Calculating B_K using HYP staggered fermions

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Collaboration

- Brookhaven National Laboratory (BNL): *Chulwoo Jung*
- Seoul National University (SNU): *Weonjong Lee, et al.*
- University of Washington, Seattle (UW): *Stephen R. Sharpe*

Computing Resources: cj20 and QCDOC

ε and $\hat{B}_K$

- $\varepsilon = (2.280 \pm 0.013) \times 10^{-3} \times e^{i\phi_\varepsilon}$, $\phi_\varepsilon = 43.5 \pm 0.05^\circ$ in experiment.
- Relation between ε and $\hat{B}_K$ in standard model.

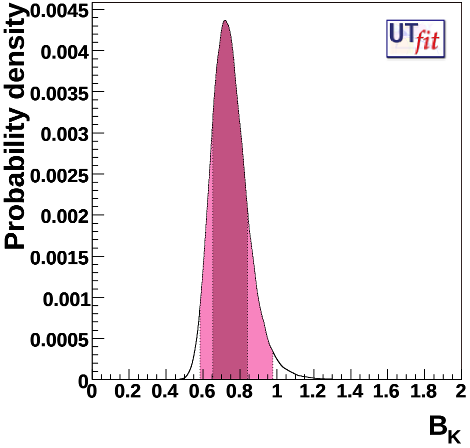
$$\begin{aligned}
\varepsilon &= \exp(i\phi_\varepsilon)\ \sqrt{2}\sin(\phi_\varepsilon)\ C_\varepsilon\ \mathrm{Im}\lambda_t\ X\ \hat{B}_K + \xi \\
X &= \mathrm{Re}\lambda_c[\eta_1 S_0(x_c) - \eta_3 S_3(x_c, x_t)] - \mathrm{Re}\lambda_t \eta_2 S_0(x_t) \\
\lambda_i &= V_{is}^* V_{id}, \qquad x_i = m_i^2/M_W^2, \qquad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} \\
\xi &= \exp(i\phi_\varepsilon)\sin(\phi_\varepsilon)\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}
\end{aligned}$$

- Definition of B_K in standard model.

$$\begin{aligned}
B_K &= \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0\rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0\rangle\langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0\rangle} \\
\hat{B}_K &= C(\mu) B_K(\mu), \qquad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}}[1 + \alpha_s(\mu) J_3]
\end{aligned}$$

CKM matrix, Unitarity triangle and $\hat{B}_K$

- Unitarity Triangle: $\hat{B}_K = 0.75 \pm 0.09$
- $\hat{B}_K$ from experiments and CKM unitarity ansatz:

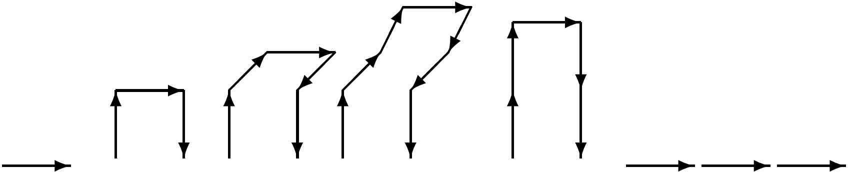


Staggered Chiral Perturbation Theory

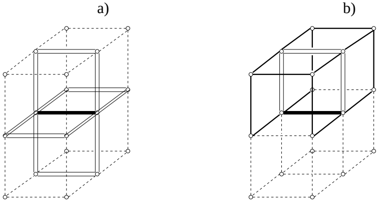
- At $a = 0$, we expect full SU(4) taste symmetry for pion multiplets: $\{\xi_T\}$ are degenerate for all T.
- At finite a, group theory (1985) predicts 8 irreps of SW_4 for pion multiplets: $\{\xi_5\}$, $\{\xi_i\}$, $\{\xi_4\}$, $\{\xi_{ij}\}$, $\{\xi_{i4}\}$, $\{\xi_{i5}\}$, $\{\xi_{45}\}$, $\{1\}$.
- Staggered Chiral Perturbation Theory (1999) predicts that
  - SU(4) is broken down to SO(4) at leading order of $\mathcal{O}(a^2) \approx \mathcal{O}(p^2)$.
  - SO(4) is broken down to SW_4 at NLO of $\mathcal{O}(a^2 p^2)$ (1999, 2005).
  - $SU(4) \xrightarrow{\mathcal{O}(a^2)} SO(4) \xrightarrow{\mathcal{O}(a^2 p^2)} SW_4$
- Hence, we expect 5 irreps of SO(4) for pion multiplets in the chiral limit: $\{\xi_5\}$, $\{\xi_\mu\}$, $\{\xi_{\mu\nu}\}$, $\{\xi_{\mu 5}\}$, $\{1\}$.
- We also expect that the slopes for pion multiplets are splitted into 8 irreps of SW_4.

Improved Staggered Actions

- Fat7 + Lepage + Naik = AsqTad



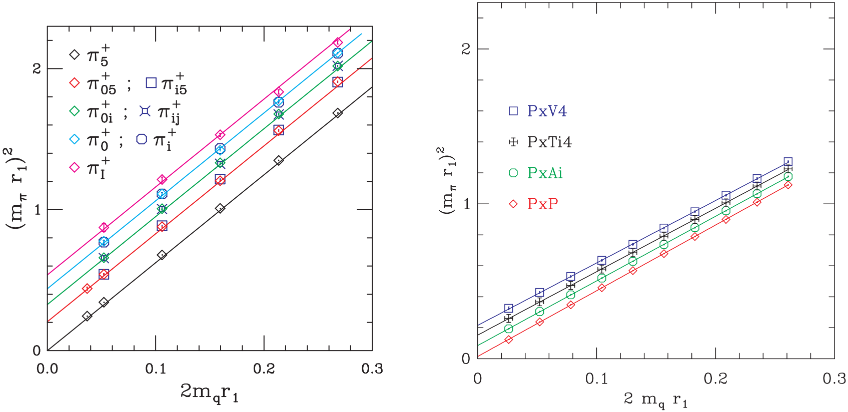
- HYP = hypercubic blocking



- $\overline{\mathrm{Fat7}}$ = Fat7 + SU(3)/U(3) Projection
- HISQ = $\overline{\mathrm{Fat7}}$ + AsqTad

Improvement Scheme [AsqTad vs. HYP]

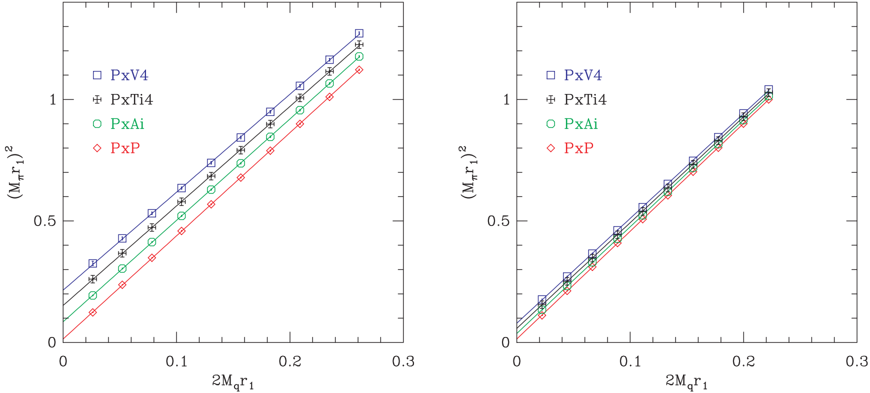
- $N_f = 2+1$, MILC, $a = 0.125$ fm, GoI, CU1, 2007:



- $\mathcal{O}(a^2) \approx \mathcal{O}(p^2)$ for AsqTad ; $\mathcal{O}(a^2) \lesssim \mathcal{O}(p^2)$ for HYP.
- $\mathcal{O}(a^2 p^2)$: negligibly small for both AsqTad and HYP.

Improvement Scheme (III) [coarse vs. fine; HYP]

- $N_f = 2 + 1$, MILC coarse (CU1) vs. MILC fine (CW).



- Coarse lattices: $\mathcal{O}(a^2) \lesssim \mathcal{O}(p^2)$ (Mixed).
- Fine lattices: $\mathcal{O}(a^2) \approx \mathcal{O}(p^4) \ll \mathcal{O}(p^2)$ (Mixed).

B_K definition in standard model.

$$
\begin{aligned}
B_K &= \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle} \\
\hat{B}_K &= C(\mu) B_K(\mu), \\
C(\mu) &= \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]
\end{aligned}
$$

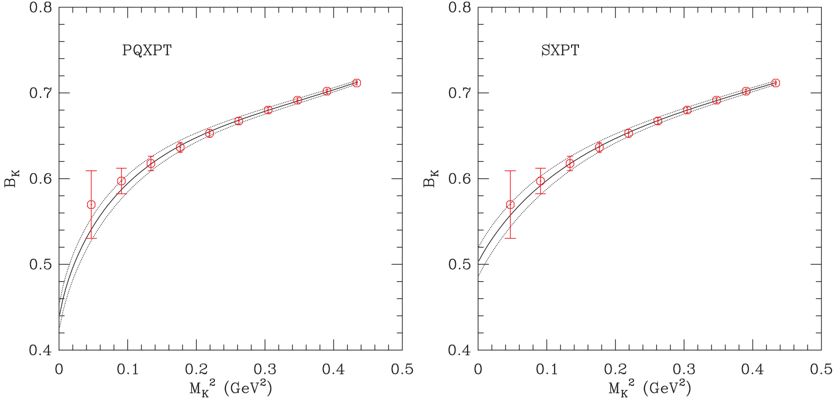
B_K and Staggered Chiral Perturbation Theory

$$
\begin{aligned}
B_K &= c_1[1 + \frac{3}{8}\frac{M_{conn}+M_{disc}}{f_\pi^2 G}] + c_2\frac{G}{\Lambda^2} + c_3(\frac{G}{\Lambda^2})^2 + c_4\frac{(X_P - Y_P)^2}{G\Lambda^2} \\
&\quad + c_5 F^{(4)}_{C^{(4)}} + c_6 F^{(4)}_{C^{(6)}} + c_7 F^{(1)} + c_8 F^{(4)}_{C^{(5)}} \\
G &= m^2_{\pi,xy}, \qquad \Lambda = \text{S}\chi\text{PT scale} \\
M_{conn} &= \frac{1}{6\pi^2}\sum_{B=I,P,V,A,T} F^{(3)}_B \\
F^{(3)}_B &= \frac{f^B}{128}[(G+X_B)I(X_B) + (G+Y_B)I(Y_B) + 2(G-K_B)I(K_B) \\
&\quad -2GK_B\tilde{I}(K_B)] \\
f^B &= \{-1,-1,-4,-4,-6\} \text{ for } B = I,P,V,A,T
\end{aligned}
$$

$$
\begin{aligned}
M_{disc} &= \frac{1}{18\pi^2} F_I \\
F_I &= l(\eta_I)(G+\eta_I)\frac{(X_I - Y_I)^2 (L_I - \eta_I)(S_I - \eta_I)}{(X_I - \eta_I)^2 (Y_I - \eta_I)^2} + \cdots \\
F^{(4)}_{C^{(4)}_B} &= N\{2G\tilde{l}(K_B) + [l(X_B) + l(Y_B) - 2l(K_B)]\} \\
F^{(4)}_{C^{(5)}} &= N\{\sum_{B=V,A} [l(X_B) + l(Y_B) - 2l(K_B)]\} \\
F^{(4)}_{C^{(6)}} &= N\{[l(X_T) + l(Y_T) - 2l(K_T)]\} \\
F^{(1)}_B &= N\{l(\eta_B)\frac{(Y_B - X_B)^2 (L_B - \eta_B)(S_B - \eta_B)}{(X_B - \eta_B)^2 (Y_B - \eta_B)^2 (\eta'_B - \eta_B)} + \cdots\} \\
N &= \frac{3}{8}\frac{1}{f_\pi^2 G}
\end{aligned}
$$

Comparison of PQ χPT and Staggered χPT

- HYP, MILC_2064f21b676m010m050 [671], $a = 0.125$ fm:

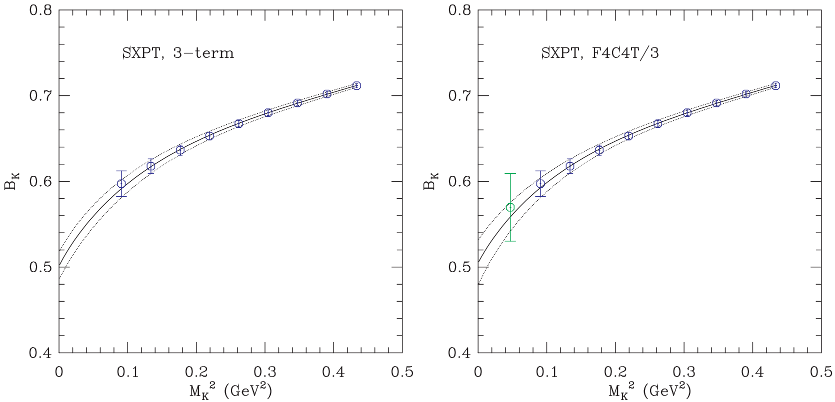


(a) $\chi^2 = 0.26(34)$ (b) $\chi^2 = 0.06(16)$

- 3 term fitting; tree-level matching (preliminary).

Bayesian method (I) for B_K data analysis

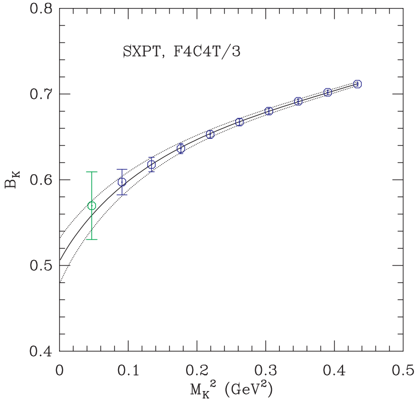
- HYP, MILC_2064f21b676m010m050 [671], $a = 0.125$ fm:



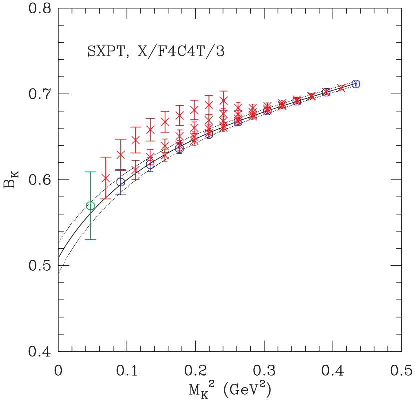
(c) $\chi^2 = 0.05(11)$ (d) $\chi^2 = 0.07(16)$

- Left: 3 term fitting, Right: $F^{(4)}_{C^{(4)}_T}/3$ fitting (Bayesian method).

Bayesian method (II) for B_K data analysis



(e) $\chi^2 = 0.07(16)$

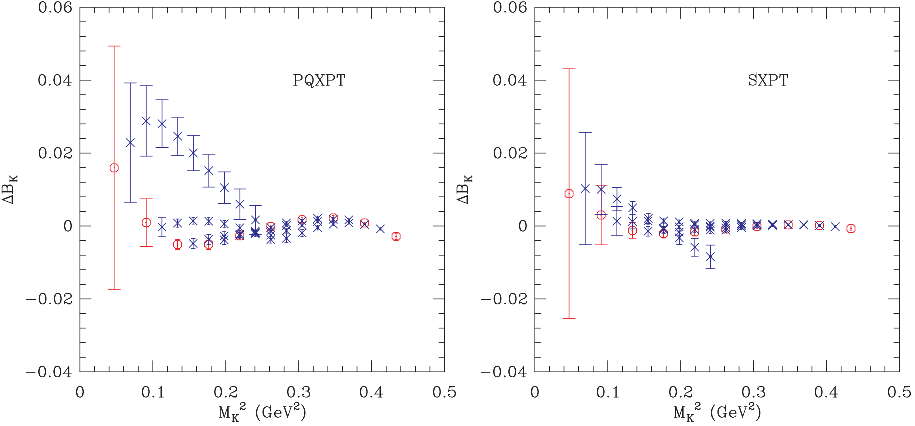


(f) $\chi^2 = 0.06(7)$

- Left: the same, Right: $F^{(4)}_{C^{(6)}}$, $F^{(1)}_{A}$, $1/\ F^{(4)}_{C^{(4)}_T}/3$ fitting (2 step Bayesian method).

Fitting quality: ΔB_K (MILC_2064f21b676m010m050)

- $\Delta B_K = B_K(\text{data}) - \text{fitting function}$



- Left: Partially Quenched χPT
- Right: Staggered χPT

B_K in $N_f = 1 + 2$ unquenched QCD

a (fm)	am_l/am_s	geometry	ens	B_K(tree,Bayes)
0.12	0.03/0.05	$20^3 \times 64$	564	0.8306(624)
0.12	0.02/0.05	$20^3 \times 64$	486	0.7162(606)
0.12	0.01/0.05	$20^3 \times 64$	671	0.7647(559)
0.12	0.01/0.05	$28^3 \times 64$	275	—
0.12	0.007/0.05	$20^3 \times 64$	651	0.8657(531)
0.12	0.005/0.05	$24^3 \times 64$	509	0.7988(442)
0.12	0.01/0.03	$20^3 \times 64$	312	—
0.09	0.0062/0.031	$28^3 \times 96$	995	0.7177(391)
0.09	0.0031/0.031	$40^3 \times 96$	500	—
0.06	0.004/0.02	$48^3 \times 144$	500	—

- Here, we quote B_K values obtained using the two step Bayesian method for fitting.
- Tree level matching (preliminary).

Theoretical Challenges

- Matching Factor: Tree $\longrightarrow$ One-loop (SNU/UW)
- Matching Factor: non-perturbative renormalization for staggered fermions (UW)
- Matching Factor: One-loop $\longrightarrow$ Two-loop (SNU)
- Staggered χPT for the mixed action: done (SNU/UW)

Numerical Challenges

- VERY PRELIMINARY.
- We need to increase statistics significantly for coarse and fine lattices in order to nail down the non-degenerate quark mass effect on B_K (SNU/BNL).
- We need to extend the measurements to MILC superfine lattices ($a = 0.06$ fm, $48^3 \times 144$) (BNL).
- Non-perturbative Renormalization: underway (UW).
- Large scale fitting of multi-tera byte data (SNU).

Tentative Goal

- We would like to determine B_K directly from the standard model with its systematic and statistical error around 2%.
- We expect to achieve this goal in a few years using QCDOC and SNU clusters.