Nucleon structure in terms of OPE with non-perturbative Wilson coefficients

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Abstract :

• Lattice calculations may contribute to the understanding of **Deep Inelastic Scattering** by evaluating **moments of the Nucleon Structure Functions.**

• To this end we study the product of electromagnetic currents between quark states. The **Operator Product Expansion (OPE)** decomposes it into matrix elements of **local** operators (depending on the quark momenta) and Wilson coefficients (as functions of the larger photon momenta).

• For consistency we evaluate a set of **Wilson coefficients non-perturbatively,** based on propagators for numerous momentum sources, on a $24^3 \times 48$ lattice. **Overlap quarks** suppress unwanted operator mixing.

• Results for the leading Wilson coefficients are extracted by means of Singular Value Decomposition.

Motivation

Deep Inelastic Scattering data are interpreted
in terms of Nucleon Structure Functions $\mathcal{M}$.

Continuum: *renormalon ambiguities !* $\Rightarrow$ Lattice formulation :

$$\mathcal{M}(q^2) = c^{(2)}(aq)A_2(a) + \frac{c^{(4)}(aq)}{q^2}A_4(a) + \ \ldots \quad \{\text{higher twists}\}$$

q : momentum transfer
$c^{(n)}$: Wilson coefficients
A_n : matrix elements

$c^{(n)}$ and A_n are "reduced" (Lorentz structure is factored out).

Traditionally $c^{(n)}$ were evaluated by cont. pert. theory.
But: $c^{(2)},\ A_4 \propto \frac{1}{a^2}$
Cancellation requires consistent non-pert. evaluation of both [1].

A_2 : determination by established procedure
based on ratio of 3-point / 2-point functions.

Focus on numerical results for $c^{(2)}$

Quenched simulation with Lüscher-Weisz gauge action,
$V = 24^3 \times 48,\ a \simeq 0.075\ \mathrm{fm}$, Landau gauge

Two flavours of **overlap valence quarks** $(\rho = 1.4)$
$\longrightarrow$ **suppresses** undesired **operator mixing** and $O(a)$ artifacts.

Quark mass $0.028 \simeq 73\ \mathrm{MeV}$.

For previous results with Wilson fermions, see Refs. [2].

OPE on the Lattice

Product of electromagnetic currents $J_\mu = \bar{\psi}\gamma_\mu\psi$

$$\begin{aligned}\mathbf{W}_{\mu\nu} &= \langle\psi(\mathbf{p})|\mathbf{J}_\mu(\mathbf{q})\mathbf{J}^\dagger_\nu(\mathbf{q})|\psi(\mathbf{p})\rangle \\ &= \sum_{\mathbf{m},\mathbf{n}} \mathbf{C}^{(\mathbf{m})}_{\mu\nu,\mu_1\ldots\mu_n}(\mathbf{qa})\ \langle\psi(\mathbf{p})|\mathcal{O}^{(\mathbf{m})}_{\mu_1\ldots\mu_n}|\psi(\mathbf{p})\rangle\end{aligned}$$

$\mathcal{O}^{(m)}$: **local** operators, characterise hadron structure

m : index for operators with same symmetry

$|\psi(p)\rangle$: quark state with (low) momentum p

$C^{(m)}$: Wilson coefficients, only depend on transfer momentum q

OPE truncation at low operator dimension requires **scale separation**

$$\underline{\mathbf{p^2 \ll q^2}} \ll (\pi/\mathbf{a})^2$$

The large lattice allows for a set of **small p^2, at fixed q.**

We consider **quark bilinears** with **up to 3 derivatives**:

$$\bar\psi\Gamma\psi\ , \quad \bar\psi\Gamma D_{\mu_1}\psi\ , \quad \bar\psi\Gamma D_{\mu_1}D_{\mu_2}\psi\ , \quad \bar\psi\Gamma D_{\mu_1}D_{\mu_2}D_{\mu_3}\psi$$

Γ : full Clifford structure $\Rightarrow$ $16\cdot\sum_{d=0}^{3}4^d = 1360$ operators. *<u>However</u> :*

we choose transfer momentum $\mathbf{aq} = \frac{\pi}{4}(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$ $(|q| \simeq 4.1\ \mathrm{GeV})$.

$\Rightarrow$ Symmetry reduces **number of indep. operators** to **67** [3].

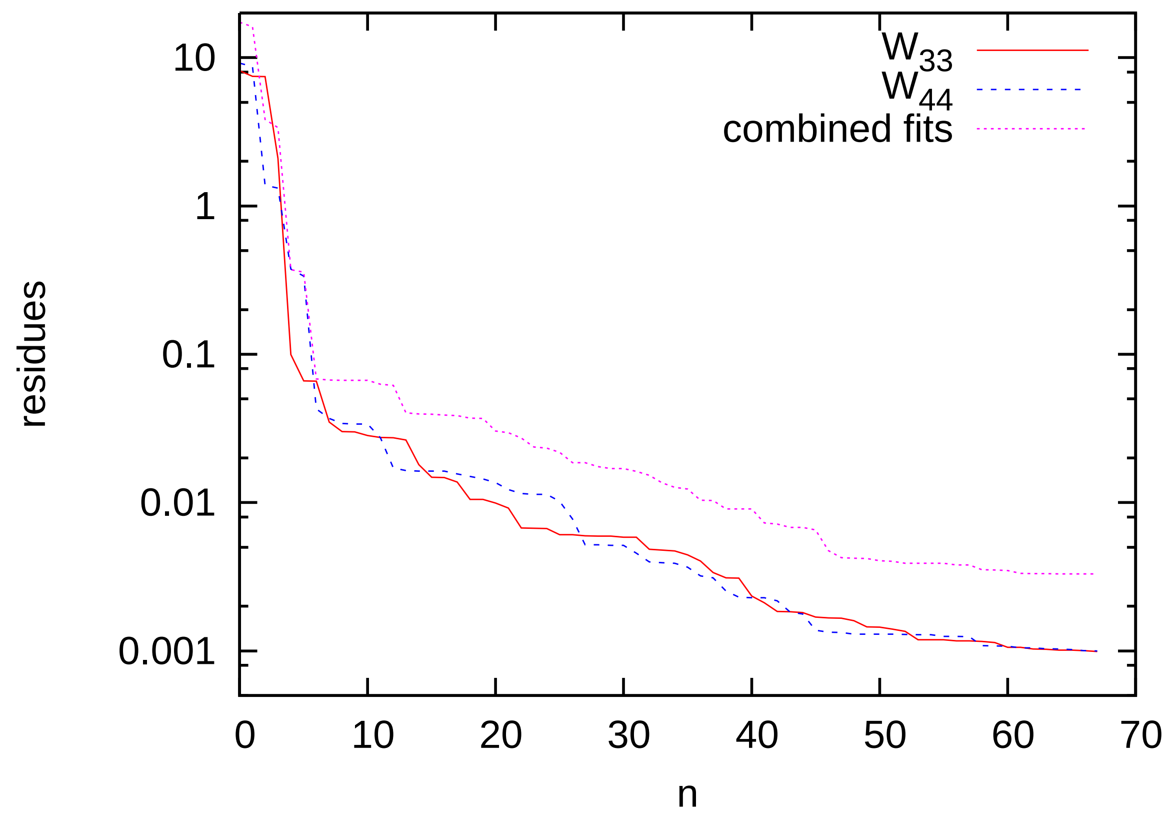
We measure $W_{\mu\nu}$ off-shell for $M = 16$ momentum sources
to determine $N = 67$ Wilson coefficients,

$$\begin{pmatrix} W^{(p_1)} \\ \cdot \\ \cdot \\ \cdot \\ W^{(p_M)} \end{pmatrix} = \begin{pmatrix} \mathcal{O}_1^{(p_1)} & \cdot & \cdot & \cdot & \mathcal{O}_N^{(p_1)} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \mathcal{O}_1^{(p_M)} & \cdot & \cdot & \cdot & \mathcal{O}_N^{(p_M)} \end{pmatrix} \begin{pmatrix} C_1 \\ \cdot \\ \cdot \\ C_N \end{pmatrix}$$

(where the elements $W^{(p_i)}$ and $\mathcal{O}_k^{(p_i)}$ are 4×4 matrices).

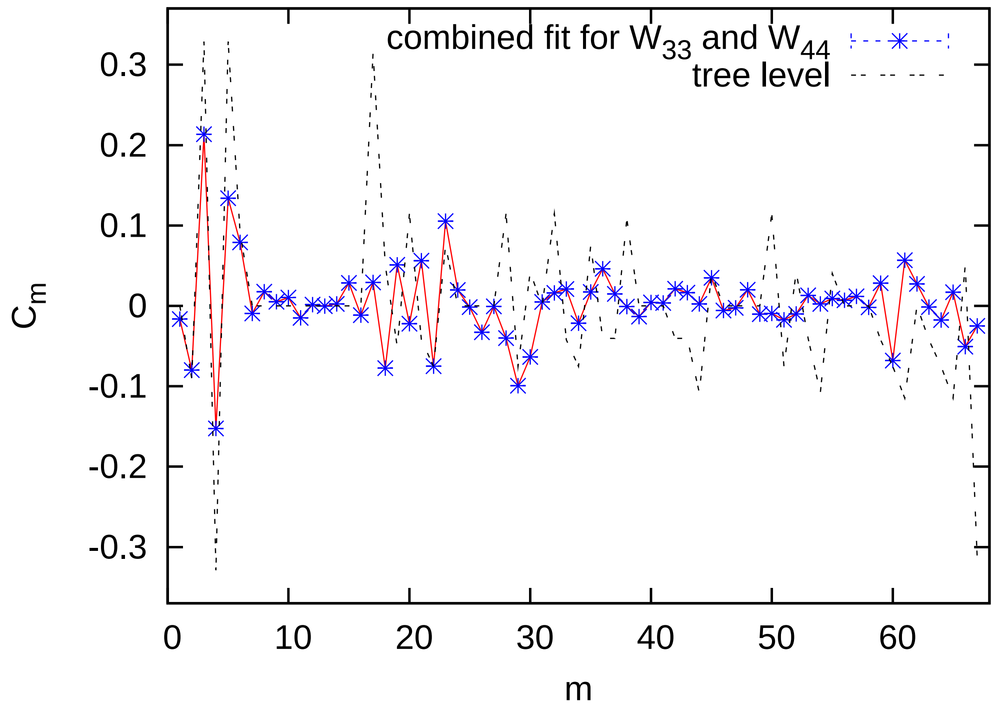
Since $16M > N$ the system is over-determined
$\longrightarrow$ we apply **Singular Value Decomposition** [4]:

Select $n \leq N$ conditions with "maximal impact" on the solution $C_1, \ldots, C_N$.
Rapid convergence for increasing n approves a reliable result.



With 12 Singular Values (analogue to eigenvalues), good convergence, $e.g.$ for operator $\bar{\psi}\vec{\gamma}\vec{D}\psi$. Precision saturates around $n \approx 50$ constraints.

Wilson coefficients obtained from W_{33} and W_{44} :



Coefficients C_1 (for $\bar{\psi}\mathbb{1}\psi$) and $C_{7\ldots16}$ vanish at $m \to 0$ (chiral symmetry).

Here *consistently small values*, unlike result with Wilson fermions [2].

Similar pattern as tree level, but reduced absolute values.

- Method for Wilson coefficients successful.

Now $\mathcal{M}$ is obtained by Nachtmann integration [5] over $W_{\mu\nu}$, *e.g.* 2^{nd} moment:

$$\begin{aligned}\mathcal{M}_2(q) &= \frac{3q^2}{(4\pi)^2}\int d\Omega_q\, n_\mu\Big[W_{\mu\nu}(q) - \frac{1}{4}\delta_{\mu\nu}W_{\rho\rho}\Big]n_\nu \\ &\to \int_0^1 dx\,\Big[F_2(x,q^2) + \frac{1}{6}F_L(x,q^2)\Big] \qquad \text{(Bjorken limit)}\end{aligned}$$

$n^2 = 1$, different projection $\rightarrow\ \ldots[F_2 - \frac{3}{2}F_L]$
$\Rightarrow$ determination of longitudinal structure function $F_L = F_2 - 2xF_1$.

• With completed data, we will obtain a fully non-perturbative and consistent evaluation of the Nucleon Structure Functions.

References

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