Proton Structure and Atomic Physics

Carl E. Carlson

The College of William and Mary in Virginia

Spin Structure at Long Distance

JLab, 12 March 2009

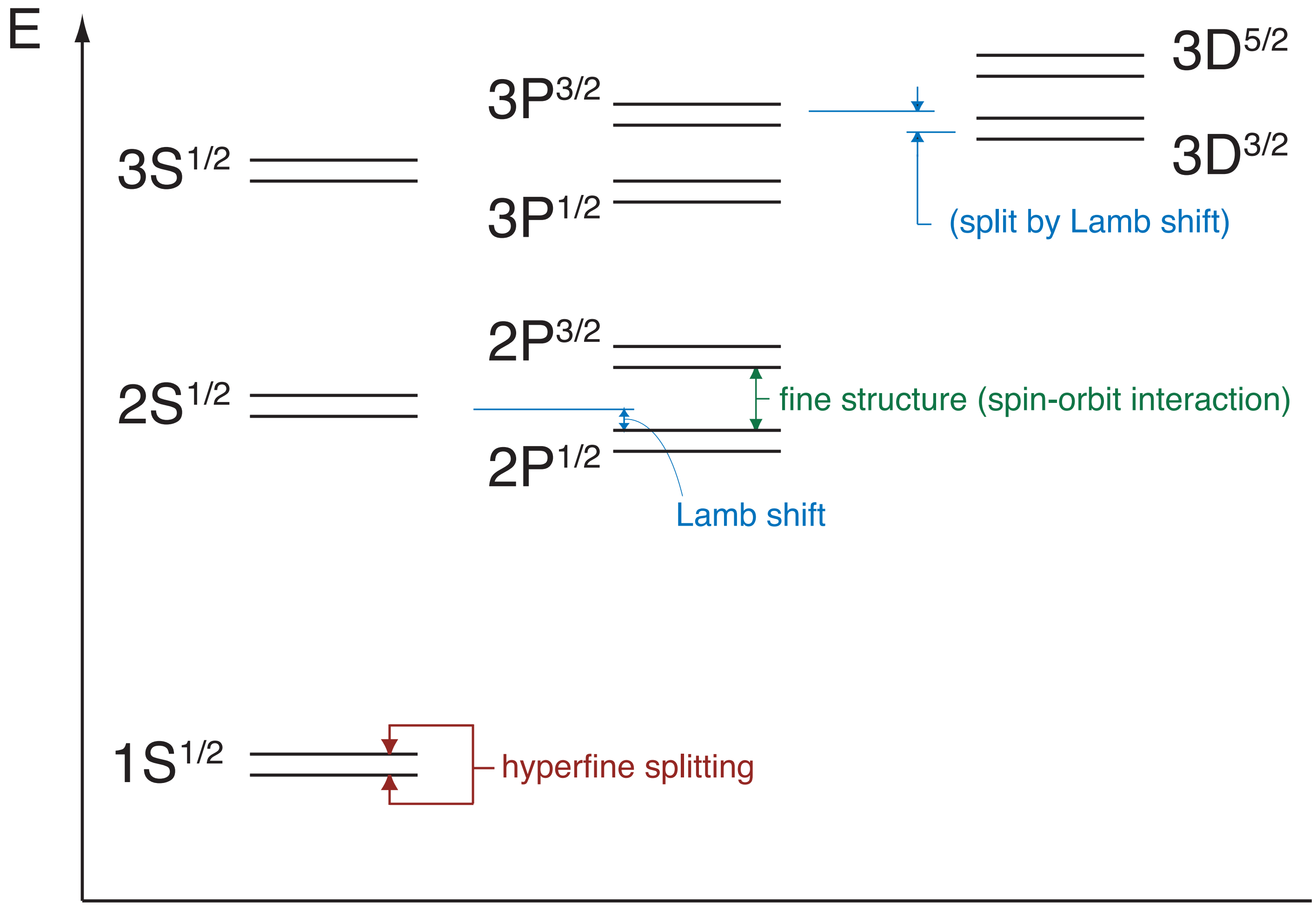
original parts done with Vahagn Nazaryan and Keith Griffioen

PRL 96, 163001 (2006) and PRA 78, 022517 (2008)

Introduction

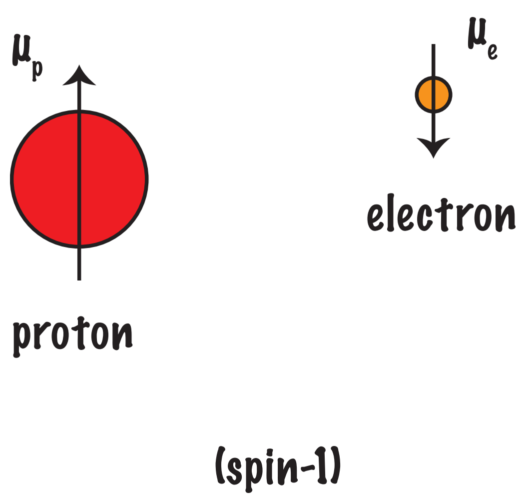
- General Subject: Proton structure effects upon precision atomic calculations.
- One direction: Proton structure needs to be understood and its effects included to calculation atomic quantities to part-per-million (ppm) level
- Reverse direction: Precise atomic measurements can constrain or even determine hadronic quantities
- Specific subject for most of this talk: Proton structure and the hydrogen hyperfine energy splitting to ppm level.

Just in case:
Hydrogen energy levels

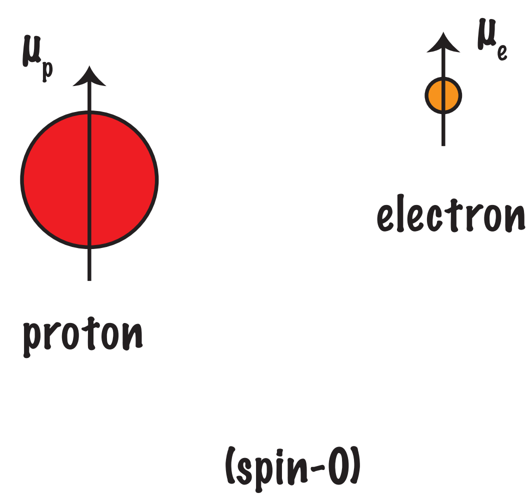


Introduction

- In spatial ground state, spin-dependent magnetic interaction gives hyperfine splitting.



← slightly higher energy than →



- Splitting known to 13 figures in frequency units,

$$E_{hfs}(e^- p) = 1\,420.405\,751\,766\,7\ (9)\ \mathrm{MHz}$$

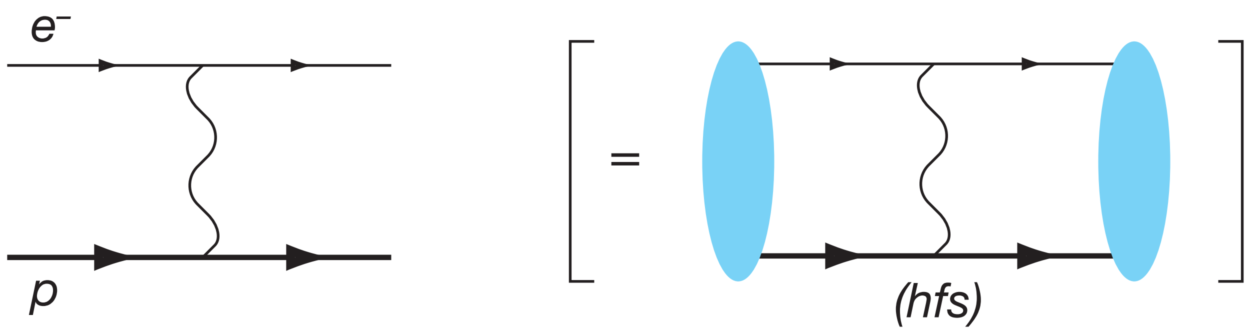
- Goal: Calculate hfs to part per million (ppm)

Introduction

- Why part per million (ppm) calculation?
  - Challenge ...
  - New physics?
    - Note: Hints of new physics in B-meson physics (BEACH 2008: Conference on Hyperons, Charm, and Beauty Hadrons)
  - Was several ppm discrepancy circa 2006
- Note: pure QED systems (e.g., muonium) easily allow ppm calculation and better. Problem is hadronic corrections --- proton structure.

Lowest order: "Fermi energy"

- Lowest order calculation can be and often is done in NR quantum mechanics course:



- LO result is "Fermi energy,"

$$E_F^p = \frac{8\alpha^3 m_r^3}{3\pi}\mu_B\mu_p = \frac{16\alpha^2}{3}\frac{\mu_p}{\mu_B}\frac{R_\infty}{(1+m_e/m_p)^3}$$

- Convention: measured μ_p for proton, and Bohr magneton μ_B for electron.

First worry: are constants well enough known to calculate lowest order to ppm or better?

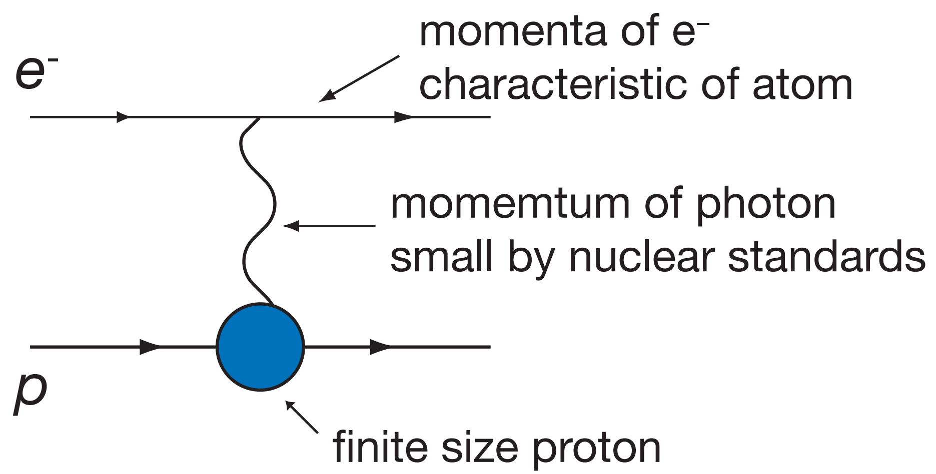
- A: Yes. Can calculate Fermi energy to 10 ppb:

$$E_F^p = \frac{8\alpha^3 m_r^3}{3\pi}\mu_B\mu_p = \frac{16\alpha^2}{3}\frac{\mu_p}{\mu_B}\frac{R_\infty}{(1+m_e/m_p)^3}$$

  - R_∞ is Rydberg constant in Hertz (6.6 ppt)
  - m_e/m_p known to ppb
  - α known to 1/2 ppb
  - μ_p/μ_B known to 10 ppb
  - Hence E_F^p known to 10 ppb level

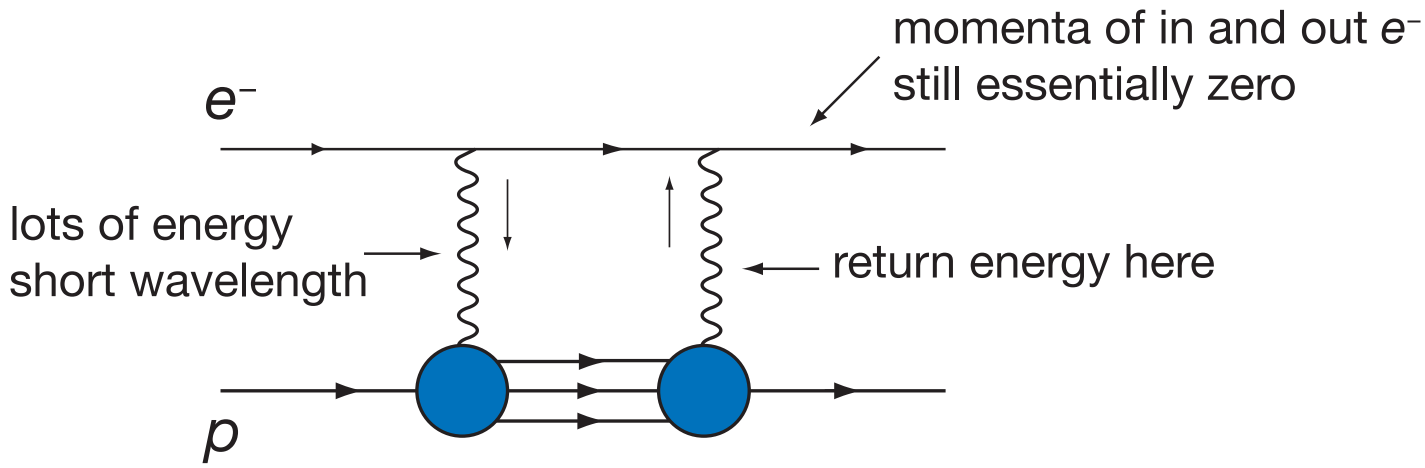
Effects of proton structure

- Proton size about 10^{-5} Ångström---enough to notice
- But not in one photon exchange:



- Fermi momentum of bound electron is order $m_e\alpha$, so Q^2 of exchanged photon is order $(m_e\alpha)^2$. Proton form factor doesn't notice until ppt level.
- Hence not mentioned in first year quantum course

Two-photon exchange



- short wavelength photon sees inside proton---effect depends on proton structure
- Inter-proton intermediate state may be proton or may be excited (inelastic) states

Corrections -- notation

$$E_{\rm hfs}(\ell^- p) = \left(1 + \Delta_{\rm QED} + \Delta^p_{\rm hvp} + \Delta^p_{\mu{\rm vp}} + \Delta^p_{\rm weak} + \Delta_{\rm S}\right) E_F$$

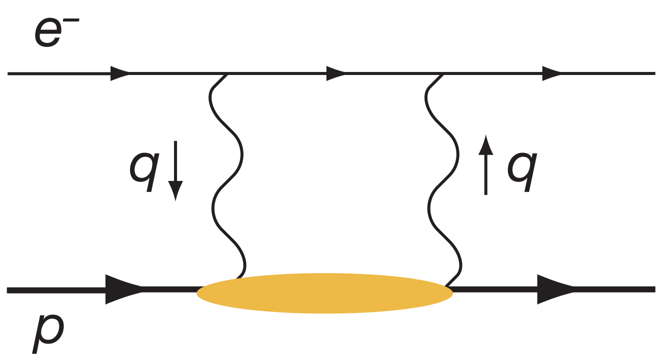
- Δ_{QED} : pure QED, well calculated
- Δ_{hvp}, $\Delta_{\mu vp}$, Δ_{weak} : some vacuum polarization terms and Z-boson exchange: small, not a problem
- Wanted here: $\Delta_S = \Delta_Z + \Delta_R + \Delta_{pol}$
  - Proton structure corrections
  - Names: Zemach, recoil, & polarizability terms
  - all 2-photon exchange

Commentary

- Δ_S (total) will be about 40 ppm, so need ca. 2% accuracy
- What we do
  - Use data from electron scattering to measure proton structure
  - Calculate proton structure effects on HHFS from results of these measurements
- What we don't do
  - We don't start from scratch, using QCD Lagrangian, or facsimile, to calculate proton structure correction. Not now possible to reach target precision calculating ab initio.
  - Cf., Chiral Lagrangian calculation by Pineda (2003) gets about 2/3 target Δ_S; or about 13 ppm accuracy

Calculation

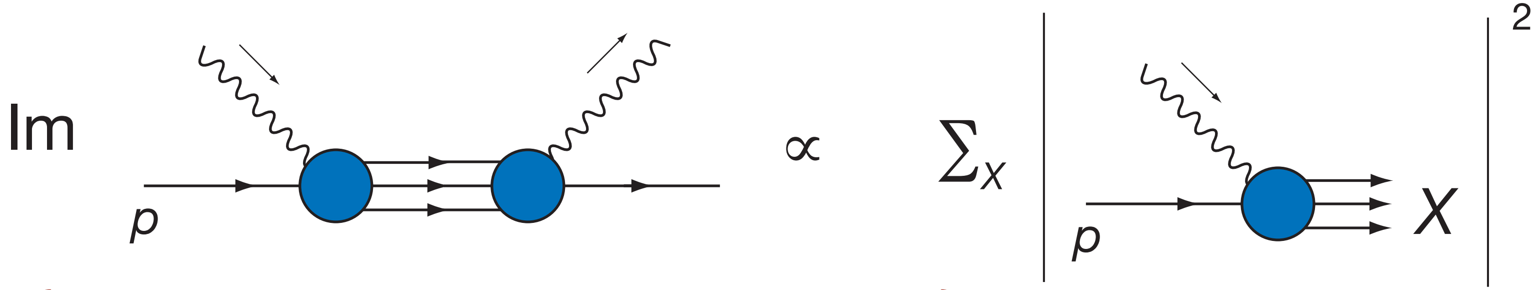
- Want



- Don't know lower line (forward off-shell Compton scattering). Note particularly that inter-proton states are not generally on shell.
- But imaginary part of diagram comes from case when intermediate electron and inter-proton states are on-shell. Can get real part by Cauchy integral formula (dispersion relation).

Optical theorem

- I.e., for lower part of diagram



$$\text{Im}\left\{\text{forward scattering amplitude}\right\} \propto \text{total cross section}$$

- RHS is cross section for e + p → e′ + X
- Codified in terms of form factors F_1, F_2 for elastic part and in terms of structure functions for inelastic part. For HFS calculation only need spin dependent g_1, g_2.
- Measured at SLAC, DESY, JLab, Mainz, ...

Will quote results--first some comments

- Forward Compton amplitude (with photon off-shell) depends on variables, ν and Q^2. Do dispersion relation in ν.
- Use unsubtracted dispersion relation
  - Depends on amplitudes falling to zero fast enough as $|\nu| \to \infty$.
  - Seems o.k. from Regge analysis of amplitudes
  - Seems o.k. from test calculations in QED
  - Correlates with "$g_p(\infty)$" = 0 from Sandorfi's talk.

Tip of the hat to the experimenters

- Jefferson Lab (Newport News, VA, USA) experiment EG1 measured spin-dependent inelastic electron-proton scattering
- $Q^2 > 0.045$ GeV^2 (earlier SLAC expt. had $Q^2 > 0.15$ GeV^2)
- Results in terms of structure functions g_i
- For reference,

Aerial view of accelerator and experimental halls

$$\frac{d\sigma_{\rightarrow\rightarrow}}{dE'\,d\Omega} - \frac{d\sigma_{\rightarrow\leftarrow}}{dE'\,d\Omega} = \frac{8\alpha^2 E'}{m_p Q^2 E}\left(\frac{E + E'\cos\theta}{m_p \nu} g_1 + \frac{Q^2}{m_p \nu^2} g_2\right)$$

$$\frac{d\sigma_{\rightarrow\uparrow}}{dE'\,d\Omega} - \frac{d\sigma_{\rightarrow\downarrow}}{dE'\,d\Omega} = \frac{8\alpha^2 E'^2}{m_p^2 Q^2 E \nu} \sin\theta \left(g_1 - \frac{2E}{\nu} g_2\right)$$

Results for structure dep. corr. Δ_S

- Recall $\Delta_S = \Delta_Z + \Delta_R + \Delta_{pol}$
- Zemach term Δ_Z is NR part of elastic contribution,

$$\Delta_Z = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa_p} - 1 \right] \equiv -2\alpha m_r r_Z$$

- Charles Zemach, 1956
- r_Z is "Zemach radius"; m_r is reduced mass

More formula results

- Recoil term Δ_R: relativistic part of elastic contribution (plus extra term to be explained)

$$
\begin{aligned}
\Delta_R^p &= \frac{2\alpha m_r}{\pi m_p^2}\int_0^\infty dQ\, F_2(Q^2)\frac{G_M(Q^2)}{1+\kappa_p} \\
&+ \frac{\alpha m_\ell m_p}{2(1+\kappa_p)\pi(m_p^2-m_\ell^2)}\left\{\int_0^\infty \frac{dQ^2}{Q^2}\left(\frac{\beta_1(\tau_p)-4\sqrt{\tau_p}}{\tau_p}-\frac{\beta_1(\tau_\ell)-4\sqrt{\tau_\ell}}{\tau_\ell}\right)F_1(Q^2)G_M(Q^2)\right. \\
&\qquad\left. +3\int_0^\infty \frac{dQ^2}{Q^2}\Big(\beta_2(\tau_p)-\beta_2(\tau_\ell)\Big)F_2(Q^2)G_M(Q^2)\right\} \\
&- \frac{\alpha m_\ell}{2(1+\kappa_p)\pi m_p}\int_0^\infty \frac{dQ^2}{Q^2}\,\beta_1(\tau_\ell)F_2^2(Q^2)
\end{aligned}
$$

- $\beta_{1,2}$ on next page; $\tau_i \equiv Q^2/4m_i^2$
- Memorize the last term

- Polarizability terms are inelastic terms with one elastic term added, and given as

$$\Delta_{\text{pol}} = \frac{\alpha m_\ell}{2(1+\kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$$

(the prefactor is about 1/4 ppm for electrons)

$$\Delta_1 = \int_0^\infty \frac{dQ^2}{Q^2}\left\{\beta_1(\tau_\ell)F_2^2(Q^2) + \frac{8m_p^2}{Q^2}\int_0^{x_{th}} dx\, \frac{x^2\beta_1(\tau) - (m_\ell^2/m_p^2)\beta_1(\tau_\ell)}{x^2 - m_\ell^2/m_p^2} g_1(x,Q^2)\right\}$$

$$\Delta_2 = -24 m_p^2 \int_0^\infty \frac{dQ^2}{Q^4}\int_0^{x_{th}} dx\, \frac{x^2\left[\beta_2(\tau) - \beta_2(\tau_\ell)\right]}{x^2 - m_\ell^2/m_p^2}\, g_2(x,Q^2)$$

with

$$\begin{aligned} \tau &= \nu^2/Q^2 \\ \beta_1(\tau) &= -3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)} \\ \beta_2(\tau) &= 1 + 2\tau - 2\sqrt{\tau(\tau+1)} \end{aligned}$$

- Massless lepton: Drell and Sullivan and others, 1960 and early 1970s
- Massive lepton: Faustov, Cherednikova, and Martynenko, 2003; Us, 2008.

Comments

- Why did the F_2^2 term included in polarizability?

  Ans: It makes Δ_1 finite in the massless lepton limit

$$\Delta_1 \quad = \quad \int_0^\infty \frac{dQ^2}{Q^2} \left\{ \frac{9}{4} F_2^2(Q^2) + 4m_p \int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2} \beta_1(\tau) g_1(\nu, Q^2) \right\}$$

  ($Q^2 \to 0$ limit of β_1 is 9/4)

- GDH sum rule states:

$$4m_p \int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2} g_1(\nu, 0) = -\kappa_p^2$$

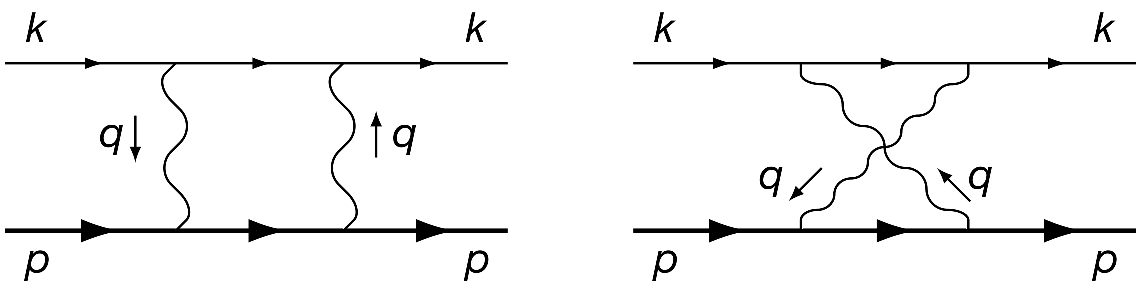
  Hence second integral by itself divergent at $Q^2 = 0$ endpoint. The F_2^2 term cancels the divergence.

- Convenience: All terms finite for $m_\ell \neq 0$

- And convention: F_2^2 multiplied by any convergent function $f(Q^2)$ with $f(0)=1$ would still work.

Not how to do calculation in 2009

- For elastic scattering only, might consider boxes



and put in photon-proton-proton vertices given by

$$\Gamma_\mu = \gamma_\mu F_1(q^2) + \frac{i}{2m_p}\sigma_{\mu\nu}q^\nu F_2(q^2)$$

- But: don't know F_1, F_2 because one proton off shell.

- Can and has been done by Bodwin-Yennie (1988) and others. Gives Zemach term + the recoil term exactly as quoted here. (!) Reason: choice of F_2^2 term in Δ_1.

- Beware: don't mix elastic contributions from Bodwin-Yennie (i.e., as quoted here) with Δ_{pol} calculated separately, with possibly different choice of F_2^2 term.

Developments

- New since 2000:
  - g_1, g_2 data good enough to give non-zero Δ_{pol}
  - (Faustov & Martynenko, 2002)
- New since 2006:
  - Final data from JLab EG1 expt. published, with systematic errors.

    [Prok et al., PLB 672, 12-16 (2009)]
  - New fits to proton form factor data (Arrington-Sick, Arrington-Melnitchouk-Tjon)

    [Albeit new low-Q^2 G_E data from Mainz (J. Bernauer, unpub.) not yet incorporated]

Results for Δ_{pol} 2008

Term	Q^2 (GeV^2)	From	Value w/AMT F_2
Δ_1	[0, 0.0452]	F_2 & g_1	1.35(0.22)(0.87) ( )
	[0.0452, 20]	F_2	7.54 ( ) (0.23) ( )
		g_1	−0.14(0.21)(1.78)(0.68)
	[20, ∞]	F_2	0.00 ( ) (0.00) ( )
		g_1	0.11 ( ) ( ) (0.01)
total Δ_1			8.85(0.30)(2.67)(0.70)
Δ_2	[0, 0.0452]	g_2	−0.22 ( ) ( ) (0.22)
	[0.0452, 20]	g_2	−0.35 ( ) ( ) (0.35)
	[20, ∞]	g_2	0.00 ( ) ( ) (0.00)
total Δ_2			−0.57 ( ) ( ) (0.57)
$\Delta_1 + \Delta_2$			8.28(0.30)(2.67)(0.90)
Δ_{pol} (ppm)			1.88(0.07)(0.60)(0.20)

- errors (statistical)(systematic from data)(modeling)
- AMT = Form factors fit by Arrington, Melnitchouk, Tjon (2007)
- Quote polarizability correction as 1.88 ± 0.64 ppm
- compatible with Faustov-Martynenko (2002).

Overall results for ordinary hydrogen 2008 (current latest)

Quantity	value (ppm)	uncertainty (ppm)
$(E_{\text{hfs}}(e^- p)/E_F^p) - 1$	1 103.48	0.01
Δ_{QED}	1 136.19	0.00
$\Delta^p_{\mu\text{vp}} + \Delta^p_{\text{hvp}} + \Delta^p_{\text{weak}}$	0.14	
Δ_Z (using AMT)	−41.43	0.44
Δ^p_R (using AMT)	5.85	0.07
Δ_{pol} (this work, using AMT)	1.88	0.64
Total	1102.63	0.78
Deficit	0.85	0.78

HHFS ending and outlook

- Our 2008 result using 2001 EG1 data (out in '08, Prok et al., PLB 672, 12-16 (2009)):

$$\Delta_{\text{pol}} = 1.88 \pm 0.64 \text{ ppm}$$

- Table of non-zero results

Authors	Δ_{pol} (ppm)
Faustov & Martynenko (2002)	1.4 ± 0.6
Us (2006)	1.3 ± 0.3
Faustov, Gorbacheva, & Martynenko (2006)	2.2 ± 0.8
Us (2008)	1.88 ± 0.64

- (Faustov et al. don't use JLab data)
- Sum of all corrections now just under 1 ppm, or about 1 standard deviation, from data

Outlook

- Have come a long way since my 1987 QM course notes claim that best calculations had 30 ppm accuracy.
- Future:
  - Better form factor fits. Uncertainties in Zemach term not now trivial. Low Q^2 elastic FF important. New data from Mainz should have useful impact.
  - Improved measurements of proton charge radius from Lamb shift expts. (not yet mentioned). Currently 1% error. May reduce by factor 10 with Lamb shift measurements (PSI, 2009) in μ hydrogen.
  - Lower systematic error in g_1. Already exists (unpublished) EG4 data ($Q^2 > 0.015$ GeV^2 instead of 0.045 GeV^2).
  - g_2 measurements for proton. Hfs less sensitive to g_2, but g_2 measurements welcome, and perhaps forthcoming (e.g., "SANE" in Hall C or "g2p" in Hall A (JLab)). Especially like low Q^2 data.
- Thinkable to have 0.3 ppm uncertainty in some years.

Extra part -- Muonic hydrogen

- Muonic hydrogen HFS may be measured at PSI along side Lamb shift measurements.
- QED corrections about same as for electron, but structure dependent corrections, e.g.,

$$\Delta_Z = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{1+\kappa_p} - 1 \right] \equiv -2\alpha m_r r_Z$$

bigger by about m_μ/m_e.

Results for Δ_{pol} 2008 --- muonic hydrogen

Term	Q^2 (GeV^2)	From	Value w/AMT F_2
Δ_1	[0, 0.0452]	F_2 and g_1	0.86(0.17)(0.67) ( )
	[0.0452, 20]	F_2	6.77 ( ) (0.21) ( )
		g_1	0.18(0.18)(1.62)(0.64)
	[20, ∞]	F_2	0.00 ( ) (0.00) ( )
		g_1	0.11 ( ) ( ) (0.01)
total Δ_1			7.92(0.25)(2.30)(0.66)
Δ_2	[0, 0.0452]	g_2	−0.12 ( ) ( ) (0.12)
	[0.0452, 20]	g_2	−0.29 ( ) ( ) (0.29)
	[20, ∞]	g_2	−0.00 ( ) ( ) (0.00)
total Δ_2			−0.41 ( ) ( ) (0.41)
$\Delta_1 + \Delta_2$			7.51(0.25)(2.30)(0.77)
Δ_{pol} (ppm)			351.(12.)(107.)(36.)

Important note

- For $m_\ell \neq 0$, previoiusly published result (Cherednikova et al.) different from ours. Difference due to different treatment of F_2^2 terms in polarizability.

$$\Delta_1 = \int_0^\infty \frac{dQ^2}{Q^2} \left\{ \frac{9}{4} \beta_0(\tau_\ell) F_2^2(Q^2) + \text{rest same} \right\}$$

$$\beta_0(\tau_\ell) = 2\sqrt{\tau(\tau+1)} - 2\tau$$

- Perfectly o.k.: just use recoil term that matches F_2^2 term added to polarizability. Ours is tuned to old Bodwin-Yennie calculation of elastic terms.

- Using Bodwin-Yennie elastic terms with Cherednikova et al. polarizability requires further correction for μHFS

$$\Delta_{\text{pol}}(\text{corr.}) = \frac{\alpha m_r}{2(1+\kappa_p)\pi m_p} \int_0^\infty \frac{dQ^2}{Q^2} \left\{ \beta_1(\tau_\ell) - \frac{9}{4} \beta_0(\tau_\ell) F_2^2(Q^2) \right\} = -128 \text{ ppm}$$

- Mentioned because 2 uncorrected examples available

Inelastic/Elastic tidbit

* Why is inelastic contribution so small? A: It isn't. Some of it got moved, using the magic of the DHG sum rule. (Motive was to remove $\ln(m_e)$ terms from inelastic contributions.)
* The pure F_2^2 term in recoil correction came from Δ_{pol}.
* This term is −22.38 ppm (!)

(AMT)	term moved		term not moved
Zemach	−41.43	Zemach	−41.43
"Recoil"	5.85	Recoil	28.22
"Total elastic"	−35.58	Actual elastic	−13.21
Polarizability	1.88	Pure inelastic	−20.49
Total proton str.	−33.70	Total proton str.	−33.70

* I.e., Actual contribution of g_1 quite large.

The end

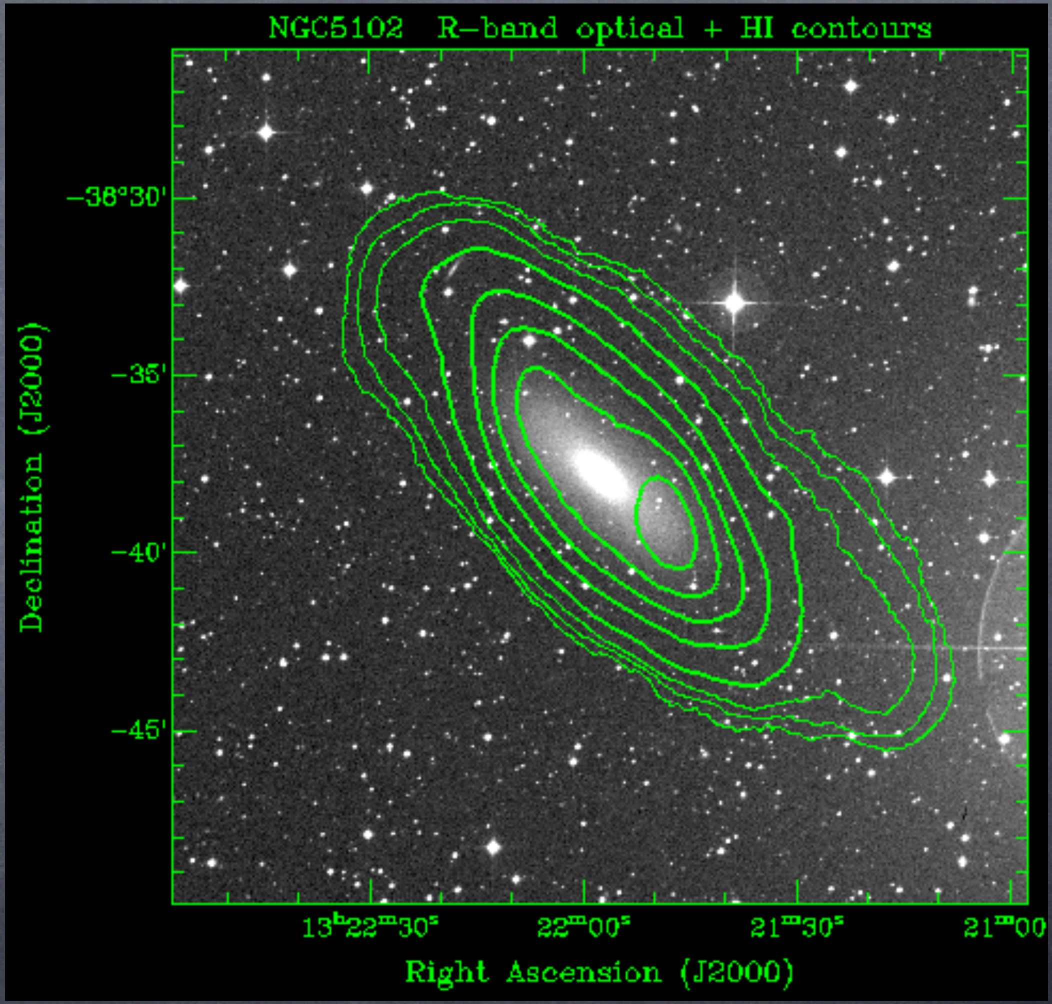
Extras

Just in case

British Columbia
Alberta
Saskatchewan
Manitoba
Canada
Ontario
Quebec
New Brunswick
Nova Scotia
Washington
Oregon
Idaho
Montana
Wyoming
North Dakota
South Dakota
Minnesota
Wisconsin
Michigan
Iowa
Nebraska
Nevada
California
Utah
Colorado
Kansas
Missouri
Illinois
Indiana
Ohio
Pennsylvania
New York
Vermont
New Hampshire
Maine
Massachusetts
Connecticut
Rhode Island
New Jersey
Delaware
Maryland
West Virginia
Virginia
Kentucky
Tennessee
North Carolina
South Carolina
Georgia
Alabama
Mississippi
Louisiana
Arkansas
Oklahoma
Texas
New Mexico
Arizona
Florida
U. S. A.
Mexico
The Bahamas
Russia
Alaska
Canada
Hawaii
Williamsburg
(or Newport News)

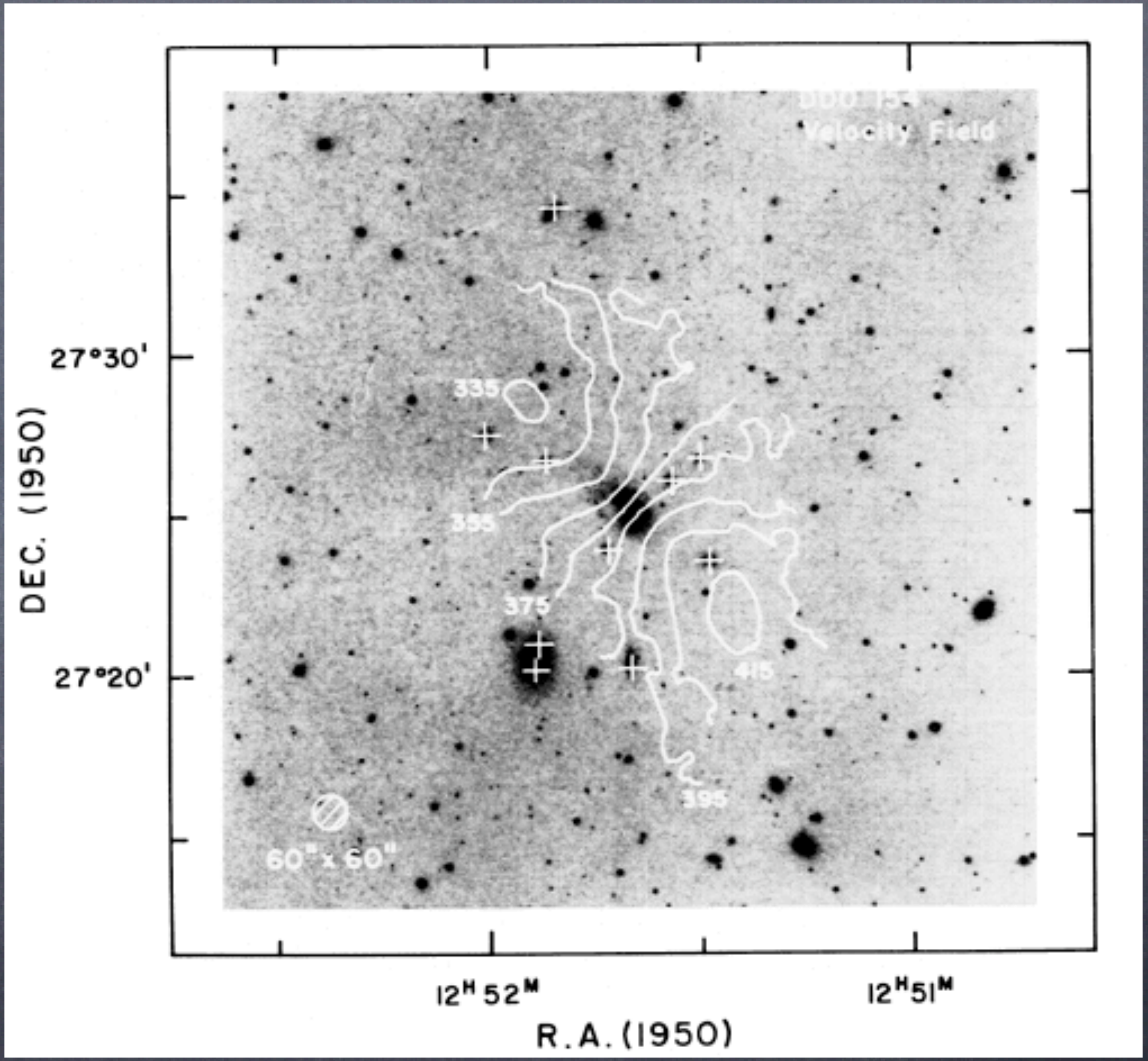
Extent of galaxies, seen in 21 cm radio light

- NGC 5102, Local Volume HI Survey
- Radio observations laid over optical photo
- 3X bigger in radio light



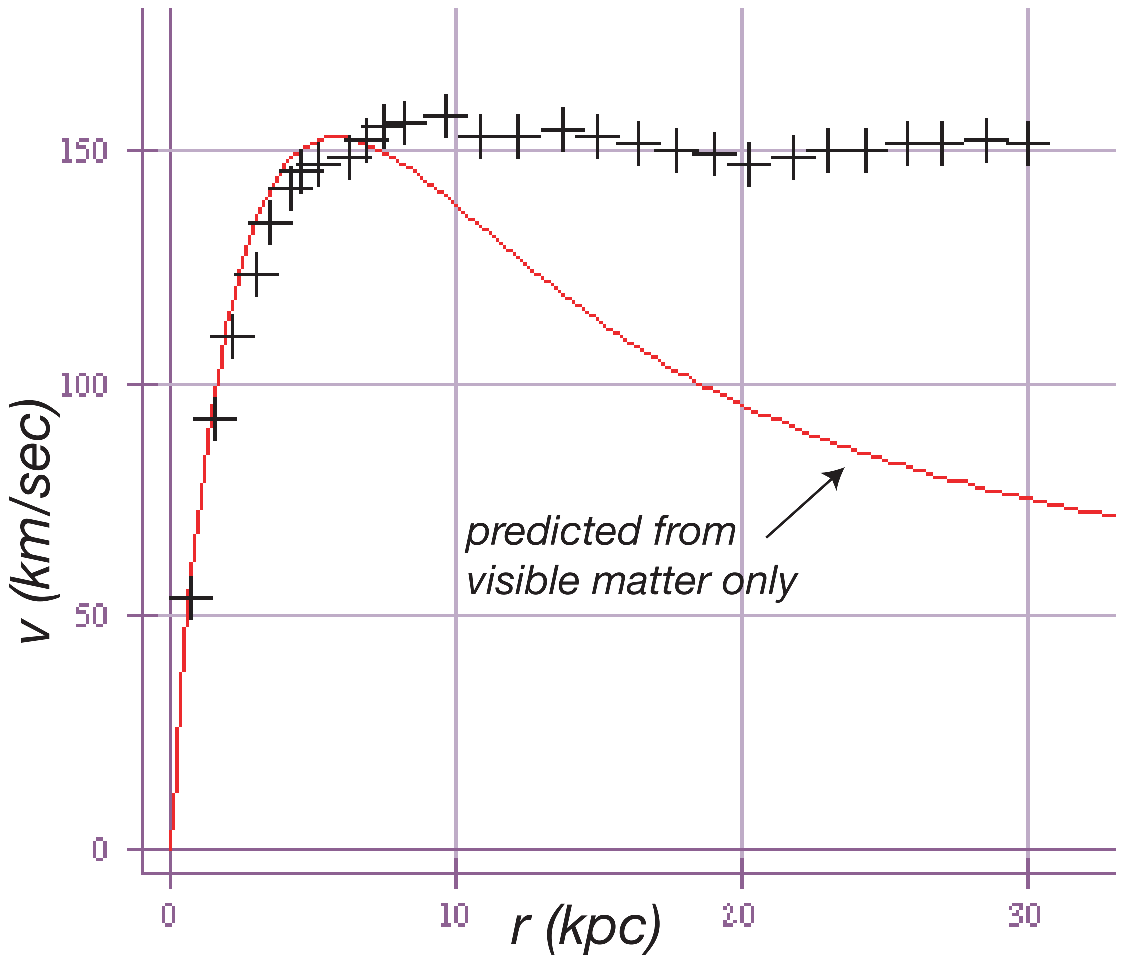
Velocity of H-gas, seen with 21 cm line

- DDO 154, Carignan et al.
- Numbers give velocities, in km/sec, from Doppler shift
- rotation curve



Sample rotation curve

- NGC 3198
- Typical of many
- Rotation curve shows need for extra (dark) matter---or change in gravitational force law at long distance

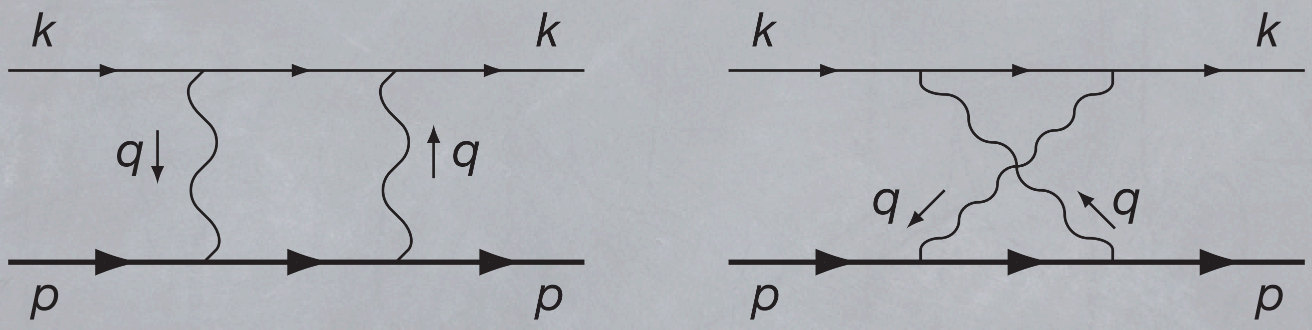


The visible NGC 3198

- Long history.
- Zemach (1956) calculates hfs from elastic contributions in terms of proton form factors.
- Iddings (1965), Drell and Sullivan (1967), deRafael (1971) calculate inelastic (polarizability) contribution to hydrogen hfs.
- Faustov and Martynenko (2002), using SLAC data, estimate numerically the polarizability contribution to hydrogen hfs. First to get result inconsistent with zero.
- Friar and Sick (2004) determine the Zemach radius [to be defined] using world form factor data.
- Dupays et al. (2003), Volotka et al. (2005), Brodsky et al. (2005) infer Zemach radius from hfs data using polarizability results of Faustov and Martynenko.
- Inconsistencies between last two called for a review of corrections.
- Newer data from JLab, esp. at lower Q^2, crucial for this purpose.

Final indelicate point

- Can we use the dispersion relation? Depends.
- E.g., do elastic box calculation



$$H_1^{el} = -\frac{2m_p}{\pi}\left(\frac{q^2 F_1(q^2) G_M(q^2)}{(q^2+i\epsilon)^2 - 4m_p^2\nu^2} + \frac{F_2^2(q^2)}{4m_p^2}\right)$$

- Pole at value of photon energy that makes the intermediate proton "real":

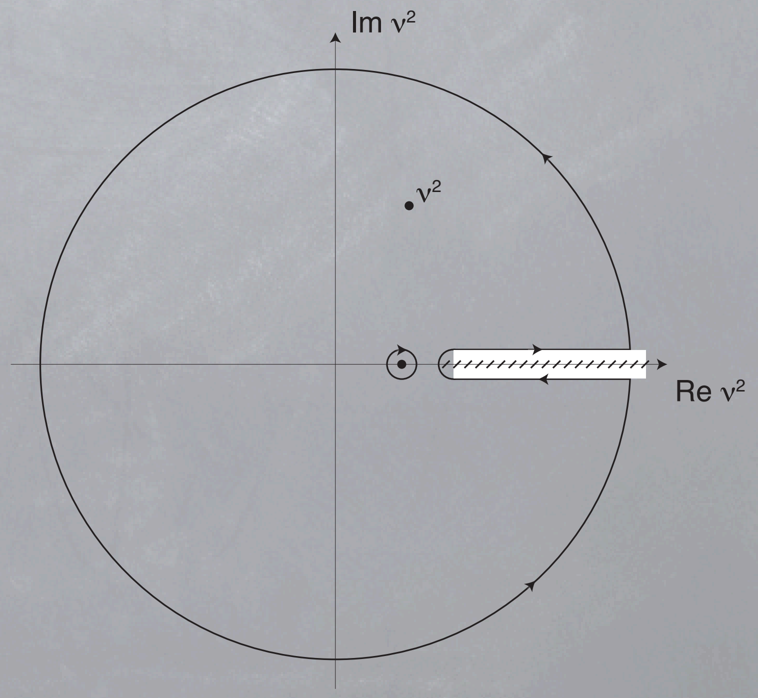
$$\nu = Q^2/(2m_p)$$

FIP

- Inelastic case similar. If total mass of intermediate state is W, pole at

$$\nu = (W^2 - m_p^2 + Q^2/(2m_p)$$

- W is continuously varying from threshold & up. Hence H_1 has elastic pole in ν plus cut,



pole/cut structure of H_1 in complex ν^2-plane

FIP

- In using Cauchy formula, pole and cut have been kept
- Infinite contour discarded: Legitimate if function falls to zero fast enough
- Fails for H_1^{el} alone, but we are dealing with composite particle
- QED models say it is o.k.
- Regge models say it is o.k.

Outlook

- Current best charge radius measurements come from Lamb shift, error 1% vs. 2% from electron scattering. Experiment "imminent" to do muonic hydrogen Lamb shift, with possible 0.1% accurate charge radius!

- New low-Q^2 G_E data from Mainz (J. Bernauer, unpub.,
    - shown at conferences, e.g. Walcher, ECT*, May 2008)

End