Spin Sum Rules and Polarizabilities

Dieter Drechsel

Institut für Kernphysik , Universität Mainz

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Introduction

- ► GDH sum rule: special case of several relations connecting real and virtual Compton scattering (RCS/VCS) to inclusive photo/electroproduction. VCS≡ VVCS
- Based on universal principles: causality, unitarity, gauge invariance, crossing symmetry.
- ▶ Unique testing ground to study internal degrees of freedom that hold the system together.
- ▶ At small photon virtuality: information about long-range phenomena, effective degrees of freedom due to interplay of quarks and mesons (Goldstone bosons, resonances).
- ► At larger virtualities: the primary degrees of freedom (quarks and gluons) become visible.
- Need better understanding of transition coherent ↔ incoherent processes generalized spin polarizabilities ↔ higher twists
- ▶ Recent experiments have collected a large body of precise and solid data and prepared the ground for theoretical activities:

Chiral Perturbation, Lattice Gauge, perturbative QCD

Gerasimov-Drell-Hearn-Hosada-Yamamoto sum rule

Unsubtracted dispersion relation relating the anomalous magnetic moment κ to spin-dependent inclusive cross sections $(\sigma_P - \sigma_A)$ for real photons with parallel/antiparallel spin w.r.t. target spin S

$$\frac{\pi e^2 \kappa^2}{M^2} S = \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \left(\sigma_P(\nu) - \sigma_A(\nu) \right) \tag{1}$$

 \heartsuit Finite $\kappa \to \text{particle has (spin-dependent)}$ excitation spectrum $\to \text{particle has spatial extension, size and shape} \Rightarrow \text{HADRONIC PHYSICS.}$

Discovery of large κ_p by Stern and collaborators (1933) rang up the curtain for hadronic physics.

♣ LHS > 0 → photon prefers absorption with parallel spins. Leads to excited state with spin S = 3/2, ruled out for absorption on single quark ($\neq DIS$).

Gyromagnetic Moment and Convergence of GDH Integral

Relation between magnetic moment $\vec{\mu}$ and spin vector \vec{S} :

$$\vec{\mu} = \frac{eg}{2M} (Q + \kappa) \vec{S} , \qquad (2)$$

gyromagnetic ratio g = 2 for particles of any spin.

- \diamond Value g=2 required for well-behaved scattering amplitude at high energy (Weinberg, 1970).
- ♠ Large a.m.m. of nucleon shows its composite structure, described by unitarity corrections from pion loops and low-energy resonance effects. Such spatially extended phenomena should fade out with increasing energy.
- ♣ Therefore, GDH integral Eq. (1) should saturate at sufficiently large energies, unsubtracted dispersion relation should exist.
- ♡ In a completely supersymmetric, high-energy world, all GDH integrals would vanish, all particles would be "truly elementary" pointlike objects .

Finite value of κ in real world: measure of broken supersymmetry.

Forward RCS and Polarizability

Forward Compton amplitude:

$$T(\nu, \theta = 0) = \vec{\varepsilon}'^* \cdot \vec{\varepsilon} f(\nu) + i \vec{\sigma} \cdot (\vec{\varepsilon}'^* \times \vec{\varepsilon}) g(\nu)$$
 (3)

Invariance under photon crossing $(\varepsilon'^* \leftrightarrow \varepsilon \text{ and } \nu \to -\nu)$ requires that $f(\nu)$ even and $g(\nu)$ odd function of ν .

Amplitudes f and g measured by double-polarization experiment:

Parallel spins: excited state has spin 3/2, transition requires a correlated 3-quark system.

Opposite spins: excited state has spin 1/2, process can take place on single quark.

$$f(\nu) = (T_{1/2} + T_{3/2})/2, \quad g(\nu) = (T_{1/2} - T_{3/2})/2$$
 (4)

Optical Theorem

Scattering amplitudes $T_{1/2}$ and $T_{3/2}$ related to helicity-dependent absorption cross sections $\sigma_A=\sigma_{1/2}$ and $\sigma_P=\sigma_{3/2}$.

Total and helicity dependent cross sections:

$$\sigma_{T} = \frac{1}{2} (\sigma_{1/2} + \sigma_{3/2}),$$

$$\sigma_{TT} = \frac{1}{2} (\sigma_{1/2} - \sigma_{3/2})$$
(5)

Unitarity ⇒ optical theorem

$$\operatorname{Im} f(\nu) = \frac{\nu}{8\pi} (\sigma_{1/2}(\nu) + \sigma_{3/2}(\nu)) = \frac{\nu}{4\pi} \, \sigma_{T}(\nu) \,,$$

$$\operatorname{Im} g(\nu) = \frac{\nu}{8\pi} (\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)) = \frac{\nu}{4\pi} \, \sigma_{TT}(\nu) \tag{6}$$

Dispersion Relations and LET

Subtracted DR for $f(\nu)$, unsubtracted DR for $g(\nu)$:

Re
$$f(\nu) = f(0) + \frac{\nu^2}{2\pi^2} \mathcal{P} \int_{\nu_0}^{\infty} \frac{\sigma_T(\nu')}{\nu'^2 - \nu^2} d\nu',$$
 (7)

$$\operatorname{Re} g(\nu) = \frac{\nu}{4\pi^2} \mathcal{P} \int_{\nu_0}^{\infty} \frac{\sigma_{1/2}(\nu') - \sigma_{3/2}(\nu')}{\nu'^2 - \nu^2} \nu' d\nu'. \tag{8}$$

Below pion production, $\nu < \nu_0$: amplitudes $f(\nu)$ and $g(\nu)$ are real, can be expanded in Taylor series and compared to low-energy theorem (Low, Gell-Mann & Goldberger, 1954),

$$f(\nu) = -\frac{e^2 e_N^2}{4\pi M} + (\alpha + \beta) \nu^2 + \mathcal{O}(\nu^4), \qquad (9)$$

$$g(\nu) = -\frac{e^2 \kappa_N^2}{8\pi M^2} \nu + \gamma_0 \nu^3 + \mathcal{O}(\nu^5). \qquad (10)$$

Leading term in Eq. (8) yields GDH sum rule, higher order terms express (forward) polarizabilities by integrals over absorption spectrum.

Sum Rules

▶ Baldin sum rule Compare $\mathcal{O}(\nu^2)$ of Eq. (7) to LET, Eq. (9):

$$\alpha + \beta = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_T(\nu)}{\nu^2} d\nu \tag{11}$$

► Gerasimov-Drell-Hearn sum rule Compare $\mathcal{O}(\nu)$ of Eq. (8) to LET, Eq. (10):

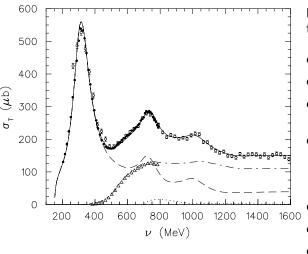
$$\frac{\pi e^2 \kappa_N^2}{2M^2} = \int_{\nu_0}^{\infty} \frac{\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)}{\nu} \, d\nu \equiv I_{GDH} \qquad (12)$$

► (Gell-Mann)-Goldberger-Thirring sum rule Compare $\mathcal{O}(\nu^3)$ Eq. (8) to LET, Eq. (10):

$$\gamma_0 = -\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)}{\nu^3} \, d\nu \tag{13}$$

▶ Higher terms of power series yield quadrupole polarizability, $\mathcal{O}(\nu^5)$, and so on.

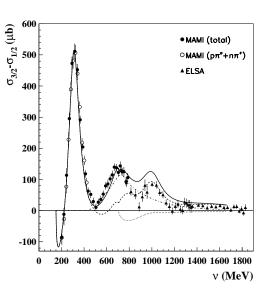
Total Photoabsorption $\sigma_T(\nu)$



phenomena from low to high lab energy ν :

- \spadesuit non-resonant S-wave π^+ (E_{0^+})
- \spadesuit P_{33} resonance (M_{1^+}/E_{1^+})
- $ightarrow F_{15} \ \text{resonance} \ (E_{3^-}/M_{3^-}) \ D_{33} \ \text{resonance} \ (E_{2^-}/M_{2^-}) \ S_{31} \ \text{resonance} \ (E_{0^+}) \ P_{13} \ \text{resonance} \ (E_{1^+}, M_{1^+})$
- $\spadesuit >$ 500 MeV: $\pi\pi$ channels
- $\spadesuit >$ 700 MeV: η channel
- ♠ > 2 GeV constant absorption slight increase in Regge region from soft pomeron exchange

Helicity Dependent Photoabsorption $\sigma_{3/2} - \sigma_{1/2}$ for proton

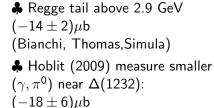


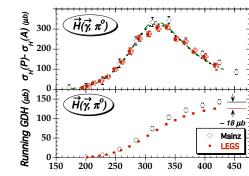
phenomena from low to high lab energy ν :

- \spadesuit non-resonant S-wave π^+ $(-|E_{0^+}|^2)$
- ♠ D_{13} resonance $(+|E_{2^-}|^2 + ...)$ S_{11} resonance $(-|E_{0^+}|^2)$
- ♠ F_{15} resonance $(+|E_{3-}|^2 + ...)$ D_{33} resonance $(+|E_{2-}|^2 + ...)$ S_{31} resonance $(-|E_{0+}|^2)$ P_{13} resonance $(-|E_{1+}|^2 + ...)$
- \spadesuit $\pi\pi$ channels positive
- \spadesuit η channel negative
- \spadesuit > 2 GeV small negative contribution from Regge tail for $\nu \to \infty$ "helicity blind" (?)

GDH and **FSP** for proton

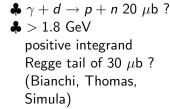
energy [GeV]	Ref.	$I^p_{GDH}[\mu b]$	$\gamma_0^p [10^{-4} \text{fm}^4]$	
$\nu_0 - 0.2$	MAID/SAID	-27.5 ± 3	0.90 ± 0.05	
0.2 - 0.8	Ahrens 2001	$226\pm5\pm12$	$-1.87 \pm 0.08 \pm 0.10$	
0.8 - 2.9	Dutz 2004	$27.5 \pm 2.0 \pm 1.2$	-0.03	
total		$226\pm 6\pm 12$	$-1.00 \pm 0.08 \pm 0.11$	
sum rule		204	_	

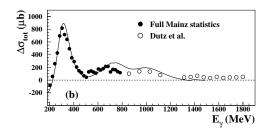




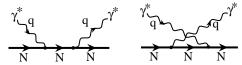
GDH for neutron

energy [GeV]	Ref.	$I^n_{GDH}[\mu extsf{b}]$
$\nu_0 - 0.2$	MAID/SAID	-30 ± 3
0.2 - 0.8	Ahrens 2006	$181\pm21\pm30$
0.8 - 1.8	Dutz 2004	$+34\pm5\pm5$
> 1.8	Regge tail	$+30\pm10$
total		$215\pm22\pm34$
sum rule		234



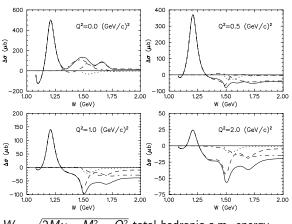


Inclusive Electroproduction and Virtual Compton Scattering



- Inclusive electroproduction cross section $\sigma(\nu,\,Q^2) = \sigma_T + \epsilon\,\sigma_L hP_x\,\sqrt{2\epsilon(1-\epsilon)}\,\sigma_{LT} hP_z\,\sqrt{1-\epsilon^2}\,\sigma_{TT}$ momentum transfer $Q^2 = 4EE^{'}\sin^2\frac{\theta}{2}$, energy transfer $\nu = E_e E_e'$, ϵ transverse photon pol., h electron helicity, $P_{x,z}$ target polarization
- Scattering amplitude $T(\nu, Q^2, \theta = 0) = (\vec{\varepsilon}''^* \cdot \vec{\varepsilon}) f_T + f_L i \vec{\sigma} \cdot (\vec{\varepsilon}''^* \times \vec{\varepsilon}) g_{TT} i (\vec{\varepsilon}''^* \vec{\varepsilon}) \cdot (\vec{\sigma} \times \vec{q}) g_{LT}$
- ► Crossing symmetry Invariance under $\vec{\varepsilon}'^* \leftrightarrow \vec{\varepsilon}$, $\vec{q} \rightarrow -\vec{q}$, $\nu \rightarrow -\nu$ g_{TT} odd and g_{LT} even function of ν .
- ▶ Optical theorem Im $\{f_T, f_L, g_{TT}, g_{LT}\} = \frac{K_H}{4\pi} \{\sigma_T, \sigma_L, \sigma_{TT}, \sigma_{LT}\}$ K_H = "equivalent photon energy"

Helicity Difference $\Delta \sigma = \sigma_{3/2}(W, Q^2) - \sigma_{1/2}(W, Q^2)$



 $W = \sqrt{2M\nu - M^2 - Q^2}$ total hadronic c.m. energy

with increasing Q^2 :

 π S-wave @ thr. rapid decrease

 $\Delta(1232)$ drops faster than dipole f.f.

2nd and 3rd resonance regions change sign near $Q^2 = 0.2 \text{GeV}^2$

above $Q^2=2{\rm GeV}^2$ resonance structures wash out, DIS regime

Spin-flip Cross Sections, Nucleon Spin Structure Functions, Asymmetries

cross sections and nucleon structure functions g_1 and g_2

Note: nucleon structure functions contain elastic and inelastic parts! $\sigma_{TT} = \frac{\pi \, e^2}{MK} \, (g_1 - \gamma^2 \, g_2), \quad \sigma_{LT} = \frac{\pi \, e^2}{MK} \, \gamma(g_1 + g_2), \quad \gamma = Q/\nu$

Asymmetries and helicity amplitudes

Experiments measure asymmetries for longitudinal electron polarization and target polarization longitudinal $(A_{||})$ and transverse $(A_{\perp}) \Rightarrow g_1, g_2$

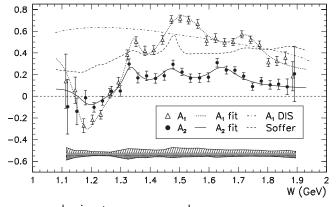
Resonance physics: Introduce polarization w.r.t. virtual photon.

$$\begin{array}{l} A_1 \sim \sigma_{TT} \sim g_1 - \gamma^2 \, g_2 \sim |A_{1/2}|^2 - |A_{3/2}|^2 \\ A_2 \sim \sigma_{LT} \sim g_1 + g_2 \sim S_{1/2}^* A_{1/2} \end{array}$$

HELICITY AMPLITUDES $A_{3/2}$, $A_{1/2}$, $S_{1/2}$ characterize resonance structure.

JLab Hall A, B, and C collaborations have provided new and precise information on both DIS and resonance structure

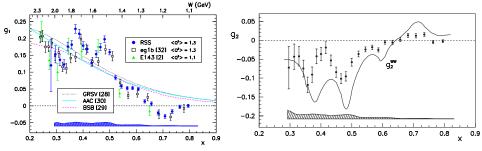
Asymmetries $A_1 \sim \sigma_{TT}$ and $A_2 \sim \sigma_{LT}$



RSS/Hall C, Wesselmann et al., PRL (2007) $Q^2 \approx 1.33 {\rm GeV}^2$

 σ_{TT} dominant, σ_{LT} suppressed distinct resonance structures \neq DIS $P_{33}(1232)$, 2nd resonance region (1500), 3rd resonance region (1700) indication of peak near 1350 MeV (Roper? mostly 2-pion?)

Spin Structure Functions



RSS/Hall C, CLAS, and SLAC data; see also new CLAS data (Bosted 2009)

g₁: fluctuations about DIS extrapolation as before

 g_2 : WW approximation right structure but differs in size

 $\sigma_{LT} \sim g_1 + g_2$ suppressed

distinct resonance structures, importance of higher twists (convergence?) $P_{33}(1232)$, 2nd resonance region (1500), 3rd resonance region (1700) indication of peak near 1350 MeV (Roper? mostly 2-pion?)

Dispersion Relations and VCS Sum Rules I

Born terms

$$\begin{split} g_{TT}^{\rm Born}(\nu,\ Q^2) &= -\frac{e^2 \nu}{8\pi M^2} \left(F_P^2 + \frac{Q^2}{\nu^2 - \nu_B^2 + i\varepsilon} \ G_M^2 \right) \\ g_{LT}^{\rm Born}(\nu,\ Q^2) &= \frac{e^2 Q}{8\pi M^2} \left(F_D F_P - \frac{Q^2}{\nu^2 - \nu_D^2 + i\varepsilon} \ G_E G_M \right) \end{split}$$

form factors Dirac: $F_D(Q^2)$, Pauli: $F_D(Q^2)$, Sachs: $G_E(Q^2)$ and $G_M(Q^2)$ pole positions: $\nu = \pm \nu_B = \pm Q^2/2M$.

RCS vs. VCS

Real photon in limit $Q^2 \to 0 \Rightarrow F_P \to \kappa_N$, $F_D \to e_N$ Pole terms vanish, Born terms real \Leftrightarrow real photon not absorbed by nucleon.

Taylor series $g(\nu)=g_{TT}(\nu,Q^2=0)$ converges for $\nu<\nu_0$

Virtual photon absorbed by nucleon $(e + N \rightarrow e' + N')$.

Pole at ν_B yields complex amplitude, fulfills dispersion relation by itself. $g_{TT}(\nu,Q^2)-g_{TT}^{\rm pole}(\nu,Q^2)$ is real for $\nu<\nu_\pi(Q^2)=\nu_0+\nu_B(Q^2)$, can be expanded in power series up to $\nu=\nu_\pi(Q^2)$ and

can be evaluated by dispersion integral over (polarized) electroproduction.

Dispersion Relations and VCS Sum Rules II

$$\begin{split} & \textit{g}_{TT}, \text{ odd power series for } \nu < \nu_{\pi}(Q^2) \\ & \textit{g}_{TT}^{\text{non-pole}}(\nu, \ Q^2) = \frac{\nu}{2\pi^2} \mathcal{P} \int_{\nu_{\pi}}^{\infty} \frac{\kappa(\nu', Q^2) \, \sigma_{TT}(\nu', Q^2)}{\nu'^2 - \nu^2} \, d\nu' \\ & \Rightarrow \frac{e^2}{2 \, \pi \, M^2} \, I_{TT}(Q^2) \, \nu + \gamma_{TT}(Q^2) \, \nu^3 + \mathcal{O}(\nu^5) \end{split}$$

generalized GDH integral, $I_{TT}(0) = -\kappa_N^2/4$ generalized spin (dipole) polarizability, $\gamma_{TT}(0) = \gamma_0$

$$\begin{array}{l} g_{LT}, \text{ even power series for } \nu < \nu_{\pi}(Q^2) \\ g_{LT}^{\mathrm{non-pole}}(\nu, \ Q^2) = \frac{1}{2\pi^2} \mathcal{P} \int_{\nu_{\pi}}^{\infty} \frac{\nu' \, K(\nu', Q^2) \, \sigma_{LT}(\nu', Q^2)}{\nu'^2 - \nu^2} \, d\nu' \\ = \frac{e^2 Q}{2\pi \, M^2} \, I_{LT}(Q^2) + Q \delta_{LT}(Q^2) \, \nu^2 + \mathcal{O}(\nu^4) \end{array}$$

longitudinal-transverse analog of generalized GDH integral longitudinal-transverse analog of generalized spin polarizability I_{LT} and $\delta_{LT}(0)=\delta_0$ obtained by extrapolation $Q^2\to 0$.

Integrals and Sum Rules

integrals

$$I_{1,2}(Q^2) = \frac{2M^2}{Q^2} \int_0^1 dx \, g_{1,2}(x, Q^2) = \frac{2M^2}{Q^2} \, \Gamma_{1,2}(Q^2)$$

limit
$$Q^2 \to \infty$$

$$\Gamma_1(Q^2) = \tilde{\Gamma}_1(Q^2) + \mathcal{O}\left(\frac{M^2}{Q^2}\right) , \quad \Gamma_2(Q^2) = \mathcal{O}\left(\frac{M^4}{Q^4}\right)$$

Bjorken sum rule, QCD prediction for isovector integral confirmed within 10%

Committee within 1076
$$\tilde{\Gamma}_{1}^{p}(Q^{2}) - \tilde{\Gamma}_{1}^{n}(Q^{2}) = \frac{1}{6} g_{A} \left\{ 1 - \frac{\alpha_{s}(Q^{2})}{\pi} + \mathcal{O}\left(\alpha_{s}^{2}\right) \right\}$$

$$\approx 0.18 \ @ \ Q^{2} = 5 \, \text{GeV}^{2}$$

Burkhard-Cottingham sum rule

$$I_2(Q^2) = 0 \Rightarrow \Gamma_2(Q^2) = 0$$
 (elastic and inelastic!)

Burkhard-Cottingham sum rule

"Superconvergence relation", assuming convergent dispersion relation for $g_2(\nu)$ and $\nu g_2(\nu)$. If these integrals converge \Rightarrow

$$\frac{M^2}{\pi e^2} \int_{\nu_{\pi}}^{\infty} \frac{K(\nu, Q^2)}{\nu^2 + Q^2} \left\{ -\sigma_{TT}(\nu, Q^2) + \frac{\nu}{Q} \sigma_{LT}(\nu, Q^2) \right\} d\nu$$

$$= I_2^{\text{inel}}(Q^2) = -I_2^{\text{el}}(Q^2) = \frac{1}{4} F_P(Q^2)(F_D(Q^2) + F_P(Q^2))$$

Integral over excitation spectrum is fully determined by electric and magnetic ground state form factors F_D and F_P .

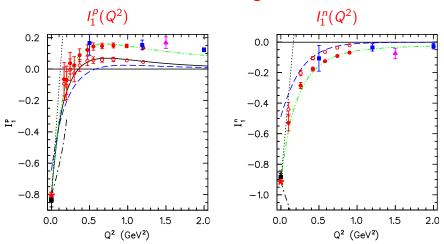
BC integral converges in QED and perturbative QCD. It also follows from Wandzura-Wilczek relation. Does it converge in strong QCD?

In limit
$$Q^2 \to \infty$$
, $I_2^{\text{inel}}(Q^2) = -I_2^{\text{el}}(Q^2) = \mathcal{O}\left(\frac{M^{10}}{Q^{10}}\right)$

$$I_1^{\text{inel}}(0) = -\frac{1}{4}\kappa_N^2, \quad I_2^{\text{inel}}(0) = \frac{1}{4}(e_N + \kappa_N)\kappa_N$$

$$I_{TT}^{\text{inel}}(0) = -\frac{1}{4}\kappa_N^2, \quad I_{LT}^{\text{inel}}(0) = \frac{1}{4}e_N\kappa_N$$

GDH-like Integrals



data: JLab CLAS and E94-010 (circles, open: W < 2 GeV, solid: plus DIS)

SLAC (diamonds), HERMES (triangles)

lines: 1-pion (- - -), 2-pion added (---), Burkert-loffe (-----)

Kao 2003 (.....), Bernard 2003 (- · - · -)

First Moments of Spin Structure Function g₁

 $\Gamma_{1}^{n}(Q^{2}) = \frac{Q^{2}}{2M^{2}} I_{1}^{n}(Q^{2})$ 0.00 -0.02 0.00

data: CLAS (circles), SLAC (diamonds)

HERMES (triangles)

lines: Burkert-Ioffe (.....), Soffer-Teryaev (- - -)

Bernard 2003 (. . .), Ji 2000 (- · - · -)

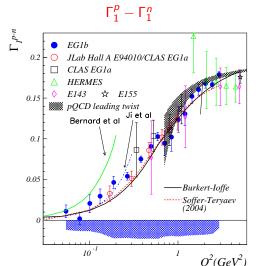
data: E94-010 (circles) SLAC (diamonds)

-0.08

lines: Burkert-loffe (-----)

MAID one-pion (---)

Isovector Moment and ChPT

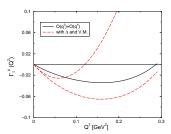


$$\Gamma_1^{p-n} = \frac{(\kappa_n^2 - \kappa_p^2)Q^2}{8M^2} + 0.62 \frac{Q^4}{M^4} - 0.77 \frac{Q^6}{M^6} + \dots$$

Ji et al.: HBChPT $\mathcal{O}(\rho^4)$

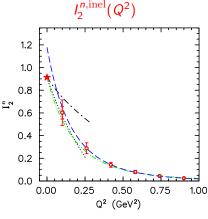
Bernard et al.: Lorentz inv. BChPT $\mathcal{O}(p^4)$

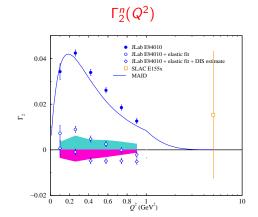
 $\Gamma_1^p(Q^2)$



loop corrections $f(Q^2/m_\pi^2)$ vector mesons (and $\Delta(1232)$) effects are large and model-dependent predictions at $Q^2=0.1{\rm GeV}^2$ differ by large factor Q^6 and 2-loop . . .

The Burkhardt-Cottingham Sum Rule: $l_2^{\text{inel}} = -l_2^{\text{el}}$???





data: E94-010

* BC sum rule at $Q^2 = 0$

lines: BC sum rule (----)MAID one-pion (---)

Bernard 2003 (····)

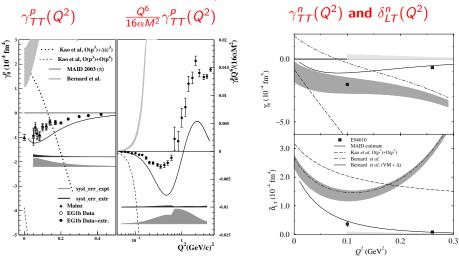
Kao 2003 (- · - · -)

data solid circles: E94-010 (inelastic contribution W < 2 GeV) open circles: elastic contribution added diamonds: DIS added

line: MAID one-pion —

K. Slifer et al, arXiv: $0812.0031 \Rightarrow \Gamma_2^p(1.33) = 0.0003 \pm 0.0007 \pm 0.0041$

Generalized Spin Polarizabilities



data: EG1b/CLAS (circles), Mainz (triangle) lines: MAID one-pion (——)

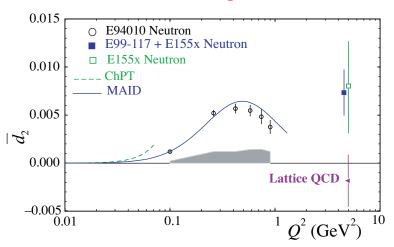
Kao (- - - - , · · · ·), Bernard (shaded)

data: E 94-010

lines: MAID one-pion (----)

Kao (---), Bernard $(-\cdot -\cdot -)$

Twist-3 d_2



surprising agreement with MAID, looked even nicer in yesterday's talk of P. Savignon!