C. CIOFI degli ATTI

NN CORRELATIONS : $(e, e'p)$ EXPERIMENTS AND THEORETICAL SPECTRAL FUNCTIONS

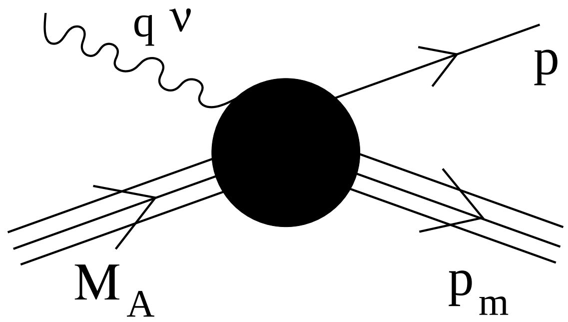
The short range structure of nuclei at 12 GeV

JLAB WORKSHOP

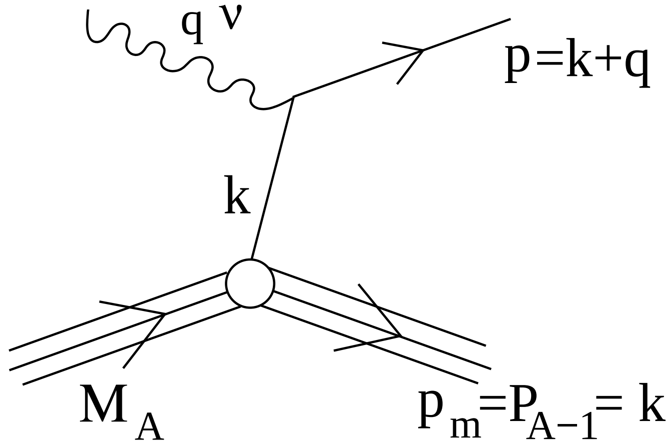
October 25-26, 2007

New dedicated experiments:

- **high** Q^2
- **back-to-back detected nucleons**
- $E_m \simeq p_m^2/2m_N$



if FSI etc. could be disregarded:



in this case the central quantity is the spectral function

$$P(k,E) = P_0(k,E) + P_1(k,E)$$

$$P(k,E) = \sum_f \Big| < \Psi^f_{A-1} | \, a_k | \Psi^0_A > \Big|^2 \delta\Big(E - \Big(E^f_{A-1} - E^0_A\Big)\Big)$$

$$P_0(k,E) = \sum_{f<F} \Big| < \Psi^f_{A-1} | \, a_k | \Psi^0_A > \Big|^2 \delta\Big(E - \Big(E^f_{A-1} - E^0_A\Big)\Big)$$

$$P_1(k,E) = \sum_{f>F} \Big| < \Psi^f_{A-1} | \, a_k | \Psi^0_A > \Big|^2 \delta\Big(E - \Big(E^f_{A-1} - E^0_A\Big)\Big)$$

$$P(k,E) = -\frac{1}{\pi} Im\, \mathcal{G}(k,E) = \frac{1}{\pi} \frac{W(k,E)}{(-E - k^2/2m - V(k,E))^2 + W(k,E)^2}$$

$$\mathcal{G}(k,E) = \frac{1}{-E - k^2/2m - V(k,E) - \imath W(k,E)}$$

DO WE KNOW IT?

Spectral Function of $A = 3$ and $A = \infty$

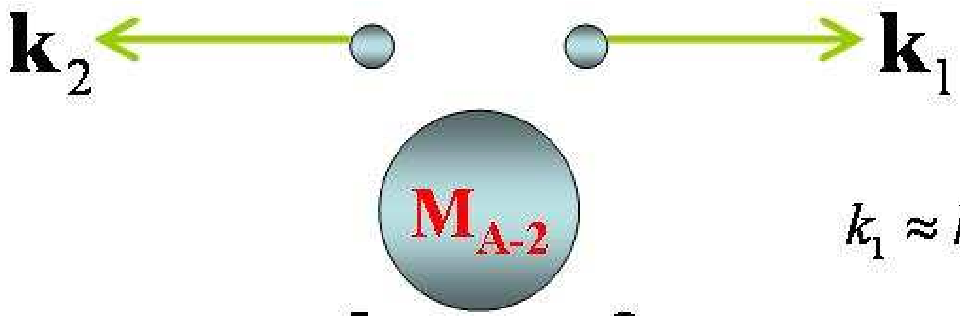
Spectral Functions calculated from many-body theory exhibit a common feature:

at high E ($> 40\ MeV$) and k ($> 1.5 - 2.0\ fm^{-1}$)

$P(k,E)$ **has maxima at** $E = \frac{(A-2)k^2}{2(A-1)m_N}$

explanation in terms of a simple and physically sound model: two-nucleon correlations (Frankfurt & Strikman 1988)

Two Nucleon Correlation Model



$$\mathbf{k}_{A-2} \approx 0$$

$$k_1 \approx k_2 \equiv k$$

Excitation energy of (A-1)

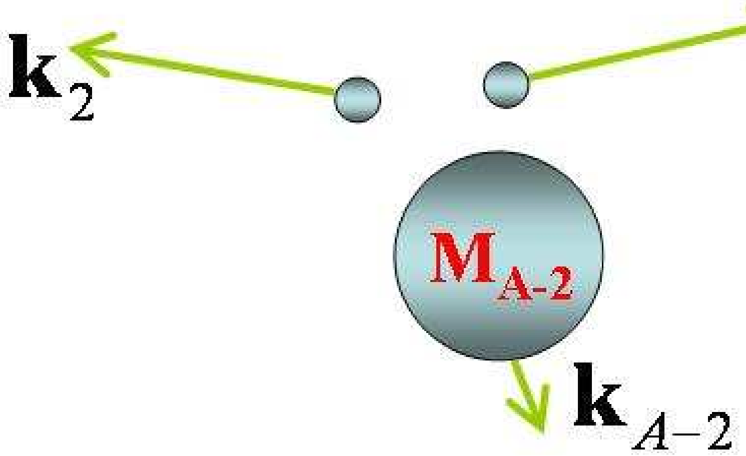
$$E^*_{A-1} + \frac{k^2}{2M_{A-1}} = \frac{k^2}{2m_N} \Rightarrow E^*_{A-1} = \frac{k^2}{2m_N} - \frac{k^2}{2M_{A-1}} \approx \frac{A-2}{A-1}\frac{k^2}{2m_N}$$

$$P(\mathbf{k}, E^*_{A-1}) = n(\mathbf{k})\,\delta\!\left(E^*_{A-1} - \frac{A-2}{A-1}\frac{k^2}{2m_N}\right)$$

Improved Two Nucleon Correlation Model

C. Ciofi, S. Simula, L.L. Frankfurt and M.I. Strikman, *Phys. Rev.* **C44**, R1(1991)

C. Ciofi and S. Simula, *Phys.Rev* **C53**, 1689(1996)



$$\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_{A-2} = \mathbf{0}$$

$$\mathbf{k}_{CM} = \mathbf{k}_1 + \mathbf{k}_2 = -\mathbf{k}_{A-2}$$

$$P(\mathbf{k}, E^*_{A-1}) = \int d\mathbf{k}_{CM}\, n_{rel}\left(\left|\mathbf{k}_1 - \frac{\mathbf{k}_{CM}}{2}\right|\right) n_{CM}(|\,\mathbf{k}_{CM}\,|) \times$$
$$\times\, \delta\left(E^*_{A-1} - \frac{A-2}{2m_N(A-1)}\left[\mathbf{k}_1 - \frac{A-1}{A-2}\mathbf{k}_{CM}\right]^2\right)$$

Spectral Function (TNC Model)

$$P(\mathbf{k},E_m)=\sum_f \left|T_{fi}\right|^2 \delta\left(E_m-E_{th}-\frac{A-2}{2m(A-1)}\left(\mathbf{k}+\frac{A-1}{A-2}\mathbf{k}_{A-2}\right)^2\right)$$

$$T_{fi}=\frac{1}{(2\pi)^{3/2}}\int d\mathbf{r}_1 \exp(i\mathbf{p}_m\cdot\mathbf{r}_1)I_{fi}(\mathbf{r}_1),$$

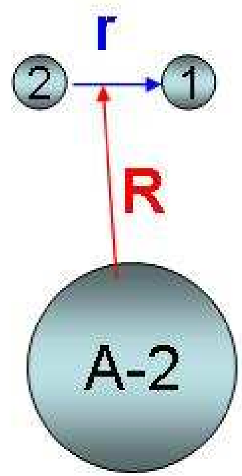
$$I_{fi}(\mathbf{r}_1)=\int d\tau_{A-1}\Psi_{A-1}^{f*}(\mathbf{r}_2,\cdots,\mathbf{r}_A)\Psi_A^i(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_A)$$

$$\Psi_A^i=\sum_{k,l}\phi_k(12)\Psi_l(3,\cdots A)\approx\sum_k\phi_k(12)\Psi_{A-2}^{(0)}(3,\cdots A)$$

$$\approx\sum_{m,n}\chi_m(\mathbf{R})\varphi_n(\mathbf{r})\Psi_{A-2}^{(0)}(3,\cdots A),$$ □@□R@ $$\frac{\mathbf{r}_1+\mathbf{r}_2}{2},\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$$

$$\approx\chi_{0S}(\mathbf{R})\sum_n\varphi_n(\mathbf{r})\Psi_{A-2}^{(0)}(3,\cdots A)\equiv\chi_{0S}(\mathbf{R})\Phi(\mathbf{r})\Psi_{A-2}^{(0)}(3,\cdots A)$$

$$\Psi_{A-1}^{f}(2,\cdots,A)=\frac{1}{(2\pi)^{3/2}}\exp(i\mathbf{k}_2\cdot\mathbf{r}_2)\Psi_{A-2}^{(f)}(3,\cdots A)$$



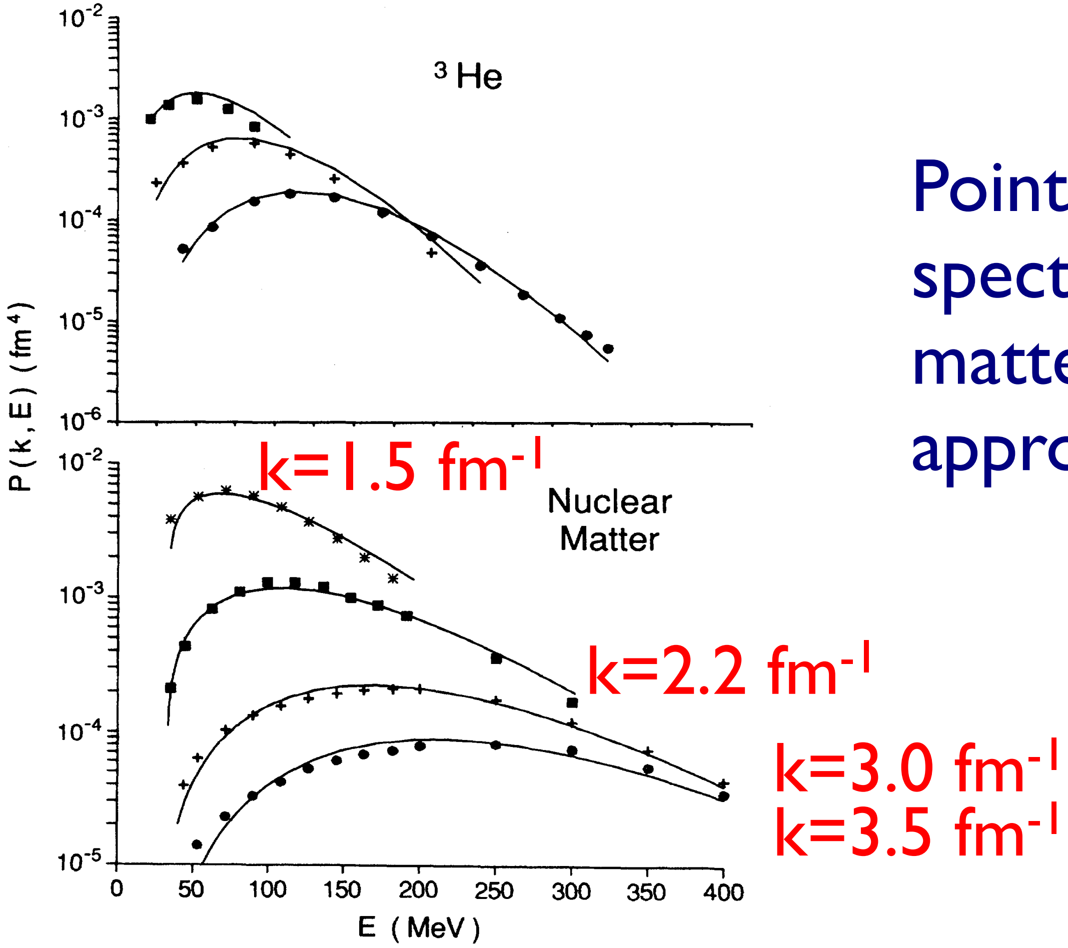
MANY BODY vs CONVOLUTION MODEL

$$W(k,E) = \frac{1}{2}\sum_{hh'p} Im \frac{|< kp\,|\,G\big(e(h)+e(h')\big)|\,hh' >_a|^2}{E - e(p) + e(h) + e(h') - \imath\eta}$$
$$= \frac{\pi}{2}\sum_{hh'p} |< kp\,|\,G\big(e(h)+e(h')\big)|\,hh' >_a|^2\,\delta\big(-E + e(p) - e(h) - e(h')\big)$$

$$P(k,E) = \frac{1}{2}\sum_{\mathbf{q},\mathbf{P}} |\,\xi\big(\mathbf{k} - \frac{1}{2}\mathbf{P},\mathbf{q}\big)\,|^2\,\theta(k_F - |\,\mathbf{q} + \frac{1}{2}\mathbf{P}|)\theta(k_F - |\,\mathbf{q} - \frac{1}{2}\mathbf{P}|)$$
$$\times\ \theta(|\mathbf{P} - \mathbf{k}| - k_F)\,\delta\big(E - e(p) + e(h) + e(h')\big)$$

$$|\xi >= |\Psi > - |\phi >$$

$$P(k,E) = \frac{m\rho^2}{32k}\int_{|k-k_0|}^{k+k_0} dP\,P\,n_{cm}^{FG}(P)\,n_{rel}\left(\sqrt{\frac{1}{2}k^2 - \frac{1}{4}P^2 + \frac{1}{2}k_0^2}\right)$$



Points are numerical calculation of the spectral functions of ^{3}He and nuclear matter - curves two nucleon approximation from CSFS 91

$$P_1^A(|\mathbf{k}|, E) = \int d^3P_{cm}\, n_{rel}^A(|\mathbf{k} - \mathbf{P_{cm}}/2|)\, n_{cm}^A(|\mathbf{P_{cm}}|)$$
$$\delta\left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)} \cdot \left(\mathbf{k} - \frac{(A-1)\mathbf{P_{cm}}}{(A-2)}\right)^2\right]$$

CdA, Simula , 1996

440 *M. Baldo et al. / Nuclear Physics A 604 (1996) 429–440*

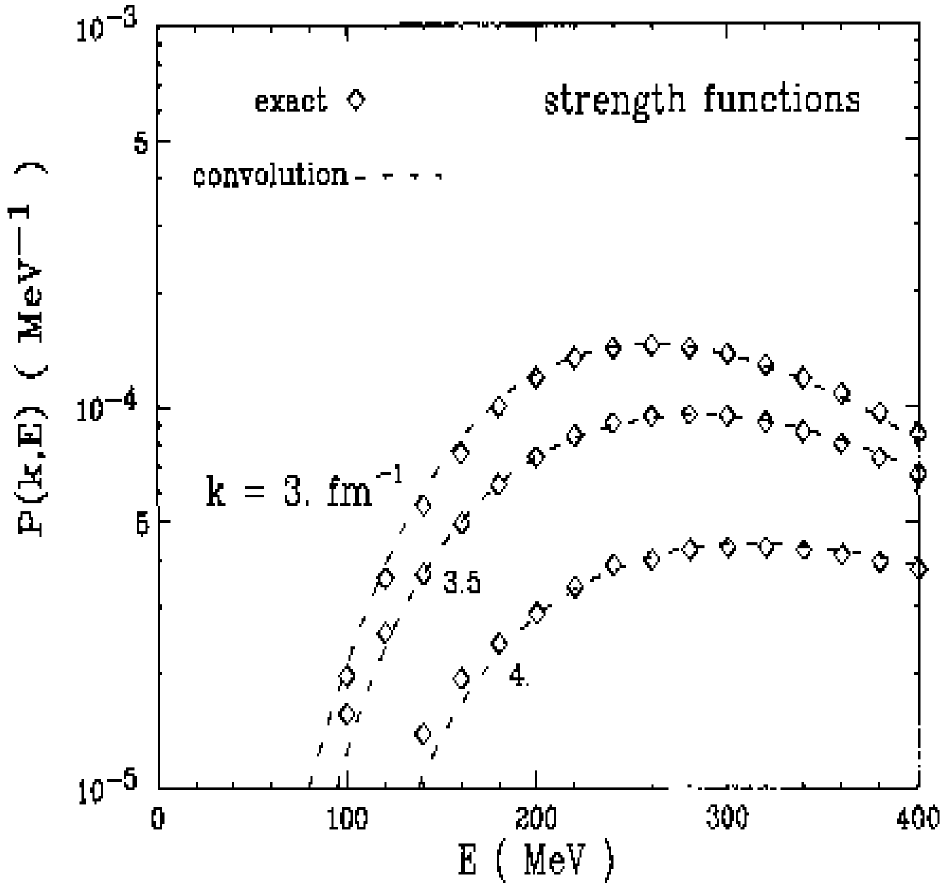
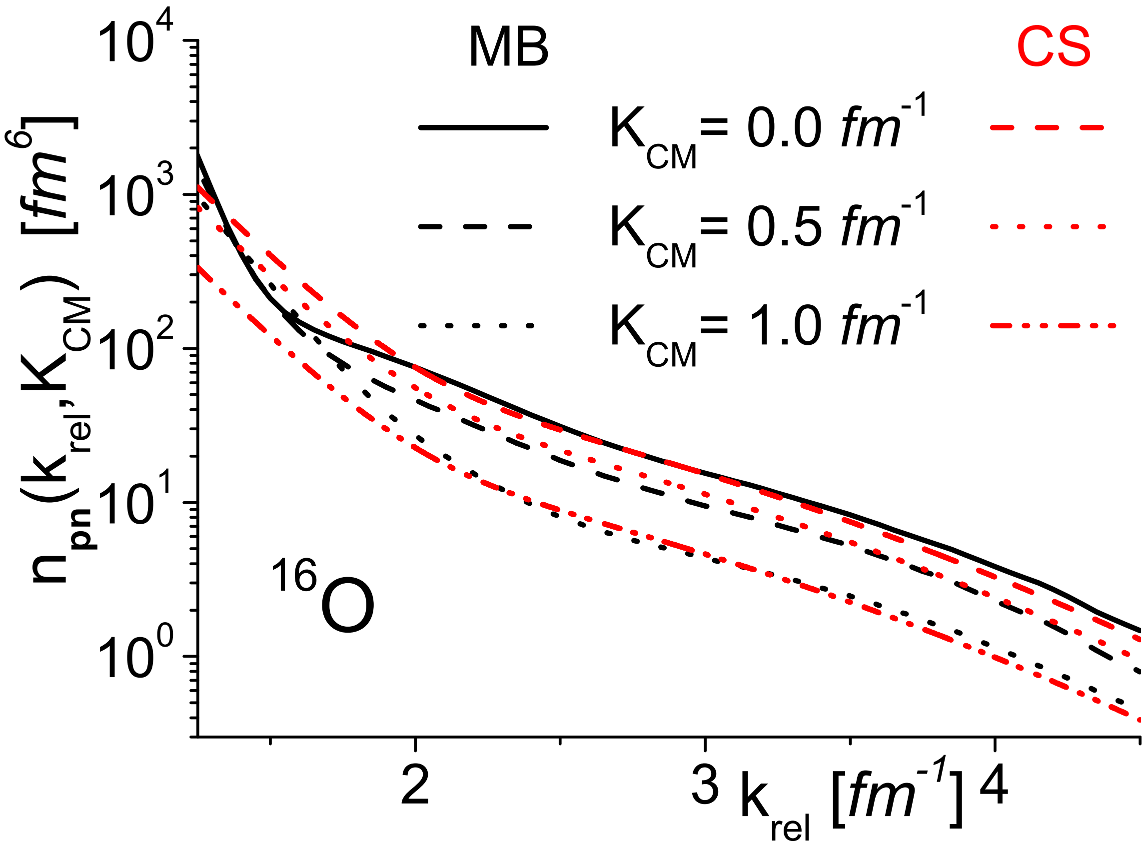


Fig. 4. Comparison between the SF obtained from the convolution model (dashed lines) and the one obtained from BBG theory (diamonds) for different values of the nucleon momentum k.

BBG Theory leads explicitly to the convolution formula

Baldo,Borromeo,CdA 1996

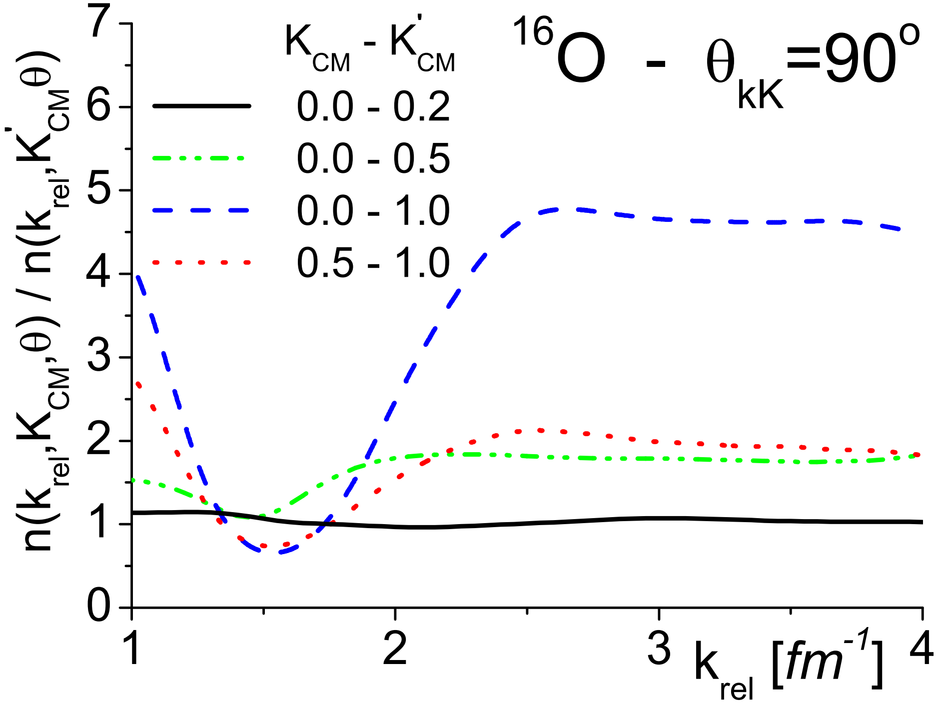
Factorization of $n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$



Many Body:
Alvioli, CdA, Morita
arXiv:0709.3989
$n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$

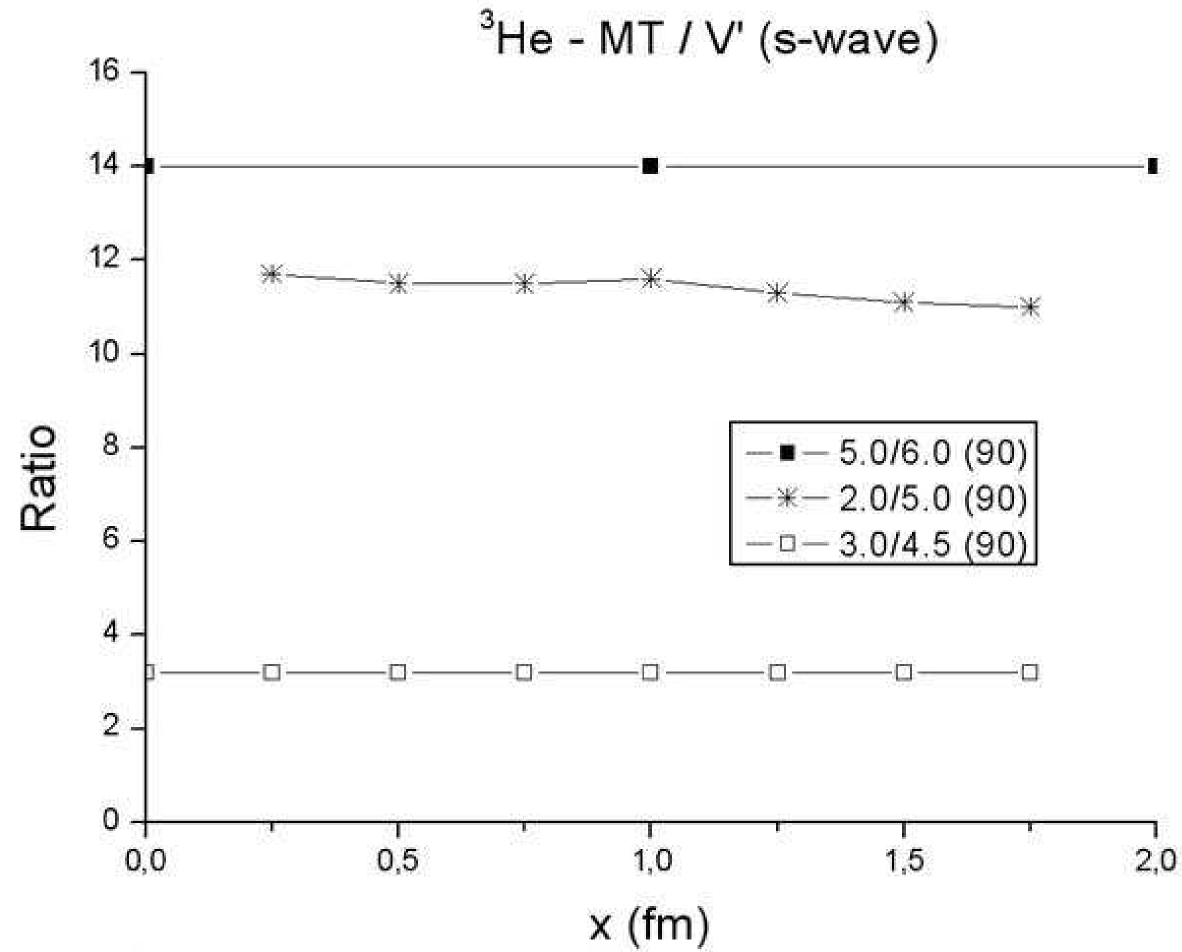
CS: Ciofi, Simula
PRC53, (1996)
$C_A \; n_{2_H}(k_{rel}) \; n_{CS}(K_{CM})$

- $n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$ **factorization in the Two-Nucleon correlation model Spectral Function implies that** $n_{NN}(k_{rel}, K_{CM}, \theta) \propto n_{NN}(k_{rel}, K'_{CM}, \theta)$



- **high** k_{rel} **and low** K_{CM} **factorization validated by many-body calculations**

Check of the “Factorization” 1

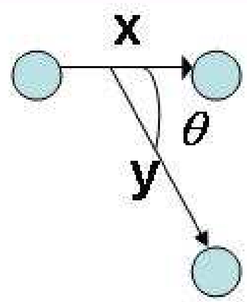


$$\frac{\Psi_{^3He}(x,y,\theta)}{\Psi_{^3He}(x,y',\theta)}$$

$$\frac{\Psi(x,y=5.0,\theta=90^\circ)}{\Psi(x,y=6.0,\theta=90^\circ)}$$

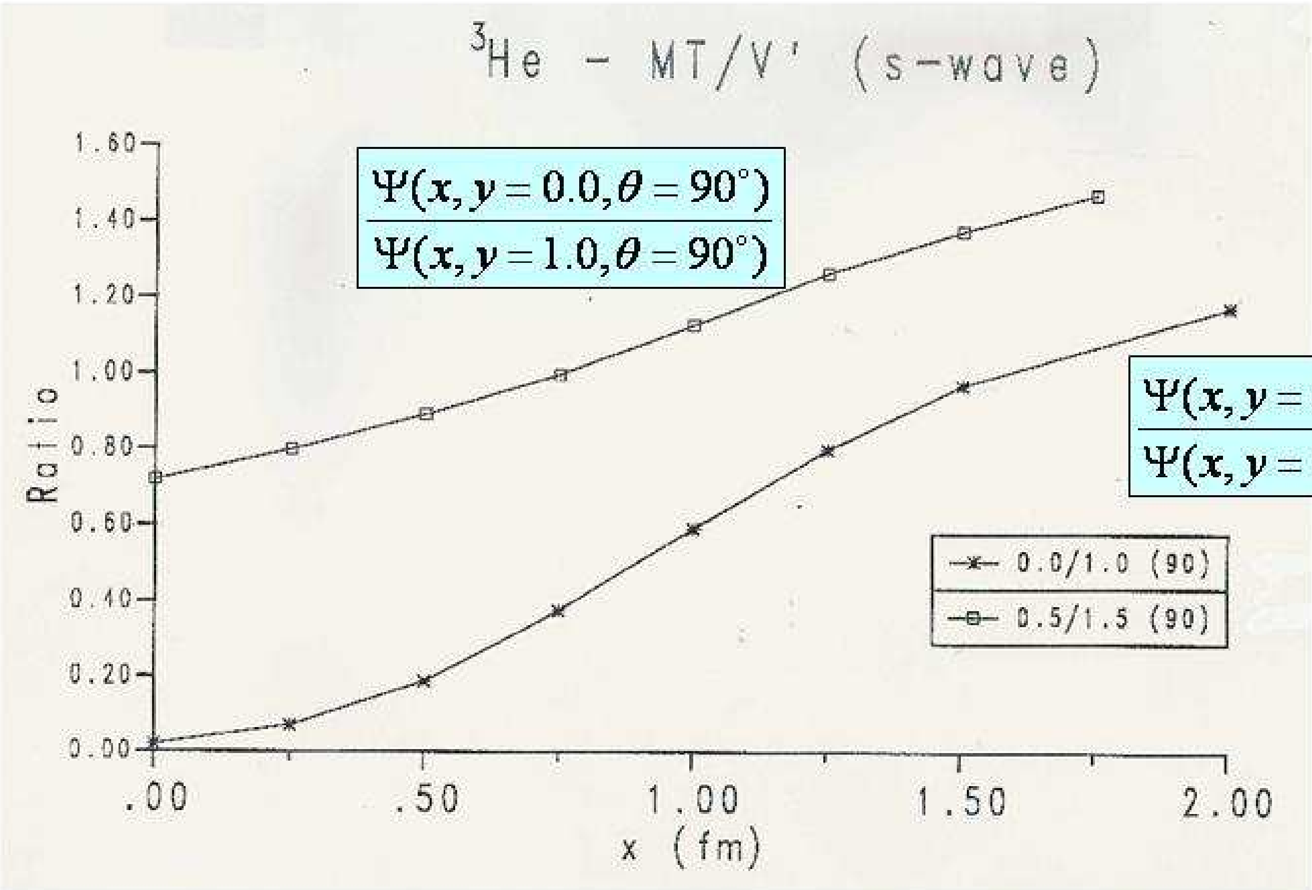
$$\frac{\Psi(x,y=2.0,\theta=90^\circ)}{\Psi(x,y=5.0,\theta=90^\circ)}$$

$$\frac{\Psi(x,y=3.0,\theta=90^\circ)}{\Psi(x,y=4.5,\theta=90^\circ)}$$

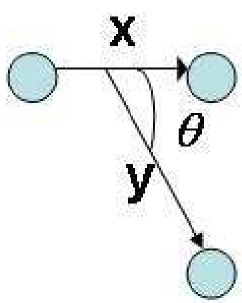


$y \rightarrow Large$: Wave function is factorized!

Check of the "Factorization" 2

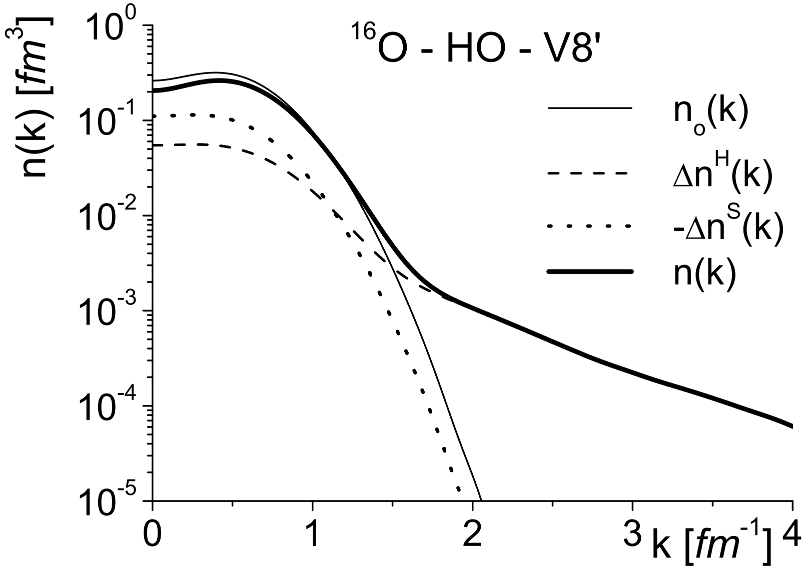


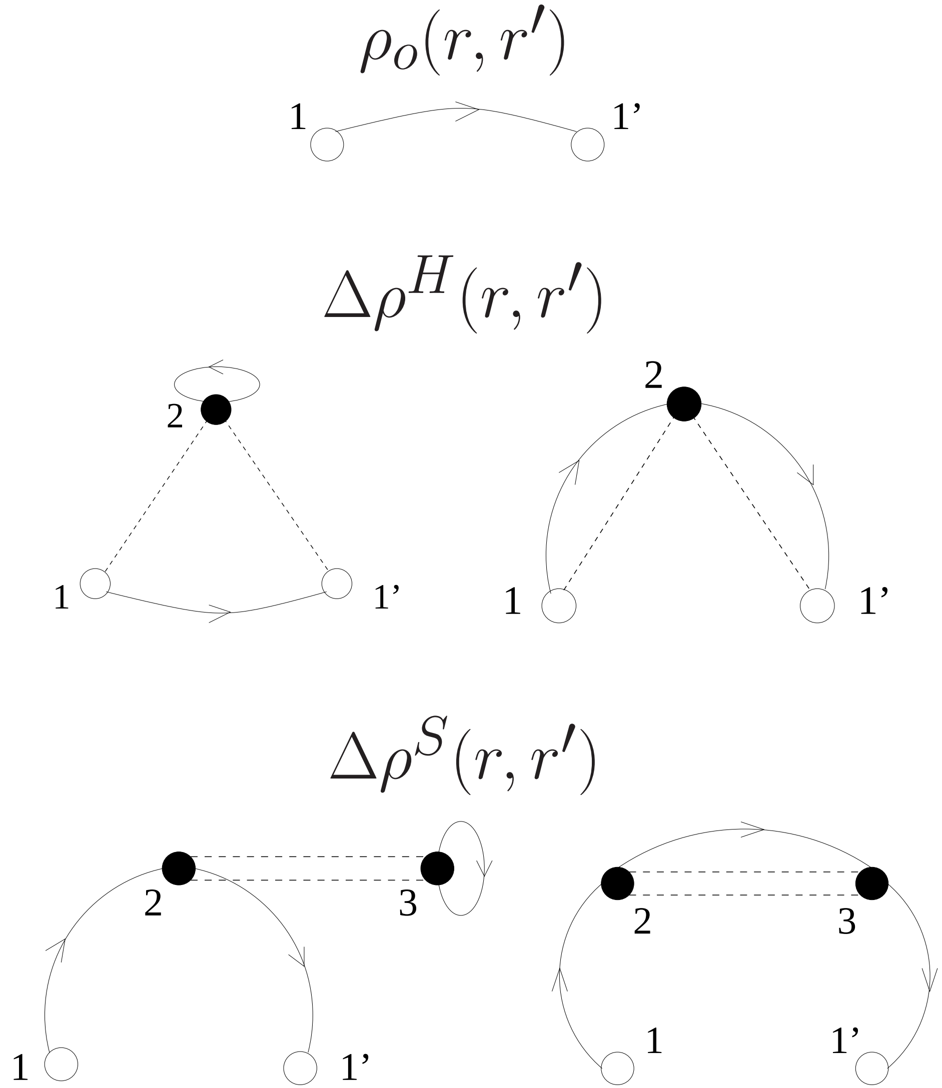
$$\frac{\Psi_{^3He}(x,y,\theta)}{\Psi_{^3He}(x,y',\theta)}$$



$y \to small$:Wave function is not factorized!

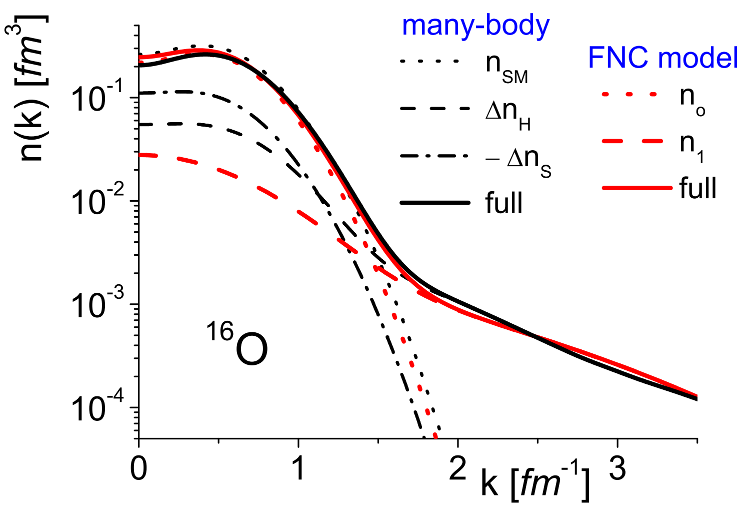
Momentum distributions and occupation numbers:
Many Body *vs* 2NC

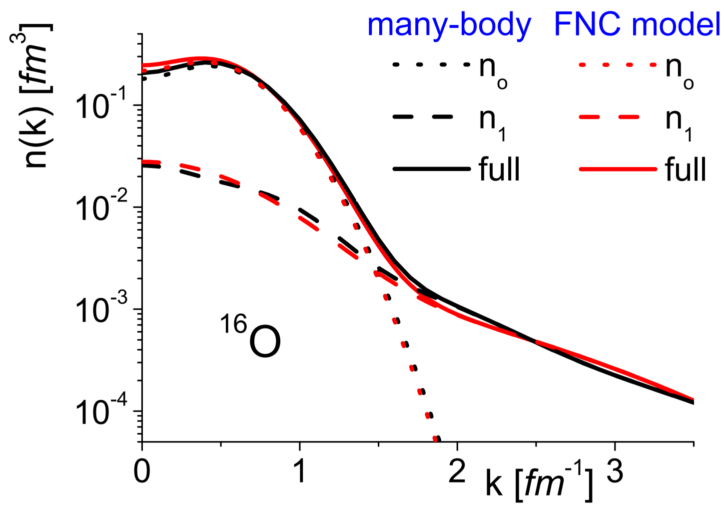




Δn^S reduces the SM occupation; Δn^H mostly generates correlations (populates states above the Fermi sea)

M. Alvioli, CdA, H. Morita , *to appear*





$$n_o(\boldsymbol{k}) = n_{SM}(\boldsymbol{k}) + \Delta n_S(\boldsymbol{k}) + C_A\, n_{SM}^H(\boldsymbol{k})$$

$$n_1(\boldsymbol{k}) = \Delta n_H(\boldsymbol{k}) - C_A\, n_{SM}^H(\boldsymbol{k})$$

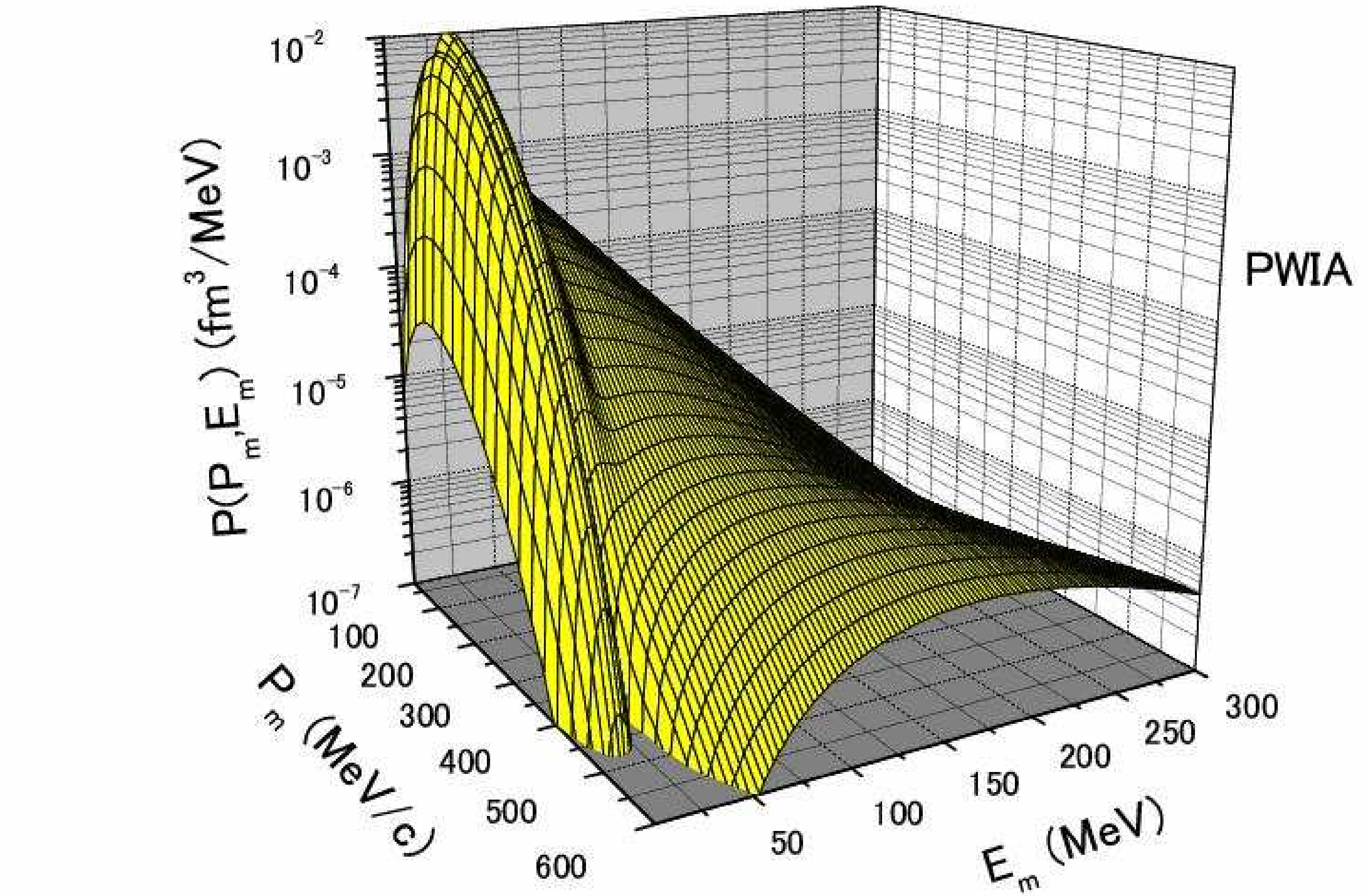
$$S_o = \int d\boldsymbol{k}\, n_o(\boldsymbol{k})$$

$$S_1 = \int d\boldsymbol{k}\, n_1(\boldsymbol{k})$$

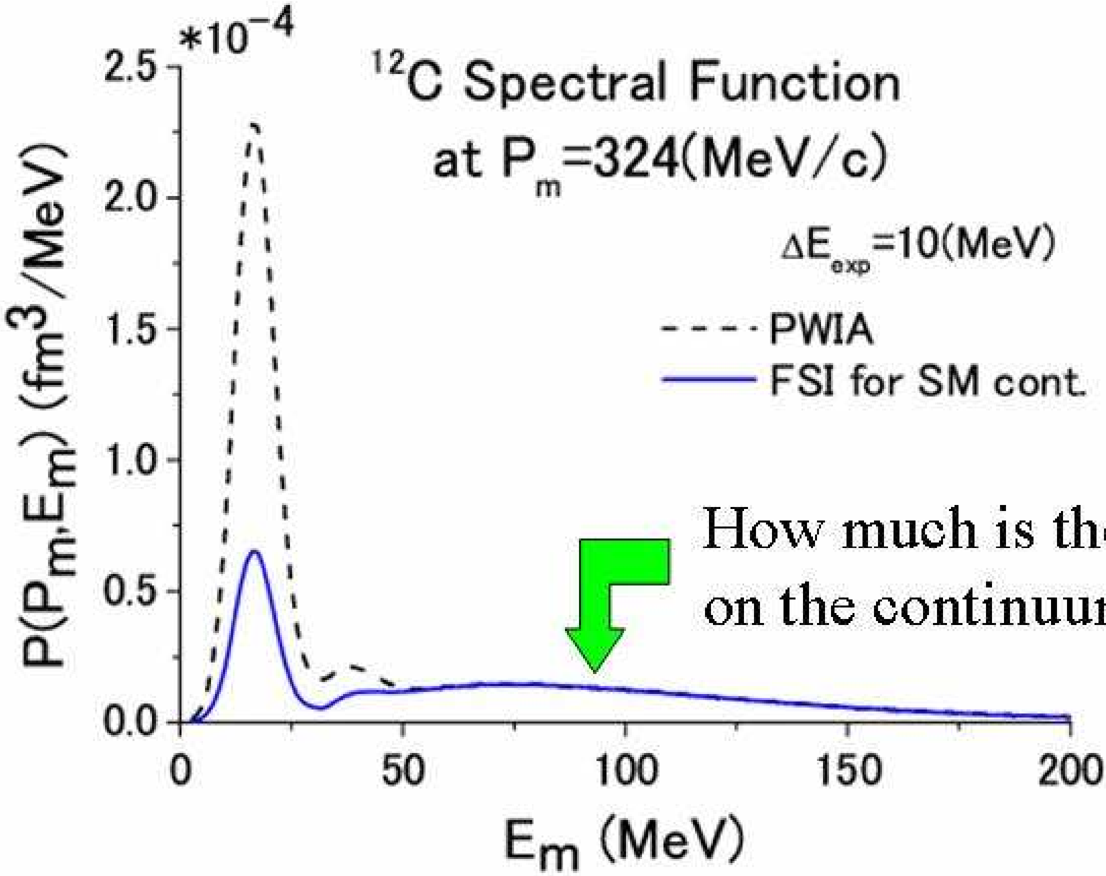
A	S_o	S_1
3	**0.65**	**0.35**
16	**0.8**	**0.2**

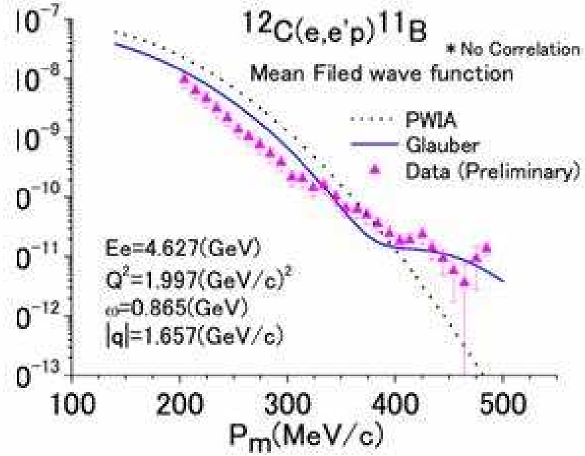
no "external" quantities in the convolution model

Spectral Function of ^{12}C



^{12}C Spectral function at JLab Kinematics(E01-E015)





How much is the effect of the FSI on the continuum part?

Tentative Summary

- **the (parameter free) convolution model is physically sound and theoretically validated by many body calculations. Any many-body approach to the spectral function should lead for $E \simeq k^2/2m_N$ to a convolution integral;**
- **representing an effective three body problem, the model can be readily extended to accommodate missing effects: relativistic effects (LC variables), three-nucleon correlations, FSI effects, isospin dependence. Work is in progress;**
- **isospin dependence is governed by the isospin dependence of n_{rel} which has been recently calculated (Schiavilla et al, Perugia (Alvioli's talk));**
- **a careful comparison with other models of SF for finite nuclei (e.g. LDA) is order; at $E \simeq k^2/2m_N$ all of them should predict the convolution model.**