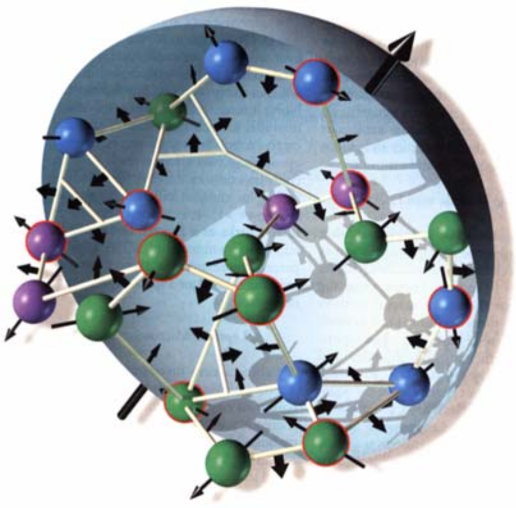
TJNAF, December 14, 2006

Theoretical and experimental challenges of SIDIS



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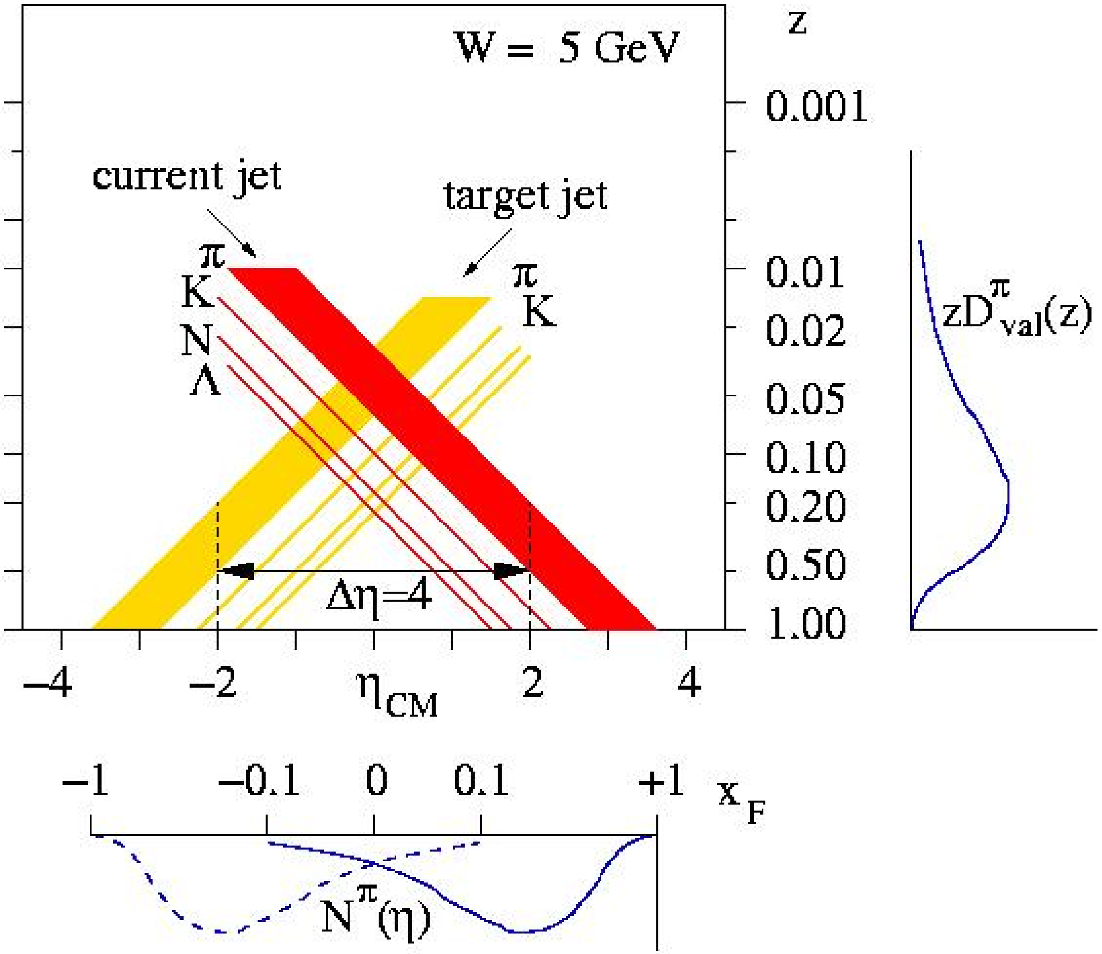
DIS

- Hard scattering processes to access partons (quarks and gluons): high energy end of duality regime
- Measure longitudinal (lightcone) momentum fraction x of struck quark and quark densities $f_1^{H\rightarrow q}(x)$
- Proton and neutron (possibly nuclei) for (limited) flavor sensitivity (q = u, d)
- Polarization to access longitudinal polarization $g_1^q(x)=\Delta q(x)$
- In principle unlimited access to higher twist contributions

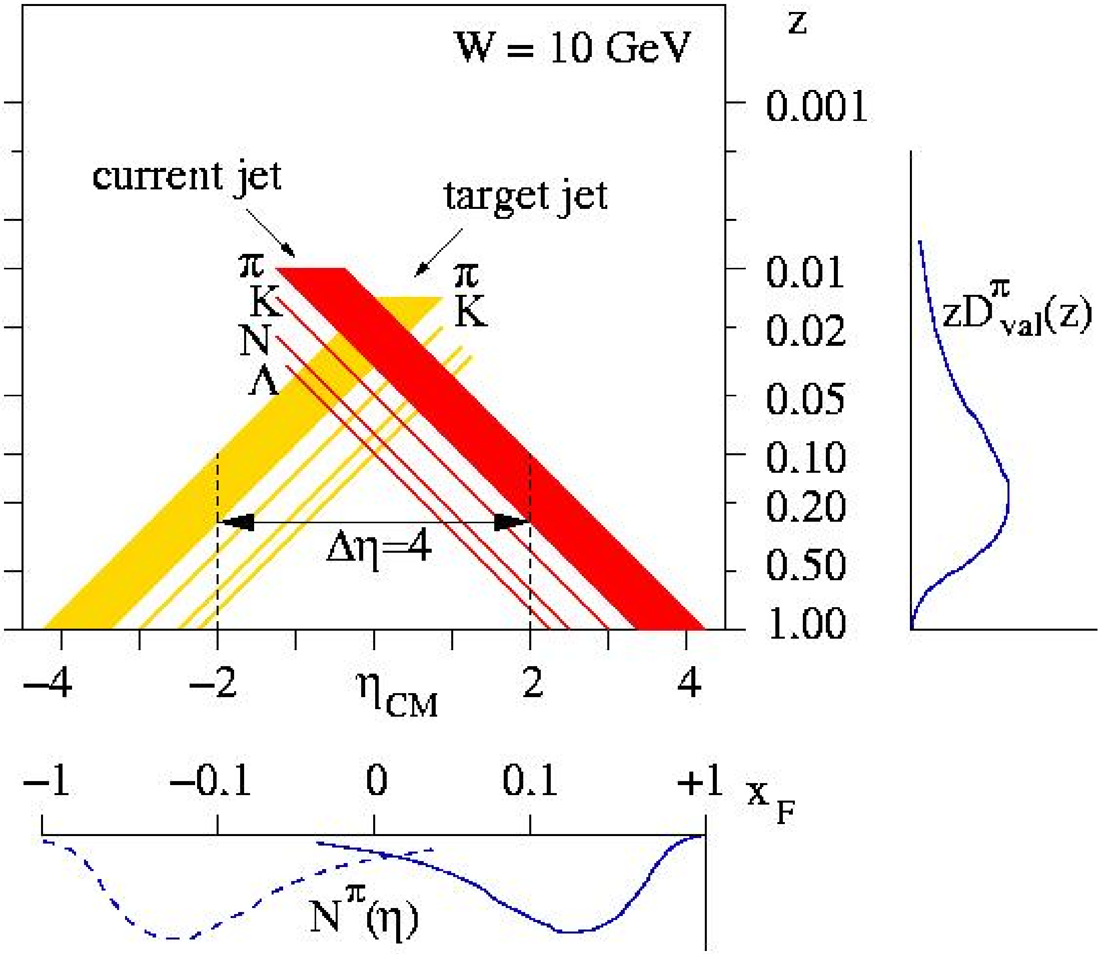
SIDIS

- ☐ Hard scattering processes to access partons (quarks and gluons): high energy end of duality regime
- ☐ Measure longitudinal (lightcone) momentum fraction x of struck quark and quark densities $f_1^{H\rightarrow q}(x)$ multiplied with decay densities $D_1^{q\rightarrow h}(z)$
- ☐ Proton and neutron (possibly nuclei) for (limited) flavor sensitivity and selection of specific hadrons (π, K, ...) for further flavor selectivity.
- ☐ Polarization to access longitudinal and transverse polarization, polarimetry for ρ or Λ-production
- ☐ Acces to transverse momenta of partons: $p = xP + p_T$
- ☐ Limited access to higher twist contributions!

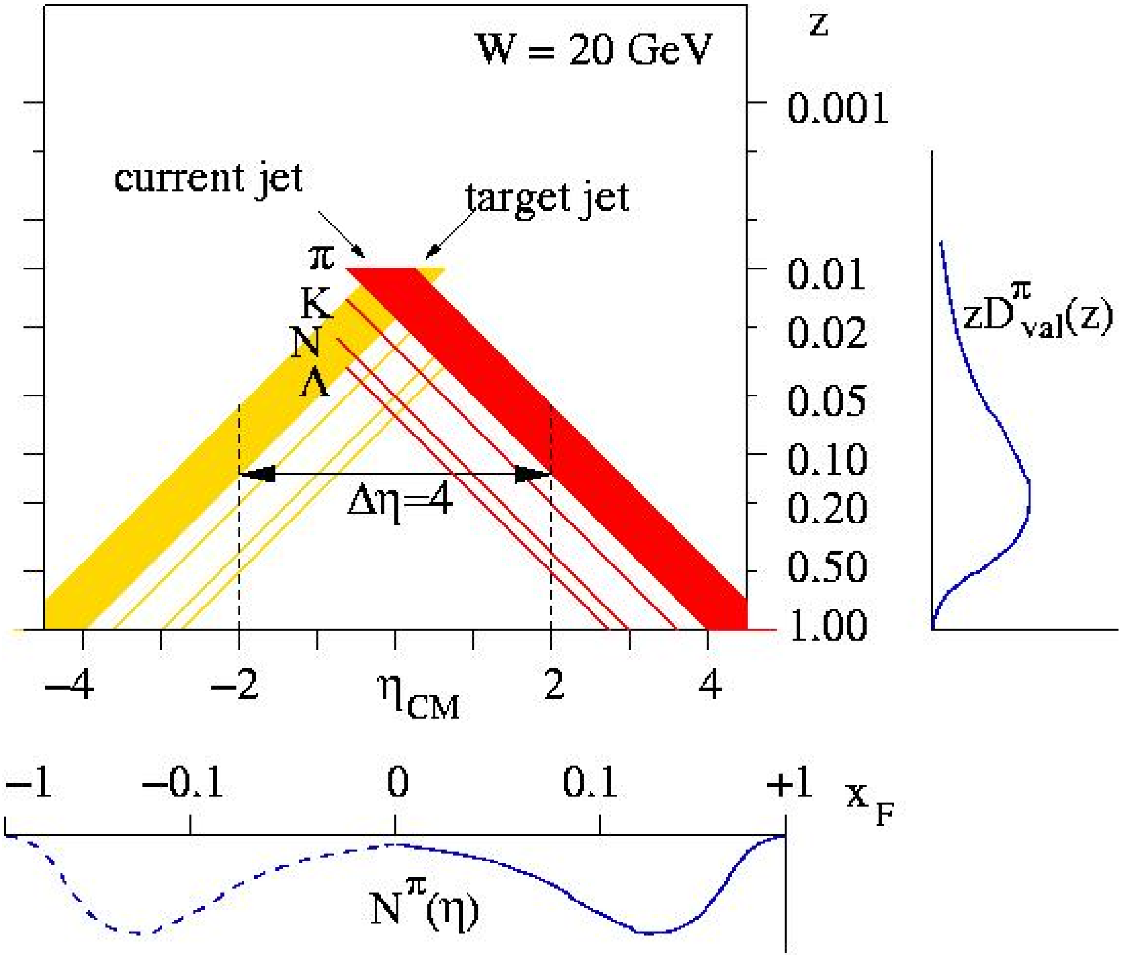
Kinematical flexibility



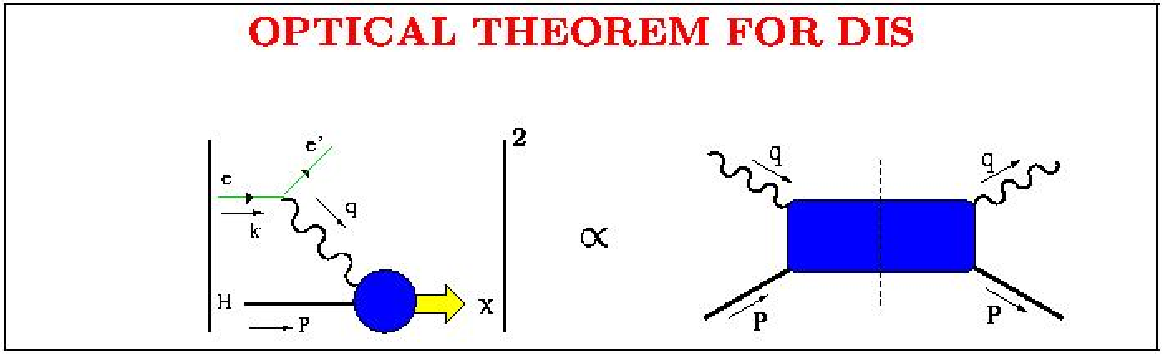
Kinematical flexibility



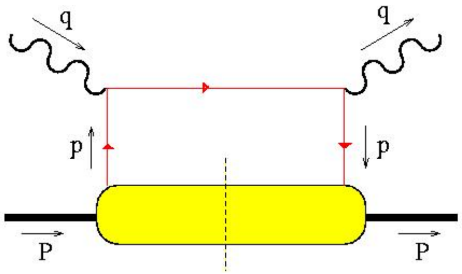
Kinematical flexibility



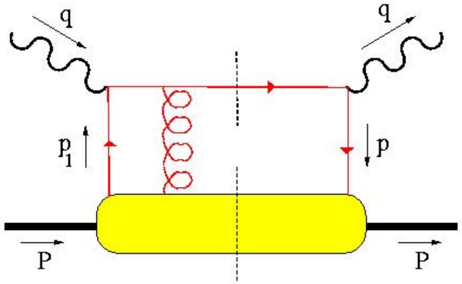
(calculation of) cross section in DIS



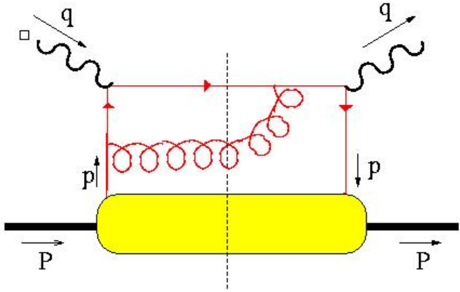
Full calculation



+

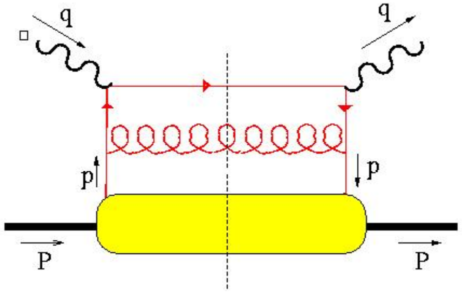


+



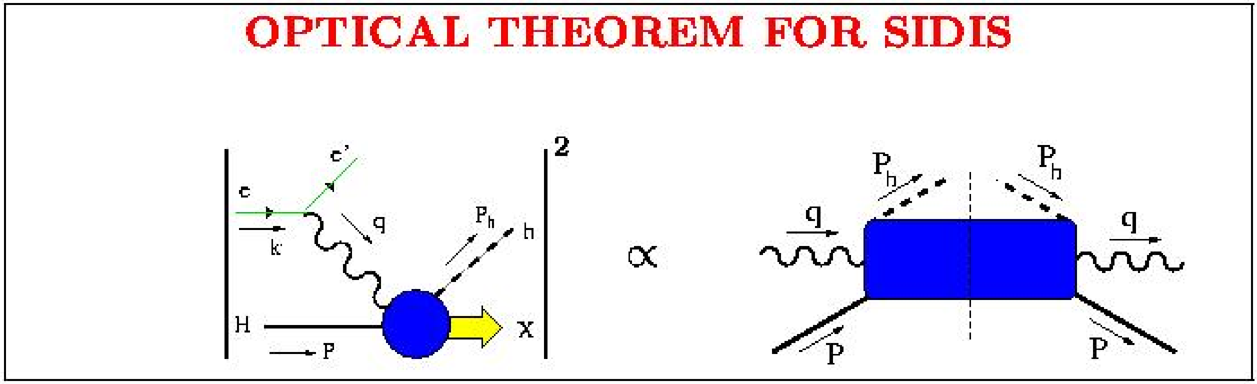
LEADING
(in 1/Q)

+

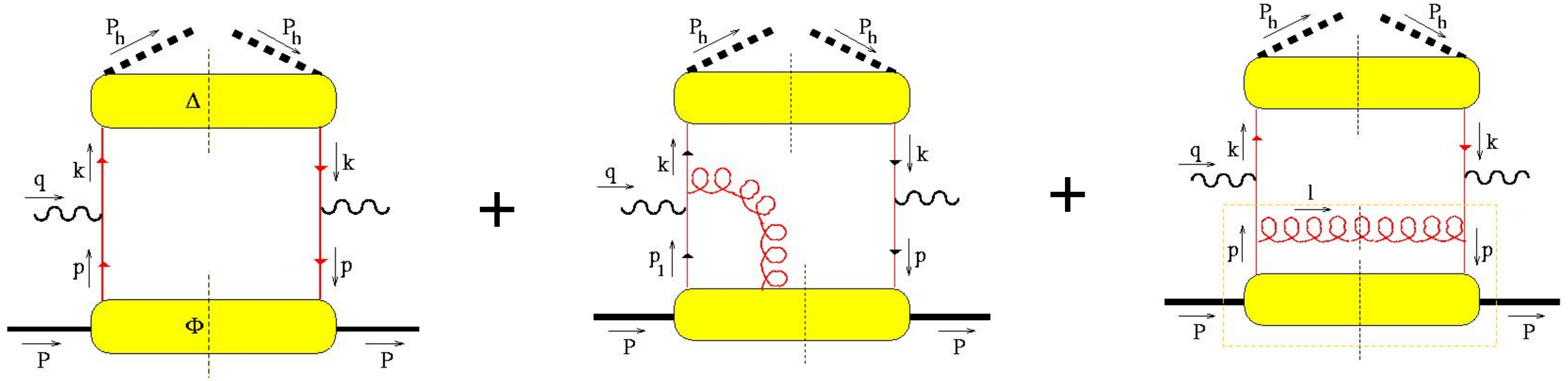


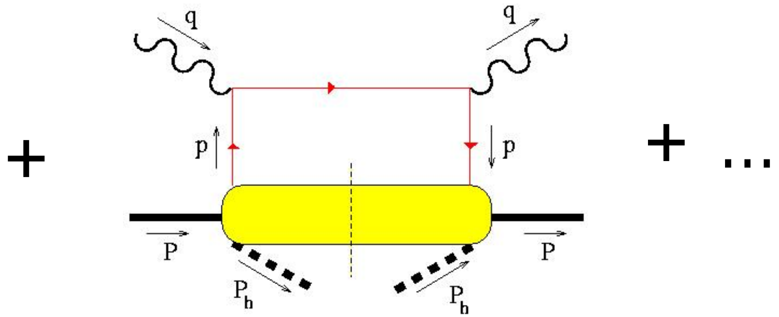
+ …

(calculation of) cross section in SIDIS



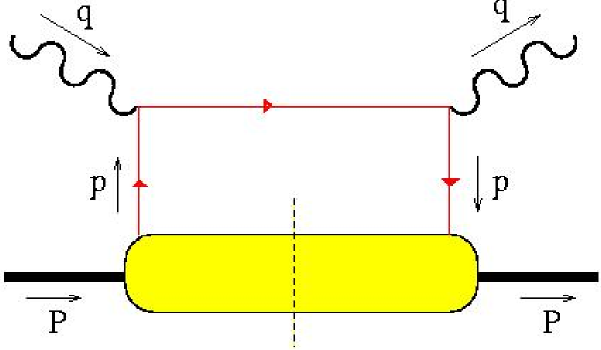
Full calculation

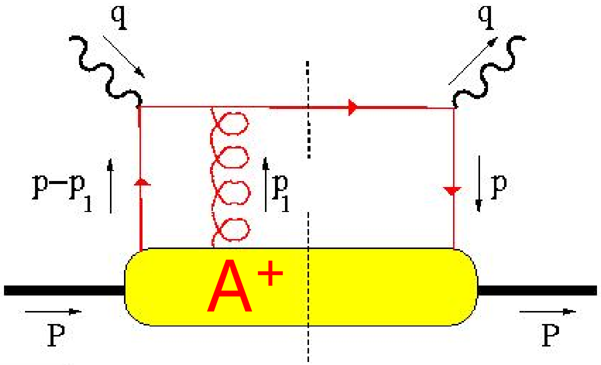




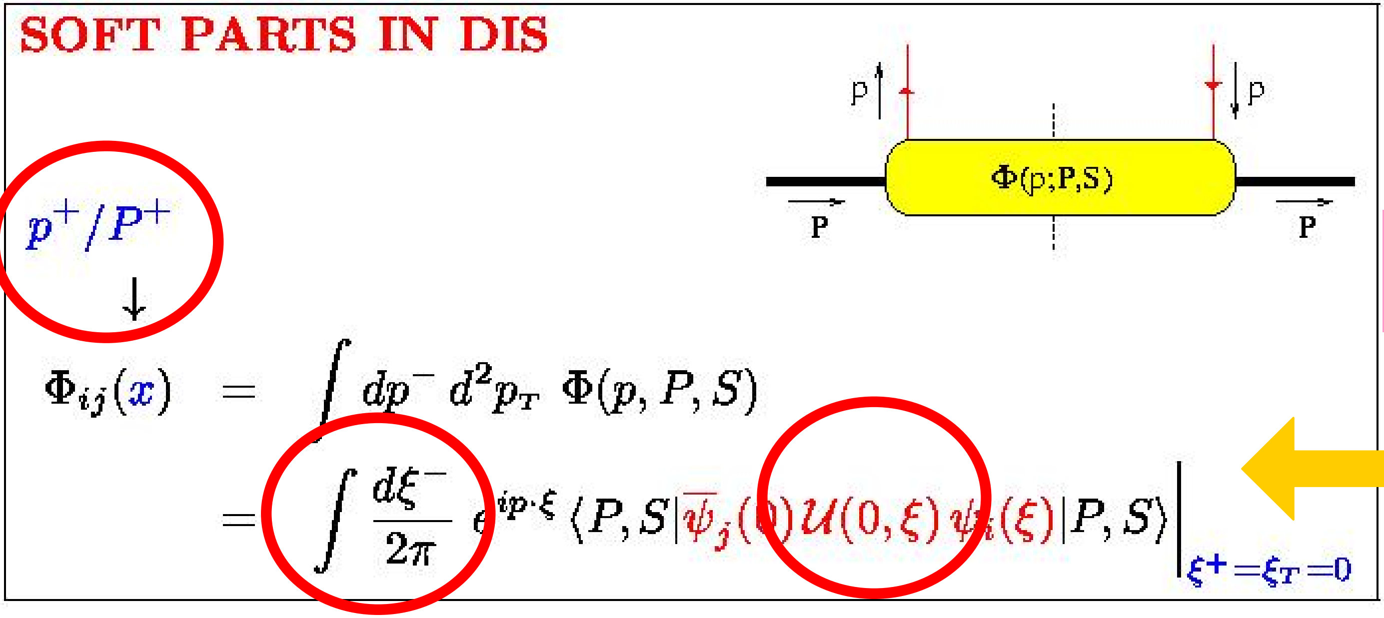
LEADING
(in 1/Q)

Soft part in (SI)DIS





- In limit of large Q^2 the result of ‘handbag diagram’ survives
- ... + contributions from A^+ gluons ensuring color gauge invariance



SOFT PARTS IN DIS

p^+/P^+

$\downarrow$

$$\Phi_{ij}(x) = \int dp^- \, d^2p_T \; \Phi(p,P,S)$$

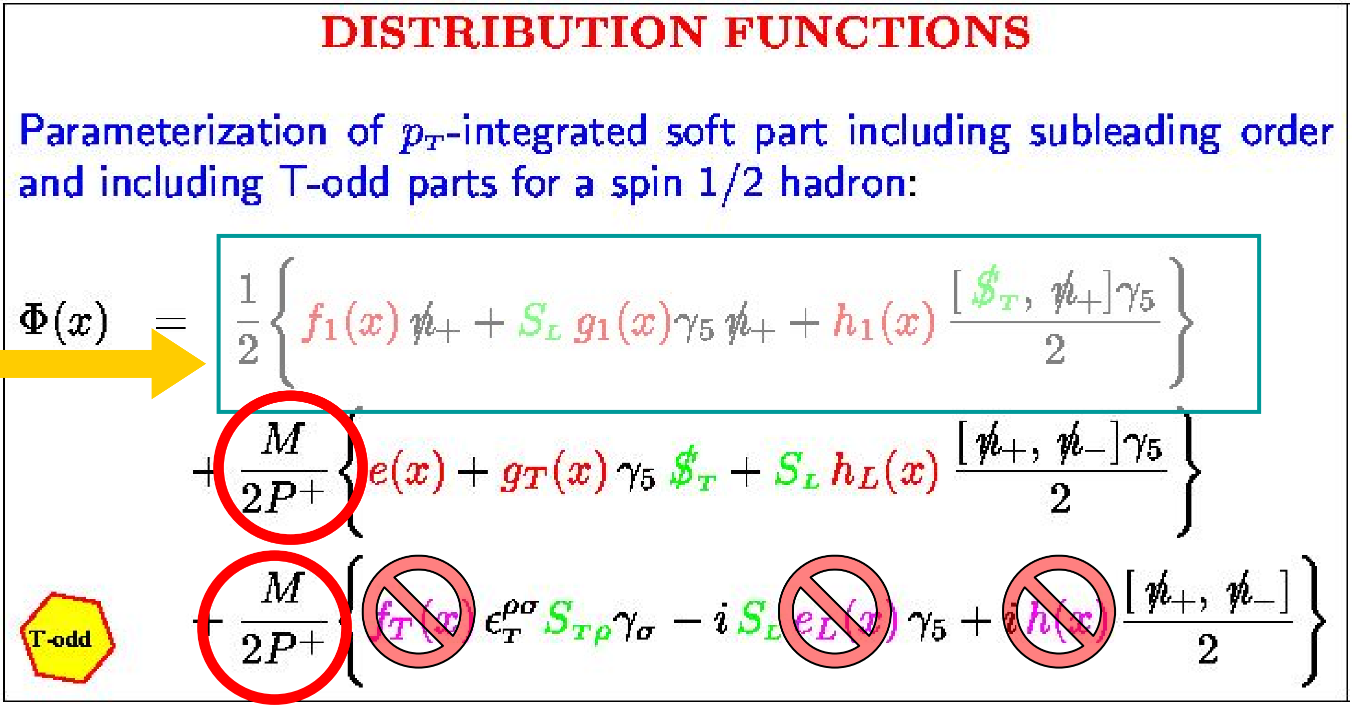
$$= \int \frac{d\xi^-}{2\pi} \, e^{ip\cdot\xi} \, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle \Big|_{\xi^+=\xi_T=0}$$

Ellis, Furmanski, Petronzio
Efremov, Radyushkin

A^+ gluons
→ gauge link

Parametrization of lightcone correlator

leading part



DISTRIBUTION FUNCTIONS

Parameterization of p_T-integrated soft part including subleading order and including T-odd parts for a spin 1/2 hadron:

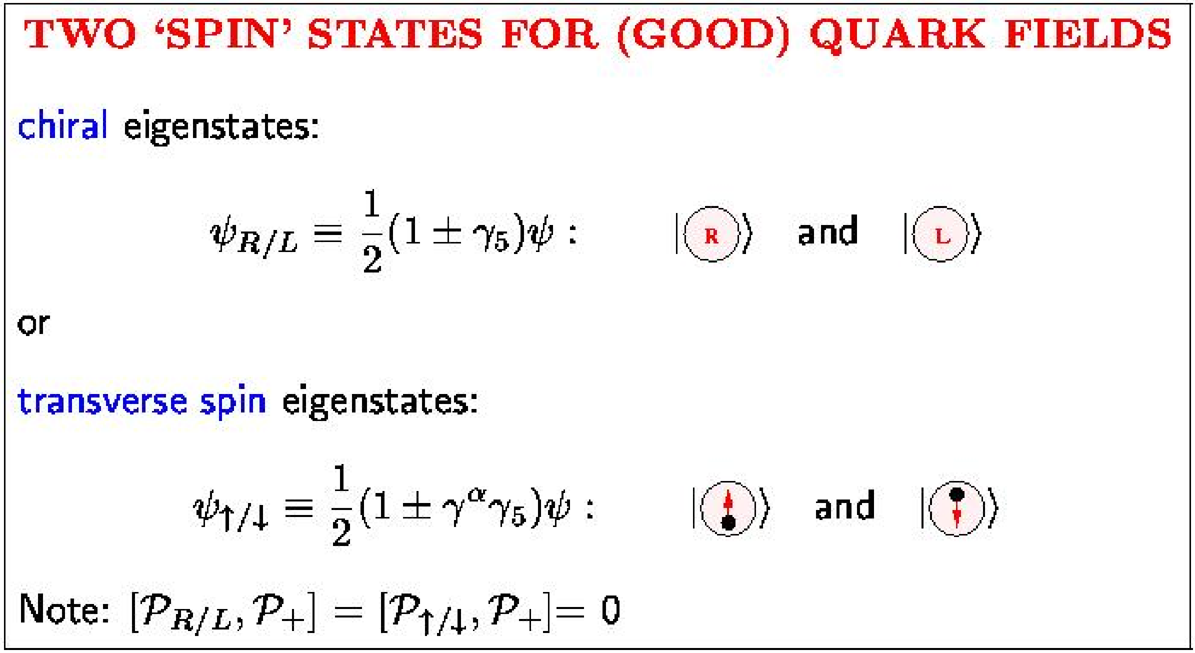
$$\Phi(x) = \boxed{\frac{1}{2}\left\{f_1(x)\,\not n_+ + S_L\,g_1(x)\gamma_5\,\not n_+ + h_1(x)\,\frac{[\not S_T,\not n_+]\gamma_5}{2}\right\}}$$
$$+\frac{M}{2P^+}\left\{e(x) + g_T(x)\,\gamma_5\,\not S_T + S_L\,h_L(x)\,\frac{[\not n_+,\not n_-]\gamma_5}{2}\right\}$$
$$+\frac{M}{2P^+}\left\{\cancel{f_T(x)}\,\epsilon_T^{\rho\sigma}S_{T\rho}\gamma_\sigma - i\,S_L\,\cancel{e_L(x)}\,\gamma_5 + \cancel{i\,h(x)}\,\frac{[\not n_+,\not n_-]}{2}\right\}$$

- M/P$^+$ parts appear as M/Q terms in cross section
- T-reversal applies to Φ(x) → no T-odd functions

Jaffe & Ji NP B 375 (1992) 527
Jaffe & Ji PRL 71 (1993) 2547

Basis of partons

- Good part of Dirac space is 2-dimensional
- Interpretation of DF's



TWO 'SPIN' STATES FOR (GOOD) QUARK FIELDS

chiral eigenstates:

$$\psi_{R/L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi : \qquad |\mathrm{R}\rangle \quad \text{and} \quad |\mathrm{L}\rangle$$

or

transverse spin eigenstates:

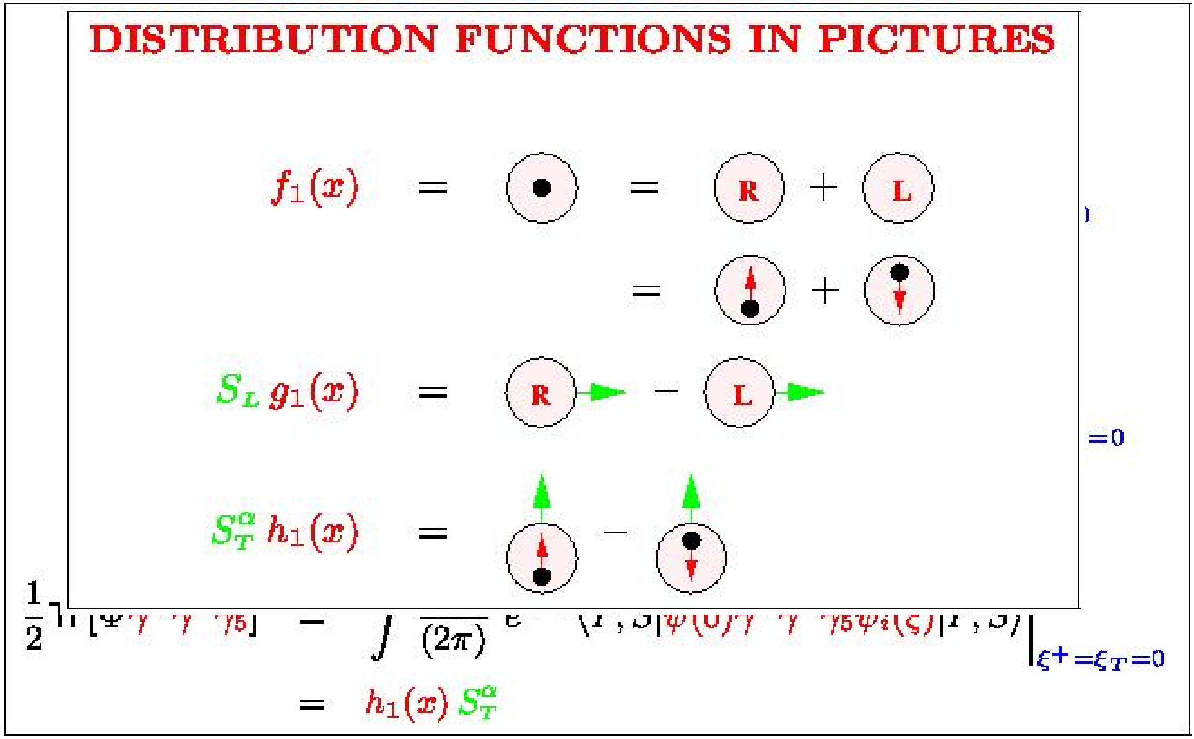
$$\psi_{\uparrow/\downarrow} \equiv \frac{1}{2}(1 \pm \gamma^\alpha \gamma_5)\psi : \qquad |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle$$

Note: $[\mathcal{P}_{R/L}, \mathcal{P}_+] = [\mathcal{P}_{\uparrow/\downarrow}, \mathcal{P}_+] = 0$

unpolarized quark distribution q(x)

helicity or chirality distribution Δq(x)

transverse spin distr. or transversity δq(x)



DISTRIBUTION FUNCTIONS IN PICTURES

$f_1(x) = ● = \mathrm{R} + \mathrm{L}$

$= \uparrow + \downarrow$

$S_L\, g_1(x) = \mathrm{R}\rightarrow - \mathrm{L}\rightarrow$

$S_T^\alpha\, h_1(x) = \uparrow - \downarrow$

$\frac{1}{2}$ $= \int$ (2π) $\Big|_{\xi^+=\xi_T=0}$

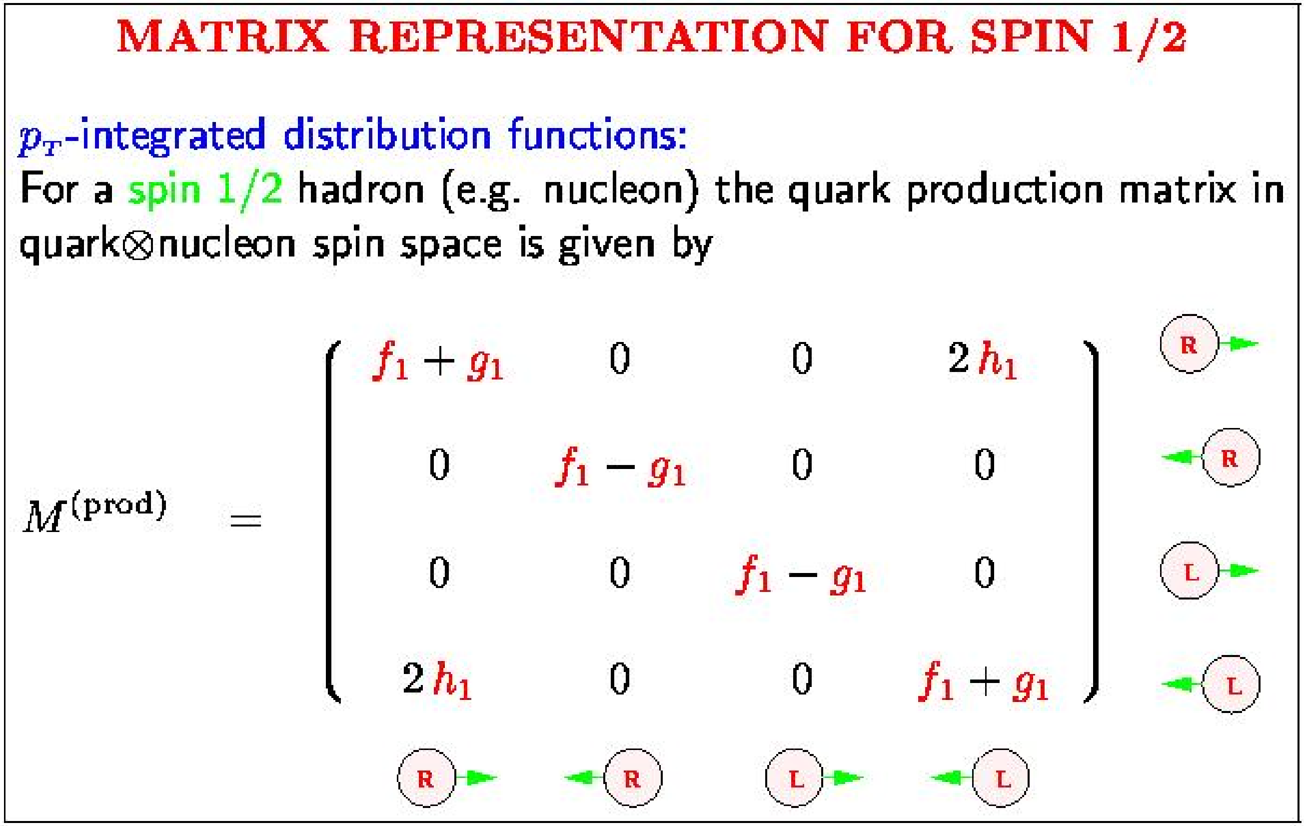
$= h_1(x)\, S_T^\alpha$

Matrix representation for M = $[\Phi(x)\gamma^+]^T$

Bacchetta, Boglione, Henneman & Mulders, PRL 85 (2000) 712

Quark production matrix, directly related to the helicity formalism

Anselmino et al.



MATRIX REPRESENTATION FOR SPIN 1/2

p_T-integrated distribution functions:

For a spin 1/2 hadron (e.g. nucleon) the quark production matrix in quark⊗nucleon spin space is given by

$$M^{(\mathrm{prod})} = \begin{array}{c} \\ \left(\begin{array}{cccc} f_1+g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1-g_1 & 0 & 0 \\ 0 & 0 & f_1-g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1+g_1 \end{array}\right) \\ \begin{array}{cccc} \textcircled{R}\rightarrow & \leftarrow\textcircled{R} & \textcircled{L}\rightarrow & \leftarrow\textcircled{L} \end{array} \end{array} \begin{array}{c} \textcircled{R}\rightarrow \\ \leftarrow\textcircled{R} \\ \textcircled{L}\rightarrow \\ \leftarrow\textcircled{L} \\ \\ \end{array}$$

- Off-diagonal elements (RL or LR) are chiral-odd functions
- Chiral-odd soft parts must appear with partner in e.g. SIDIS, DY

Theory

- ☐ Acces to transverse momenta of partons: $p = xP + p_T$
- ☐ Limited access to higher twist contributions!

Measuring transverse structure

$p \approx x\,P + p_T$

$k \approx z^{-1}\,K + k_T$

- In a hard process one probes partons (quarks and gluons)
- Momenta fixed by kinematics (external momenta)

  DIS $x = x_B = Q^2/2P.q$

  SIDIS $z = z_h = P.K_h/P.q$
- Also possible for transverse momenta

  SIDIS $q_T = k_T - p_T$

  $= q + x_B P - K_h/z_h \approx -K_{h\perp}/z_h$

  2-particle inclusive hadron-hadron scattering

  $q_T = p_{1T} + p_{2T} - k_{1T} - k_{2T}$

  $= K_1/z_1 + K_2/z_2 - x_1 P_1 - x_2 P_2 \approx K_{1\perp}/z_1 + K_{2\perp}/z_2$
- Sensitivity for transverse momenta requires ≥3 momenta

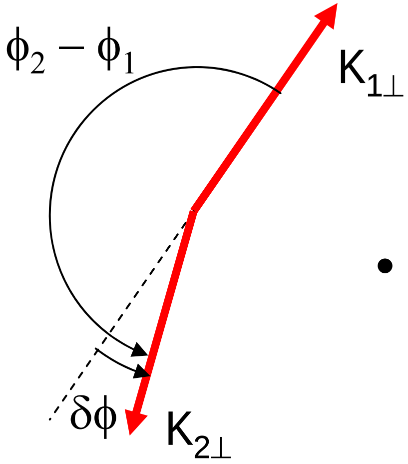
  SIDIS: $\gamma^* + H \rightarrow h + X$

  DY: $H_1 + H_2 \rightarrow \gamma^* + X$

  e^+e^-: $\gamma^* \rightarrow h_1 + h_2 + X$

  hadronproduction: $H_1 + H_2 \rightarrow h + X$

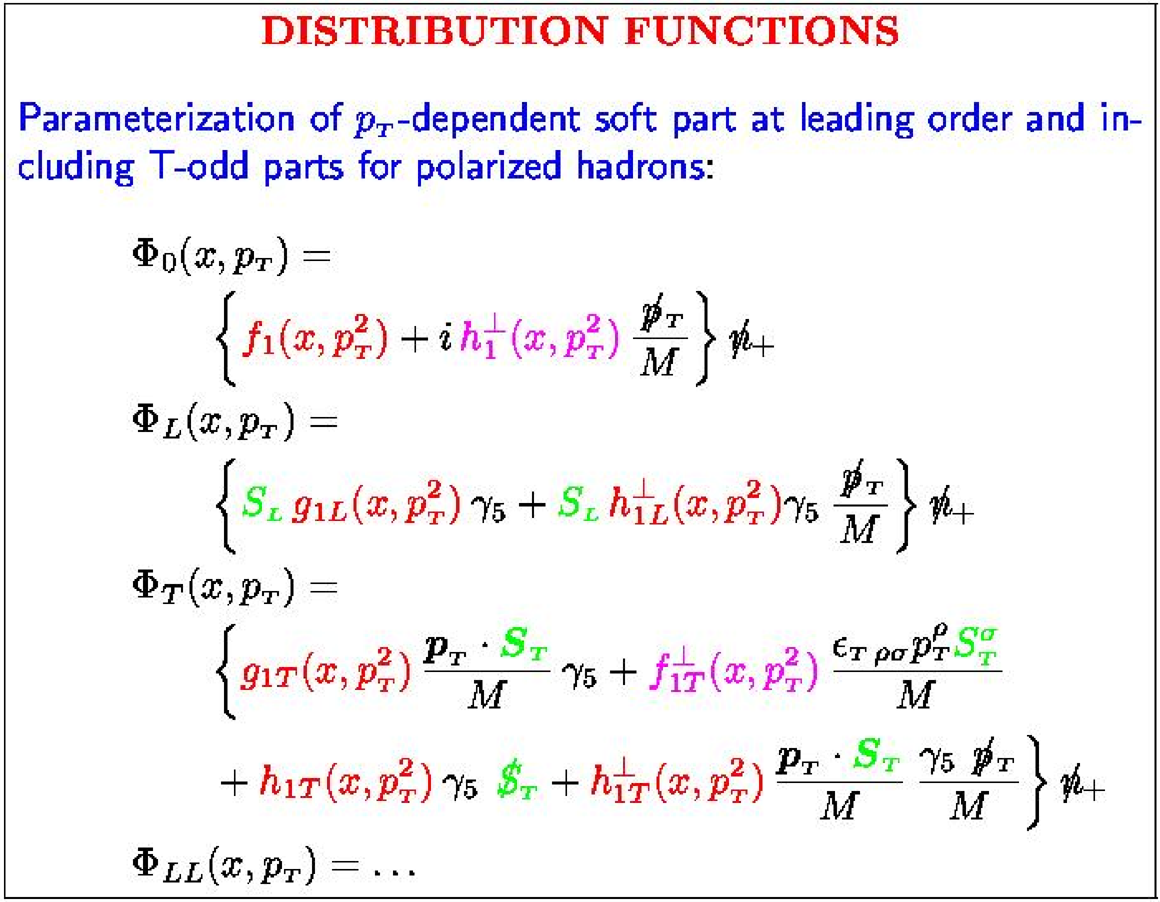
  $\rightarrow h_1 + h_2 + X$



pp-scattering

Parametrization of $\Phi(x,p_T)$

- Additional TMD distribution functions, terms ~ p_T
- Link dependence allows also T-odd distribution functions since T U[0,∞] $T^\dagger$ = U[0,-∞]
- Functions $h_1^\perp$ and $f_{1T}^\perp$ (Sivers) nonzero! They come from gauge link (i.e. involve gluon fields)
- Similar functions (of course) exist as fragmentation functions (no T-constraints) $H_1^\perp$ (Collins) and $D_{1T}^\perp$
- For spin 0 and spin ½ T-odd effects require p_T



DISTRIBUTION FUNCTIONS

Parameterization of p_T-dependent soft part at leading order and including T-odd parts for polarized hadrons:

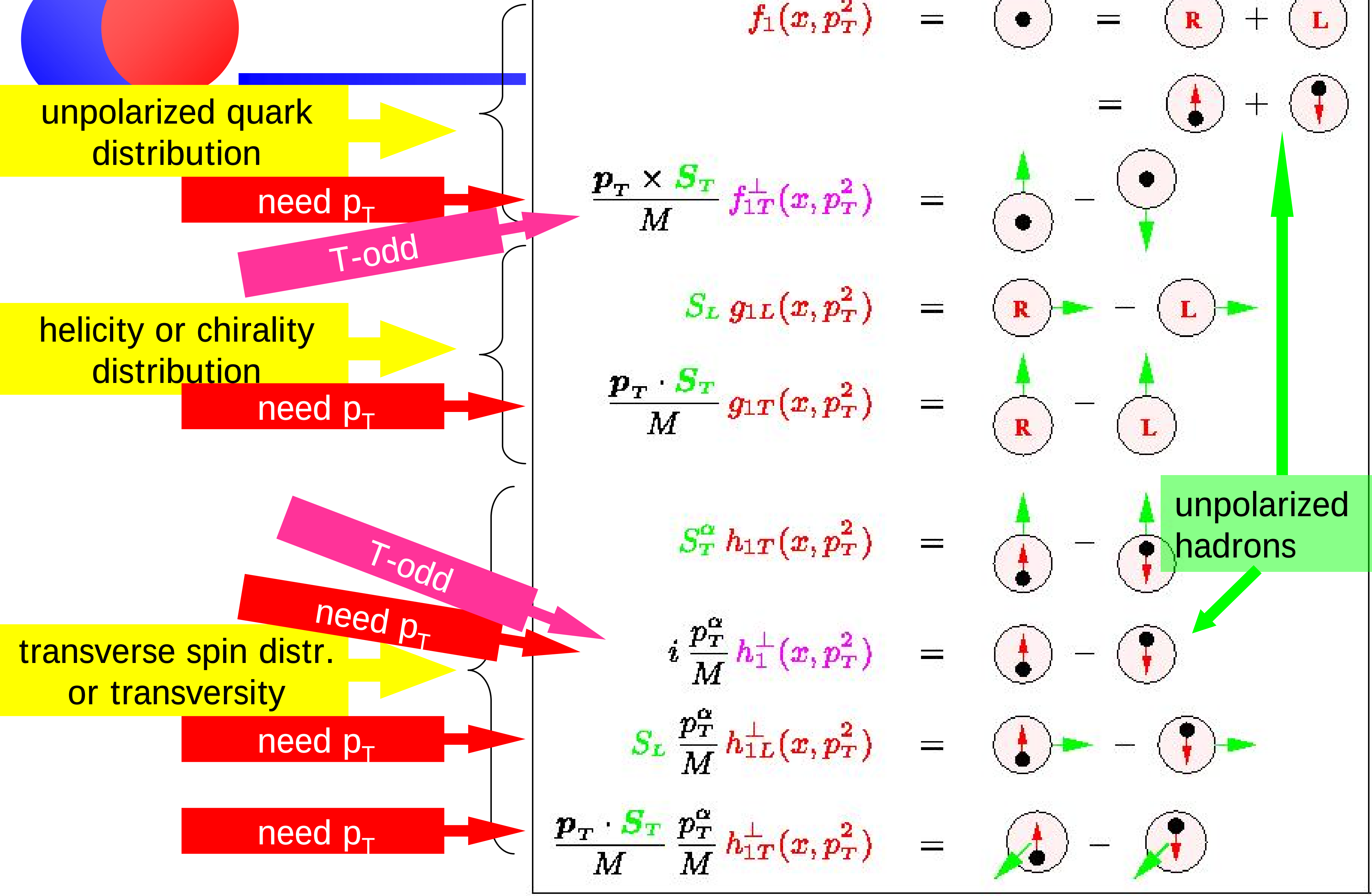
$$\Phi_0(x,p_T) = \left\{ f_1(x,p_T^2) + i\, h_1^\perp(x,p_T^2)\, \frac{\not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_L(x,p_T) = \left\{ S_L\, g_{1L}(x,p_T^2)\, \gamma_5 + S_L\, h_{1L}^\perp(x,p_T^2) \gamma_5\, \frac{\not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_T(x,p_T) = \left\{ g_{1T}(x,p_T^2)\, \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M}\, \gamma_5 + f_{1T}^\perp(x,p_T^2)\, \frac{\epsilon_{T\,\rho\sigma} p_T^\rho S_T^\sigma}{M} + h_{1T}(x,p_T^2)\, \gamma_5\, \not{S}_T + h_{1T}^\perp(x,p_T^2)\, \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M}\, \frac{\gamma_5\, \not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_{LL}(x,p_T) = \ldots$$

Interpretation



unpolarized quark distribution

need p_T

T-odd

helicity or chirality distribution

need p_T

T-odd

need p_T

transverse spin distr. or transversity

need p_T

need p_T

unpolarized hadrons

DISTRIBUTION FUNCTIONS IN PICTURES

$$f_1(x, p_T^2) =$$

$$\frac{\boldsymbol{p}_T \times \boldsymbol{S}_T}{M} f_{1T}^{\perp}(x, p_T^2) =$$

$$S_L\, g_{1L}(x, p_T^2) =$$

$$\frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M} g_{1T}(x, p_T^2) =$$

$$S_T^{\alpha}\, h_{1T}(x, p_T^2) =$$

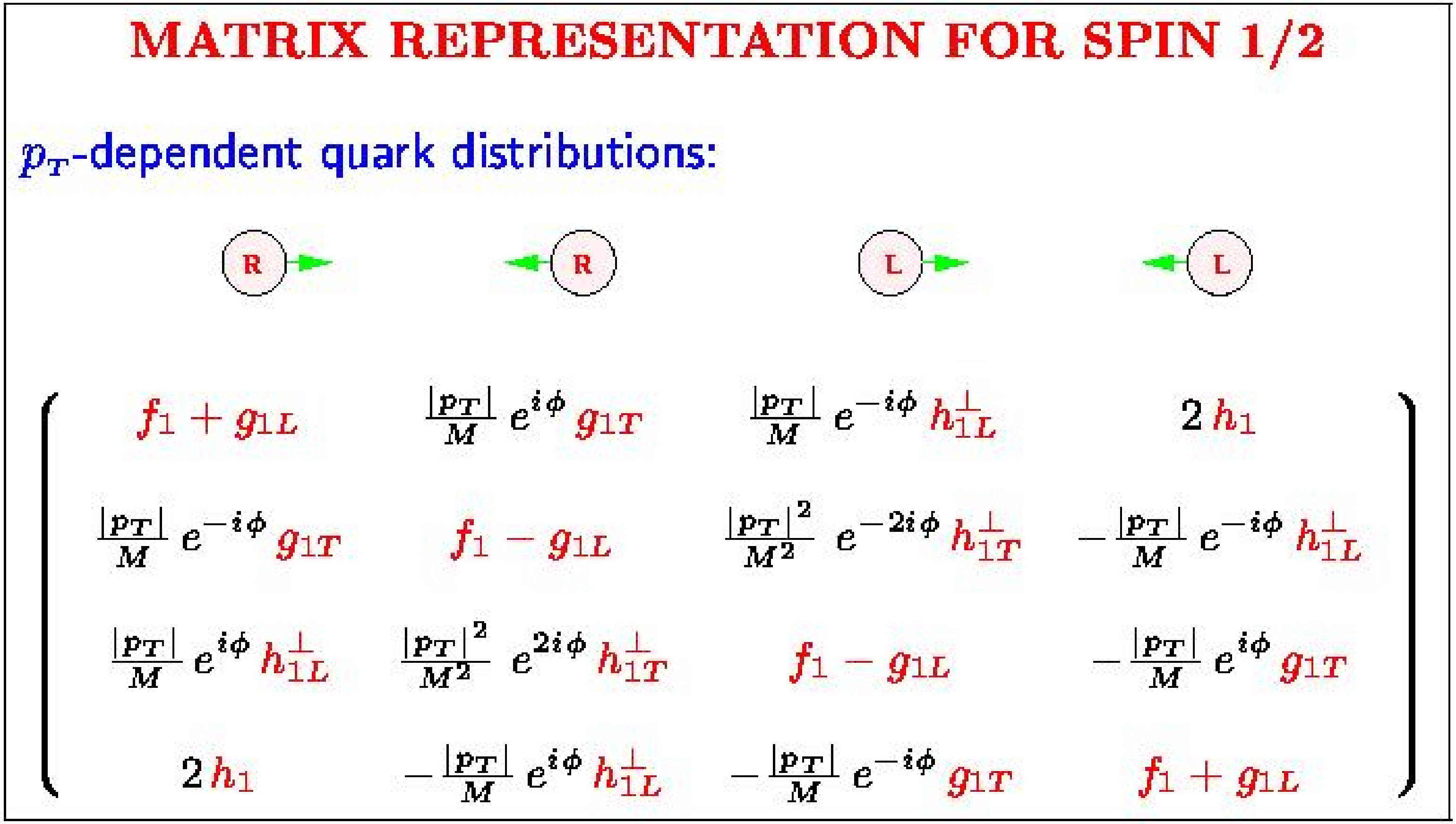
$$i\,\frac{p_T^{\alpha}}{M} h_1^{\perp}(x, p_T^2) =$$

$$S_L \frac{p_T^{\alpha}}{M} h_{1L}^{\perp}(x, p_T^2) =$$

$$\frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M} \frac{p_T^{\alpha}}{M} h_{1T}^{\perp}(x, p_T^2) =$$

Matrix representation for M $= [\Phi^{[\pm]}(x,p_T)\gamma^+]^T$

- p_T-dependent functions



MATRIX REPRESENTATION FOR SPIN 1/2

p_T-dependent quark distributions:

$$\begin{array}{cccc}
R \rightarrow & \leftarrow R & L \rightarrow & \leftarrow L \\
\end{array}$$

$$\begin{pmatrix}
f_1 + g_{1L} & \frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} & \frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} & 2\, h_1 \\
\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2}\, e^{-2i\phi}\, h_{1T}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} \\
\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & \frac{|p_T|^2}{M^2}\, e^{2i\phi}\, h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} \\
2\, h_1 & -\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 + g_{1L}
\end{pmatrix}$$

T-odd: $g_{1T} \rightarrow g_{1T} - i\, f_{1T}^{\perp}$ and $h_{1L}^{\perp} \rightarrow h_{1L}^{\perp} + i\, h_1^{\perp}$ (imaginary parts)

Bacchetta, Boglione, Henneman & Mulders
PRL 85 (2000) 712

Possibilities in leptoproduction of pions

Asymmetries are (theoretically) most clean in terms of transverse moments (p_T-weighted functions)

MATRIX REPRESENTATION FOR SPIN 0

p_T-dependent quark fragmentation functions:

$$M^{(\mathrm{dec})} = \begin{array}{cc} \begin{pmatrix} D_1 & i\,\frac{|k_T|\,e^{-i\phi}}{M_h}\,H_1^{\perp} \\ -i\,\frac{|k_T|\,e^{+i\phi}}{M_h}\,H_1^{\perp} & D_1 \end{pmatrix} & \begin{matrix} \text{Ⓡ} \\ \text{Ⓛ} \end{matrix} \\ \begin{matrix} \text{Ⓡ} & \qquad\qquad \text{Ⓛ} \end{matrix} & \end{array}$$

SIDIS: $\ell + H^{\uparrow} \to \ell + h + X$

$$\left\langle \frac{Q_T}{M}\,\sin(\phi_h^\ell - \phi_S^\ell) \right\rangle_{OTO} = \frac{2\pi\alpha^2\,s}{Q^4}\,|S_T|\left(1 - y + \frac{1}{2}\,y^2\right)\sum_{a,\bar{a}} e_a^2\,x_B\,f_{1T}^{\perp(1)a}(x_B)\,D_1^a(z_h)$$

$$\left\langle \frac{Q_T}{M_h}\,\sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OTO} = \frac{2\pi\alpha^2\,s}{Q^4}\,|S_T|\,2(1-y)\sum_{a,\bar{a}} e_a^2\,x_B\,h_1^a(x_B)\,H_1^{\perp(1)a}(z_h)$$

$H_1^{\perp}$ is T-odd and chiral-odd

Factorization studies in SIDIS

- Measurements of TMD distribution (and fragmentation functions)
- Investigate the p_T-dependence
- Important as experimental input on factorization behavior
- Normal situation

$$\Phi_2(x, p_T) \rightarrow \frac{\alpha(p_T^2)}{p_T^2} K \otimes \Phi_2(x)$$

- TMD functions

$$\Phi_3(x, p_T) \rightarrow \frac{\alpha(p_T^2)}{|\, p_T \,|} K \otimes \Phi_2(x)$$

After integration: subleading in 1/Q → NLO (in α_s)

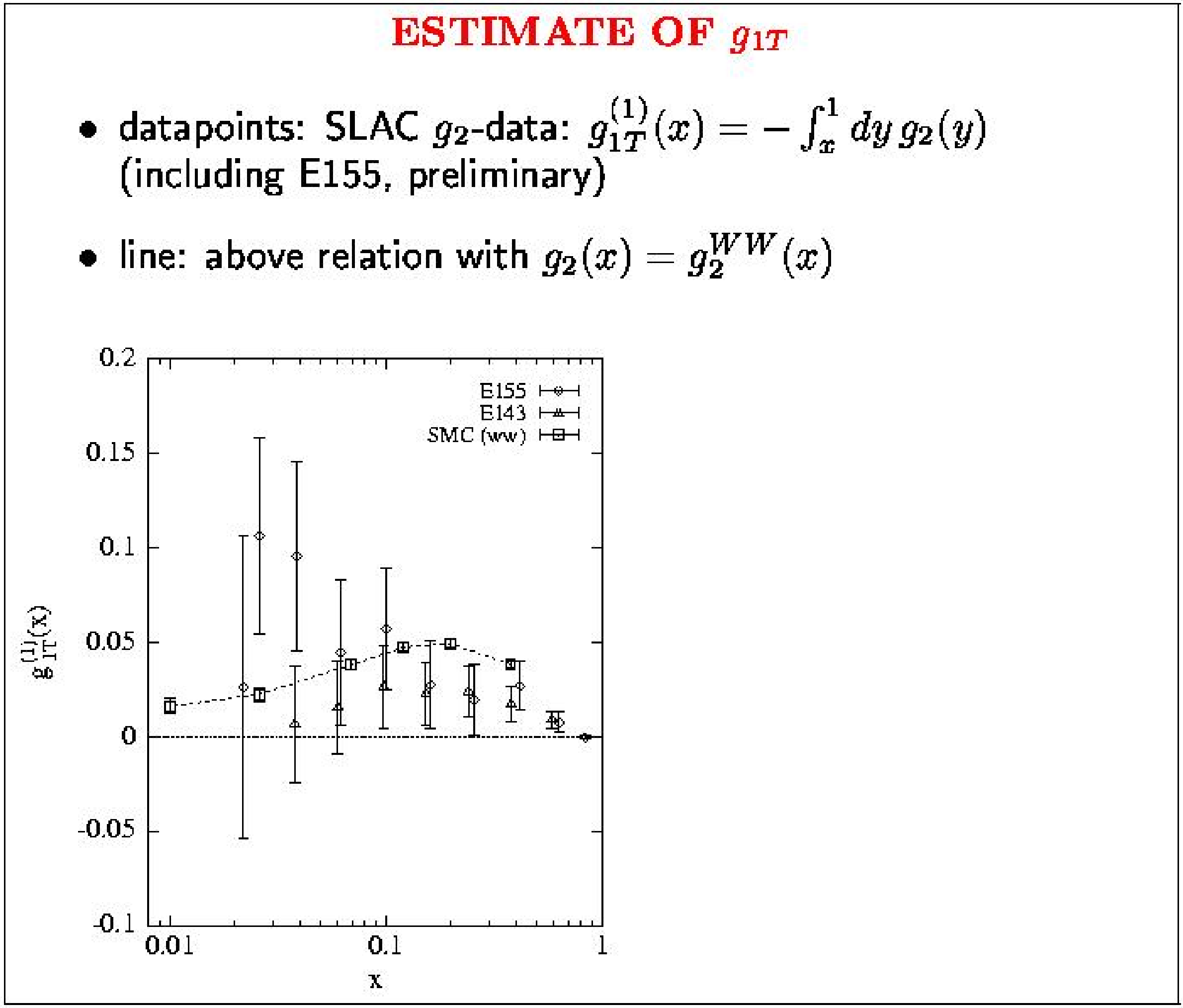
- Where: look for $<\cos \phi_h>$, $<\sin \phi_h>$ azimuthal asymmetries.

Transverse momenta and higher twist

- Link between weighted TMD functions (transverse moments) and higher twist functions (QCD eom)

- $g_{1T}^{(1)}(x,p_T^2)$ from SIDIS $\langle q_T \cos(\phi_s-\phi_h)\rangle$
- g_2 from DIS at 1/Q $\langle \cos(\phi_s)\rangle$

- Role of gauge link needs to be (further) investigated.



ESTIMATE OF g_{1T}

- datapoints: SLAC g_2-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy\, g_2(y)$ (including E155, preliminary)
- line: above relation with $g_2(x) = g_2^{WW}(x)$

Single spin asymmetries
Color gauge invariance

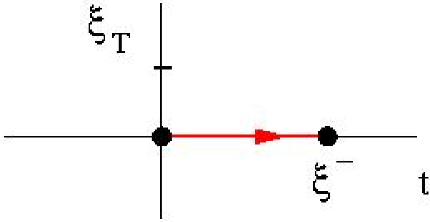
- Nonlocal combinations of colored fields must be joined by a gauge link:

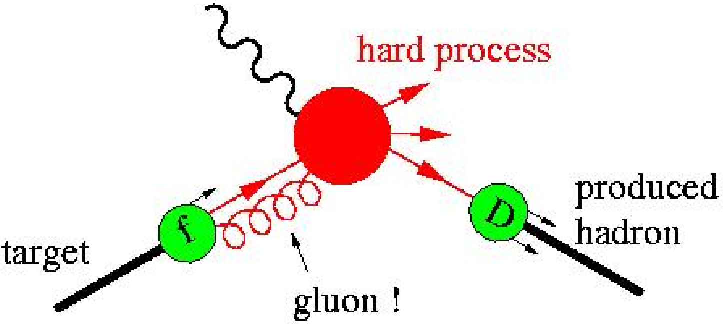
$$\bar{\psi}(0)\psi(\xi) \to \bar{\psi}(0)U(0,\xi)\psi(\xi)$$

$$\hookrightarrow U(0,\xi) = \mathscr{P}\exp\left(-ig\int_0^{\xi} ds^{\mu} A_{\mu}\right)$$

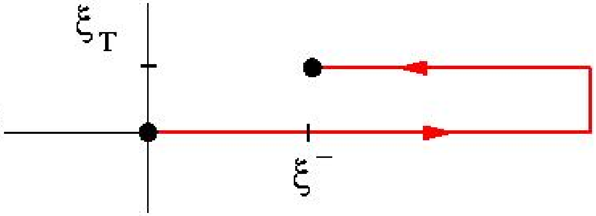
- Gauge link structure is calculated from collinear A.n gluons exchanged between soft and hard part

DIS → $\Phi^{[U]}$

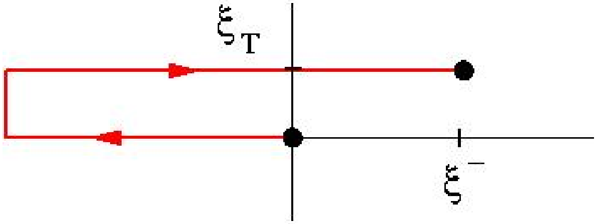




SIDIS → $\Phi^{[U^+]} = \Phi^{[+]}$



DY → $\Phi^{[U^-]} = \Phi^{[-]}$



- Link structure for TMD functions depends on the hard process!
  (Pijlman, Bomhof, PM
  unifies Brodsky, Schmidt, Ji, Yuang, ...)

Summary

- Access to the (transverse) partonic structure of hadrons is still in a very preliminary stage (azimuthal asymmetries by themselves or combined with single or double spin asymmetries)
- Theoretical interesting parts are contained in α_s, p_T and 1/Q structure of cross section putting demands on experimental detection capabilities
- Like the spin puzzle, interplay of theory and experiment is essential