

Final State Interaction in (e,e') Reactions at large x and Q^2

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Outlook of the talk

FSI in inclusive reactions

Within Generalized Eikonal Approximation

several issues

- Setting up kinematics relevant for SRC
- Setting up kinematics relevant for GEA
- the space time properties of FSI in GEA
- Conservation law for alpha in GEA and why it is good for SRC



in light and medium nuclei, FSI can be
localized within SRC

Correlation Parameter

$$\alpha_i = A \frac{E_i - p_i^z}{E_A - p_A^z}$$

Momentum Fraction of Nucleus
carried by the constituent

$\alpha_i > j$ corresponds to j -nucleon correlation

In Electroproduction Reaction

$$x = \frac{\alpha - \frac{m_f^2 - m_i^2}{2mq_0}}{1 + \frac{2p_i^z}{q_0 + q}}$$

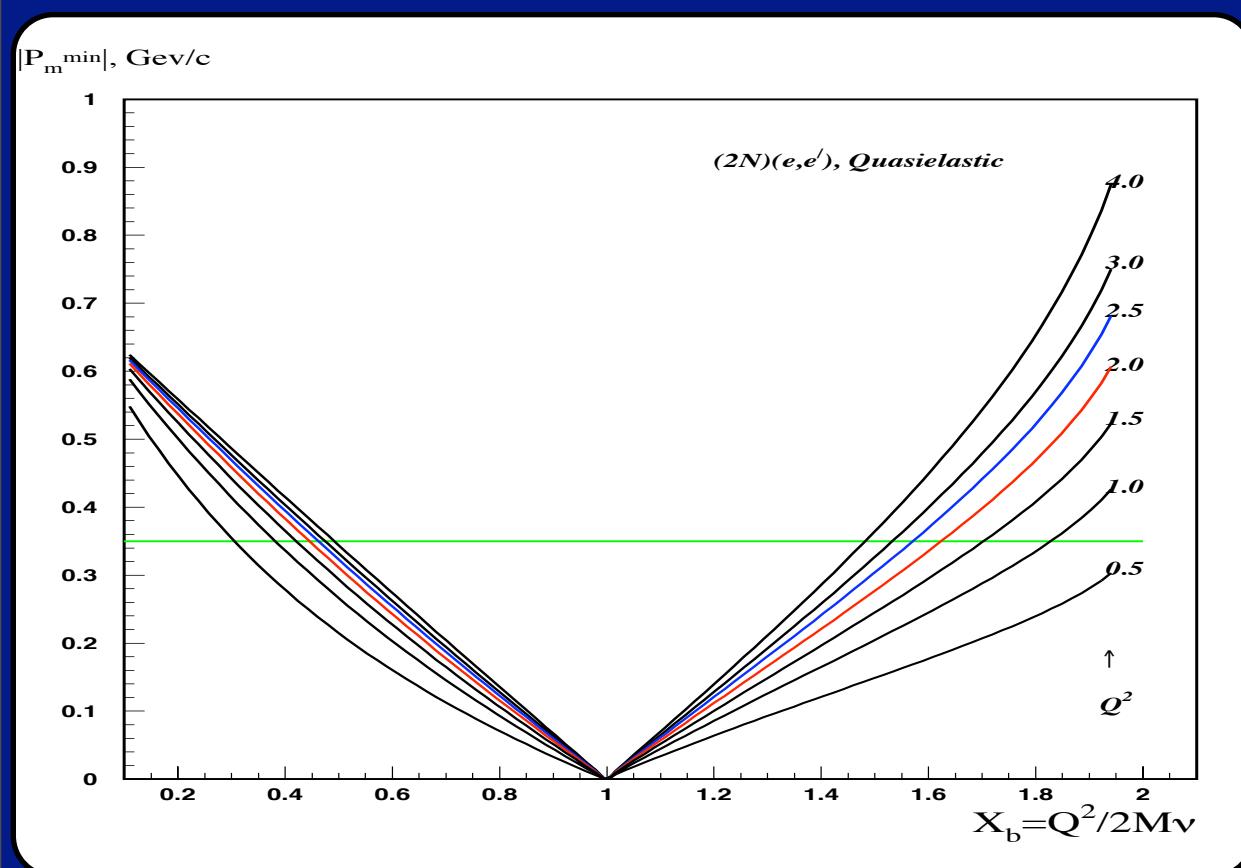
Introduction to $x > 1$ inclusive $A(e,e')X$ processes

Frankfurt & Strikman Phs. Rep. 81

Bjorken x as a correlation index

For quasielastic scattering at $x > j$

Corresponds to the scattering from a minimum $j + 1$ nucleon system at rest



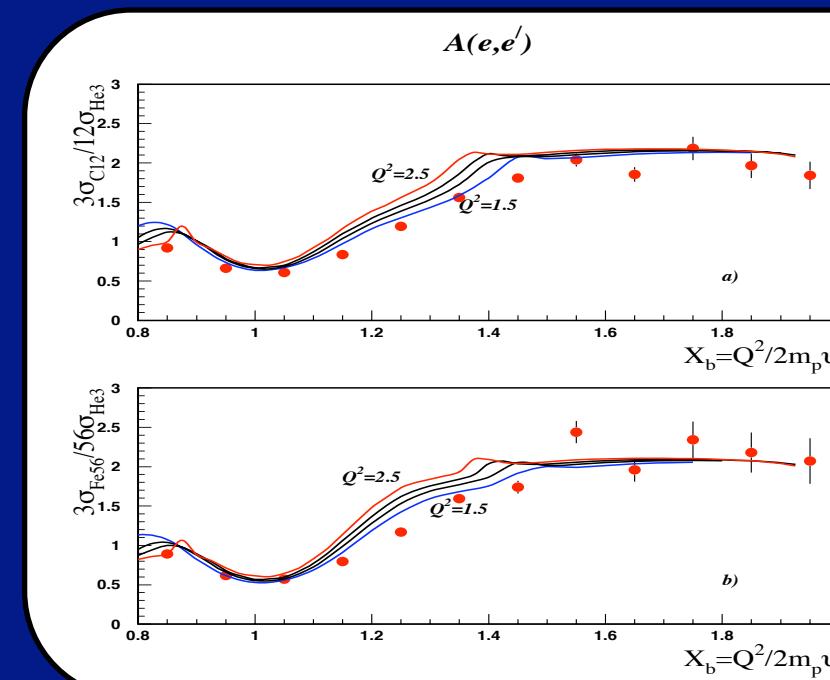
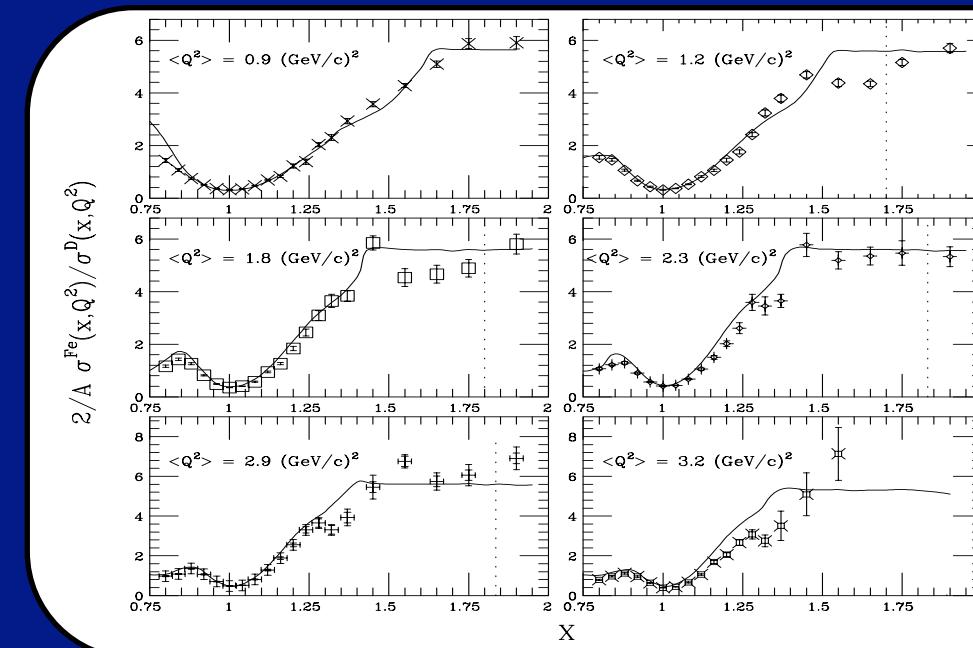
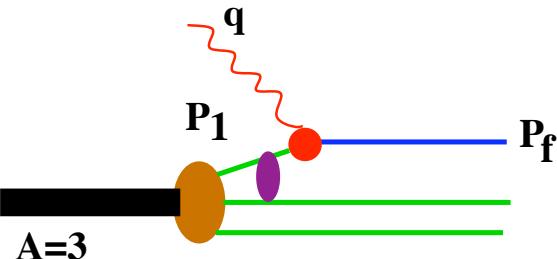
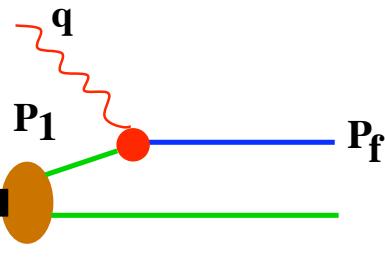
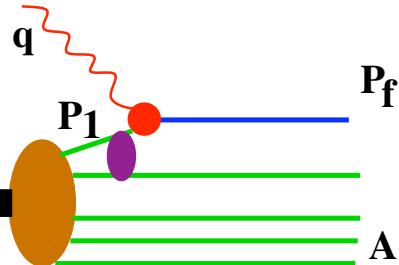
$$\vec{p}_i = -\vec{p}_m = \vec{p}_f - \vec{q}$$

$$R = \frac{A_2 \sigma [A_1(e,e')X]}{A_1 \sigma [A_2(e,e')X]}$$

Two Nucleon Correlations

$$x > 1$$

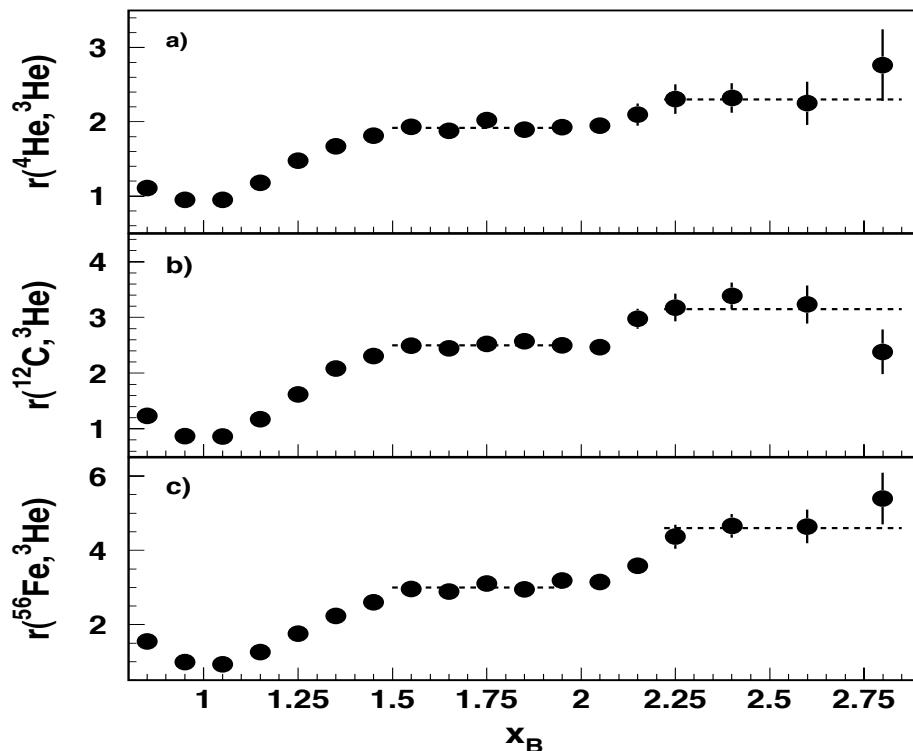
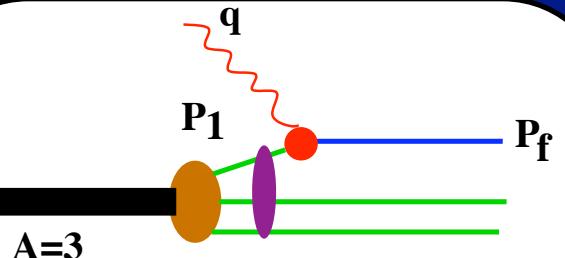
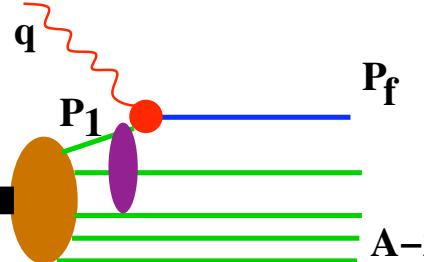
$$R = \frac{A_2 \sigma [A_1(e, e') X]}{A_1 \sigma [A_2(e, e') X]}$$



Three Nucleon Correlations

$$R = \frac{A_2 \sigma [A_1(e, e') X]}{A_1 \sigma [A_2(e, e') X]}$$

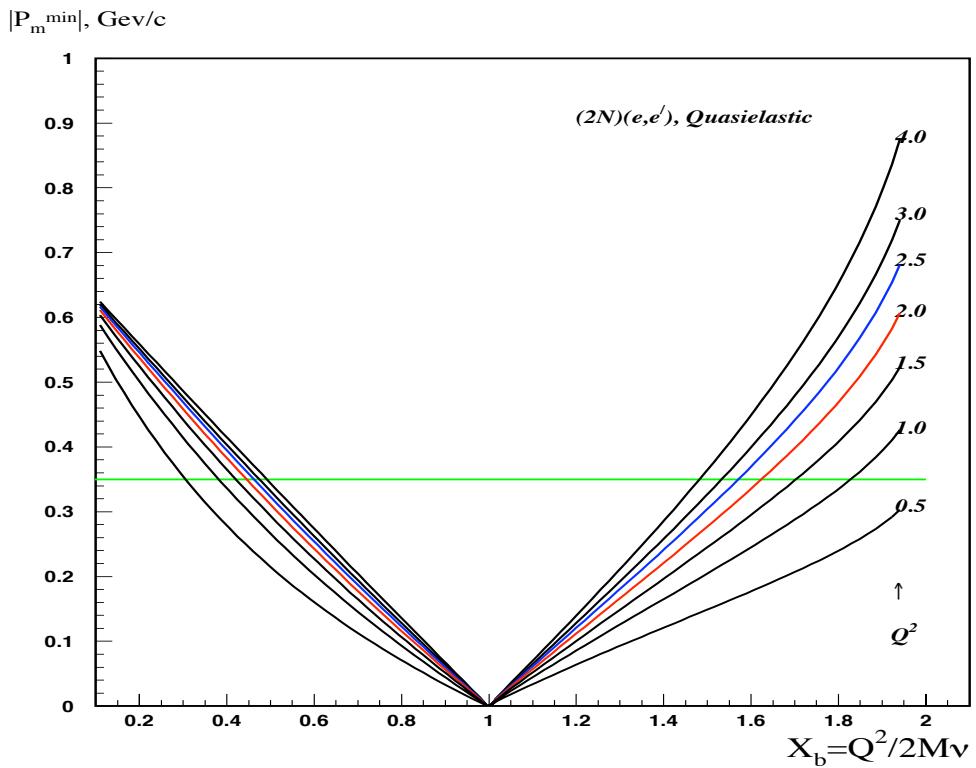
$$x > 2$$



Egiyan, et al PRL
2006

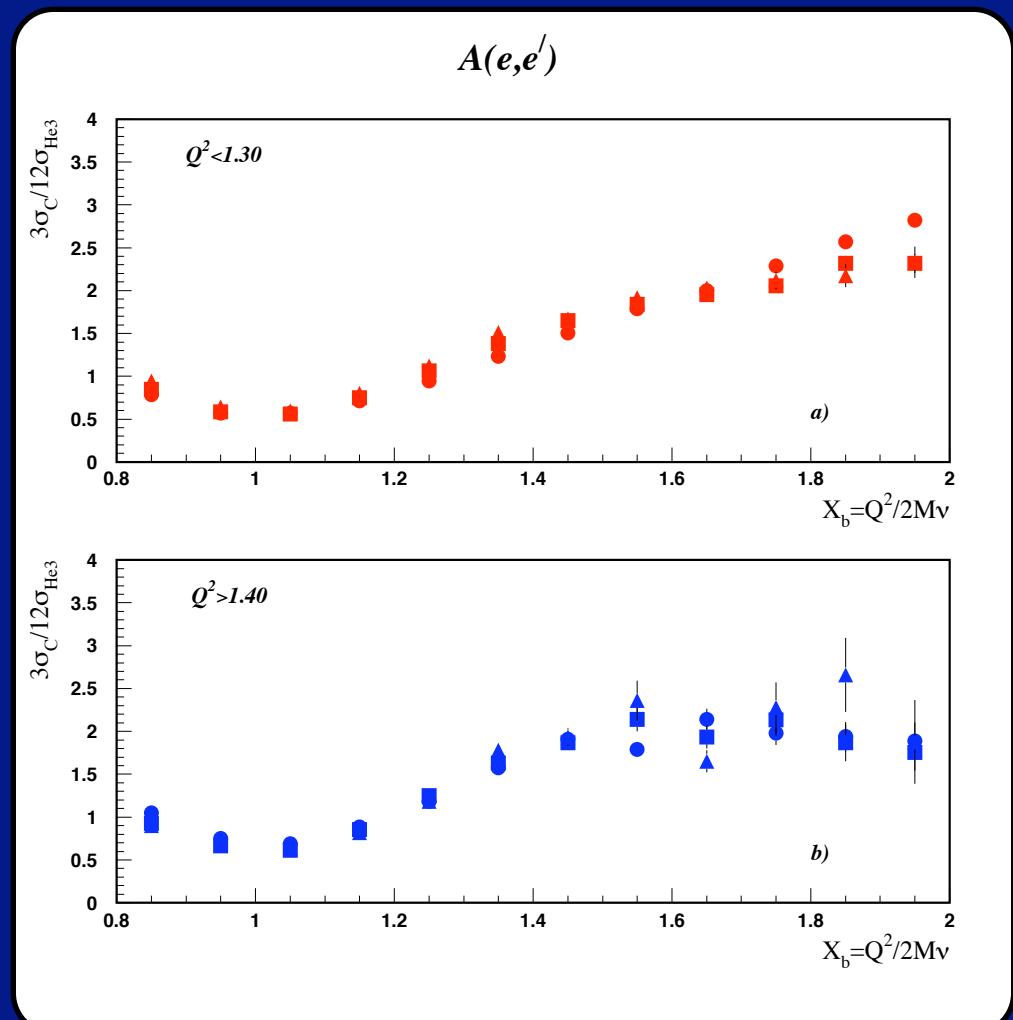
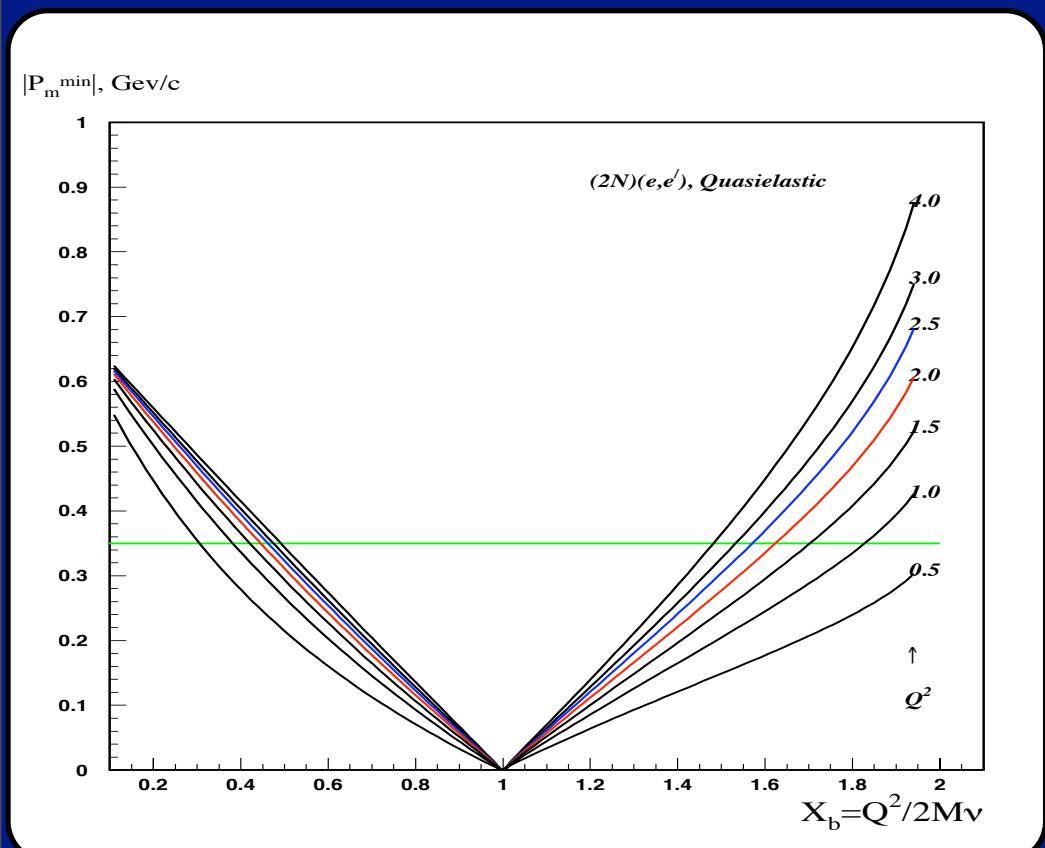
Is the scaling accidental ?

Onset of the scaling is Q^2 dependent in agreement with SRC picture



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Onset of the scaling is Q^2 dependent in agreement with SRC picture

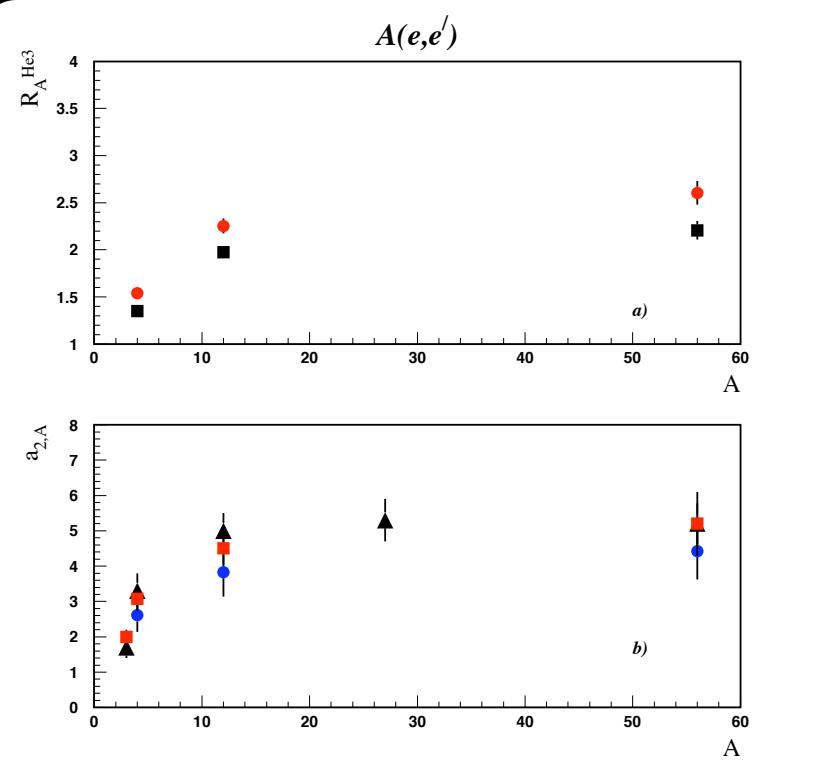


Is the scaling accidental ?

Within SRC model

$$R \mid_{1 < x < 2} \sim \frac{a_2(A_1)}{a_2(A_2)}$$

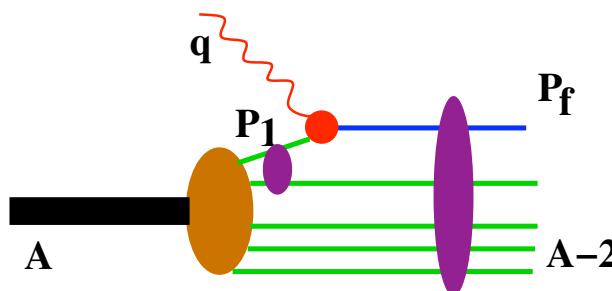
$$R \mid_{2 < x < 3} \sim \frac{a_2(A_1)}{a_2(A_2)}$$



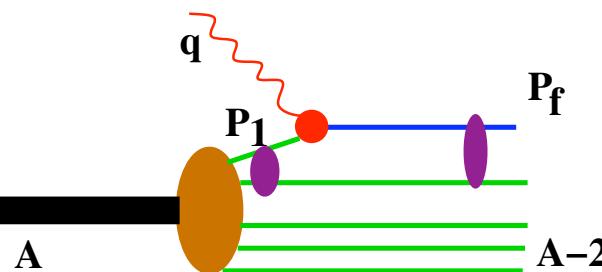
$a_2(^3\text{He})$	=	1.7(0.3)
$a_2(^4\text{He})$	=	3.3(0.5)
$a_2(^{12}\text{C})$	=	5.0(0.5)
$a_2(^{27}\text{Al})$	=	5.3(0.6)
$a_2(^{56}\text{Fe})$	=	5.2(0.9)
$a_2(^{197}\text{Au})$	=	4.8(0.7)

	$a_{2N}(A)$	$a_{8q}(A)$	$a_{3N}(A)$	$a_{9q}(A)$
${}^3\text{He}$	$0.080 \pm 0.000 \pm 0.004$	0.134	$0.0018 \pm 0.0000 \pm 0.0006$	0.022
${}^4\text{He}$	$0.154 \pm 0.002 \pm 0.033$	0.166	$0.0042 \pm 0.0002 \pm 0.0014$	0.047
${}^{12}\text{C}$	$0.193 \pm 0.002 \pm 0.041$	0.125	$0.0055 \pm 0.0003 \pm 0.0017$	0.026
${}^{56}\text{Fe}$	$0.227 \pm 0.002 \pm 0.047$	0.146	$0.0079 \pm 0.0003 \pm 0.0025$	0.036

Biggest question how come FSI is not distorting this picture?

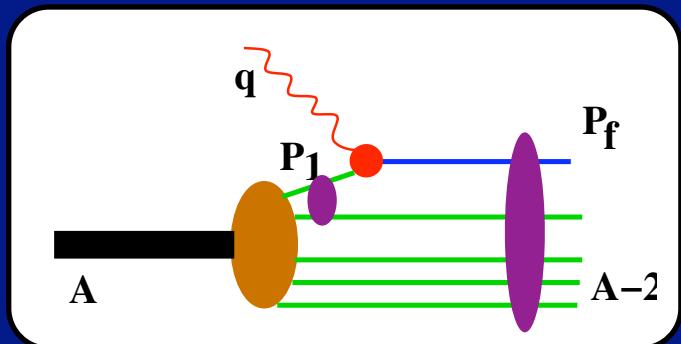


Our prediction is that
FSI is confined within SRC



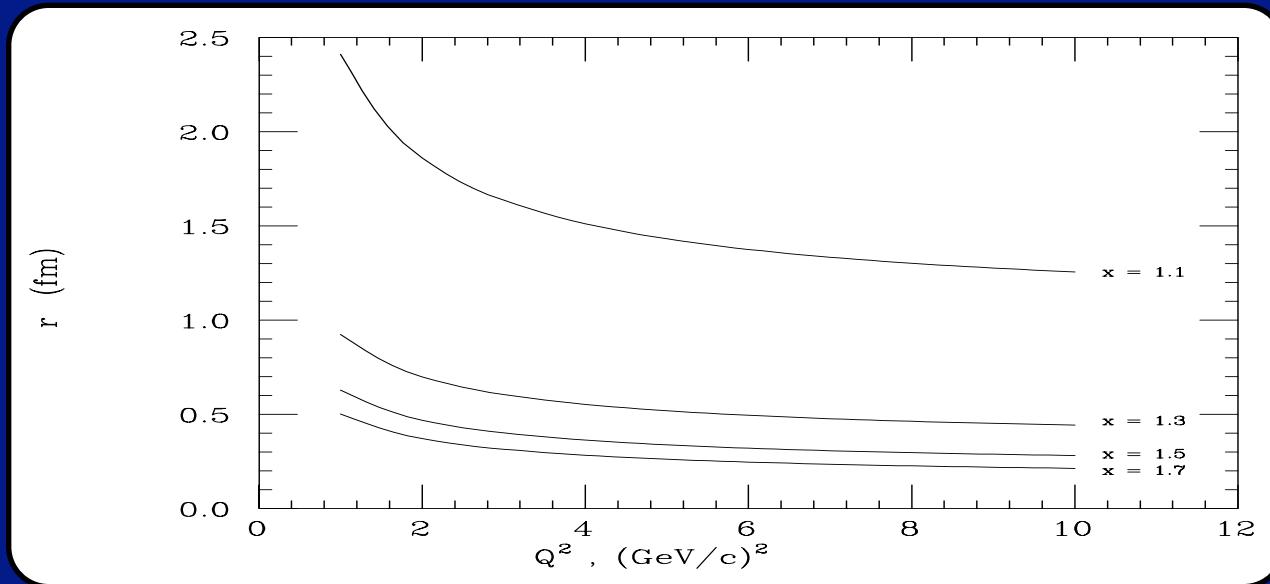
We made this observation
based on the estimates of the characteristic
distances that highly virtual struck nucleon
propagates

Day, Frankfurt, MS,
Strikman, PRC 1993



$$r \approx \frac{1}{\Delta E v}$$

$$\Delta E = -q_0 - M_A + \sqrt{m^2 + (p_i + q)^2} + \sqrt{M_{A-1}^2 + p_i^2}$$

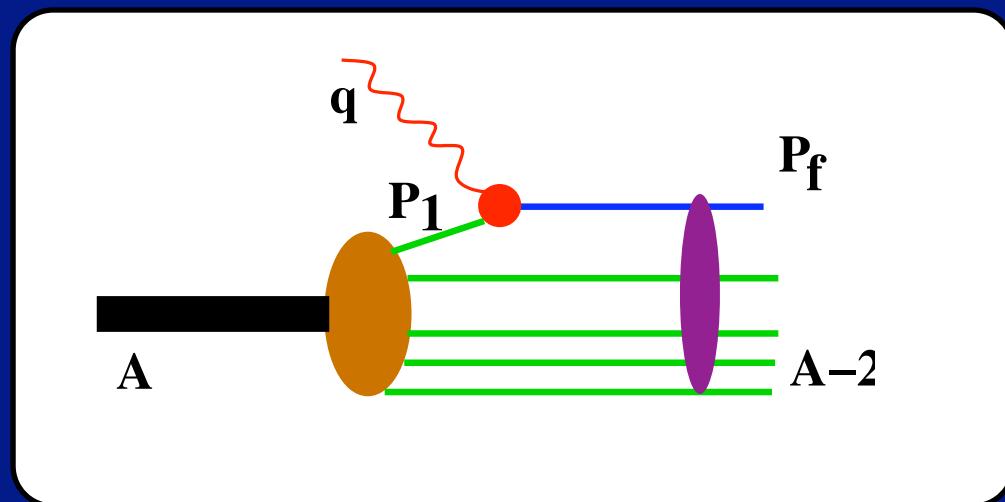


Generalized Eikonal Approximation

Frankfurt,
Greenberg, Miller,
MS, Strikman, ZPhys
1995 ,

Frankfurt, MS,
Strikman, PRC1997 ,

MS, Int. J. Mod. Phys
2001,

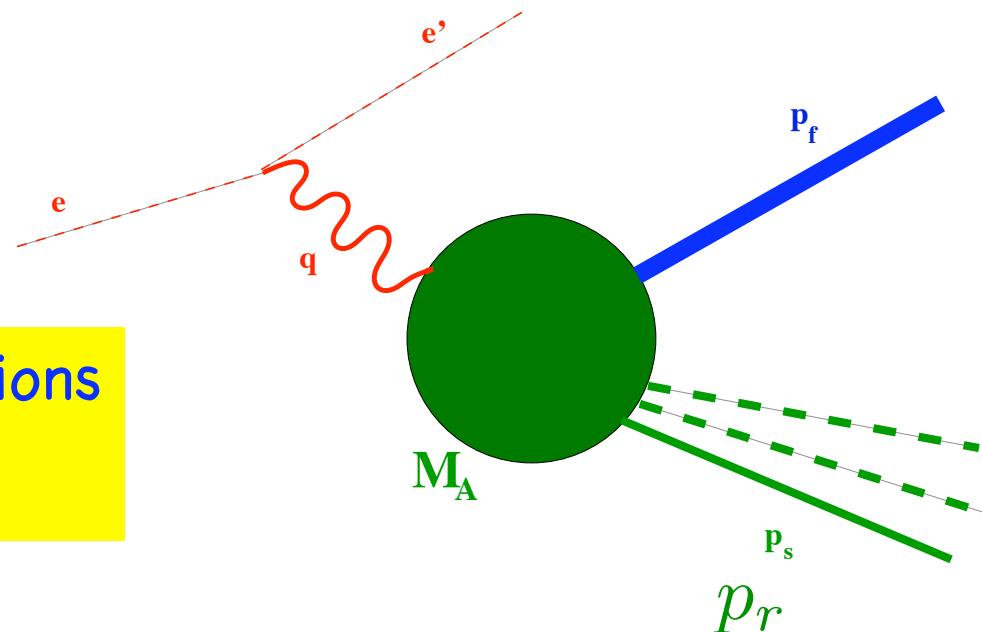


High Energy Photo/Electro-Nuclear Reactions

Kinematics

I. Momenta involved in the reactions

$$q \approx p_f > \text{few GeV}/c.$$

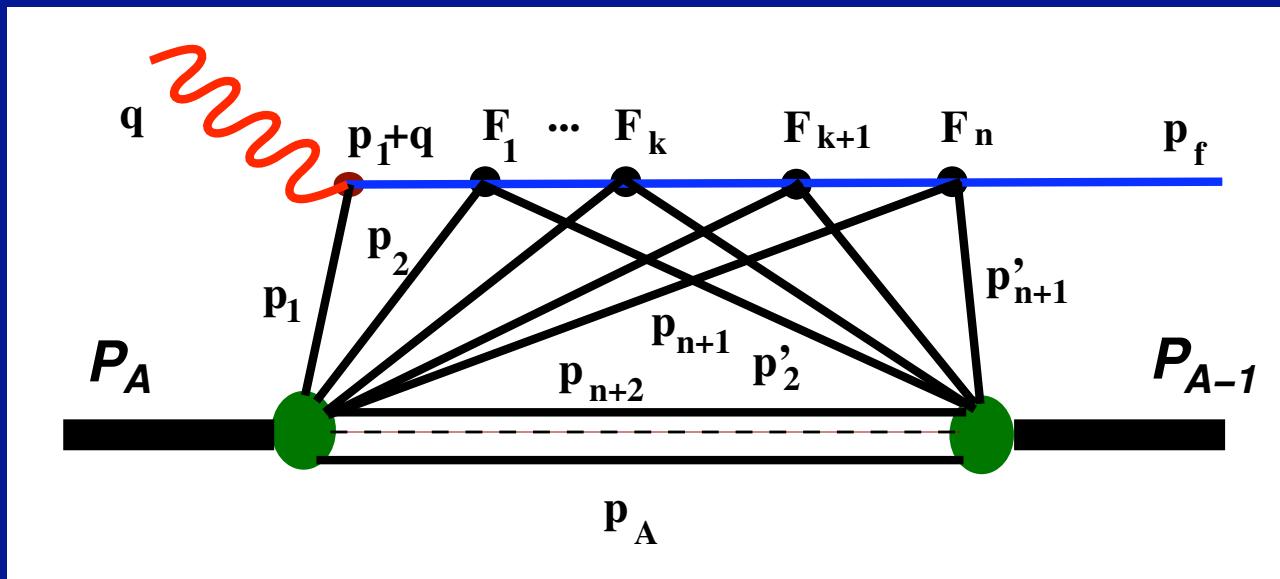
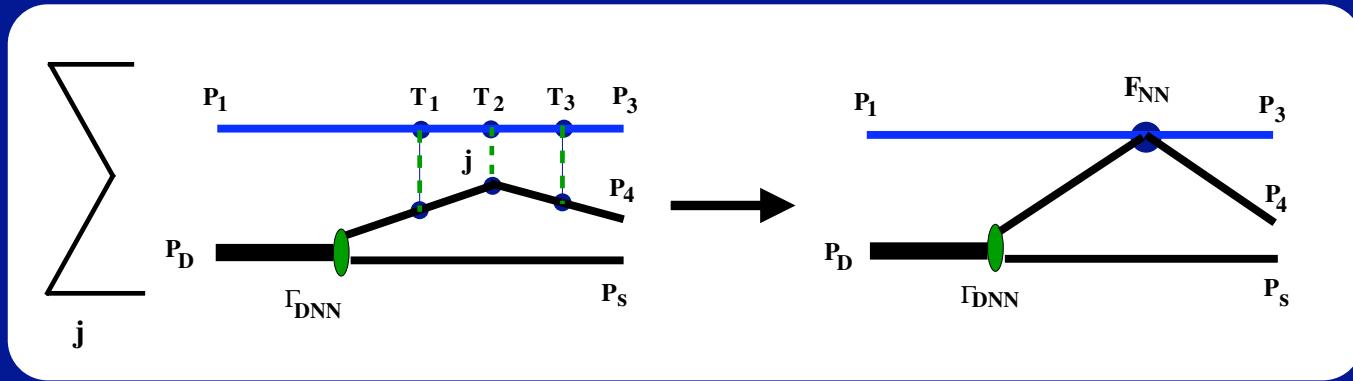


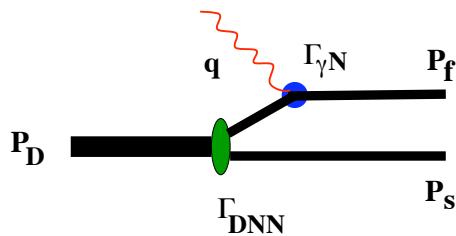
A new small parameter

$$\frac{p_-^f}{p_+^f} \equiv \frac{E^f - p_z^f}{E^f + p_z^f} \approx \frac{m^2}{4p_z^f} \ll 1$$
$$\frac{q_-}{q_+} \approx \frac{x_{Bj}^2 m^2}{Q^2} \ll 1$$

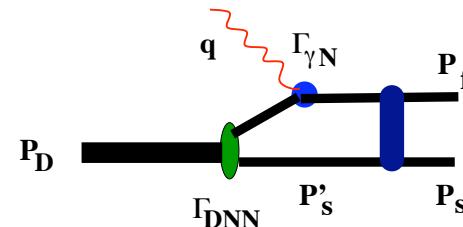
For inclusive (e, e') reaction

$$\sqrt{\frac{Q^2(2-x)}{x}} \geq \frac{1}{2}$$

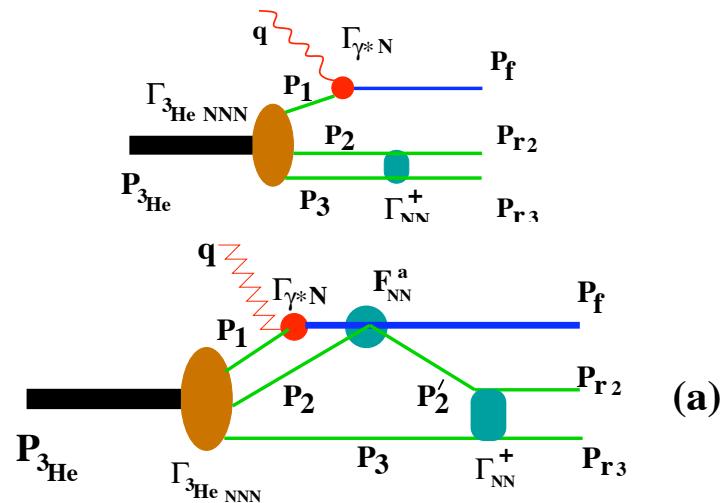




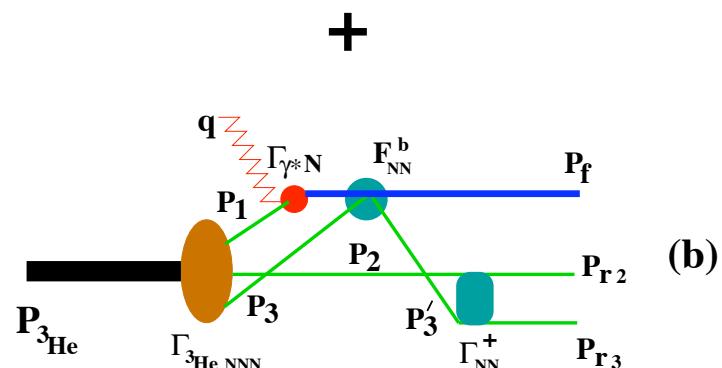
(a)



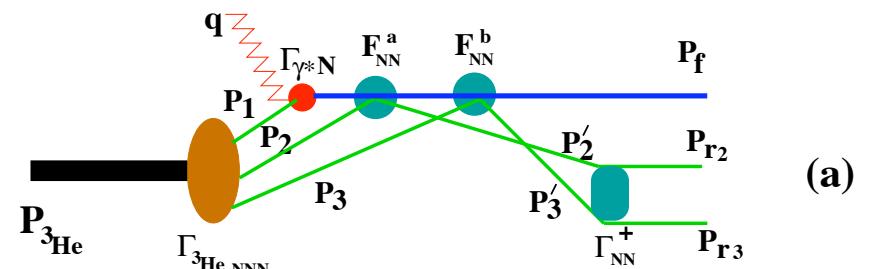
(b)



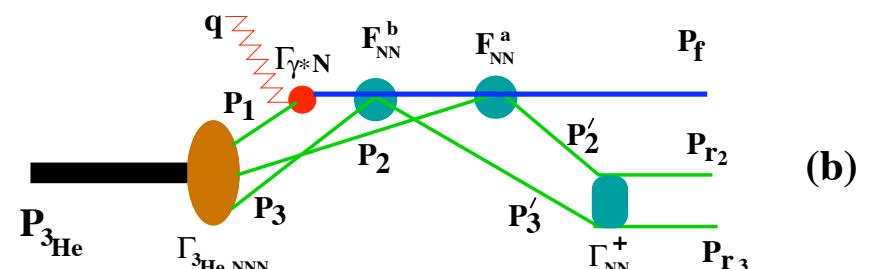
(a)



(b)

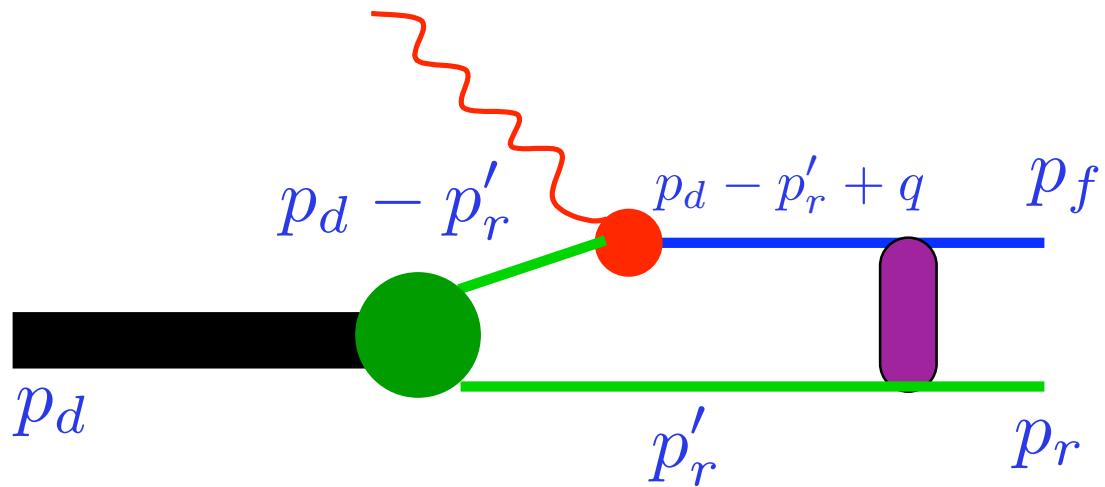


(a)



(b)

$$e + d \rightarrow e + p + n$$



$$A_1^\mu = - \int \frac{d^4 p'_r}{i(2\pi)^4} \frac{\bar{u}(p_f) \bar{u}(p_r) F_{NN} [\not{p}'_r + m] [\not{p}_D - \not{p}'_r + \not{q} + m]}{(p_D - p'_r + q)^2 - m^2 + i\epsilon} \\ \frac{\Gamma_{\gamma^* N}^\mu [\not{p}_D - \not{p}'_r + m] \Gamma_{D NN}}{((p_D - p'_r)^2 - m^2 + i\epsilon)(p'^2_r - m^2 + i\epsilon)}.$$

$$\int \frac{d^0 p'_r}{p'^2_r - m^2 + i\epsilon} = - \frac{i(2\pi)}{2E'_r}$$

$$A_1^\mu = -\sqrt{2}(2\pi)^{\frac{3}{2}} \sum_{\lambda_1} \int \frac{d^3 p'_r}{(2\pi)^3} \frac{1}{\sqrt{2E'_r}} \frac{\sqrt{s(s-4m^2)} f_{pn}(p_{r\perp} - p'_{r\perp})}{(p_D - p'_r + q)^2 - m^2 + i\epsilon} \\ \times J_{\gamma^* N}^\mu(\lambda_f, p_D - p'_r + q; \lambda_1, p_D - p'_r) \cdot \frac{\psi_D(p'_r)}{N(p'_r)}.$$

$$(p_D - p'_r + q)^2 - m^2 + i\epsilon = m_D^2 - 2p_D p'_r + p'^2_r + 2q(p_D - p'_r) - Q^2 - m^2 + i\epsilon.$$

From Energy-Momentum conservation

$$(p_D - p_r + q)^2 = m^2 = m_D^2 - 2p_D p_r + m^2 + 2q(p_D - p_r) - Q^2$$

$$(p_D - p'_r + q)^2 - m^2 + i\epsilon = 2|\mathbf{q}| \left[p'_{rz} - p_{rz} + \frac{q_0}{|\mathbf{q}|} (E_r - E'_r) + \frac{m_D}{|\mathbf{q}|} (E_r - E'_r) \right].$$

$$A_1^\mu = -(2\pi)^{\frac{3}{2}} \sum_{\lambda_1} \int \frac{d^3 p'_r}{(2\pi)^3} \frac{1}{2|q|\sqrt{E'_r}} \frac{\sqrt{s(s-4m^2)} f_{pn}(p_{r\perp} - p'_{r\perp})}{p'_{rz} - p_{rz} + \Delta + i\epsilon}$$

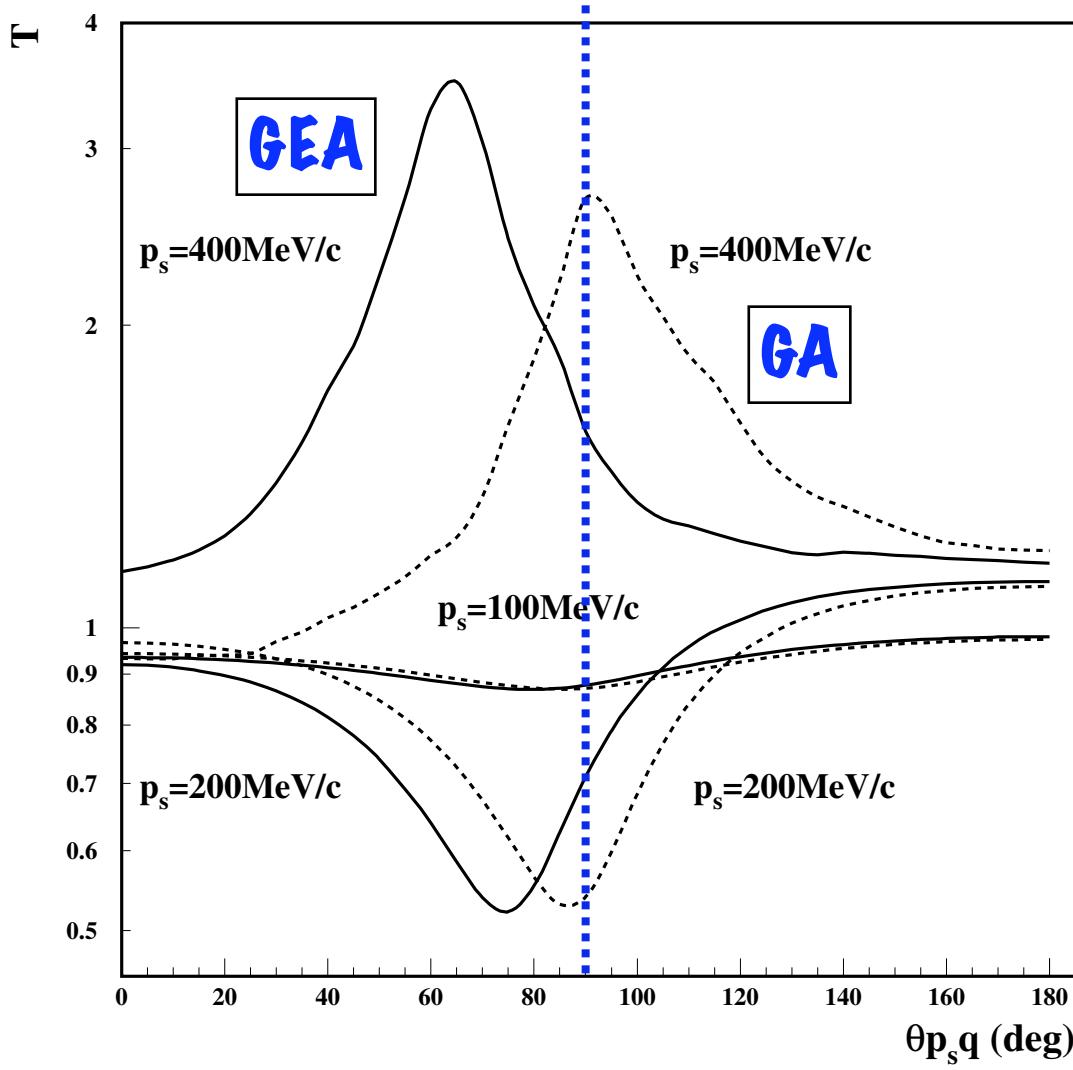
$$\times J_{\gamma^* N}^{\mu}(\lambda_f, p_D - p'_r + q; \lambda_1, p_D - p'_r) \cdot \frac{\psi_D(p'_r)}{N(p'_r)}.$$

where $\boxed{\Delta = \frac{q_0}{|q|}(E_r - E'_r) + \frac{M_d}{|q|}(E_r - E'_r)}$ $\approx m \frac{q_0}{|q|}(1-x)$

$$\int \frac{dp'_{rz}}{(2\pi)} \frac{\Psi_d(p'_r)}{p'_{rz} - (p_{rz} - \Delta) + i\varepsilon} = -\frac{i}{2} \left[\Psi_d(\tilde{p}_r) + i\tilde{\Psi}_d(\tilde{p}_r) \right]$$

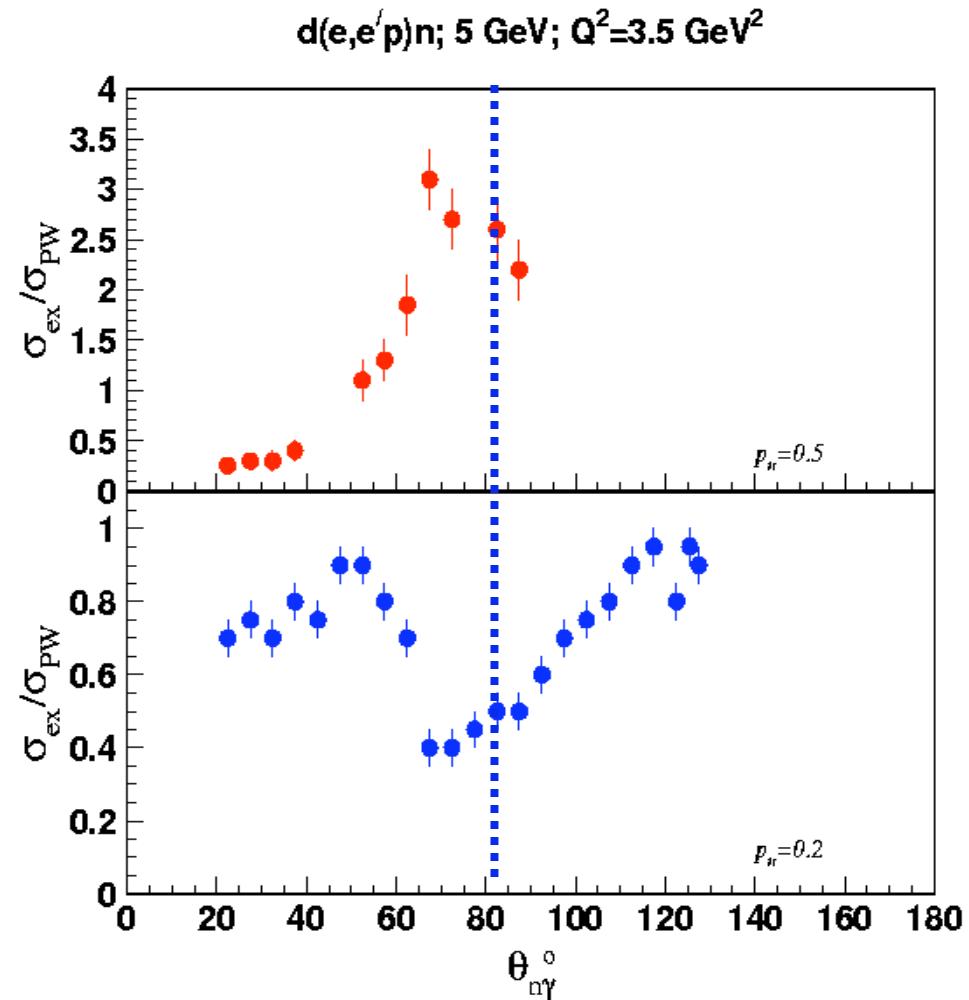
where $\tilde{p}_r \equiv (p'_{r\perp}, p_{rz} - \Delta)$

$$A_1^\mu = -\frac{\sqrt{2}(2\pi)^{\frac{3}{2}}}{4i} \sum_{\lambda_1} \int \frac{d^2 k_\perp}{(2\pi)^2} \sqrt{2E'_r} \frac{\sqrt{s(s-4m^2)}}{2|q|E'_r} \\ \left[f_{pn}^{on}(k_\perp) J_{\gamma^* N}^{\mu, on} \Psi_D(\tilde{p}_r) + i f_{pn}^{off}(k_\perp) J_{\gamma^* N}^{\mu, off} \tilde{\Psi}(\tilde{p}_r) \right] \frac{1}{N(p'_r)}$$



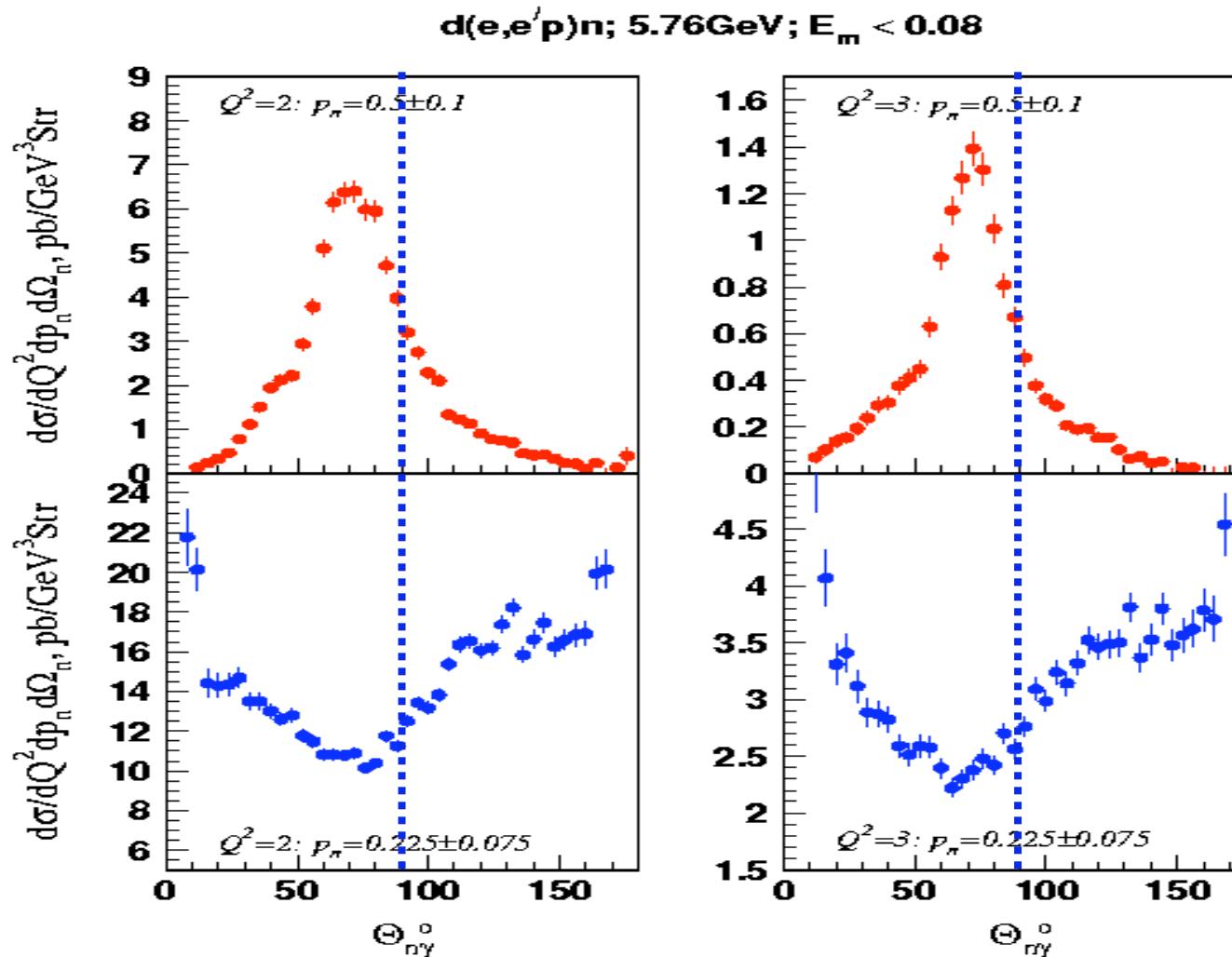
Frankfurt, MS,
Strikman, PRC1997 ,

Recoil-Neutron Angular Distributions; Hall A Exp.

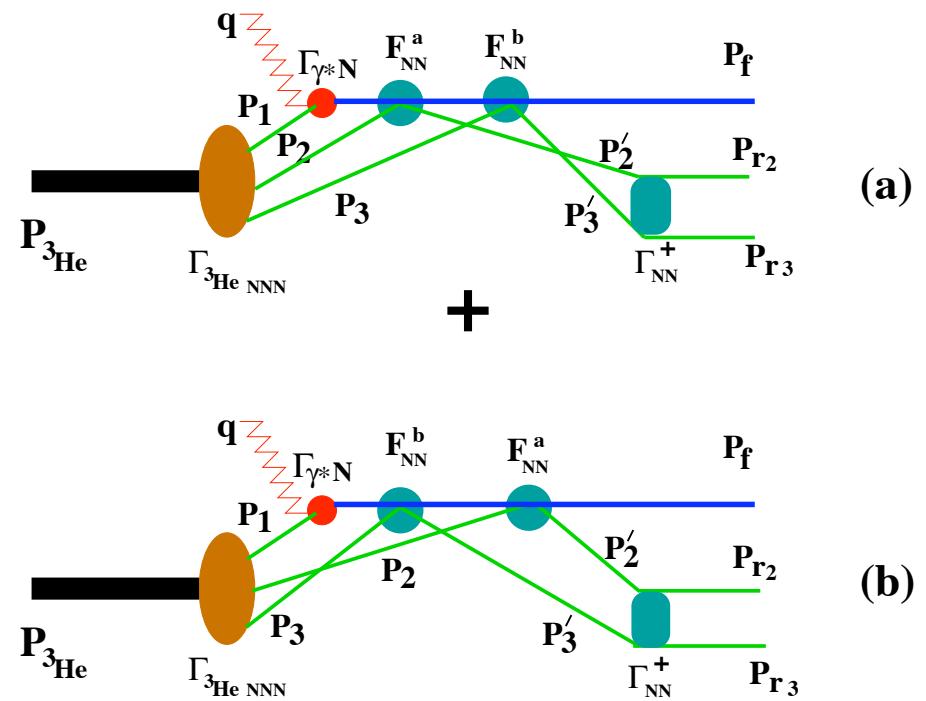
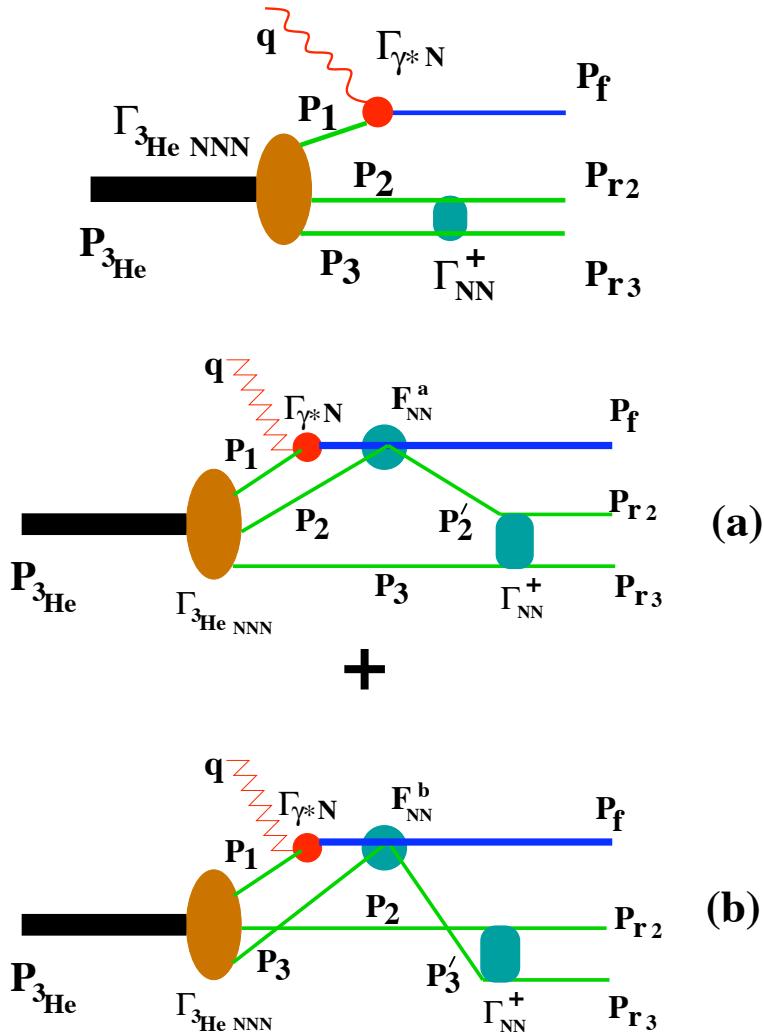


PRELIMINARY

Recoil-Neutron's Angular Distributions - I

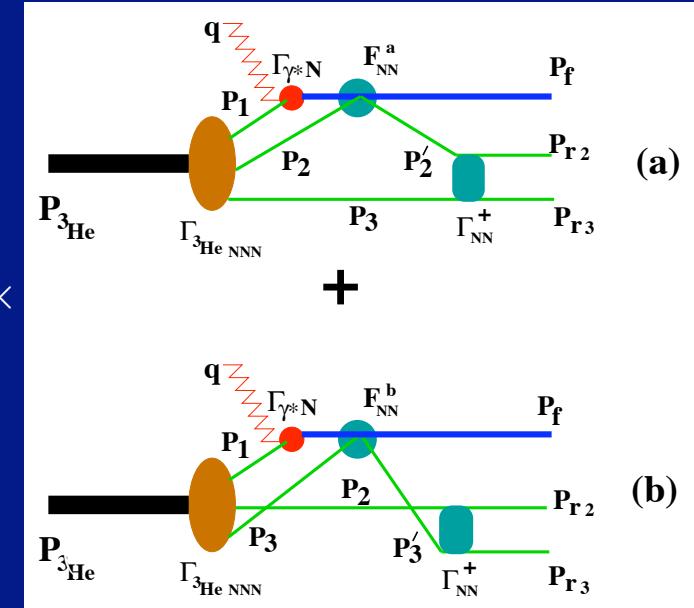


P
R
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Single Rescattering

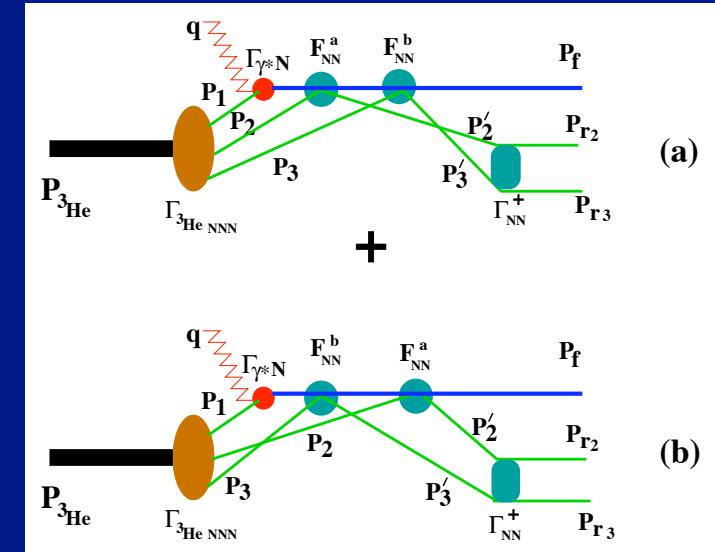
$$\begin{aligned}
A_{1a}^\mu &= - \int \frac{d^4 p_2}{i(2\pi)^4} \frac{d^4 p_3}{i(2\pi)^4} \bar{u}(p_{r3}) \bar{u}(p_{r2}) \bar{u}(p_f) \frac{\Gamma_{NN}^+(p'_2, p_3)(\hat{p}'_2 + m)}{p'^2_2 - m^2 + i\varepsilon} \times \\
&\times \frac{F_{NN}^a(p'_2 - p_2)(\hat{p}_1 + \hat{q} + m)}{(p_1 + q)^2 - m^2 + i\varepsilon} \cdot \Gamma_{\gamma^* N}^\mu \cdot \frac{\hat{p}_3 + m}{p'^2_3 - m^2 + i\varepsilon} \times \\
&\times \frac{\hat{p}_2 + m}{p'^2_2 - m^2 + i\varepsilon} \cdot \frac{\hat{p}_1 + m}{p'^2_1 - m^2 + i\varepsilon} \cdot \Gamma_{^3\text{He}NNN}(p_1, p_2, p_3) \chi^A.
\end{aligned}$$



$$\begin{aligned}
A_{1a}^\mu &= -\frac{F}{2} \sum_{s_{1'}, s_{2'}, s_1, s_2, s_3} \sum_{t_1, t_2', t_2, t_3} \int \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p'_2, s_{2'}, t_{2'}; p_3, s_3, t_3) \\
&\times \frac{\sqrt{s_2^{NN}(s_2^{NN} - 4m^2)}}{2qm} \frac{f_{NN}(p'_2, s_{2'}, t_{2'}, p_f, s_f, t_f; |p_2, s_2, t_2, p_1 + q, s_{1'}, t_1|)}{p_{mz} + \Delta^0 - p_{1z} + i\varepsilon} \\
&\times j_{t_1}^\mu(p_1 + q, s_{1'}; p_1, s_1) \cdot \Psi_A^{s_A}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3).
\end{aligned}$$

$$\Delta^0 = \frac{q_0}{q} (T_{r2} + T_{r3} + |\epsilon_A|)$$

Double Rescattering



$$\begin{aligned}
 A_{2a}^\mu &= \frac{F}{4} \sum_{s_1, s_2, s_3} \sum_{t_1, t_2, t_3, t_{1'}, t_{2'}, t_{3'}} \int \frac{d^3 p'_3}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p'_2, s_2, t_{2'}; p'_3, s_3, t_{3'}) \times \\
 &\times \frac{\chi_2(s_{b3}^{NN}) f_{NN}^{t_{3'}, t_f | t_3, t_{1'}}(p'_{3\perp} - p_{3\perp})}{\Delta_3 + p'_{3z} - p_{3z} + i\varepsilon} \frac{\chi_1(s_{a2}^{NN}) f_{NN}^{t_{2'}, t_{1'} | t_2, t_1}(p'_{2\perp} - p_{2\perp})}{\Delta^0 + p_{mz} - p_{1z} + i\varepsilon} \\
 &\times j_{t1}^\mu(p_1 + q, s_f; p_1, s_1) \cdot \Psi_A^{s_A}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3),
 \end{aligned}$$

$$\Delta^3 \approx \frac{E_f}{p_{fz}} T_{r3}$$

Dynamics of Reinteraction within GEA

Comparing with Glauber theory - Single Rescattering

GEA in coordinate space

$$A_1^\mu \sim \int d^3r \psi_{A-1}^\dagger e^{-ip_i r} \Theta(z) \Gamma_{GEA}^{NN}(\Delta_0, z, b) \Psi_A(r)$$

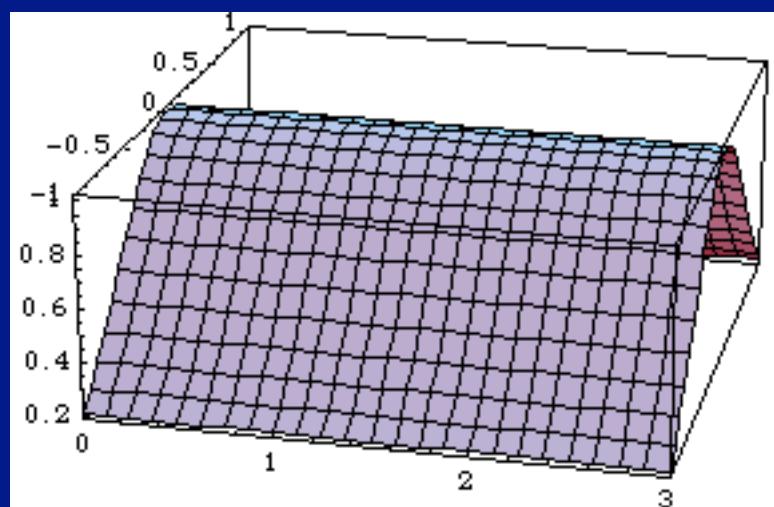
$$\Gamma_{GEA}^{NN}(\Delta_0, z, b) = e^{i\Delta_0 z} \Gamma_{Glauber}^{NN}(z, b)$$

$$\Gamma_{Glauber}^{NN}(z, b) = \frac{1}{2i} \int f^{NN}(k_\perp) e^{-ik_\perp b} \frac{d^2 k_\perp}{(2\pi)^2}$$

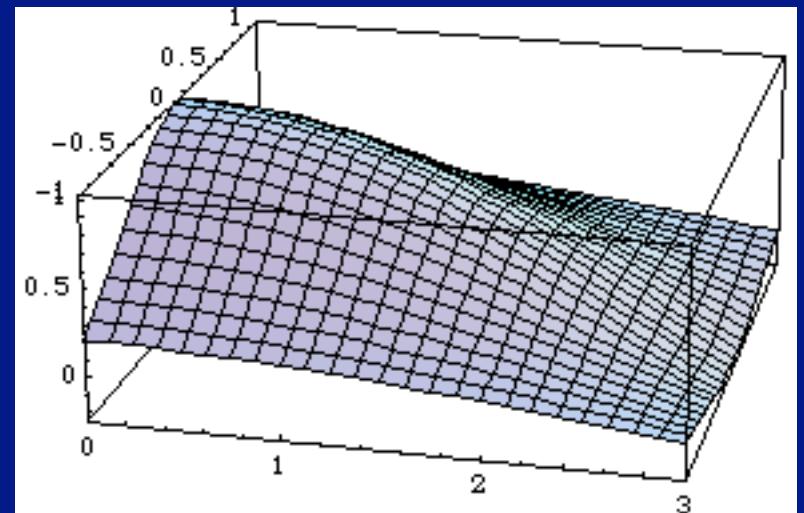
$$\Delta^0 = \frac{q_0}{q} (T_{r2} + T_{r3} + |\epsilon_A|)$$

Impulse Approximation

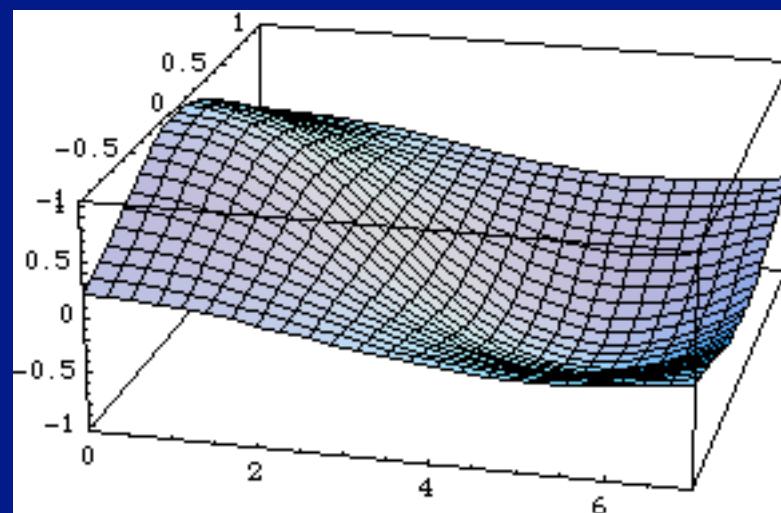
$\Gamma_{Glauber}(z, b)$



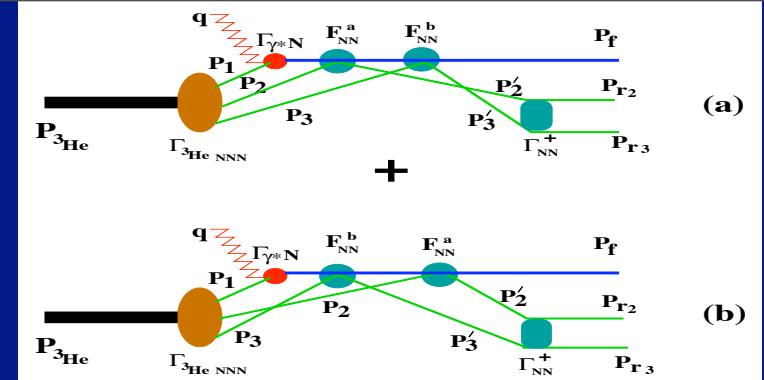
$\Gamma_{GEA}(\Delta_0, z, b)$



$\Gamma_{GEA}(\Delta_0, z, b)$

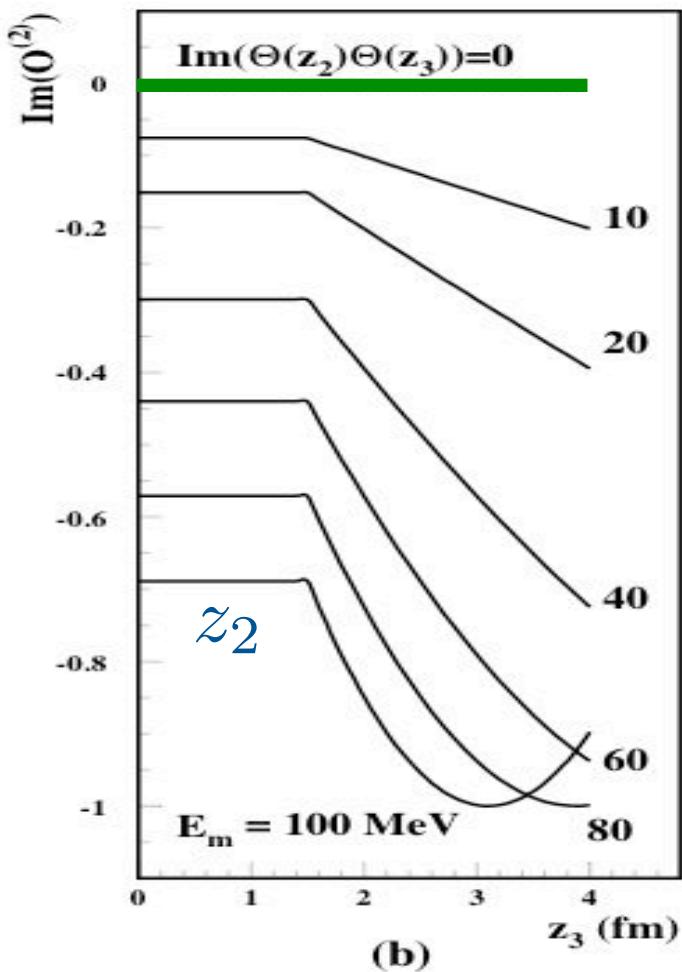
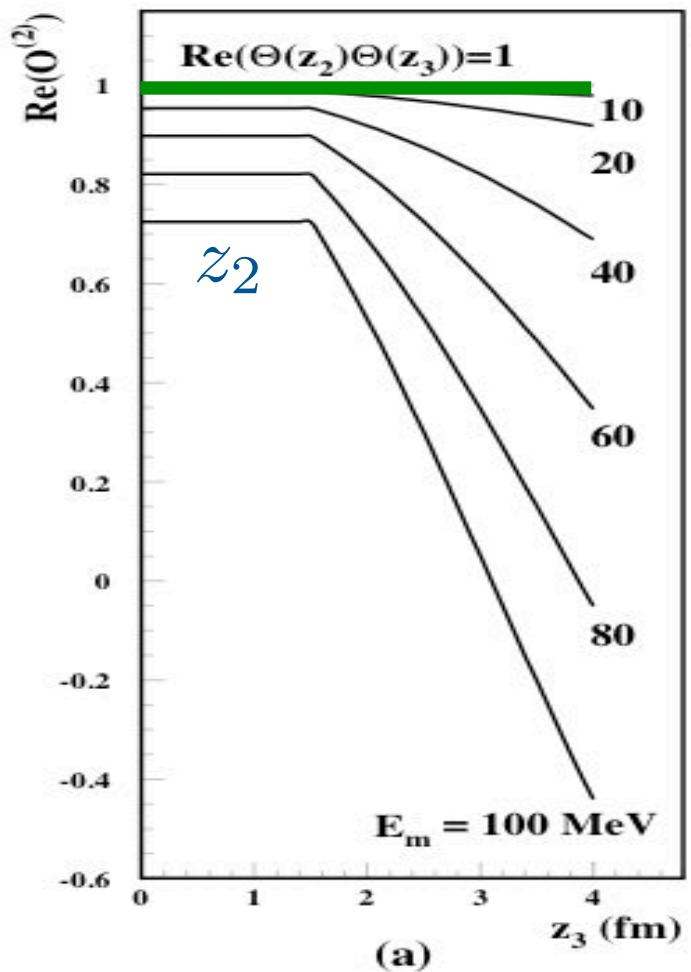


Double Rescattering



$$\begin{aligned}
 A_2^\mu &\sim \int d^3x_1 d^3x_2 d^3x_3 \psi^\dagger(x_2 - x_3), \mathcal{O}^{(2)}(z_1, z_2, z_3, \Delta_0, \Delta_2, \Delta_3) \\
 &\quad \Gamma^{NN}(x_2 - x_1, \Delta_0) \Gamma^{NN}(x_3 - x_1, \Delta_0) e^{-i\vec{r}_1 \cdot \vec{p}_m} \psi_A(x_1, x_2, x_3) \\
 \mathcal{O}^{(2)}(z_1, z_2, z_3, \Delta_0, \Delta_2, \Delta_3) &= \\
 &\quad \Theta(z_2 - z_1) \Theta(z_3 - z_2) e^{-i\Delta_3(z_2 - z_1)} e^{i(\Delta_3 - \Delta_0)(z_3 - z_1)} \\
 &\quad + \Theta(z_3 - z_1) \Theta(z_2 - z_3) e^{-i\Delta_2(z_3 - z_1)} e^{i(\Delta_2 - \Delta_0)(z_2 - z_1)}. \tag{1}
 \end{aligned}$$

$$\mathcal{O}|_{\Delta, \Delta_2, \Delta_3 \rightarrow 0} \rightarrow \Theta(z_2 - z_1) \Theta(z_3 - z_1)$$



Conservation of α

$$\begin{aligned}
A_{1a}^\mu &= -\frac{F}{2} \sum_{s_{1'}, s_{2'}, s_1, s_2, s_3} \sum_{t_1, t_{2'}, t_2, t_3} \int \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p'_2, s_{2'}, t_{2'}; p_3, s_3, t_3) \\
&\times \frac{\sqrt{s_2^{NN}(s_2^{NN} - 4m^2)}}{2qm} \frac{f_{NN}(p'_2, s_{2'}, t_{2'}, p_f, s_f, t_f; |p_2, s_2, t_2, p_1 + q, s_{1'}, t_1|)}{p_{mz} + \Delta^0 - p_{1z} + i\varepsilon} \\
&\times j_{t_1}^\mu(p_1 + q, s_{1'}; p_1, s_1) \cdot \Psi_A^{s_A}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3). \tag{1}
\end{aligned}$$

$$\frac{1}{[p_z^m + \Delta_0 - p_{1z} + i\epsilon]} = \frac{1}{m[\alpha_1 - \alpha_i - \frac{Q^2}{2q^2} \frac{E_m}{m} + i\epsilon]}.$$

$$E_m = q_0 - T_f$$

$$\alpha_i = \alpha_f - \frac{q_-}{m}$$

$$\frac{Q^2}{2|q|^2} \frac{E_m}{m} = \frac{1}{2(1 + \frac{q_0}{2mx})} \frac{E_m}{m} \rightarrow 0$$

Conservation of α

$$A_1^\mu \sim - \int \psi_A(\alpha_1, p_{1t}, \alpha_2, p_{2t}, \alpha_3, p_{3t}) J_1^{em,\mu}(Q^2) \frac{f^{NN}}{\underbrace{[\alpha_1 - \alpha_m - \frac{Q^2}{2q^2} \frac{E_m}{m} + i\epsilon]}_{\text{red line}}} \psi_{A-1}(\alpha'_2, p'_{2t}, \alpha_3, p_{3t}) \frac{d\alpha_1 d^2 p_{1t}}{(2\pi)^3} \frac{d\alpha_3 d^2 p_{3t}}{(2\pi)^3}.$$

$$\begin{aligned} A_2^\mu &\sim \int \psi_A(\alpha_1, p_{1t}, \alpha_2, p_{2t}, \alpha_3, p_{3t}) J_1^{em,\mu}(Q^2) \times \\ &\quad \frac{f^{NN}(p_{1t} - p_{mt} - (p'_{3t} - p_{3t}))}{\underbrace{[\alpha_1 - \alpha_m - \frac{Q^2}{2q^2} \frac{E_m}{m} + i\epsilon]}_{\text{red line}}} \frac{f^{NN}(p'_{3t} - p_{3t})}{\underbrace{[\alpha_3 - \alpha'_3 - \frac{Q^2}{2q^2} \frac{k_{3t}^2}{2m^2} + i\epsilon]}_{\text{red line}}} \psi_{A-1}(\alpha_2, p'_{2t}, \alpha_3, p'_{3t}) \\ &\quad \frac{d\alpha d^2 p_{1t}}{(2\pi)^3} \frac{d\alpha_3 d^2 p_{3t}}{(2\pi)^3} \frac{d\alpha'_3 d^2 p'_{3t}}{(2\pi)^3}. \end{aligned} \tag{1}$$

Conservation of α

Therefore if the kinematics is chosen such that $\alpha_i = \alpha_f - \frac{q_-}{m} > j$

The α_1 which enters in FSI amplitude is $\alpha_1 \geq j$

and therefore FSI amplitude will be dominated by SRC

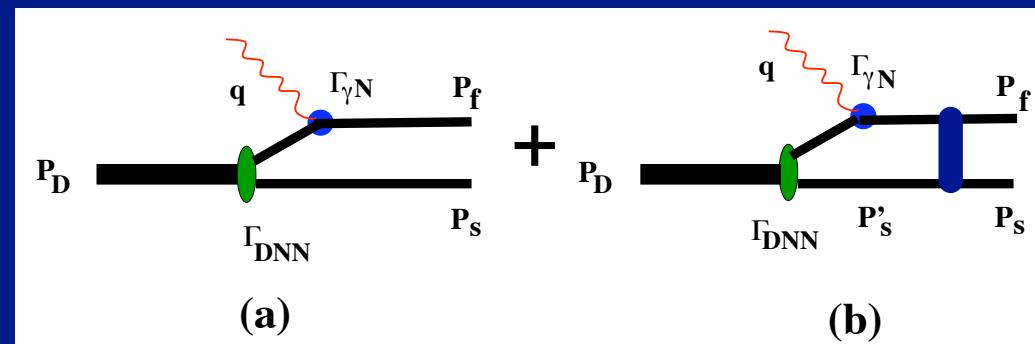
Which experimental signatures will indicate the suppression of long-range FSI ?

- ★ Naturally will explain the scaling at $x > 1$
- ★ $E_m \approx \frac{p_m^2}{2m}$ - relation survives FSI
- ★ CM momentum distribution is not affected by FSI

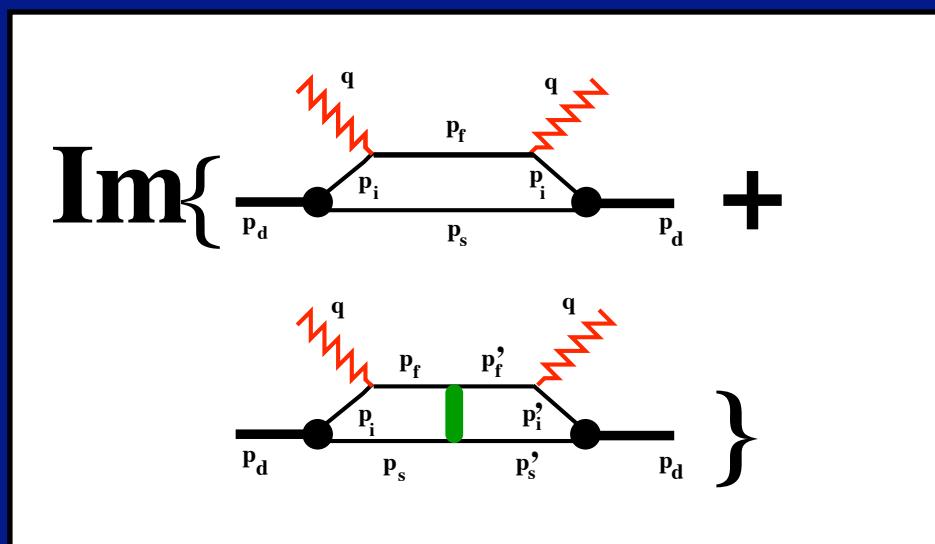
Note on applying Eikonal/Glauber Theory to Inclusive Reactions

*Eikonal/Glauber Theory Violates Unitarity

$$\sigma_{incl} \neq$$



$$\sigma_{incl} \sim$$

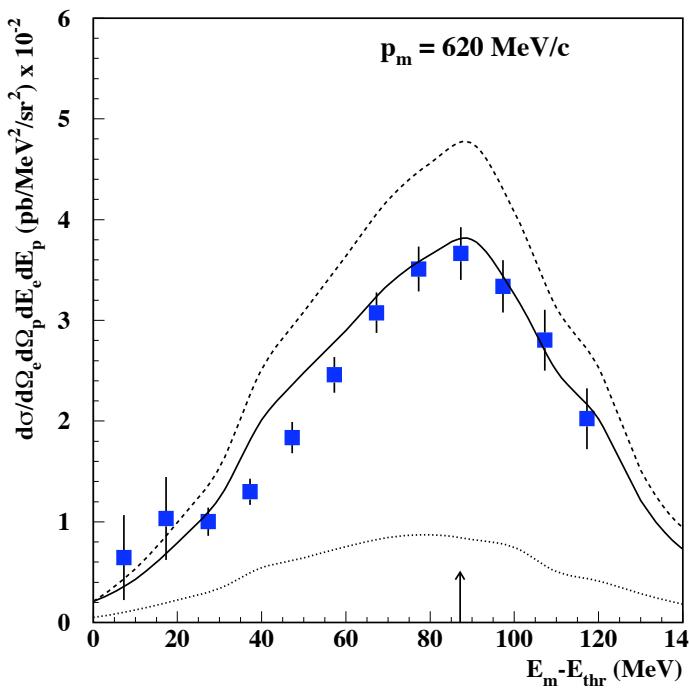
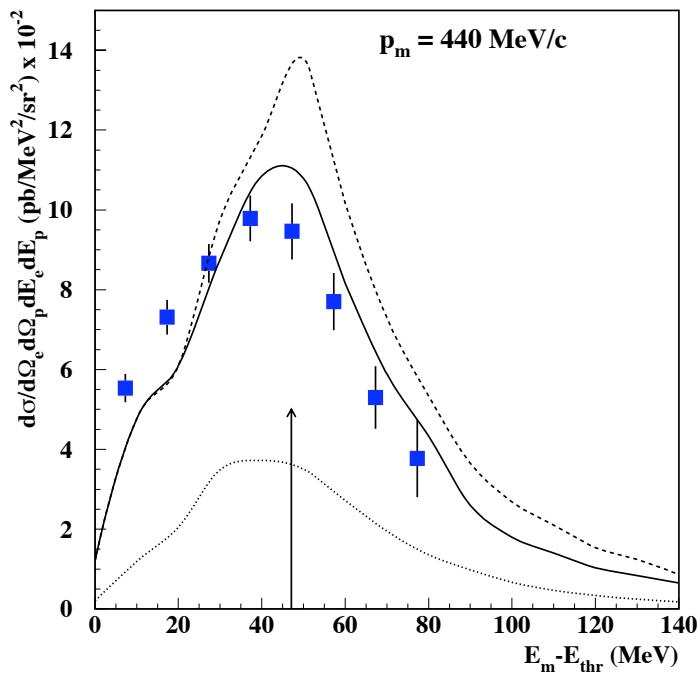
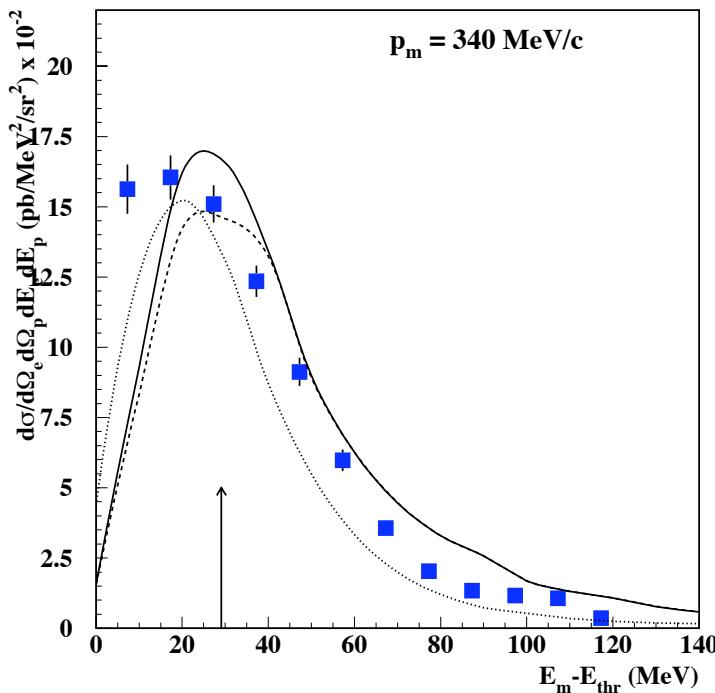
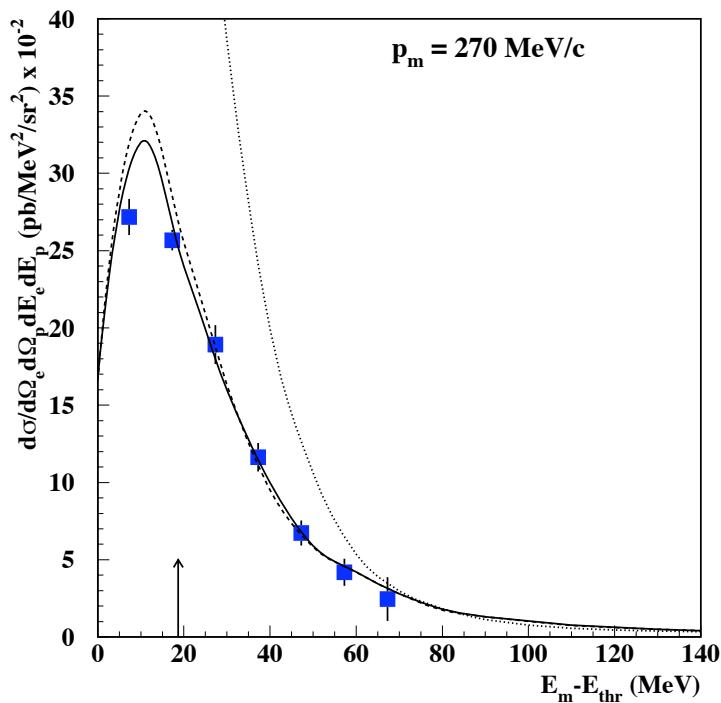


2

Three Body Break-up He3(e,e'p)pn Reaction

$Q^2 = 1.55 \text{ GeV}^2$

Benmokhtar, et al PRL 2005



$$E_m \approx \frac{p_m^2}{2m}$$

He3 WF
Bochum Group
Andreas Nogga

Conclusion

- Generalized Eikonal Approximation provides adequate theoretical framework for understating the effect of reduced long range FSI
- As well as confinement of FSI within Short Range Correlations
- It will allow to explain the scaling properties of the inclusive cross section ratios at $x>1$
- It will allow also to explain why observed pp/pn ratios consistent with PWIA predictions
- Possibility to confine FSI in SRC may open new ways of exploring 2- and 3- nucleon short range correlations