Short-Range Structure of Nuclei*

- Tensor forces and ground-state structure
- Observing tensor correlations: (e,e′pp) and (e,e′np), ...
- Observing pp short-range correlations: Coulomb sum rule
- Summary

* Dedicated to the memory of

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Correlations in Nuclei

Outstanding features of v_{ij}:

- short-range repulsion

- tensor character (from OPE)

These produce strongly anisotropic femtometer structures in T=0,S=1 channel in all nuclei:

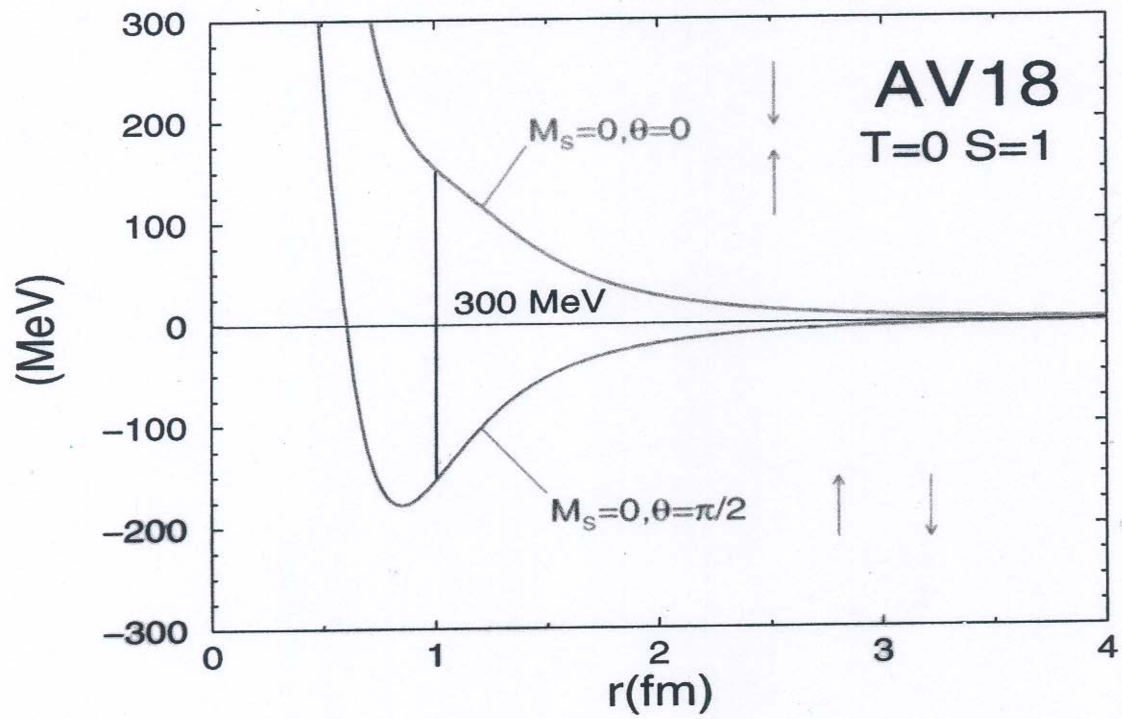
$$\rho^{M_S}_{T=0,S=1}(\mathbf{r}) \;\propto\; \rho^{M_S}_{d}(\mathbf{r})$$
$$\rho^{M_S=0}_{T=0,S=1}(\mathbf{r}) \;\neq\; \rho^{M_S=\pm1}_{T=0,S=1}(\mathbf{r})$$

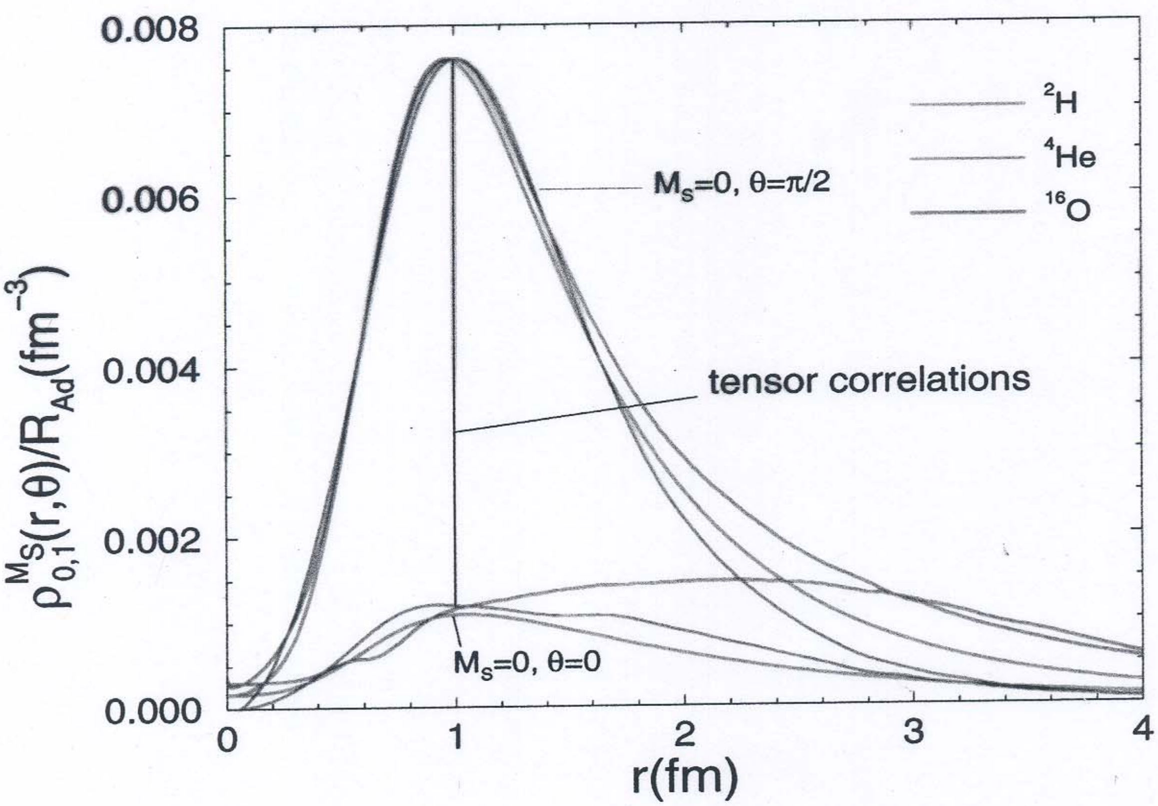
Two-nucleon density function:

$$\rho^{M_S}_{T,S}(\mathbf{r}) = \frac{1}{2J+1}\sum_{M_J}\langle JM_J|\sum_{i<j} P^{T,SM_S}_{ij}(\mathbf{r})|JM_J\rangle$$

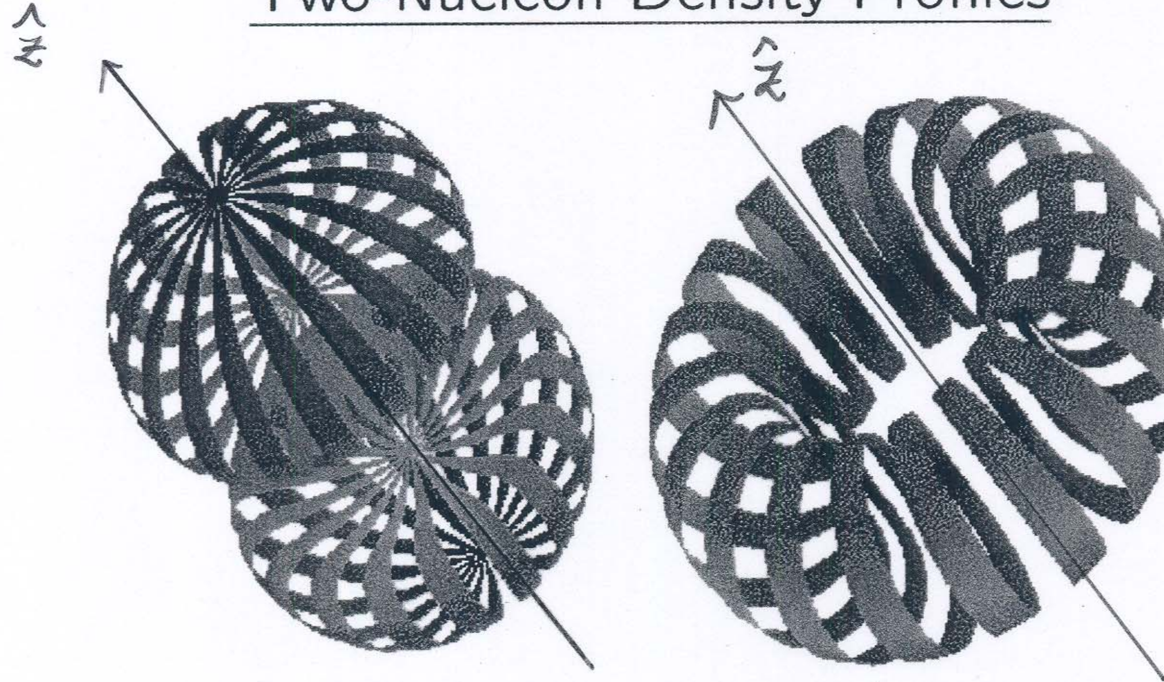
$$P^{T,SM_S}_{ij}(\mathbf{r}) \equiv \delta(\mathbf{r}-\mathbf{r}_{ij})\, P^{T}_{ij}\, |SM_S,ij\rangle\langle SM_S,ij|$$

COUPLING OF SPATIAL AND SPIN VARIABLES





Two-Nucleon Density Profiles



$M_S = \pm 1$ $\qquad\qquad$ $M_S = 0$

- Hole due to short-range repulsion

- Angular confinement due to tensor force

- Size of torus: $d \simeq 1.4$ fm
  $t \simeq 0.9$ fm

- At small separation, np relative w.f. in a nucleus $\propto$ deuteron w.f. (Levinger and Bethe conjecture)

- $\langle O \rangle_A \simeq R \langle O \rangle_d$, where O is any short-range operator effective in the $T = 0, S = 1$ channel

Scaling

	R	$\langle v^\pi \rangle_A / \langle v^\pi \rangle_d$	$\sigma_A^\pi / \sigma_d^\pi$	$\sigma_A^\gamma / \sigma_d^\gamma$
^{3}He	2.0	2.1	2.4(1)	$\simeq 2$
^{4}He	4.7	5.1	4.3(6)	$\simeq 4$
^{6}Li	6.3	6.3		
^{7}Li	7.2	7.8		$\simeq 6.5(5)$

Evidence for Tensor Correlations in ^{2}H

- Deuteron is a special case:

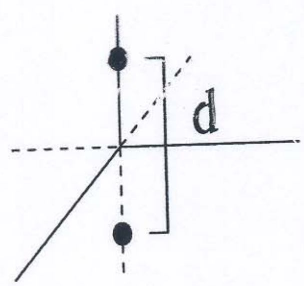
$$\rho^{M_S}_{T=0,S=1}(\mathbf{r})|_d \propto \rho^M_d(\mathbf{r}' = \mathbf{r}/2)$$

where $\rho^M_d(\mathbf{r}')$ is the one-nucleon density

- $\rho^M_d(\mathbf{r}')$ "measured" in elastic e-scattering:

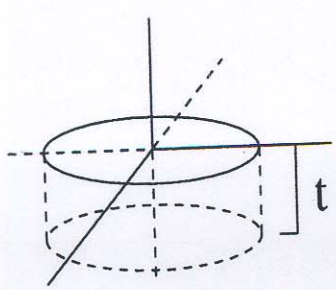
$$F_M(q) = \int d\mathbf{r}' e^{iqz'} \rho^M_d(\mathbf{r}')$$

$\rho_d^{M=\pm 1} \simeq$



$$F_{M=\pm 1}(q) = \cos\left(\frac{qd}{2}\right)$$

$\rho_d^{M=0} \simeq$



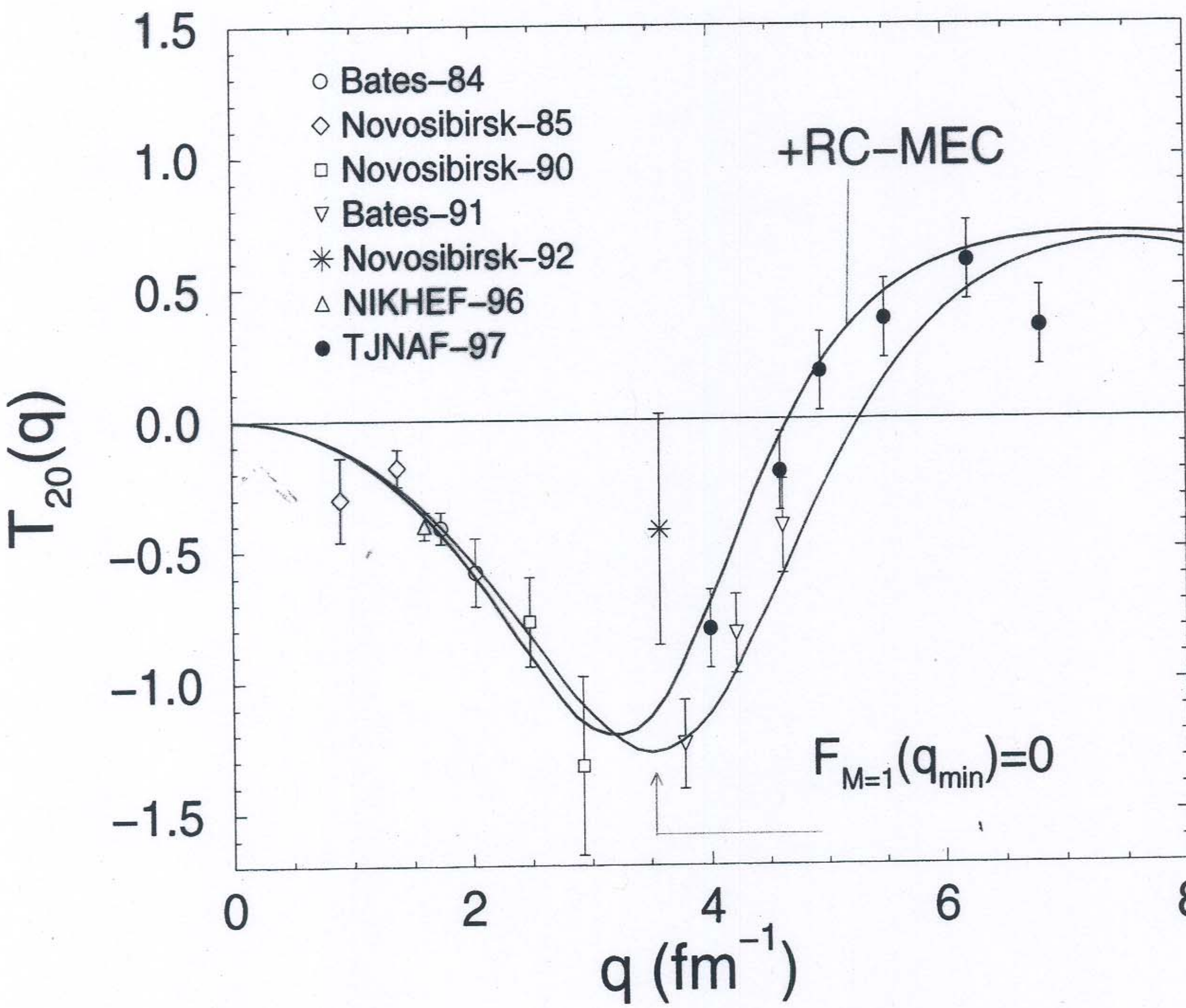
$$F_{M=0}(q) = \frac{\sin(qt/2)}{(qt/2)}$$

- Map out $M = 0$ and $M = \pm 1$ densities:

$$A(q) \simeq |F_{M=0}(q)|^2 + 2\,|F_{M=1}(q)|^2$$

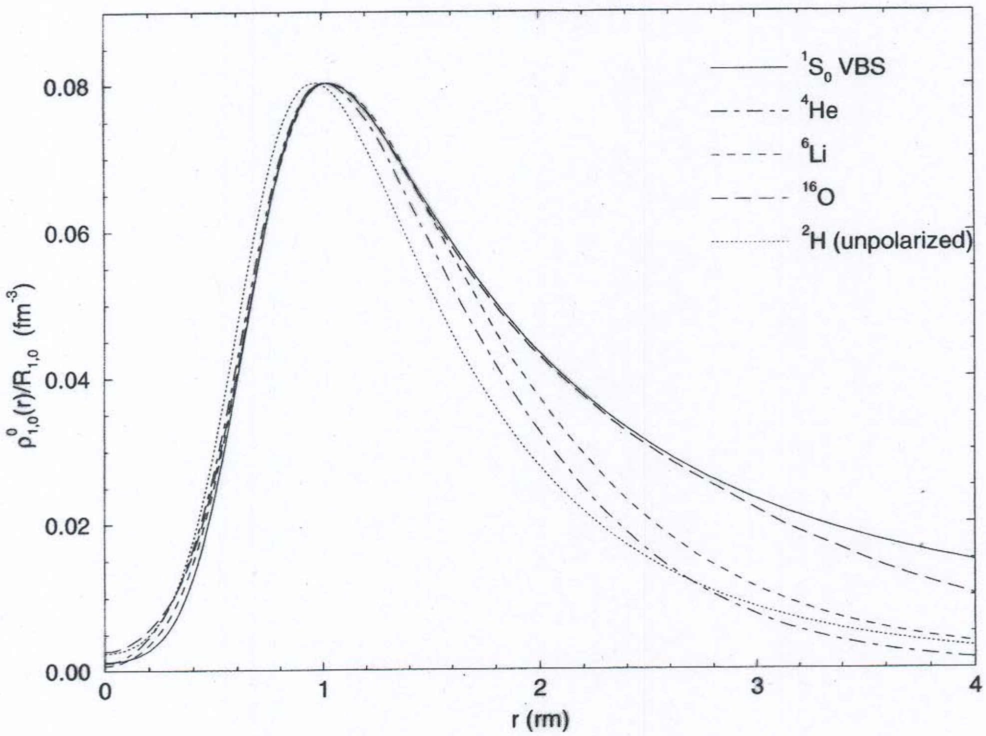
$$T_{20}(q) \simeq -\sqrt{2}\frac{|F_{M=0}(q)|^2 - |F_{M=1}(q)|^2}{|F_{M=0}(q)|^2 + 2\,|F_{M=1}(q)|^2}$$

- There are RC and MEC corrections, but gross features confirmed by experiment

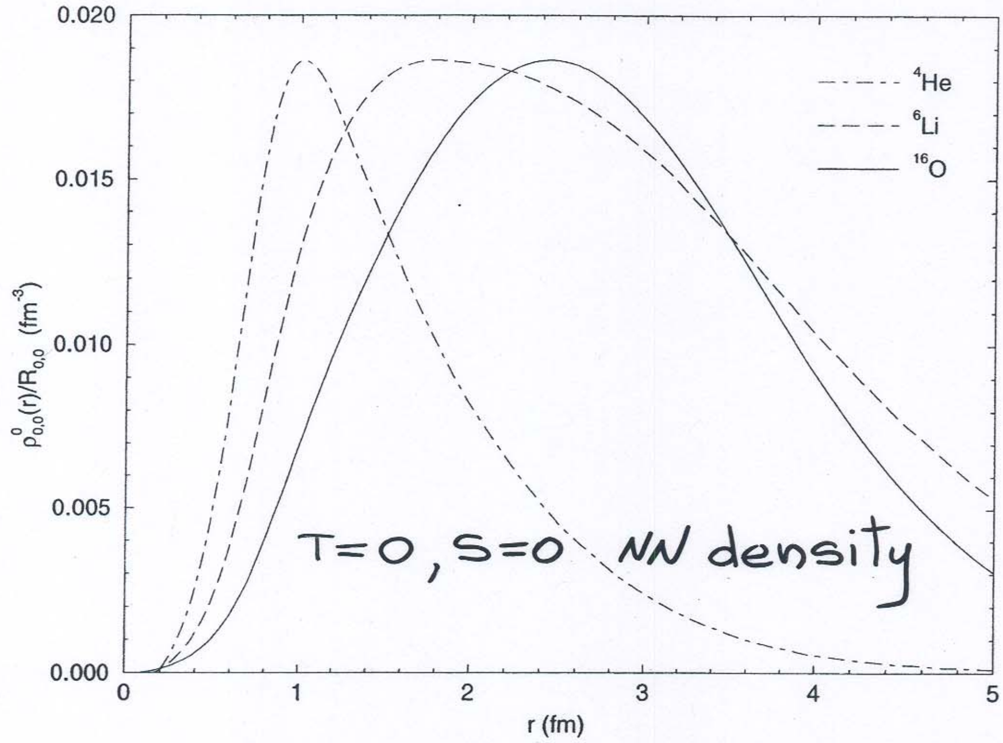


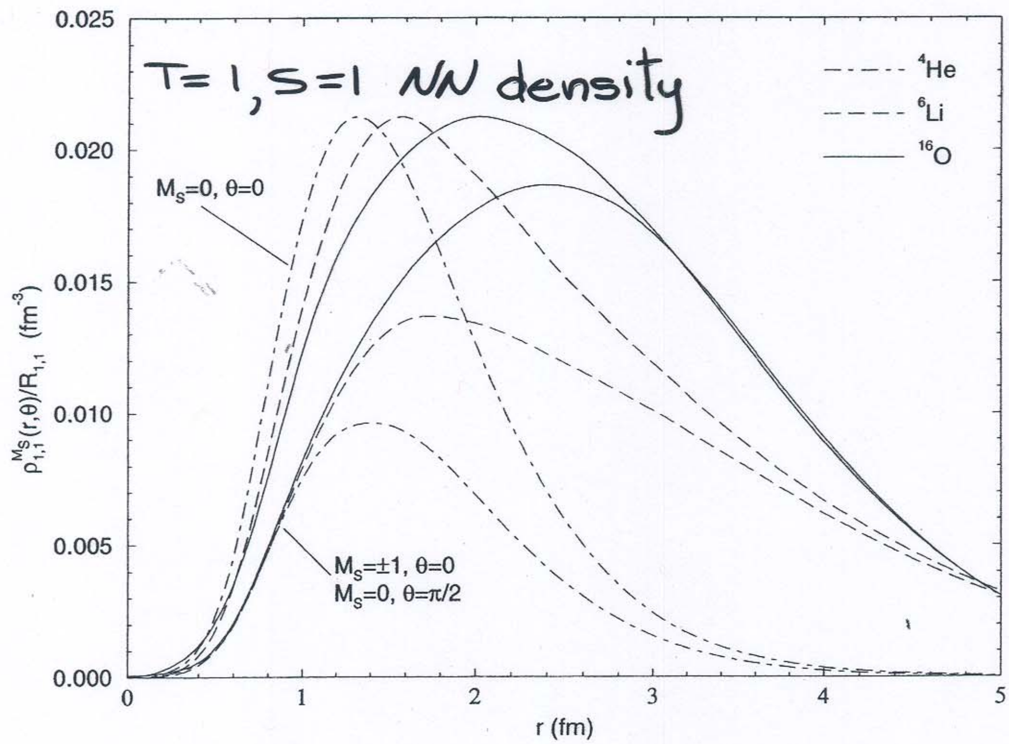
NN density distribution in $^1S_0$ state*

- it also scales in nuclei ($r \lesssim 2$ fm)



* Forest et al., PRC 54, 646 (1996)


0.020
0.015
0.010
0.005
0.000
0
1
2
3
4
5
r (fm)
$\rho^0_{0,0}(r)/R_{0,0}$ (fm^{-3})
^4He
^6Li
^{16}O
T=0, S=0 NN density


0.025
0.020
0.015
0.010
0.005
0.000
0
1
2
3
4
5
r (fm)
$\rho^{M_S}_{1,1}(r,\theta)/R_{1,1}$ (fm^{-3})
T=1, S=1 NN density
^4He
^6Li
^{16}O
M_S=0, θ=0
M_S=±1, θ=0
M_S=0, θ=π/2

Observing the short-range structure I

Tensor correlations strongly influence two-nucleon momentum distributions

$$\rho_{NN}(\vec{p}, \vec{P}) = \frac{A(A-1)}{2} \int d\vec{r}\,'_{12}\, d\vec{R}\,'_{12}\, d\vec{r}_{12}\, d\vec{R}_{12}\, d\vec{r}_{3} \cdots d\vec{r}_{A}$$

$$\times\, \psi^{\dagger}(\vec{r}\,'_{12}, \vec{R}\,'_{12}, \ldots)\, e^{-i\vec{p}\cdot(\vec{r}_{12}-\vec{r}\,'_{12})}\, e^{-i\vec{P}\cdot(\vec{R}_{12}-\vec{R}\,'_{12})}\, P_{NN}(12)\, \psi(\vec{r}_{12}, \vec{R}_{12}, \ldots)$$

probability to find two nucleons with relative momentum $\vec{p}$ and total momentum $\vec{P}$ in ground state

$$\int \frac{d\vec{p}}{(2\pi)^3} \frac{d\vec{P}}{(2\pi)^3}\, \rho_{NN}(\vec{p}, \vec{P}) = \begin{cases} Z(Z-1)/2 & pp \text{ pairs} \\ Z(A-Z) & np \text{ pairs} \end{cases}$$

ρ_{NN} can be calculated exactly with QMC

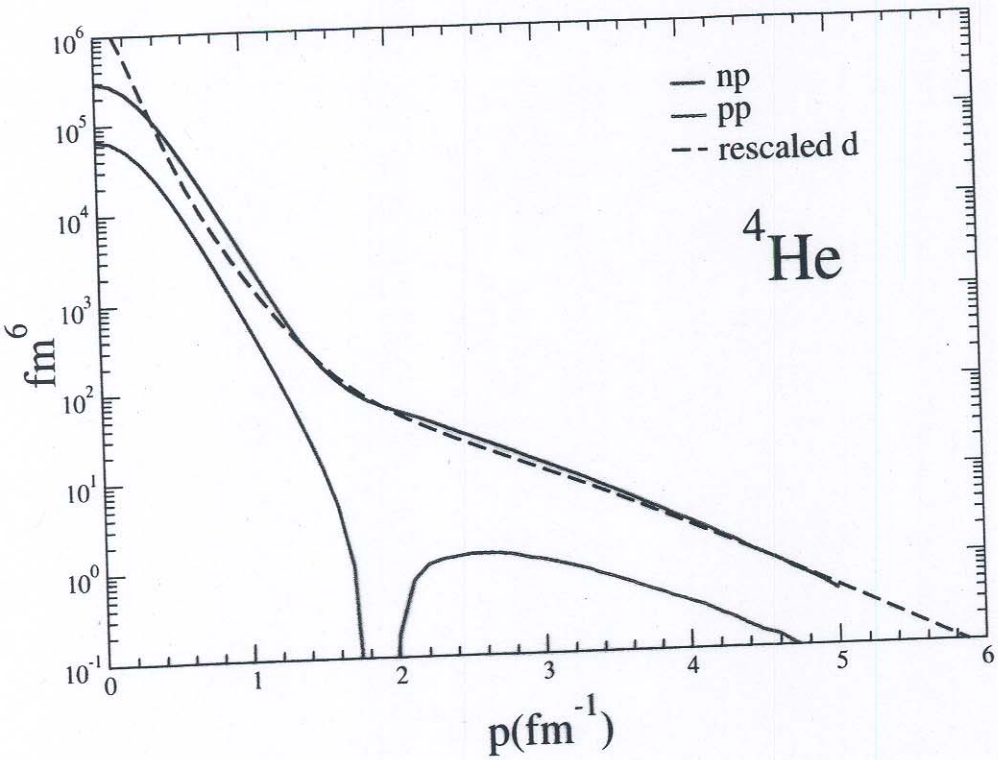
np pairs predominantly in $T=0$ $^3S_1 - {}^3D_1$ state (deuteron-like); pp pairs predominantly in $T=1$ $^1S_0$ state

$\downarrow$

this fact produces large difference between ρ_{np} and ρ_{pp}

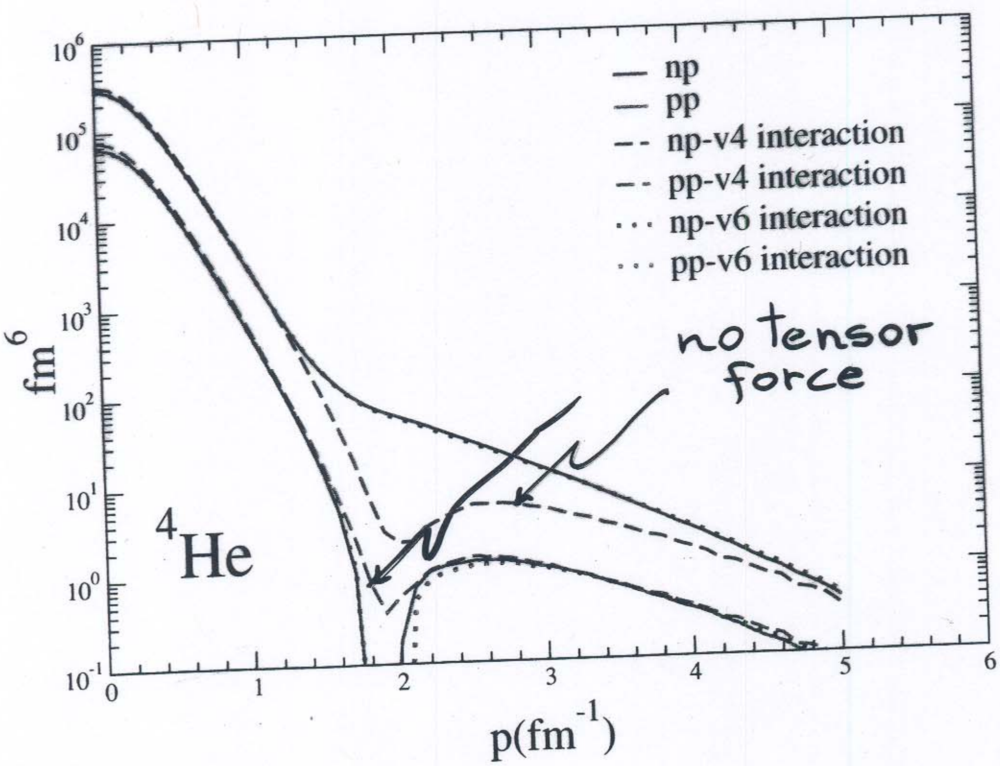
"seen" in $A(e,e'np)$ and $A(e,e'pp)$ (back-to-back kinematics)

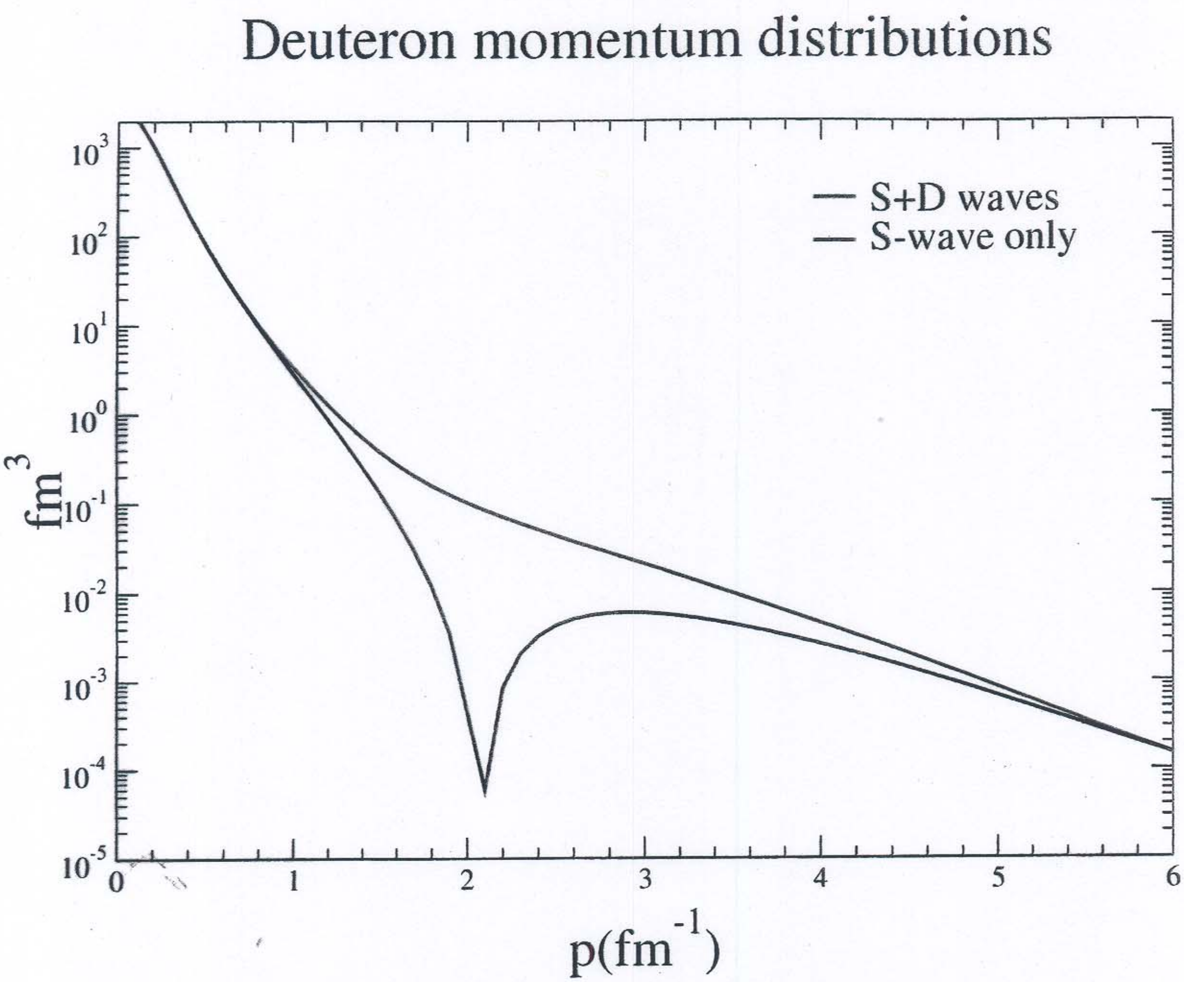
NN momentum distributions at P=0

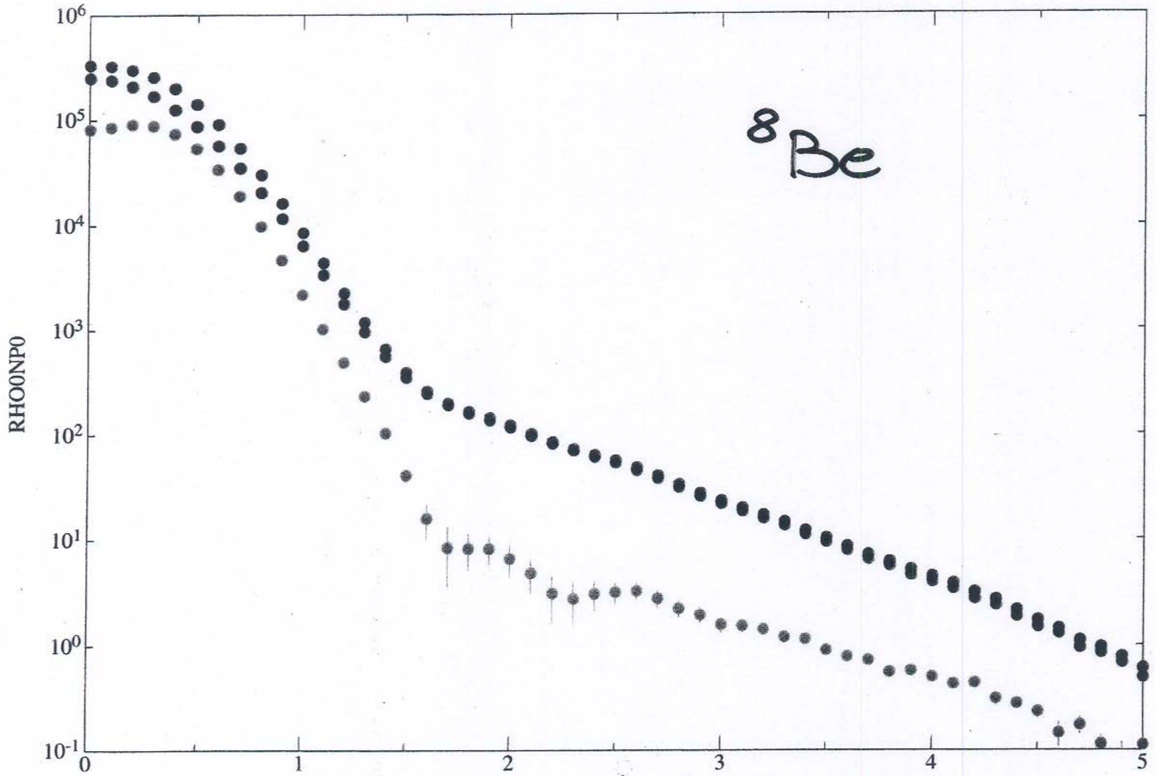


t low p np/pp ~ # of np pairs/(# of pp pairs) = 4

NN momentum distributions at P=0

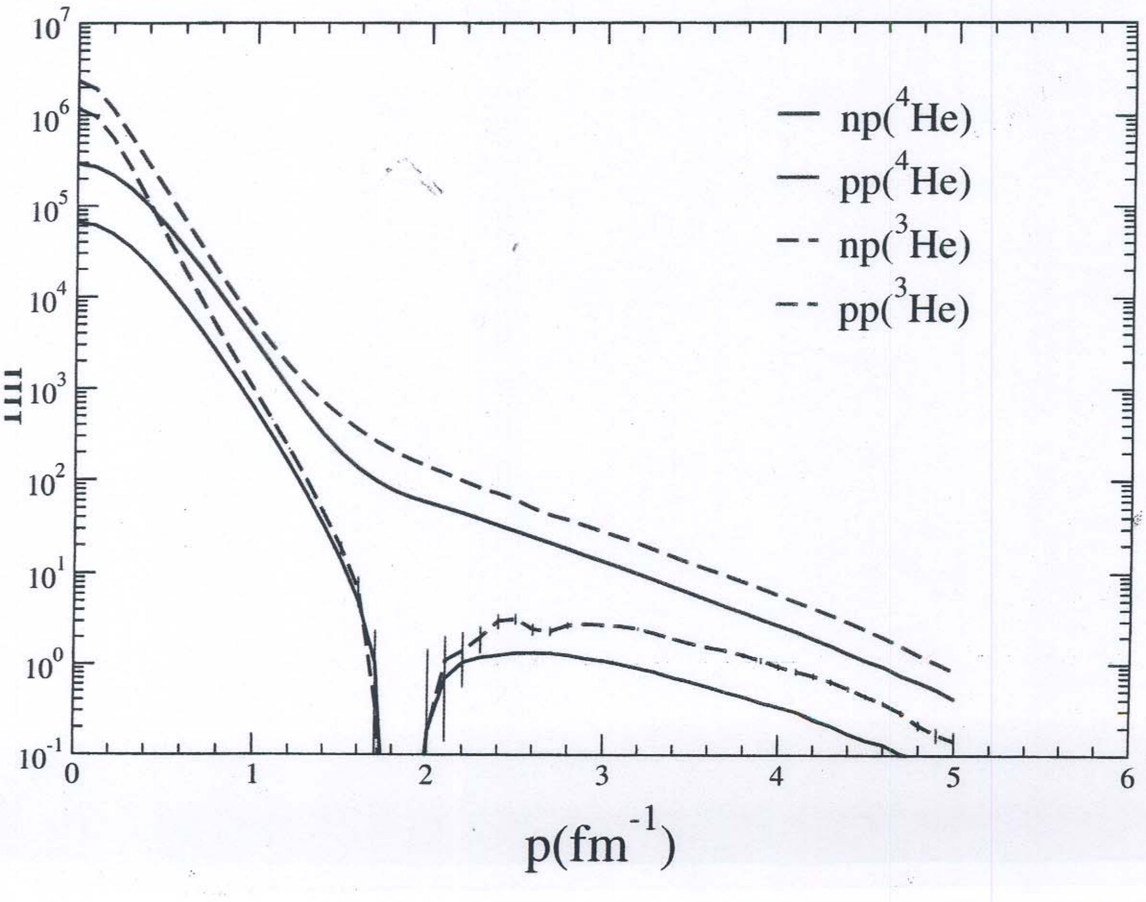



Deuteron momentum distributions
— S+D waves
— S-wave only
fm^3
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
1
2
3
4
5
6
p(fm^-1)



persistent feature in nuclei

NN momentum distributions at P=0



Observing the short-range structure II

Short-range and tensor correlations influence cluster amplitudes:

$d\vec{p}$ in $^3\vec{He}$, $\vec{d}\vec{d}$ in ^4He, ...

$$A_{ab}(M_a, M_b, M; \underbrace{\vec{r}_{ab}}_{\text{intercluster separation}}) = \langle \mathcal{A}[\underbrace{\psi_{a,M_a}\,\psi_{b,M_b}}_{\text{cluster ground states}}, \vec{r}_{ab}] \,|\, \underbrace{\psi_M}_{\text{ground state}} \rangle$$

$$= c_0 R_0(r_{ab}) Y_{00} + c_2 R_2(r_{ab}) Y_{2M_L}(\hat{r}_{ab})$$

only S- and D-wave allowed

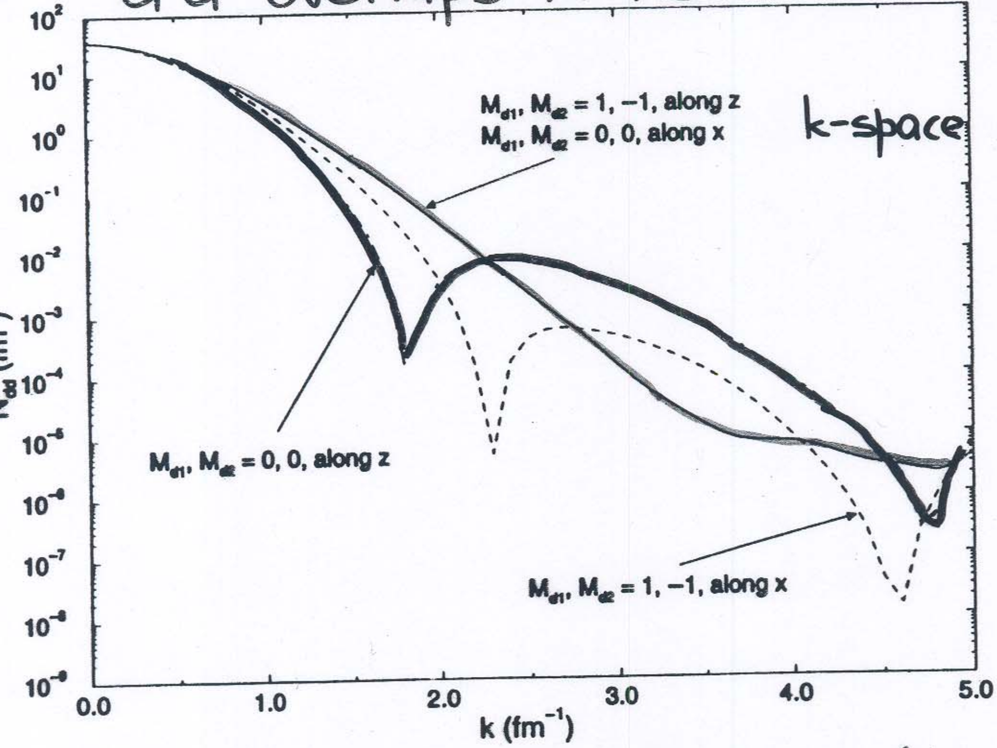
In PWIA x-section $\vec{A}(e, e'\vec{a})b$

$$\sigma \propto |\tilde{A}_{ab}(M_a, M_b, M; \vec{k})|^2$$

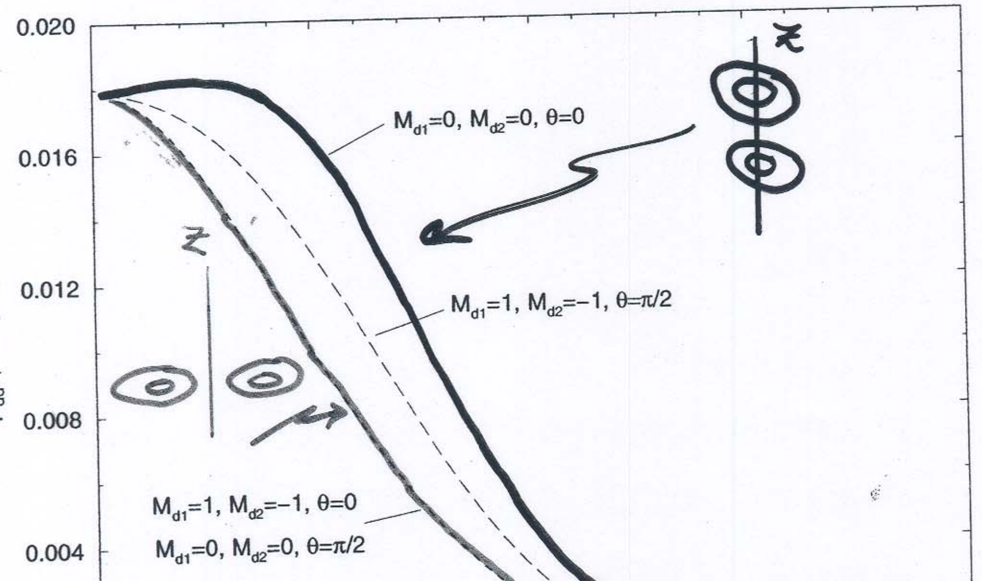
$\vec{k}$: missing momentum

experimentally accessible" (FSI, MEC, ...)

$\vec{d}\,\vec{d}$ overlaps in ^{4}He



large asymmetries expected in $^4He(\vec{e},e'\vec{d})d$



influenced by $\vec{d}$ structure: $|\rho_{dd}(\vec{r})|^2$ larger corresponding to most efficient "packing"

Coulomb Sum Rule

$$S_L(q) = \frac{1}{Z}\int_{\omega_{th}}^{+\infty} d\omega\ R_L(q,\omega)/G_{Ep}^2(q,\omega) \xrightarrow[q \gtrsim 500\ \mathrm{MeV}/c]{} 1$$

measured by (e,e')
for $\omega < q$

small ($\lesssim 10\%$) contribution to $S_L(q)$ in time-like region ($\omega > q$) estimated via:

i) model calculations of $R_L(q,\omega)$

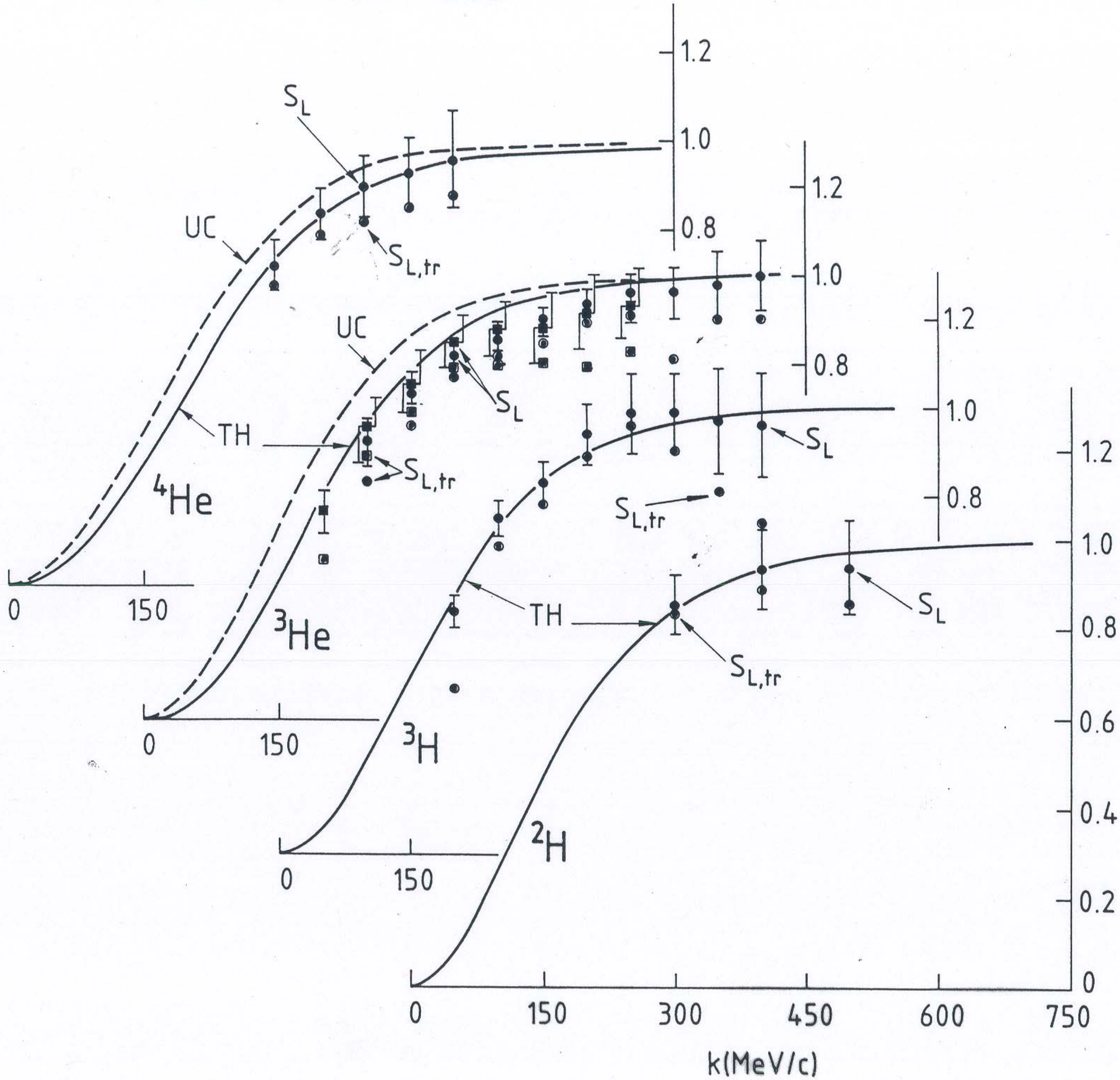
ii) energy-weighted sum rules,

$$W_L(q) = \frac{1}{Z}\int_{\omega_{th}}^{+\infty} d\omega \cdot \omega \cdot R_L(q,\omega)/G_{Ep}^2(q,\omega)$$

$$= \frac{1}{2Z}\langle 0 | \left[\rho_L^\dagger(\vec{q}), \left[H, \rho_L(\vec{q}) \right] \right] | 0 \rangle$$

can be accurately calculated

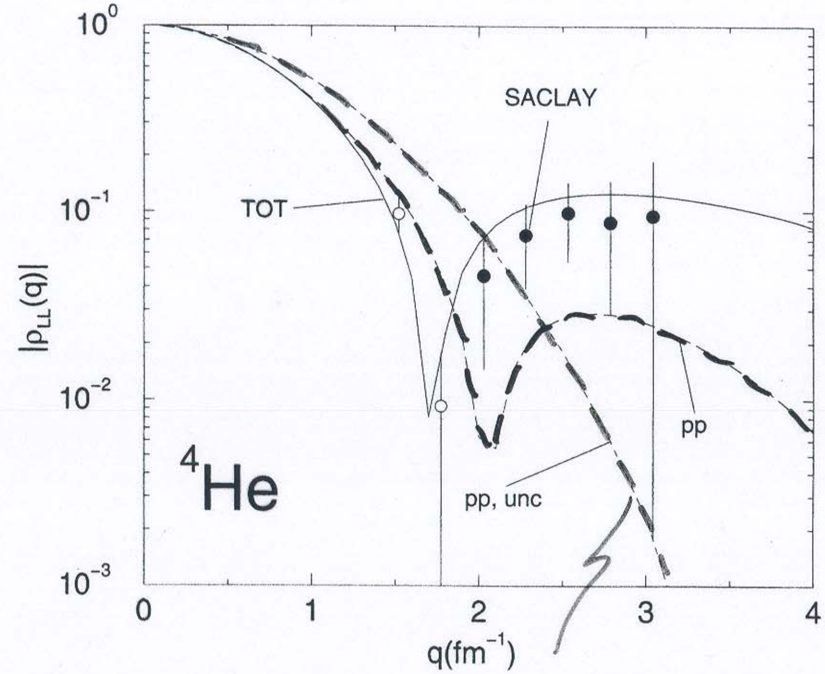
COULOMB SUM RULE



Short-range correlations

$$\rho_{LL}(q) = S_L(q) - 1 + Z\,|F_c(q)|^2$$ charge f.f.

$$= \frac{1}{Z}\langle 0|\rho_L^{\dagger}(\vec{q})\,\rho_L(\vec{q})|0\rangle - 1$$

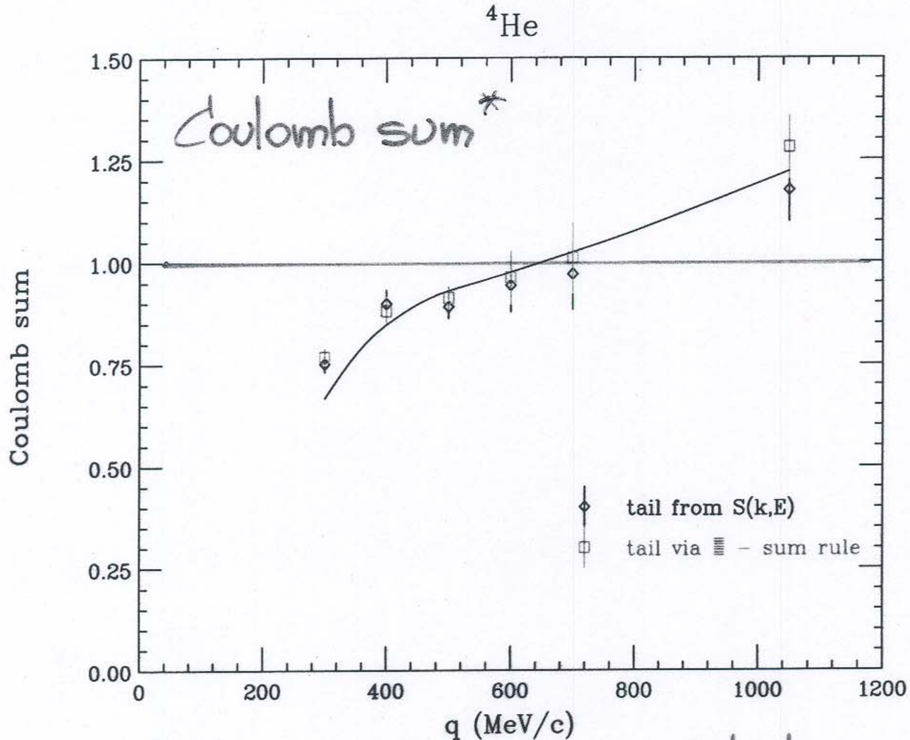


uncorrelated pp

$$\rho_L(\vec{q}) = \sum_i e^{i\vec{q}\cdot\vec{r}_i}\,\frac{1+\tau_{z,i}}{2} + RC$$

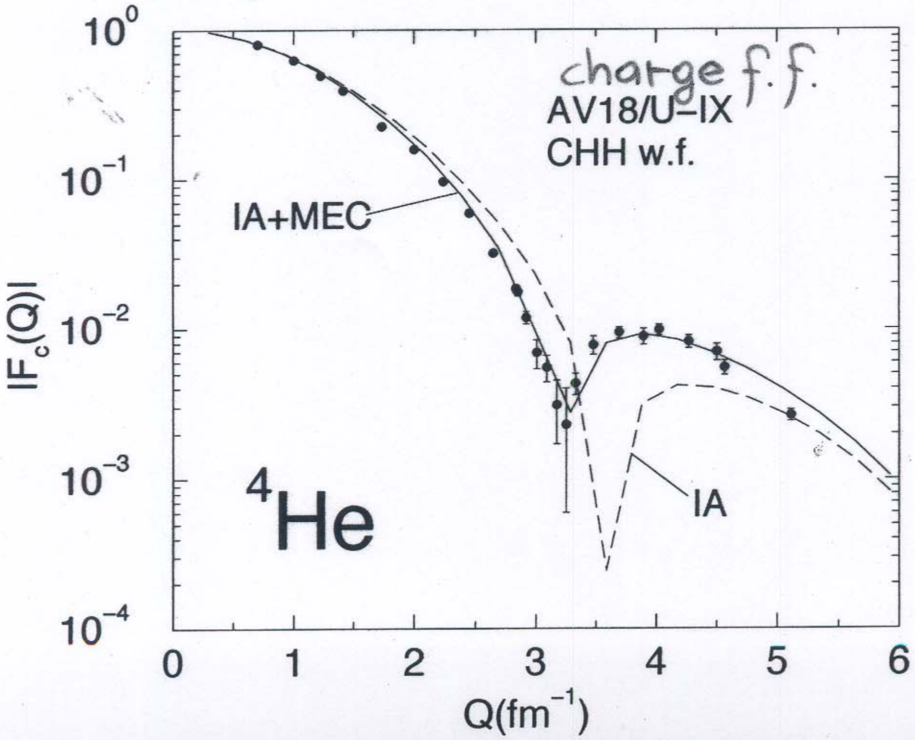
if these were negligible, then

$$\rho_{LL}(q) = (Z-1)\int d\vec{r}\; e^{-i\vec{q}\cdot\vec{r}}\,\rho_{pp}(r)$$ pp distribution function



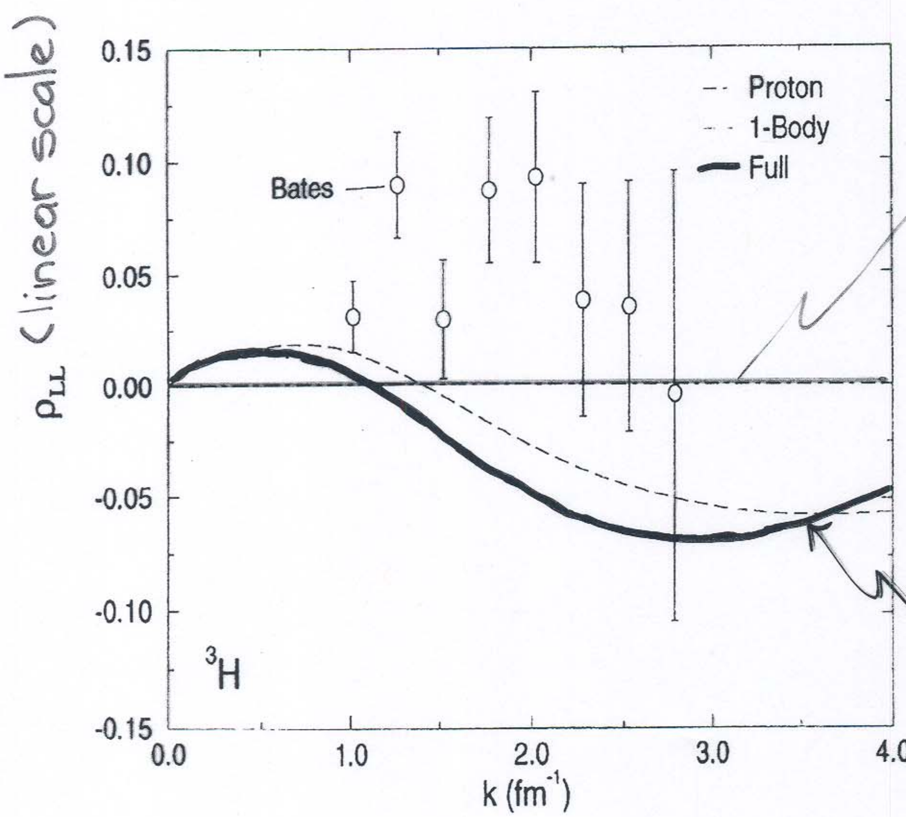
$$S_L = \frac{1}{Z}\int d\omega \, \frac{R_L}{\tilde{G}_E^2} \quad \text{note normalization} \qquad S_L^{\text{1-body}} \underset{q \text{ large}}{\sim} 1$$

$$\tilde{G}_E^2 = \left[G_{Ep}^2 + \frac{(A-Z)}{Z} G_{En}^2 \right] / (1+\tau)$$



* Carlson, Jourdan, Schiavilla, and Sick, PLB 553 (2003)

An interesting case: ^3H*



$\rho_{LL}(q) = 0$ if RC in charge operator ignored

RC contributions in charge operator

FIG. 3. Same as in Fig. 1 but for ^{3}H.

* Schiavilla, Wiringa, and Carlson, PRL 70, 3856 (1993)

Summary

- Tensor correlations strongly influence (e,e'NN) x-sections

$$\sigma(e,e'pn) \gg \sigma(e,e'pp)$$

They also produce large asymmetries in $(e,e'\vec{p})$ and $(e,e'\vec{d})$ knock-out processes

- Coulomb sum rule is a "clean" probe of pp short-range correlations ("contaminations" from MEC, etc. under control)

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