Demonstrating 6D Cooling. Guggenheim Channel Simulations

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Ionization cooling

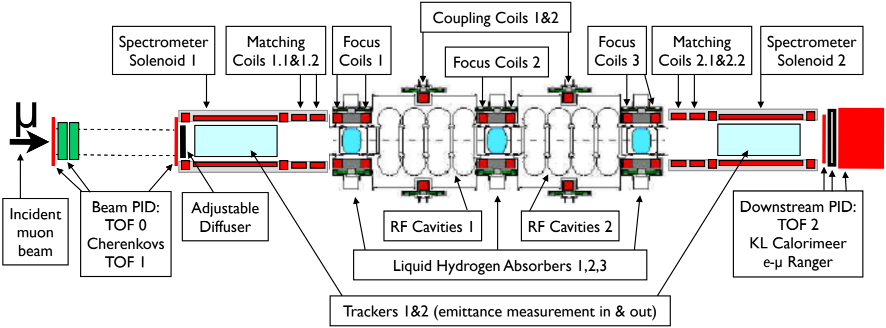
- The only efficient technique to cool within a muon lifetime.
- Passing through material reduces all three components of momentum, only the longitudinal is restored, $\Rightarrow$ transverse cooling.
- Need emittance exchange to cool in 6D.

6D cooling

- Transverse part: will be demonstrated in MICE.
- 6D cooling (via transverse cooling + emittance exchange):
  - Initial test using a wedge absorber in MICE (emittance exchange).
  - Down-selection of the cooling channel.
  - 6D experiment planning and design (implementation is not part of MAP).

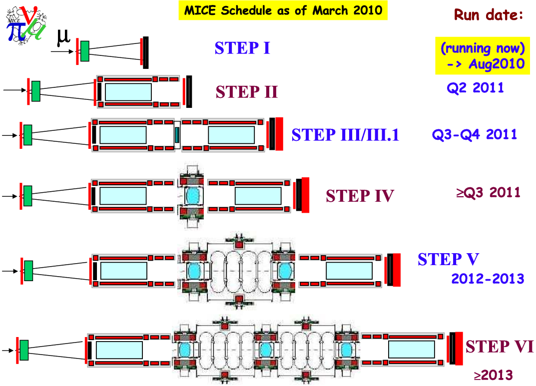
Wedge absorber in MICE
Step IV

MICE layout



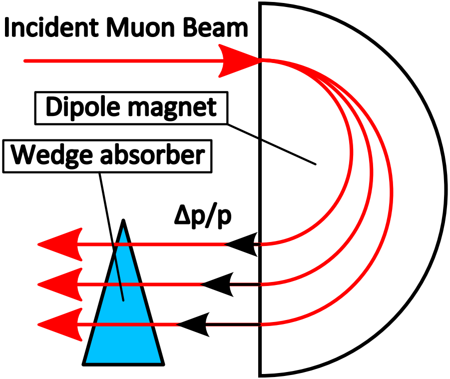
MICE layout scheme

MICE step-wise implementation



MICE implementation schedule. Now it looks like Step II/III might be skipped due to spectrometer solenoid issues (see Kaplan's talk on MICE).

Emittance exchange

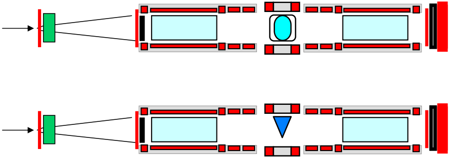


Based on the image by Muons, Inc.

- Introduce dispersion (in MICE: by careful beam selection).
- Let particles pass through a wedge absorber in such a way that particles with larger momentum lose more energy.
- Longitudinal emittance is reduced at the expense of deliberately increasing transverse emittance (emittance exchange).

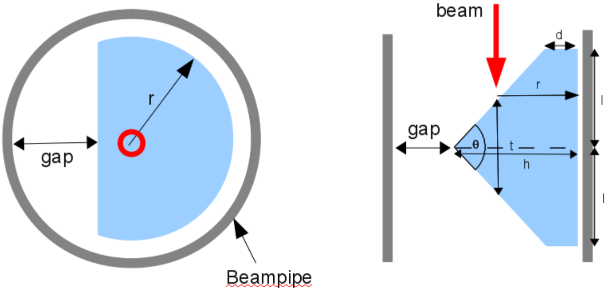
MICE Step IV with wedge

- Top: MICE Step IV with a liquid hydrogen absorber. MICE is a 4D cooling experiment: transverse emittance is reduced while longitudinal emittance stays the same or increases slightly due to stochastic processes in the energy loss.



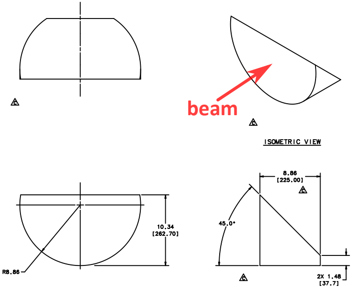
- Bottom: LH_2 absorber is replaced with a solid wedge absorber. This way emittance exchange can be observed if the beam is properly matched (dispersion is introduced).

Wedge schematic



- Wedge absorber = cylinder intersected with a triangular prism.
- Opening angle = 90°, on-axis length = 75.4 mm (corresp. to 12 MeV energy loss at p=200 MeV/c), radius=225 mm, gap=187.3 mm.

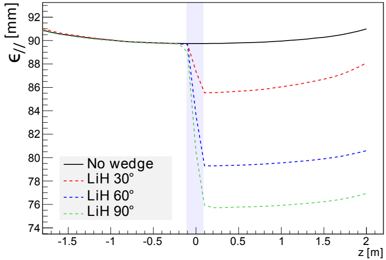
LiH wedge

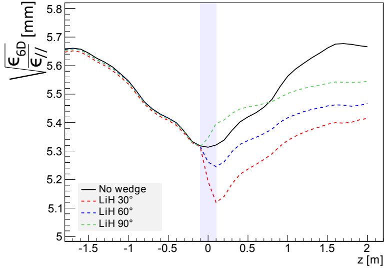


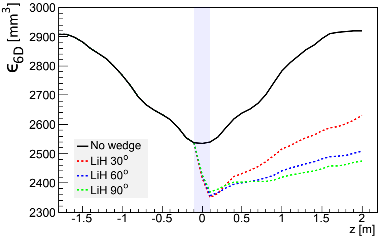
- 90° LiH wedge ordered (consisting of two parts, only one part is shown).

- 90° wedge provides best longitudinal cooling / emittance exchange.
- 45° half-wedge needs to be simulated.
- In addition to the LiH wedge a set (90°, 60°, and 30°) of plastic wedges would be useful to test properties of different materials (time permitting).
- Wedge support design is underway.

Cooling performance







- Cooling effect observed for different angles (red – 30°, blue – 60°, green – 90°)

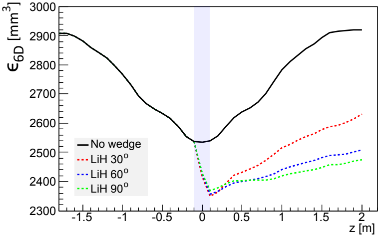
$$\epsilon_{\|} = \frac{c}{m^3}\sqrt{\det(\mathrm{V}(\mathrm{ct},\mathrm{E}))},$$

$$\epsilon_{6D} = \frac{c}{m}\sqrt{\det(\mathrm{V}(\mathrm{ct},\mathrm{E},\mathrm{x},\mathrm{p_x},\mathrm{y},\mathrm{p_y}))},$$

V – covar. matrix of the specified space.

Concern: 6D emittance change with no material

- With no material in the channel the system is Hamiltonian.
- According to the graph, the 6D emittance changes.
- Is Liouville safe?

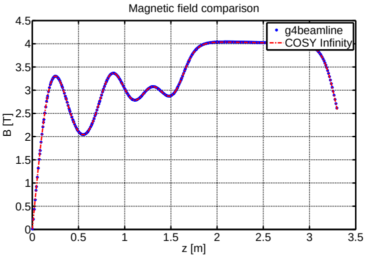


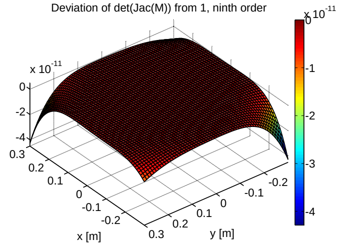
Phase volume conservation

- Hamiltonian system: $\det(\mathrm{Jac}(\mathcal{M})) = 1$, where $\mathrm{Jac}(\mathcal{M})$ is the Jacobian of the transformation of phase space: $\mathcal{M}(\vec{q}, \vec{p}) = (\vec{Q}, \vec{P})$, (q, p) are phase space coordinates before the transformation, (Q, P)—after the transformation.
- Let S_1 be a subset of phase space, $S_2 = \mathcal{M}(S_1)$.
- Then, $V_2 = \int_{S_2} d^n\vec{Q}d^n\vec{P} = \int_{S_1} \det(\mathrm{Jac}(\mathcal{M}))d^n\vec{q}d^n\vec{p} = \int_{S_1} d^n\vec{q}d^n\vec{p} = V_1$.
- $V_2 = V_1 = \mathrm{const}$.
- Need to check that the determinant in question is indeed equal to 1.

COSY Infinity analysis

- Implemented MICE magnets in COSY, compared to g4beamline, very good agreement.
- Calculated a high-order transformation map.
- Obtained the determinant of the Jacobian as a high-order polynomial.
- $\det(\mathrm{Jac}(M)) = 1$, deviation from 1 in (x,y) is shown on right.



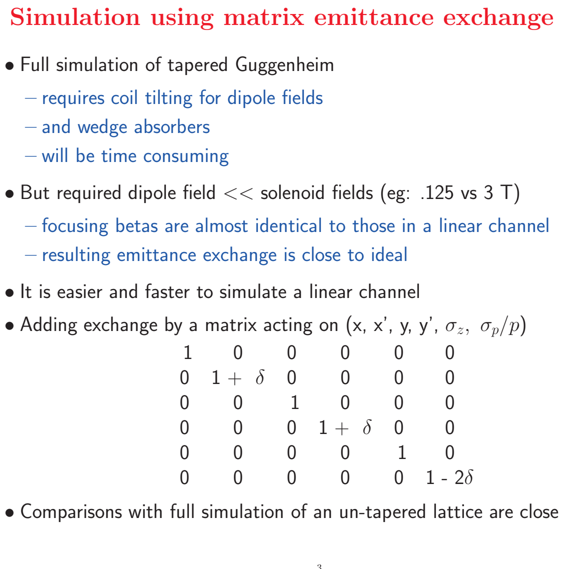


COSY analysis summary

- 6D emittance is conserved.
- Change in emittance observed is due to approximation:
  $\epsilon_{6D} = \frac{c}{m}\sqrt{\det(\mathrm{V}(\mathrm{ct}, \mathrm{E}, \mathrm{x}, \mathrm{p_x}, \mathrm{y}, \mathrm{p_y}))}$,
  V – covar. matrix of the specified space.
- Need a better 6D emittance estimate.
- Two ideas:
  - find 6D phase space volume using Voronoi tesselation (computationally challenging in 6D);
  - reconstruct emittance immediately upstream and downstream of the wedge from tracker measurements.

Tapered Guggenheim simulations (step towards down-selection)

From Bob Palmer's talk in April 2010:



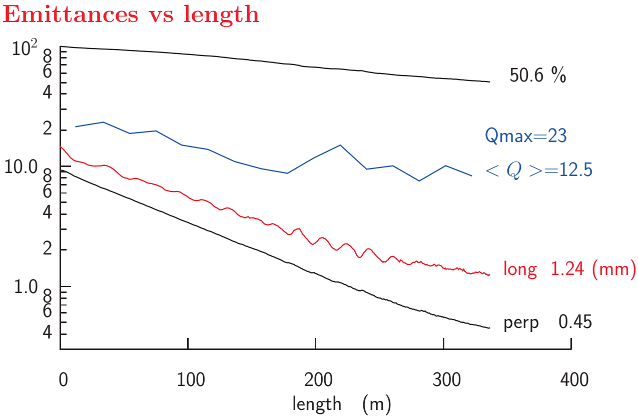
Simulation using matrix emittance exchange

- Full simulation of tapered Guggenheim
  - requires coil tilting for dipole fields
  - and wedge absorbers
  - will be time consuming
- But required dipole field << solenoid fields (eg: .125 vs 3 T)
  - focusing betas are almost identical to those in a linear channel
  - resulting emittance exchange is close to ideal
- It is easier and faster to simulate a linear channel
- Adding exchange by a matrix acting on (x, x', y, y', σ_z, σ_p/p)

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1+\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\delta \end{matrix}$$

- Comparisons with full simulation of an un-tapered lattice are close

From Bob Palmer’s talk in April:



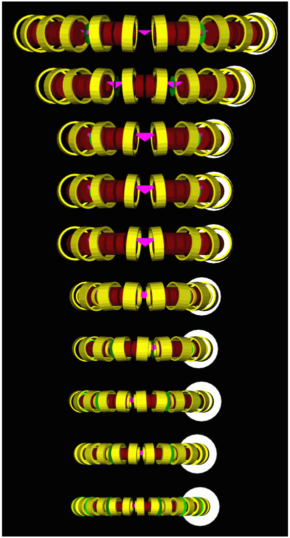
Emittances vs length

- No transverse emittance growth at matches is observed
- Initial Q is better than in un-tapered lattice (23 vs. 15)
- Final Q is better than in un-tapered lattice (12 vs. 8)

G4beamline simulation

For each step:

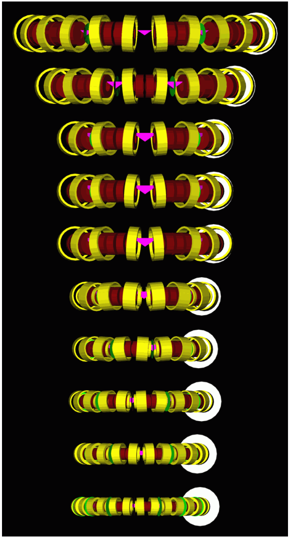
- Earlier studies show that simulation results for the helix and the ring are very similar.
- Hence, I simulate a set of rings rather than the helix.
- This allows for staged simulations, and reduces the complexity of the model.
- Results will be more realistic than simulating a linear channel + linear emittance exchange map.



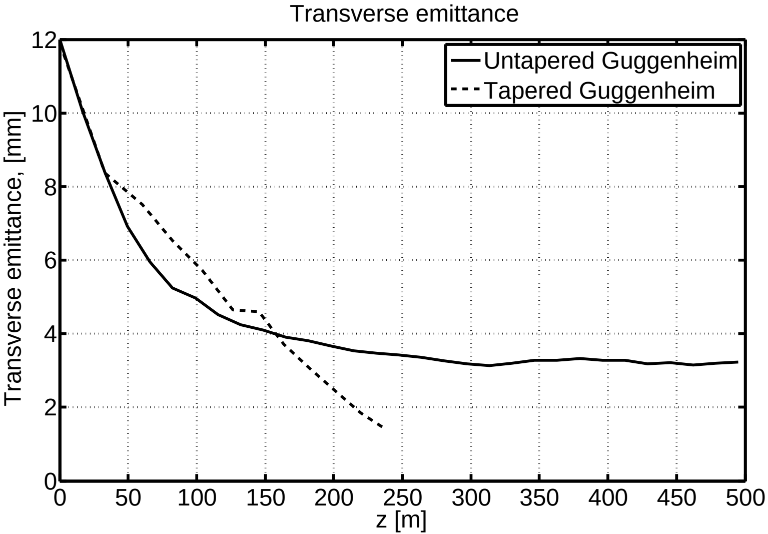
G4beamline simulation

For each stage:

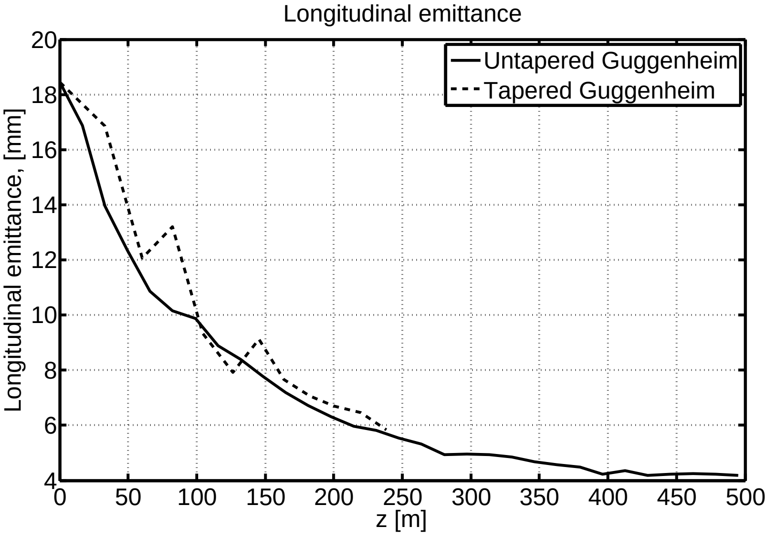
1. Coil tilting (to generate bending field).
2. Coil displacement (to minimize vertical orbit excursion).
3. Geomety issues (placement along the arc rather than a straight line).
4. Wedge absorbers (size, position, tilt, edge cut).
5. Closed orbit.



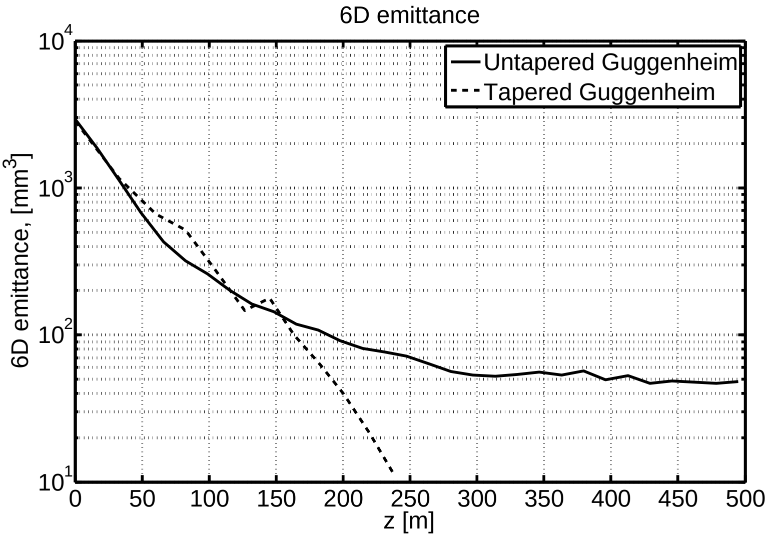
Tapered Guggenheim simulation



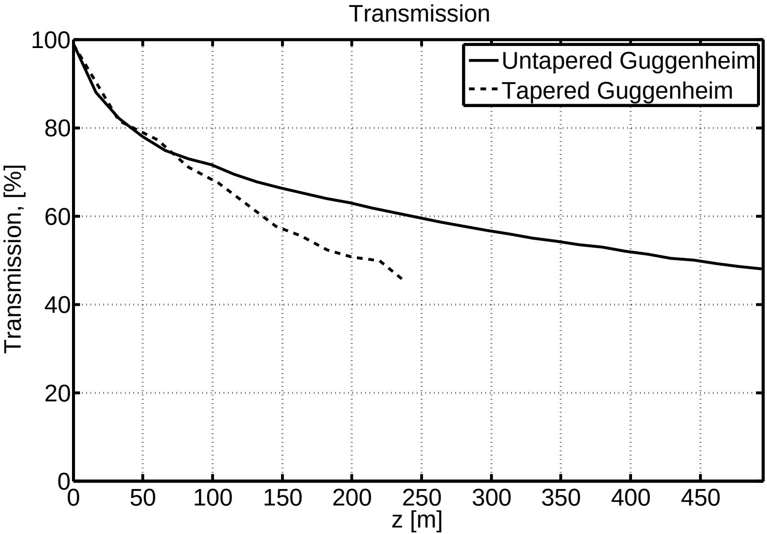
Tapered Guggenheim simulation



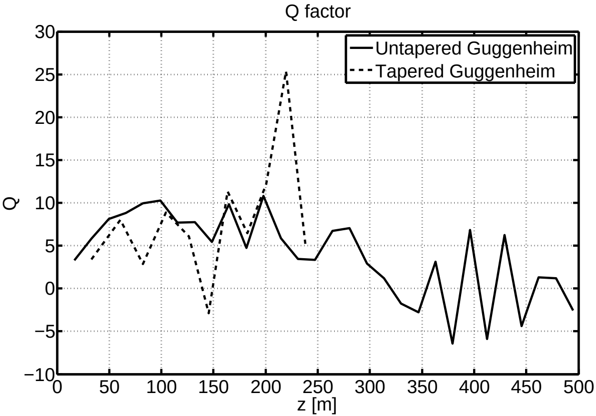
Tapered Guggenheim simulation



Tapered Guggenheim simulation



Tapered Guggenheim simulation



$Q = \frac{d\epsilon_{6D}^N/ds}{dN/ds}\frac{N(s)}{\epsilon_{6D}^N(s)}$, $\epsilon_{6D}^N(s)$—normalized six-dimensional emittance of the beam, $N(s)$—number of surviving particles. Q factor compares the rate of change of emittance to the particle loss.

6D demo strategy

- MICE is both technology demo and beam experiment. Once MICE demonstrates transverse cooling and emittance exchange, most of remaining 6D-cooling-channel risk is believed to be technological (i.e., can we build and operate the channel as designed).
- Bench test: cooling channel section should be long enough to address key integration issues (cavities in B field, spatial compatibility issue,
- Bench-tested channel section may be different than that needed for a beam test (but try to maintain compatibility).

- Cooling experiment design:
    - Simulations to clarify appropriate performance + needed precision.
    - Diagnostics/detector study to determine how to measure the muon beam to required precision.
    - Design/integration study to specify and lay out experiment: coordinate to ensure bench-test hardware also suitable for beam test, find suitable location, design needed muon beam line (unless MICE hall and beam suitable and available).
- Many details undefined until baseline channel is selected.

Thank You!