$P - P$ and $P - N$ CORRELATIONS IN MEDIUM-WEIGHT NUCLEI

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1. Introduction

$$\hat{\mathbf{H}}\,\Psi_n = E_n\,\Psi_n\,, \qquad with: \qquad \hat{\mathbf{H}} = -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{i<j}\hat{v}_{ij}$$

where

$$\hat{v}_{ij} = \sum_n v^{(n)}(r_{ij})\,\hat{\mathcal{O}}_{ij}^{(n)}$$

$$\hat{\mathcal{O}}_{ij}^{(n)} = \Big[1\,,\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\,,\,\hat{S}_{ij}\,,\,(\boldsymbol{L}\cdot\boldsymbol{S})_{ij}\,,\,\ldots\Big]\otimes\big[1\,,\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\big]\,.$$

The same operatorial dependence is cast onto Ψ_o:

$$\Psi_o = \hat{\mathbf{F}}\,\phi_o$$

where ϕ_o is the *mean-field* wave function and

$$\hat{\mathbf{F}} = \hat{S}\prod_{i<j}\hat{f}_{ij} = \hat{S}\prod_{i<j}\sum_n f^{(n)}(r_{ij})\,\hat{\mathcal{O}}_{ij}^{(n)}$$

is a *correlation* operator.

2. Ground State Properties: Cluster Expansion

- The ground state energy E_0 is given by:

$$E_o = -\frac{\hbar^2}{2\,m}\int d\boldsymbol{r}\left[\hat{\nabla}^2\,\rho^{(1)}(\boldsymbol{r},\boldsymbol{r}')\right]_{\boldsymbol{r}=\boldsymbol{r}'} + \sum_n \int d\boldsymbol{r}_1 d\boldsymbol{r}_2\,\hat{v}^{(n)}\rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2)$$

$$\longrightarrow\ \rho^{(1)}(\boldsymbol{r},\boldsymbol{r}') = A\int\prod_{j=2}^{A} d\boldsymbol{r}_j\,\Psi_o^\dagger(\boldsymbol{r},\boldsymbol{r}_2\ldots,\boldsymbol{r}_A)\,\Psi_o(\boldsymbol{r}',\boldsymbol{r}_2\ldots,\boldsymbol{r}_A)$$

$$\longrightarrow\ \rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{A(A-1)}{2}\int\prod_{j=3}^{A} d\boldsymbol{r}_j\,\Psi_o^\dagger(\boldsymbol{r}_1\ldots,\boldsymbol{r}_A)\,\hat{O}^{(n)}_{12}\,\Psi_o(\boldsymbol{r}_1\ldots,\boldsymbol{r}_A)$$

- $\rho^{(1)}(\boldsymbol{r},\boldsymbol{r}')$ and $\rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ are *cluster expanded*;
- expansion truncated at **1st order** in $\eta_{ij} = \hat{f}^2_{ij}-1$; (Mean Field is recovered at 0th order; normalization is conserved)
- the wave functions and correlation functions which minimize the ground-state energy used for the *expectation value of any operator at same order*

• at **first order** of the $\eta-$expansion, the **full correlated** **`one-body mixed`** density matrix expression is as follows:

$$\rho^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) \;=\; \rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) \,+\, \rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) \,+\, \rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,,$$

with

$$\rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) \;=\; \int d\boldsymbol{r}_2\left[H_D(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2) \;-\; H_E(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\,\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\right]$$

$$\rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) \;=\; -\int d\boldsymbol{r}_2 d\boldsymbol{r}_3\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\left[H_D(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\rho_o(\boldsymbol{r}_3) \;-\; H_E(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}_3)\rho_o^{(1)}(\boldsymbol{r}_3,\boldsymbol{r}'_1)\right]$$

and the functions H_D and H_E are defined as:

$$H_{D(E)}(r_{ij},r_{kl}) \;=\; \sum_{p,q=1}^{6} f^{(p)}(r_{ij})\,f^{(q)}(r_{kl})\,C_{D(E)}^{(p,q)}(r_{ij},r_{kl}) - C_{D(E)}^{(1,1)}(r_{ij},r_{kl})$$

with $C_{D(E)}^{(p,q)}(r_{ij},r_{kl})$ proper functions arising from spin-isospin traces;

(*Alvioli, Ciofi degli Atti, Morita, PRC72 (2005)*)

• at **first order** of the $\eta-$expansion, the **full correlated** `two-body mixed` density matrix expression is as follows:

$$\rho^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2)$$

with:

$$\rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = C_D\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - C_E\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)$$

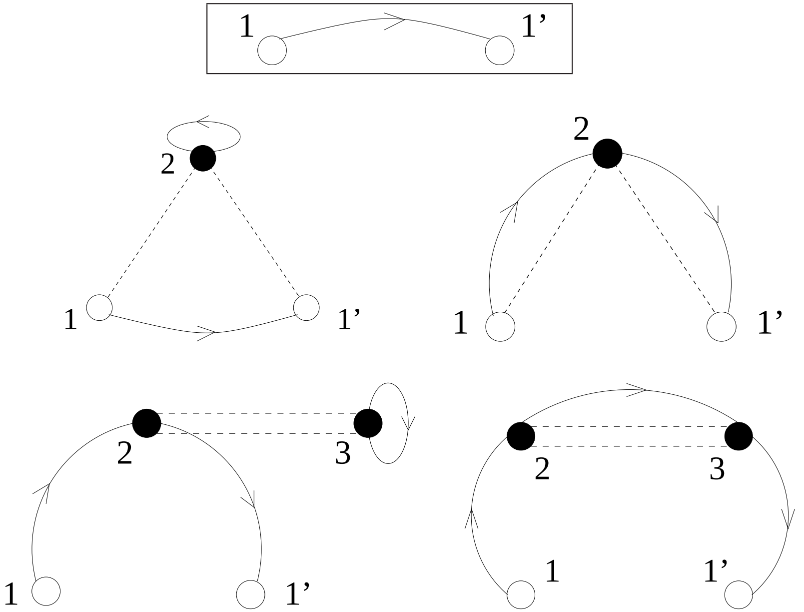
$$\rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)$$

$$\begin{aligned}\rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \int d\boldsymbol{r_3}\,\hat{\eta}(r_{13},r_{1'3})\,[\,&\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3) + \\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2) + \\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2) + \\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1) + \\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1) + \\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3)\,]\end{aligned}$$

$$\begin{aligned}\rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= \frac{1}{4}\int d\boldsymbol{r_3}d\boldsymbol{r_4}\,\hat{\eta}(r_{34})\,\cdot \\ &\cdot\sum_{\mathcal{P}\in\mathcal{C}}(-1)^{\mathcal{P}}\,[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_{\mathcal{P}1'})\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_{\mathcal{P}2'})\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_{\mathcal{P}3})\,\rho_o(\boldsymbol{r}_4,\boldsymbol{r}_{\mathcal{P}4})\,]\end{aligned}$$

(*Alvioli, Ciofi degli Atti, Morita, PRC72 (2005)*)

(*Alvioli, Ciofi degli Atti, Morita,* `arXiv:0709.3989 [nucl-th]`)

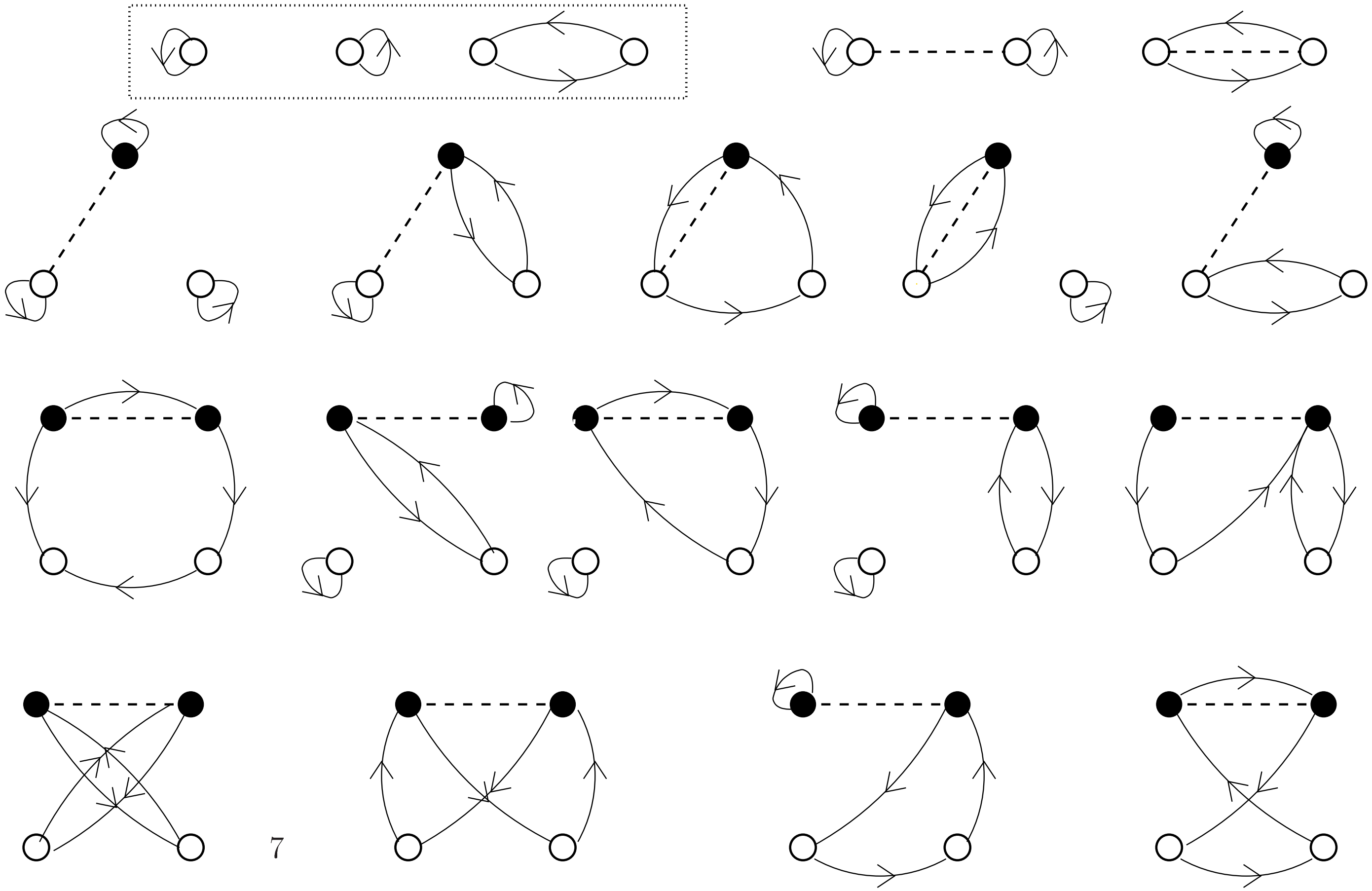


one-body, *non-diagonal*

$\longleftarrow \rho(\boldsymbol{r}_1, \boldsymbol{r}'_1)$ *diagrams*

two-body, *diagonal*

$\rho(\boldsymbol{r}_1, \boldsymbol{r}_2)$ *diagrams* $\longrightarrow$



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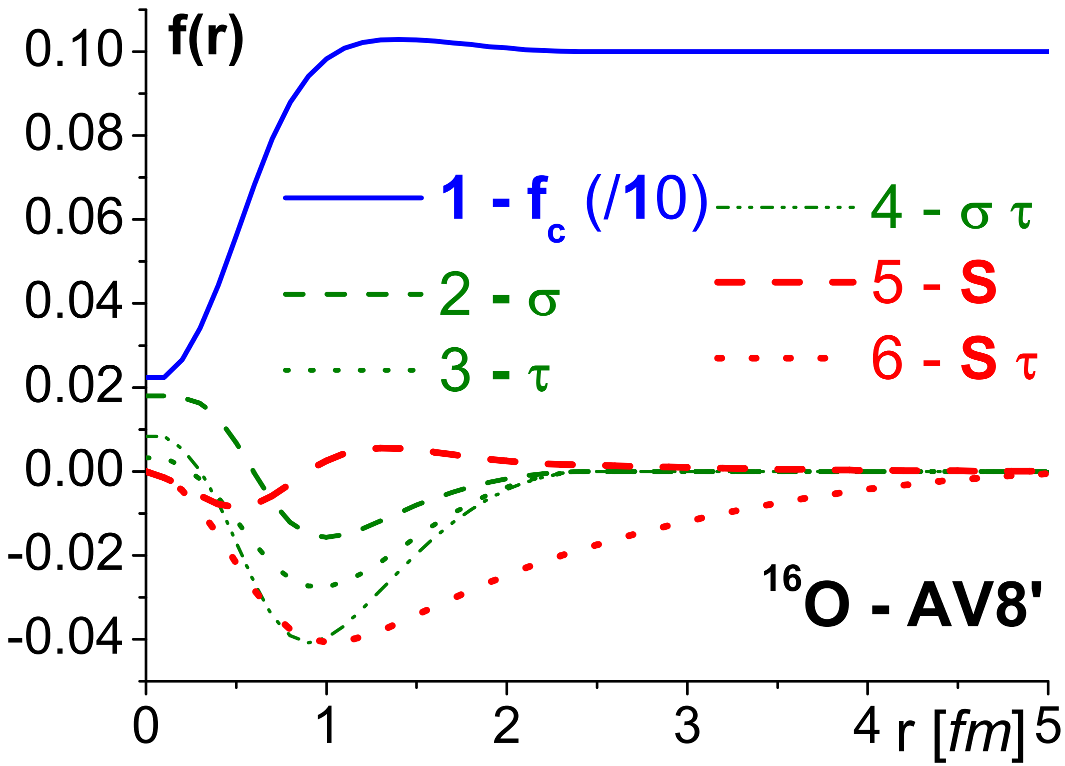
Ground state energy: ^{16}O - Argonne $V8'$

	$< V_c >$	$< V_\sigma >$	$< V_\tau >$	$< V_{\sigma\tau} >$	$< V_S >$	$< V_{S\tau} >$	$< \mathbf{V} >$	$< \mathbf{T} >$	**E**	**E/A** MeV
$\eta-exp$	0.19	-35.88	-9.47	-171.32	-0.003	-172.89	-389.40	323.50	**-65.90**	-4.12
FHNC	0.694	-40.13	-10.61	-180.00	-0.07	-160.32	-390.30	325.18	**-65.12**	-4.07

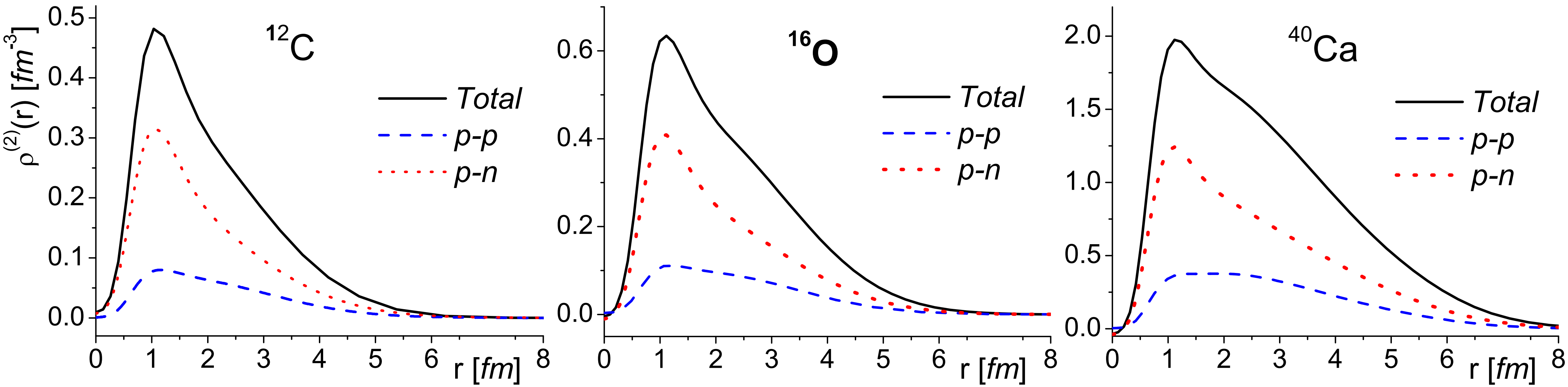
correlation functions: *Central*, *Spin-Isospin*, *Tensor*



3. TWO-BODY DENSITIES & TWO-BODY MOMENTUM DISTRIBUTIONS of COMPLEX NUCLEI

Two-Body Densities

$$\rho^{(2)}(r) = \int d\boldsymbol{R}\, \rho^{(2)}\left(\boldsymbol{R}+\frac{1}{2}\boldsymbol{r}\,,\, \boldsymbol{R}-\frac{1}{2}\boldsymbol{r}\,;\, \boldsymbol{R}+\frac{1}{2}\boldsymbol{r}\,,\, \boldsymbol{R}-\frac{1}{2}\boldsymbol{r}\right)$$



- normalization (**number of pairs**) conserved by the expansion
- **isospin** separation feasible
- closed **j-shell** nuclei included in the formalism

Two-Body Densities: isospin separation

in each of the terms of our cluster-expansion expression of two body density:

$$\rho^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) \;=\; \rho^{(2)}_{\rm SM}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\bf 2b}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\bf 3b}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\bf 4b}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2)$$

the contributions from proton-proton, proton-neutron and neutron-neutron can be separated:

$$\rho^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) \;= \\ = \rho^{pp}_{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{pn}_{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{nn}_{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2)$$

by inserting the proper *isospin projection operators* in the cluster expansion for particle 1 and 2;
as a consequence, the same holds for the two body momentum distributions:

$$n^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) \;=\; n_{pp}(\boldsymbol{k}_1,\boldsymbol{k}_2) + n_{pn}(\boldsymbol{k}_1,\boldsymbol{k}_2) + n_{nn}(\boldsymbol{k}_1,\boldsymbol{k}_2)$$

which is defined in the next slide.

Two-Body Momentum Distributions

$$\boldsymbol{k}_{rel} \equiv \boldsymbol{k} = \frac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2) \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 \qquad \boldsymbol{r}' = \boldsymbol{r}'_1 - \boldsymbol{r}'_2$$

$$\boldsymbol{K}_{CM} \equiv \boldsymbol{K} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \qquad \boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2) \qquad \boldsymbol{R}' = \frac{1}{2}(\boldsymbol{r}'_1 + \boldsymbol{r}'_2)$$

we have

$$n(\boldsymbol{k},\boldsymbol{K}) = \frac{1}{(2\pi)^6}\int d\boldsymbol{r}\,d\boldsymbol{r}'\,d\boldsymbol{R}\,d\boldsymbol{R}'\, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')}\, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{(2)}(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{R},\boldsymbol{R}')$$

and

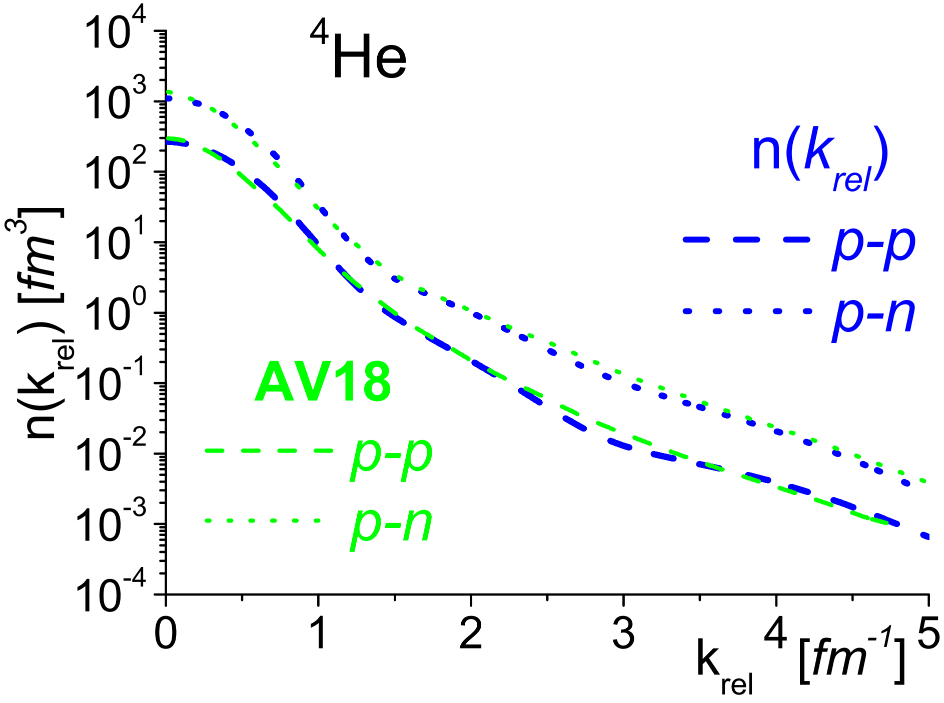
$$n(\boldsymbol{k}) = \int d\boldsymbol{K}\, n(\boldsymbol{k},\boldsymbol{K}) = \frac{1}{(2\pi)^3}\int d\boldsymbol{r}\,d\boldsymbol{r}'\,d\boldsymbol{R}\, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{(2)}(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{R},\boldsymbol{R})$$

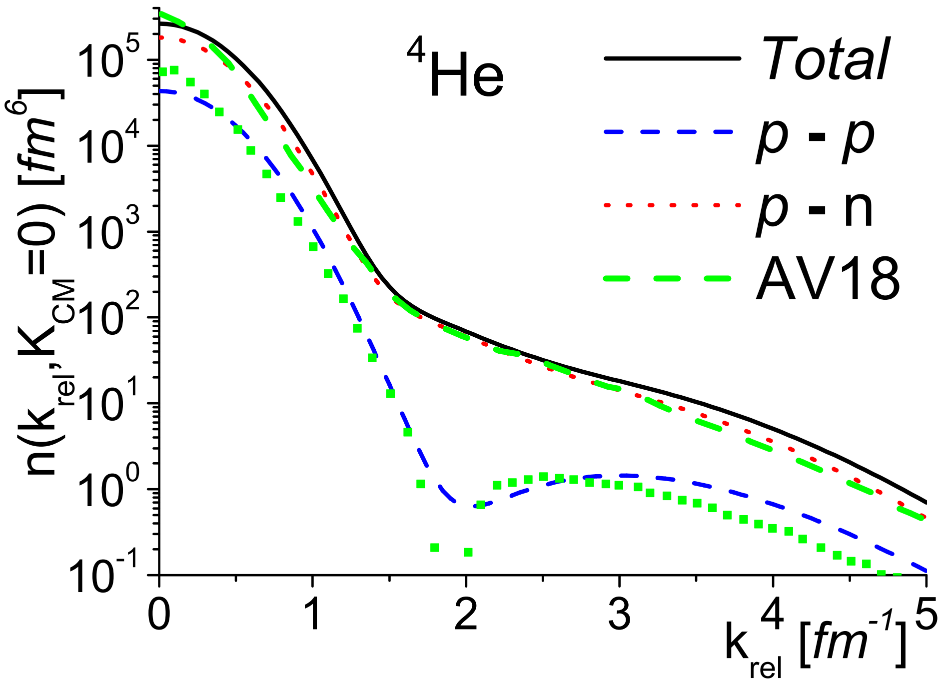
$$n(\boldsymbol{K}) = \int d\boldsymbol{k}\, n(\boldsymbol{k},\boldsymbol{K}) = \frac{1}{(2\pi)^3}\int d\boldsymbol{r}\,d\boldsymbol{R}\,d\boldsymbol{R}'\, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')}\rho^{(2)}(\boldsymbol{r},\boldsymbol{r};\boldsymbol{R},\boldsymbol{R}')$$

$\boldsymbol{K}_{CM} = 0$ corresponds to $\boldsymbol{k}_2 = -\boldsymbol{k}_1$, *i.e. back-to-back* nucleons

^4He: comparison with VMC

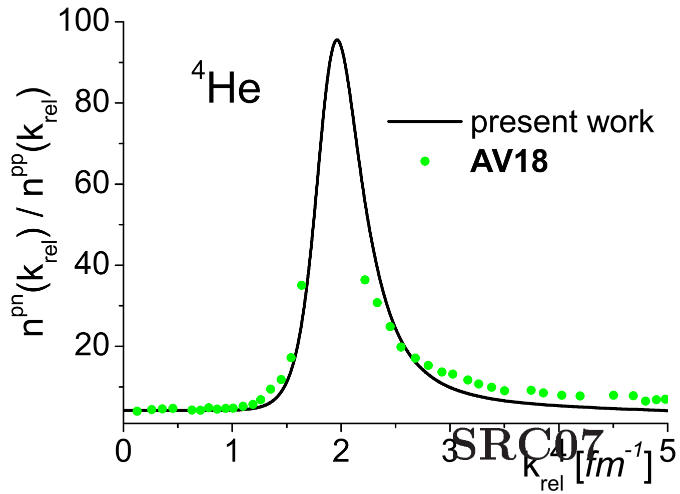
$$n_{pN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\; n_{pN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) \qquad n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}=0)$$





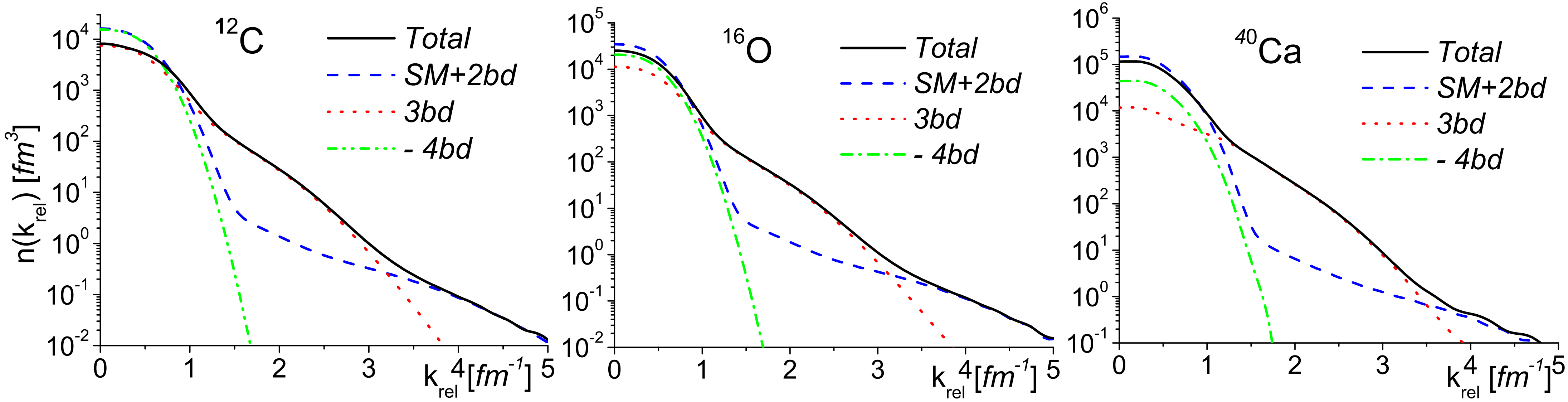
- good agreement with VMC calculations
- $n_{pn}(k_{rel},0)/n_{pp}(k_{rel},0)$ `peak location ok` $\longrightarrow$

(**AV18**: *Schiavilla at al.* **PRL98** (2007))



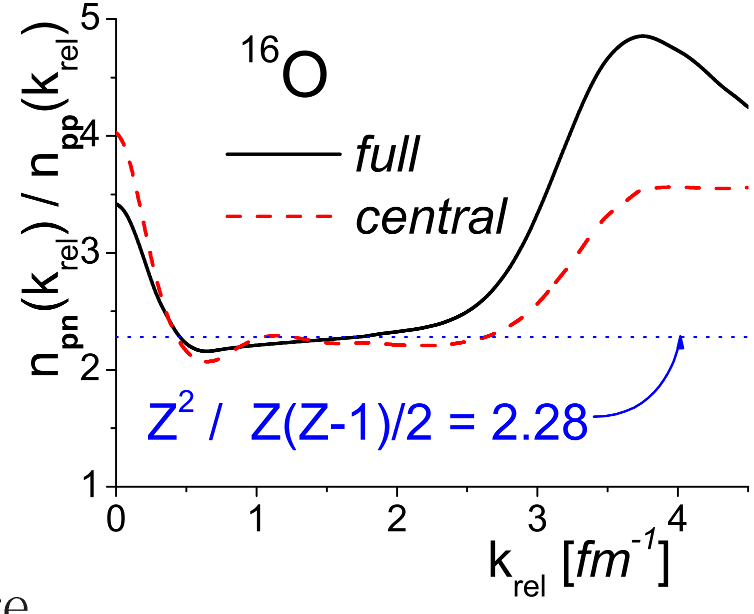
$n_{NN}(k_{rel})$ for Complex Nuclei

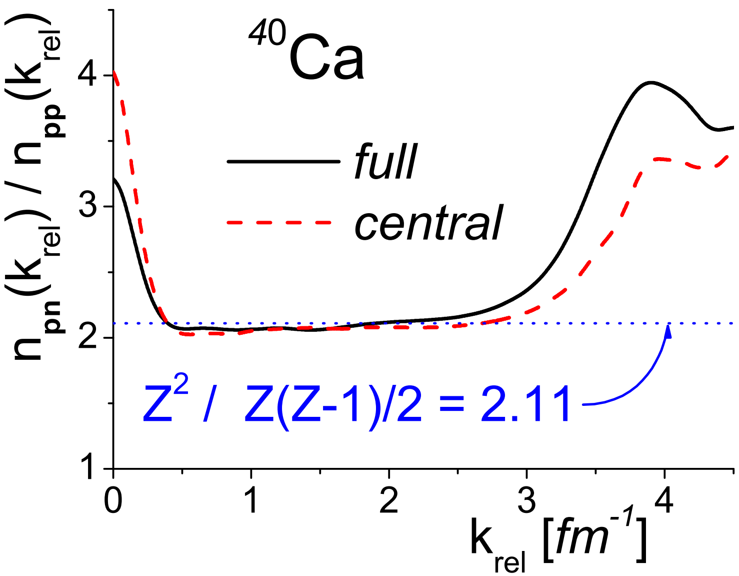
$$n_{NN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\, n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$$



- normalization (**number of pairs**) conserved by the expansion
- **isospin** separation feasible
- closed **j-shell** nuclei included in the formalism
- *three* and *four-body* diagrams **essential**

$n_{\mathbf{pn}}(k_{rel})\ /\ n_{\mathbf{pp}}(k_{rel})$: *central* vs. *full* correlations





where

$$n_{\mathbf{pN}}(\boldsymbol{k}_{rel}) = \int d\boldsymbol{K}_{CM}\, n_{\mathbf{pN}}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) =$$

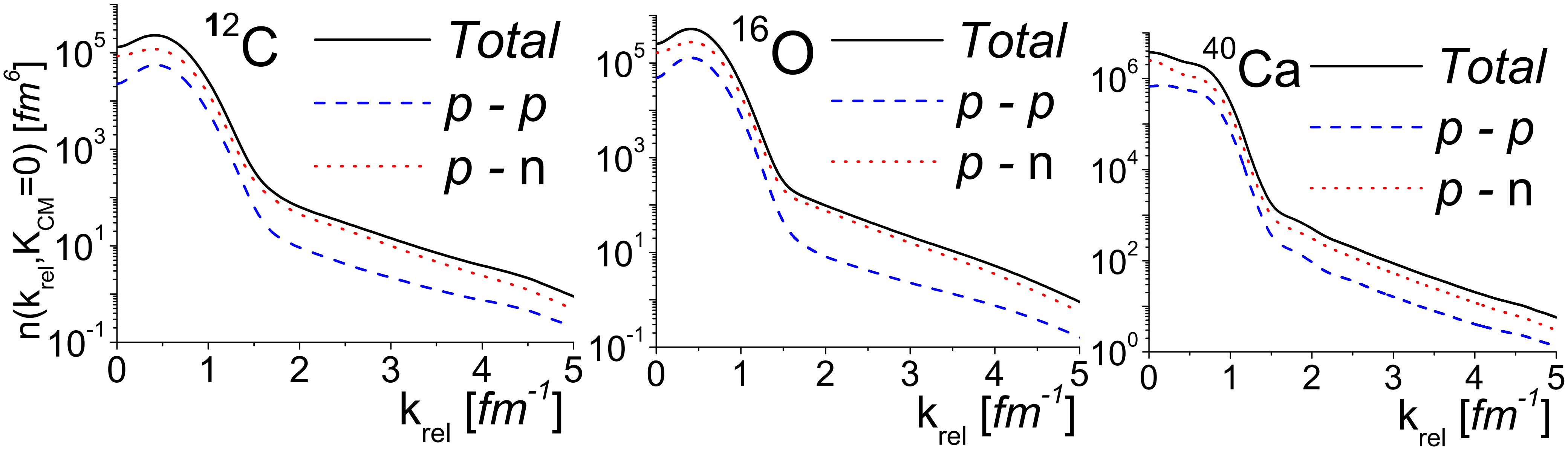
$$= \frac{1}{(2\pi)^3}\int d\boldsymbol{r}\, d\boldsymbol{r}'\, d\boldsymbol{R}\, e^{-i\,\boldsymbol{k}_{rel}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{\mathbf{pN}}_{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R})$$

and the *blue line* is the number of *pn* to *pp* pairs ratio $(Z = N = A/2)$:

$$\left(Z^2\right)\ /\ \left(\frac{Z(Z-1)}{2}\right).$$

$n_{NN}(k_{rel})$ for Complex Nuclei: $K_{CM} = 0$

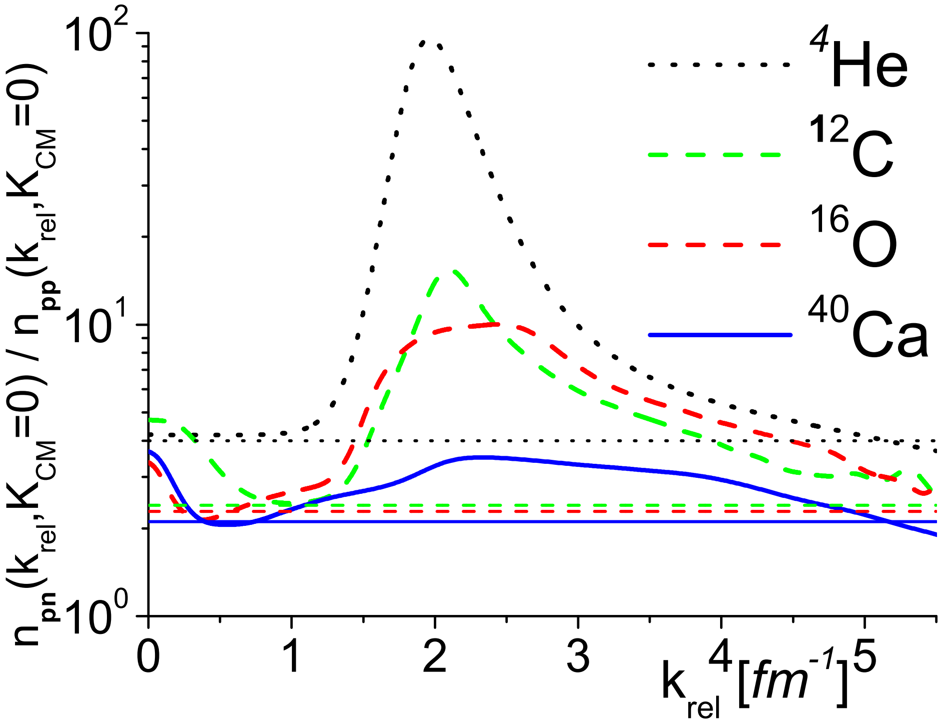
$$n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM} = 0)$$



- ^{12}C three and four-body diagrams evaluation is still preliminary - relevant for k_{rel} around 2 fm^{-1}
- pn to pp ratio decreases with increasing A - due to decreasing number of pairs ratio

$n_{NN}(k_{rel})$ for Complex Nuclei: $K_{CM} = 0$

- $n_{pn}(k_{rel}, 0)/n_{pp}(k_{rel}, 0)$ is a measure of relative **pn** to **pp** correlation strenghts



- *pn* to *pp* ratio decreases with increasing A - due to decreasing number of pairs ratio

- peak gets filled with increasing A

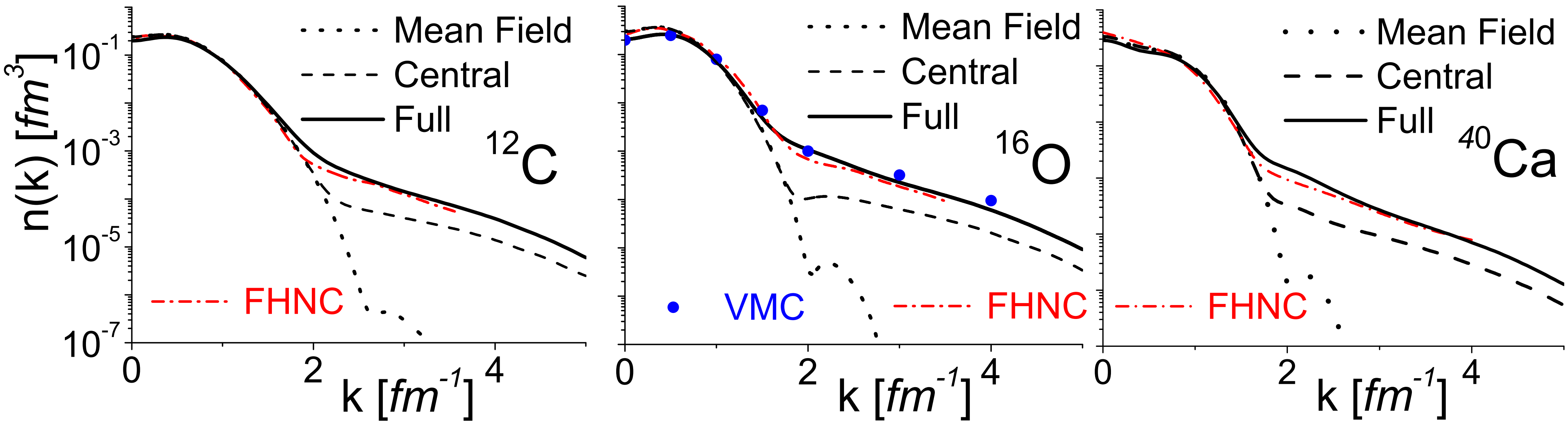
Conclusions

- We have calculated one- and two-body ground state properties of complex nuclei in the framework of cluster expansion
- The cluster expansion method proved to be relatively easy to use and computationally affordable; comparison with acccurate many-body calculations is very satisfactory
- We have checked our two-body $n_{pN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$ against the prediction of Two-Nucleon correlation model
- Tensor correlations appear to be an essential ingredient for the correct description of (one- and two-body) high-momentum distributions

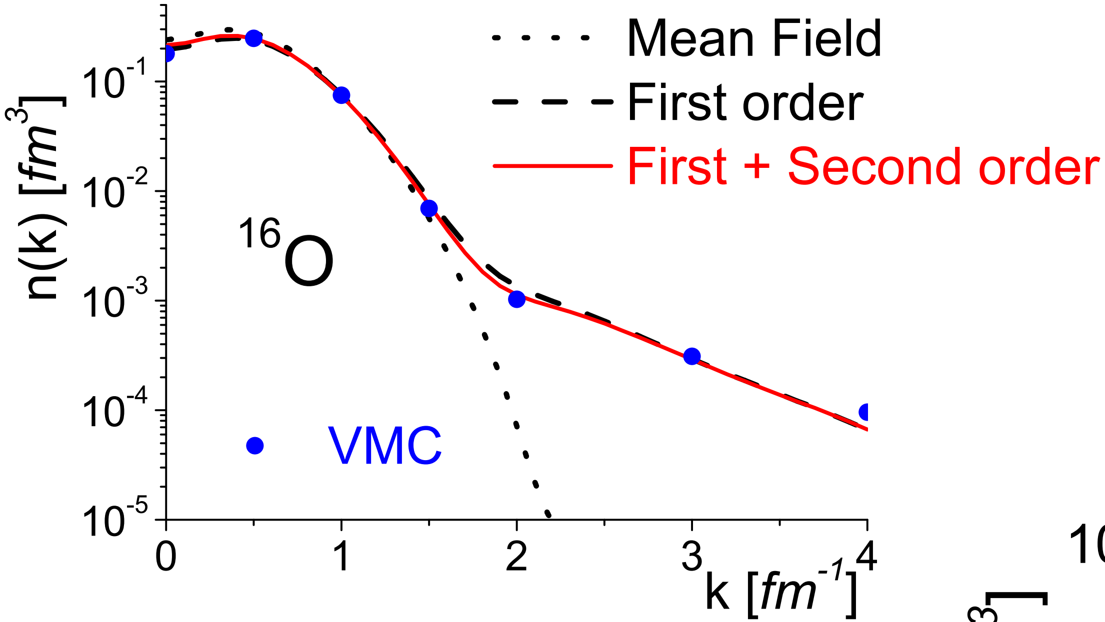
Additional Slides

One-Body Momentum Distributions

$$n(\boldsymbol{k}) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r}_1 d\boldsymbol{r}'_1 \, e^{i\,\boldsymbol{k}(\boldsymbol{r}_1 - \boldsymbol{r}'_1)} \, \rho^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}'_1)$$

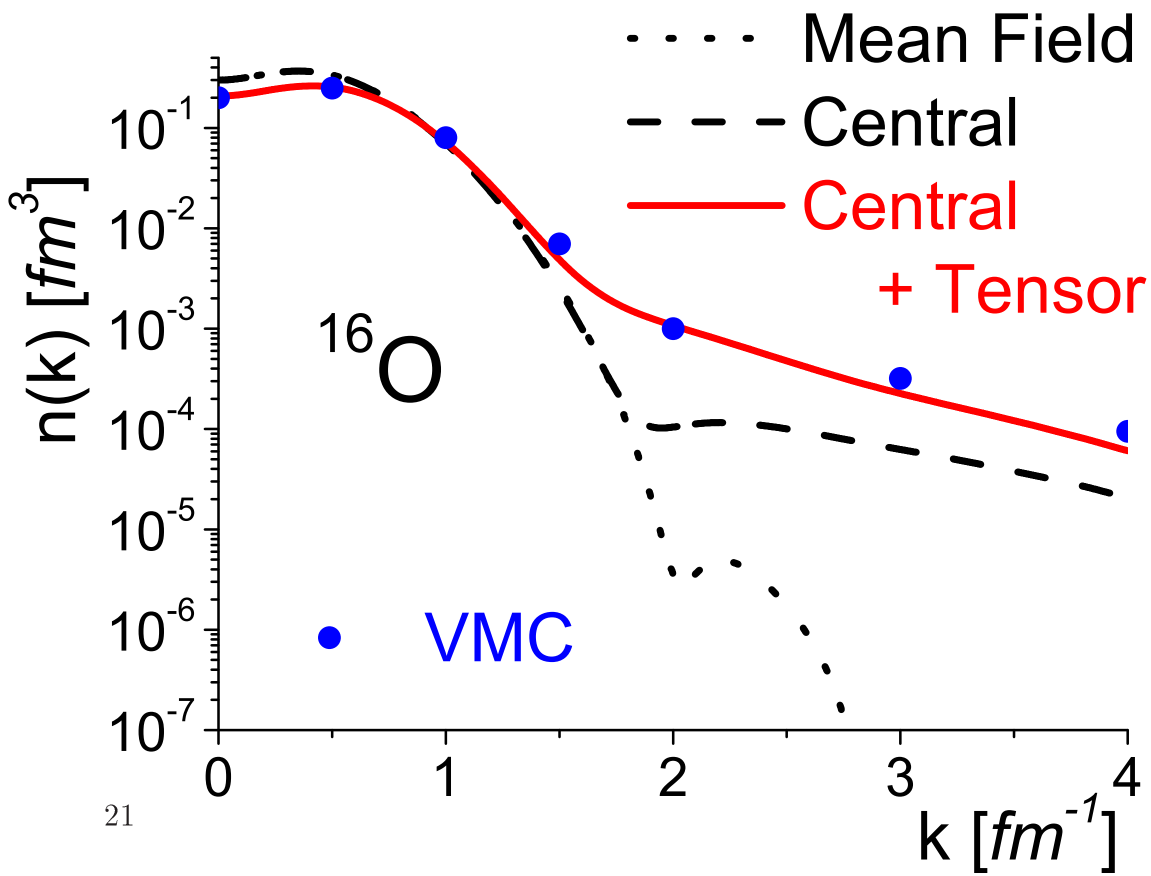


and



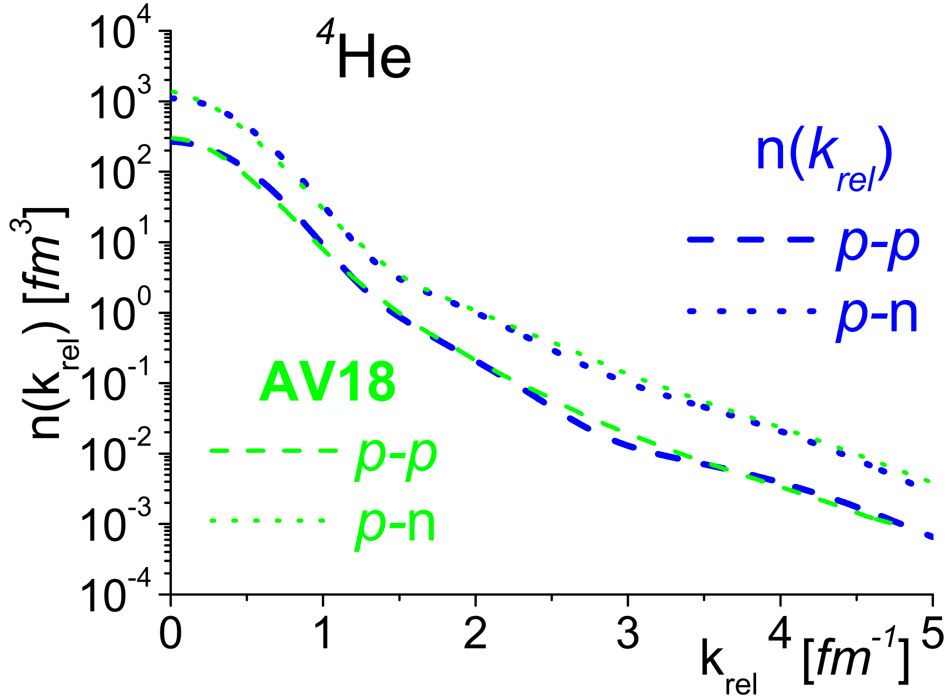
$\longleftarrow$ **cluster expansion**

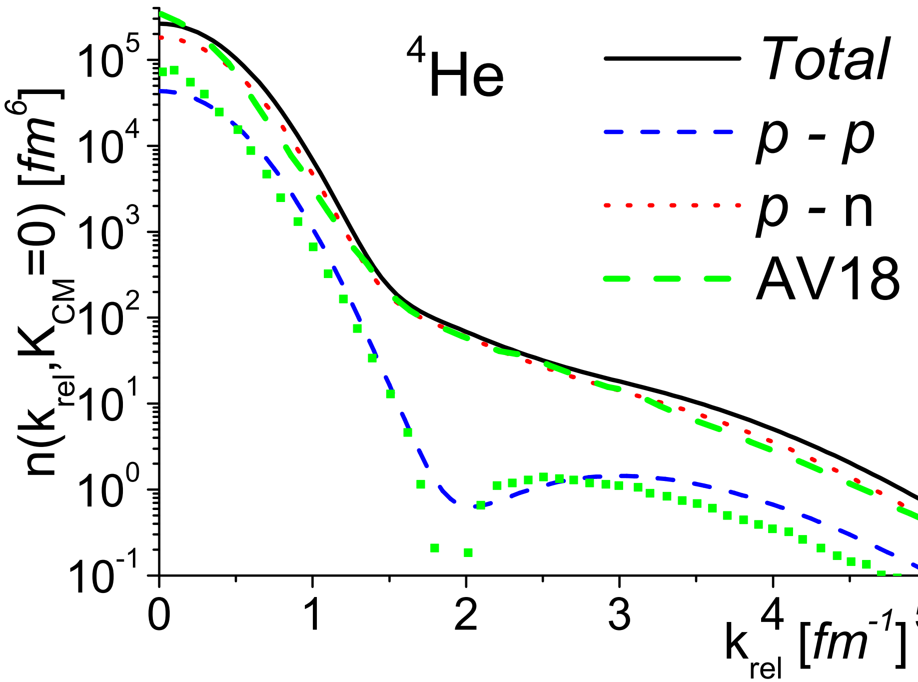
k > 1.5 fm^{-1}: $\longrightarrow$



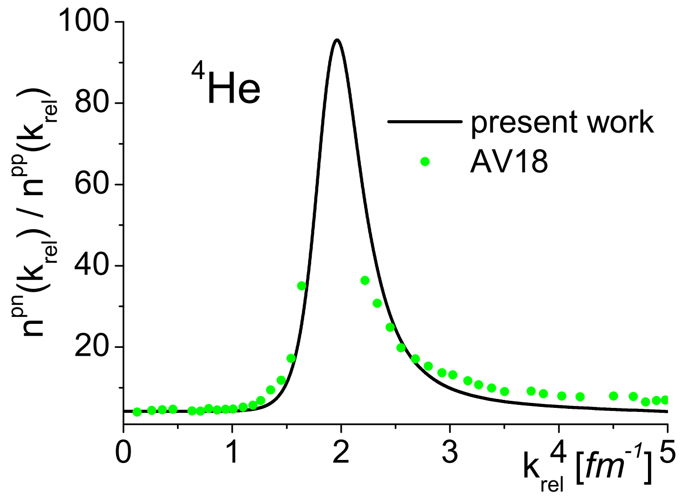
^4He: comparison with VMC

$$n_{pN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\; n_{pN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) \qquad\qquad n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM} = 0)$$





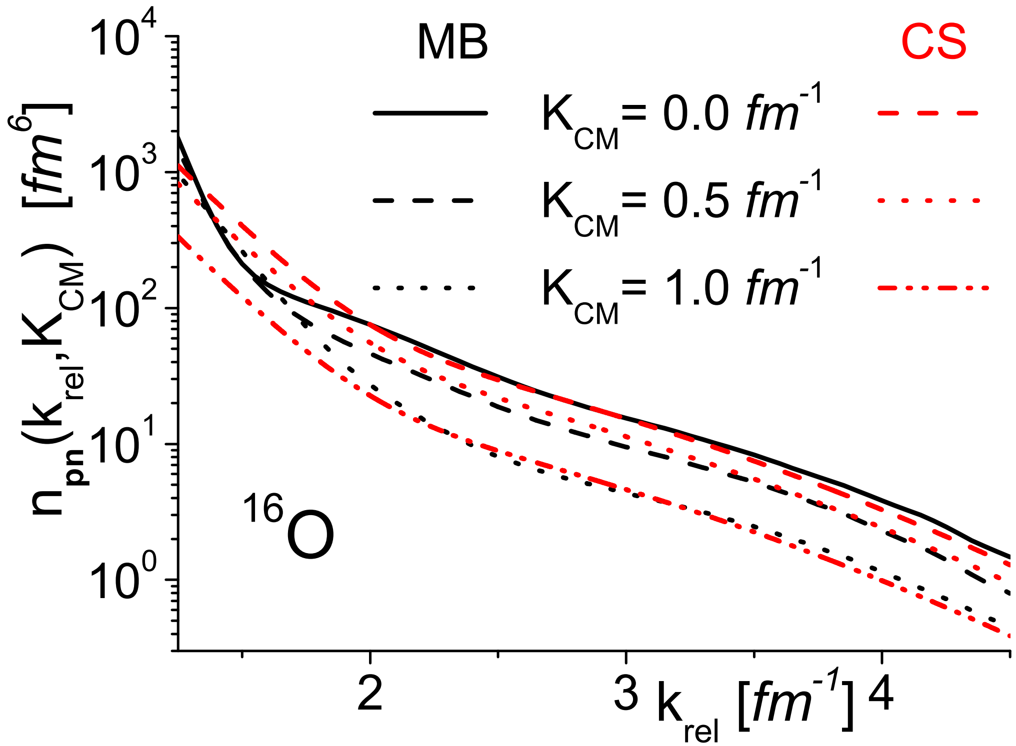
- good agreement with VMC calculations
- $n_{pn}(k_{rel}, 0)/n_{pp}(k_{rel}, 0)$ `peak location ok` $\longrightarrow$



(**AV18**: *Schiavilla at al.* **PRL98** (2007))

Spectral Function properties at low K_{CM} and high k_{rel}

$$P_1^A(|\mathbf{k}|, E) = \int d\mathbf{P}_{cm}\, n_{rel}^A(|\mathbf{k} - \mathbf{P}_{cm}/2|)\, n_{cm}^A(|\mathbf{P}_{cm}|) \cdot \delta\left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)} \cdot \left(\mathbf{k} - \frac{(A-1)\mathbf{P}_{cm}}{(A-2)}\right)^2\right]$$

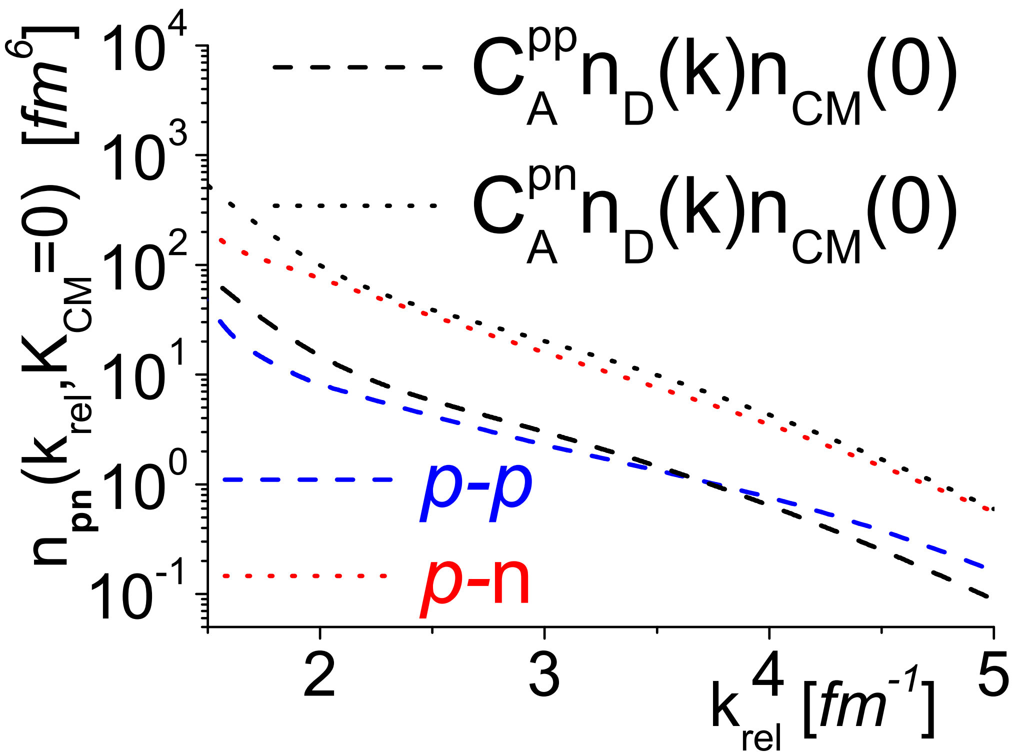


MB: $n(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$

CS: Ciofi, Simula
PRC53, (1996)
$C_A\, n_{2H}(k_{rel})\, n_{CS}(K_{CM})$

Spectral Function properties at $K_{CM} = 0$ and high k_{rel}

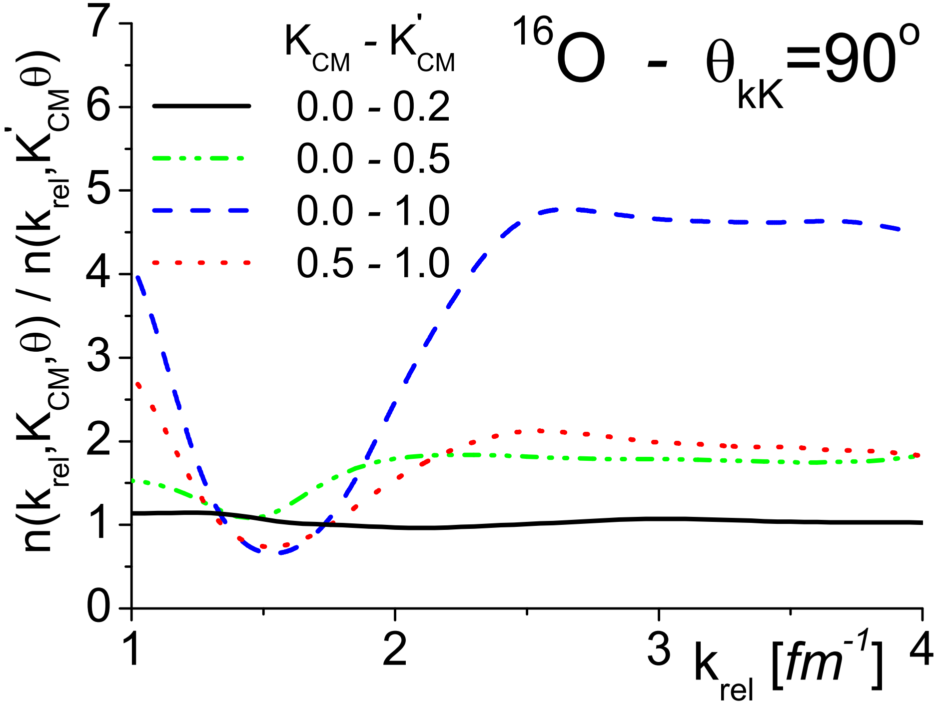
$$P_1^A(|\mathbf{k}|, E) = \int d\mathbf{P}_{cm}\, n_{rel}^A(|\mathbf{k} - \mathbf{P}_{cm}/2|)\, n_{cm}^A(|\mathbf{P}_{cm}|) \cdot$$
$$\cdot\, \delta\left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)} \cdot \left(\mathbf{k} - \frac{(A-1)\mathbf{P}_{cm}}{(A-2)}\right)^2\right]$$



pp and pn: **MB** $n(\boldsymbol{k}_{rel}, 0)$

CS: Ciofi, Simula
PRC53, (1996)
$C_A\, n_{^2H}(k_{rel})\, n_{CM}(0)$

- $n(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$ factorization in the Two-Nucleon correlation model Spectral Function requires $n(k_{rel}, K_{CM}, \theta) \propto n(k_{rel}, K'_{CM}, \theta)$



- high k_{rel} and low K_{CM} `factorization verified` by many-body calculation