Final state interaction effects on the inclusive x-section: what (I think) we understand

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- Bottom line:
  - ⋆ Final state interactions (FSI) do affect the inclusive x-section, even at large momentum transfer $\mathbf{q}$
  - ⋆ The inclusive x-section carries information on FSI taking place within a distance $\sim |\mathbf{q}|^{-1}$ of the primary interaction vertex
  - ⋆ At large momentum transfer, the probability of FSI within a distance $\sim |\mathbf{q}|^{-1}$ is small. However, FSI move strength from the quasi free peak, where the inclusive cross section is is large, to the region of low energy loss, $\omega \ll Q^2/2m$, where the x-section is very small. As a consequence, their effects may become appreciable, in fact even dominant.

Inclusive cross section & nuclear response

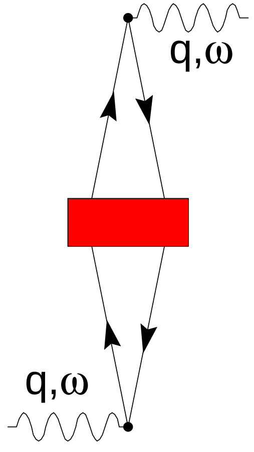
- ⋆ Consider scattering of a scalar probe, for simplicity

$$\frac{d\sigma}{d\omega d\Omega} \propto \sigma_{el}\, S(\mathbf{q},\omega)$$

$$S(\mathbf{q},\omega) = \sum_n |\sum_k \langle n|a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}}|0\rangle|^2\, \delta(\omega + E_0 - E_n)$$

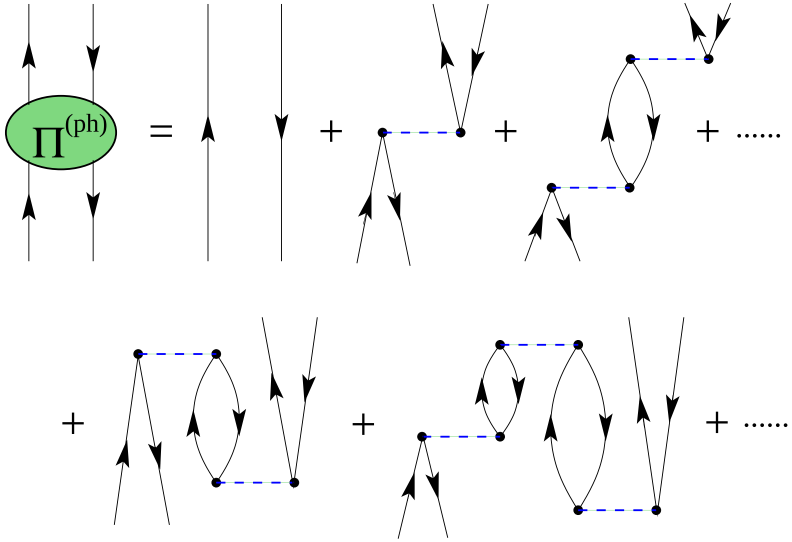
$$= \int \frac{dt}{2\pi}\, \mathrm{e}^{i(\omega+E_0)t} \sum_{p,k} \langle 0|a_{\mathbf{p}+\mathbf{q}} a^\dagger_{\mathbf{p}}\, \mathrm{e}^{-iHt}\, a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}}|0\rangle$$

$=$



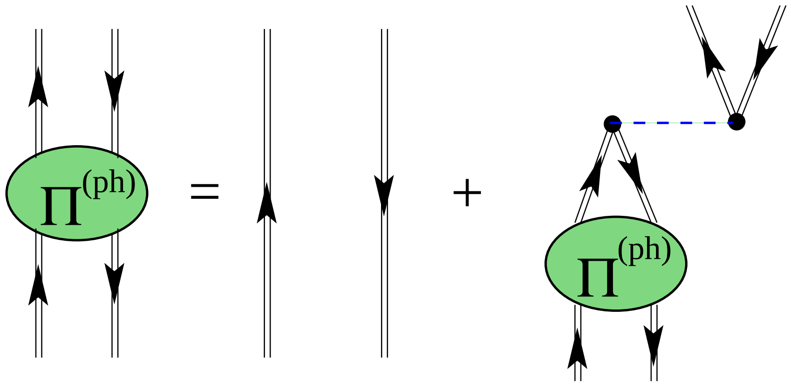
⋆ The nuclear response can be expressed in terms of Green functions, i.e. spectral functions, in a variety of approximation schemes

RPA



dressing all particle and hole lines leads to

DRPA



Large |q|: the impulse approximation (IA) regime

- Consider the leading term of to the DRPA series

$$S(\mathbf{q},\omega) = \int d^3k\, dE\ P_h(\mathbf{k},E) P_p(\mathbf{k}+\mathbf{q},\omega - E)$$

- The hole and particle spectral functions describe initial and final state effects, respectively
- Problem: at large $|\mathbf{q}|$, P_h and P_p cannot be consistently obtained from nonrelativistic many-body theory $\rightarrow$ approximations needed for P_p
- Neglecting FSI (PWIA) amounts to approximating the particle spectral function according to the Fermi gas model ($\epsilon^0_{|\mathbf{k}+\mathbf{q}|} = \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}$)

$$S_0(\mathbf{q},\omega) = \int d^3k\, dE\ P_h(\mathbf{k},E) \theta(k_F - |\mathbf{k}+\mathbf{q}|) \delta(\omega - \epsilon^0_{|\mathbf{k}+\mathbf{q}|} - E)$$

- Note: the generalization to the case of inelastic scattering is straightforward

The folding approximation

- ⋆ Write the response in the form

$$S(\mathbf{q},\omega) = \int d\omega' S_0(\mathbf{q},\omega') f_{\mathbf{q}}(\omega-\omega')$$

implying ($|\mathbf{q}| \ll |\mathbf{k}|$)

$$P_p(\mathbf{k}+\mathbf{q},E') \approx f_{\mathbf{q}}(E' - \epsilon^0_{|\mathbf{k}+\mathbf{q}|})$$

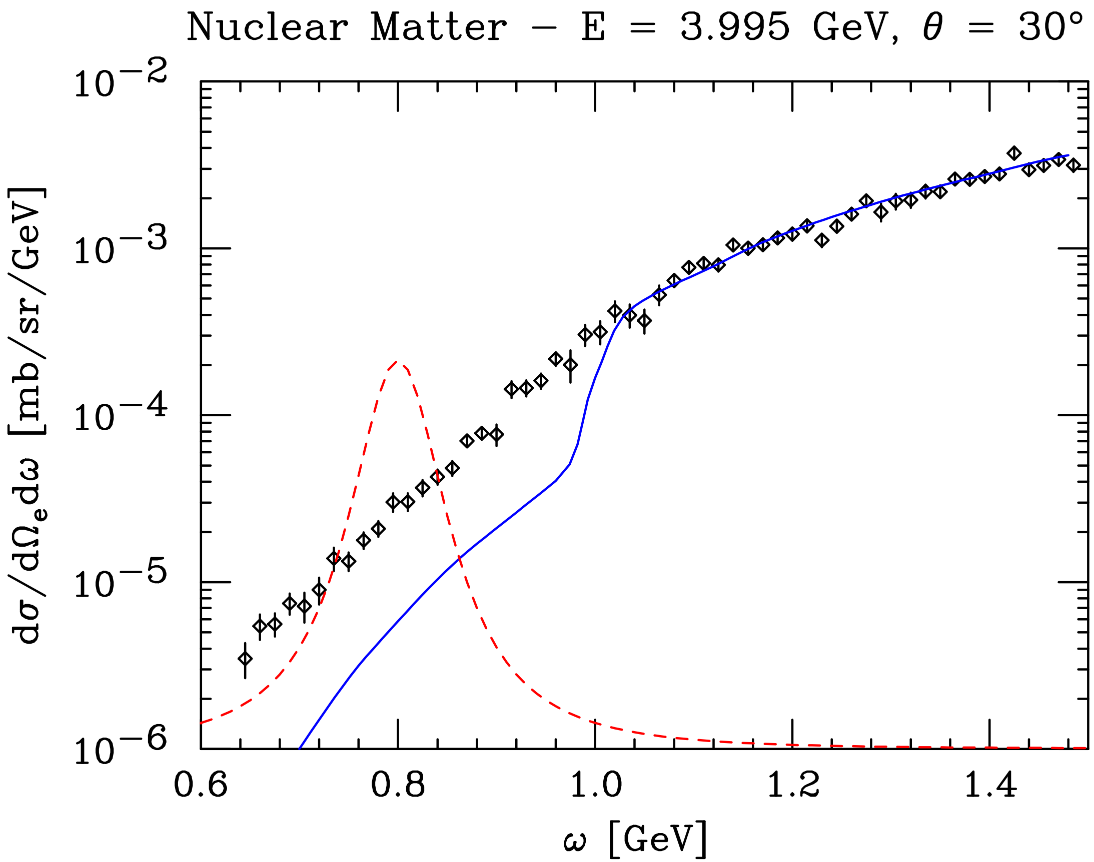
- ⋆ Within the *eikonal* + *frozen spectators* approximation

$$f_{\mathbf{q}}(\omega) = \int \frac{dt}{2\pi}\, \mathrm{e}^{i\omega t} \langle\, \mathrm{e}^{-i\int_0^t dt' H_{FSI}}\, \rangle$$

$$\langle\, \mathrm{e}^{-i\int_0^t dt' H_{FSI}}\, \rangle = 1 - i \int d^3r_1 d^3r_2\, \rho(\mathbf{r}_1)\rho(\mathbf{r}_2) g(\mathbf{r}_1,\mathbf{r}_2) \times \int_0^t dt' \Gamma_{\mathbf{q}}(\mathbf{r}_1 + \mathbf{v}t' - \mathbf{r}_2)\; + \;\ldots$$

How the folding approximation works

⋆ PWIA inclusive x-section and folding function corrisponding to scattering off uniform nuclear matter (data extrapolated by D. Day & I. Sick)

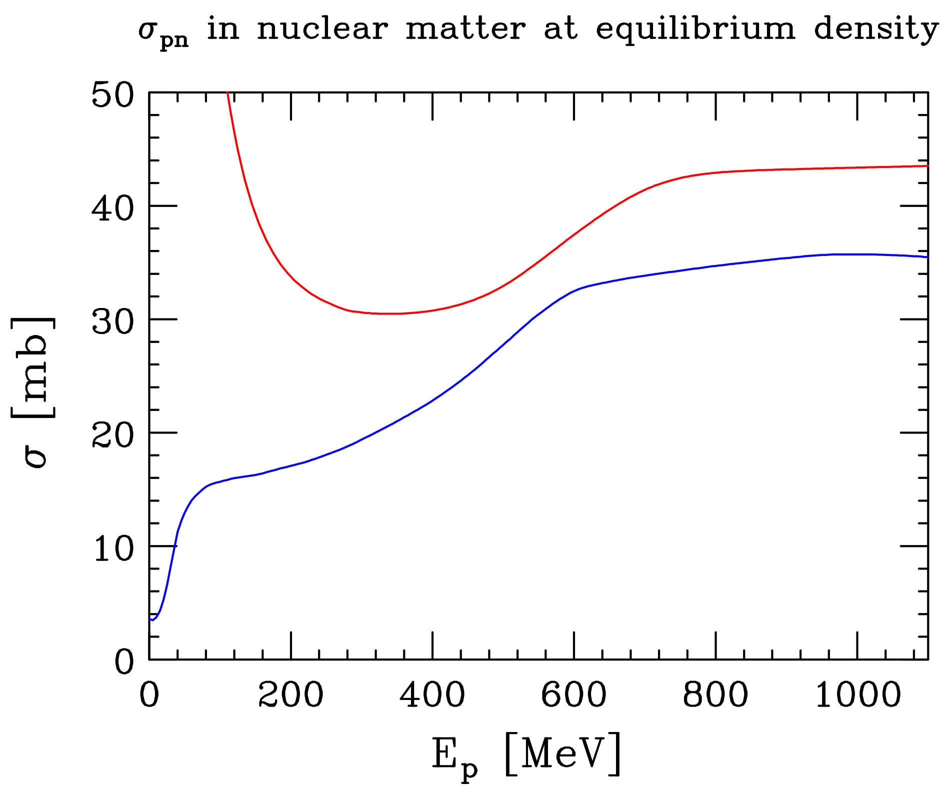


▷ Warning: the folding function is shown in linear scale, and multiplied by a factor 10^{-3}

Basic elements of the folding approximation

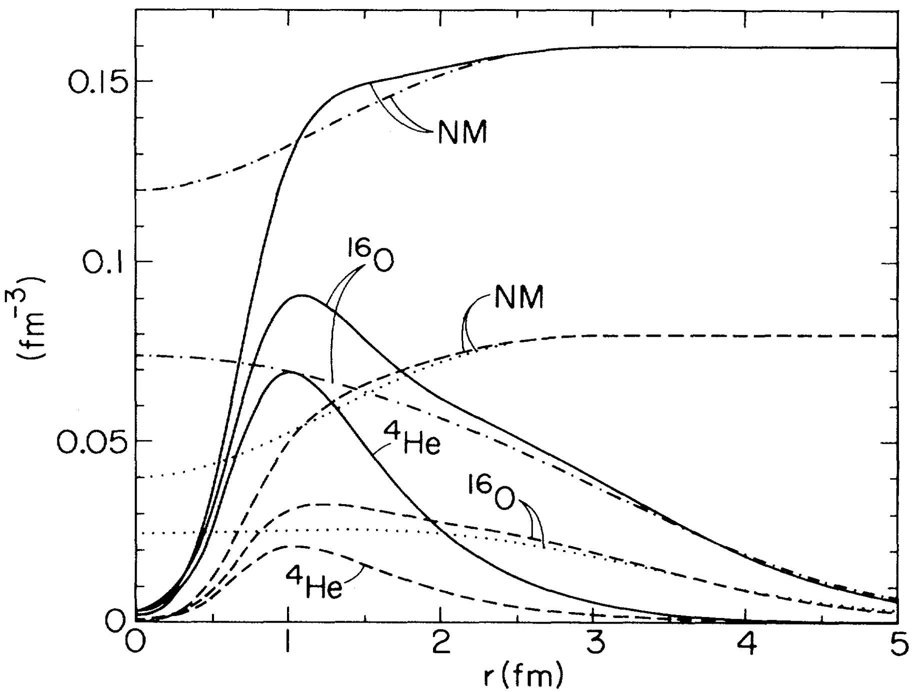
- ⋆ The coordinate space NN scattering t-matrix, $\Gamma_{\mathbf{q}}$, written in terms of three parameters extracted from fits to the data: σ (total x-section), β (slope), and α (ratio between real and imaginary part of $\Gamma_{\mathbf{q}}$)

- ▷ Warning: theoretical results suggest that medium modifications of σ are significant, and persist even at large energy (Pieper & Pandharipande)



Basic elements of the folding approximation (continued)

- ⋆ Bottom line: the probablity of NN rescattering depends upon the *joint* probability of finding the the struck particle at position $\mathbf{r}_1$ *and* a spectator at position $\mathbf{r}_2$: $\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)g(\mathbf{r}_1, \mathbf{r}_2)$
- ⋆ As $\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) \ll 1$ @ $r = |\mathbf{r}_1 - \mathbf{r}_2| \lesssim 1$ fm, FSI are strongly suppressed



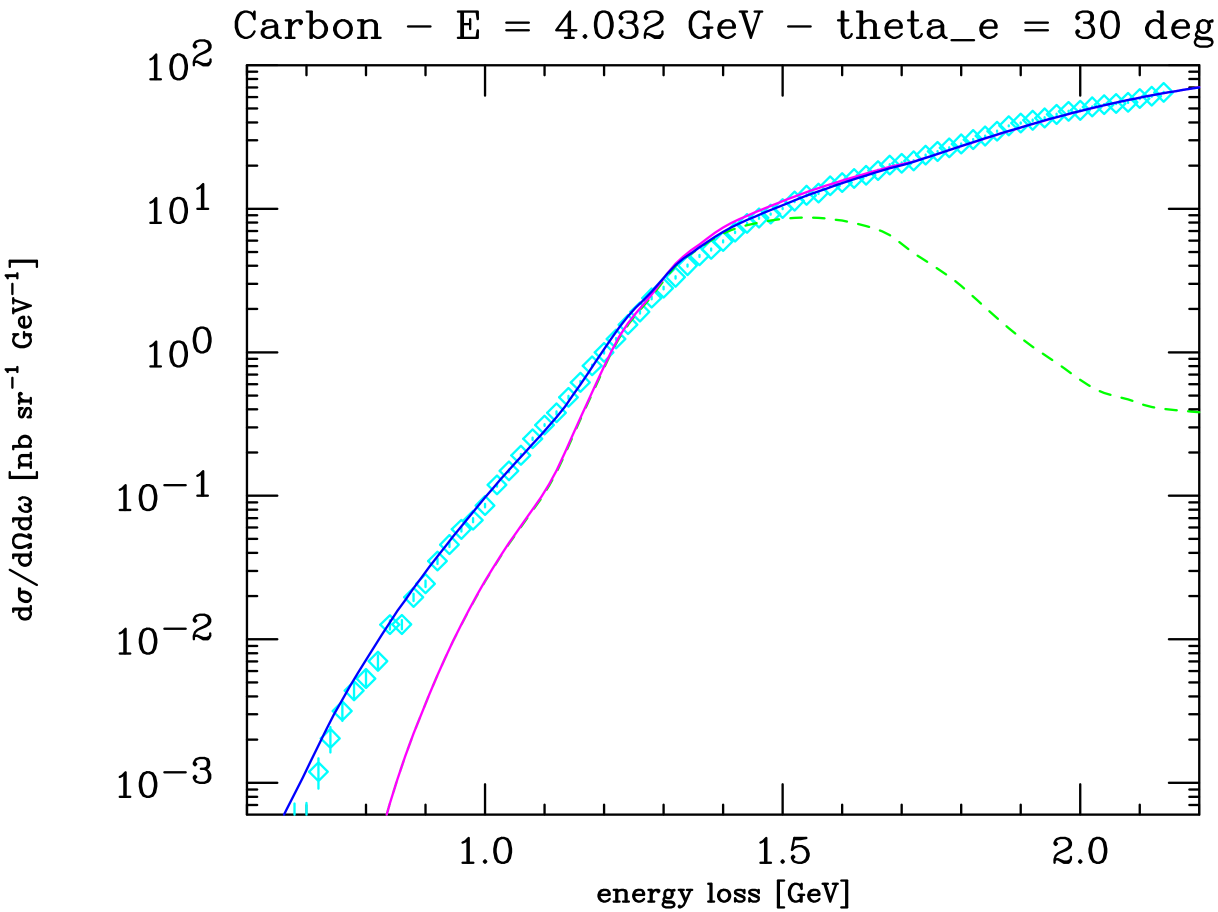
- ⋆ Additional constraint on the folding function, not taken into account in early applications
- ⋆ The relation between folding function and particle spectral function implies that

$$f_{\mathbf{q}}(\omega) \geq 0$$

- ⋆ Oscillations of the folding functions to (very small) negative values are unphysical and must be removed
- ⋆ Implementation of the above constraint requires some modeling. However, what we know on the behavior of the spectral functions provides guidance

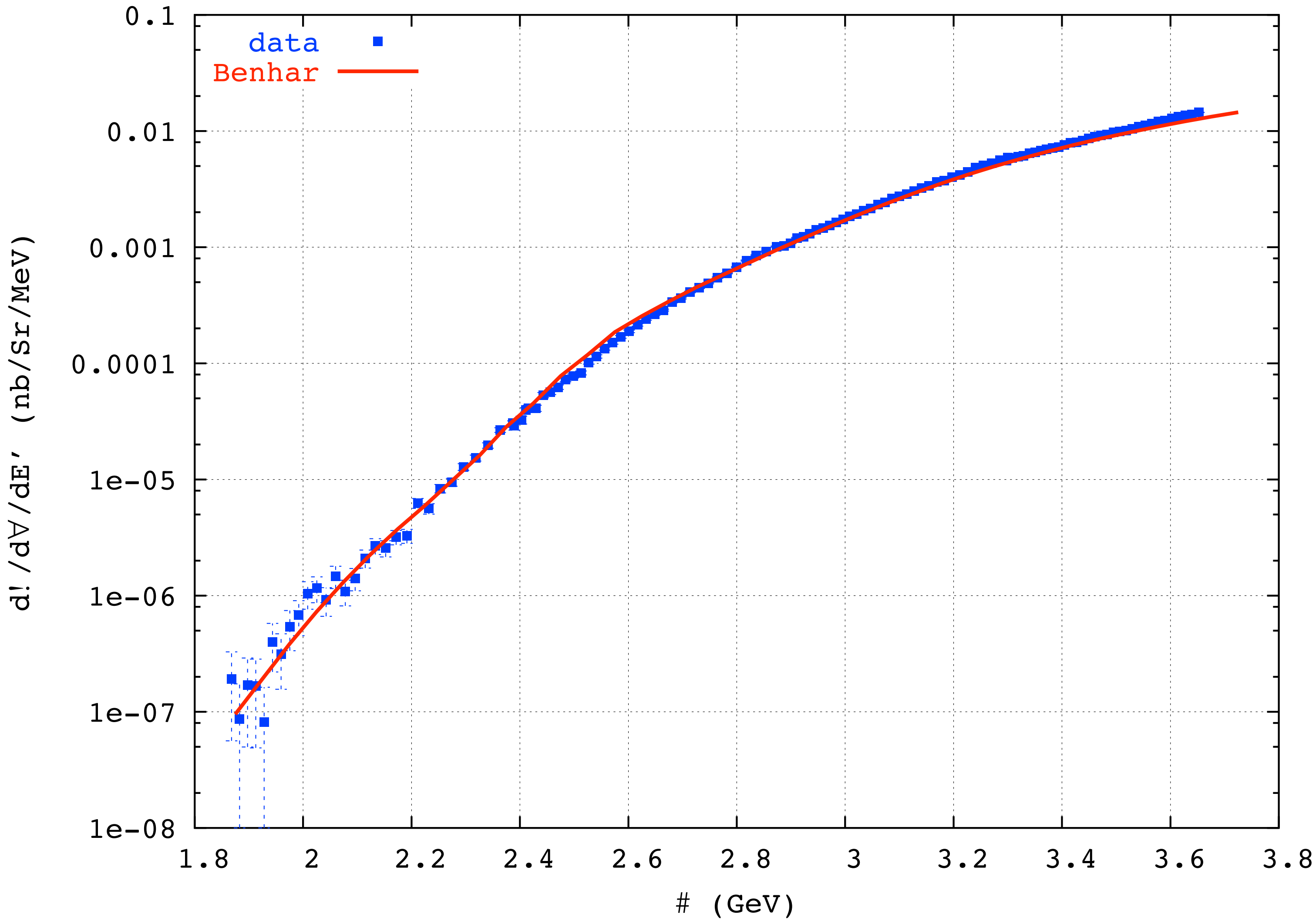
Results

- ⋆ Comparison to JLab E89-008 data



Results

⋆ Comparison to JLab E02-019 data



Issues for future work

- ⋆ NN scattering in the nuclear medium: is the transition probability also modified ?
- ⋆ Does the internal structure of the nucleon play a role ? Can its effects be unambiguously identified ?
- ⋆ How good is the frozen spectator approximation ? Calculations for nonrelativistic systems suggest that the motion of the spectators may have appreciable effects even at large $|\mathbf{q}|$
- ⋆ Any theoretical description of FSI in inclusive processes should fulfill two minimal requirements:
  - be consistent with the decription of the initial state
  - be also applicable to exclusive processes, where FSI have much larger effects (e.g. nuclear transparency)