# Correlations in asymmetric nuclear matter


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T. Frick *et al.* Phys. Rev. C 71, 014313 (2005).
A. Rios *et al.* Phys. Rev. C 73, 024305 (2006).

# Outline

1. Asymmetric Nuclear Matter at Finite Temperature
2. Self-Consistent Green's Functions at Finite Temperature
3. SCGF results for asymmetric nuclear matter
4. Connection to experimental results

# Motivation: nuclear matter

## Nuclear Matter

- Infinite system of nucleons
- High densities $\rho \sim 10^{14}$ g cm$^{-3}$ $\Rightarrow$ strong interaction
- Model heavy nuclei cores and neutron stars
- Short range effects close to finite nuclei

## Asymmetric Nuclear Matter

- Symmetric ($Z = N$) vs. asymmetric ($Z \neq N$)
- Measured by $x_p = \alpha = \frac{Z}{N+Z}$ or $\beta = \frac{N-Z}{N+Z}$
- Isospin asymmetric systems in nature:
  - (Heavy) Nuclei: $^{208}$Pb, $\alpha = 0.39$, $\beta = 0.2$
  - Neutron Stars: $\alpha \sim 0.05$, $\beta \sim 0.9$
- How to extrapolate safely? RIB's, drip line physics...

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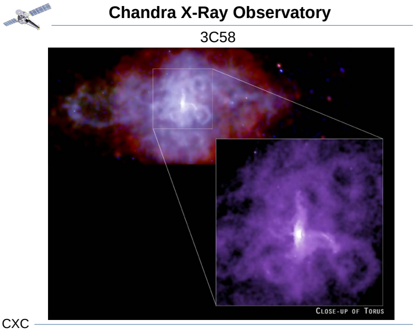
# Motivation: "hot" nuclear systems

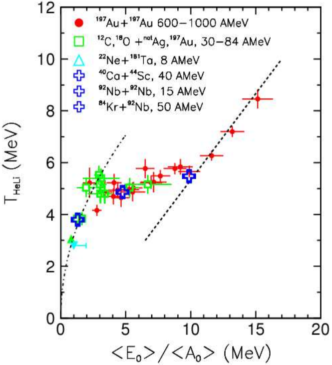
$$E \sim 1 \text{ MeV} \Rightarrow T \sim 10^{10} \text{ K}$$

## Proto-neutron stars




SN 1181 remnant (SNR3C58) and Pulsar PSRJ0205+6449

## $AA$ collisions




Nuclear caloric curve

# SCGF: Ingredients

- Based on many-body Green's functions formalism at $T \neq 0$
- Main approximation: ladder decoupling at the level of $\mathcal{G}_{III}$
- Includes short-range and tensor correlations
- Full off-shell energy dependence is considered
- Thermodynamically consistent (conserving) theory
- Ladder includes hole-hole propagation (beyond BHF), which leads to a pairing instability for $T = 0$ ...
- Finite temperature actually solves theoretical problems!

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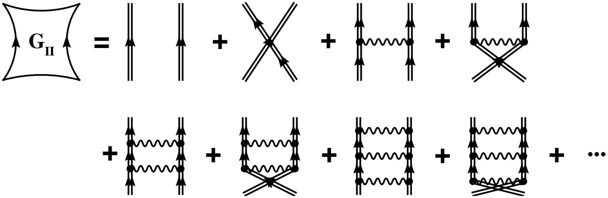
# Ladder approximation




- Valid for strong interactions and low densities
- Self-consistency is imposed at each step
- Solved in terms of Dyson's equation
- Ladder self-energy
- In-medium interaction accounts for “Pauli" effects
- Off-shell behavior, beyond quasi-particle approximation

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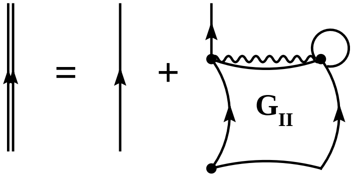


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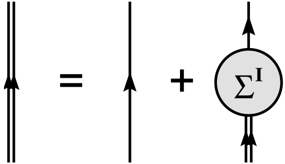


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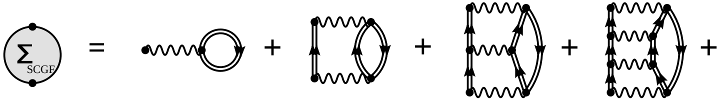


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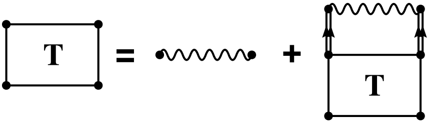


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# Ladder approximation

$$\langle \mathbf{k}_1 \mathbf{k}_2 | T(Z_\nu) | \mathbf{k}_3 \mathbf{k}_4 \rangle = \langle \mathbf{k}_1 \mathbf{k}_2 | V | \mathbf{k}_3 \mathbf{k}_4 \rangle$$
$$+ \mathcal{V} \int \frac{\mathrm{d}^3 k_5}{(2\pi)^3} \mathcal{V} \int \frac{\mathrm{d}^3 k_6}{(2\pi)^3} \langle \mathbf{k}_1 \mathbf{k}_2 | V | \mathbf{k}_5 \mathbf{k}_6 \rangle \, \mathcal{G}^0_{II}(Z_\nu; k_5 k_6) \, \langle \mathbf{k}_5 \mathbf{k}_6 | T(Z_\nu) | \mathbf{k}_3 \mathbf{k}_4 \rangle$$

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# Lehmann's representation $T = 0$

Spectral decomposition:

$$\mathcal{G}(k,\omega)=\int_{-\infty}^{\epsilon_F}\frac{d\omega}{2\pi}\frac{\mathcal{A}_h(k,\omega')}{\omega-\omega'-i\eta}+\int_{\epsilon_F}^{\infty}\frac{d\omega}{2\pi}\frac{\mathcal{A}_p(k,\omega')}{\omega-\omega'+i\eta}$$

Hole spectral function

$$\mathcal{A}_h(k,\omega)=2\pi\sum_n\left|\left\langle\Psi_n^{A-1}\middle|a_{\boldsymbol{k}}\middle|\Psi_0^A\right\rangle\right|^2\delta\left(w-(E_0^A-E_n^{A-1})\right)$$

$$\omega<\mu$$

Particle spectral function

$$\mathcal{A}_p(k,\omega)=2\pi\sum_n\left|\left\langle\Psi_n^{A+1}\middle|a_{\boldsymbol{k}}^\dagger\middle|\Psi_0^A\right\rangle\right|^2\delta\left(w-(E_n^{A+1}-E_0^A)\right)$$

$$\omega>\mu$$

# Lehmann's representation $T \neq 0$

Spectral decomposition:

$$\mathcal{G}(k,\omega) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\mathcal{A}^<(k,\omega')}{\omega - \omega' + i\eta} + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\mathcal{A}^>(k,\omega')}{\omega - \omega' - i\eta}$$

"Hole" spectral function

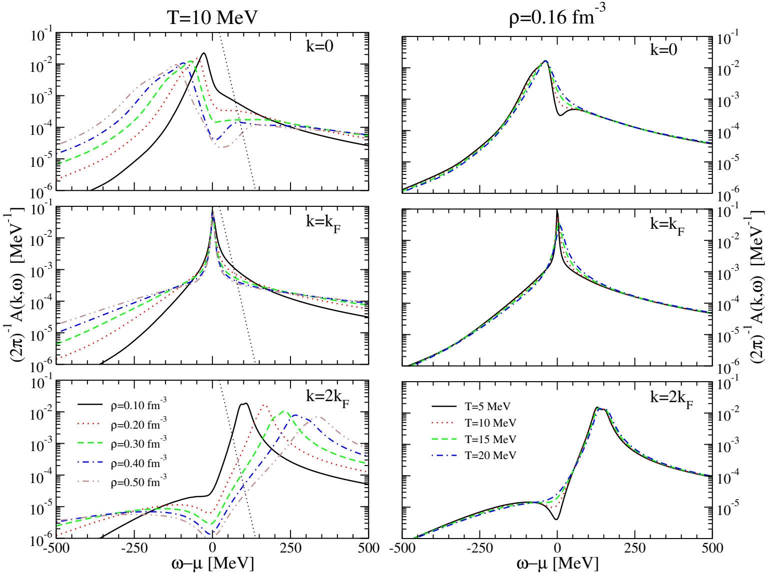
$$\mathcal{A}^<(k,\omega) = 2\pi \sum_{m,n} \frac{e^{-\beta(E_n - \mu N_n)}}{Z} \left| \left\langle \Psi_m \middle| a_{\mathbf{k}} \middle| \Psi_n \right\rangle \right|^2 \delta\big(w - (E_n - E_m)\big)$$

"Particle" spectral function

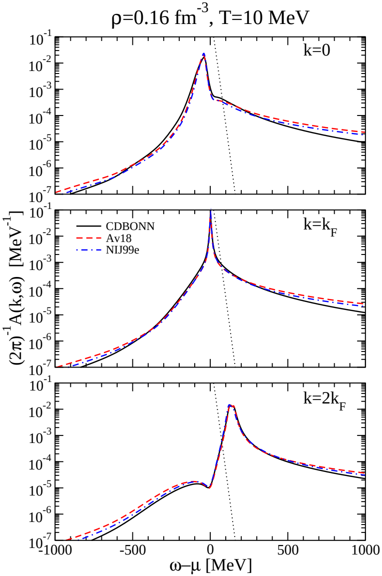
$$\mathcal{A}^>(k,\omega) = 2\pi \sum_{m,n} \frac{e^{-\beta(E_n - \mu N_n)}}{Z} \left| \left\langle \Psi_m \middle| a^\dagger_{\mathbf{k}} \middle| \Psi_n \right\rangle \right|^2 \delta\big(w - (E_m - E_n)\big)$$

Defined for all $\omega$!

# Spectral functions



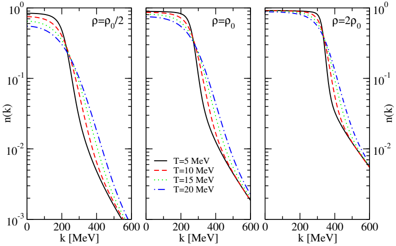
# Spectral functions: NN potentials



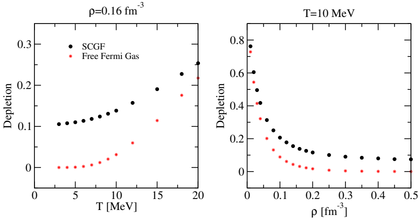

- Effects on high energy tails for all $k$
- CDBONN nonlocal and softer tensor
- Av18 local and more tensor
- Tensor correlations $\Rightarrow$ higher tails

# Other nuclear matter results...

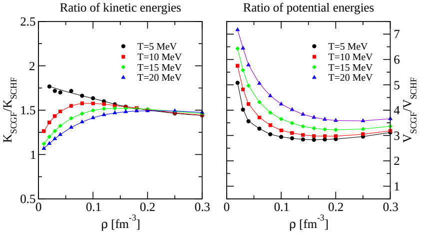
## Momentum distributions



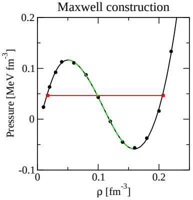

## Depletion



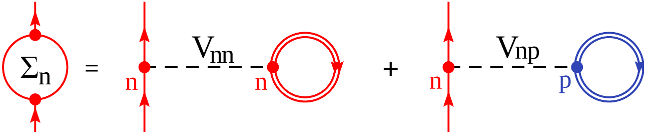

## Energy per particle




## Thermodynamical properties



# Ladder approximation in asymmetric matter




- Neutron to neutron contribution

$$\Sigma_n^n(k) = \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \langle n\boldsymbol{k}, n\boldsymbol{k}' | V | n\boldsymbol{k}, n\boldsymbol{k}' \rangle_A \, n_n(k')$$

- Proton to neutron contribution

$$\Sigma_n^p(k) = \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \langle n\boldsymbol{k}, p\boldsymbol{k}' | V | n\boldsymbol{k}, p'\boldsymbol{k}' \rangle_A \, n_p(k')$$

$$\Sigma_n^{HF}(k) = \Sigma_n^n(k) + \Sigma_n^p(k)$$

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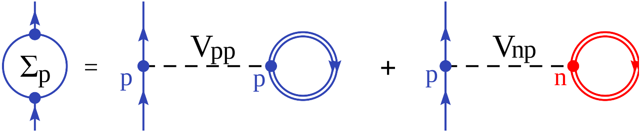



- Proton to proton contribution

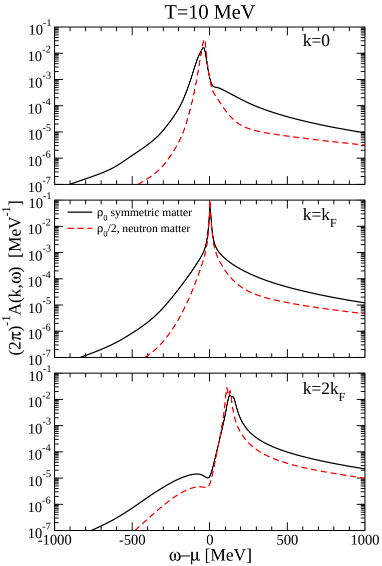
$$\Sigma_p^p(k) = \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \langle p\boldsymbol{k}, p\boldsymbol{k}' | V | p\boldsymbol{k}, p\boldsymbol{k}' \rangle_A \, n_p(k')$$

- Neutron to proton contribution

$$\Sigma_p^n(k) = \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \langle p\boldsymbol{k}, n\boldsymbol{k}' | V | p\boldsymbol{k}, n\boldsymbol{k}' \rangle_A \, n_n(k')$$

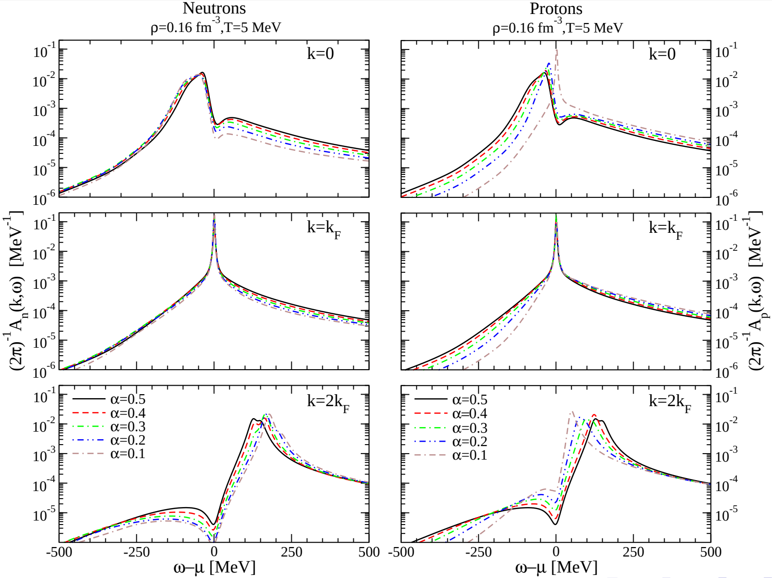
$$\Sigma_p^{HF}(k) = \Sigma_p^p(k) + \Sigma_p^n(k)$$

# Nuclear vs. neutron matter

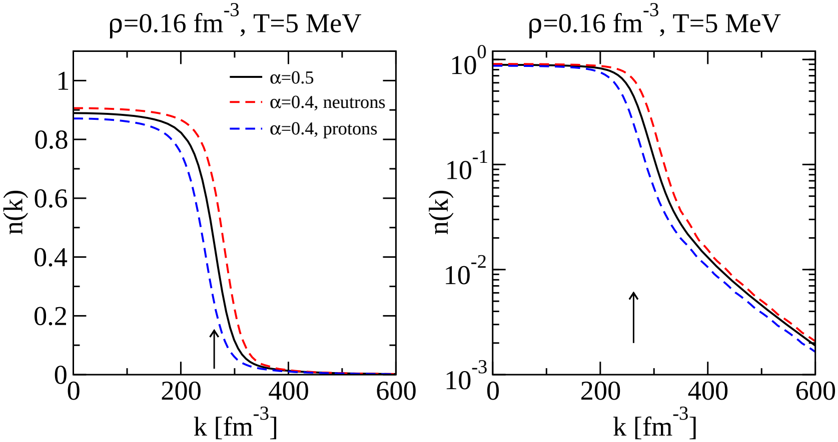



- Different $\rho$, same $k_F$
- Neutron matter $\Rightarrow$ lower tails
- $T=1 \Rightarrow$ inactive ${}^3S_1 -{}^3D_1$ tensor
- Tensor correlations $\Rightarrow$ higher tails

# Spectral functions: asymmetric matter

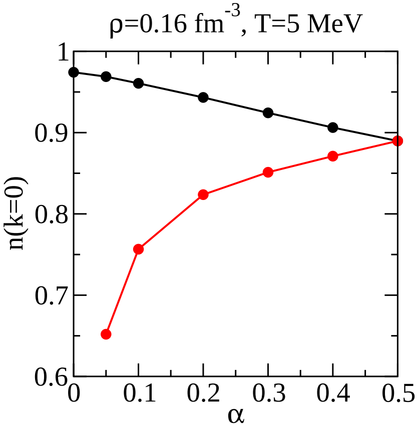


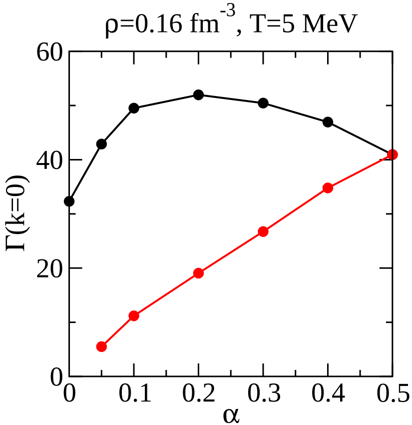
# Momentum distribution: asymmetric matter




- Protons more depleted
- Important splitting already for finite nuclei!

# Depletion and width at $k = 0$



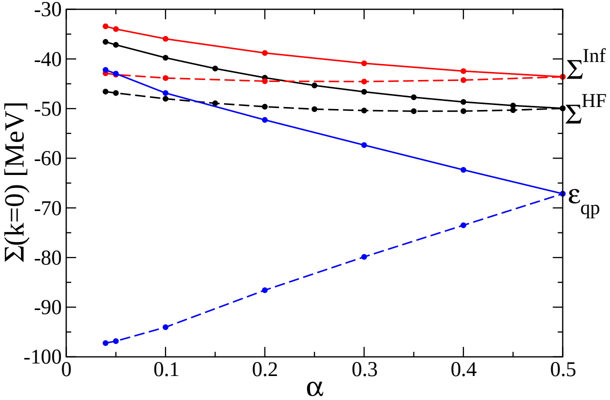




- Proton depletion also due to thermal effects
- Competition between $nn$ and $np$ correlations

# Sum rules and tensor correlations

$$m_1 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \, \omega \, \mathcal{A}(k,\omega) = \frac{k^2}{2m} + \Sigma^{\infty}(k)$$

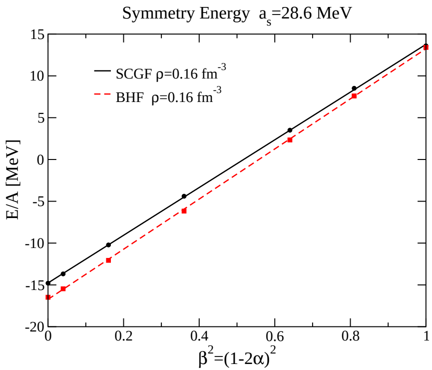
$$\varepsilon_{qp}(k) = \frac{k^2}{2m} + \mathrm{Re}\,\Sigma[k, \omega = \varepsilon_{qp}(k)]$$



# Symmetry energy

$$e = \frac{\nu}{2} \sum_{\tau} \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[ \frac{k^2}{2m_\tau} + \omega \right] \mathcal{A}_\tau(k,\omega) f_\tau(\omega)$$

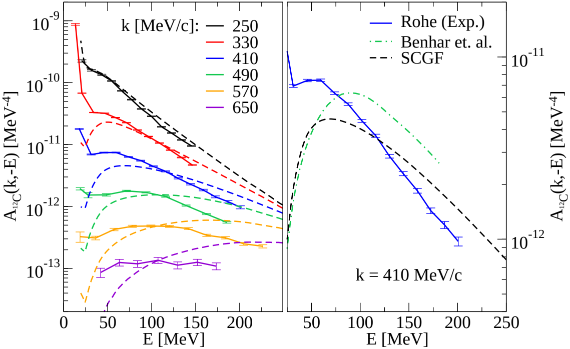
$$e(\rho,\beta) \sim e(\rho,\beta=0) + a_s(\rho)\beta^2$$




- Energy from GMK sum rule
- Low value due to lack of TBF
- Determines the pressure in NS
- Correlated with neutron skin thickness
- High energy components?

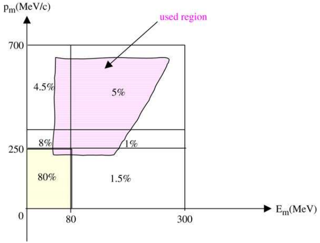
# Measures of SRC correlations

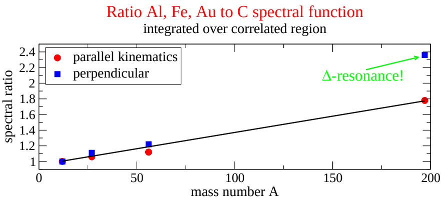
- Knock-out reactions and spectroscopic factors
- $(e, e'p)$ experiments




D. Rohe *et al.* PRL 93, 182501 (2004).
T. Frick *et al.* PRC 70, 024309 (2004).

# Integrated strength



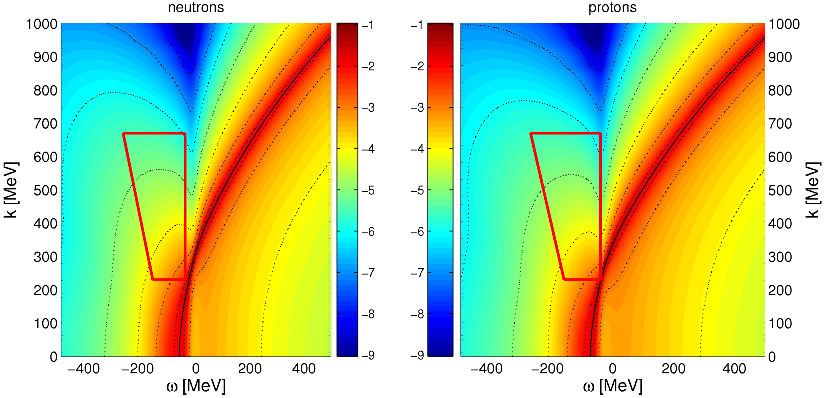

| | $\kappa$ |
|---|---|
| Experiment | $0.61 \pm 0.06$ |
| CBF theory | $0.64$ |
| SCGF theory | $0.61$ |

- Effect of isospin?
  $\frac{\kappa(\alpha=0.4)}{\kappa(\alpha=0.5)} \sim 1.2$
- Effect of density?
  $\frac{\kappa(\rho_0)}{\kappa(\rho_0/2)} \sim 1.5$

D. Rohe *et al.*, Eur. Phys. Jour. A 17, 439 (2003).
I. Sick, Prog. Part. Nucl. Phys 59, 447 (2004).

# Spectral functions: asymmetric matter

$$\rho = 0.16 \text{ fm}^{-3},\ T = 5 \text{ MeV},\ \alpha = 0.4$$



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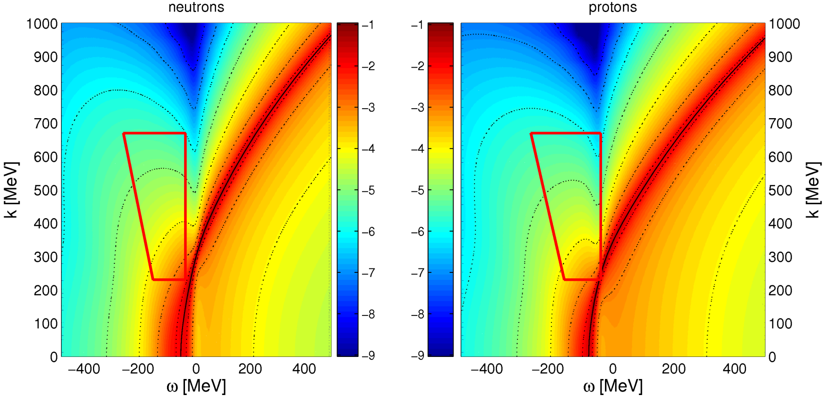

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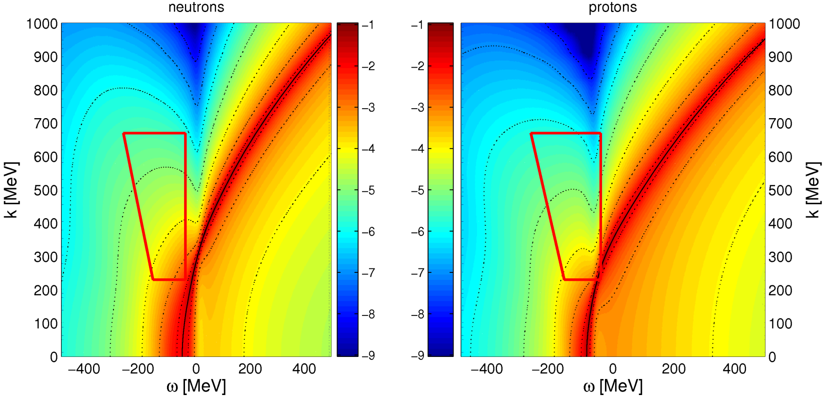
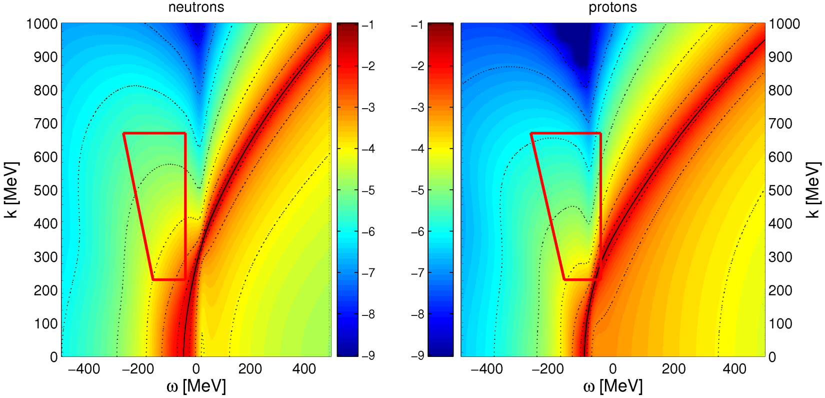


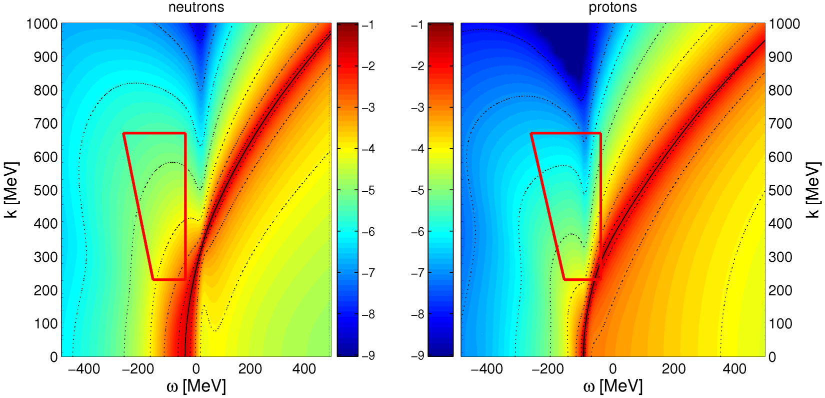
# Spectral functions: asymmetric matter

$\rho = 0.16$ fm$^{-3}$, $T = 5$ MeV, $\alpha = 0.1$



# Spectral functions: asymmetric matter

$$\rho = 0.16 \text{ fm}^{-3},\ T = 5 \text{ MeV},\ \alpha = 0.04$$



# Conclusion

- Isospin asymmetry affects substantially the microscopic properties of neutrons and protons in infinite matter
- Protons
  1. Larger particle (lower hole) energy tails
  2. Larger quasiparticle peaks and lower depletion
  3. More “correlated" due to $np$ tensor correlations
- Neutrons
  1. Less affected by asymmetry
  2. Neutron matter is less correlated than nuclear matter
  3. Competition between $np$ and $nn$ correlations

# Outlook

- Dependence on the 2-body NN potential
- Inclusion of 3-body effects
- $\rho$, $T$, $\alpha$ dependences of microscopic properties
- $\alpha$ dependence of TD properties of the system
- Pairing phase transition beyond quasi-particle approach
- Extension to time-dependent systems (HIC)

# Thank you!

# For further reading I


- T. Frick and H. Müther,
  *Self-consistent solution to the nuclear many-body problem at finite temperature,*
  Physical Review C **68**, 034310 (2003).
- T. Frick, H. Müther, A. Rios, A. Polls and A. Ramos,
  *Correlations in hot asymmetric nuclear matter,*
  Physical Review C **71**, 014313 (2005).
- A. Rios, A. Polls and H. Müther,
  *Sum rules of single-particle spectral functions in hot asymmetric nuclear matter,*
  Physical Review C **73**, 024305 (2006).