Spin Structure Functions
at Lower Energies

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Overview

- Introduction – what do we measure?
- What do we want to learn? - QCD, effective theories and models
- Status of World Data
- Experiments with CLAS - EG1 and EG4
- JLab results
- Outlook: Future Experiments…

Orbital Angular Momentum

A_1 (x→1)

Reso-
nance
Structure

GDH,
ChPT

OPE,
twist >2

Duality

TMDs

Bjorken
Sum Rule

Δq, ΔG

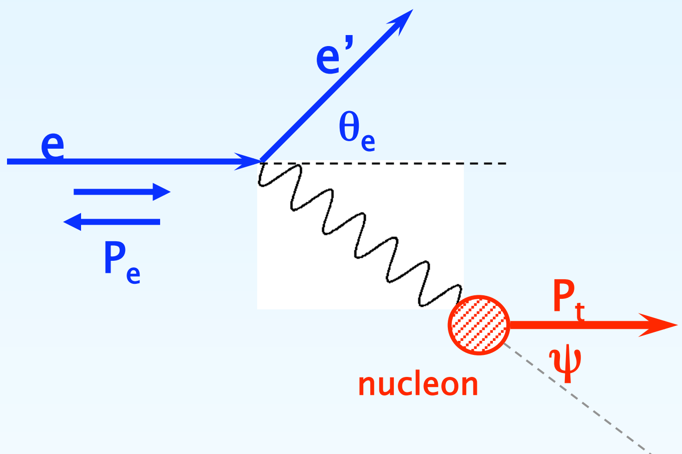
PDFs

DVCS

Virtual Photon Asymmetries - Measurement

$$\frac{d\sigma}{dE'\,d\Omega} = \Gamma_v\left[\sigma_T + \varepsilon\sigma_L + P_e P_t\left(\sqrt{1-\varepsilon^2}A_1\sigma_T\cos\psi + \sqrt{2\varepsilon(1-\varepsilon)}A_2\sigma_T\sin\psi\right)\right]$$

$$\mathbf{A_1} = \frac{\sigma_{1/2}-\sigma_{3/2}}{\sigma_T} \qquad \mathbf{A_2} = \frac{\sigma_{LT'}}{\sigma_T}$$



the asymmetries $\mathbf{A_1}$ and $\mathbf{A_2}$ can be extracted by varying the *direction of the nucleon polarization* or by varying the *beam energy* at fixed Q^2, ν

$$A_{\|} = D(A_1 + \eta A_2)$$
$$A_{\perp} = d(A_1 + \zeta A_2)$$

[where D, η, d, ζ are functions of Q^2, E′, E, R, e.g.:

$$D = \frac{1-\varepsilon E'/E}{1+\varepsilon R}$$

$$\eta = \frac{\varepsilon\sqrt{Q^2}}{E-\varepsilon E'} \qquad R = \frac{\sigma_L}{\sigma_T} \quad]$$

$$A_1 \approx \frac{\sum_i e_i^2 \Delta q_i(x)}{\sum_i e_i^2 q_i(x)}$$

EG1 used parametrization of world data on A_2 to extract A_1 (η is usually small)

Spin Structure Functions

$$\frac{d\sigma}{dE'd\Omega}\downarrow\Uparrow - \frac{d\sigma}{dE'd\Omega}\uparrow\Uparrow = \frac{4\alpha^2 E'}{M\nu Q^2 E}\left[\left(E + E'\cos\theta\right)\mathbf{g_1} - 2xM\mathbf{g_2}\right]$$

Unpolarized: $F_1(x,Q^2)$ and $F_2(x, Q^2)$

Polarized: $g_1(x,Q^2)$ and $g_2(x, Q^2)$

Parton model:

$$F_1(x) = \tfrac{1}{2}\sum_i e_i^2 q_i(x) \text{ and } F_2(x) = 2xF_1(x)$$

$$g_1(x) = \tfrac{1}{2}\sum_i e_i^2 \Delta q_i(x) \text{ and } g_2(x) = 0$$

i = quark flavor

e_i = quark charge

the structure functions $\mathbf{g_1}$ and $\mathbf{g_2}$ are linear combinations of $\mathbf{A_1}$ and $\mathbf{A_2}$

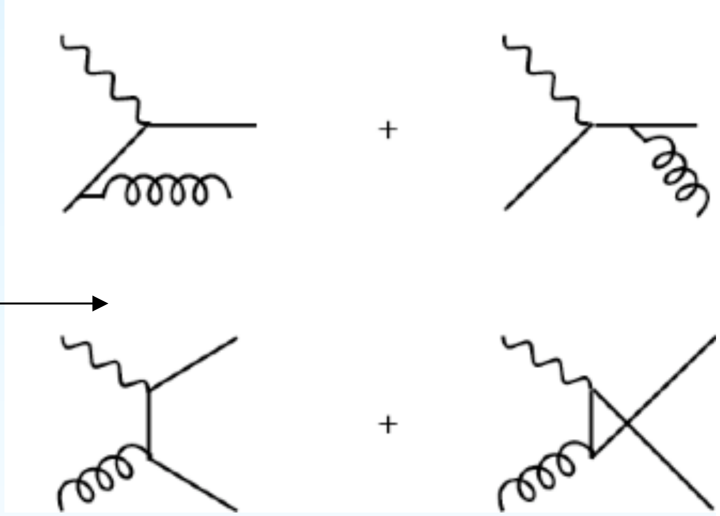
$$g_1(x,Q^2) = \frac{\tau}{1+\tau}\left(A_1 + \tfrac{1}{\sqrt{\tau}}A_2\right)F_1 = \frac{\tau}{1+\tau}\left(\frac{A_\parallel}{D} + \left(\tfrac{1}{\sqrt{\tau}} - \eta\right)A_2\right)F_1$$

$$g_2(x,Q^2) = \frac{\tau}{1+\tau}\left(\sqrt{\tau}A_2 - A_1\right)F_1$$

$$\tau = \frac{\nu^2}{Q^2}$$

Parton Distribution Functions and NLO pQCD

Two effects modify simple parton picture:



1) (Gluon) radiative corrections change elementary cross section

$$g_1(x,Q^2)_{pQCD} = \frac{1}{2}\sum_q^{N_f} e_q^2[(\Delta q + \Delta\bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \frac{\delta C_G}{N_f}]$$

$\delta C_q, \delta C_G - Wilson$ coefficient functions

2) pQCD evolution makes PDFs Q^2-dependent (NLO DGLAP equations) → we can extract information on the gluon from DIS

Moments of spin structure functions

- Related to matrix elements of local operators - in principle accessible to lattice QCD calculations
- Sum rules relate moments to the total spin carried by quarks in the nucleon and to axial vector coupling g_A of the nucleon

1st moment

$$\Gamma_1{}^p\left(Q^2\right) = \int_0^1 g_1{}^p\left(x,Q^2\right)dx = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right) \quad \text{+ pQCD corrections}$$

$$= \left(\frac{g_A{}^{(3)}}{12} + \frac{g_A{}^{(8)}}{36}\right)C_{NS}\left(Q^2\right) + \frac{\Delta\Sigma}{9}C_S\left(Q^2\right)$$

$$g_A{}^{(3)} = \Delta u - \Delta d \text{ (n} \rightarrow \text{p axial form factor)}$$

$$g_A{}^{(8)} = \Delta u + \Delta d - 2\Delta s \text{ (hyperon decay)}$$

non-singlet and singlet Wilson Coeff.

$$C_{NS} = 1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.22\left(\frac{\alpha_s}{\pi}\right)^3 \ldots$$

$$C_S = 1 - \frac{\alpha_s}{\pi} - 1.096\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots(\overline{\text{MS}})$$

Bjorken Sum Rule (fundamental)

$$\Gamma_1^{p-n} = \int g_1^p\,dx - \int g_1^n\,dx = \frac{g_A{}^{(3)}}{6}C_{NS}$$

The 2nd SSF g_2

In parton model, $g_2 = 0$ for massless quarks

In DIS, Wandura-Wilczek (no higher twist):

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{g_1(x,Q^2)}{y} dy$$

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \overline{g}_2(x,Q^2)$$

Higher Twist

Burkardt-Cottingham Sum Rule:

$$\Gamma_2(Q^2) = \int_0^1 g_2(x,Q^2)dx = 0$$

expected to be valid at all Q^2

Higher Twist contributions

Further modification of the first moment of g_1 due to quark-gluon and quark-quark correlations:

$$\Gamma_1(Q^2) = \mu_2(\ln Q^2) + \frac{\mu_4(\ln Q^2)}{Q^2} + \cdots; \qquad \mu_4 = \frac{M^2}{9}\left(a_2 + 4d_2 + 4f_2\right)$$

twist-2
targ. mass

$$d_2\left(Q^2\right) = \int_0^1 x^2\left[2g_1\left(x,Q^2\right) + 3g_2\left(x,Q^2\right)\right]dx$$

twist-3

$$f_2(Q^2)\ M^2 S^\mu \ = \ \frac{1}{2}\sum_q e_q^2\ \langle N|g\ \bar{\psi}_q\ \widetilde{G}^{\mu\nu}\gamma_\nu\ \psi_q|N\rangle$$

Twist-4; related to the "Color-polarizability" of the nucleon - accessible through Q^2-dependence of $\Gamma_1(Q^2)$

Valence Region and moderate Q^2: SFs for $x \rightarrow 1$

- SU(6)-symmetric wave function of the proton in the "naïve" quark model:

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\left(3u\uparrow[ud]_{S=0} + u\uparrow[ud]_{S=1} - \sqrt{2}u\downarrow[ud]_{S=1} - \sqrt{2}d\uparrow[uu]_{S=1} - 2d\downarrow[uu]_{S=1}\right)$$

- In this model: d/u = 1/2, Δu/u = 2/3, Δd/d = -1/3 for all $x \Rightarrow$

$$\sum_q \Delta q = 1 \quad \Rightarrow \quad S_p = \frac{1}{2}\sum_q \Delta q = \frac{1}{2}\Delta\Sigma; \quad g_A^{(3)} = \Delta u - \Delta d = 5/3; \quad g_A^{(8)} = \Delta u + \Delta d - 2\Delta s = 1$$

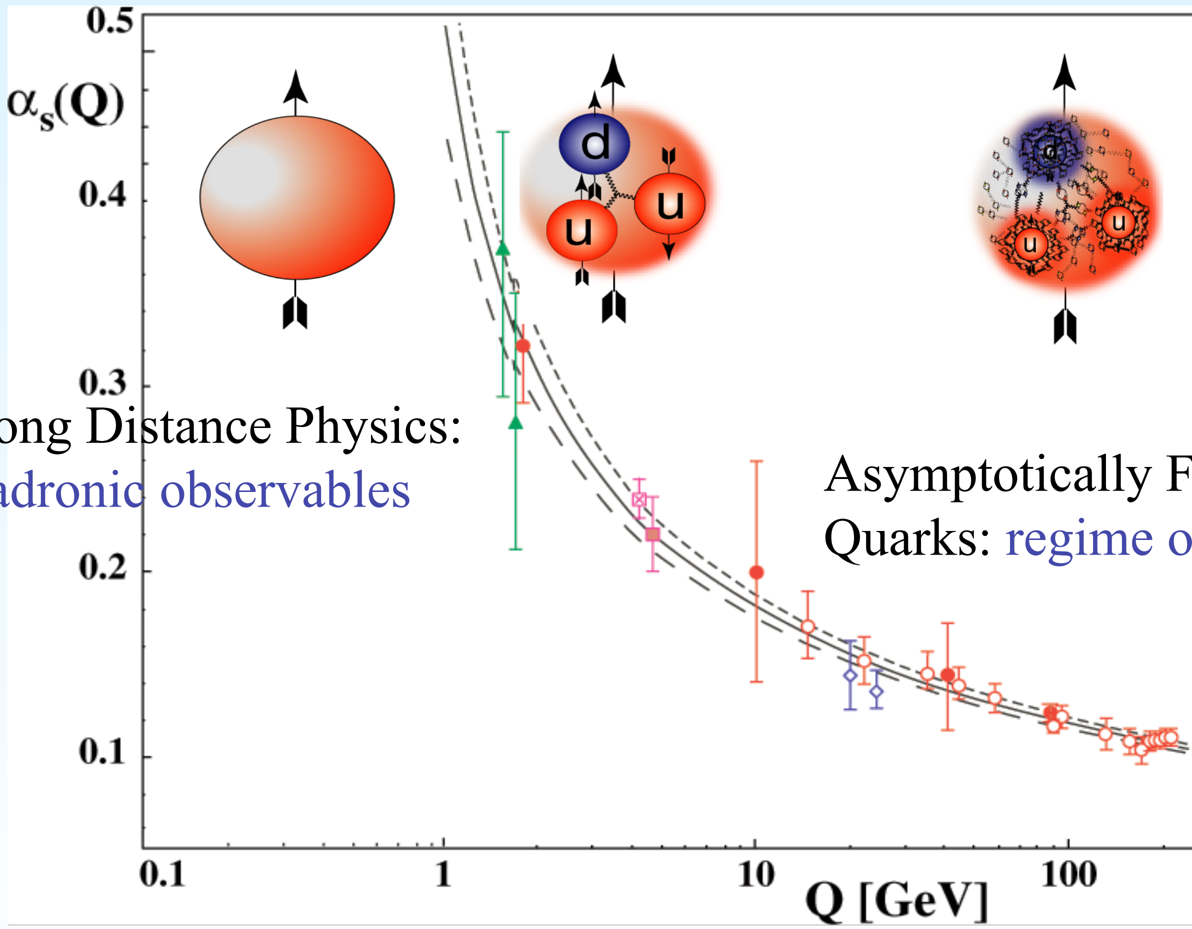
- Relativistic Correction: lower component reduces axial charge, adds to orbital angular momentum (p-wave) $\Rightarrow$

$$\sum_q \Delta q = \Delta\Sigma \approx 60\%; \quad g_A^{(3)} = \Delta u - \Delta d \approx 1.26; \quad g_A^{(8)} = \Delta u + \Delta d - 2\Delta s \approx 0.6$$

- Hyperfine structure effect: S=1 suppressed => d/u = 0, Δu/u = 1, Δd/d = -1/3 for $x \rightarrow 1$ => $A_{1p} = 1$, $A_{1n} = 1$, $A_{1D} = 1$

- pQCD: helicity conservation ($q\uparrow\uparrow p$) => d/u =2/(9+1) = 1/5, Δu/u = 1, Δd/d = 1 for $x \rightarrow 1$

Duality



Long Distance Physics:
hadronic observables

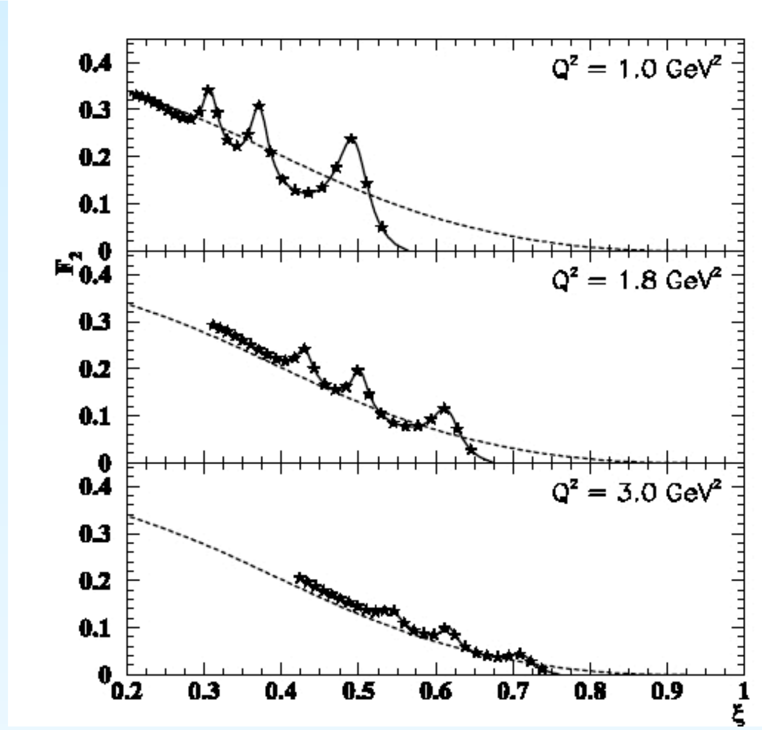
Asymptotically Free
Quarks: regime of pQCD

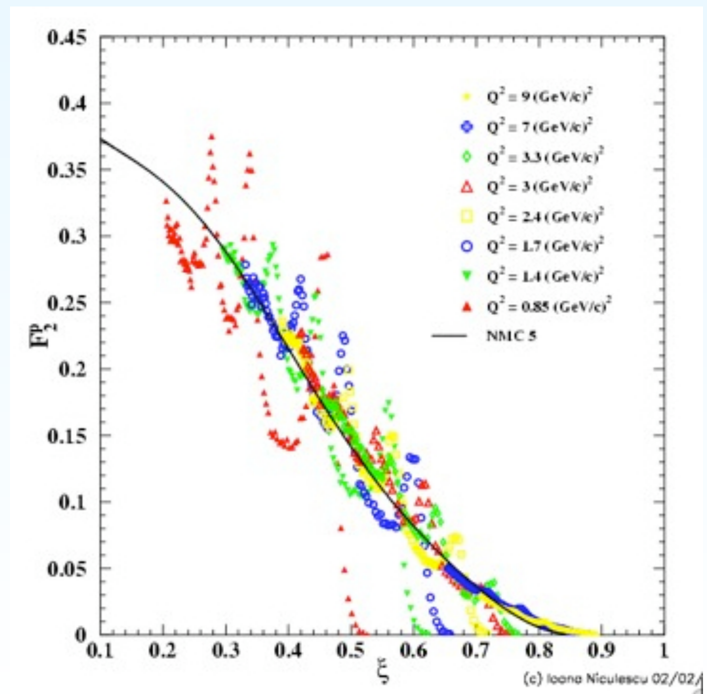
κ g_A

- Nucleon resonances at low Q^2 average to the scaling curve measured in DIS
  - Bloom and Gilman, PRL **25**, 1140 (1970); PRD **4**, 2901 (1971)
- Observed with high precision in the unpolarized F_2^p structure function in Hall C, Jlab
  - I. Niculescu *et al.,* PRL **85**, 1182, 1186 (2000)
- Local duality also observed (*i.e.,* average over a smaller range in W)
- Related to the absence of higher twist strength in structure function moments
- Also valid for spin structure functions? Not so obvious - can change in sign:

$$A_1^{DIS}(x \to 1) \to 1$$

$$A_1^{\Delta}(\text{low } Q^2) \approx -\frac{1}{2} \quad \left(\sigma_{\frac{3}{2}} > \sigma_{\frac{1}{2}}\right)$$





The Limit $Q^2 \to 0$: GDH Sum Rule

$$I_{GDH} = \frac{M^2}{8\alpha\pi^2}\int_{thr}^{\infty}\left(\sigma_{1/2}-\sigma_{3/2}\right)\frac{d\nu}{\nu} = -\frac{1}{4}\kappa^2$$

- relates the difference of the photo-absorption cross section for helicity 1/2 and 3/2 to the nucleon magnetic moment, i.e. a connection between dynamic and static properties

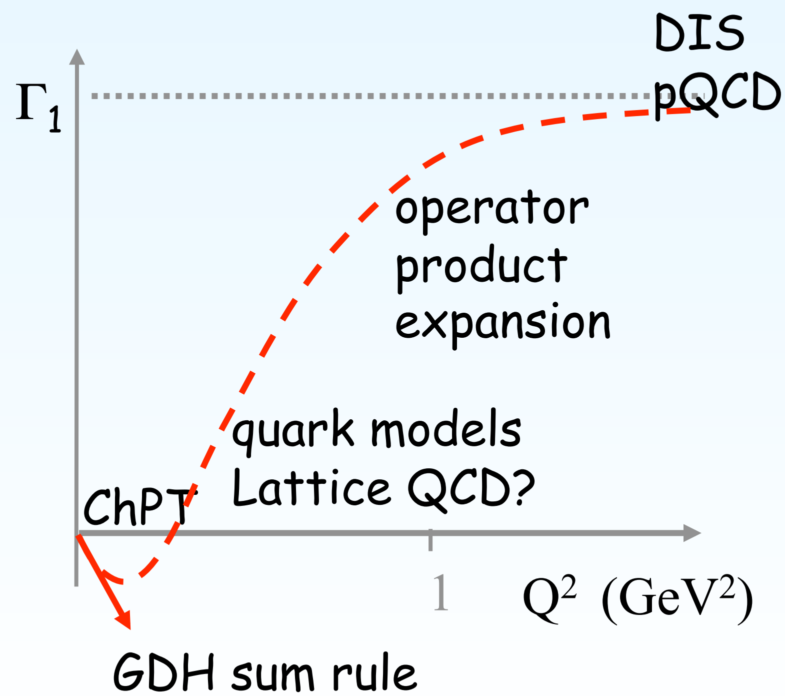
- based on very general principles, as gauge invariance, dispersion relation, low energy theorem

- at finite Q^2 can be related to the integral of the spin structure function g_1

$$\Gamma_1 = \int g_1\left(x,Q^2\right)dx \xrightarrow[Q^2\to 0]{} \frac{Q^2}{2M^2} I_{GDH}$$

- strong variation of nucleon spin properties as a function of Q^2

- Q^2-dependence described by Chiral Perturbation Theory (χPT) at low Q^2



The Limit $Q^2 \to 0$: Spin Polarizability

$$\int_{thr}^{\infty} (\sigma_{1/2} - \sigma_{3/2}) \frac{d\nu}{\nu^3} = 4\pi^2 \gamma_0$$

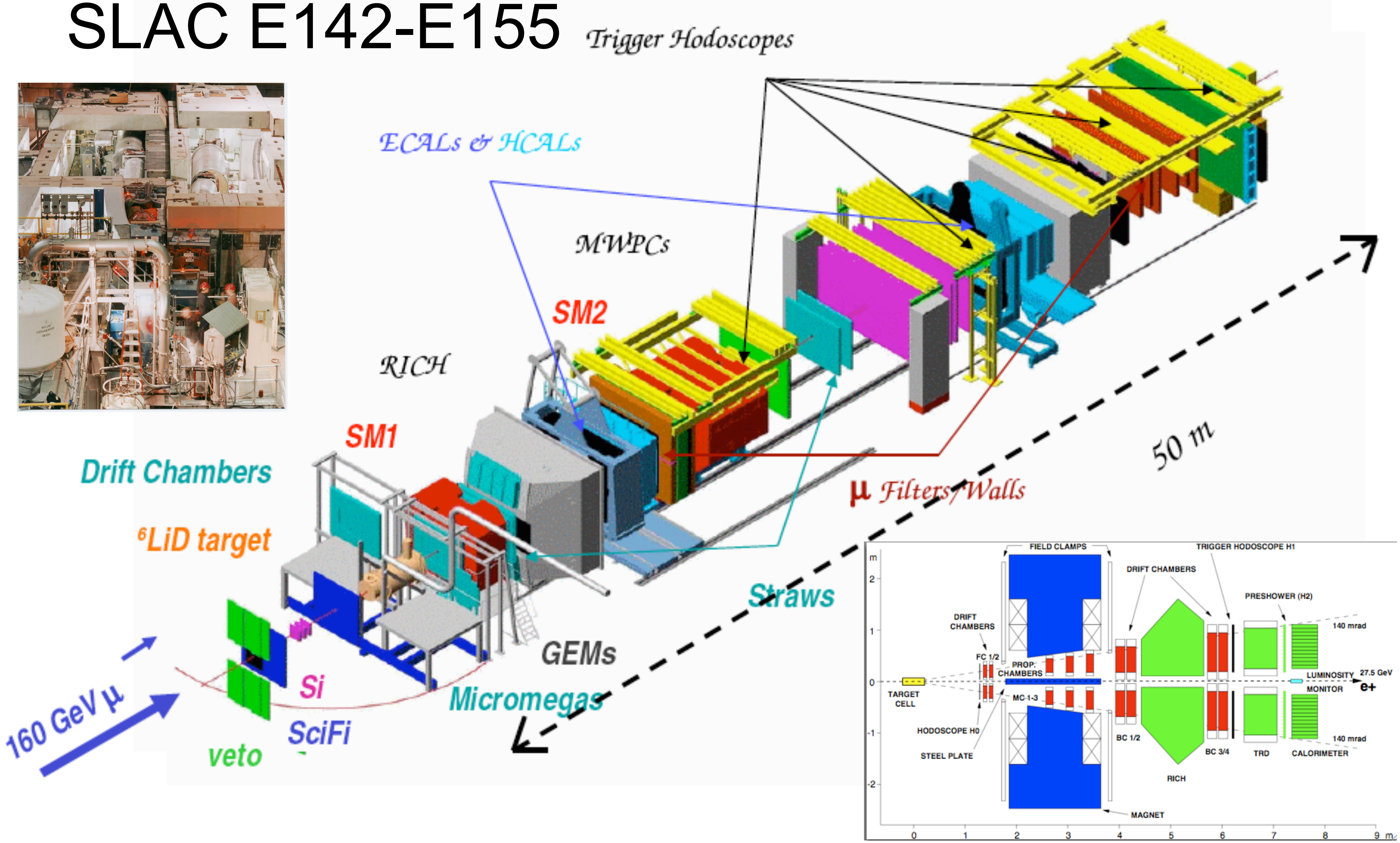
- γ_0 measures the response ("stiffness") of the nucleon spin against electromagnetic deformations along the spin axis
- Follows from same dispersion relation and low energy theorem (limit of forward Compton scattering) as GDH sum rule
- can also be extended to finite Q^2:

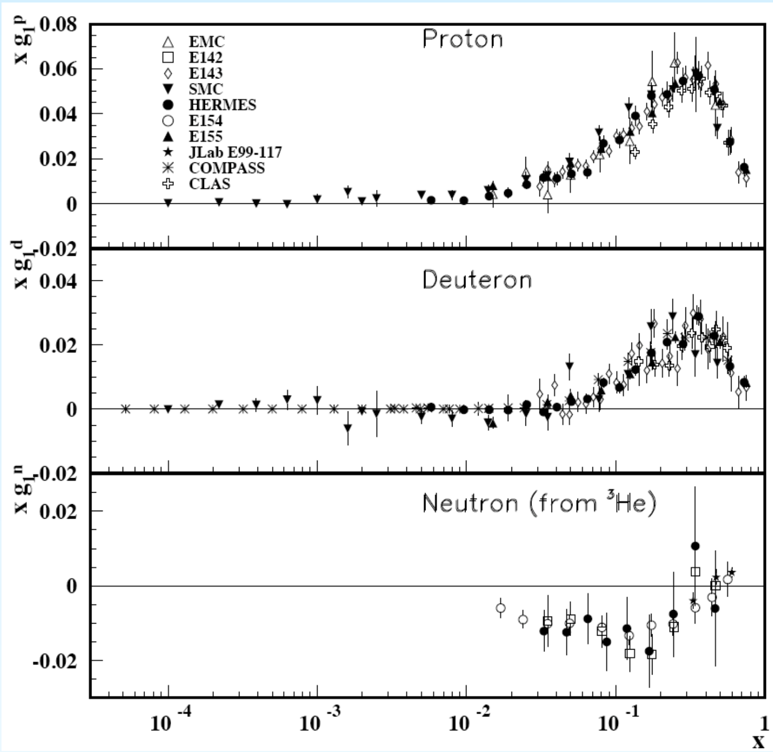
$$\Gamma_3^N = \int x^2 g_1{}^N\left(x,Q^2\right) dx \xrightarrow[Q^2 \to 0]{} \frac{Q^6}{16\alpha M^2} \gamma_0{}^N$$

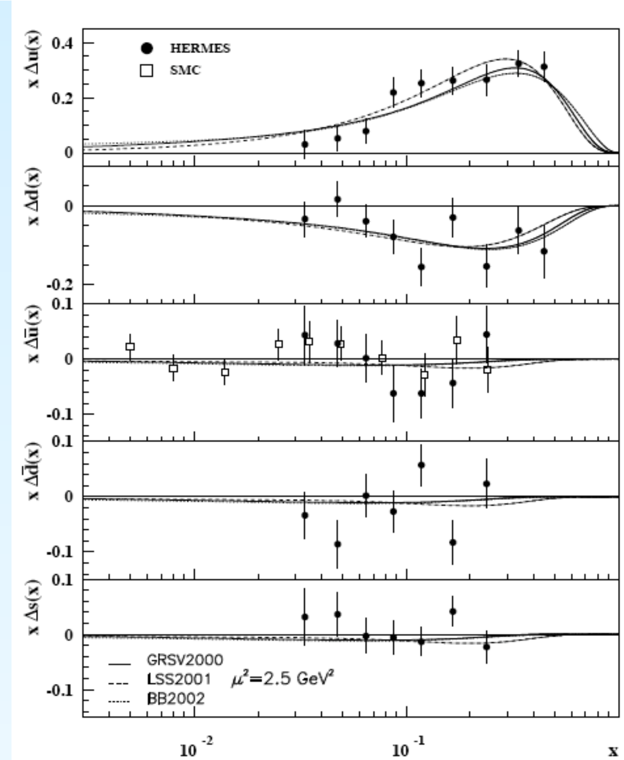
- much more sensitive to low-energy (high x) part of the integral -> ideal for Jlab
- plus other polarizabilities: δ_{LT}
- $\Rightarrow$ Chiral Perturbation Theory should be able to predict $\gamma_0(Q^2)$, $\delta_{LT}(Q^2)$ and **b** in

$$\Gamma_1\left(Q^2\right) = -\frac{\kappa^2}{8M^2} Q^2 + bQ^4 \ldots$$

Results from High-Energy Experiments: EMC, SMC, COMPASS, HERMES, SLAC E142-E155



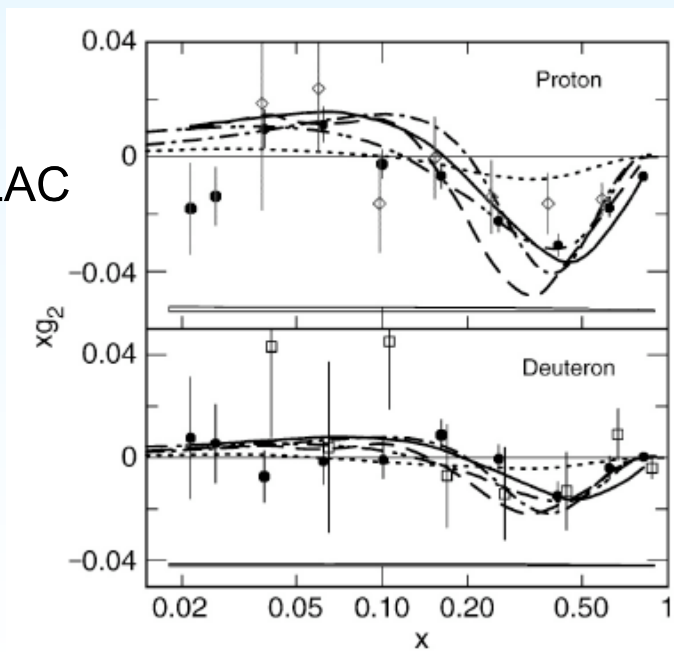


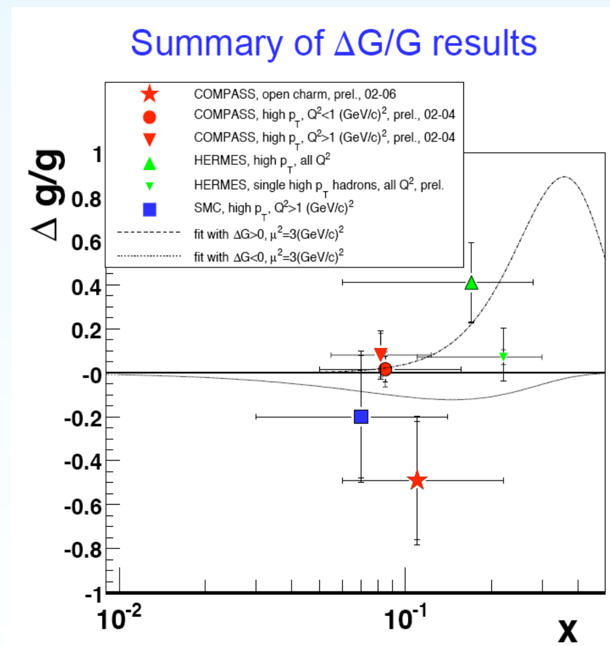


inclusive

semi-inclusive

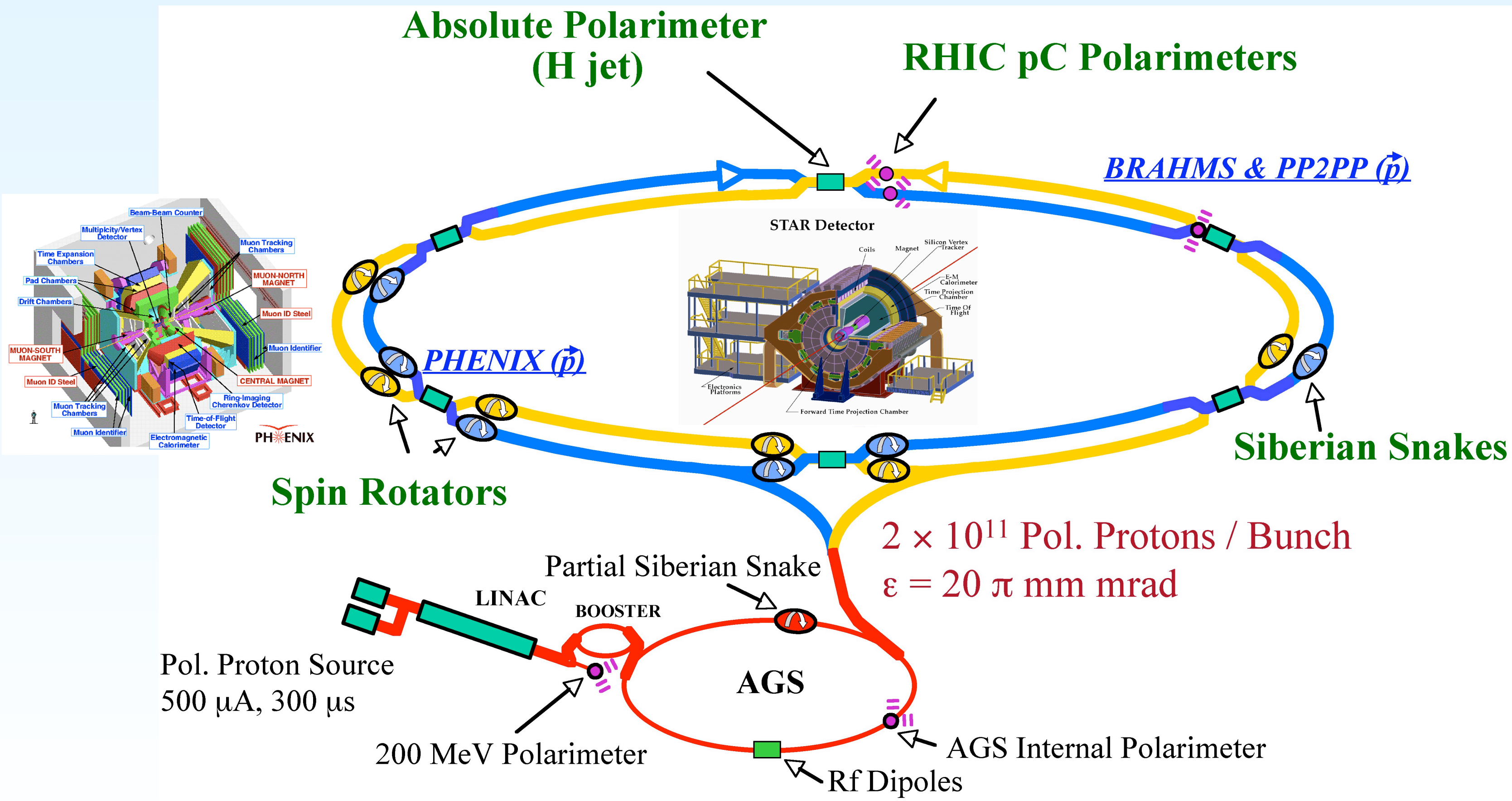
SLAC



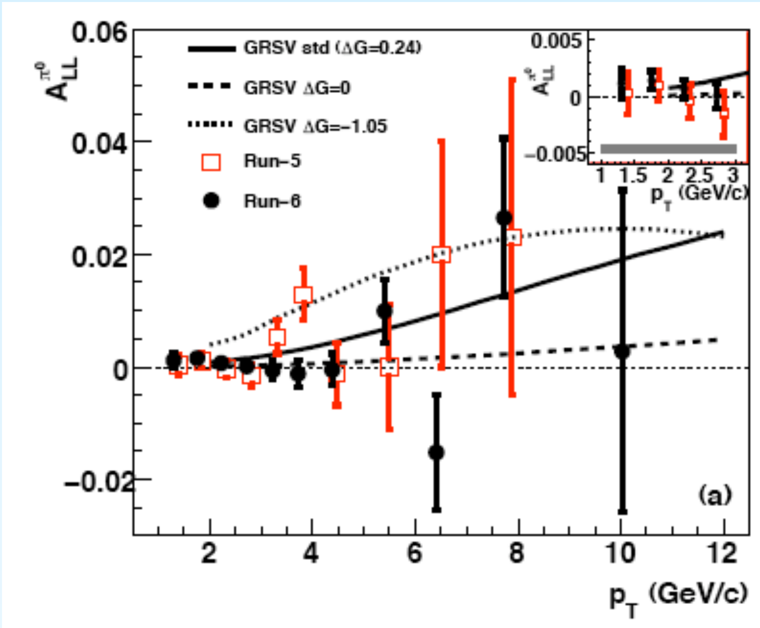


Spin Program at RHIC

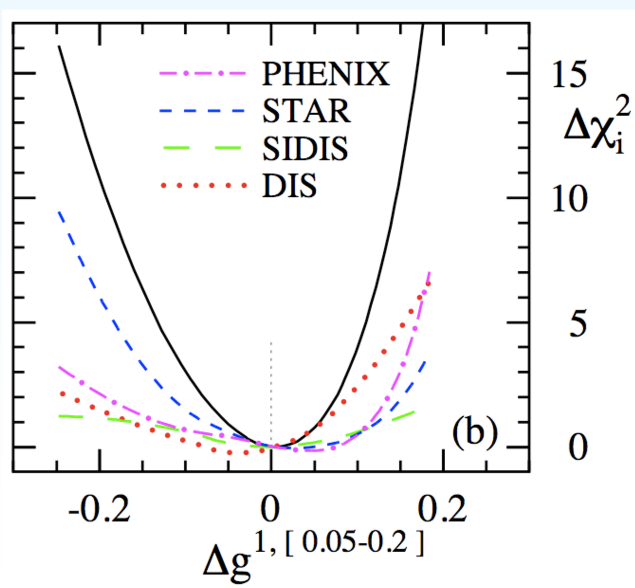
Proton-Proton collisions at $\sqrt{s}$ = 64,200, 500 GeV
qq, qg and gg elementary interactions

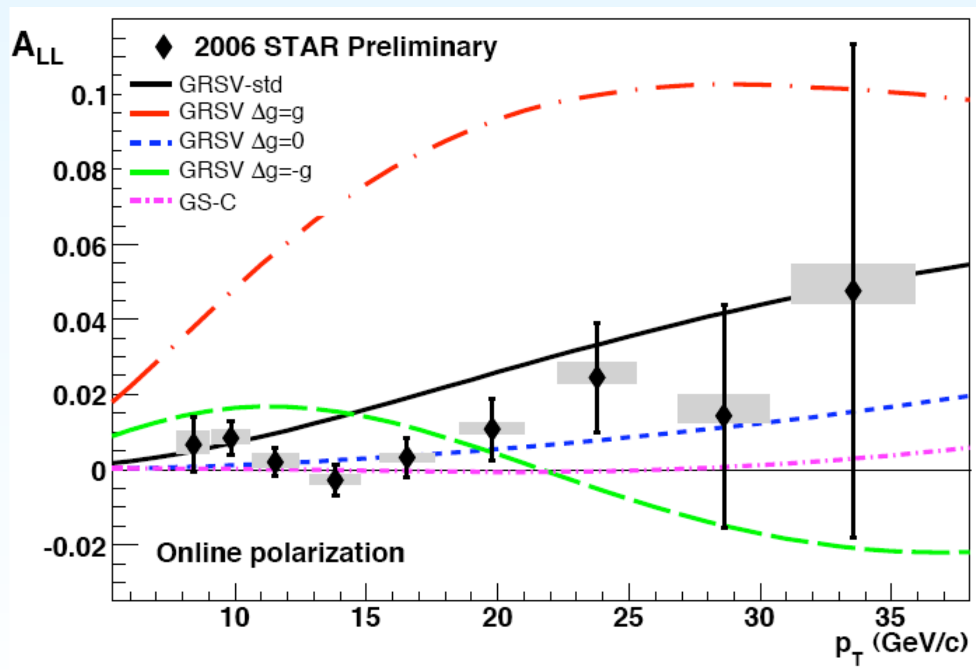


Preliminary Result:
ΔG appears small
in measured region



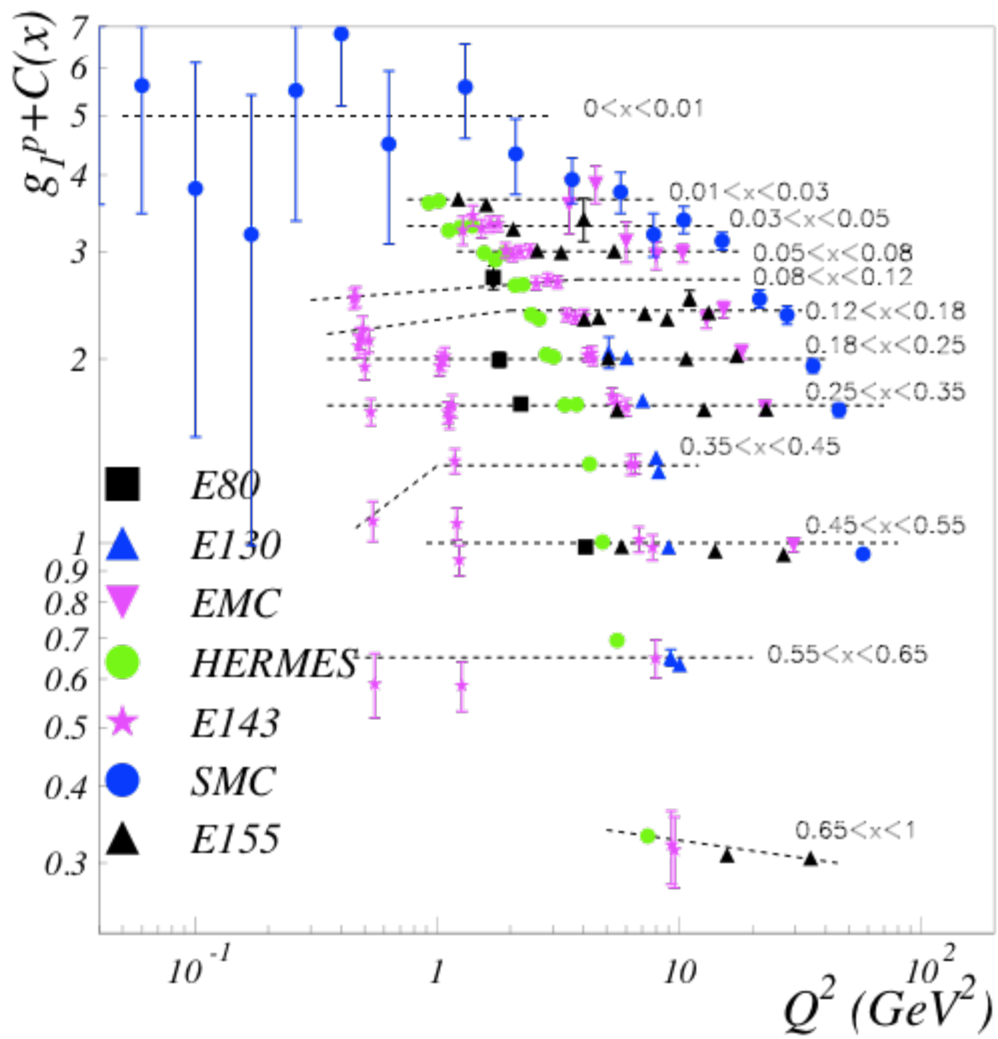
PHENIX $\vec{p} + \vec{p} \rightarrow \pi^0 + X$





Contributions from Jefferson Lab

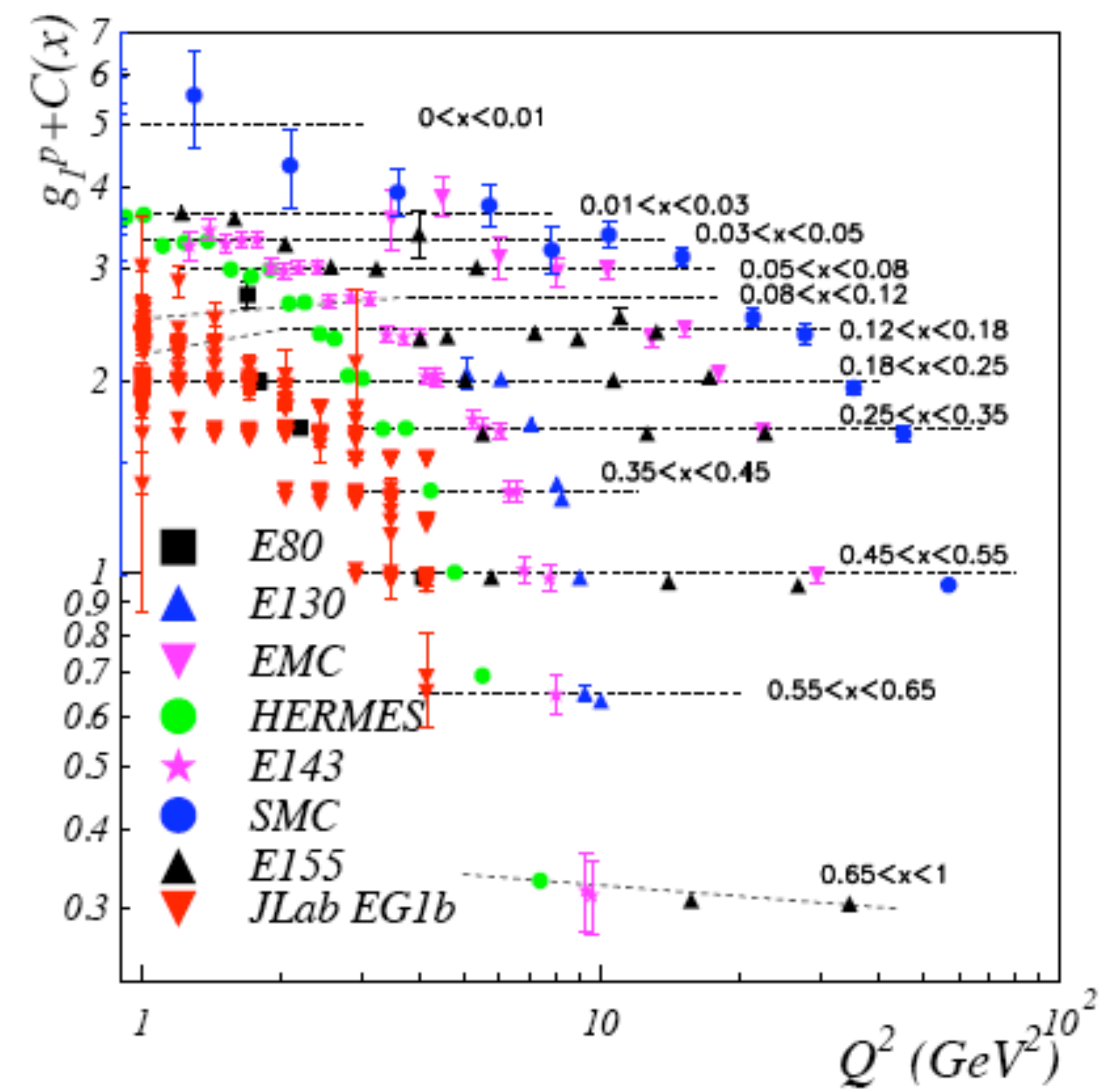
World data on the proton before JLab
(without COMPASS)



Contributions from Jefferson Lab

World data on the proton before JLab
(without COMPASS)

World data on the proton including EG1

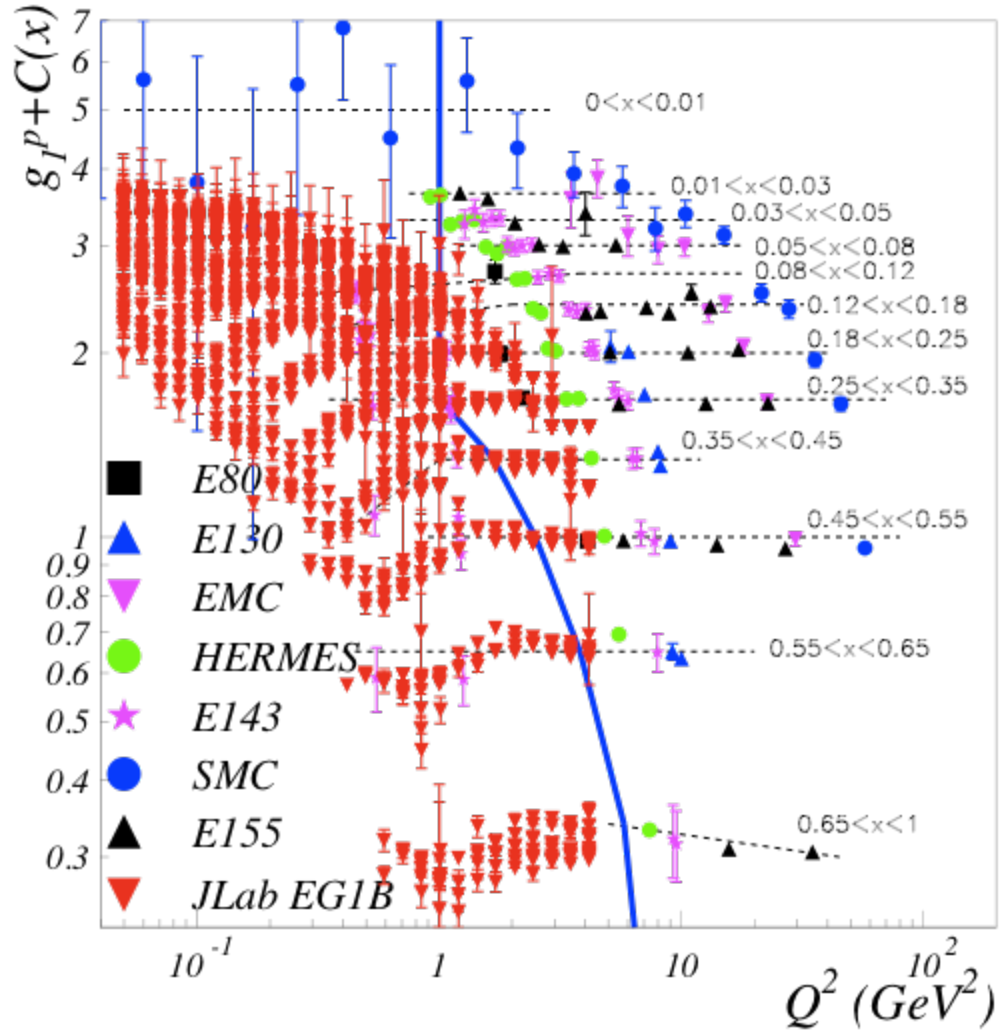


Contributions from Jefferson Lab

World data on the proton before JLab
(without COMPASS)

World data on the proton including EG1

…including resonance region data!



JLab Experiments - Kinematic Coverage

"Everything"

Sum Rules at low Q^2

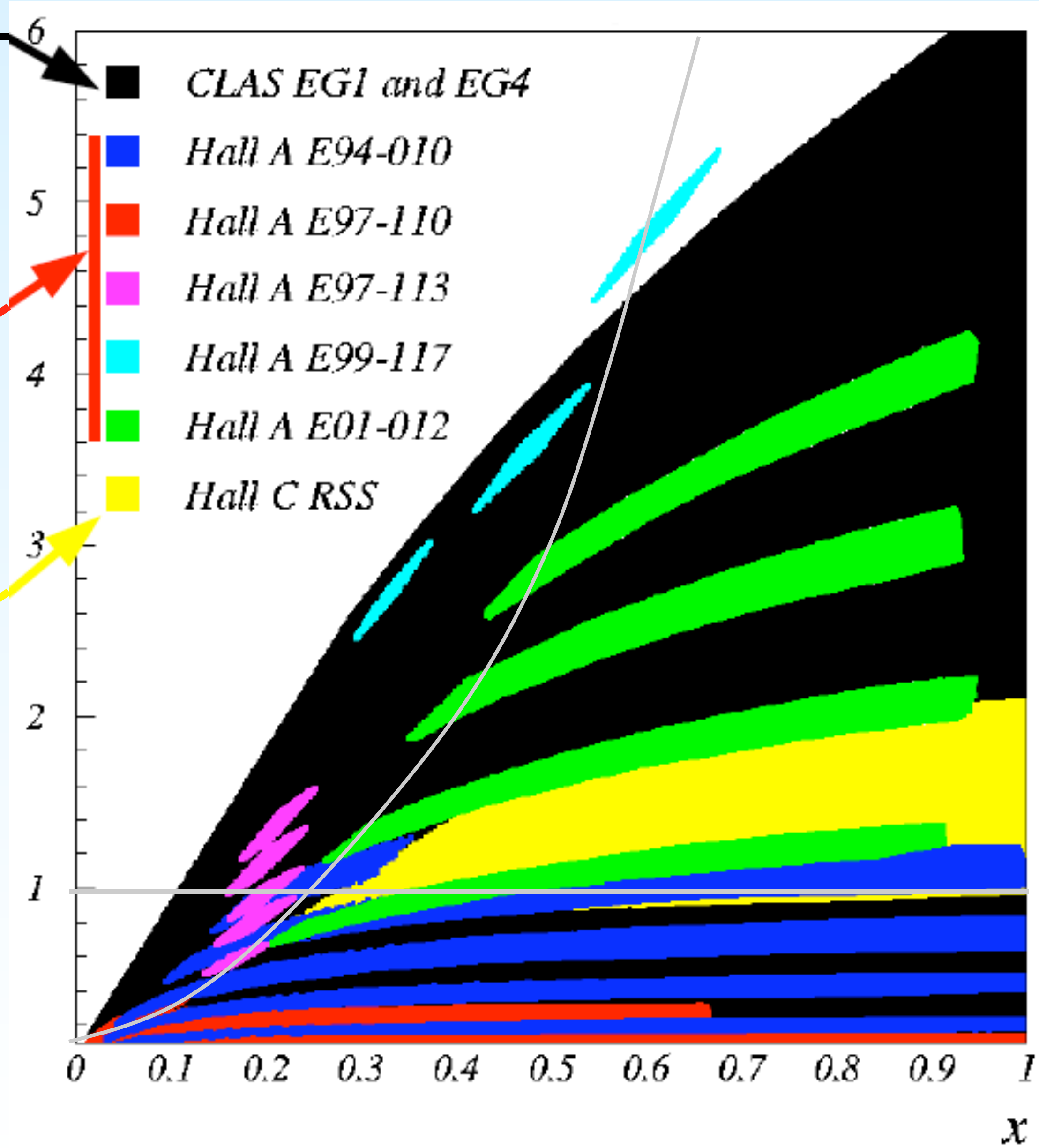
very low Q^2 - χPT

Q^2-dep. of g_2

A_{1n} at high x

Duality

Res. Region, Duality



✓8 completed experiments

3 (+3) approved with 6 GeV JLab

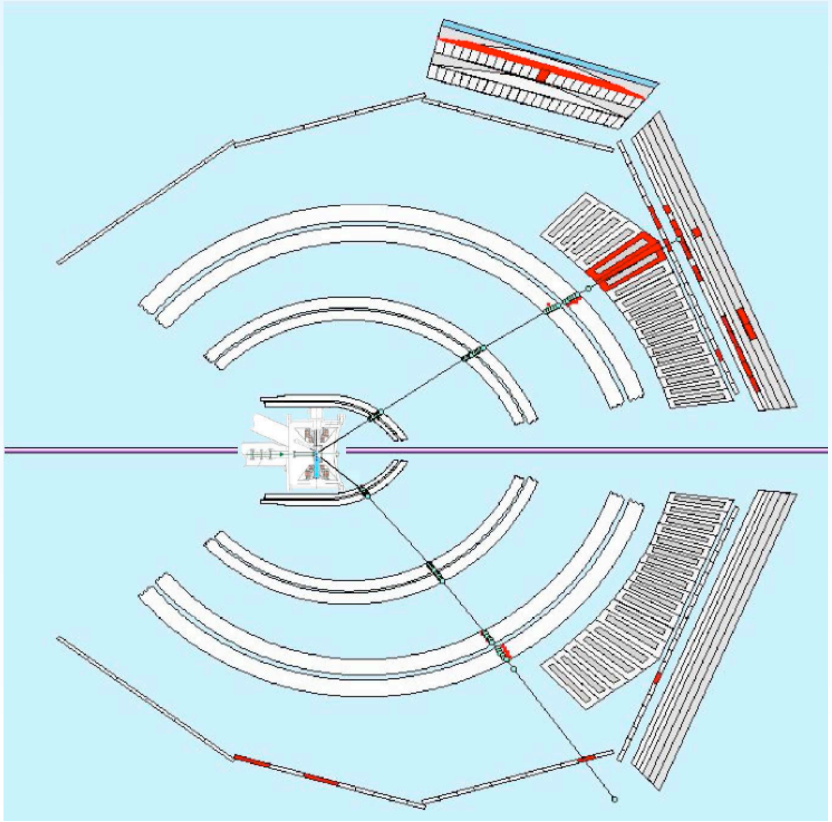
3 (+1) approved with 12 GeV (A/B/C)

Experiments EG1 and EG4 with CLAS

EG1: $Q^2 = 0.05 \ldots 5\ \mathrm{GeV}^2$

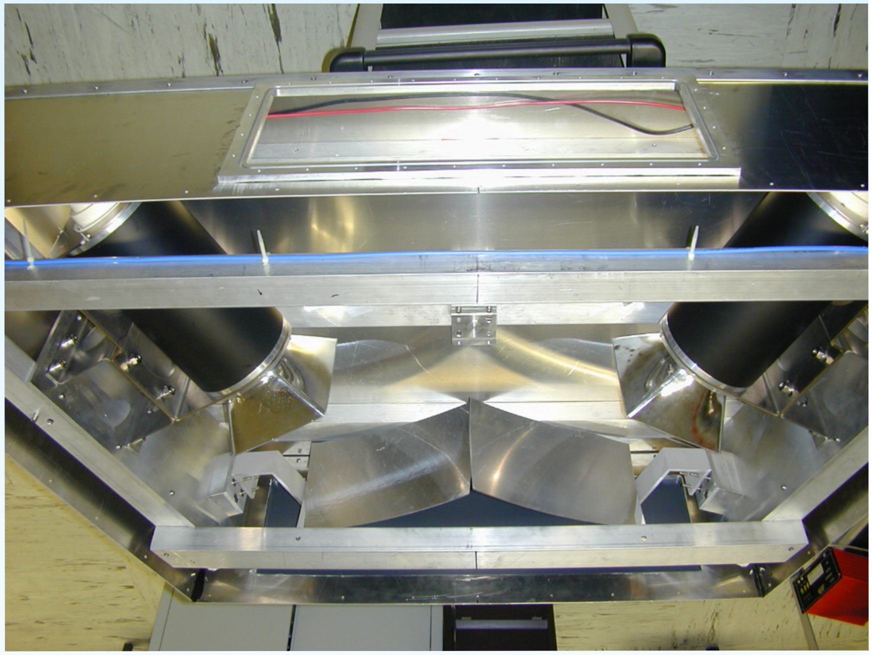
Largest possible kinematic coverage
→ inbending and outbending configuration, E = 1.6…5.8 GeV

1998 - 2001



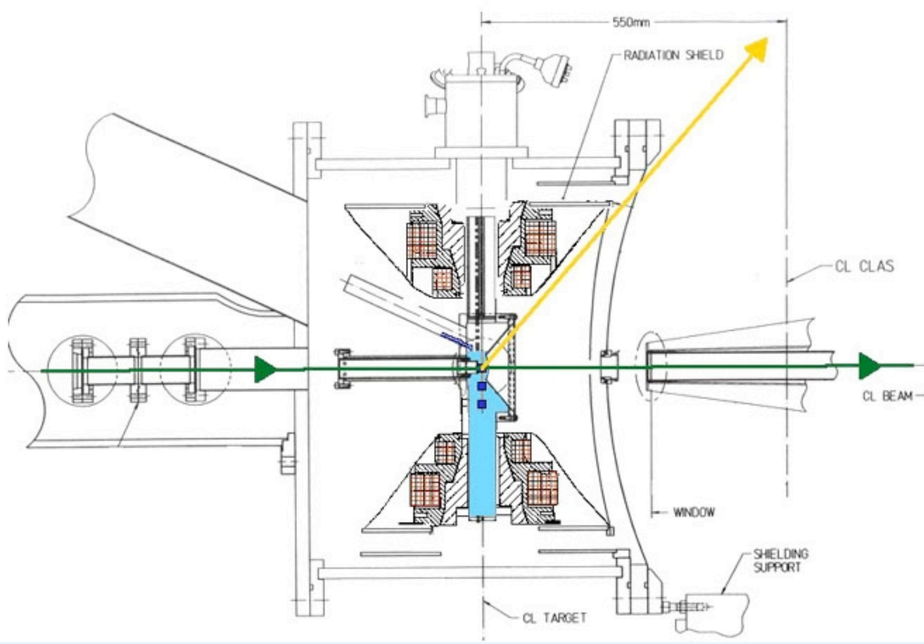
EG4: $Q^2_{min}=0.015\ \mathrm{GeV}^2$

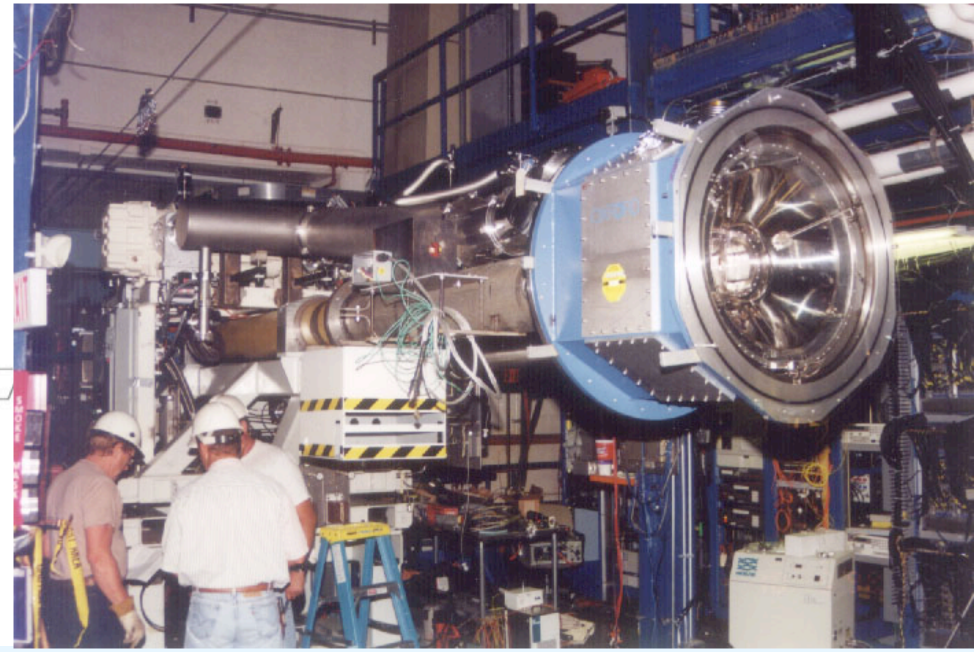
note: $m_\pi^2 = 0.02\ \mathrm{GeV}^2$



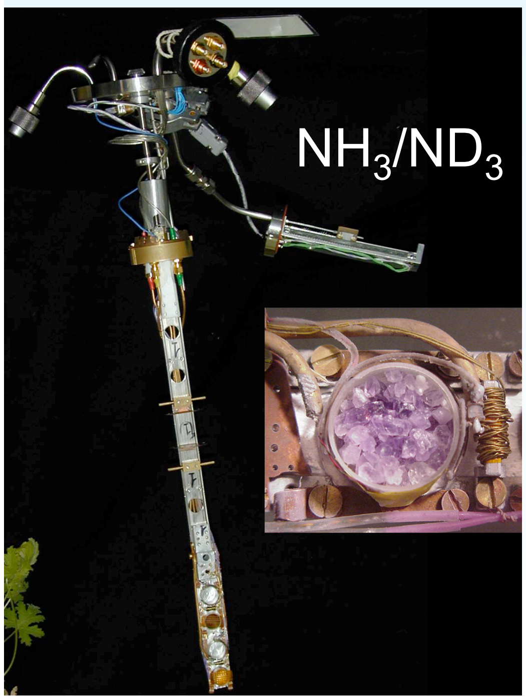
Focus on low Q^2 (GDH, χPT) => lower beam energies, new Cherenkov for optimal acceptance in outbending configuration, θ_e as small as 6 degrees

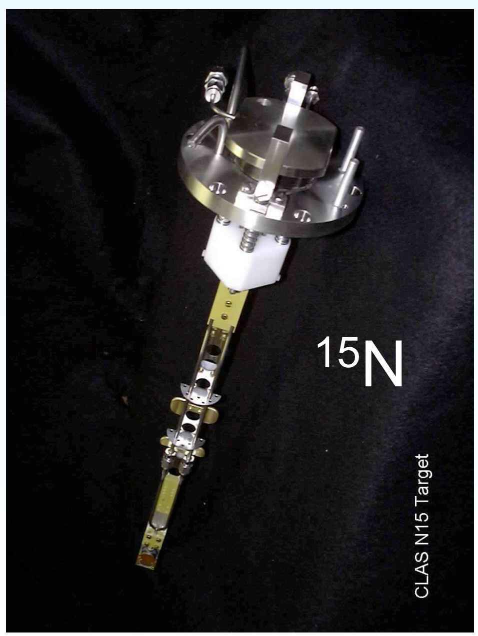
2006

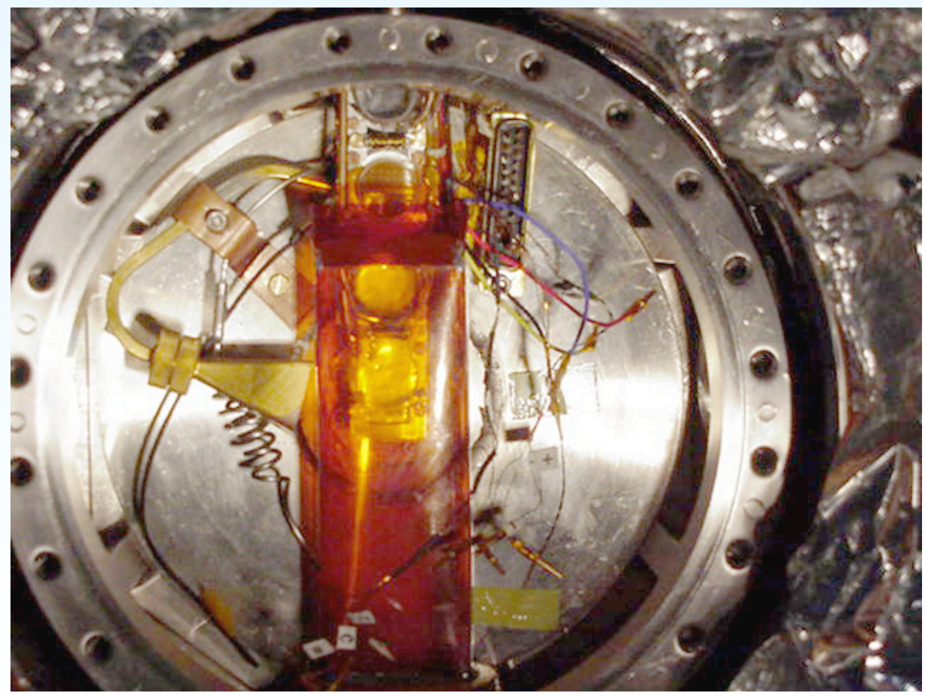




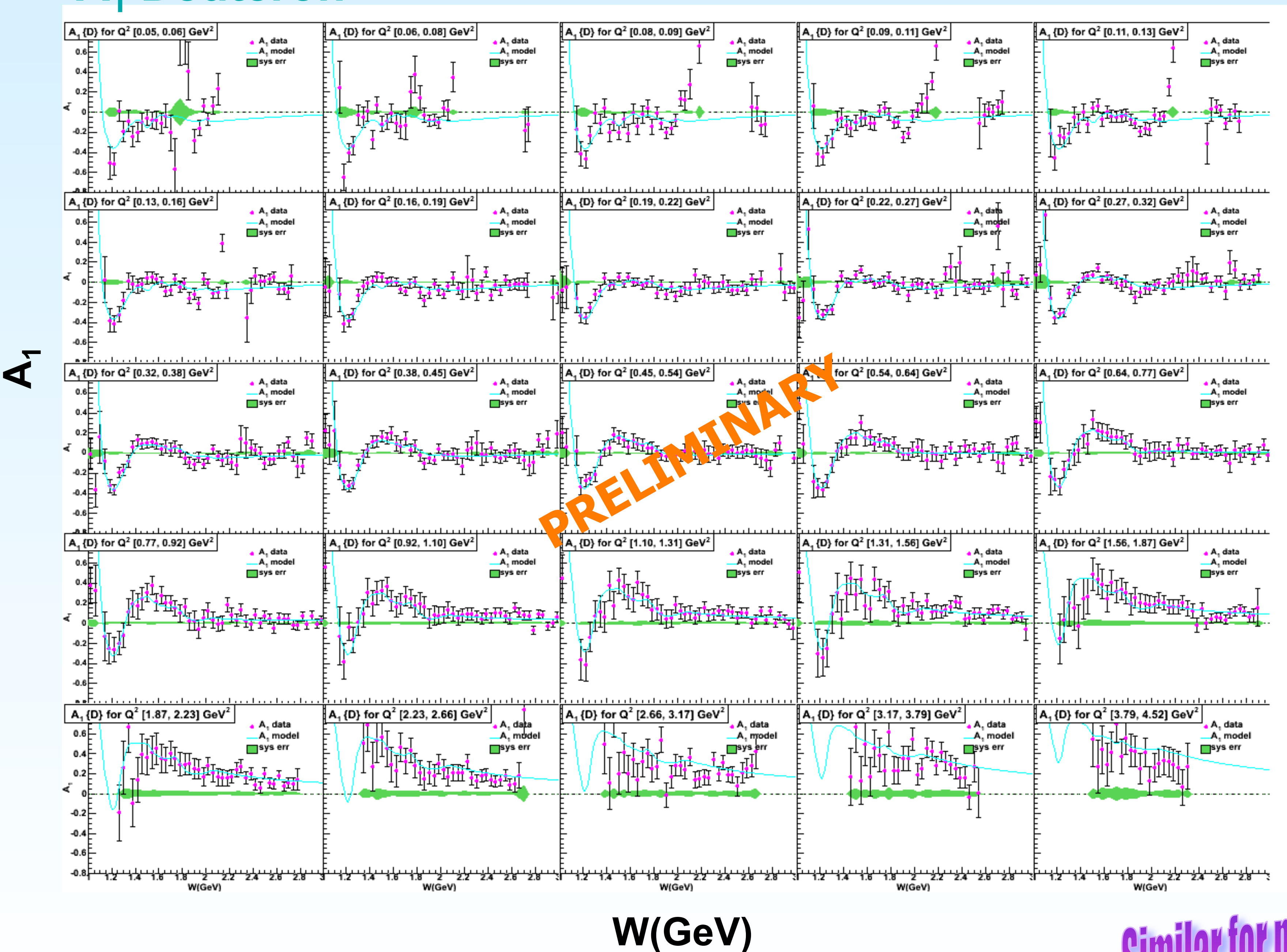
EG1/EG4 target (CLAS): Polarization up to 0.9 (p) or 0.4 (d)





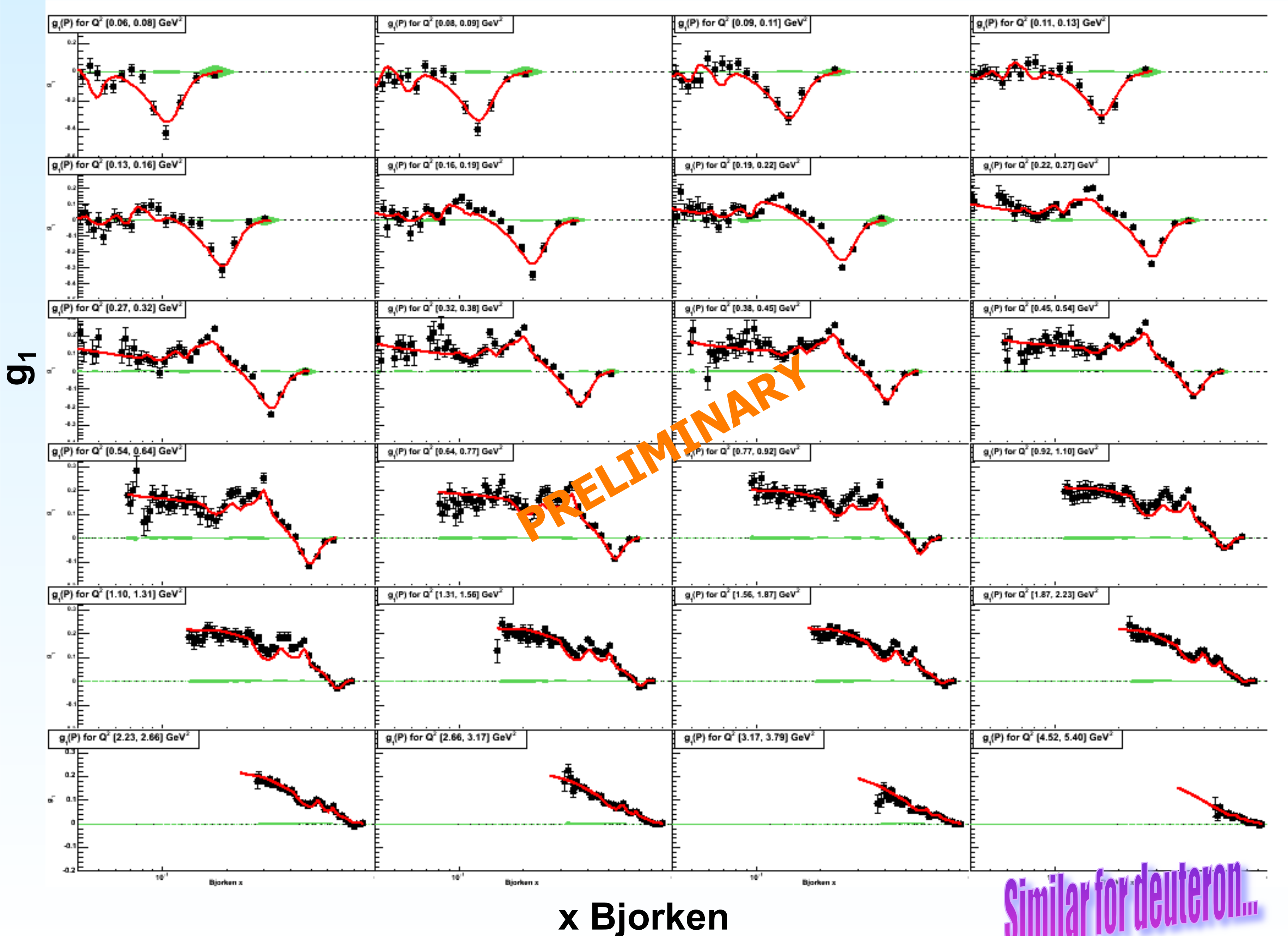


A_1 Deuteron



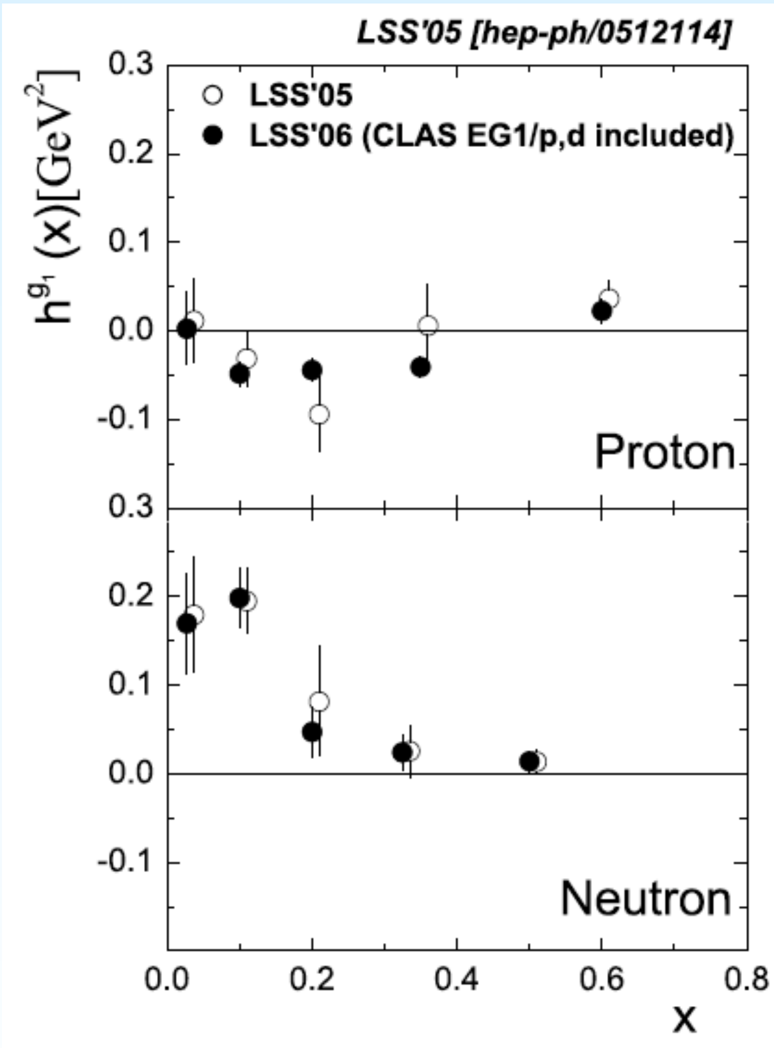
Similar for proton...

g_1 Proton for different Q^2 bins

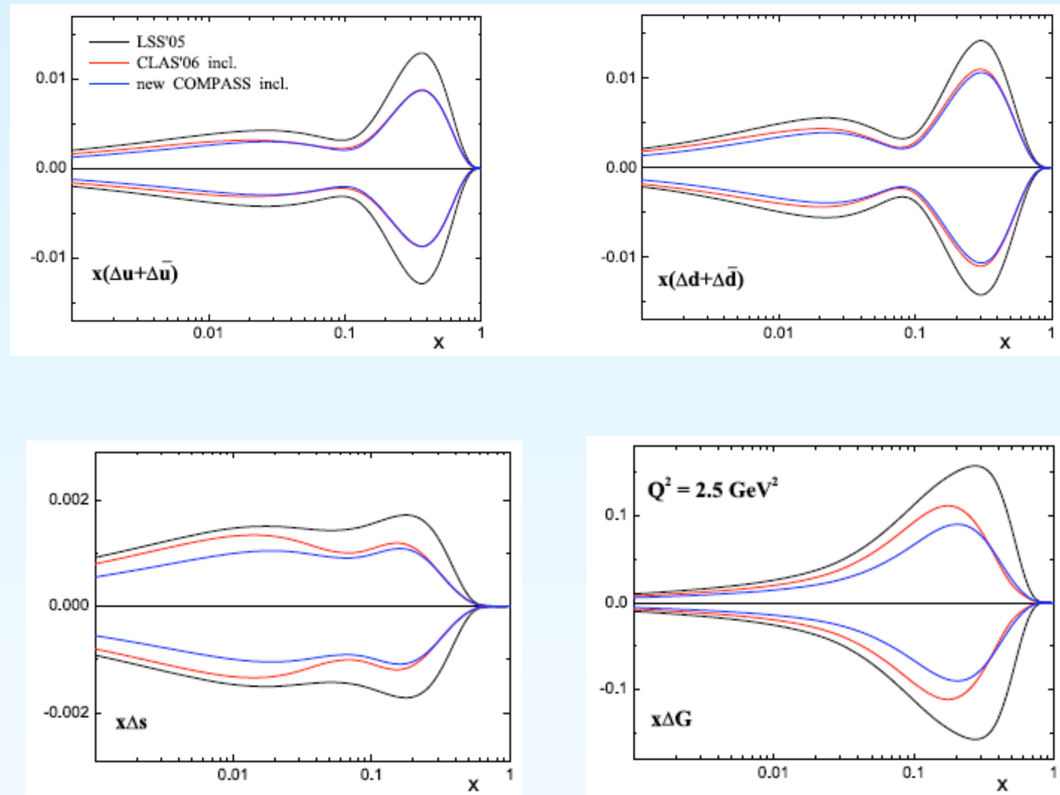


Similar for deuteron...

Effect of CLAS data on NLO fits of PDFs

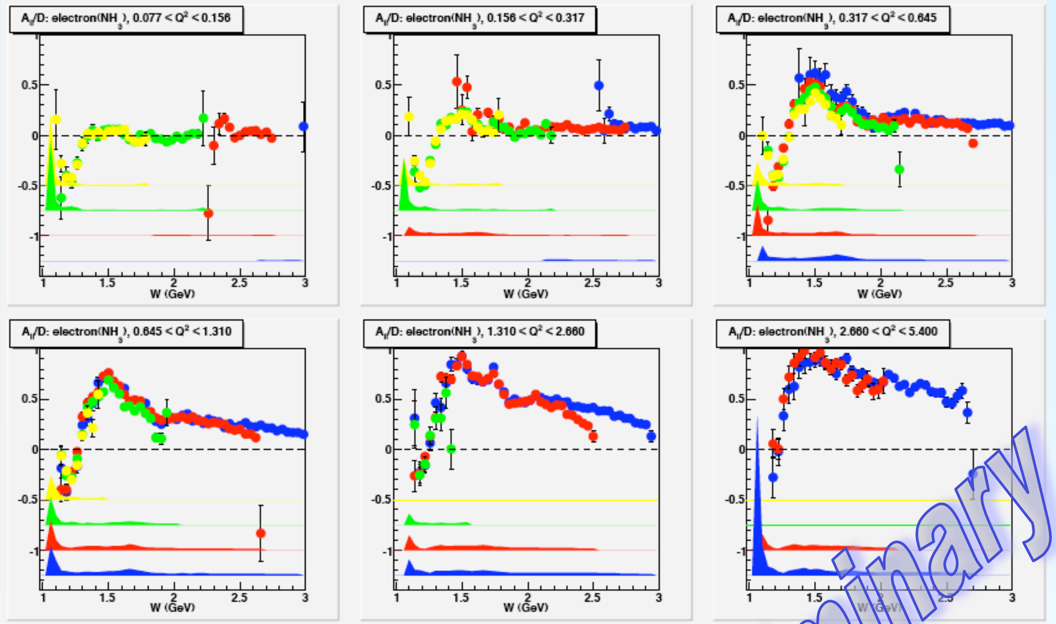


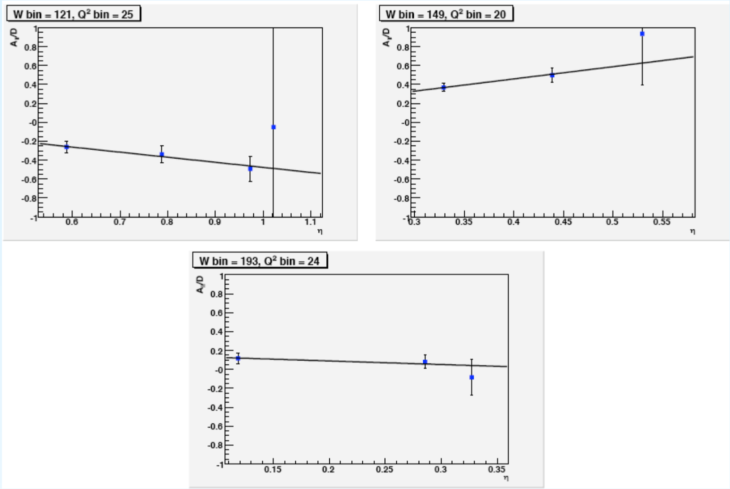
Higher Twist contribution to g_1

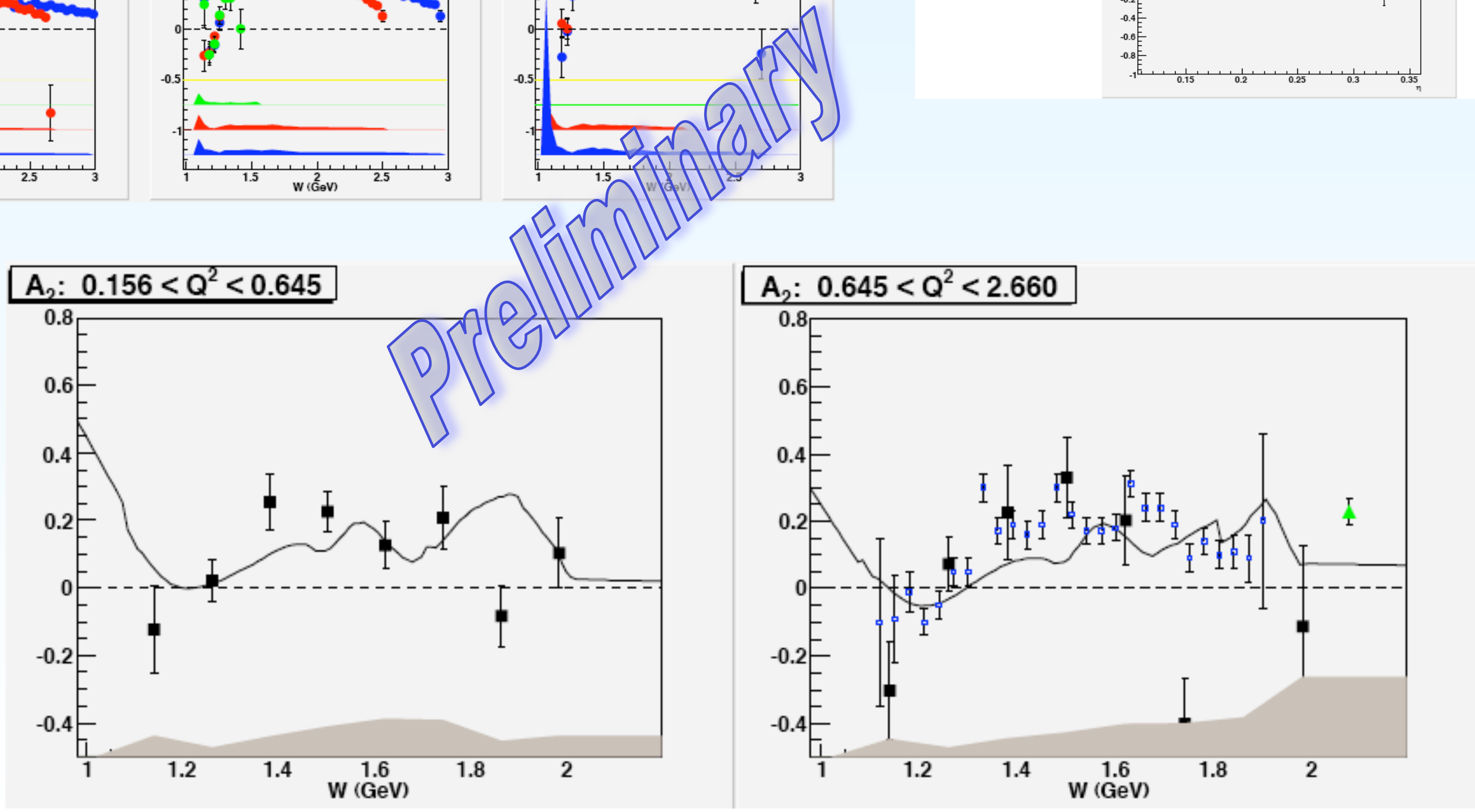


NLO fit by Leader, Stamenov and Siderov, including both CLAS data and new COMPASS data on the deuteron

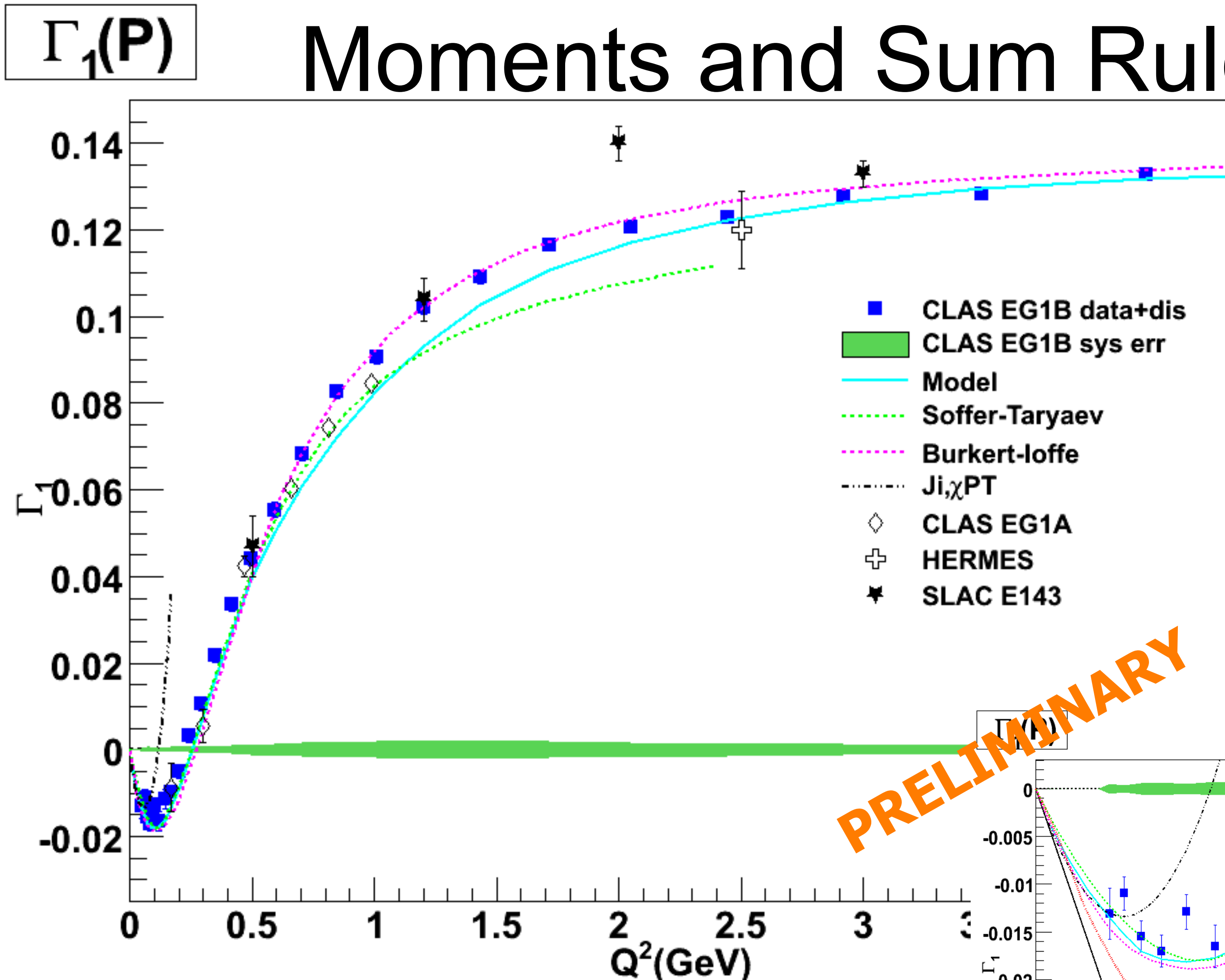
Extracting A_2 / g_2

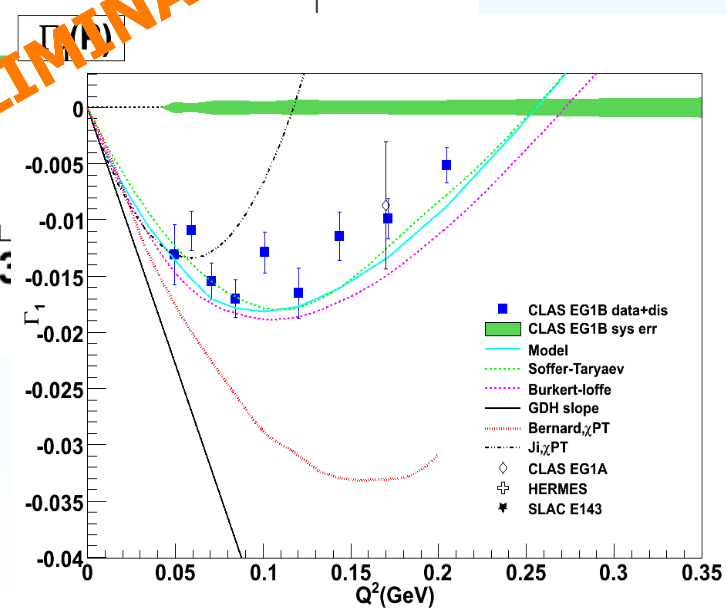






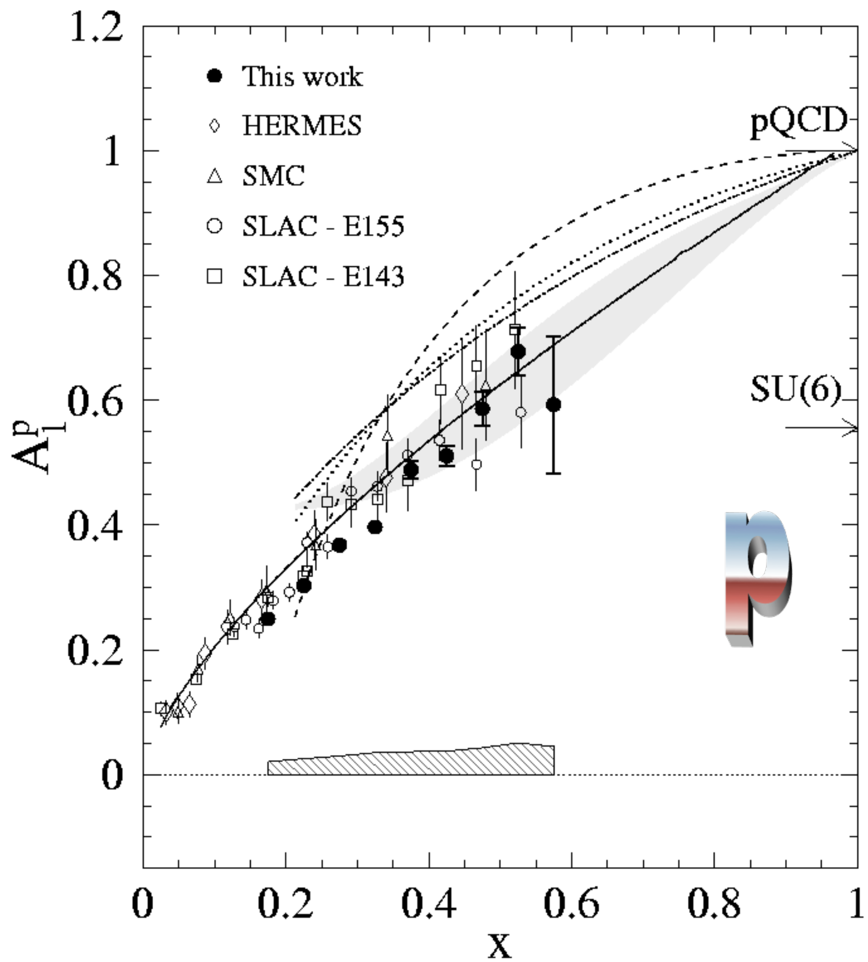
Moments and Sum Rules

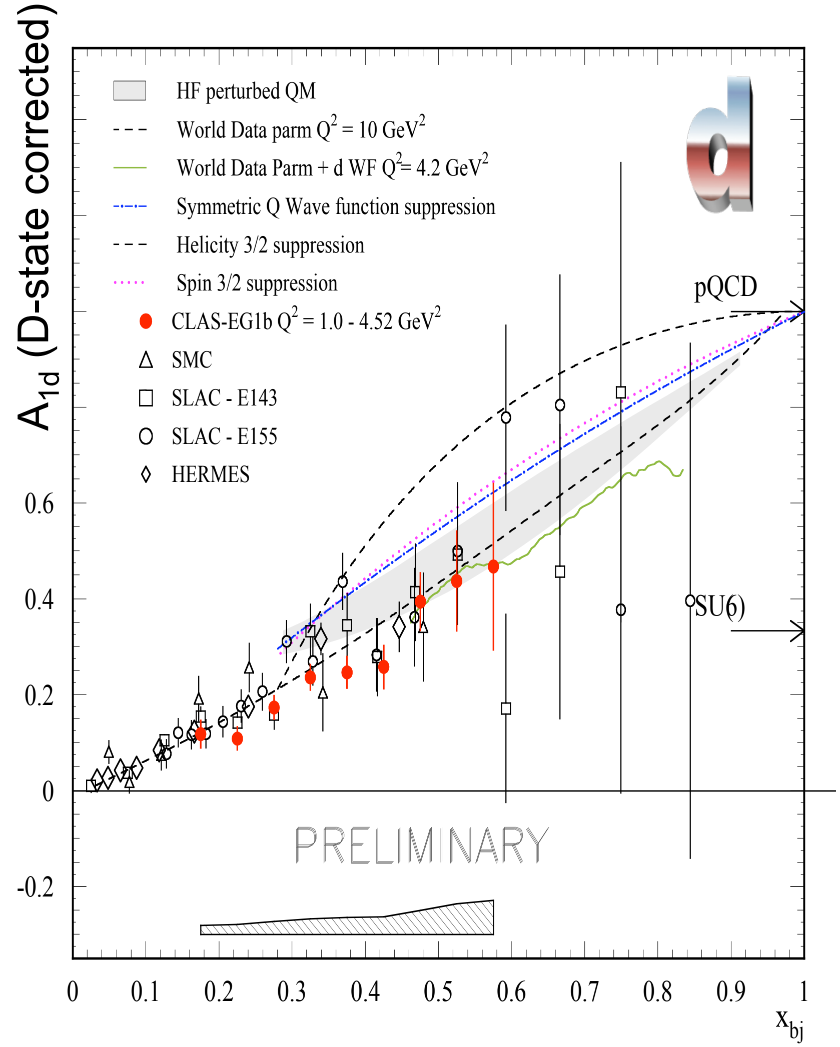




Similar for deuteron...

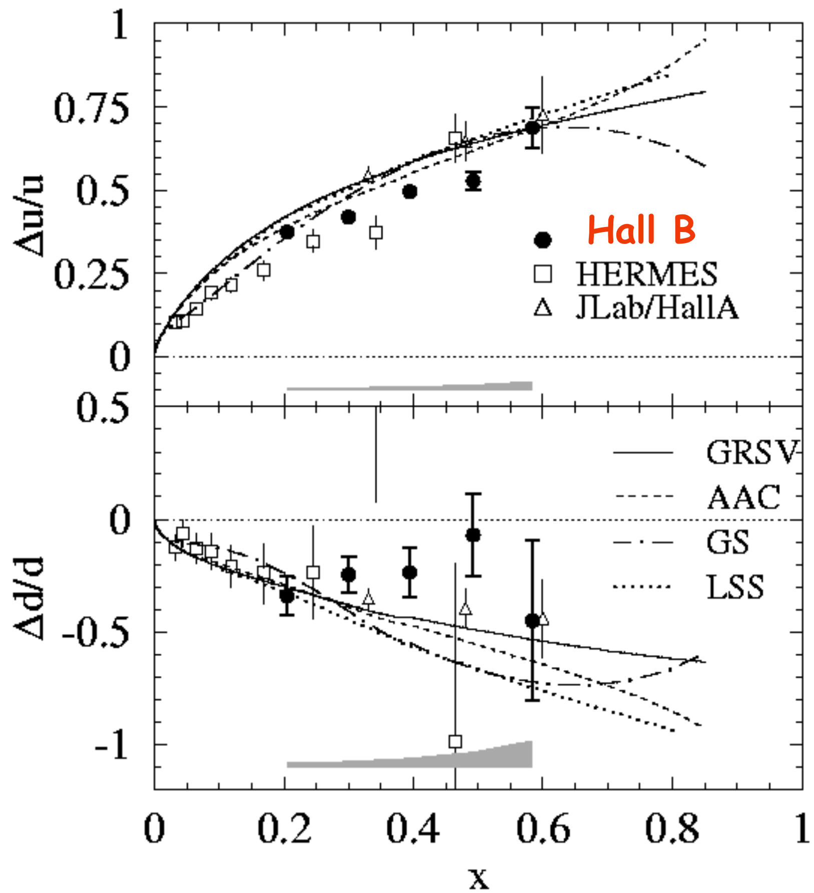
Virtual photon asymmetry A_1

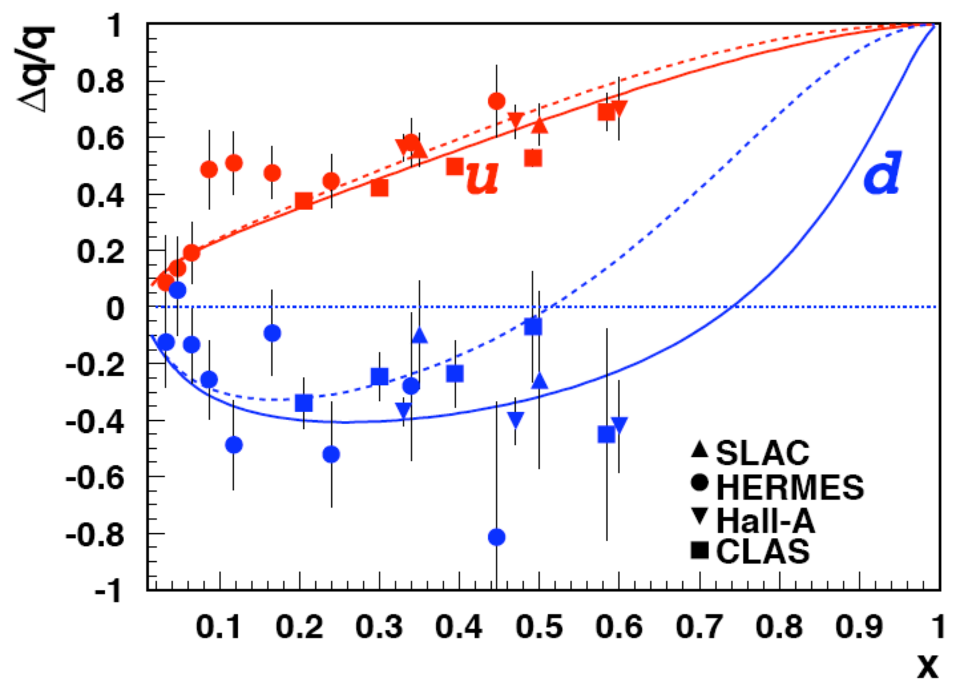




N. Isgur, Phys. Rev. D 59, 34013 *F. Close and W. Melnitchouk, Phys. Rev. C 68, 035210*

Combined analysis: “naïve” quark polarizations





Avakian et al., Phys.Rev.Lett.99:082001,2007

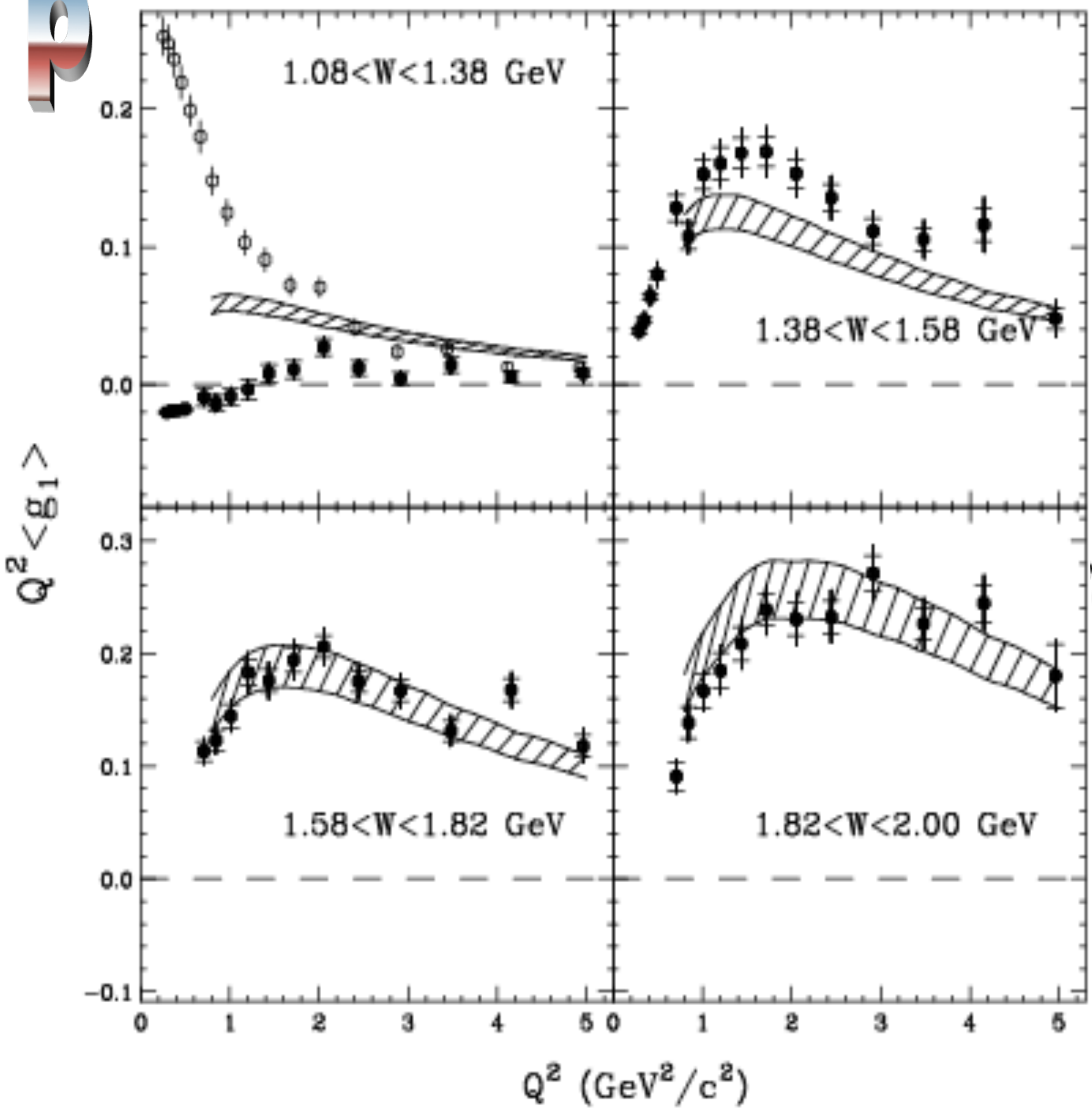
Disagrees with pQCD expectations (helicity conservation)

$$\frac{\Delta u}{u} \approx \frac{5g_1^p - 2g_1^d/(1-1.5\omega_D)}{5F_1^p - 2F_1^d}$$

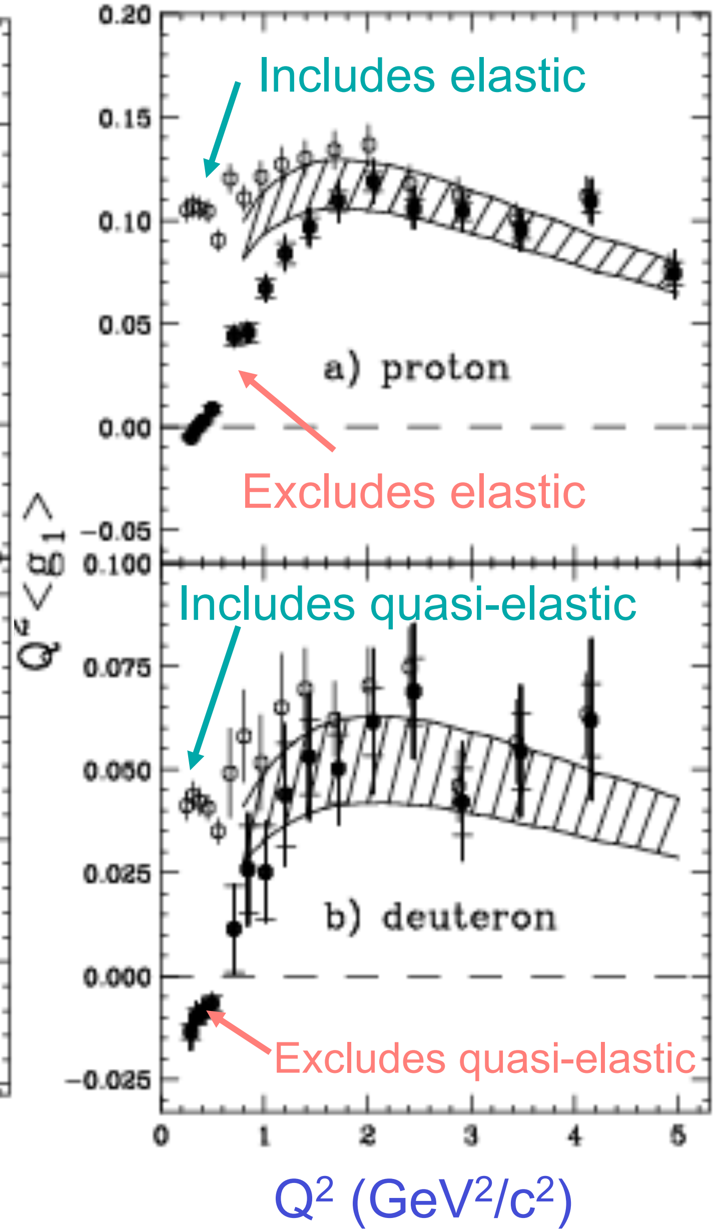
$$\frac{\Delta d}{d} \approx \frac{8g_1^d/(1-1.5\omega_D) - 5g_1^p}{8F_1^d - 5F_1^p}$$

Orbital angular momentum may change this picture

Local and "global" Duality

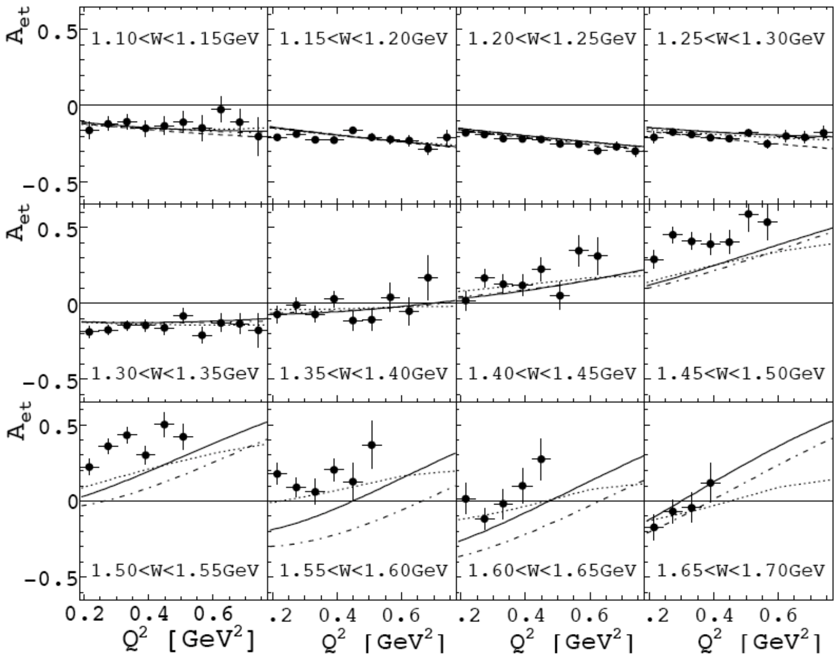


1<W<2 GeV

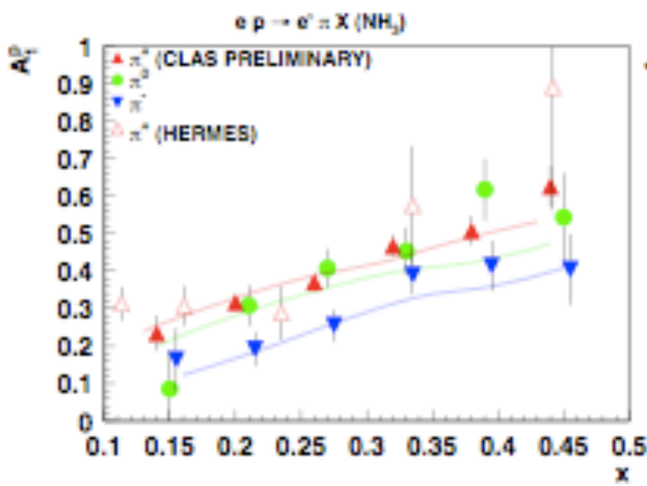


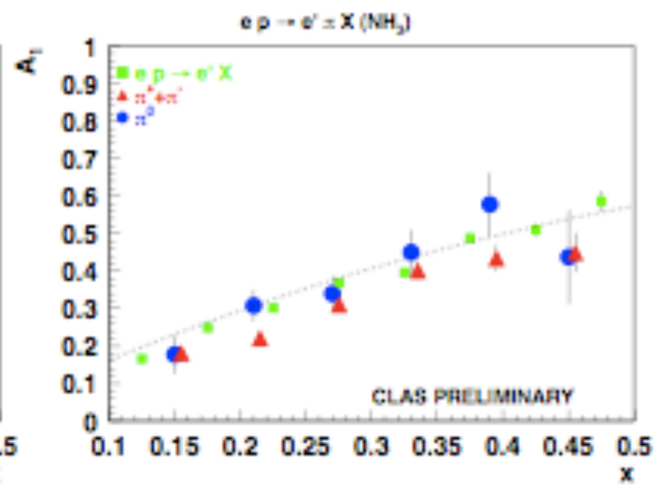
2-particle final states in CLAS

$H(e,e'\pi^0)p$



…and
$H(e,e'\pi^+)n$
$D(e,e'p\pi^-)p$
rho production
eta production
…





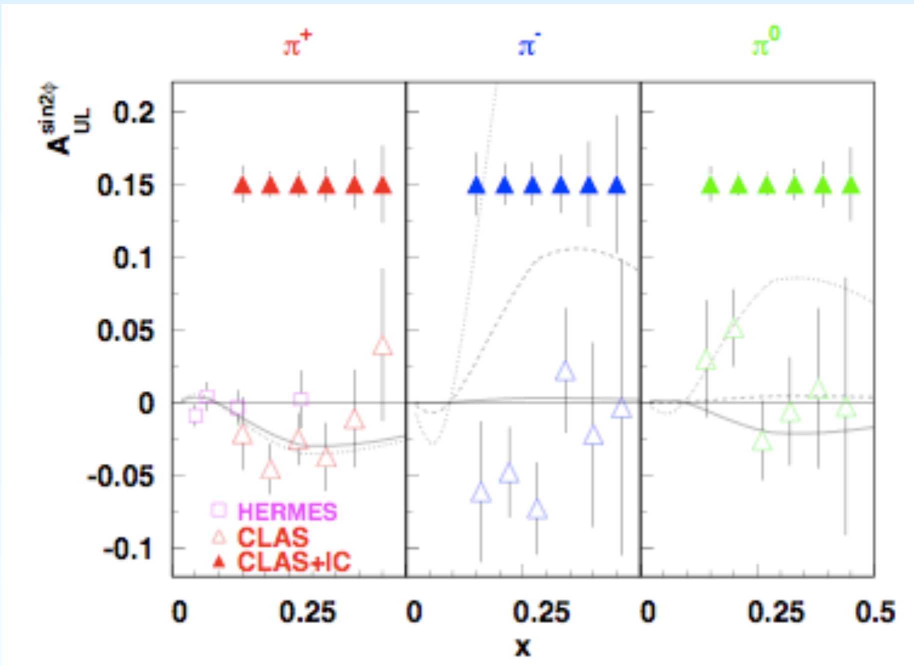
semi-
inclusive
DIS

Outlook: The Future at JLab

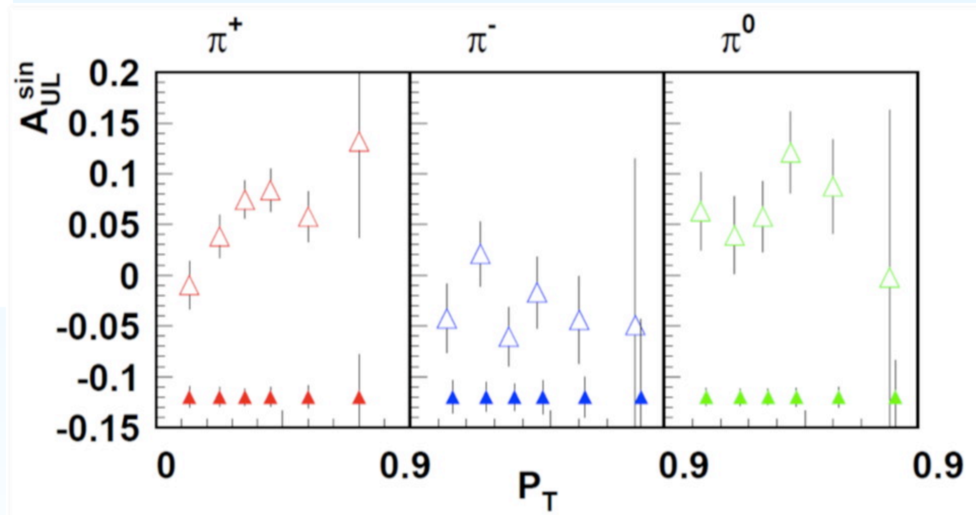
- Remaining experiments at 6 GeV
  - Hall A
    - E-06-010: Transverse target single spin asymmetry in n↑(e,e'π^-)
    - E-06-011: Transverse target single spin asymmetry in n↑(e,e'π^+)
    - E-06-014: Precision measurement of d_2 on the neutron
    - E-08-027: g_{2p} and δ_{LT}
  - Hall B
    - E-05-113: Semi-inclusive pion production (and DVCS) on p→
    - E-08-015: Semi-inclusive pion production (and DVCS) on p↑
  - Hall C
    - E-07-011: High precision g_{1d} in DIS region
    - E-07-003: SANE (SSFs on p, with emphasis on g_2)
- Approved experiments for 12 GeV
  - Hall A/C
    - E12-06-122: A_{1n} at high x with 8.8 GeV and 6.6 GeV beam in Hall A
    - E12-06-121: Precision measurement of g_2 and d_2 on the neutron
  - Hall B
    - E12-06-10: SSFs on longitudinal target with CLAS12
    - E12-07-107: Semi-inclusive pion production on p→

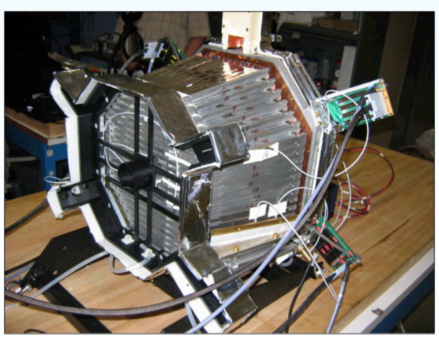
Running now!

E-05-113 with CLAS and longitudinal target
Study semi-inclusive pion production, TMDs and Collins fragmentation function as well as DVCS



Expected Precision for $\sin\phi$ and $\sin2\phi$ moments of target SSA

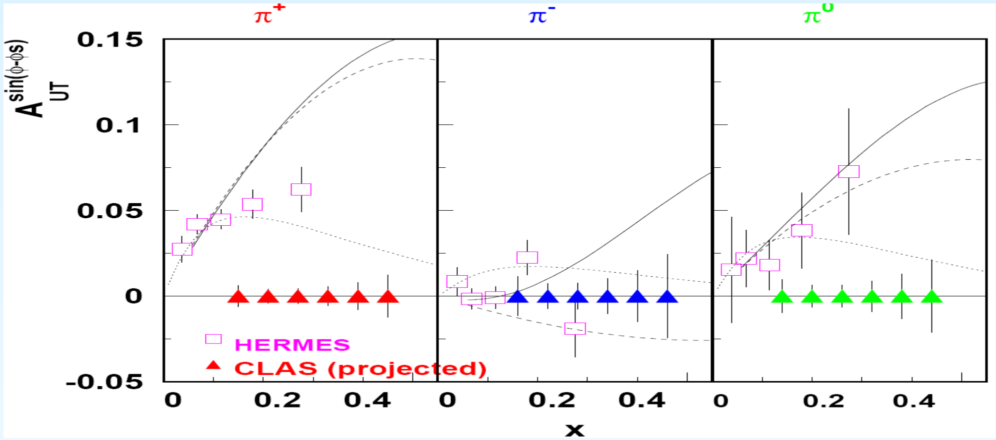


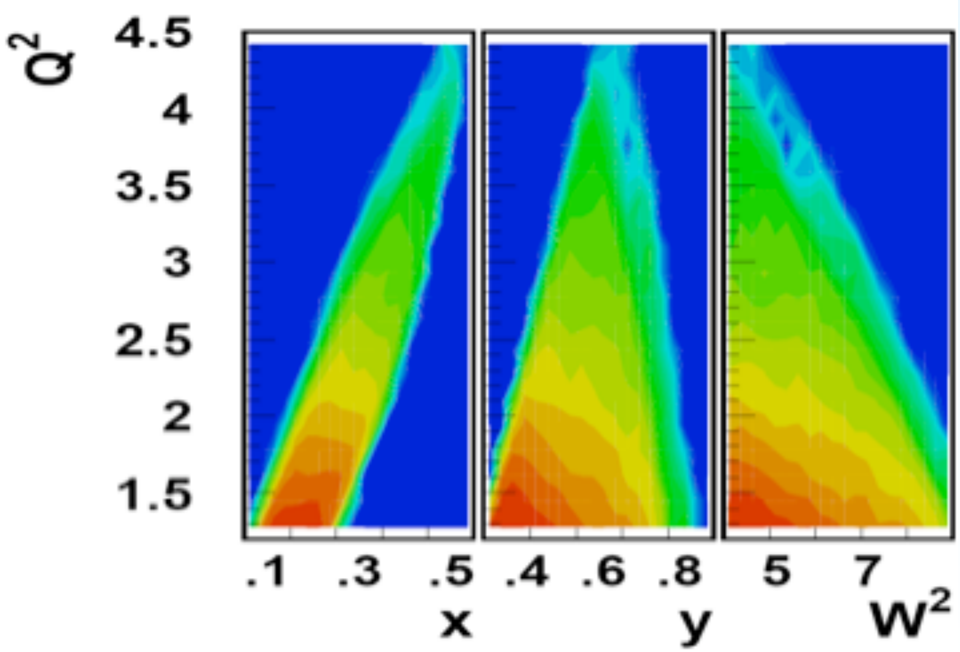


60 days (P_H=75%)

uses 2π inner calorimeter for γ/π^0 coverage

E-08-015 with CLAS and transverse HD ice target
Study Spin-Orbit correlations in Semi-Inclusive DIS and Sivers distribution function



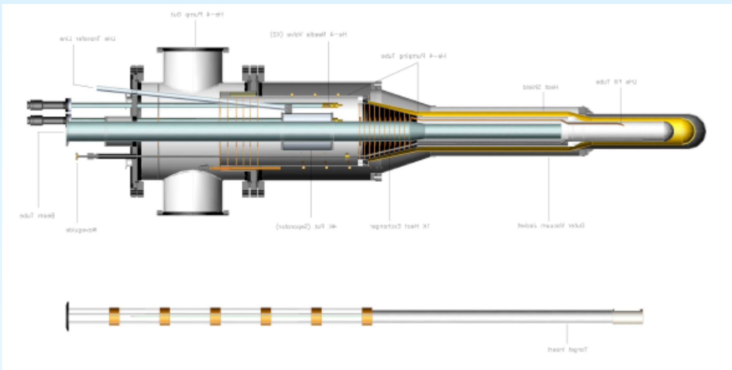


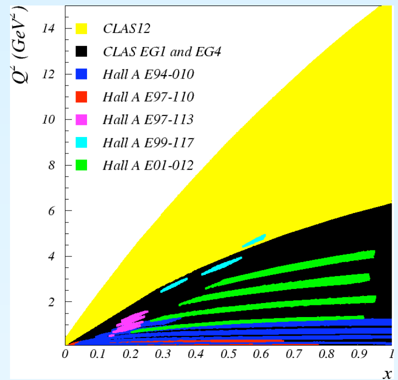
25 days (P_H=75% P_D=25%)

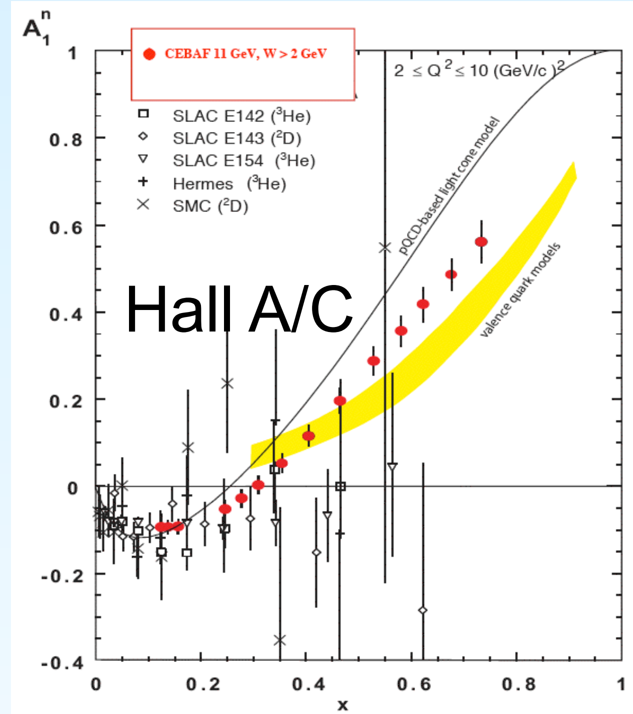
Potential to add to world data on g_2 and A_2

The Future with 12 GeV

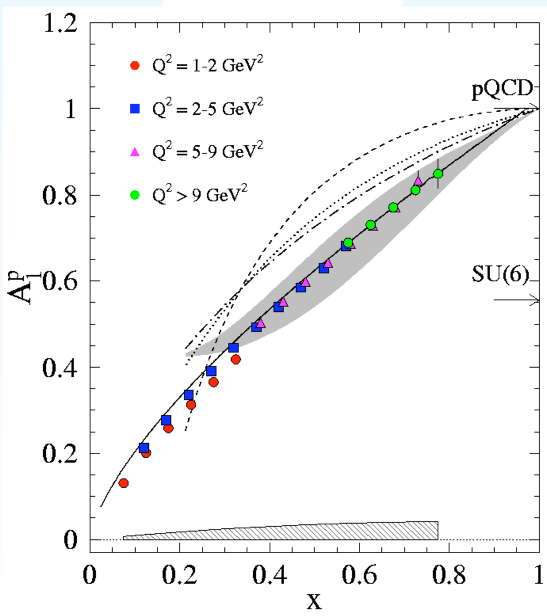
CLAS12

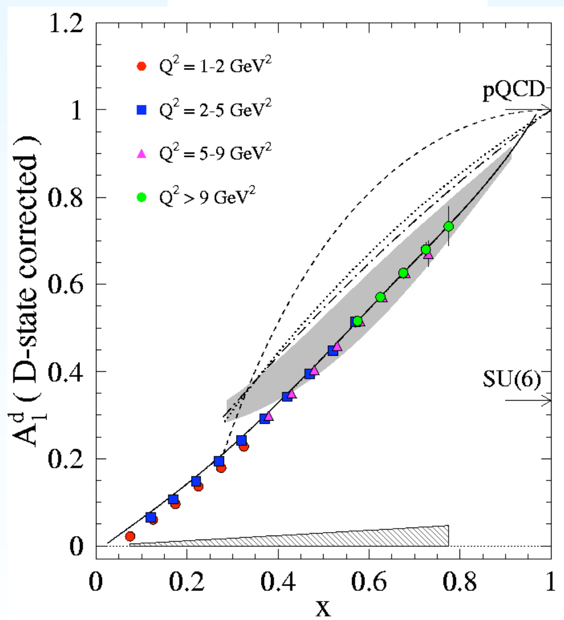


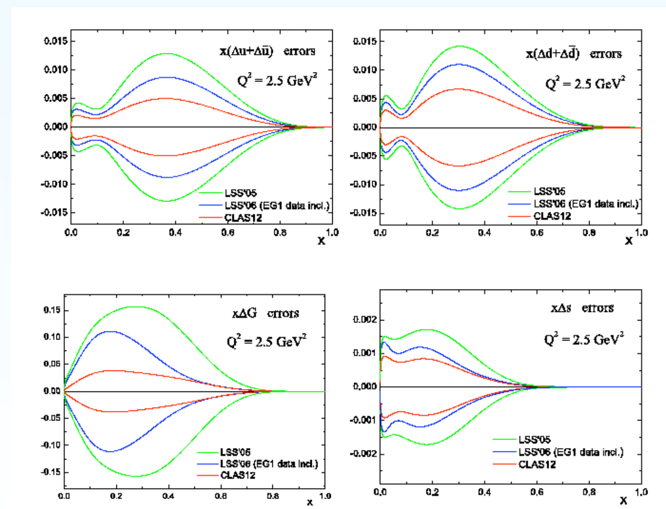




Proton W > 2; Q^2 > 1 Deuteron







Conclusions

- Nucleon Spin Structure has gotten very complicated!
- Data from SLAC, CERN, HERA, MAMI, ELSA, LEGS, **JLab**, RHIC,…
- Sum rules, Moments, OPE, Duality, PDFs, Transversity, TMD PDFs, OAM, GPDs…
- Much to come: COMPASS+RHIC, Spring8, **JLab @ 12 GeV**, J-PARC, FAIR, … EIC?

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