Update on heavy meson spectroscopy

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MILC and Fermilab Collaborations

Abstract

We present an update on the heavy meson mass spectrum resulting from clover-Wilson heavy quarks and antiquarks on $N_f = 2 + 1$ improved staggered (Asqtad) configurations. Emphasis is placed on using ratios of correlators to resolve spin splittings, in particular those of P-wave charmonium states (χ_{cJ} and h_c). We also show some preliminary results from an excited-state analysis.

* Rotten tomato recipient.

Configurations and Run Parameters

β	am_l/am_s	a (fm)	$N_s^3 \times N_t$	κ_c [†]
6.503	0.0492/0.082	≈ 0.18	$16^3 \times 48$	0.120
6.485	0.0328/0.082			
6.467	0.0164/0.082			
6.458	0.0082/0.082			
6.600	0.0290/0.0484	≈ 0.15	$16^3 \times 48$	0.122
6.586	0.0194/0.0484			
6.572	0.0097/0.0484			
6.566	0.00484/0.0484		$20^3 \times 48$	
6.81	0.03/0.05	≈ 0.125	$20^3 \times 64$	0.122
6.79	0.02/0.05			
6.76	0.01/0.05			
6.76	0.007/0.05			
6.76	0.005/0.05		$24^3 \times 64$	
7.11	0.0124/0.031	≈ 0.09	$28^3 \times 96$	0.127
7.09	0.0062/0.031			
7.08	0.0031/0.031		$40^3 \times 96$	

[†] New tuning runs for κ_c and κ_b in progress.

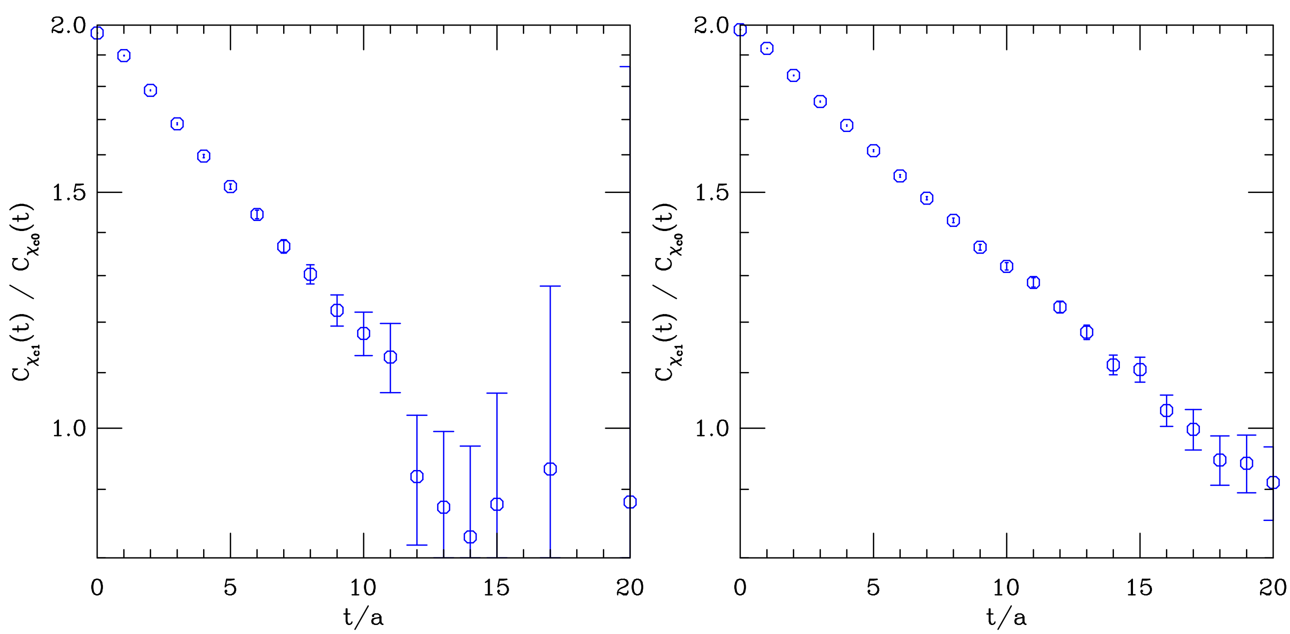
P-wave Correlator Ratios

1P spatial wavefunction to reach $J^{PC} = 0^{++}, 1^{++}, 2^{++}, 1^{+-}$ states: $\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c$.

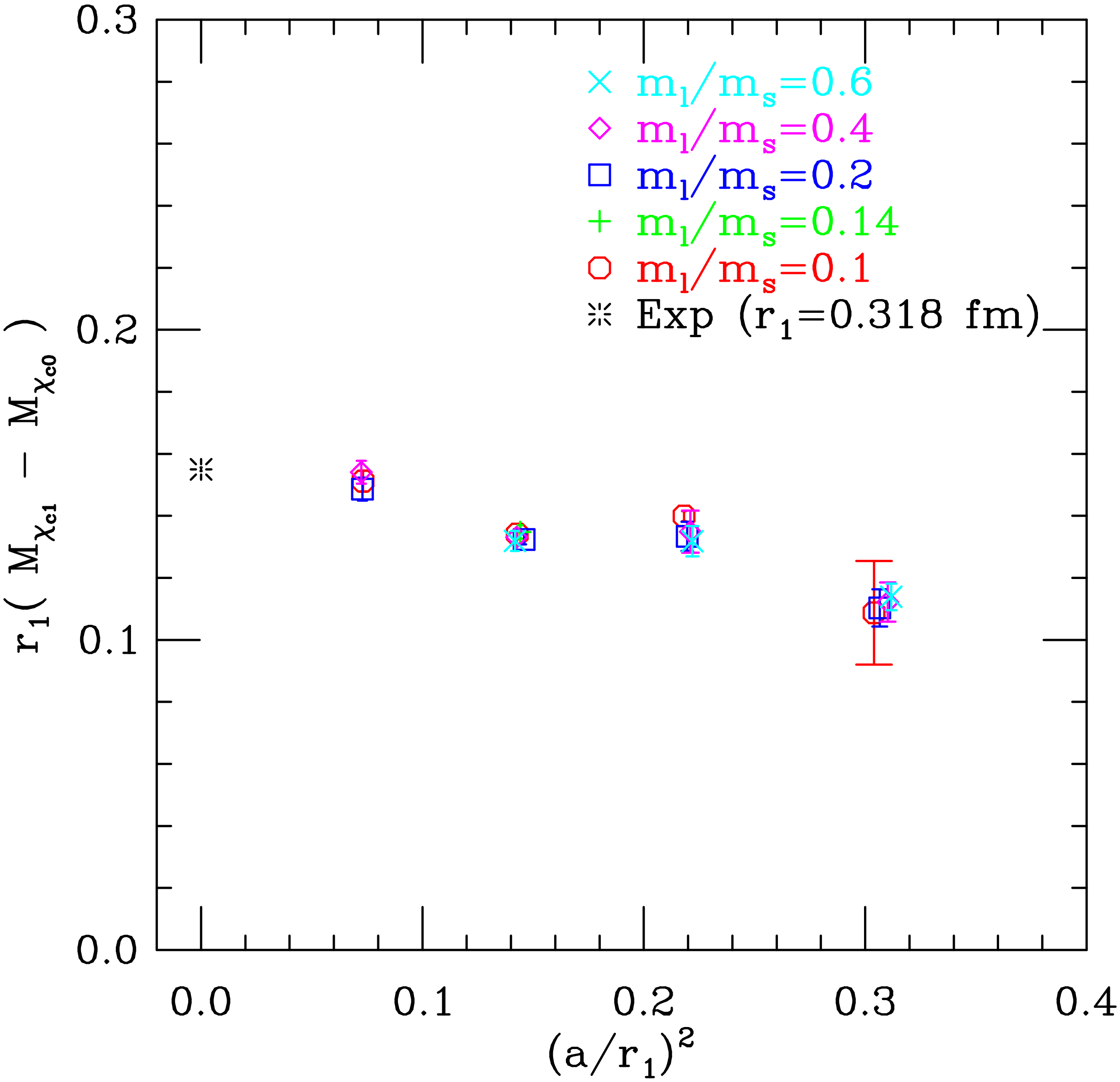
We use ratios of correlators to resolve the ground-state mass splittings:

$$C_2(t)/C_1(t) = Ae^{-(M_2-M_1)t} + \ldots$$

E.g., jackknifed $C_{\chi_{c1}}(t)/C_{\chi_{c0}}(t)$ ratios on the $\beta = 6.572$ and $\beta = 7.11$ lattices:

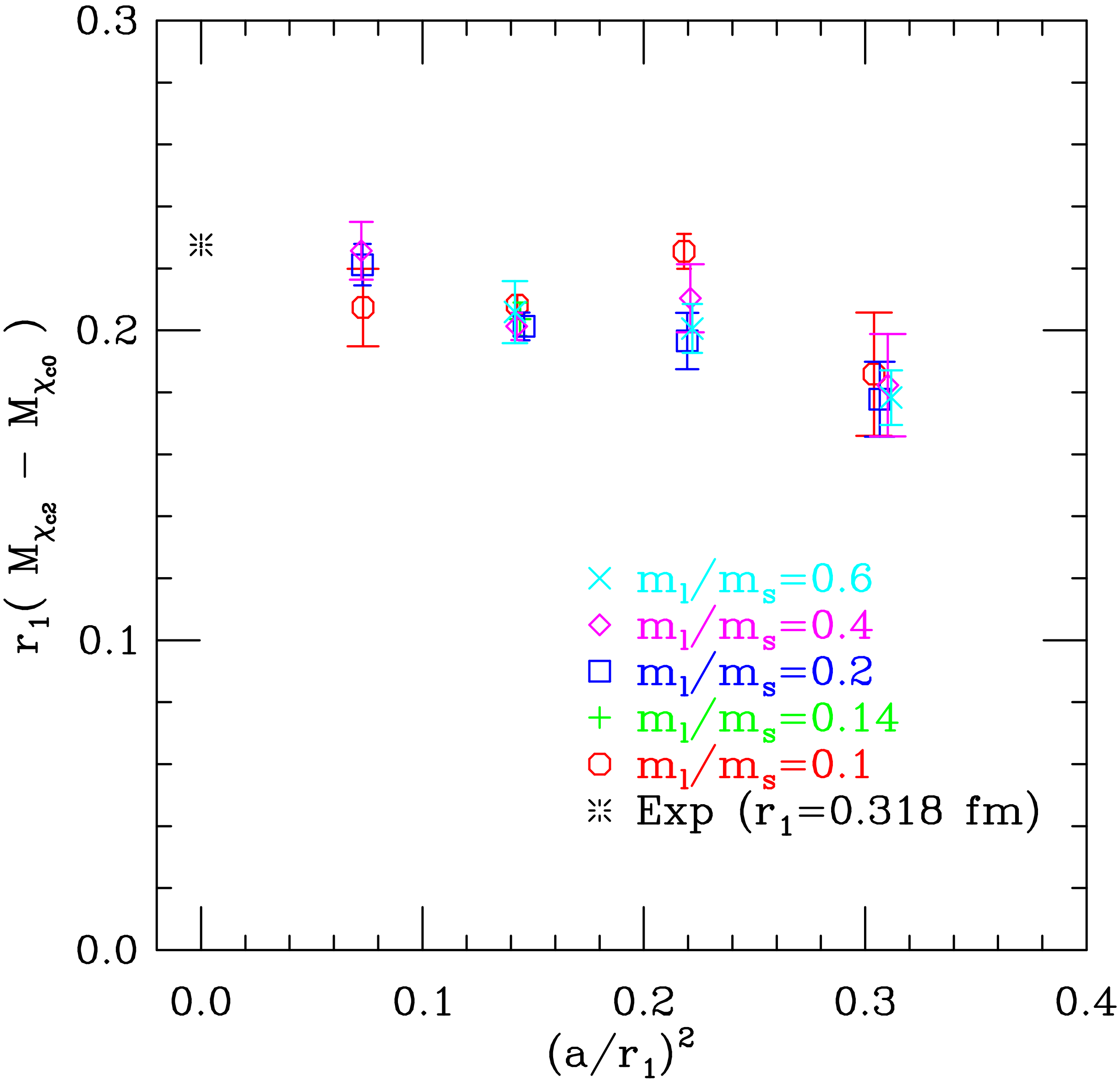


$\chi_{c1} - \chi_{c0}$ Mass Splitting

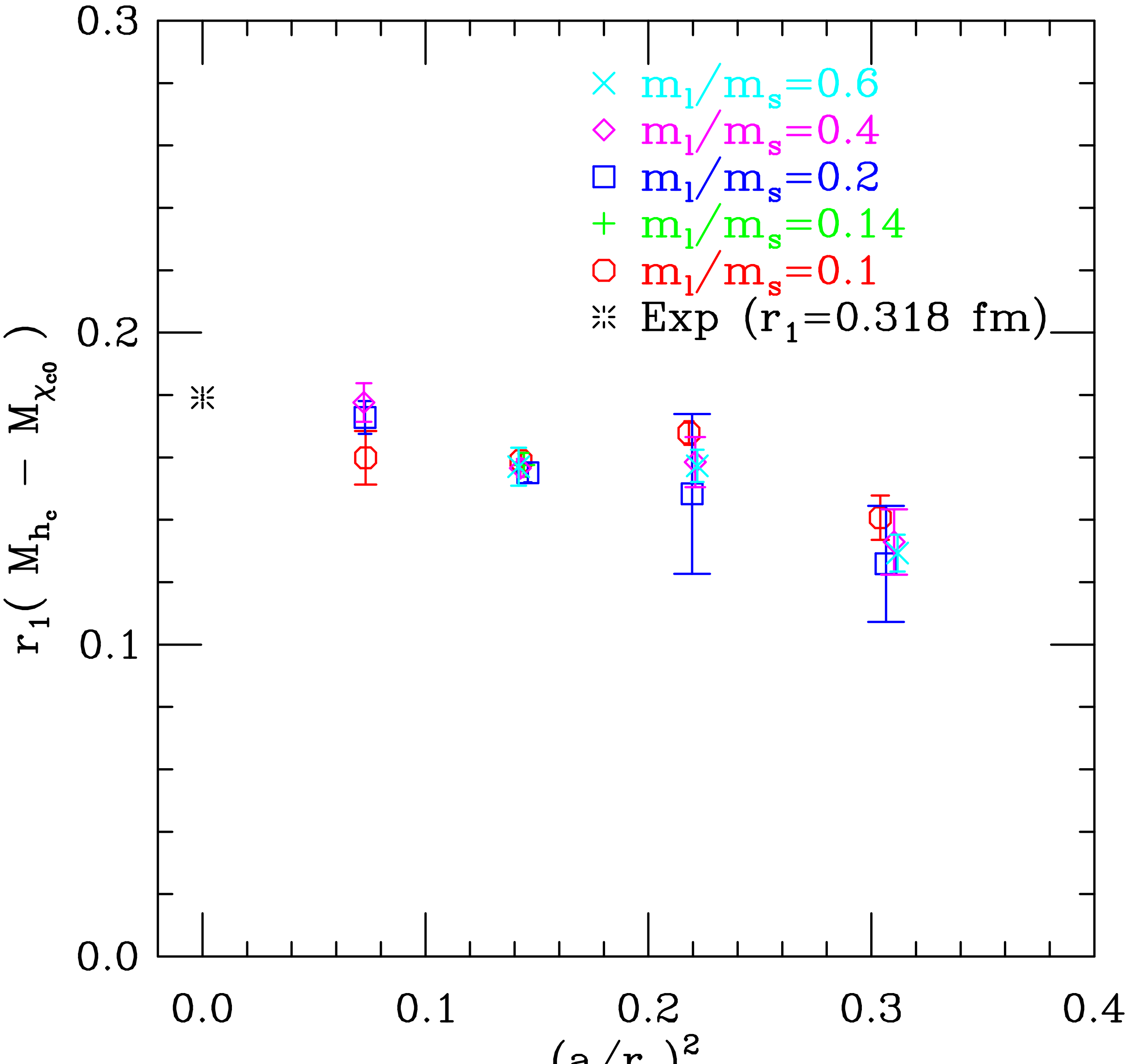


*The $r_1 = 0.318$ fm value arises from high-precision Υ splittings by HPQCD.

$\chi_{c2} - \chi_{c0}$ Mass Splitting



$h_c - \chi_{c0}$ Mass Splitting



Variational Method

Using different source and sink operators, construct a matrix of correlators:

$$\begin{aligned} C_{ij}(t) &= \langle O_j O_i^\dagger \rangle \\ &= \sum_k a_j^{(k)} a_i^{(k)*} e^{-t\,M_k} \end{aligned}$$

Generalized eigenvalue problem:

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t,t_0)\, C(t_0)\vec{\psi}^{(k)} \ ,$$

$$\lambda^{(k)}(t,t_0) \propto e^{-t\,M_k}[1 + \mathcal{O}(e^{-t\,\Delta M_k})]$$

C. Michael, NPB259, 58 (1985)

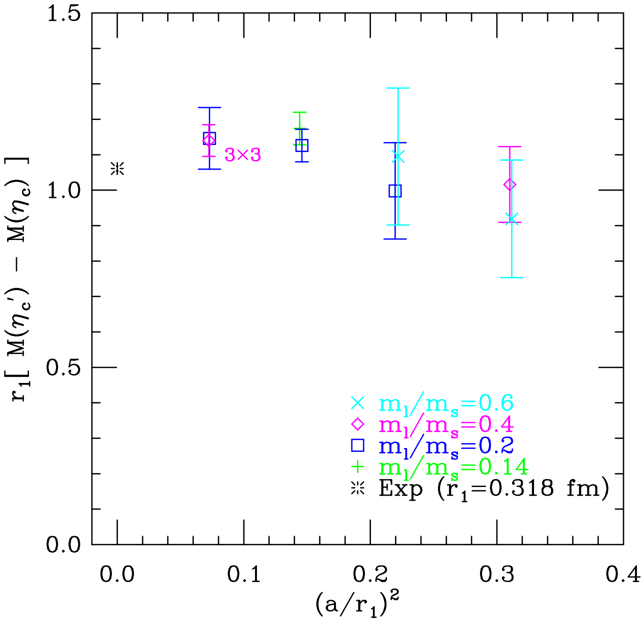
M. Lüscher & U. Wolff, NPB339, 222 (1990)

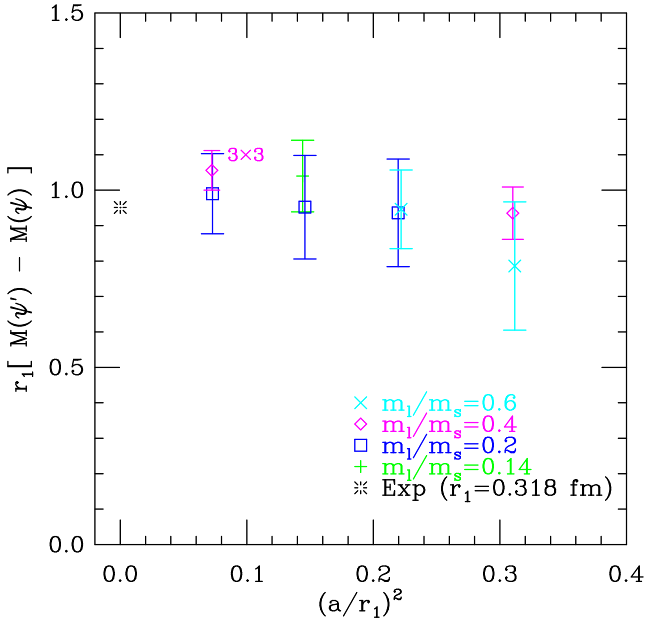
T.B., C. Gattringer, L. Glozman, C. Hagen, & C.B. Lang, PRD73, 017502 (2006)

Having cross-correlators from **point**, **1S**, and **2S** (for one ensemble: $\beta = 7.11$) sources:

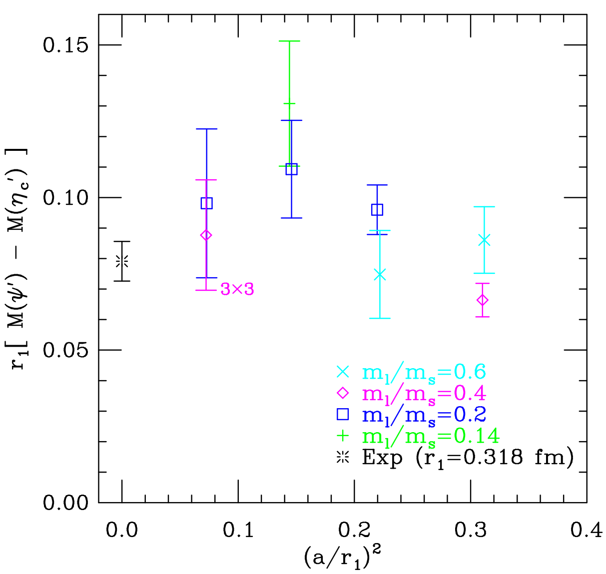
For 2×2 basis, perform single-mass fits to first eigenvalue and two-mass fits to second eigenvalue. For 3×3 basis, perform single-mass fits to first and second eigenvalues...

Mass Splittings Involving η_c', η_c, ψ', ψ



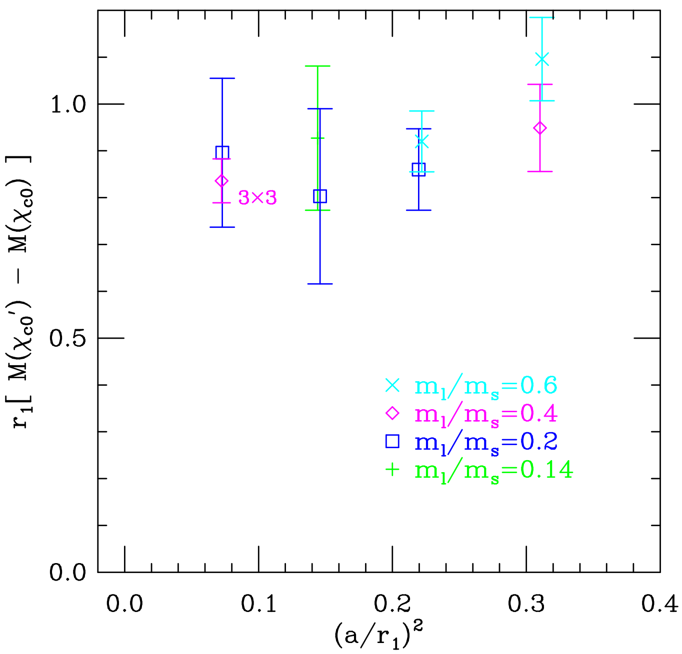


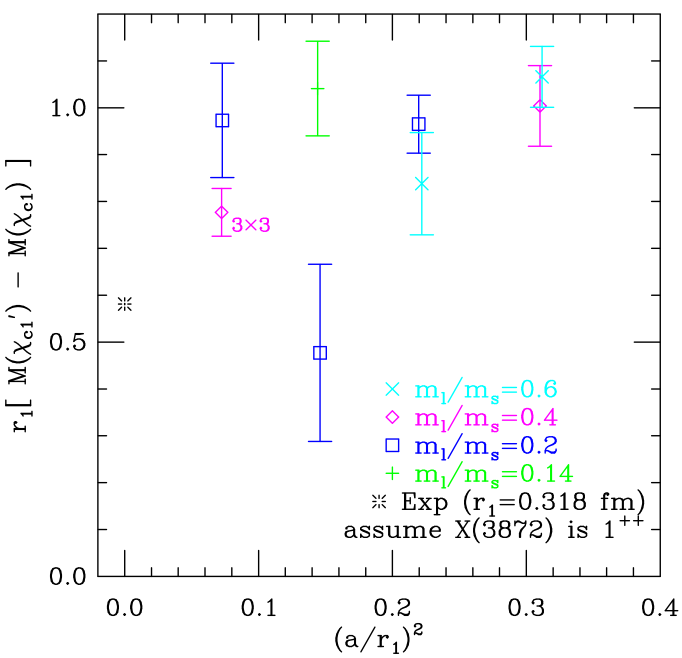
From ratios of second eigenvalues, $\lambda^{(2)}_{1--}/\lambda^{(2)}_{0-+}$, the $\psi' - \eta_c'$ mass difference:



For details on contributions to the $J/\psi - \eta_c$ mass splitting, see parallel talk by L. Levkova.

$\chi'_{cJ} - \chi_{cJ}$ Mass Splittings





Conclusions

splitting	@ $\frac{m_l}{m_s} = 0.2$	@ $\frac{m_l}{m_s} = 0.1$	Exp. (MeV)
$\chi_{c1} - \chi_{c0}$	95.6(2.7)(?)	88.1(2.4)(??)	96.0(4)
$\chi_{c2} - \chi_{c0}$	142.6(5.3)	121.0(6.6)(?)	141.0(4)
$h_c - \chi_{c0}$	116.0(5.5)	101.6(3.8)(??)	111.0(5)
$\psi' - \eta_c'$	69(19)	–	49(4)
$\eta_c' - \eta_c$	760(90)	–	657(4)
$\psi' - \psi$	630(110)	–	589.17(4)
$\chi_{c0}' - \chi_{c0}$	550(140)	–	–
$\chi_{c1}' - \chi_{c1}$	530(110)(??)	–	360.0(6)†

† Assuming the X(3872) is $J^{PC} = 1^{++}$.

(?) Extrapolations with $\chi^2/dof > 2$. (??) Extrapolations with $\chi^2/dof > 5$.

- The ratios of correlators (and correlator matrix eigenvalues) provide us with precise determinations of $c\bar{c}$ spin splittings.
- The "P-wave" splittings at $m_l/m_s = 0.2$ show surprising agreement with experiment, while those at $m_l/m_s = 0.1$ appear worse. This suggests we have further systematic effects: e.g., better statistics are needed to choose more definitive fit ranges and to achieve a finer tuning of κ_c (in progress).
- The Point–1S 2×2 basis clearly needs expansion to better resolve excited states. The 3×3 results show reduced statistical errors, but are (so far) only done for $m_l/m_s = 0.4$ at $a \approx 0.09$ fm...

3×3 Basis Results / Outlook

splitting	@ $\frac{m_l}{m_s} = 0.4$, $a \approx 0.09$ fm	Exp. (MeV)
$\psi' - \eta_c'$	54(11)	49(4)
$\eta_c' - \eta_c$	706(28)	657(4)
$\psi' - \psi$	654(35)	589.17(4)
$\chi_{c0}' - \chi_{c0}$	518(29)	–
$\chi_{c1}' - \chi_{c1}$	481(32)	360.0(6)†

† Assuming the X(3872) is $J^{PC} = 1^{++}$.

For the future...

- Expand the basis of operators (e.g., Liao & Manke, hep-lat/0210030), including cross-correlations, to better isolate the low-lying excitations, and construct ratios of correlator matrix eigenvalues to resolve small splittings.
- Include different states: e.g., exotics (1^{-+}, 0^{+-}, 2^{+-}, 0^{--}).
- Boost statistics by running on extensions of existing configurations and more ensembles with different m_l/m_s ratios and lattice spacings ($a \to 0.06$ fm).