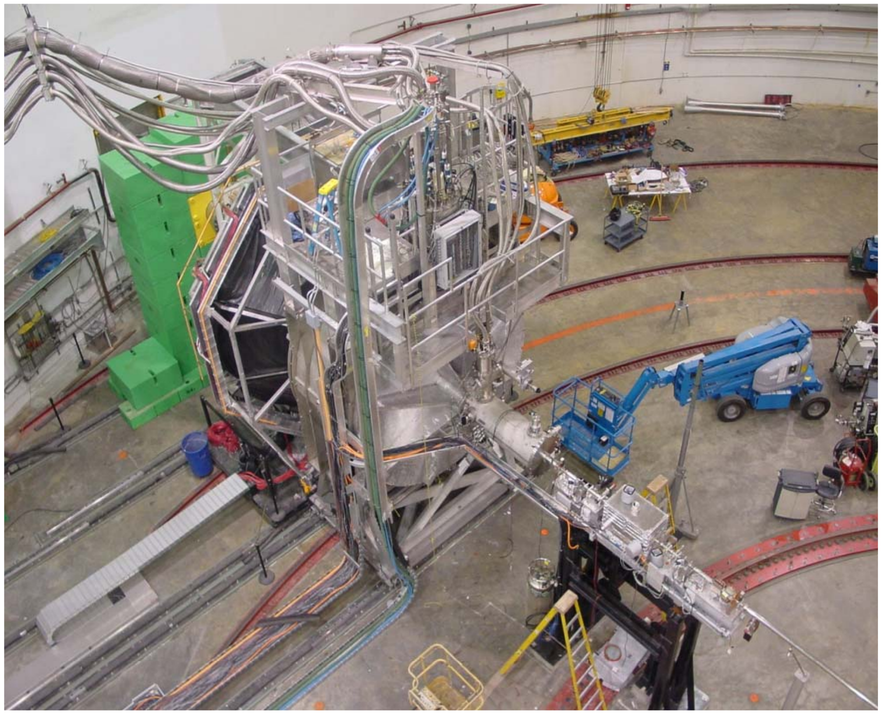
As much as 10% of the spin of the proton

- **HOW MUCH OF THE ELECTRIC and MAGNETIC FORM FACTORS ?**

MIT-Bates & A4 at Mainz

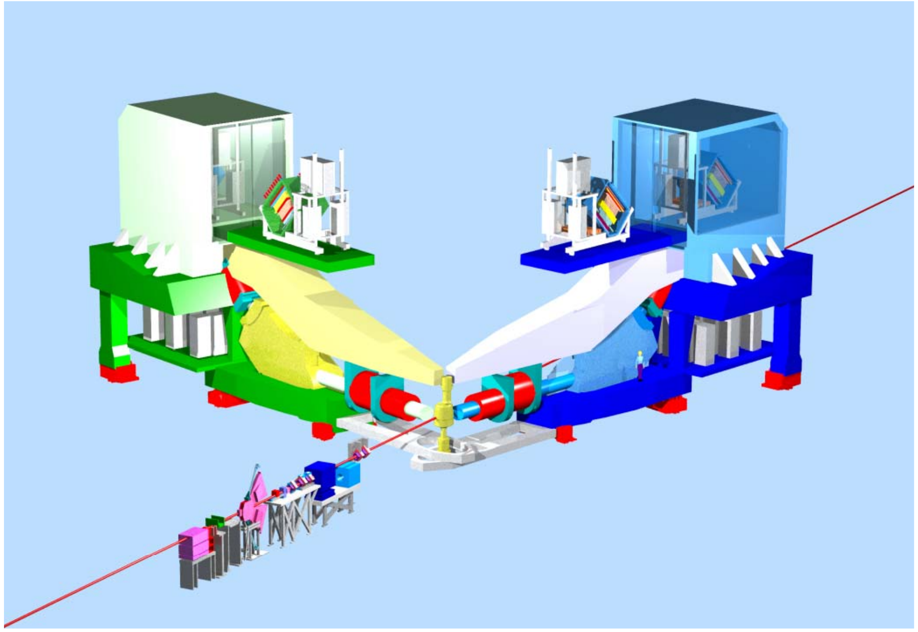


G0 and HAPPEx at Jlab



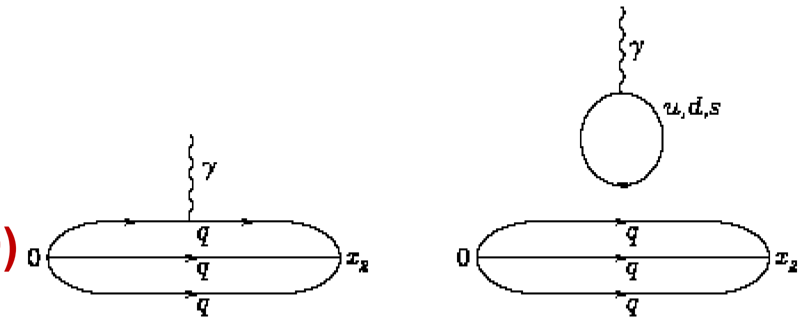
Direct calculation pioneered by K-F Liu and collaborators BUT very difficult

Instead try indirect method…



Magnetic Moments within QCD

(Leinweber and Thomas, Phys Rev D62 (2000)



CS

$p = 2/3\, u^p - 1/3\, d^p + O_N$

$n = -1/3\, u^p + 2/3\, d^p + O_N$

$2p + n = u^p + 3\, O_N$

(and $p + 2n = d^p + 3\, O_N$)

$\Sigma^+ = 2/3\, u^\Sigma - 1/3\, s^\Sigma + O_\Sigma$

$\Sigma^- = -1/3\, u^\Sigma - 1/3\, s^\Sigma + O_\Sigma$

$\Sigma^+ - \Sigma^- = u^\Sigma$

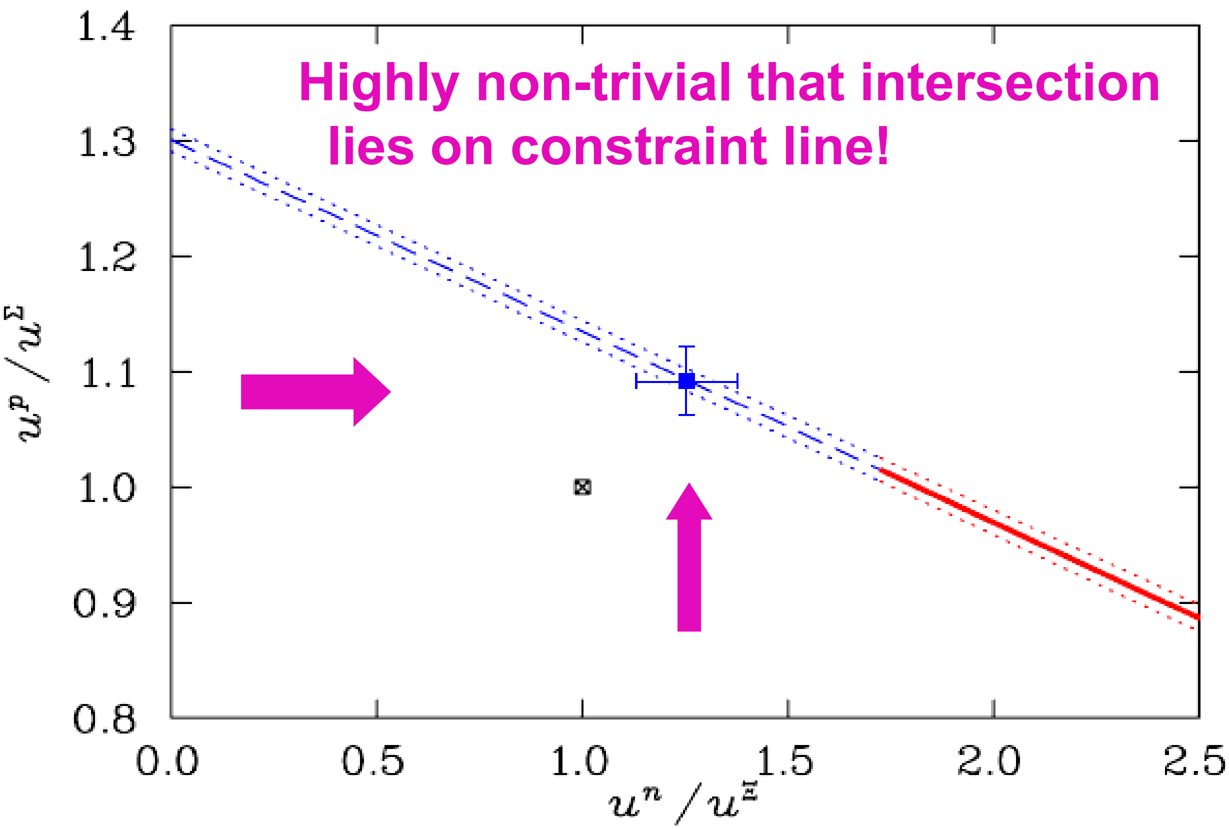
HENCE: $O_N = 1/3\, [\, 2p + n - (\, u^p / u^\Sigma\,)\, (\Sigma^+ - \Sigma^-)\,]$

Just these ratios from Lattice QCD

OR

$O_N = 1/3\, [\, n + 2p - (\, u^n / u^\Xi\,)\, (\Xi^0 - \Xi^-)\,]$

Accurate Final Result for G_M^s



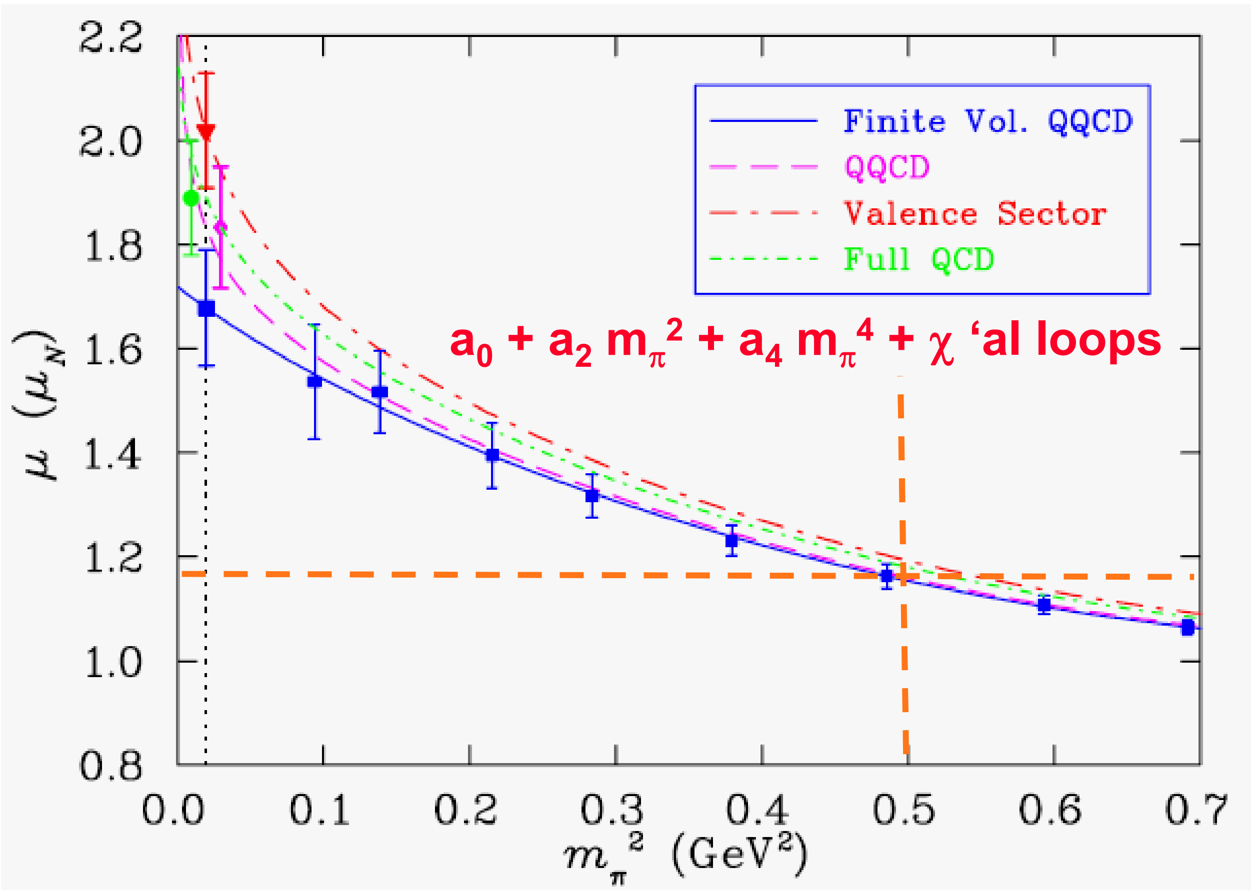
1.10±0.03

1.25±0.12

Yields : $G_M^s = -0.046 \pm 0.019\ \mu_N$

Leinweber et al., (PRL June '05) hep-lat/0406002

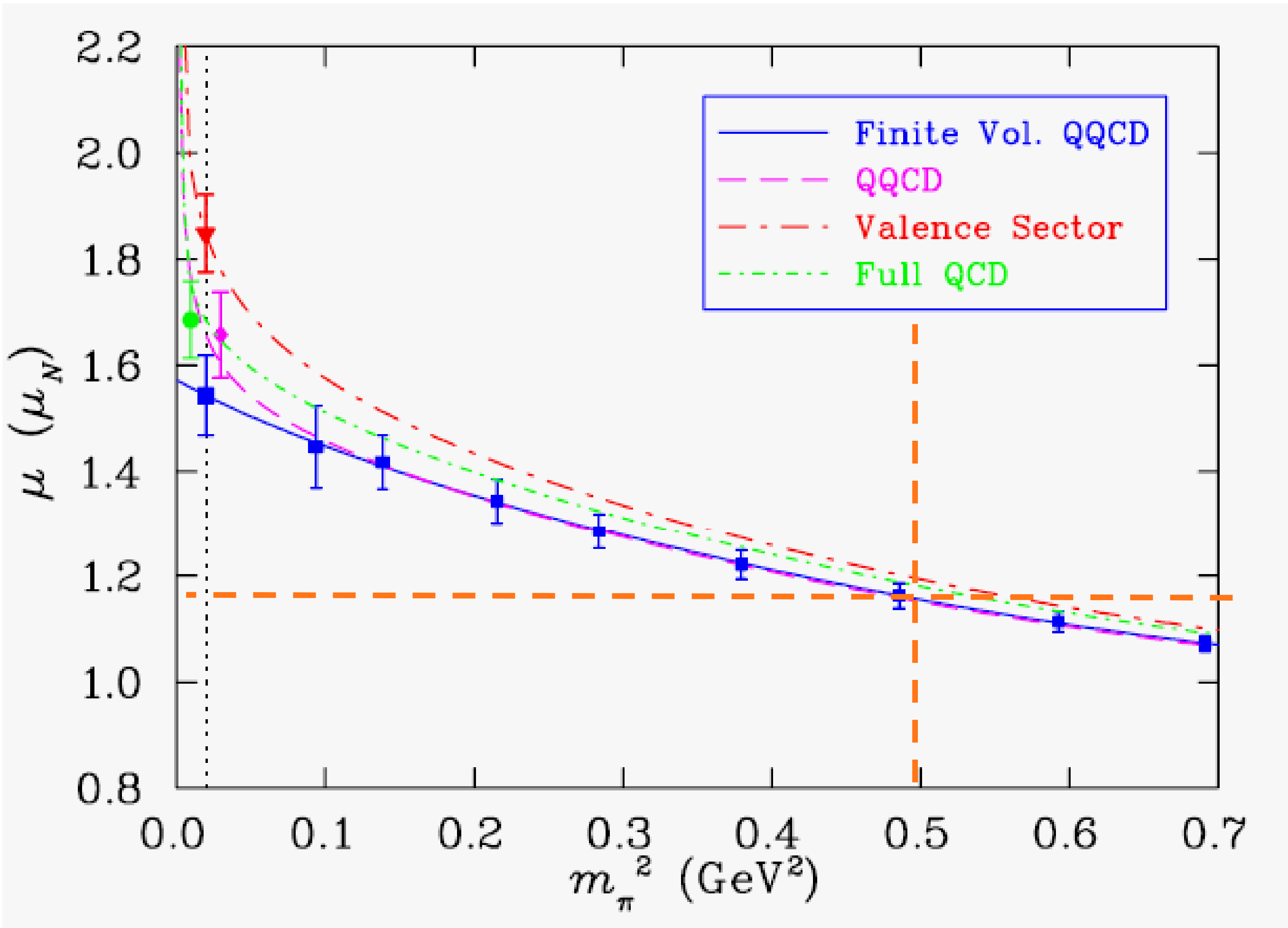
$u^p_{valence}$: QQCD Data Corrected for Full QCD Chiral Coefficients



c.f. CQM
2/3 940/540
$\sim$ 1.18

Lattice data from Zanotti et al. ; Chiral analysis Leinweber et al.

$u^{\Sigma}{}_{valence}$

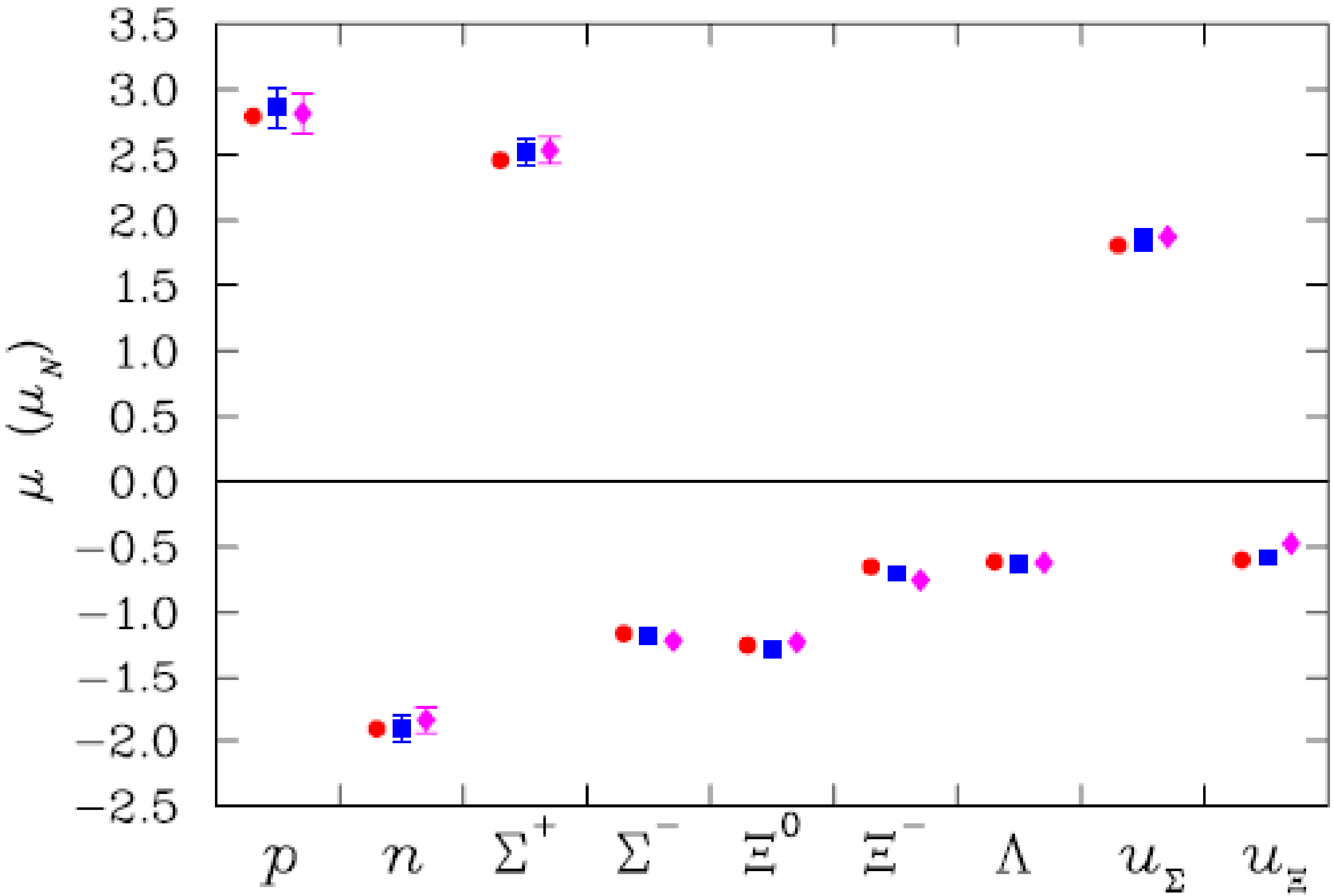


Universal Here!

State of the Art Magnetic Moments

	QQCD	Valence	Full QCD	Expt.
p	2.69 (16)	2.94 (15)	2.86 (15)	2.79
n	-1.72 (10)	-1.83 (10)	-1.91 (10)	-1.91
Σ^+	2.37 (11)	2.61 (10)	2.52 (10)	2.46 (10)
Σ^-	-0.95 (05)	-1.08 (05)	-1.17 (05)	-1.16 (03)
Λ	-0.57 (03)	-0.61 (03)	-0.63 (03)	-0.613 (4)
Ξ^0	-1.16 (04)	-1.26 (04)	-1.28 (04)	-1.25 (01)
Ξ^-	-0.65 (02)	-0.68 (02)	-0.70 (02)	-0.651 (03)
u^p	1.66 (08)	1.85 (07)	1.85 (07)	1.81 (06)
u^Ξ	-0.51 (04)	-0.58 (04)	-0.58 (04)	-0.60 (01)

Convergence LNA to NLNA Again Excellent (Effect of Decuplet)



G_E^s by similar technique

In this case only know Σ^- radius (and p and n)

hence use absolute values of u and d radii:

$2p + n = u^p + 3\,O_N$ $\qquad$ $p + 2n = d^p + 3\,O_N$

$\Rightarrow \langle r^2 \rangle_s = 0.000 \pm 0.006 \pm 0.007\ \text{fm}^2$; $0.002 \pm 0.004 \pm 0.004\ \text{fm}^2$

(c.f. using Σ^- : $-0.007 \pm 0.004 \pm 0.007 \pm 0.021\ \text{fm}^2$)

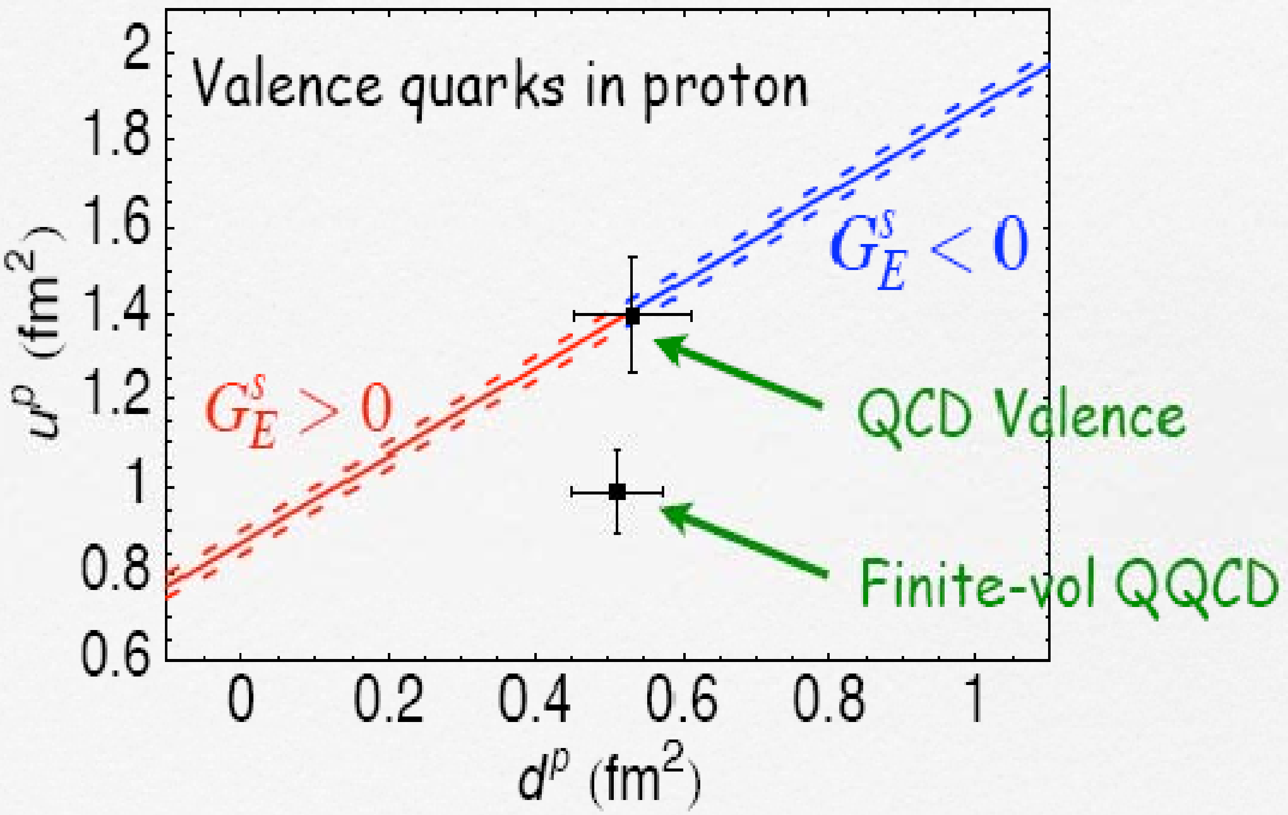
$$G_E^s(0.1\,\text{GeV}^2) = +0.001 \pm 0.004 \pm 0.004$$

(up to order Q^4)

Note consistency and level of precision!

Leinweber, Young et al., hep-lat/0601025 (Jan 2006)

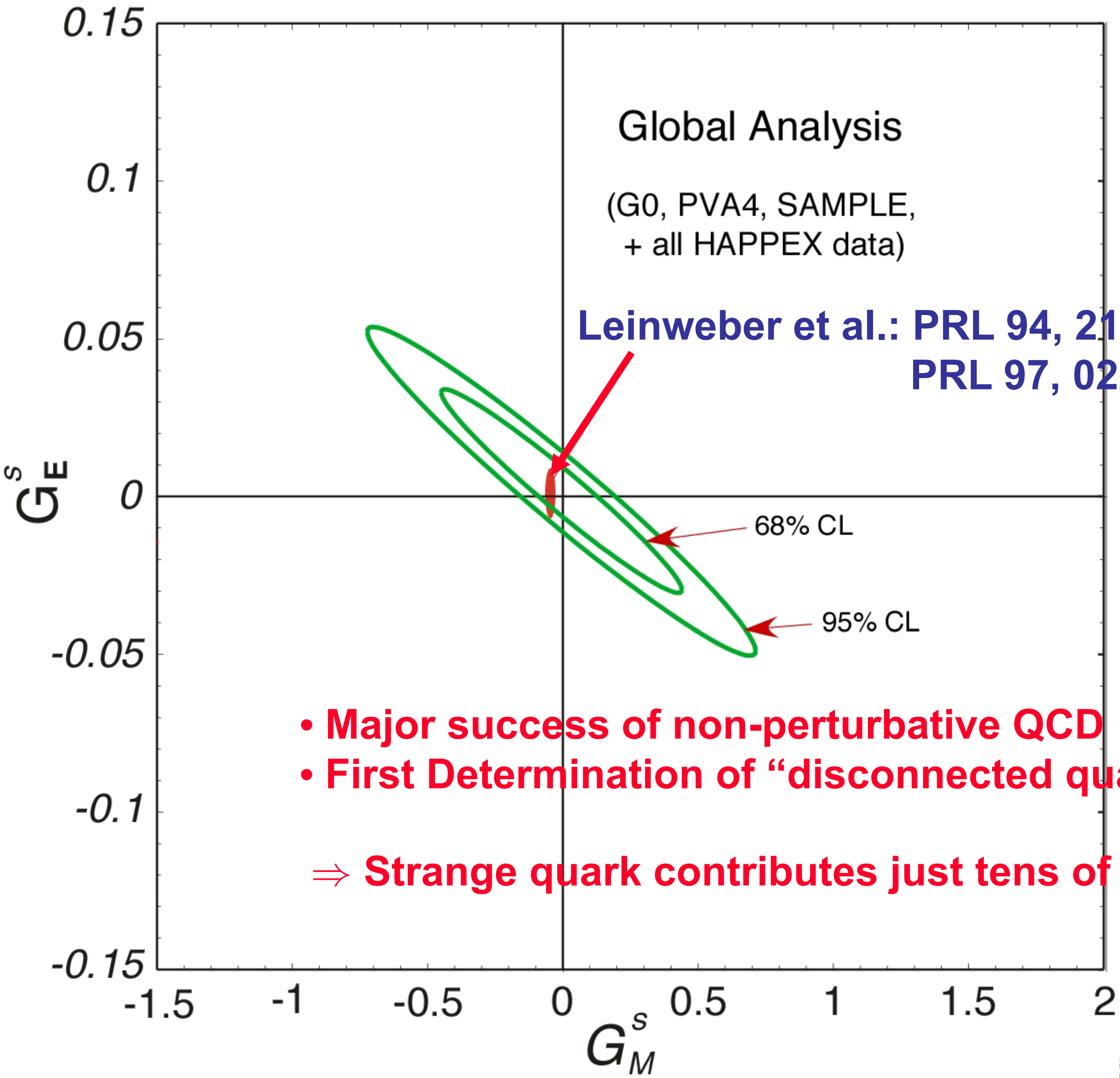
Model Independent Constraint Again Satisfied



$$G_E^s(Q^2 = 0.1) = +0.001 \pm 0.004 \pm 0.004$$

Leinweber, RDY *et al.* hep-lat/0601025

Include new HAPPEx data : halves errors of previous world data !



Leinweber et al.: PRL 94, 212001 (2005)
PRL 97, 022001 (2006)

- **Major success of non-perturbative QCD**
- **First Determination of "disconnected quark loop"**

$\Rightarrow$ Strange quark contributes just tens of MeV to M_N

Octet Charge Radii

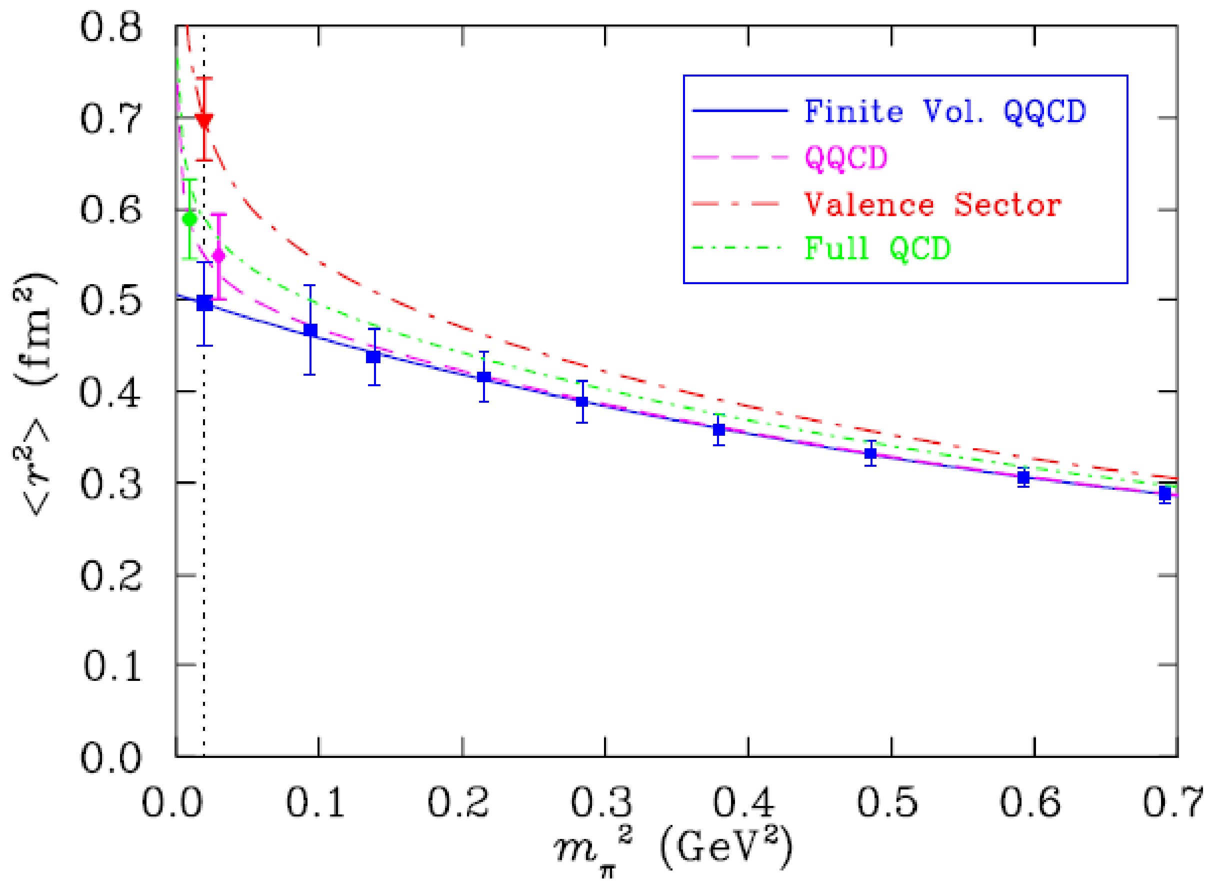


FIG. 4: The contribution of a single u quark with unit charge to the proton charge radius versus pion mass. The blue, purple, red and green curves are for the finite volume quenched QCD, infinite volume quenched QCD, valence sector and full QCD results, respectively.

Wang et al., arXiv: 0810-1021 (hep-ph)

Octet Radii - Summary

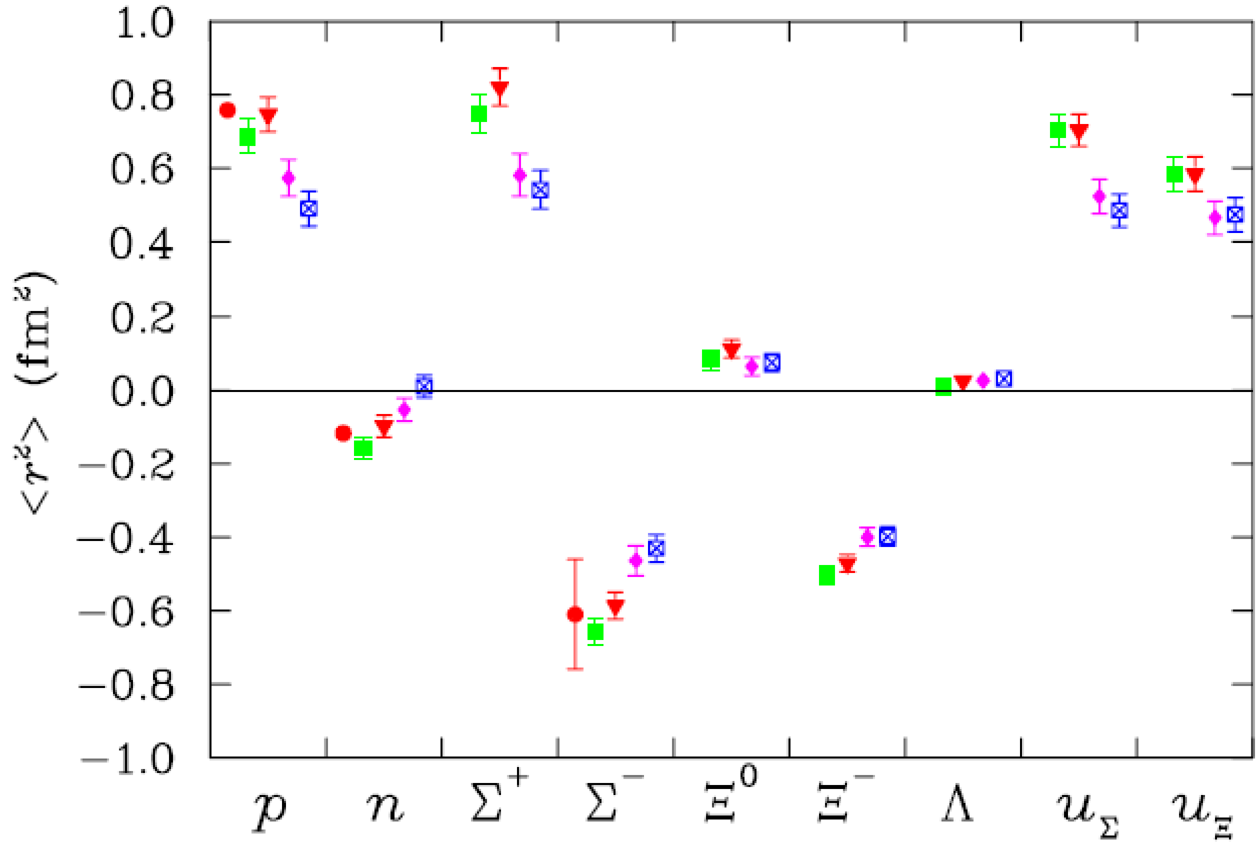


FIG. 14: Octet-baryon charge radii at the physical pion mass. The blue, purple, red and green symbols are for the finite volume quenched QCD, infinite volume quenched QCD, valence sector and full QCD results, respectively. The experimental data for proton, neutron and Σ^- is shown with the left-most bullet.

Return to Sigma Commutator

- Of broader importance – not only role of s-quark in N
  BUT also related to K-condensation and dense matter

- From fit to LHPC data on octet masses directly evaluate this:

  $\sigma \sim 40$ MeV (detailed error analysis underway)

- At physical strange quark mass (COMPLETELY different from chiral limit!) variation of nucleon mass with m_s is very small.

  Reason is familiar: m_K is above 0.4 GeV where we have learnt that chiral loops are highly suppressed!

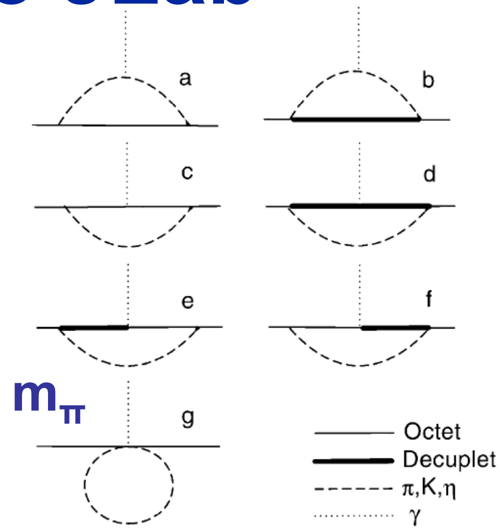
- Confirms result of Flambaum et al. : $$\frac{\delta M_N}{M_N} = 0.011 \frac{\delta m_s}{m_s} \Rightarrow y = 0.023$$
  PHYSICAL REVIEW D 69, 115006 (2004)

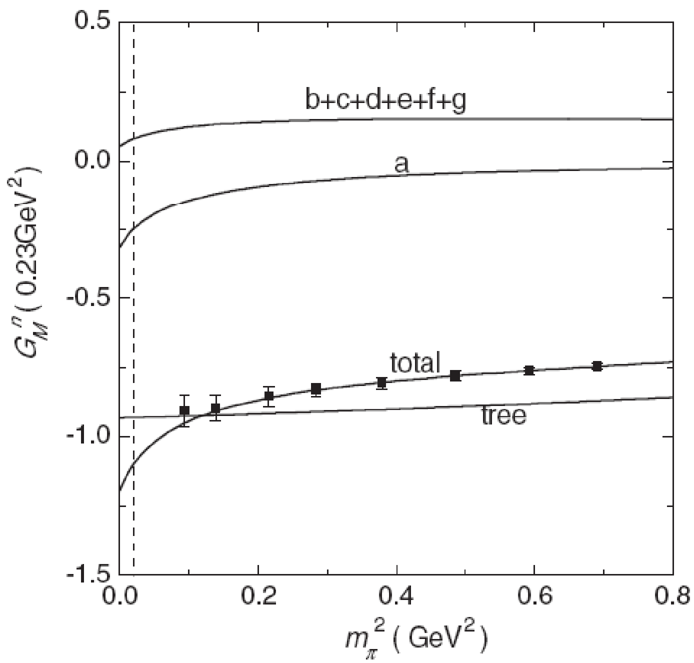
- Recent direct LQCD calculation of σ by Okhi et al.
  (arXiv:0806.4744 [hep-lat]) $y = 0.030 \pm 0.016 \pm 0.007$

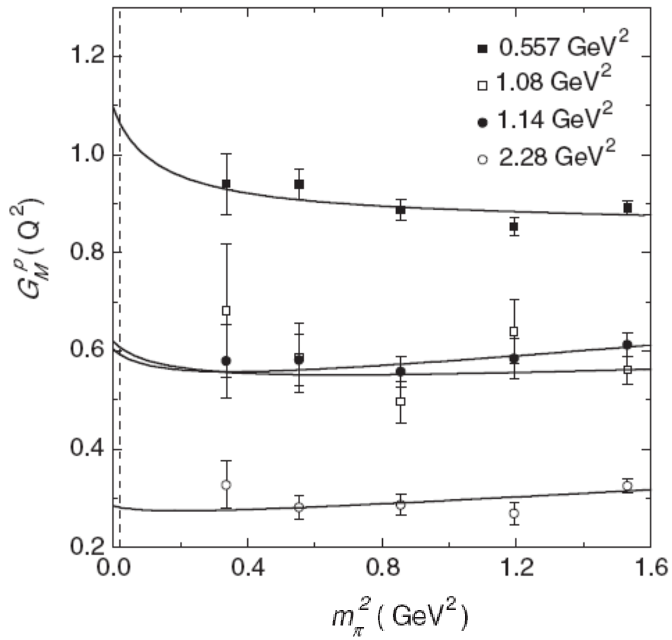
Form Factors at Q^2 relevant to JLab

Wang et al., PR D75, 073012 (2007)

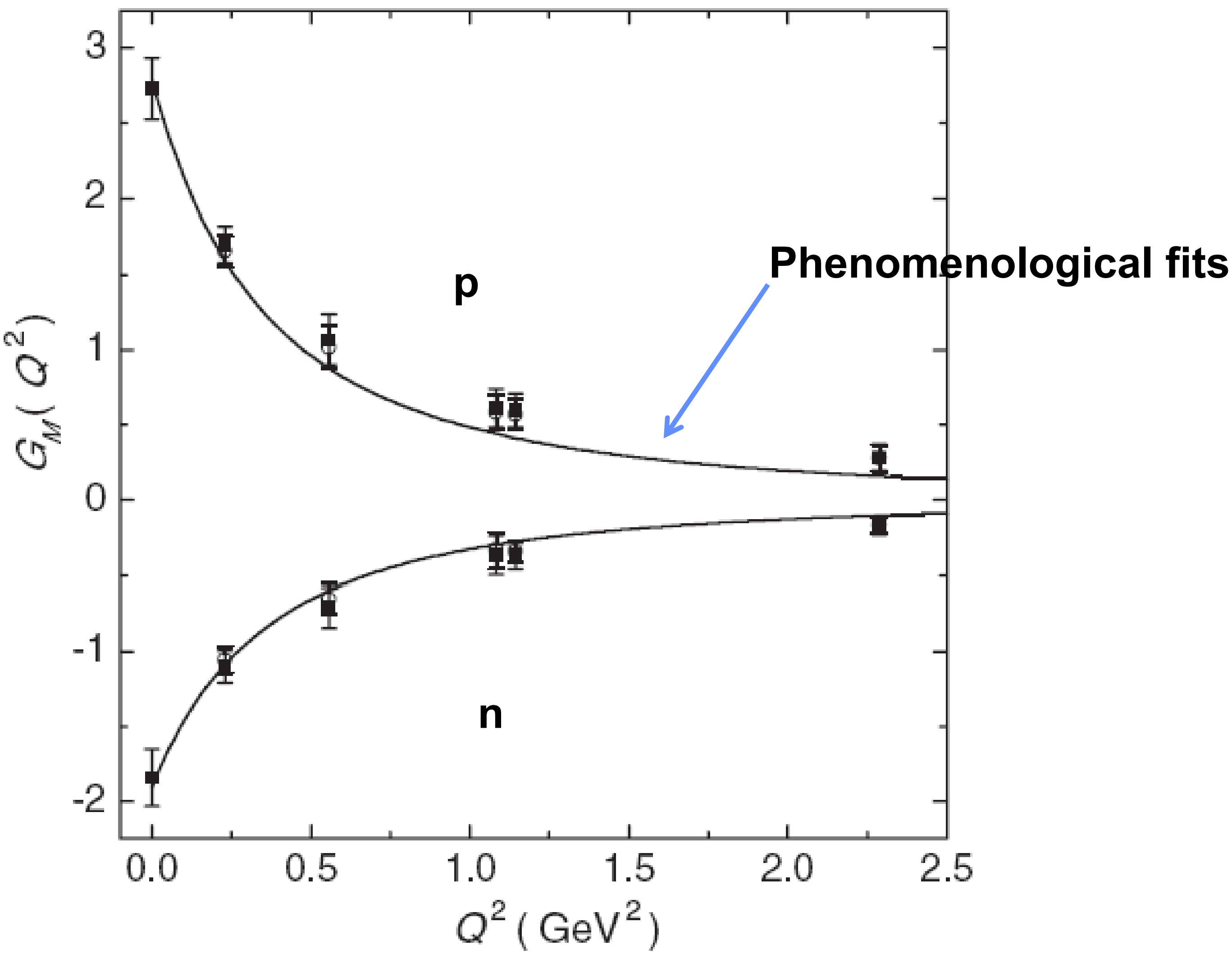
- Data from QCDSF Collaboration and CSSM
- Expand in m_π^2 at fixed Q^2 - finesse radius of conv.
- π cloud contribution dies rapidly at both large Q^2 and m_π







Approach seems promising



Conclusions

- **Wonderful synergy between experimental advances at Jlab and progress using Lattice QCD to solve QCD**
- **Study of hadron properties as function of m_q using data from lattice QCD is extremely valuable….. (major qualitative advance in understanding)**
- **Inclusion of model independent constraints of χ PT to get to physical quark mass is essential**

FRR χPT resolves problem of convergence

- **Insight enables: accurate, controlled extrapolation of all hadronic observables….**

(e.g. m_H , μ_H , $G_{E,M}^s$, $\langle r^2 \rangle_{ch}$, G_E,G_M, $\langle x^n \rangle$……..)

- **Apologise for not discussing spin and angular momentum**

Conclusions…. $_2$

- In case where chiral coefficients are known, FRR enables accurate extrapolation to physical point

- Without chiral coefficients (e.g. spectroscopy of baryons and mesons) need data at very low pion mass
  (several points below $\sim$ 0.25 GeV)

- It is a major challenge to obtain a reliable signal for "disconnected" loops directly in lattice QCD
  — this is a very important challenge

- For future there is a wonderful synergy with 12 GeV program at JLab and work on GPDs, form factors at high Q^2 , and higher moments of PDFs just beginning…..

The Open Questions

- Does lattice QCD precisely reproduce the best experimental data
  - spectroscopy, form factors, DIS, GPDs?
- Are some observables more likely to yield interesting constraints than others?
- What insight can LQCD yield into how QCD works?
- Can it give us physical insight?
- Are we able to take the lessons learnt in hadron structure and use them to understand nuclear structure better?

Open questions (cont.)

- LQCD opens a new axis in QCD, namely varying quark masses: what does this teach us?
- QQCD is typically within 10% of data, why?
- Chiral corrections grow with quark mass (e..g. LNA for $M_N \sim -5.6\ m_{GB}^3$) yet QQCD is limit of infinite sea quark mass?
- The nucleon electromagnetic form factors behave like $1/(1 + Q^2/\Lambda^2)$ with $\Lambda \sim 0.7$ GeV $\Rightarrow$ radius of convergence well below 1 GeV. What replaces the traditional power series expansion in Q^2 ?