Nucleon (and Δ) Momentum Distributions in Nuclei

Robert B. Wiringa (ANL)
Rocco Schiavilla (JLab & ODU)
Steven C. Pieper (ANL)
Joe Carlson (LANL)

Argonne Laboratory Computing Resource Center
National Energy Research Supercomputing Center

Argonne NATIONAL LABORATORY
Physics Division

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Theoretical Framework

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

Variational Monte Carlo: Minimize expectation value of H

$$E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} \geq E_0$$

$$|\Psi_T\rangle = [1 + \sum_{i<j<k} U_{ijk}][\mathcal{S} \prod_{i<j} (1 + U_{ij})] \prod_{i<j} f_{ij} |\Phi(J M_J T M_T)\rangle$$

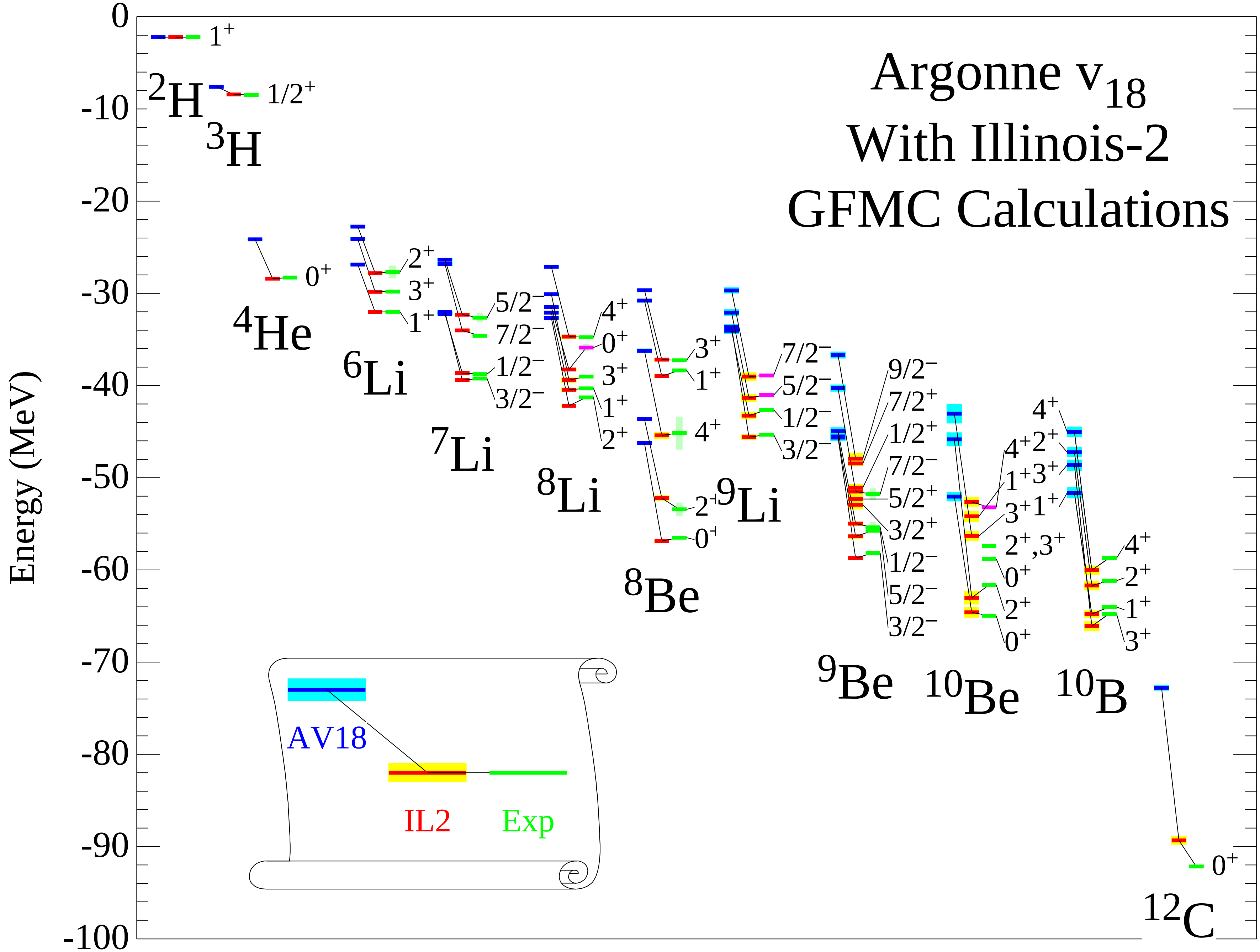
U_{ij} and U_{ijk} are non-commuting 2- and 3-body correlations from v_{ij} and V_{ijk}
f_{ij} are central correlations; Φ is antisymmetric $1\hbar\omega$ shell-model wave function

Green's function Monte Carlo: Ψ_T propagated to imaginary time τ:

$$\Psi(\tau) = e^{-(H-E_0)\tau} \Psi_T \quad ; \quad \Psi_T = \Psi_0 + \sum \alpha_i \Psi_i$$

$$\Psi(\tau) = [\Psi_0 + \sum \alpha_i e^{-(E_i - E_0)\tau} \Psi_i] \quad ; \quad \Psi_0 = \lim_{\tau \to \infty} \Psi(\tau)$$

$$E(\tau) = \frac{\langle \Psi_T | H | \Psi(\tau) \rangle}{\langle \Psi_T | \Psi(\tau) \rangle} \geq E_0$$



Argonne v18
With Illinois-2
GFMC Calculations
Energy (MeV)
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
2H
3H
4He
6Li
7Li
8Li
8Be
9Li
9Be
10Be
10B
12C
AV18
IL2
Exp

Momentum Distributions

Probability of finding a nucleon in a nucleus with momentum k in a given spin-isospin state:

$$\rho_{\sigma\tau}(k) = \int d\mathbf{r}'_1 d\mathbf{r}_1 d\mathbf{r}_2 \cdots d\mathbf{r}_A \, \psi^\dagger_A(\mathbf{r}'_1, \mathbf{r}_2, \ldots, \mathbf{r}_A) \, e^{-i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}'_1)} \, P_{\sigma\tau} \, \psi_A(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$$

with normalization

$$\int \frac{d\mathbf{k}}{(2\pi)^3} \, \rho_{\sigma\tau}(k) = N_{\sigma\tau}$$

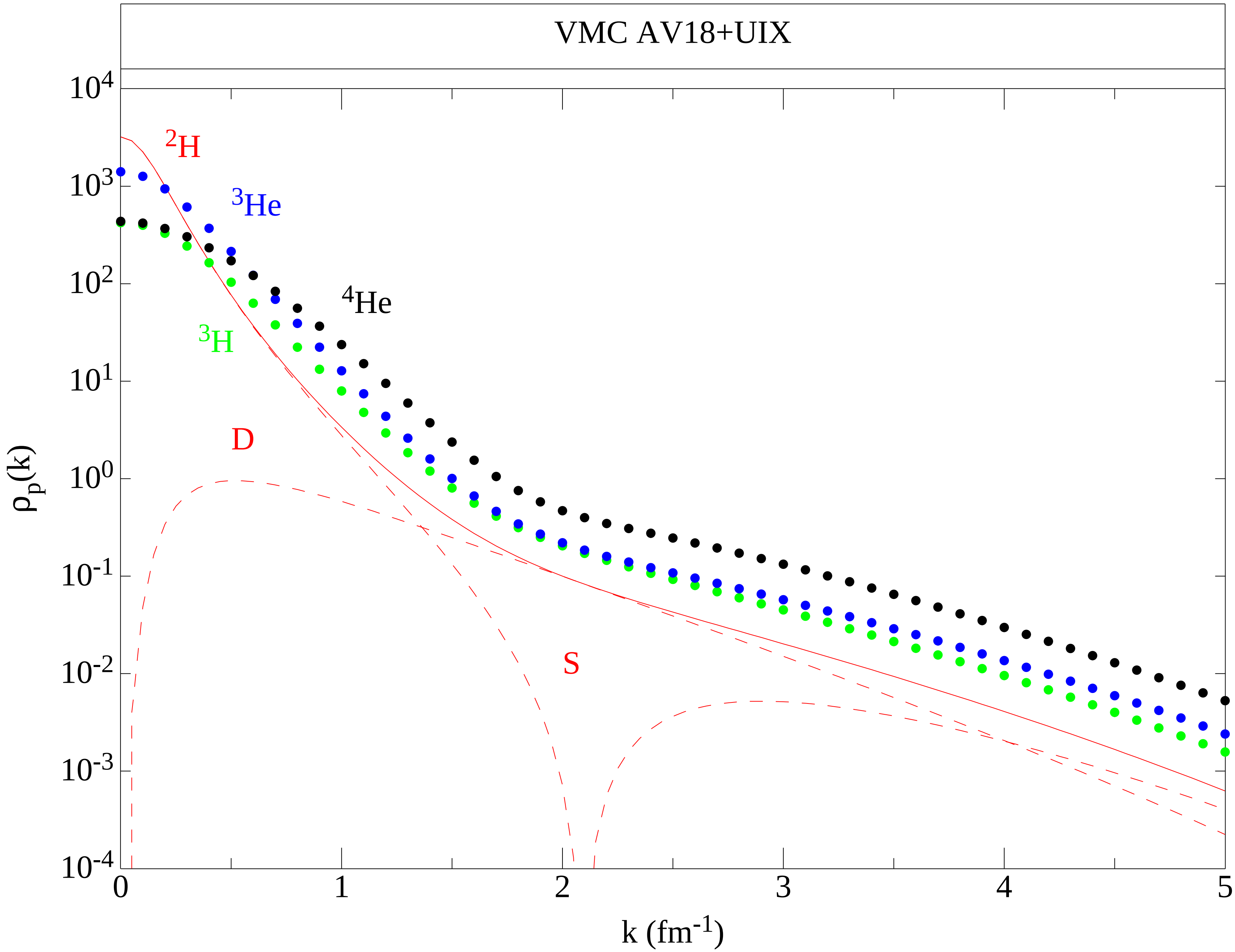
For two nucleons with relative momentum $\mathbf{q}$ and total momentum $\mathbf{Q}$ in pair state S, T:

$$\rho_{ST}(\mathbf{q}, \mathbf{Q}) = \int d\mathbf{r}'_1 d\mathbf{r}'_2 d\mathbf{r}_1 d\mathbf{r}_2 \cdots d\mathbf{r}_A \, \psi^\dagger_A(\mathbf{r}'_1, \mathbf{r}'_2, \ldots, \mathbf{r}_A)$$

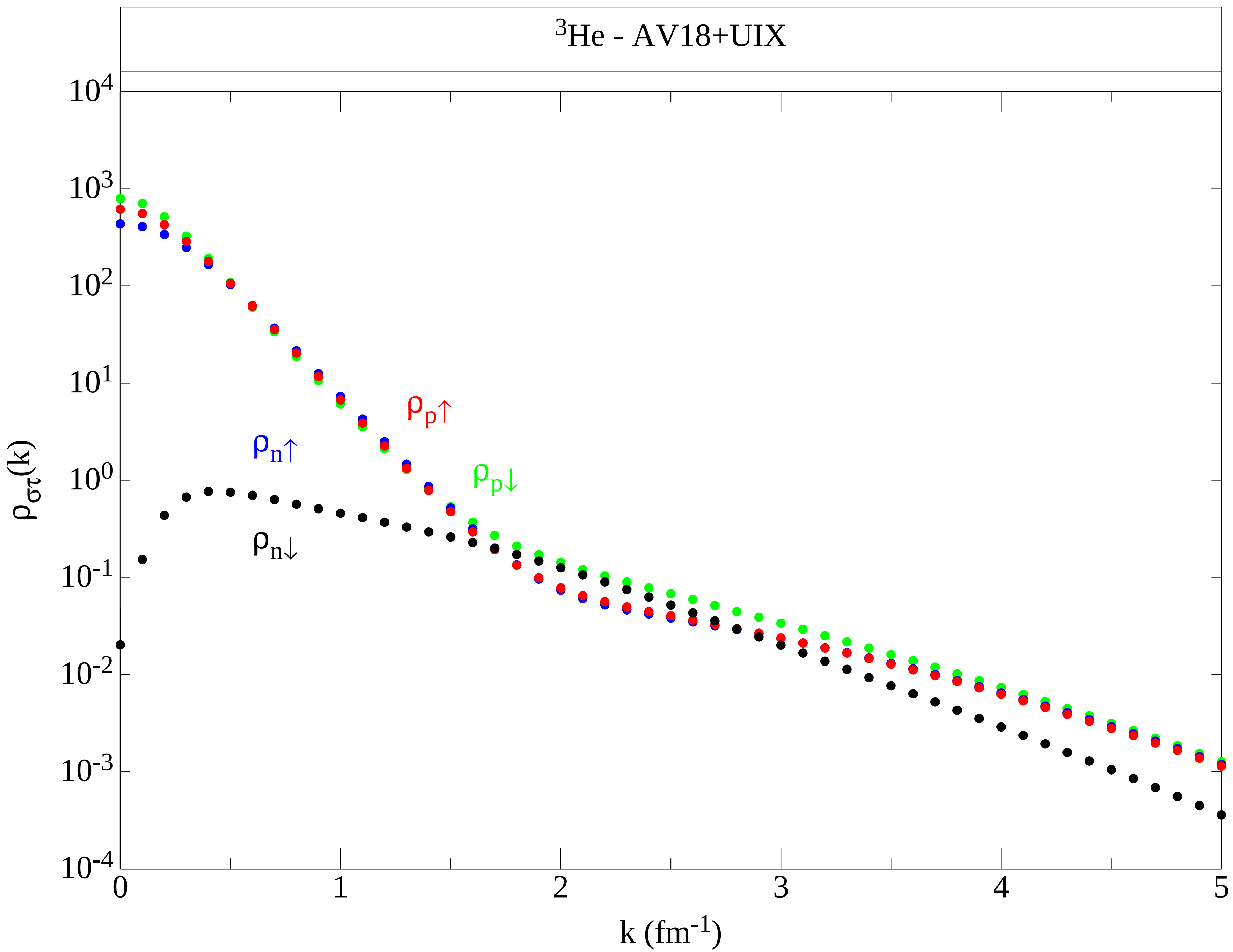
$$e^{-i\mathbf{q}\cdot(\mathbf{r}_{12} - \mathbf{r}'_{12})} \, e^{-i\mathbf{Q}\cdot(\mathbf{R}_{12} - \mathbf{R}'_{12})} \, P_{ST} \, \psi_A(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$$

with normalization

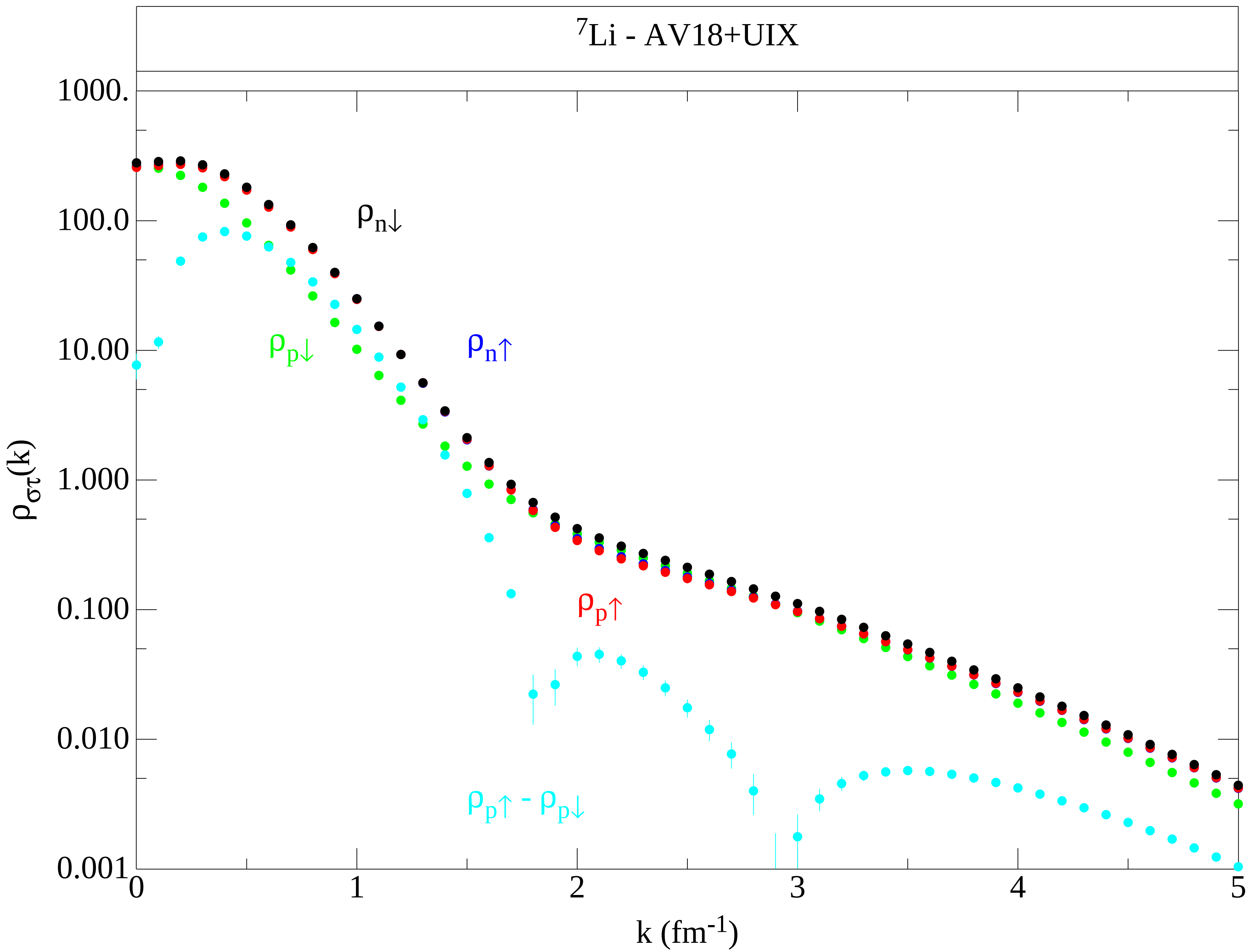
$$\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{d\mathbf{Q}}{(2\pi)^3} \, \rho_{ST}(\mathbf{q}, \mathbf{Q}) = N_{ST}$$



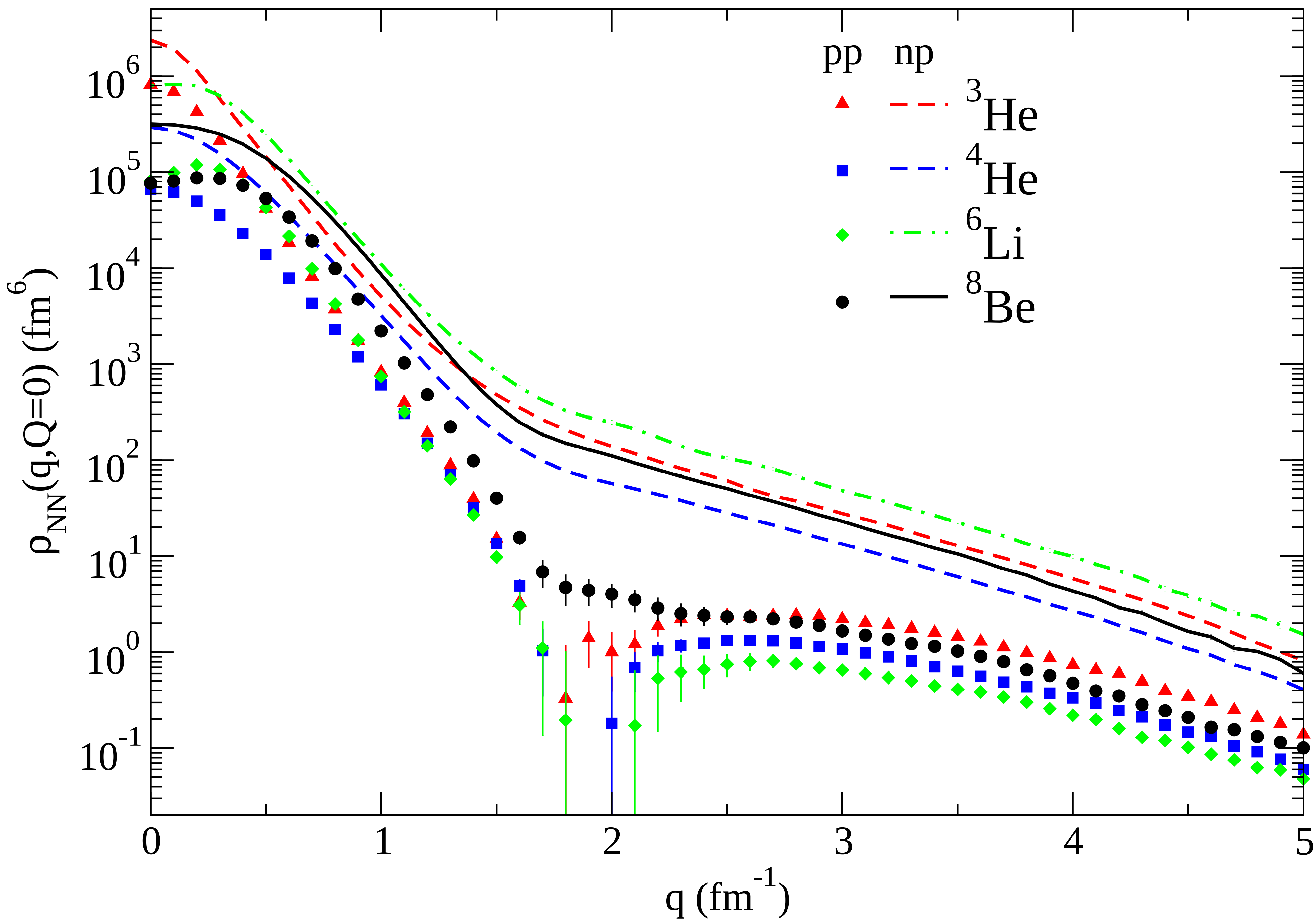
VMC AV18+UIX
^{2}H
^{3}He
^{4}He
^{3}H
D
S
$\rho_p(k)$
k (fm^{-1})



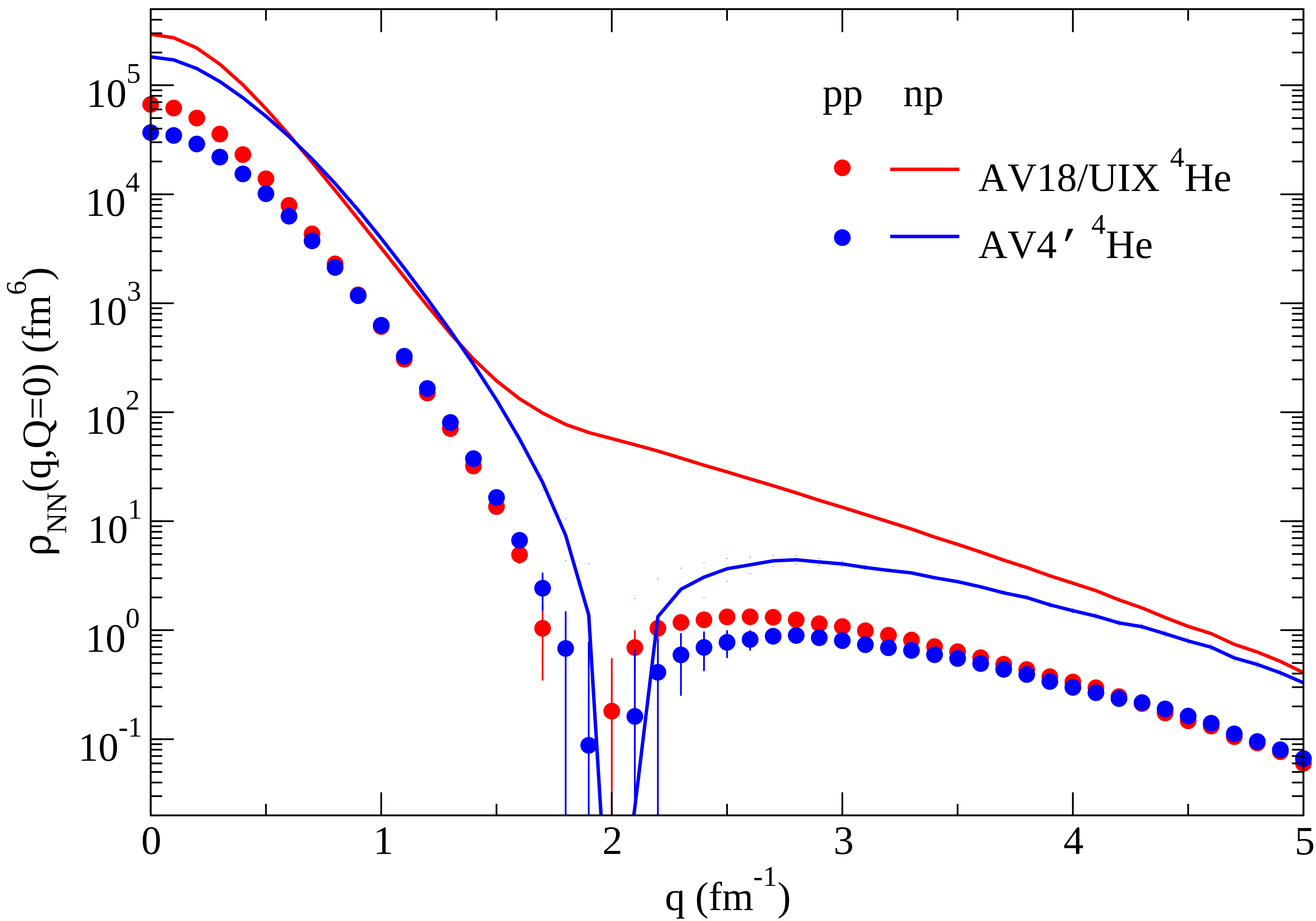
^{3}He - AV18+UIX
$\rho_{\sigma\tau}(k)$
k (fm^{-1})
$\rho_{p\uparrow}$
$\rho_{n\uparrow}$
$\rho_{p\downarrow}$
$\rho_{n\downarrow}$
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
1
2
3
4
5



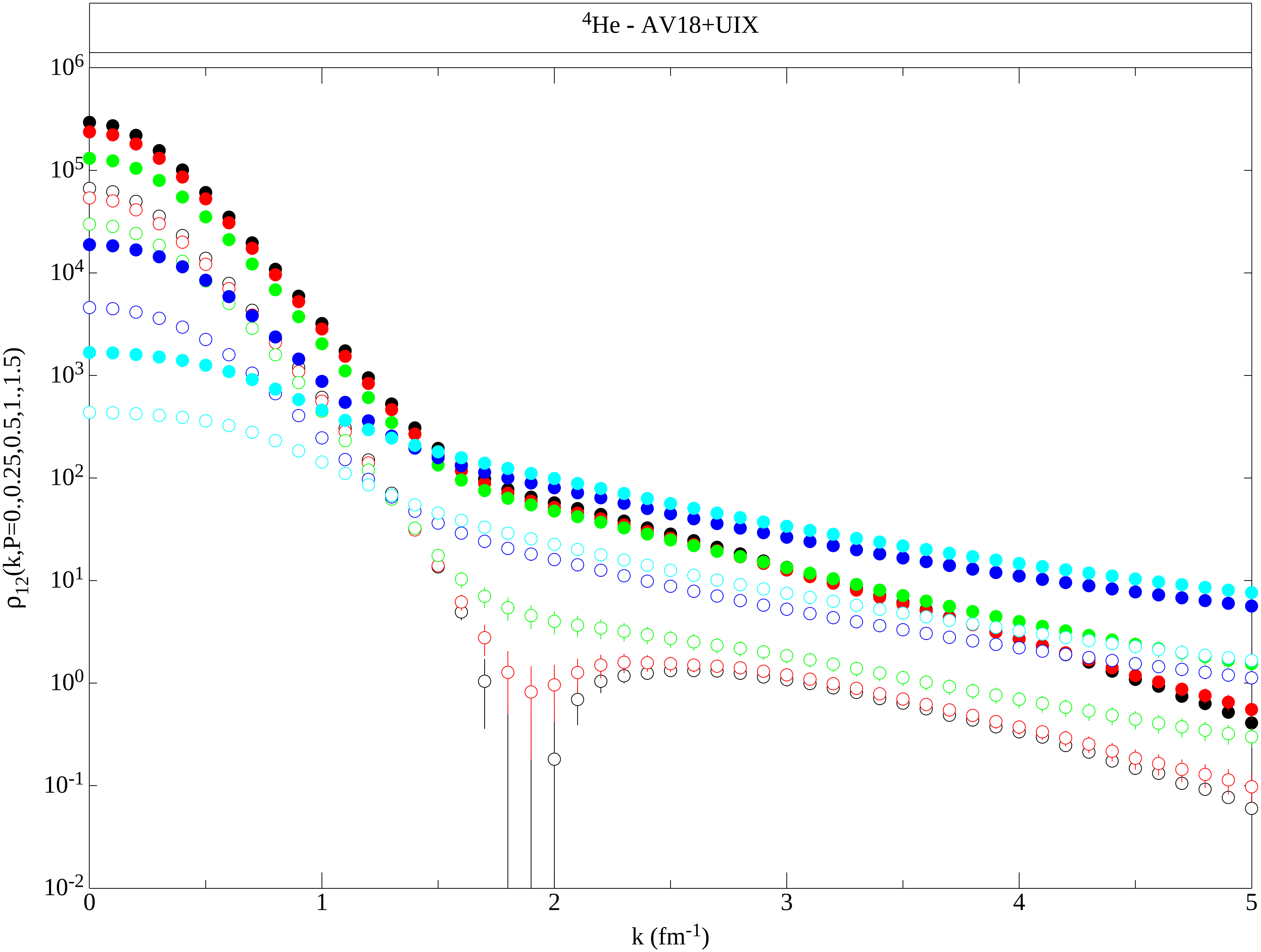
^{7}Li - AV18+UIX
1000.
100.0
10.00
1.000
0.100
0.010
0.001
0
1
2
3
4
5
$\rho_{\sigma\tau}(k)$
k (fm^{-1})
$\rho_{n\downarrow}$
$\rho_{p\downarrow}$
$\rho_{n\uparrow}$
$\rho_{p\uparrow}$
$\rho_{p\uparrow} - \rho_{p\downarrow}$



$\rho_{NN}(q,Q=0)$ (fm^6)
q (fm^{-1})
pp np
^{3}He
^{4}He
^{6}Li
^{8}Be
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
0
1
2
3
4
5



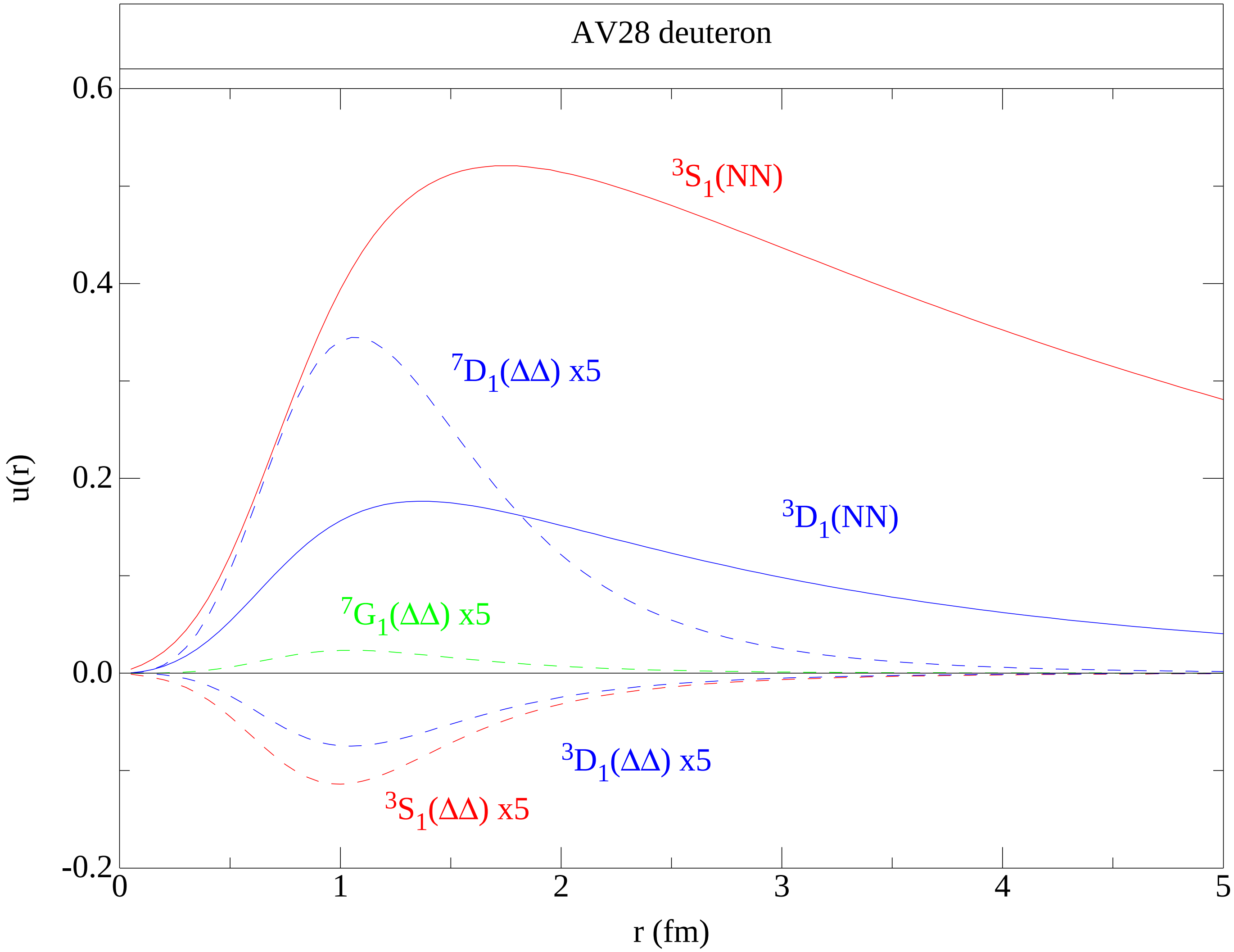
$\rho_{NN}(q,Q=0)$ (fm^6)
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
0
1
2
3
4
5
q (fm^{-1})
pp np
AV18/UIX ^{4}He
AV4′ ^{4}He



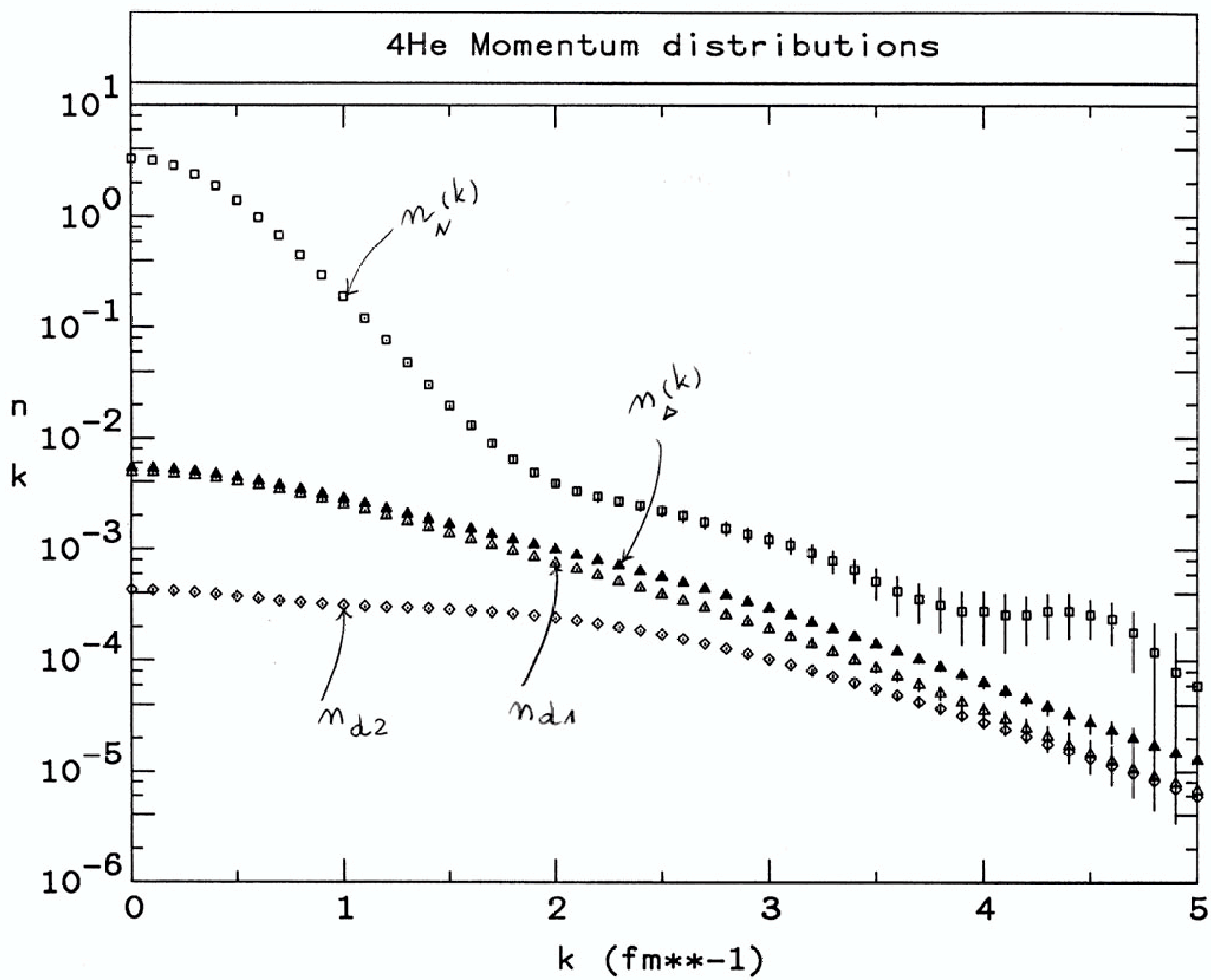
^{4}He - AV18+UIX
ρ_{12}(k,P=0.,0.25,0.5,1.,1.5)
k (fm^{-1})
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0
1
2
3
4
5

Δ COMPONENTS

Argonne v_{28} potential (1984) and unpublished v_{28q} have explicit Δ degrees of freedom.



Elastic scattering data does not constrain the Δ content; deuteron P_Δ = 0.5% or 0.25%.



4He Momentum distributions
n
k
k (fm**-1)
0
1
2
3
4
5
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
n_N(k)
n_Δ(k)
n_d1
n_d2

CONCLUSIONS

We have the capability to calculate a variety of 1- and 2-nucleon momentum distributions in light $A \leq 10$ nuclei which we believe are fairly accurate.

We can make some crude predictions for Δ momentum distributions, but these are much more model-dependent.