Nucleon Spin Structure at low energies

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Outline

- ChPT and low energy QCD
- Nucleon spin structure and sum rules
- V^2CS within ChPT and the role of Δ-isobar
- Δ-isobar in ChPT
- Preliminary results for polarizabilities
- Summary & Outlook

ChPT and low energy QCD

Spontaneous + explicit (by small quark masses) breaking of chiral symmetry in QCD

$\Downarrow$

Existence of light weakly interacting Goldstone bosons

$\Downarrow$

Chiral Perturbation theory (ChPT)
Expansion in small momenta and masses of Goldstone bosons

$\Downarrow$

Systematic description of QCD by ChPT in low energy sector
(low momenta $q \ll \Lambda_\chi \simeq 1\,\mathrm{GeV}$)

Nucleon spin structure and sum rules

- Spin-dependent forward Compton-Amplitude (V²CS)

$$\frac{1}{4\pi}\int d^4x e^{iq\cdot x}\langle PS|TJ^\mu(x)J^\nu(0)|PS\rangle = -i\epsilon^{\mu\nu\alpha\beta}q_\alpha\left[S_\beta S_1(\nu,Q^2) + (M\nu S_\beta - S\cdot qP_\beta)S_2(\nu,Q^2)\right]$$

$S_1(\nu,Q^2)$ and $S_2(\nu,Q^2)$ related to spin-structure functions g_1 and g_2

  - ν - Expansion at low energies of reduced amplitudes $\bar{S}_1(\nu,Q^2)$ and $\bar{S}_2(\nu,Q^2)$

$$\bar{S}_{12}(\nu,Q^2) = S_{12}(\nu,Q^2) - S_{12}^{\text{elastic}}(\nu,Q^2)$$

- Generalized Gerasimov-Drell-Hearn (GDH) sum rule

$$\bar{S}_1^{(0)}(0,Q^2) = 4e^2\int_{\nu_0}^{\infty} d\nu\,\frac{G_1(\nu,Q^2)}{\nu} = \frac{4e^2}{m^2}I_1(Q^2)$$

Ji, Osborne '00

Two different limits

$Q^2 \to 0$

$I_1(Q^2) = -\frac{1}{4}\kappa^2$ GDH S.R.

$Q^2 \to \infty$

$I_1(Q^2) = \frac{2m^2}{Q^2}\text{const}$ Ellis-Jaffe S.R.

ChPT calculations of V^{2}CS up to q^4

- $S_{12}^{(i)}(0,Q^2)$ for low virtualities calculated within ChPT

HBChPT: Ji, Kao, Osborne, Spitzenberg, Vanderhaeghen, Birse, McGovern, Kumar

- Expansion of amplitude in one over nucleon mass

IRChPT: Bernard, Hemmert, Meißner

- Systematic resumation of one over nucleon mass contributions

Contributions up to $O(q^4)$

- All tree diagrams with insertion from $\mathcal{L}_{\pi N}^{(i)}, i=1,2,3,4$
- All loop diagrams with insertion from at most $\mathcal{L}_{\pi N}^{(2)}$
- Two <u>well known</u> low energy constants: $c_6 \& c_7 \Longleftrightarrow \kappa_p \& \kappa_n$
- Elastic contributions to $S_{12}^{(i)}(0,Q^2)$ subtracted

Validity range of IRChPT

- Change in integral boundaries of Feynman-parameter integration

Two-point function examplified

$$H(Q^2) = \frac{1}{i}\int \frac{d^dl}{(2\pi)^d}\frac{1}{l^2 - M_\pi^2}\frac{1}{(l+q)^2 - m^2} = \frac{1}{i}\int \frac{d^dl}{(2\pi)^d}\int_0^1 dz \frac{1}{[(1-z)(l^2 - M_\pi^2) + z((l+q)^2 - m^2)]^2}$$

$$H(Q^2) = \int_0^\infty dz \ldots - \int_1^\infty dz \ldots = I - R$$ Becher, Leutwyler '99

Change in integral boundaries $\Longrightarrow$ Unphysical cuts at finite Q^2

Restriction in range of Q^2

- In V²CS unphysical cut at $Q^2 = 1\mathrm{GeV}^2$ far beyond the range of validity: However

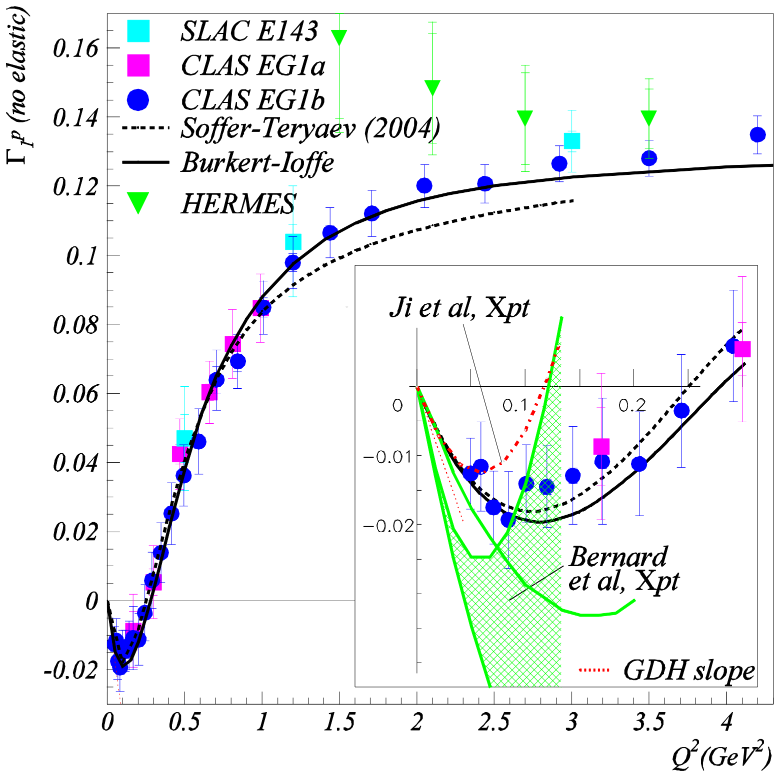
Calculations of derivatives of loop functions

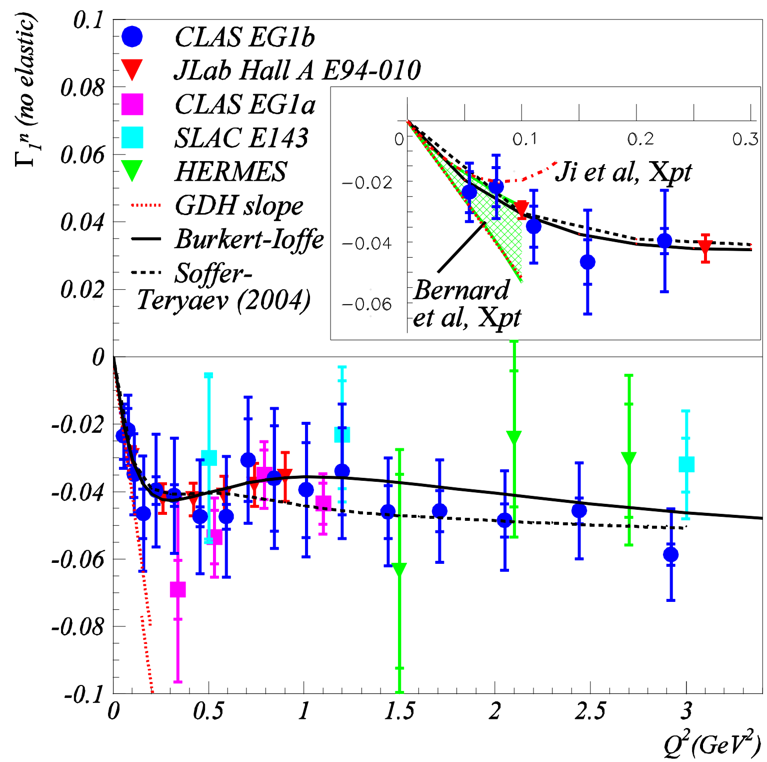
Effects at $Q^2 \sim 0.2 - 0.5\mathrm{GeV}^2$

Numerical Results

First moment $\Gamma_1(Q^2) = \dfrac{Q^2}{2m^2} I_1(Q^2)$ for proton and neutron

Kuhn, Chen, Leader '08





- Reasonable agreement of IRChPT results with data at $Q^2 = 0.05 - 0.1\mathrm{GeV}^2$
- Inclusion of the Δ and vector meson makes the agreement worse. However this is just an estimate: systematic ϵ^3 calculation needed

Generalized spin-polarizabilities

Bernard, Hemmert, Meißner '03

- Forward spin polarizabilities at $Q^2 = 0$: $\gamma_0(Q^2) = \frac{1}{8\pi}\left(\bar{S}_1^{(2)}(0,Q^2) - \frac{Q^2}{m}\bar{S}_2^{(3)}(0,Q^2)\right)$

Chiral expansion $\mu = \frac{M_\pi}{m}$

- Poor convergence of γ_0^p due to large prefactors $\sim \kappa_v \simeq 3.7$
- Much better convergence of γ_0^n

$$\gamma_0^n = \underset{O(\mu^{-2})}{4.45} - \underset{O(\mu^{-1})}{5.25} + \underset{O(\mu^{0})}{2.00} + \underset{O(\mu^{1})}{0.68} + O(\mu^2) = 1.82$$

$$\gamma_0^p = \underset{O(\mu^{-2})}{4.45} - \underset{O(\mu^{-1})}{8.31} + \underset{O(\mu^{0})}{6.03} + \underset{O(\mu^{1})}{3.22} + O(\mu^2) = 4.64$$

- Longit.-transv. spin-polarizabilities at $Q^2 = 0$: $\delta_0(Q^2) = \frac{1}{8\pi}\left(\bar{S}_1^{(2)}(0,Q^2) + \frac{1}{m}\bar{S}_2^{(1)}(0,Q^2)\right)$

- Well behaved chiral expansion of $\delta_0^{p,n}$
- No large prefactors in $\delta_0^{p,n}$

$$\delta_0^n = \underset{O(\mu^{-2})}{2.23} + \underset{O(\mu^{-1})}{0.93} - \underset{O(\mu^{0})}{0.26} - \underset{O(\mu^{1})}{0.24} + O(\mu^2) = 2.66$$

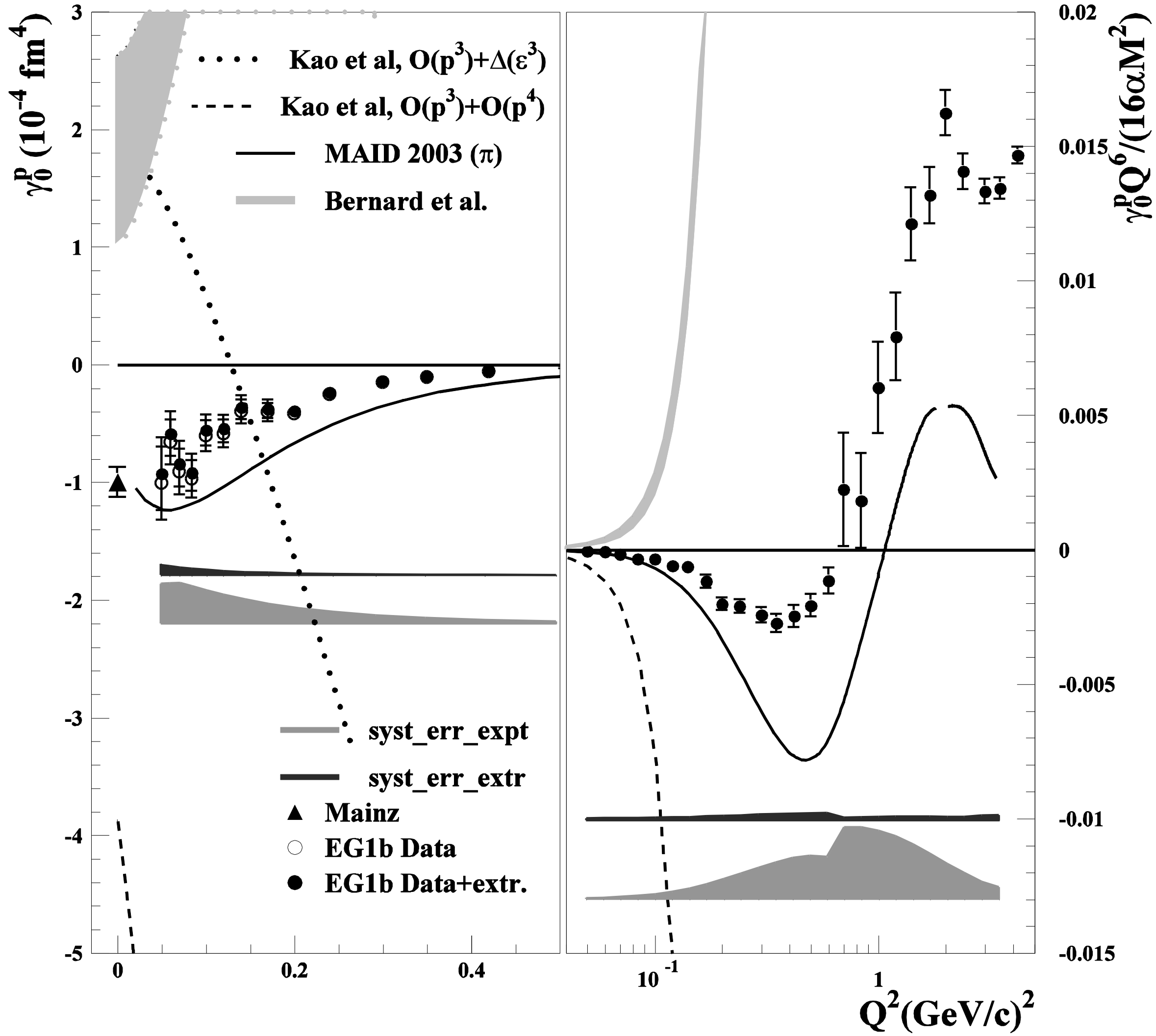
$$\delta_0^p = \underset{O(\mu^{-2})}{2.23} - \underset{O(\mu^{-1})}{0.75} + \underset{O(\mu^{0})}{0.53} + \underset{O(\mu^{1})}{0.12} + O(\mu^2) = 2.04$$

All numbers given in $10^{-4}\mathrm{fm}^4$

Generalized spin-polarizabilities

Forward spin-polarizability of the proton

CLAS Colaboration '09



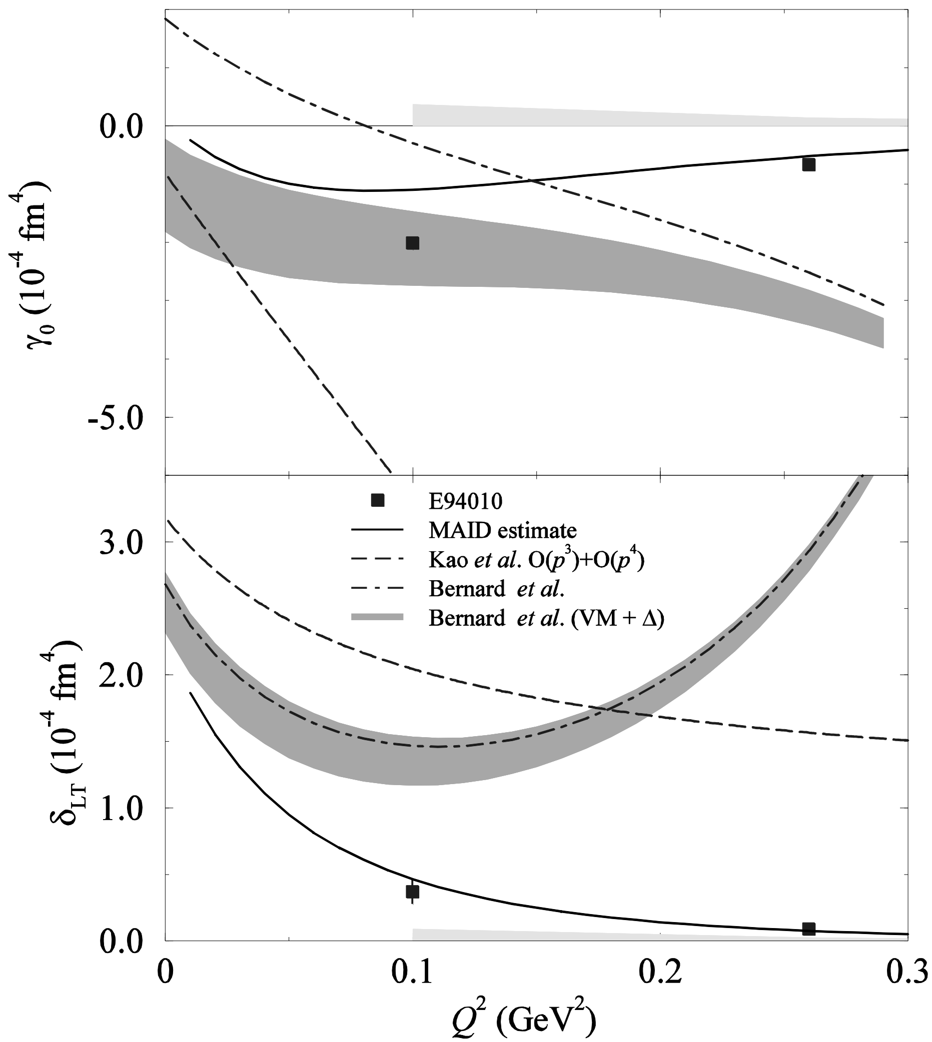
IRChPT contributions

- Positive chiral loop contr. increasing with Q^2
- Δ Born-graph contr. Is Negative and flat
- Serious disagreement with the data for γ_0^p
- Hope for improvement From Δ loop-contr.

Generalized spin-polarizabilities

Spin-polarizabilities for the neutron

Bernard's review PPNP 60 (2008) 82



- γ_0 is sensitive to Δ dof
- Nice agreement with exp. point at $Q^2 = 0.1\mathrm{GeV}^2$

- δ_{LT} is insensitive to Δ dof
- Very well behaved chiral Expansion of δ_{LT} at $Q^2 = 0$
- Strong curvature due to unphysical cut in IR results
- Comparisons with exp. data at lower Q^2 are desirable

Theory with explicit Δ dof

- Small scale expansion (SSE) is a systematic inclusion of Δ dof in ChPT

Hemmert, Holstein, Kambor ‘98

Expansion in low momenta, pion-masses and nucleon-delta mass splitting

$$\Delta = m_\Delta - m$$

Two on mass-shell equivalent representations for covariant Δ - fields

Pascalutsa ’01, H.K., Epelbaum, Meißner ‘08

- All interactions are invariant under $\psi_\mu \to \psi_\mu + \partial_\mu \epsilon$ → No unphysical spin-1/2 contr. But Gauge-invariance only modulo higher order

- All interactions are Gauge-invariant → Gauge-invariance exact But Unphysical spin-1/2 contr. to be absorbed by LECs of higher order

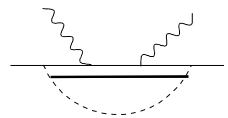
We calculated V²CS up to third order with both methods to get an error-estimate of higher order effects

Calculation of SSE loops

Computational Effort Estimate

$$S^{\mu\nu}(p) = \frac{\not{p} + m}{p^2 - m^2}\left(-g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3m}(\gamma^\mu p^\nu - \gamma^\nu p^\mu) + \frac{2}{3m^2}p^\mu p^\nu\right)$$

- Δ propagator includes 5 times more terms than nucleon propagator

  Box-diagram  has $5^3 = 125$ times more terms

- More complicated Spin-structures

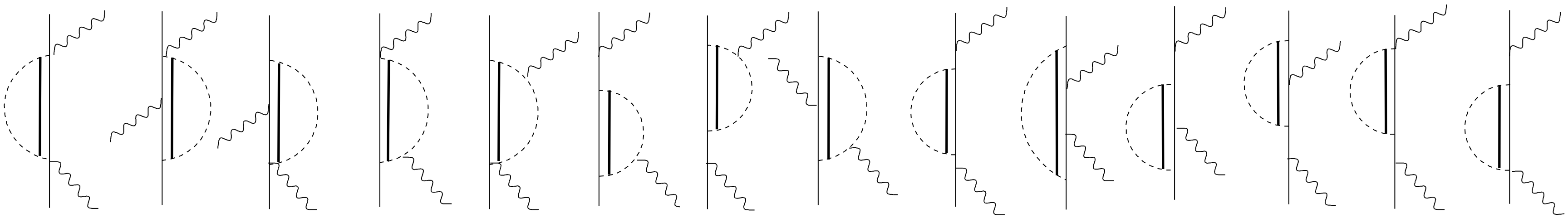
Automatization of calculation is necessary

Mathematica & FORM used for tensor reduction in current calc.

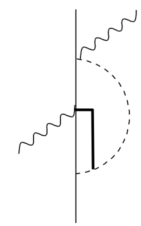
We developed our own code which is able to reduce tensor integrals of any rank in relativistic and Heavy Baryon formalism

- FORM used to reduce tensor integrals to series of scalar integrals with higher power of propagators and shifted dimensions Davydychev ‘91
- Mathematica used for further Passarino-Veltman reduction

SSE calculation of V²CS



14 diagrams contribute to the order ϵ^3

$\gamma N \Delta$-vertex starts to contribute to the order ϵ^4 ⇒ Diagrams like  suppressed

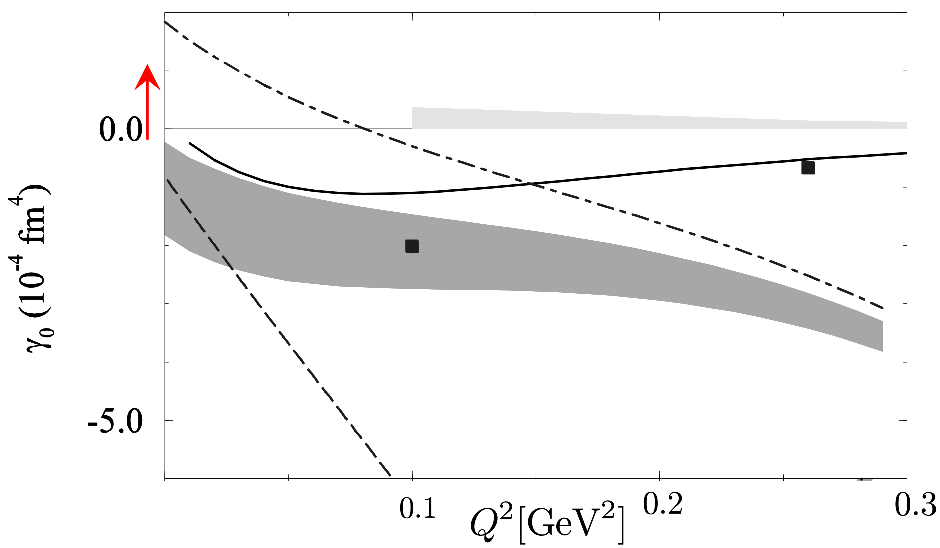
- Like in the nucleon case all ϵ^3 divergencies from loop-diagrams cancel
- No counter-term contributions to the spin-dependent part of V²CS
- For additional $\pi N \Delta$ coupling we use large - N_c relation $h_A = \frac{3g_A}{2\sqrt{2}} \sim 1.34$
- We explicitly checked the amplitudes for gauge-invariance

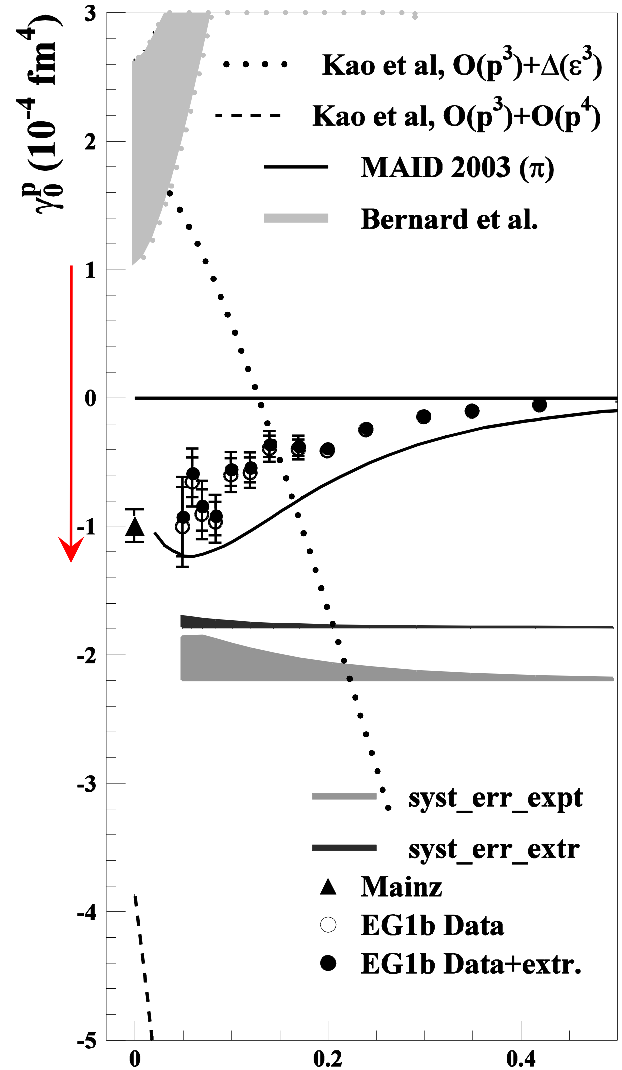
First numerical results

Preliminary

Generalized spin-polarizabilities at photon point
(Δ - loop contributions in $10^{-4}\mathrm{fm}^4$)

Proton	Neutron
$\delta_{LT}^{p,\Delta-\mathrm{loop}} = -0.82$	$\delta_{LT}^{n,\Delta-\mathrm{loop}} = 0.02$
$\gamma_0^{p,\Delta-\mathrm{loop}} = -2.27$	$\gamma_0^{n,\Delta-\mathrm{loop}} = 1.31$





Summary

- Spin-structure functions analyzed within ChPT upto order q^4
- No counterterms appear at this order $\Longrightarrow$ Parameterfree predictions
- Qualitative agreement between Theory & Experiment
- Systematic inclusion of Δ - isobar for V^2CS upto order ϵ^3
- First promising preliminary results at photon point for gen. spin-polarizabilities

Outlook

- Recheck of the calc. and numerical studies at finite virtualities
- Looking forward to more data at low Q^2