Charmed Hadron Interactions

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Introduction

We study the scattering lengths of charmed mesons with light hadrons in full QCD. We use Fermilab formulation[1] for charm quark and domain wall fermions for light quarks and staggered sea quarks. In addition, the charmed baryon spectrum is also presented.

Fermion Action

- Fermilab formulation (for charm quark)

$$S = S_0 + S_B + S_E$$
$$S_0 = \sum_x \bar{q}(x)[m_0 + (\gamma_0\nabla_0 - \frac{a}{2}\triangle_0) + \nu\sum_i(\gamma_i\nabla_i - \frac{a}{2}\triangle_i)]q(x)$$
$$S_B = -\frac{a}{2}c_B\sum_x \bar{q}(x)(\sum_{i<j}\sigma_{ij}F_{ij})q(x)$$
$$S_E = -\frac{a}{2}c_E\sum_x \bar{q}(x)(\sum_i \sigma_{0i}F_{0i})q(x)$$

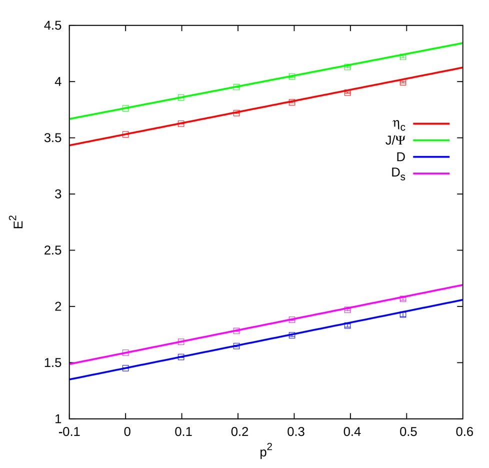
  - Incorporate interactions from both the small- and large-mass limits.
  - Without imposing axis-interchange invariance.
  - Coefficients must be mass dependent to eliminate lattice artifact for heavy quarks.
- Domain wall fermions(for light quark)
  - Preserve chiral symmetry well.
  - Expensive in computation time.
- Staggered fermions(for sea quark)
  - Relatively cheap.
  - Tastes mixing.
- Tuning the coefficients in Fermilab formulation
  - Using the spin average mass of Charmonium ($\eta_c, J/\Psi$) to tune the charm quark mass.
  - Tuning the anisotropic to restore the dispersion relations. In S_0, the value of ν was tuned to be 1.265.
  - Tuning the clover coefficients.
    The tree level tadpole estimate of the clover coefficients is $C_B = C_E = 1/u_0^3$. Where u_0 is the tadpole coefficients. We use the clover terms that depend on the bare velocity of light ν as suggested by Chen [3] :

$$C_B = \frac{\nu}{u_0^3}, \qquad C_E = \frac{1}{2}(1+\nu)\frac{1}{u_0^3}$$

- Heavy quark action test
  The mass of Charmonium and hyperfine splitting compared with the experimental values:

	Numerical	Experimental
$\frac{1}{4}m_{\eta_c} + \frac{3}{4}m_{J/\Psi}$	3056.54(1.15)	3067.67
$m_{J/\Psi} - m_{\eta_c}$	97.3(1.5)	117.061

The dispersion relations of mesons D,D_s,η_c,J/Ψ:



	c^2
η_c	0.989(0.005)
J/Ψ	0.965(0.009)
D	1.012(0.017)
D_s	1.006(0.009)

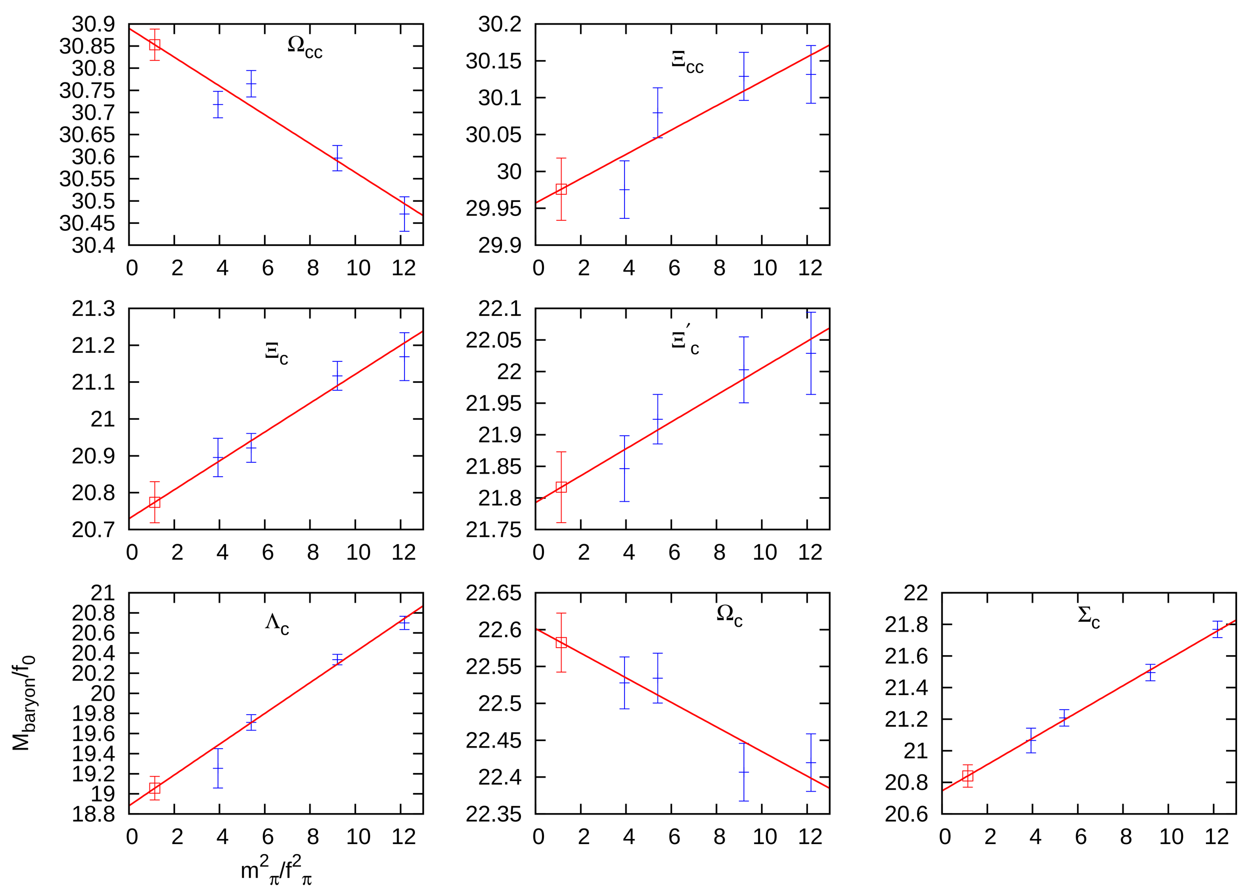
Numerical Ensembles

We employ the gauge configurations generated by the MILC collaboration. We use the $20^3 \times 64$ lattices generated at four values of light-quark masses. The lattice spacing $b = 0.12406fm$. The details of the ensembles are listed below.

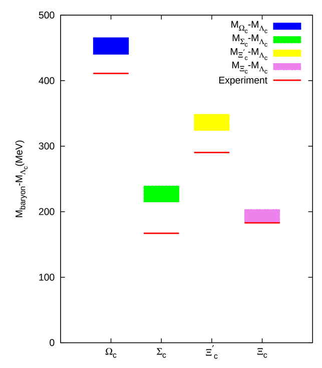
Ensemble	bm_l	bm_s	bm_l^{dwf}	bm_s^{dwf}	♯ of props
2064f21b676m007m050	0.007	0.050	0.0081	0.081	450
2064f21b676m010m050	0.010	0.050	0.0138	0.081	650
2064f21b679m020m050	0.020	0.050	0.0313	0.081	550
2064f21b781m030m050	0.030	0.050	0.0478	0.081	380

Charmed Baryon Spectrum

- The masses of single charmed and double charmed baryon at four different light quark masses are shown in the following figure. The masses are extrapolated linearly to the physical point.



- The mass difference between single charmed baryons are shown in the right figure. Similar work has been done using staggered light quark action[6]. We got comparable results. The deviation from experiment probably due to lattice artifacts.



Charmed Hadron Scattering

- Two hadrons in a finite box
  The total energy of two hadrons is obtained from the four-point correlation function:

$$G^{h_1-h_2}(t) = \langle \mathcal{O}^{h_1}(t)\mathcal{O}^{h_2}(t)(\mathcal{O}^{h_1}(0)\mathcal{O}^{h_2}(0))^\dagger\rangle$$

Lüscher has shown that the scattering phase shift is related to the energy shift ($\triangle E$) of the total energy of the two hadrons relative to the total energy of individual hadron[4]. To extract ($\triangle E$),let's define a ratio $R^{h_1-h_2}(t)$:

$$R^{h_1-h_2}(t) = \frac{G^{h_1-h_2}(t,0)}{G^{h_1}(t,0)G^{h_2}(t,0)} \longrightarrow \exp(-\triangle E \cdot t)$$

where $G^{h_1}(t,0)$ and $G^{h_2}(t,0)$ are corresponding two-point functions.
The momentum p was related to $\triangle E$ by

$$\triangle E = \sqrt{p^2 + m_{h_1}^2} + \sqrt{p^2 + m_{h_2}^2} - m_{h_1}^2 - m_{h_2}^2$$

The phase shift is obtained from the following relation:

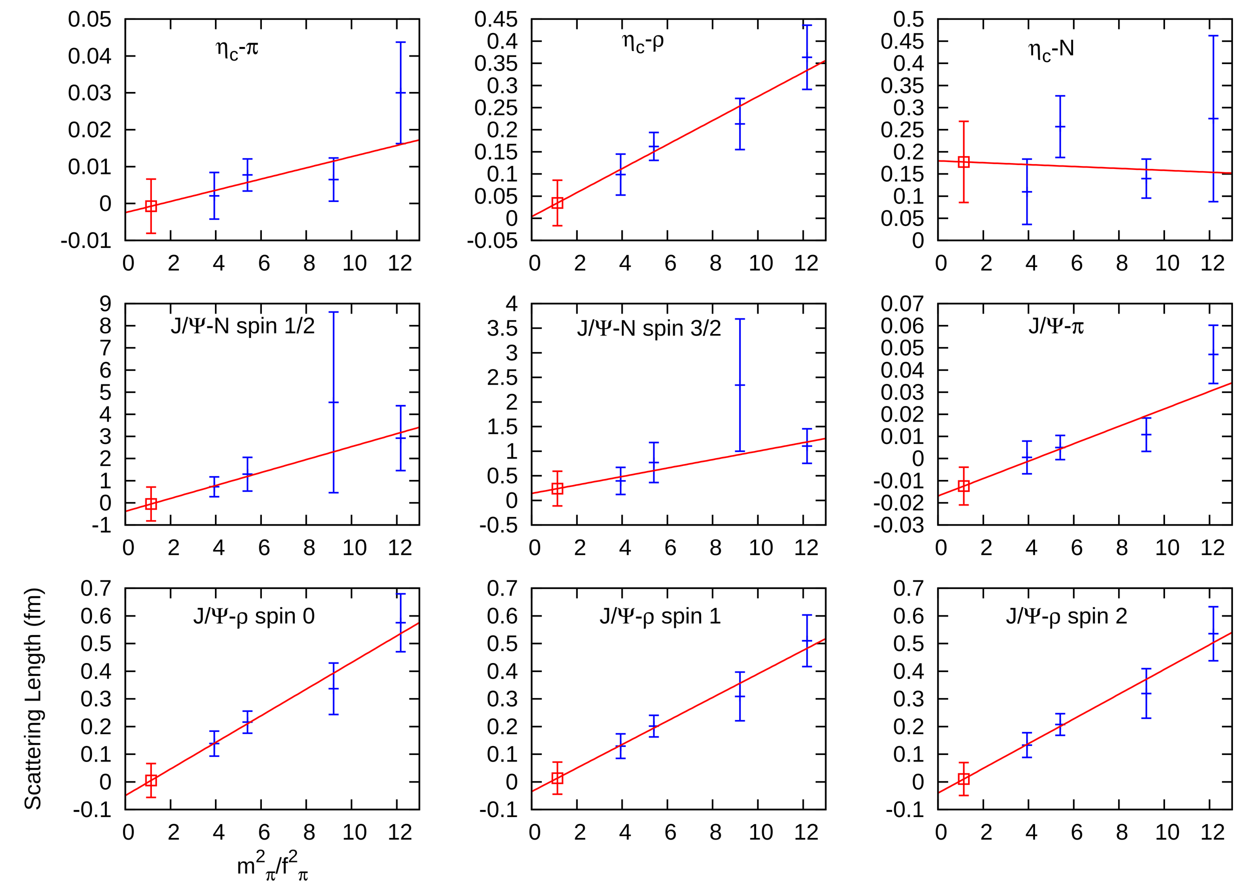
$$p\cot\delta(p) = \frac{1}{\pi L}\mathbf{S}\Big((\frac{pL}{2\pi})^2\Big)$$

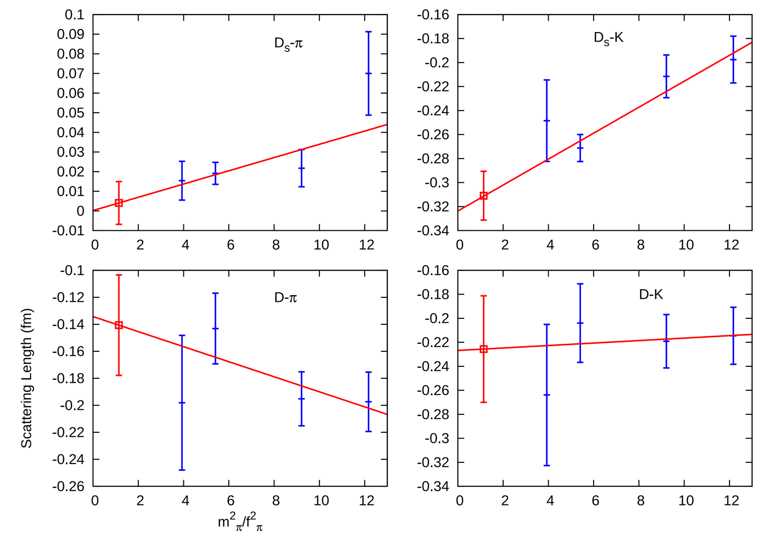
If the interaction range is smaller than half of the lattice size, the s-wave phase shift can be written as

$$p\cot\delta(p) = \frac{1}{a} + \mathcal{O}(p^2)$$

where a is the scattering length. We will use this relation to get the scattering lengths.

- Numerical results
  The scattering lengths of each channel at four values of light quark masses as well as linear extrapolations are shown below.





Extrapolated values of scattering lengths:

Channel	Scattering lengths(fm)
$\eta_c - \pi$	0.00(0.01)
$\eta_c - \rho$	0.03(0.05)
$\eta_c - N$	0.18(0.09)
$J/\Psi - \pi$	−0.01(0.01)
$J/\Psi - N$ spin1/2	−0.05(0.77)
$J/\Psi - N$ spin3/2	0.24(0.35)
$J/\Psi - \rho$ spin0	0.00(0.06)
$J/\Psi - \rho$ spin1	0.01(0.06)
$J/\Psi - \rho$ spin2	0.01(0.06)
$D - \pi$	−0.14(0.04)
$D - K$	−0.23(0.04)
$D_s - \pi$	0.00(0.01)
$D_s - K$	−0.31(0.02)

Conclusions

- For the channels of charmonium with light hadrons and $D_s - \pi$ channel, the scattering lengths are zero or close to zero. The interactions are weak due to the fact that there is no quark exchange diagram. Gluon exchange plays essential role in these channels.
- For the $D - \pi, D_s - \pi, D_s - K$ channels, we found relatively strong repulsive interactions.
- Need to improve statistics.

References

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