Spin sum rules
and
the strong coupling constant
at large distances

A. Deur
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Moments of spin structure functions and spin sum rules

Nth-moments: $\left\{ \begin{array}{l} \int g_1 x^{n-1} dx \\ \int g_2 x^{n-1} dx \end{array} \right.$

First moments: Γ_1, Γ_2

- ✲ Γ_1^N: $\left\{ \begin{array}{l} \text{Ellis-Jaffe sum rule (large } Q^2\text{)} \\ \text{Gerasimov-Drell-Hearn (GDH) sum rule (} Q^2=0\text{)} \end{array} \right.$
- ✲ Γ_1^{p-n}: Bjorken sum rule (large Q^2)
- ✲ Γ_2^N: Burkhardt–Cottingham (BC) sum rule (any Q^2)

- ✲ d_2 ”sum rule”
- ✲ Spin polarizability sum rules

} No low-x extrapolation

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- d_2 ”sum rule”
- Spin polarizability sum rules

} No low-x extrapolation

The generalized Bjorken Sum Rules

Bjorken sum rule (large Q^2):

$$\int g^p_1 - g^n_1 \, dx = \Gamma_1^{p-n} = \frac{1}{6} g_A \left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \frac{M^2}{9Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right] + \ldots$$

Nucleon triplet
axial charge
(Bjorken limit)

pQCD radiative
corrections

Higher
Twists
(+rad. corr.)

Fundamental test of the pQCD Q^2-evolution and OPE in the spin sector

Individual nucleon:

$$\int g_1^N dx = \left(\pm 12 g_A + \frac{a_8}{36}\right)\left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \frac{a_0}{9}\left(1 - \frac{\alpha_s}{\pi} - 1.10\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \text{Higher Twists}$$

Octet axial charge

singlet axial charge

(Assuming SU(3) symmetry and no strange quark polarization leads to the (violated) Ellis-Jaffe sum rule)

Here: $\overline{\text{MS}}$ (no gluon contribution to Γ_1) and a_0 is Q^2-independent

The generalized Gerasimov-Drell-Hearn sum

Original GDH sum rule ($Q^2 = 0$):

$$\boxed{\int_{\nu_{thr}}^{\infty} (\sigma_A - \sigma_P) \frac{d\nu}{\nu} = \frac{-4\alpha\pi^2 S\kappa^2}{M^2}}$$

σ_A, σ_P : photoproduction cross sections

κ: anomalous magnetic moment

S: Spin

Generalized GDH sum: $Q^2 > 0$:

photoproduction → electroproduction $\sigma_A - \sigma_P = f(g_1, g_2)$

One possible generalization:

$$\boxed{\frac{8}{Q^2}\int g_1 dx = S_1(0,Q^2)}$$

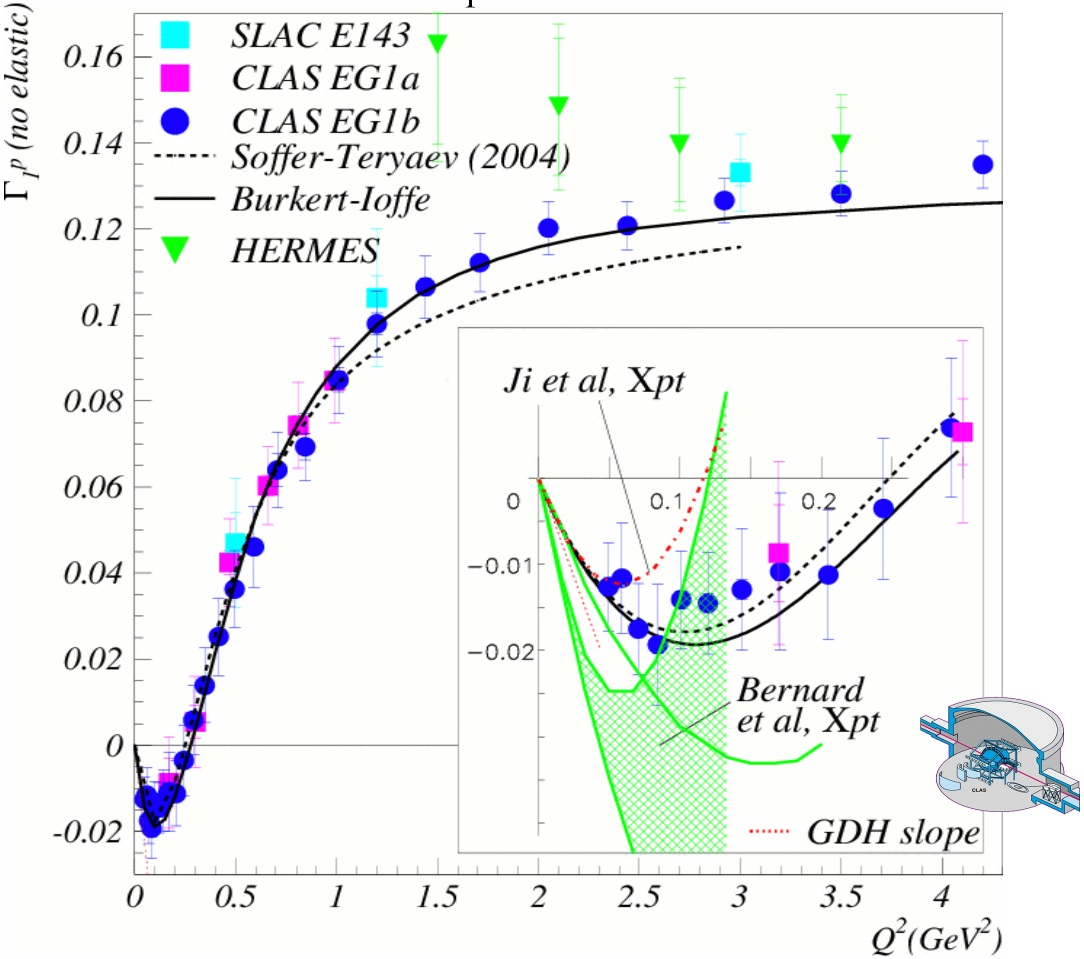
(Ji and Osborne, 1999)

$S_1(\nu,Q^2)$: spin dependent Compton amplitude

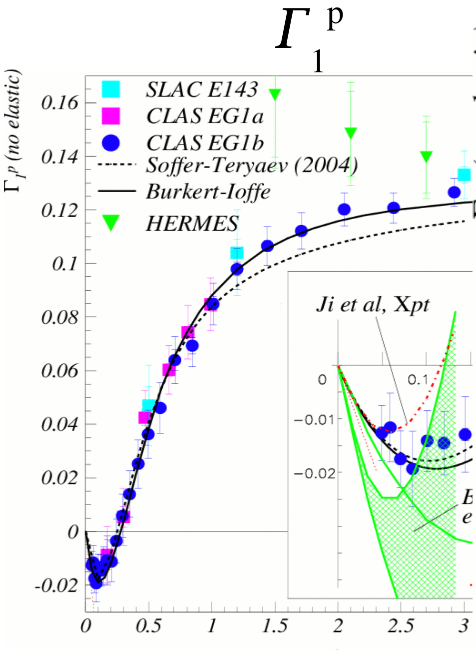
Connection allows to study the pQCD sum rules at low Q^2.

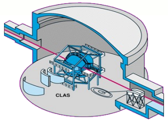
Results on sum rules

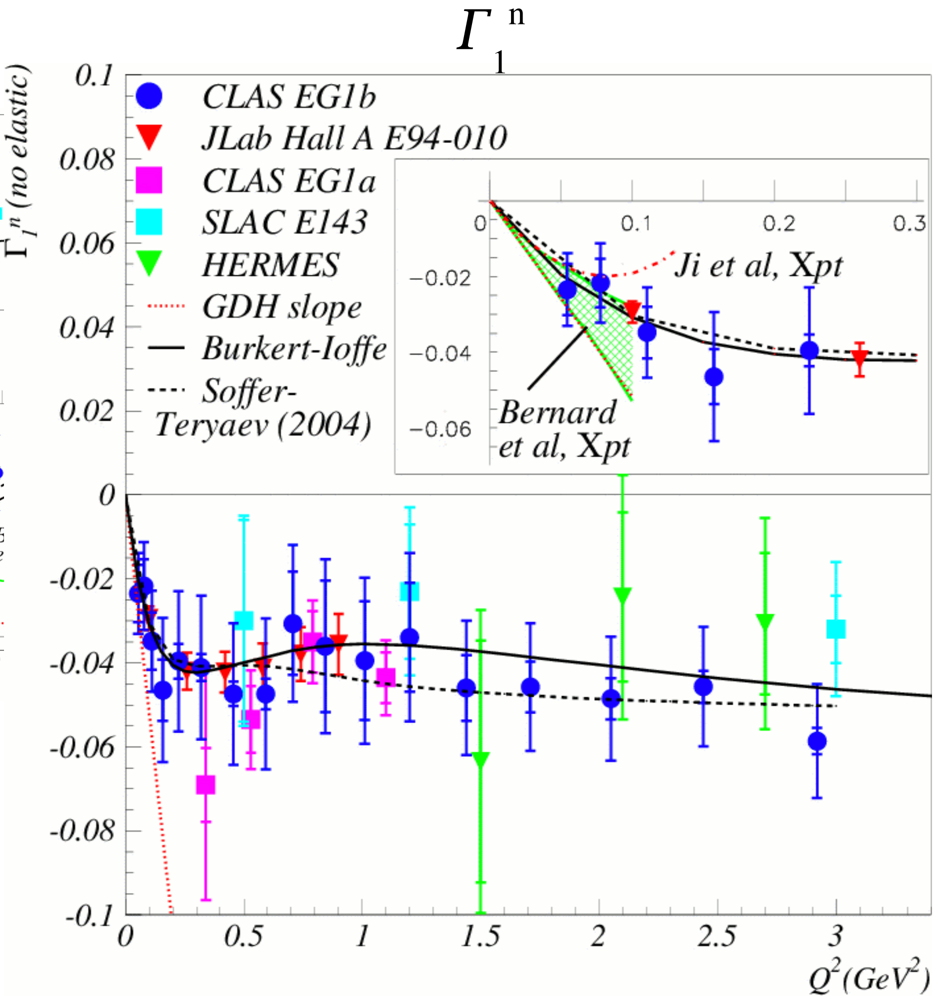
$\Gamma_1^{\,p}$

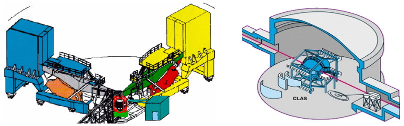


Results on sum rules









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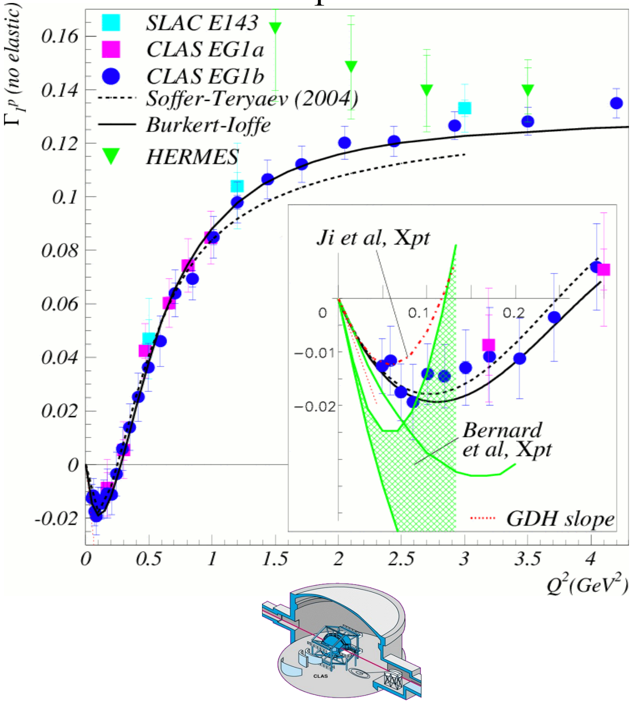
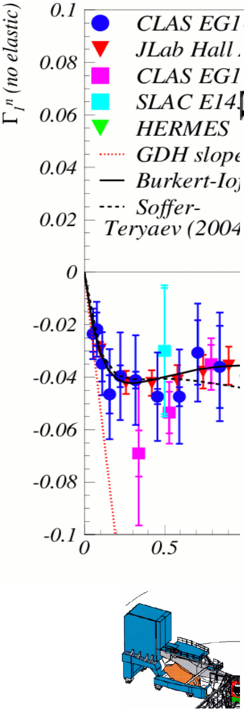


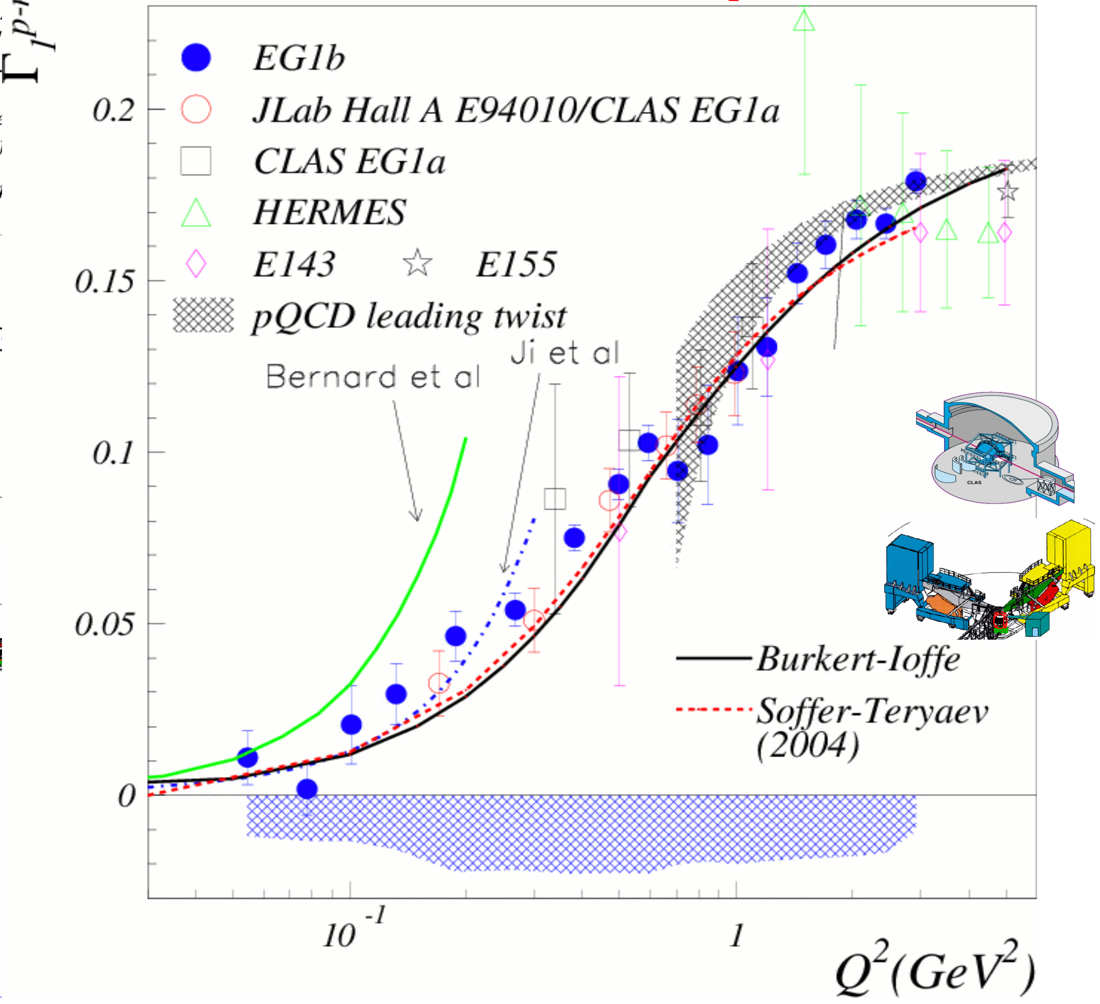
$\Gamma_1^{\,n}$



Bjorken sum: $\int g_1^{\,p}-g_1^{\,n}dx$

Δ contribution suppressed

⇒Easier check of χpT.



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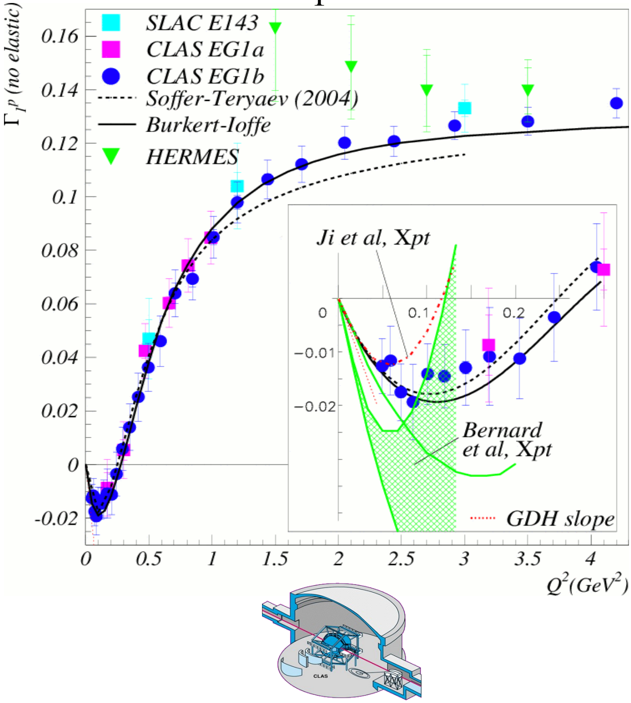

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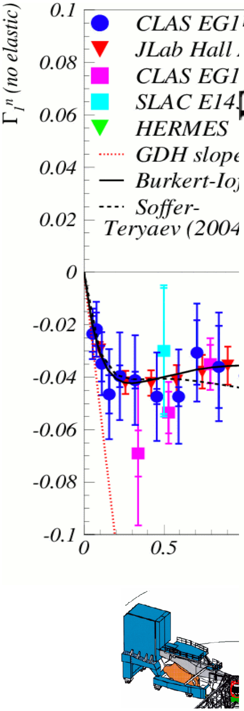

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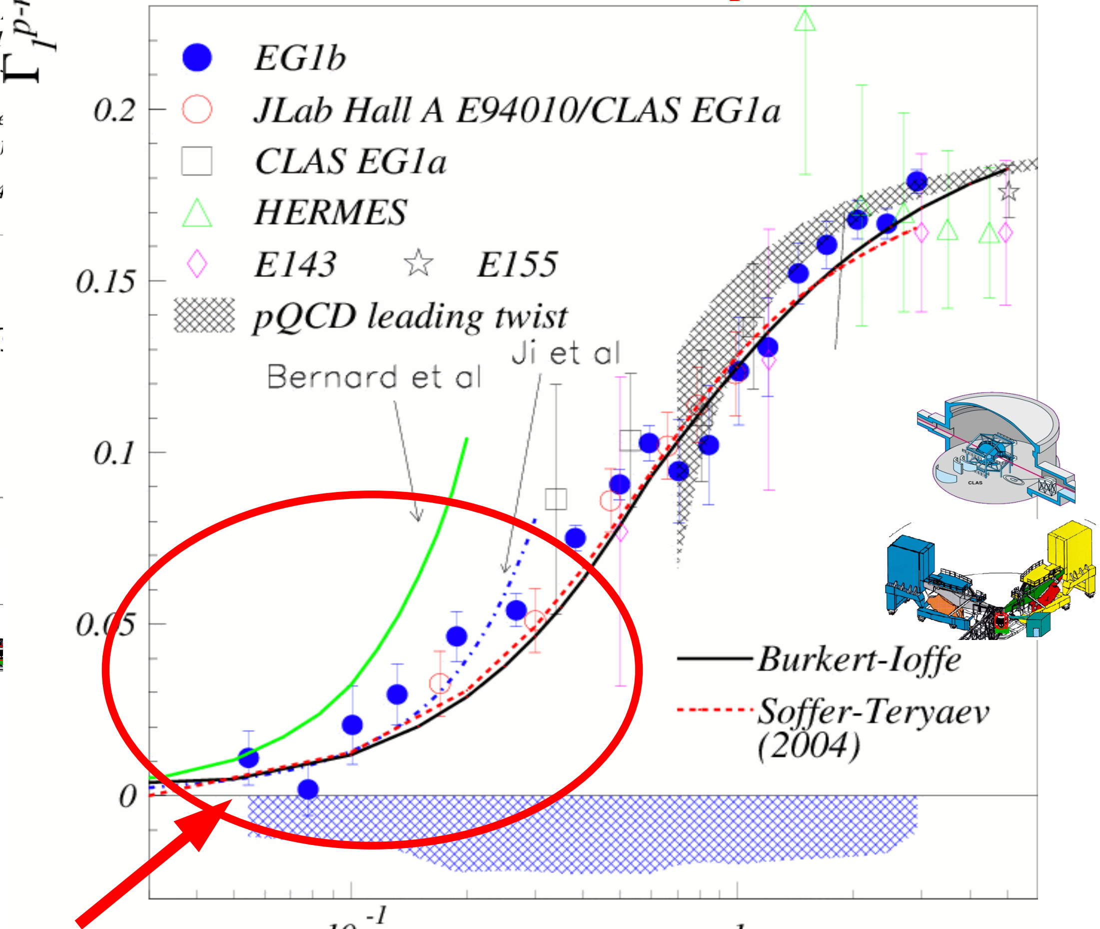
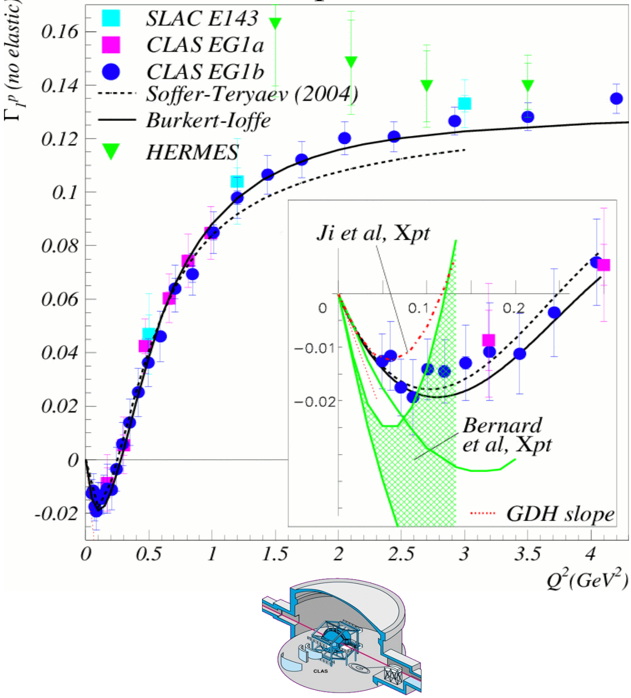


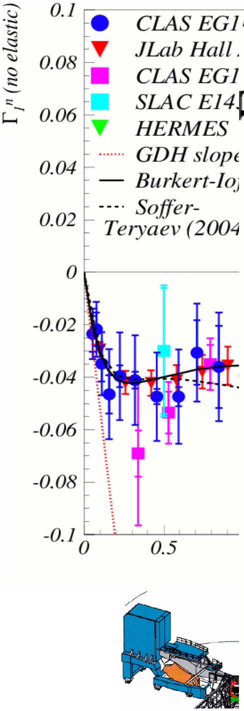
Nice agreement with χpT (Δ suppressed?)

Results on sum rules

Γ_1^p



Γ_1^n



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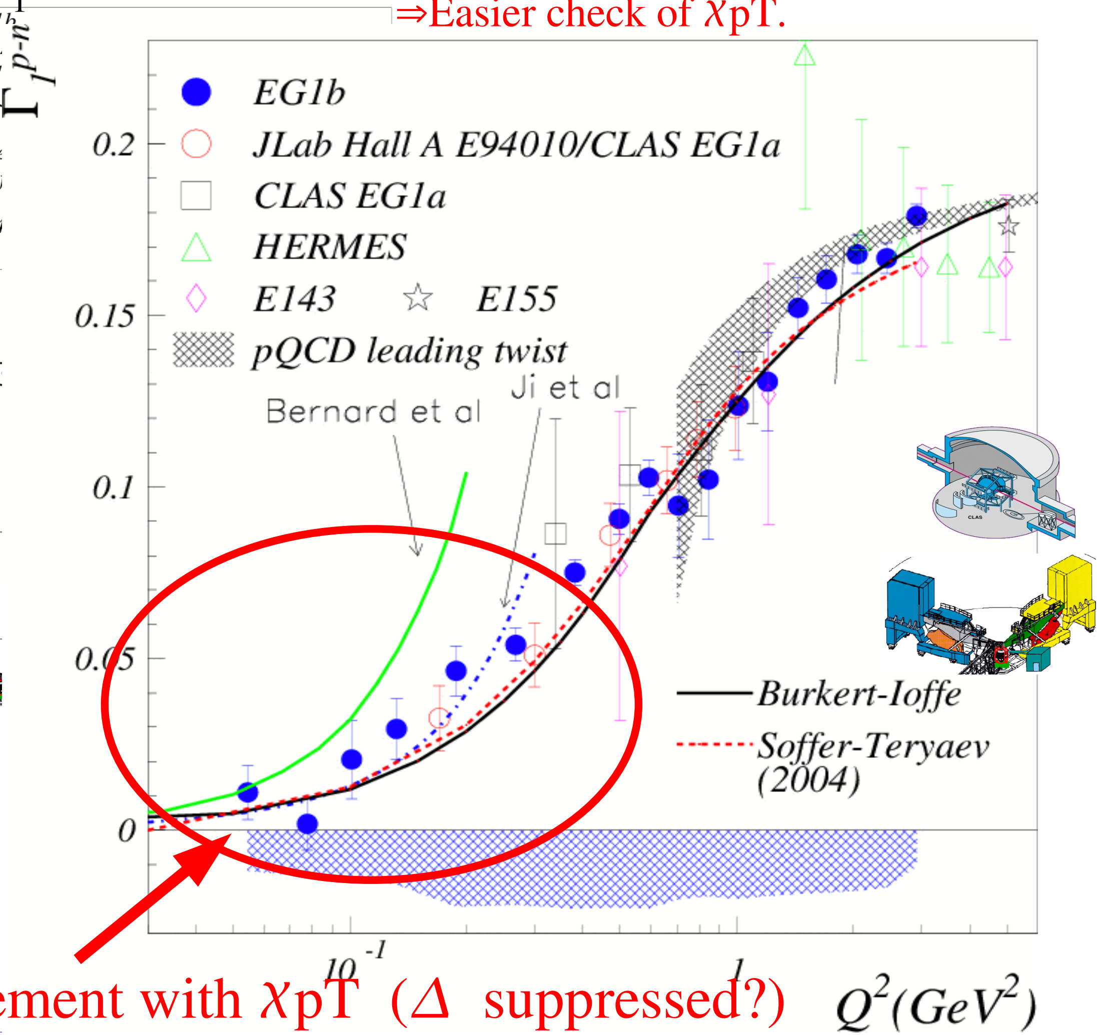

Low Q^2 fit:

$$\Gamma_1^{p-n} = \frac{\kappa_n^2-\kappa_p^2}{8M^2} Q^2 + aQ^4 + bQ^6$$

a=0.80±0.07±0.23, b=-1.13±0.16±0.39

$a^{\chi pT,Ji}=0.74$, $a^{\chi pT,B.}=2.4$

Nice agreement with χpT (Δ suppressed?)

Results on sum rules (higher moments)

Generalized forward spin polarizability:

$$\gamma_0 = \frac{4e^2M^2}{\pi Q^6} \int x^2(g_1 - \frac{4M^2}{Q^2} \; x^2 g_2)dx$$

For Neutron

Longitudinal-Transverse polarizability:

$$\delta_{LT} = \frac{4e^2M^2}{\pi Q^6} \int x^2(g_1 + g_2)dx$$

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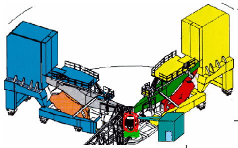

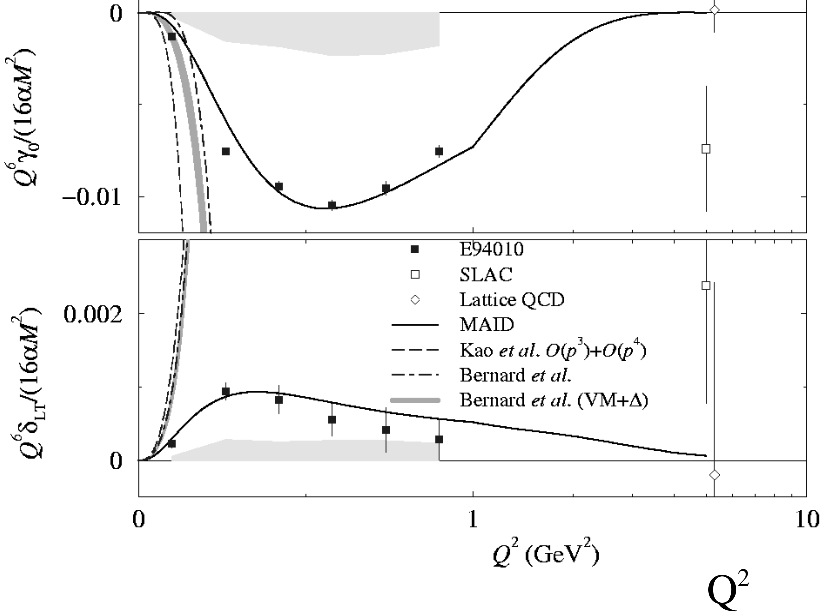

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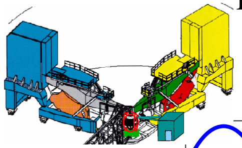
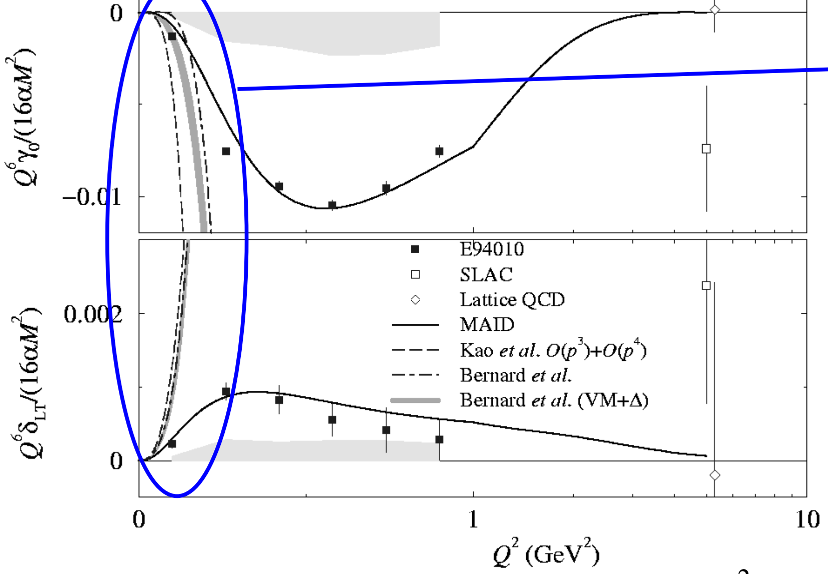
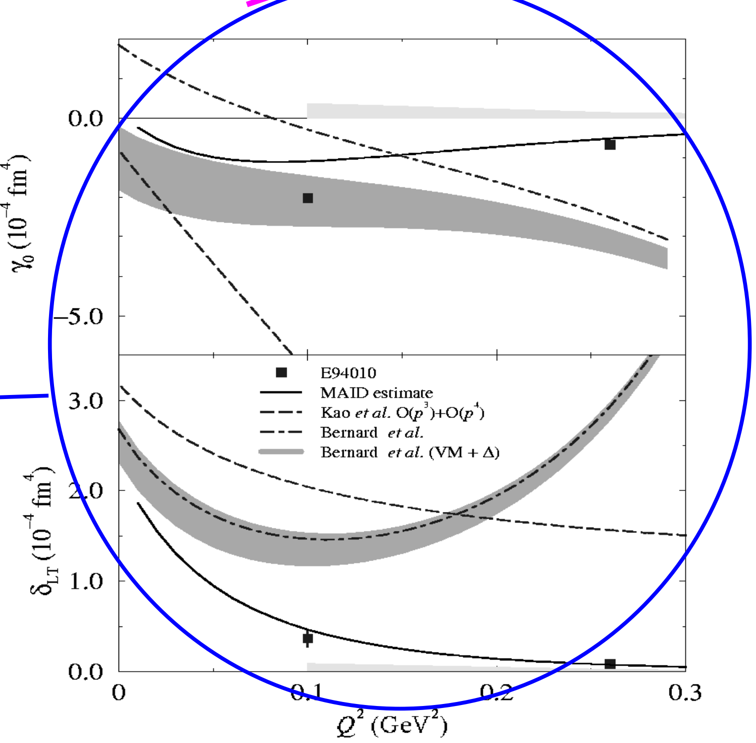




Q^2

For Neutron



Results on sum rules (higher moments)

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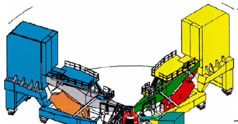

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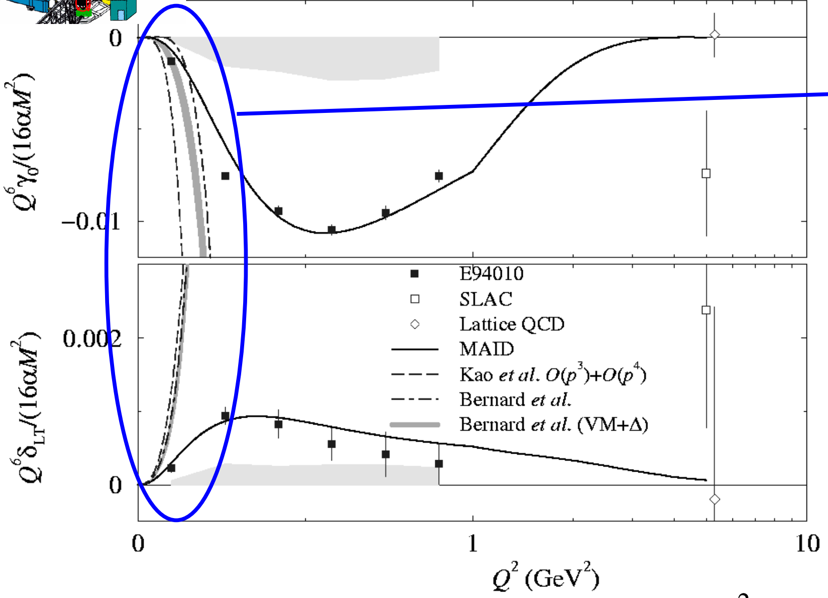

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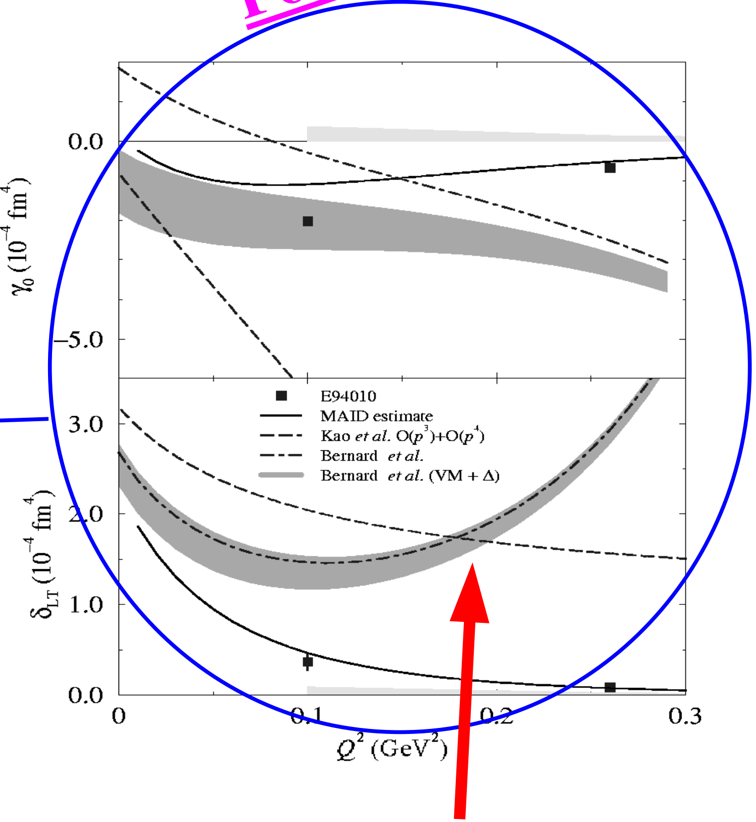
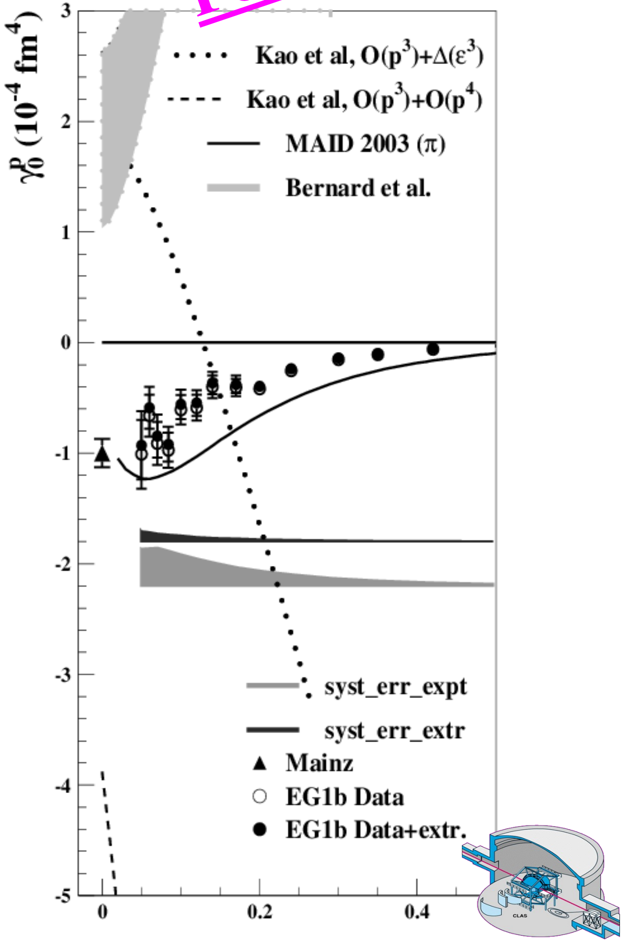


Failure of χpT in spite of Δ suppression.

Results on sum rules (higher moments)

For Proton



Results on sum rules (higher moments)

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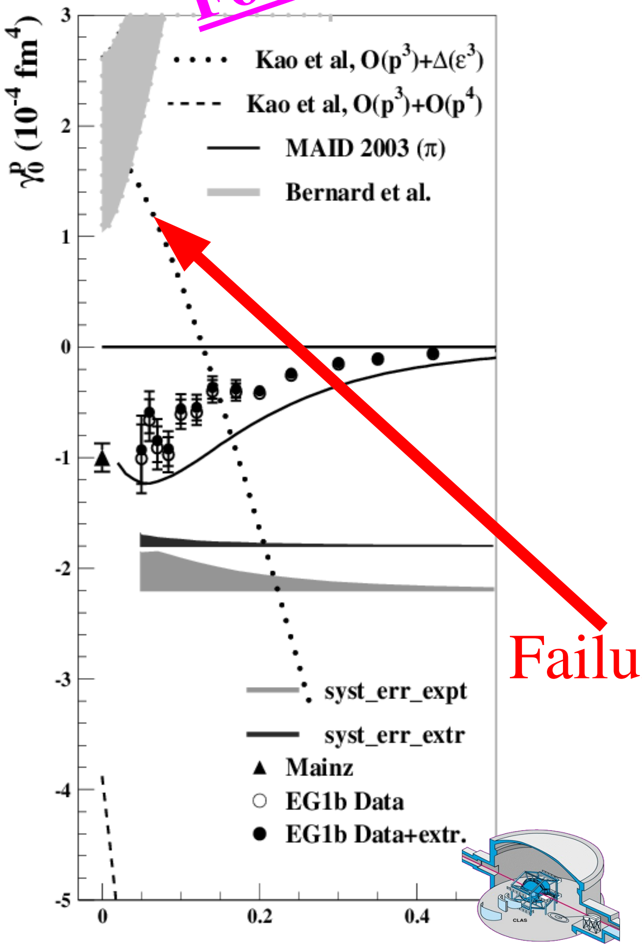
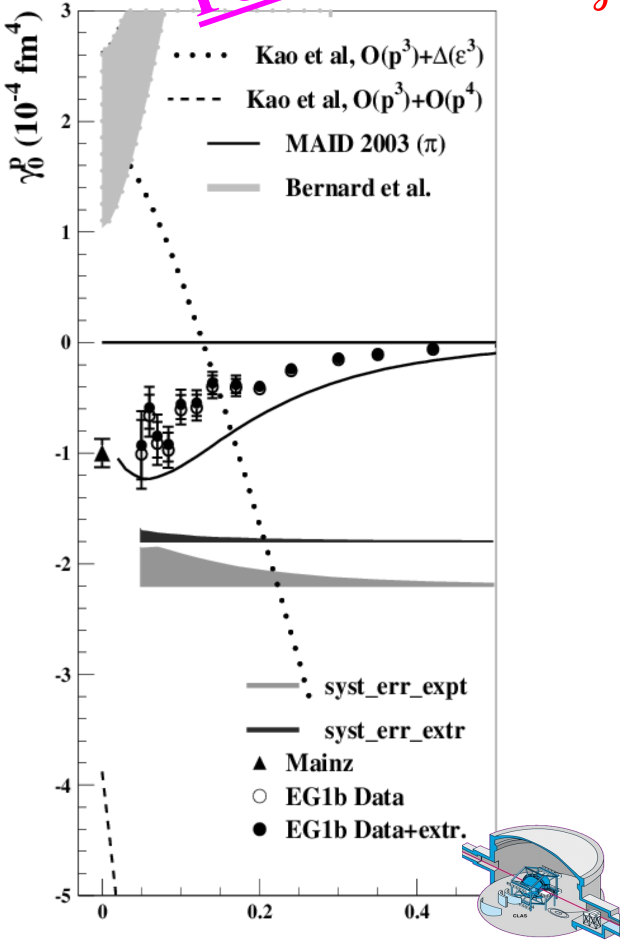


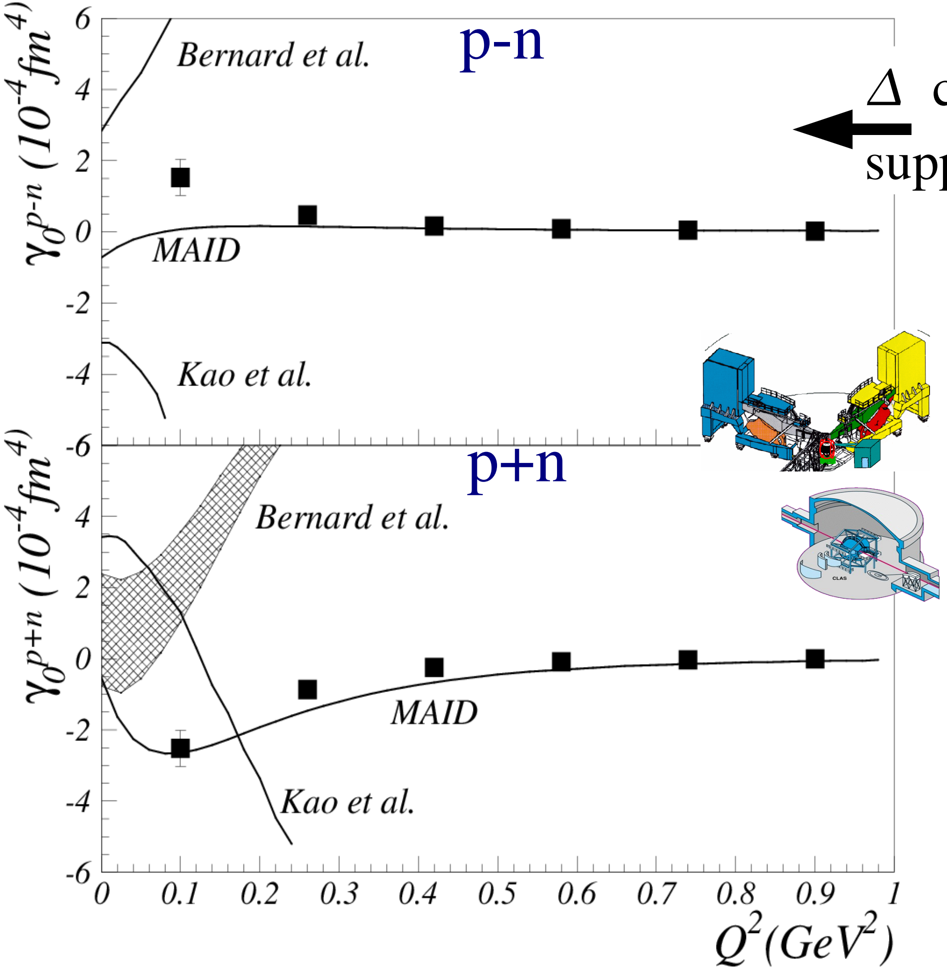
Failure of χpT

Results on sum rules (higher moments)

For Proton

To study the influence of the Δ: Isospin decomposition of γ_0 using the Hall A and B data.





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Results on sum rules (higher moments)

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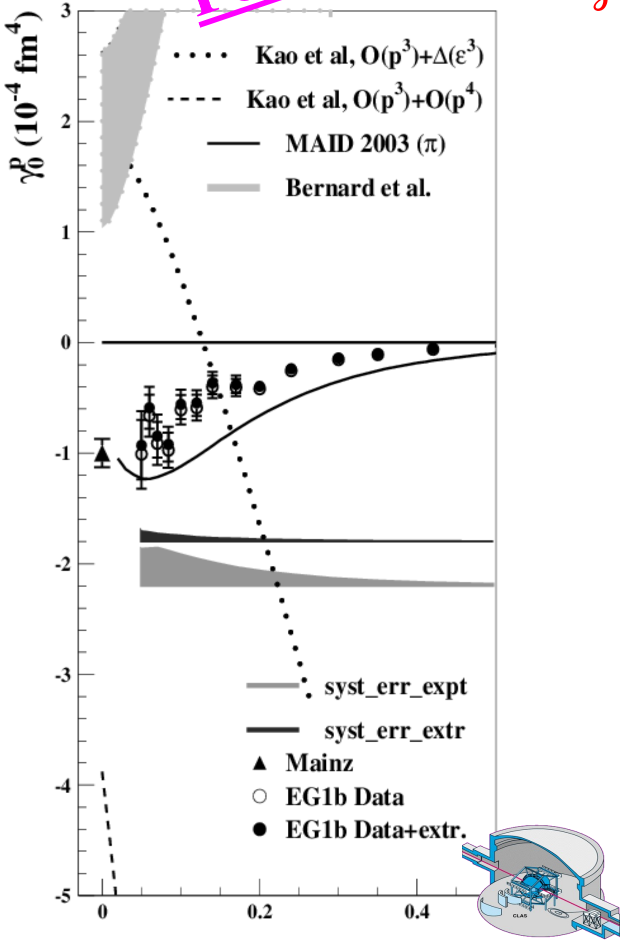

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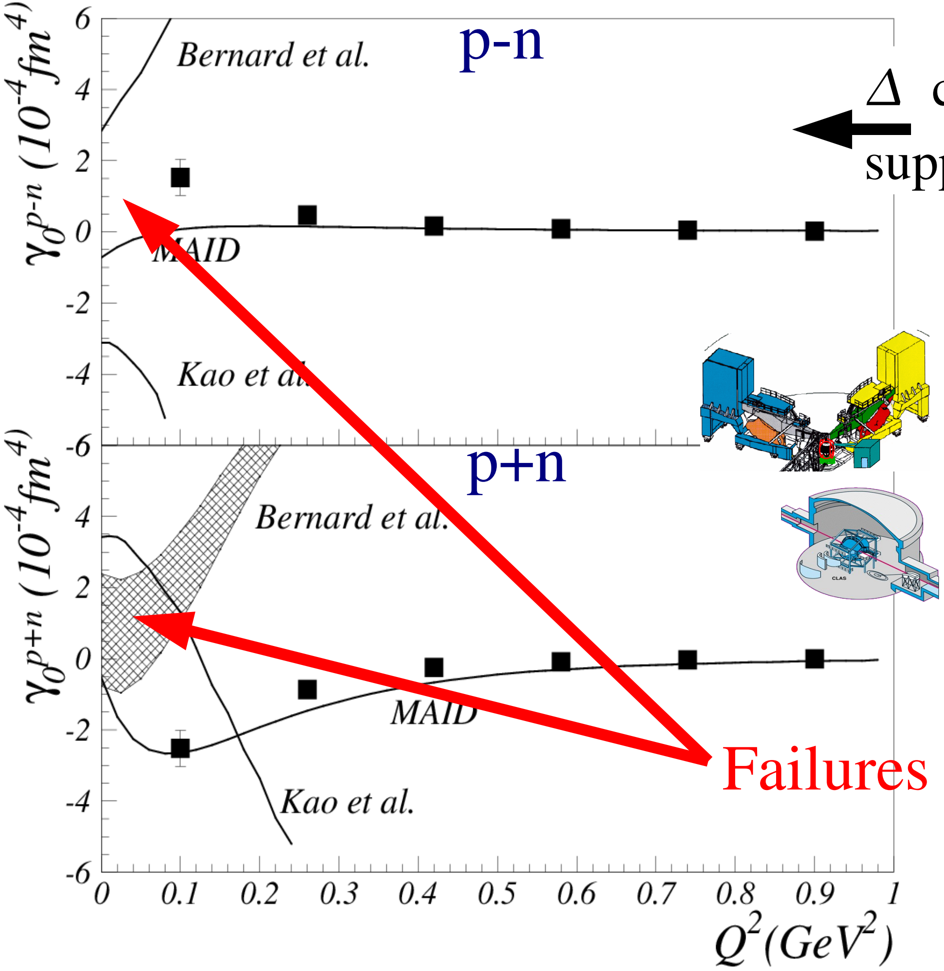


Δ contribution suppressed

Failures of χpT

Results on sum rules

Summary:

No low-x → γ_0

No low-x, No Δ → δ_{LT}

No low-x → d_2

No Δ → P-N

χpT:

	Γ_1	γ_0	δ_{LT}	d_2
Proton	$a^{exp}=4.31\pm0.31\pm1.36$ $a^{Ji}=3.89$ Up to $Q^2\sim0.08$ GeV^2	🟥	*No low Q^2 data*	*No low Q^2 data*
Neutron	🟨	Up to $Q^2\sim0.1$ GeV^2 (Bernard *et al.* only)	🟥	🟥
P-N	$a^{exp}=0.80\pm0.07\pm0.23$ $a^{Ji}=0.74$, $a^{B.}=2.4$ Up to $Q^2\sim0..3$ GeV^2	🟥	*No low Q^2 data*	*No low Q^2 data*
P+N	$a^{exp}=6.97\pm0.96\pm1.48$ $a^{Ji}=7.11$ Up to $Q^2\sim0.1$ GeV^2	🟥	*No low Q^2 data*	*No low Q^2 data*

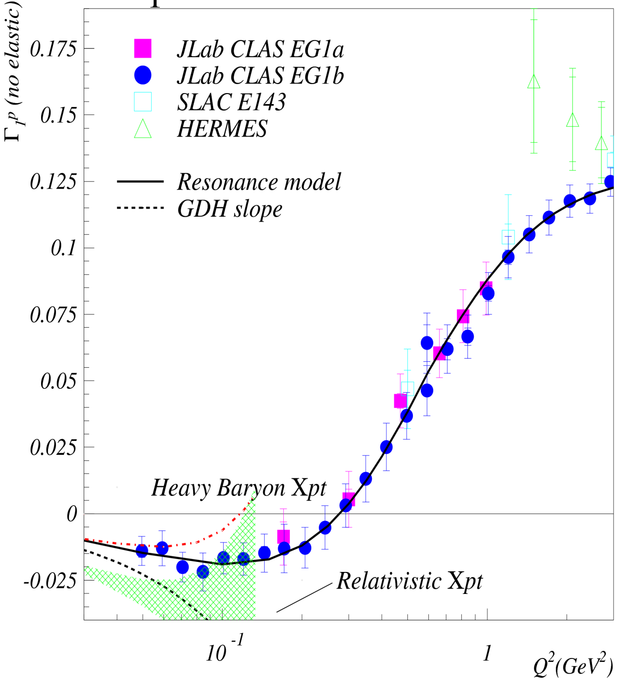
Models (MAID, Burkert-Ioffe, Soffer-Teryaev) are generally doing very well

Spin structure studies in the pQCD →npQCD transition region

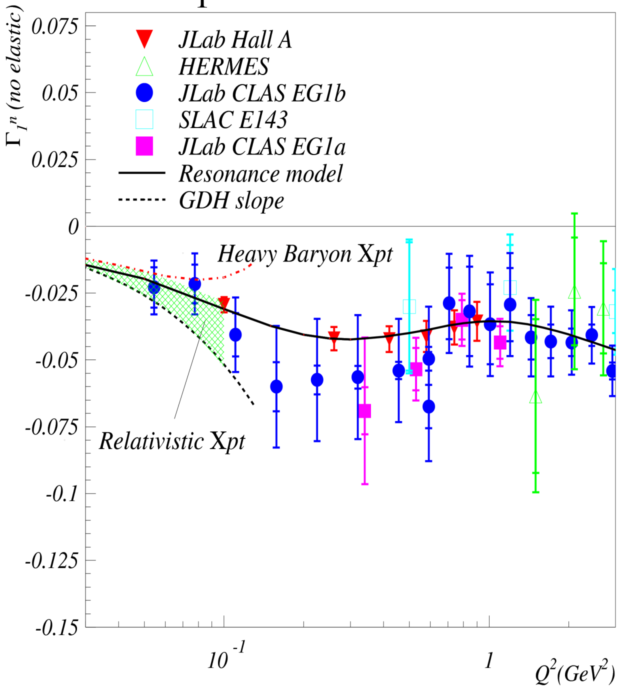
Smooth transition, nothing special happens near Λ_{QCD}^{2}.

JLab Halls A&B results:

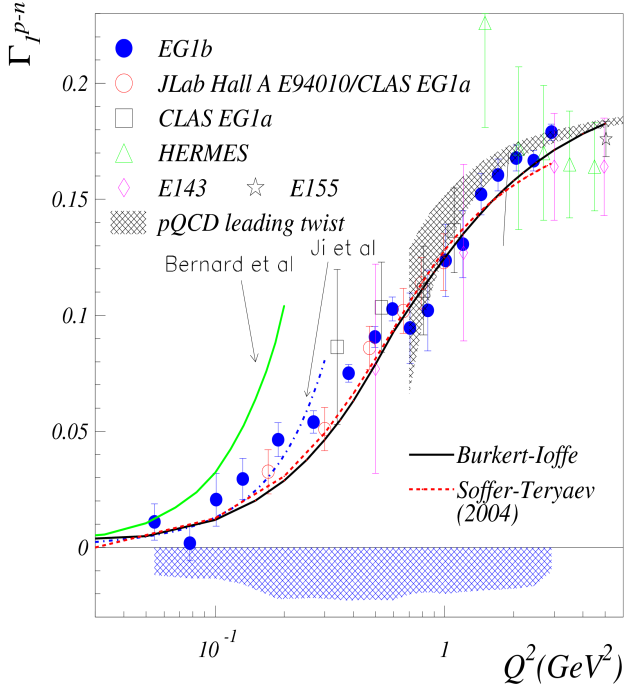
Γ_1^{p}



Γ_1^{n}

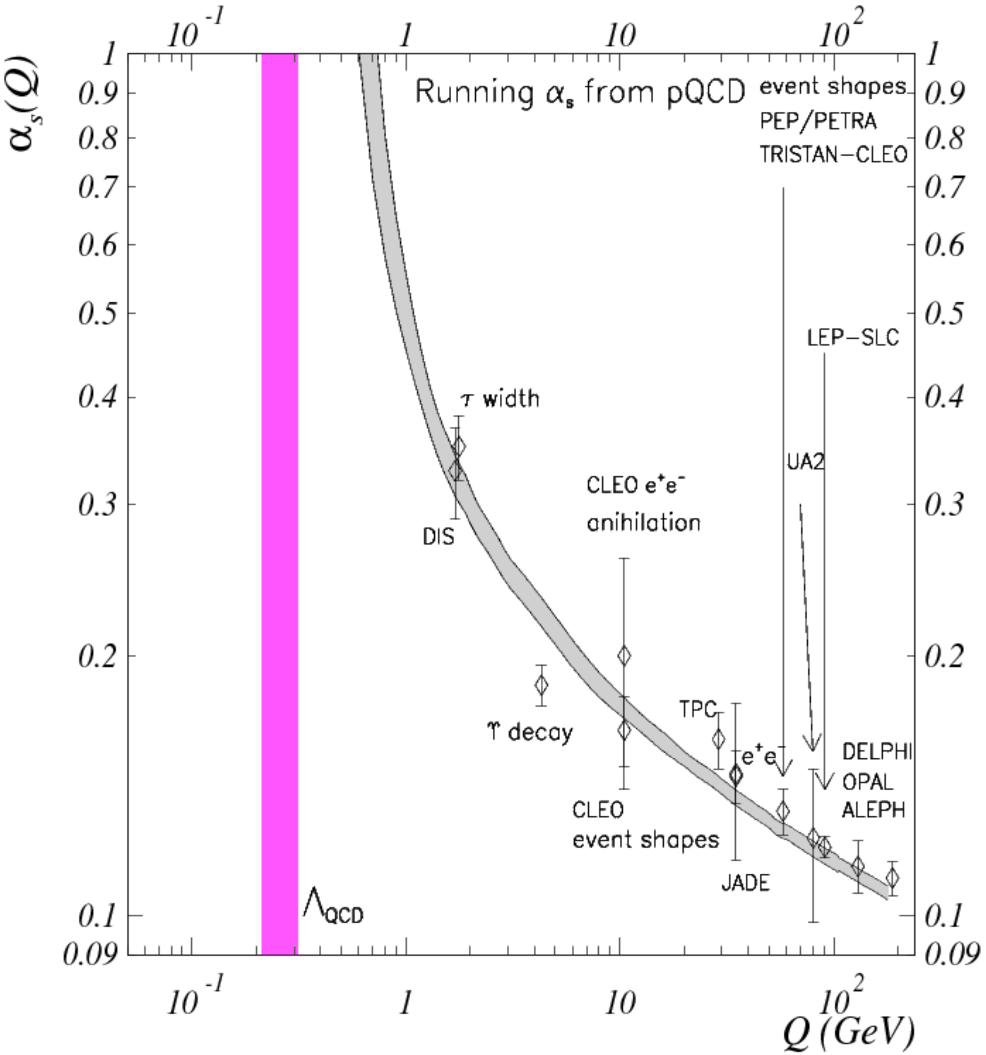


$\Gamma_1^{p\text{-}n}$



⇒ Can be used to define a QCD effective coupling at large distance.

The strong coupling constant from pQCD



α_s (Q) is well defined in pQCD at large Q^2.

Can be extracted from data

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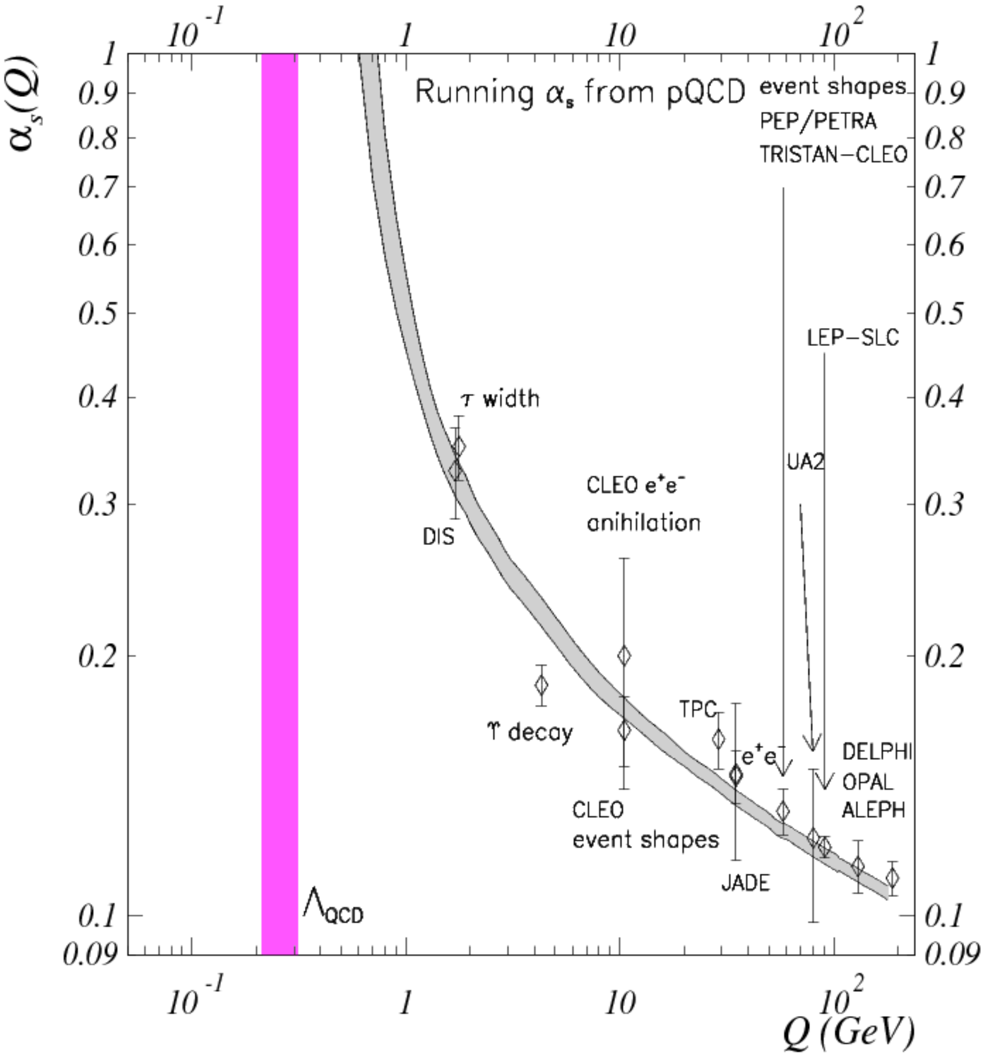

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Can be extracted from data (e.g. Bjorken Sum Rule).

$$\int g^p_1 - g^n_1 \, dx = \frac{1}{6} g_A\left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - ...\right)$$

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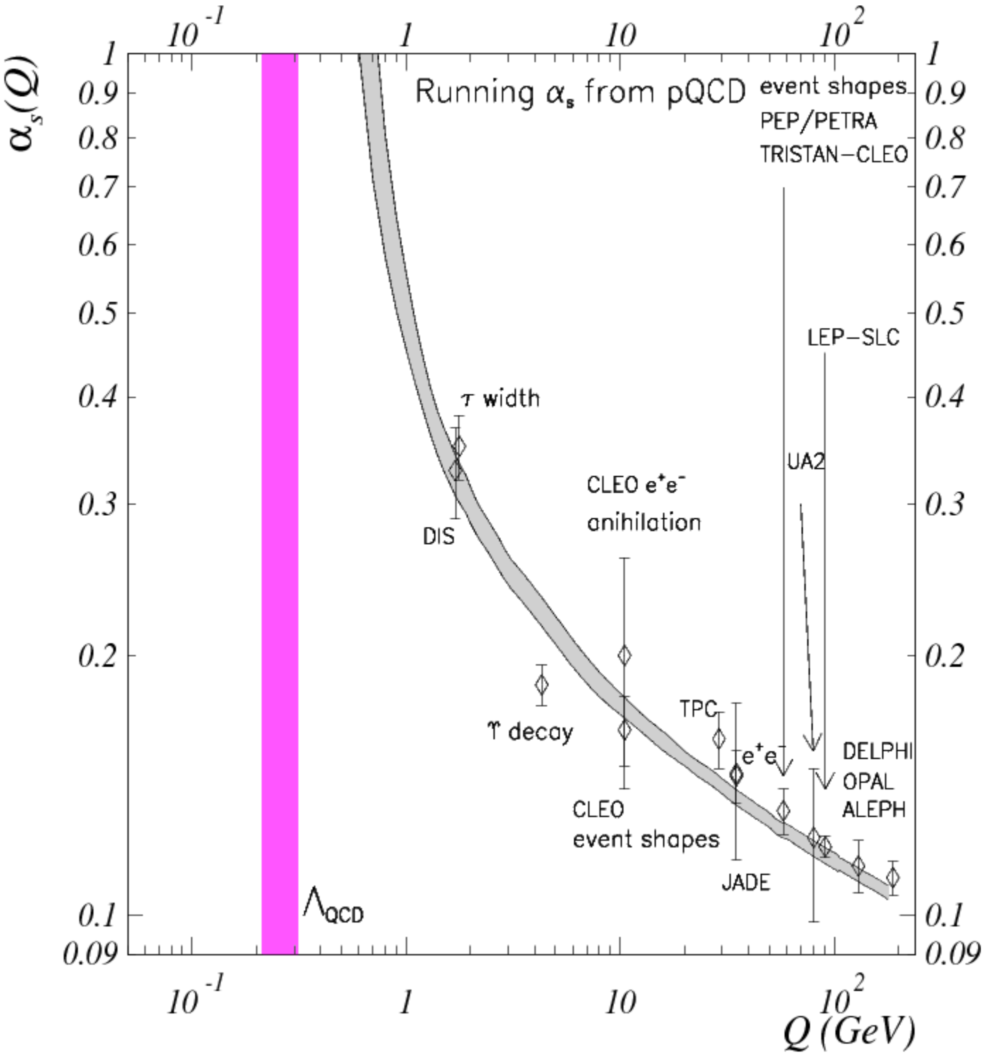


α_s (Q) is well defined in pQCD at large Q^2.

Can be extracted from data (e.g. Bjorken Sum Rule).

At low Q^2 (~GeV^2) , pQCD cannot be used to define α_s : *If* pQCD is trusted,

$\alpha_s \rightarrow \infty$ for $Q \rightarrow \Lambda_{QCD}$.

Definition of effective QCD couplings

G. Grunberg, PLB B95 70 (1980); PRD 29 2315 (1984); PRD 40 680(1989).

Prescription:

Define effective couplings from a perturbative series truncated to the first term in α_s.

Generalized Bjorken sum rule:

$$\int g^p_1 - g^n_1 \, dx = \Gamma_1^{p-n} = \frac{1}{6} g_A \left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \frac{M^2}{9Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right] + \ldots$$

$$\Rightarrow \boxed{\Gamma_1^{p-n} \mathrel{\hat{=}} \frac{1}{6} g_A\left(1 - \frac{\alpha_{s,g1}}{\pi}\right)}$$

$\alpha_{s,g1} \mathrel{\hat{=}} \alpha_s^{eff}$ extracted from Γ_1^{p-n}

By doing so we obtain a coupling constant that is:

- Extractable at any Q^2.
- Free of divergence.
- Not renormalization scheme dependent.
- Analytic when crossing quark thresholds.

But that is:

- Process dependent

$\Rightarrow$There is a priori a different α_s^{eff} for each different process.

<u>However</u> these α_s^{eff} can be related, so they are not useless quantities.

"Commensurate scale relations"

S.J. Brodsky & H.J Lu, PRD 51 3652 (1995)
S.J. Brodsky, G.T. Gabadadze, A.L. Kataev, H.J Lu, PLB 372 133 (1996)

Advantages of extracting $\alpha_{s,g1}$ from the Bjorken Sum Rule

- Bjorken sum: simple Q^2-dependence.

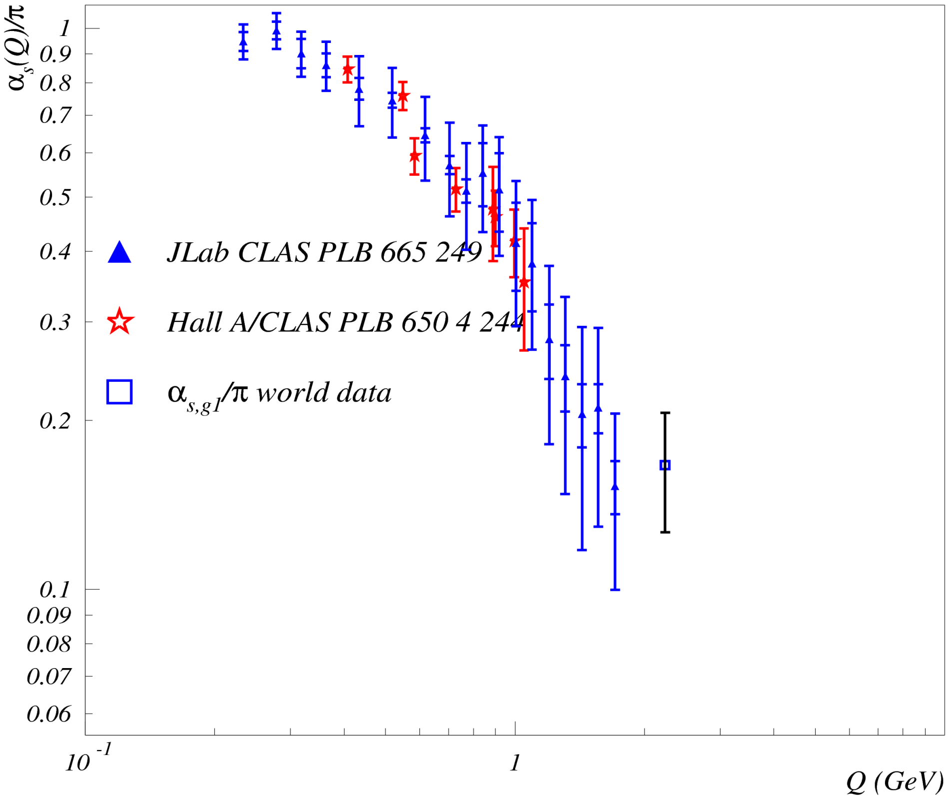
- Data exist at low, intermediate, and high Q^2.

- Sum rules (generalized GDH and Bjorken sum rules) complement the data in the unmeasured regions $Q^2 \to 0$ and $Q^2 \to \infty$.

  ⇒We can obtain $\alpha_{s,g1}$ at any Q^2.

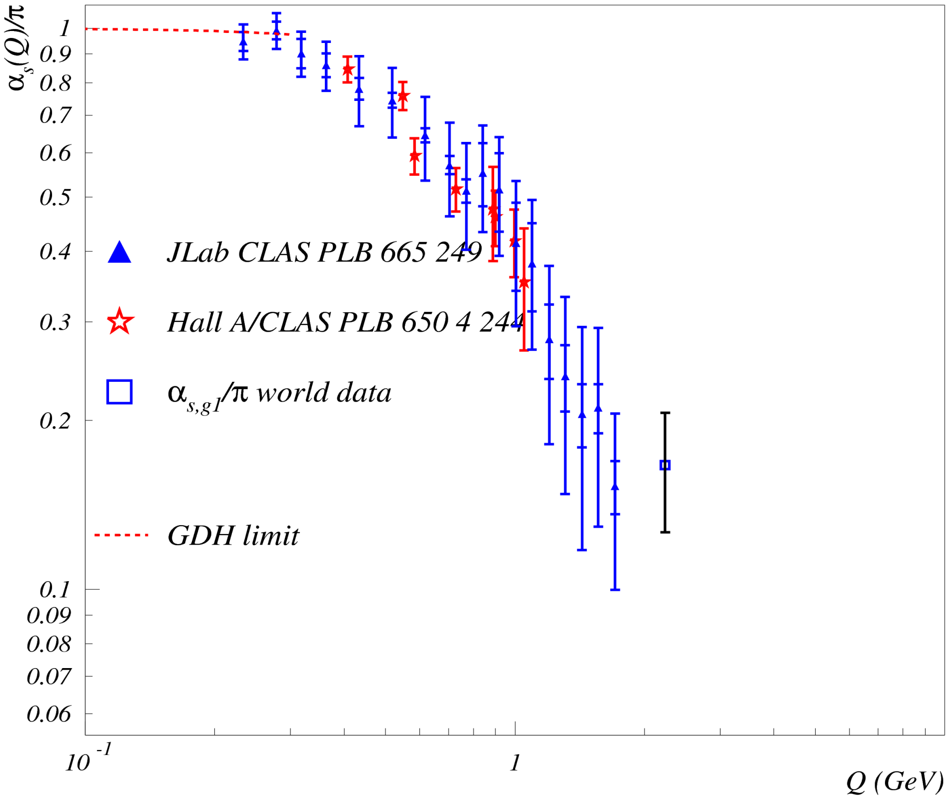
- Coherent contribution partly suppressed in the Bjorken sum. ⇒ Definition of $\alpha_{s,g1}$ may be closest to α_s^{pQCD} definition ? Argument is stronger if global duality works (excluding the Δ and the elastic contributions).

$\alpha_{s,g1}$ from the Bjorken Sum data



$$\Gamma_1^{p-n} \hat{=} \frac{1}{6}\ g_A\left(1-\frac{\alpha_{s,g1}}{\pi}\right)$$

Low Q^2 limit



Bjorken and Gerasimov-Drell-Hearn sums are related:

⇒ $Q^2 = 0$ constraints:

$$\Gamma_1^{p\text{-}n} = \frac{Q^2}{16\alpha\pi^2}\left(\mathrm{GDH}^p\text{-}\mathrm{GDH}^n\right)$$

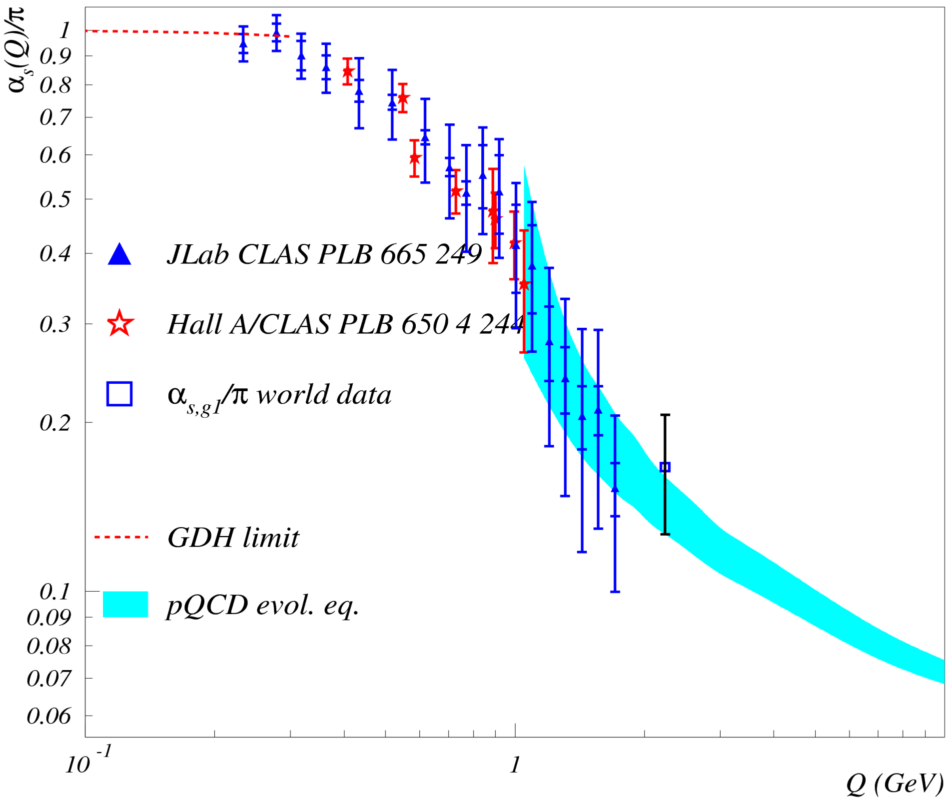
$$\boxed{\Rightarrow\begin{cases}\alpha_{s,g1} = \pi \\ \dfrac{d\alpha_{s,g1}}{dQ^2} = \dfrac{3\pi}{4g_A}\left(\dfrac{\kappa_n^2}{M_n^2} - \dfrac{\kappa_p^2}{M_p^2}\right)\end{cases}}$$

$Q^2=0$

First experimental evidence of *conformal behavior* (i.e. no Q^2-dependence) of α_s at low Q^2.

Large Q^2 limit

$$\Gamma_1^{p-n} = \frac{g_A}{6}\left[1- \frac{\alpha_s^{pQCD}}{\pi} -3.58\left(\frac{\alpha_s^{pQCD}}{\pi}\right)^2 - \dots\right] = \frac{g_A}{6}\left(1-\frac{\alpha_{s,g1}}{\pi}\right)$$

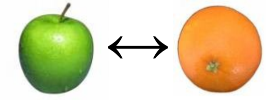


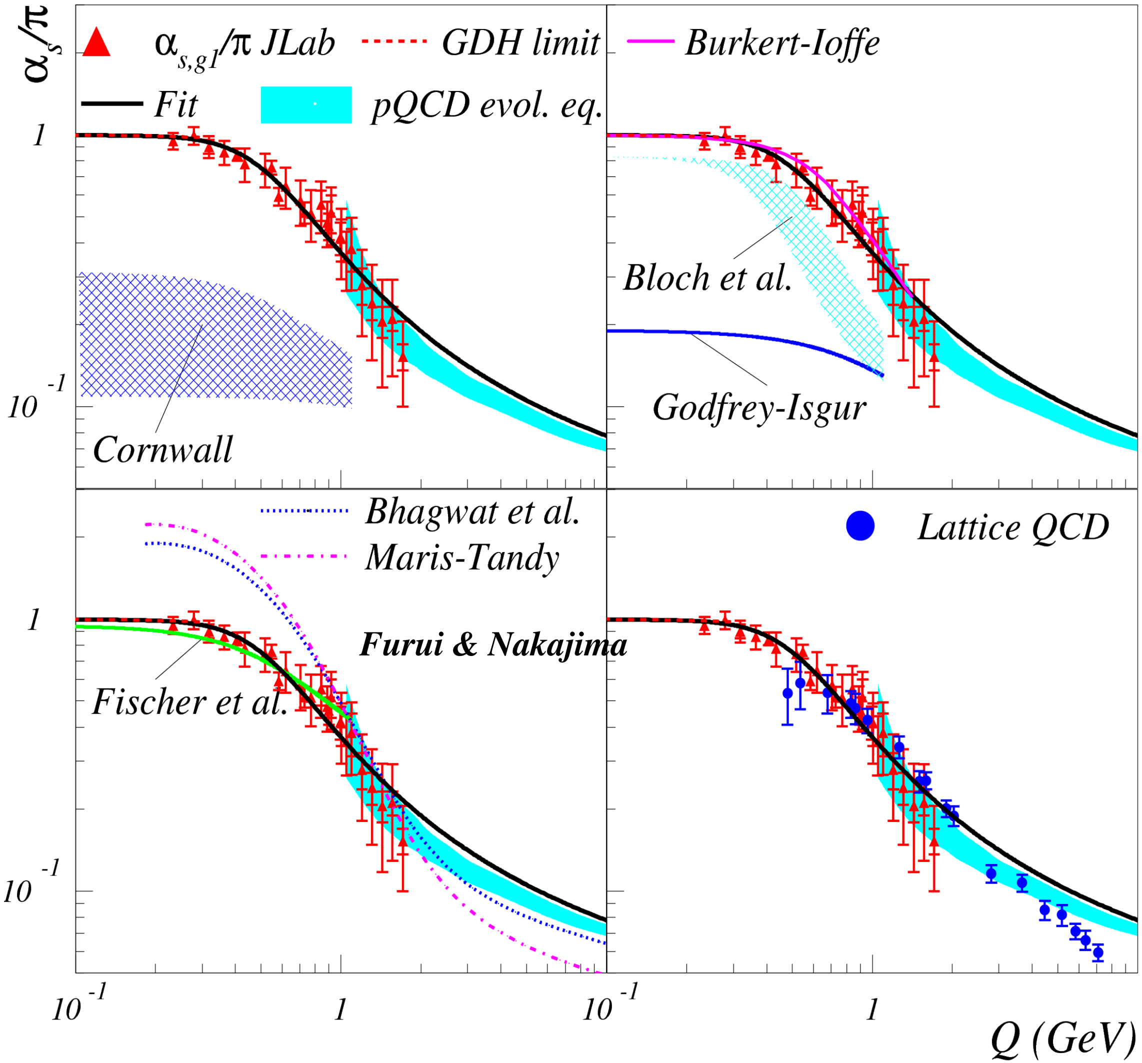
$$\boxed{\Rightarrow \alpha_{s,g1} = \alpha_s^{pQCD}}$$

$Q^2 \to \infty$

$\Rightarrow$ We know $\alpha_{s,g1}$ at any Q^2.

"Comparison" with theory



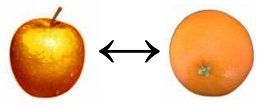


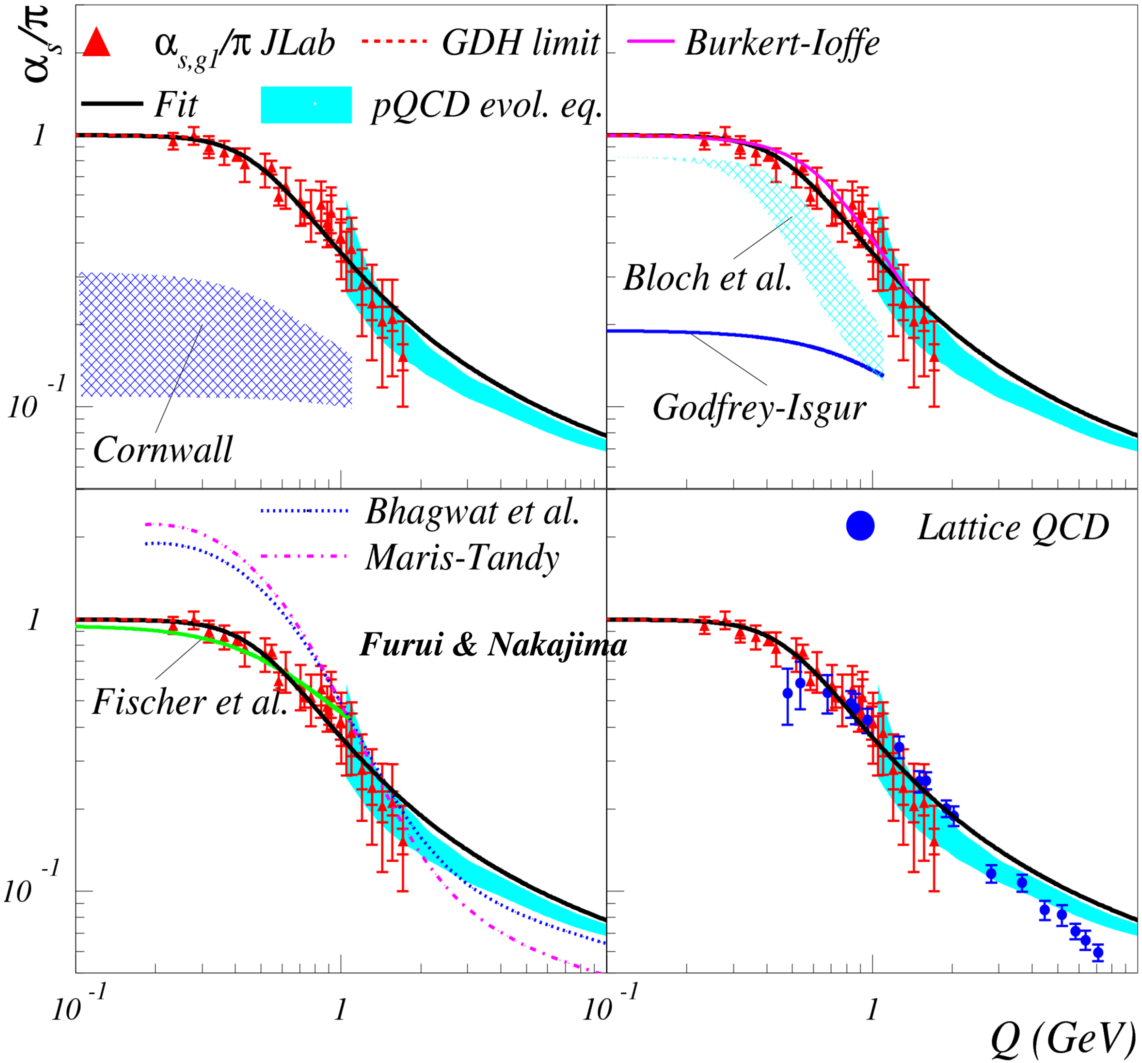
Fisher *et al.*
Bloch *et al.*
Maris-Tandy
Bhagwat *et al.*
Cornwall
} Schwinger -Dyson

Godfrey-Isgur: Constituant Quark Model

Furui & Nakajima: Lattice

“Comparison” with theory





Fisher *et al.*
Bloch *et al.*
Maris-Tandy
Bhagwat *et al.*
Cornwall

} Schwinger -Dyson

Godfrey-Isgur: Constituant Quark Model

Furui & Nakajima: Lattice

$\alpha_{s,g1}$ and the AdS/CFT correspondance

Anti de Sitter/ Conformal Field Theory correspondence (AdS/CFT, or Maldacena duality):

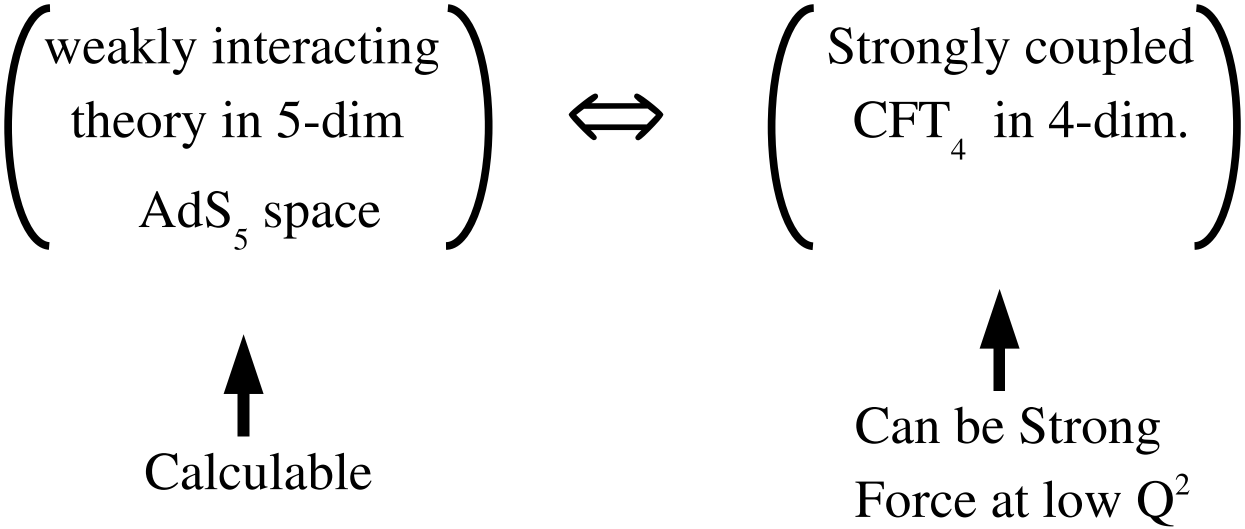
Anti de Sitter space: ~Space with constant negative curvature.

Conformal Field Theory: ~Field theory without scale dependence.

Correspondence: a weakly interactive, gravity-like, theory in
N-dimentional anti de Sitter space can be mapped on the
boundary of the anti de Sitter space (⇒N-1 dim.) into a
strongly interacting, QCD-like, conformal field theory.

$\alpha_{s,g1}$ and the AdS/CFT correspondance

Important fact: Strong force is conformal at low Q^2.



$$\begin{pmatrix}\text{weakly interacting} \\ \text{theory in 5-dim} \\ \text{AdS}_5 \text{ space}\end{pmatrix} \Leftrightarrow \begin{pmatrix}\text{Strongly coupled} \\ \text{CFT}_4 \text{ in 4-dim.} \\ \end{pmatrix}$$

Calculable

Can be Strong
Force at low Q^2

⇒New possibilities of QCD analytical calculations in non-perturbative domain (S. J. Brodsky, G. de Teramond,...)

PRL 94 201601 (2005); PRL 96 201601(2006)

Conclusions

- Data on SSF moments at low Q^2 and χpT do not consistently agree (or disagree).
- Δ cannot be the explanation for some disagreement.
- Low-Q^2 fits provide a quantitative comparisons. Importance of Q^6 terms.
- Need high precision data at lower Q^2. Transverse data on proton is especially missing. New experiments are fulfilling these needs:
  - E97110: $\parallel$ and $\perp$ on neutron (ran in 2003 in Hall A)
  - EG4: $\parallel$ on proton and deuteron (ran in 2006 in Hall B)
  - E08027: $\parallel$ and $\perp$ on proton (approved for Hall A)

  Possibility for $\perp$ data on P and D in Hall B is opening (Hdice target)

- Effective QCD couplings can be defined over the whole Q^2 domain.
- Bjorken Sum is advantageous to define an effective coupling.
- Data and Sum rules allow to obtain the effective coupling at all Q^2.
- Comparison with low-Q^2 calculation shows similar features, same Q^2-dependence and similar size. In particular α_s "freezes" at low Q^2.
- QCD conformal at low $Q^2 \Rightarrow$ Application of AdS/CFT correspondence to non-perturbative QCD.

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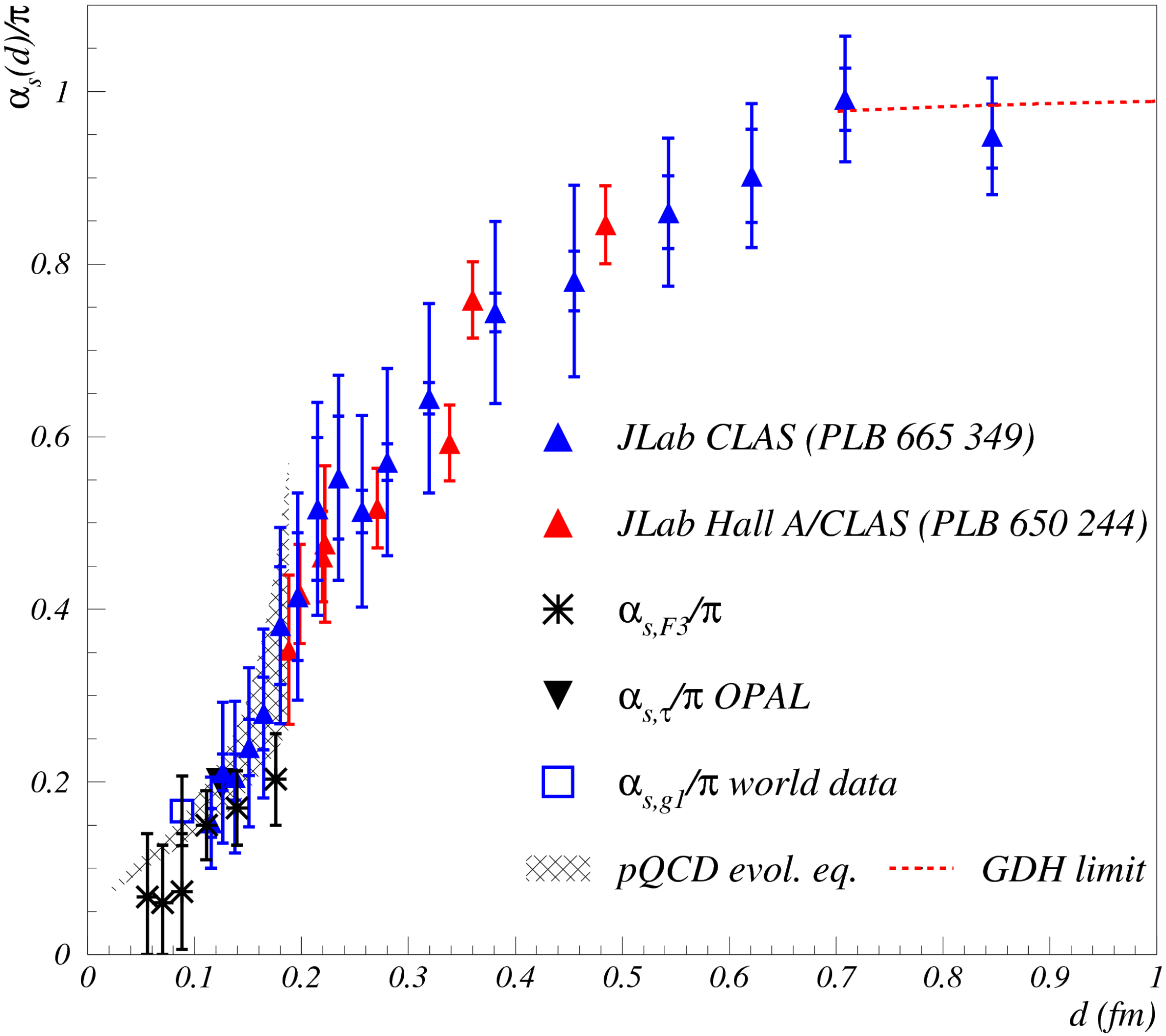
$\alpha_{s,g1}$(d)



$\alpha_s(d)/\pi$
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
d (fm)
JLab CLAS (PLB 665 349)
JLab Hall A/CLAS (PLB 650 244)
$\alpha_{s,F3}/\pi$
$\alpha_{s,\tau}/\pi$ OPAL
$\alpha_{s,g1}/\pi$ world data
pQCD evol. eq.
GDH limit