EQUATION OF STATE AT FINITE DENSITY IN TWO-FLAVOR QCD WITH IMPROVED WILSON QUARKS

WHOT-QCD Collaboration:

S. Aoki, S. Ejiri, T. Hatsuda, N. Ishii, K. Kanaya, Y. Maezawa, N. Ukita and T. Umeda

We study the equation of state in two-flavor QCD at finite temperature and density. Simulations are made with the RG-improved gluon action and the clover-improved Wilson quark action. Along the lines of constant physics for m_{PS}/m_V = 0.65 and 0.80, we compute the derivatives of the quark determinant with respect to the quark chemical potential μ_q up to the fourth order at μ_q =0. We adopt several improvement techniques in the evaluation. We study thermodynamic quantities and quark number susceptibilities at finite μ_q using these derivatives. We find enhancement of the quark number susceptibility at finite μ_q, in accordance with previous observations using staggered-type quarks. This suggests the existence of a nearby critical point.

1. Introduction

Finite density QCD has been studied on the lattice mainly using staggered-type quarks. However, because the expected O(4) universality of the deconfining transition in two-flavor QCD has not been confirmed with staggered-type quarks, the results may contain sizable lattice artifacts. Therefore, we need to crosscheck the results with different lattice actions. We study it with Wilson-type quarks, with which the O(4) scaling has been confirmed on current lattices.

Because Wilson-type quarks are numerically more intensive, we have to adopt/develop several improvement techniques. We adopt a hybrid method of Taylor expansion and spectral reweighting, and apply a couple of improvement tricks.

2. Formulation

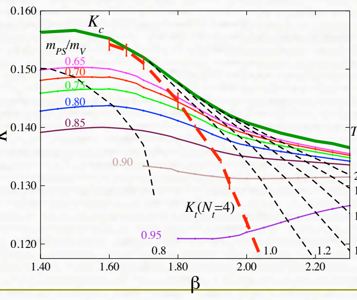
We extend the study by the CP-PACS Collaboration at μ_q=0 [Phys. Rev. D63, 034502 (2001); D64, 074510 (2001)] to finite densities. Preliminary reports have been presented at Lattice 2006 and 2007.

Lattice action:

2-flavor clover-improved Wilson quarks coupled with RG-improved Iwasaki glue.

Lattice size: 16^3 x 4

Simulations are carried out along the lines of constant physics for m_{PS}/m_V = 0.65 and 0.80.



3. Taylor expansion up to μ_q^4

Grand canonical potential: $$\omega \equiv \frac{1}{VT^3}\ln \mathcal{Z} = \frac{p}{T^4} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)$$

$$c_n(T) = \frac{1}{n!}\frac{N_t^3}{N_s^3}\frac{\partial^3}{\partial(\mu_q/T)}\ln \mathcal{Z}\Big|_{\mu_q=0}$$

$$c_2 = \frac{N_t}{2N_s^3}\mathcal{A}_2, \quad c_4 = \frac{1}{4! N_s^3 N_t}(\mathcal{A}_4 - 3\mathcal{A}_2^2),$$

$$\mathcal{A}_2 = \langle \mathcal{D}_2\rangle + \langle \mathcal{D}_1^2\rangle, \quad \mathcal{A}_4 = \langle \mathcal{D}_4\rangle + 4\langle \mathcal{D}_3\mathcal{D}_1\rangle + 3\langle \mathcal{D}_2^2\rangle + 6\langle \mathcal{D}_2\mathcal{D}_1^2\rangle + \langle \mathcal{D}_1^4\rangle$$

$$\mathcal{D}_1 = N_f \mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial \mu}\right), \qquad \mathcal{D}_n := N_f\frac{\partial^n \ln\det M}{\partial(\mu_q a)^n}$$

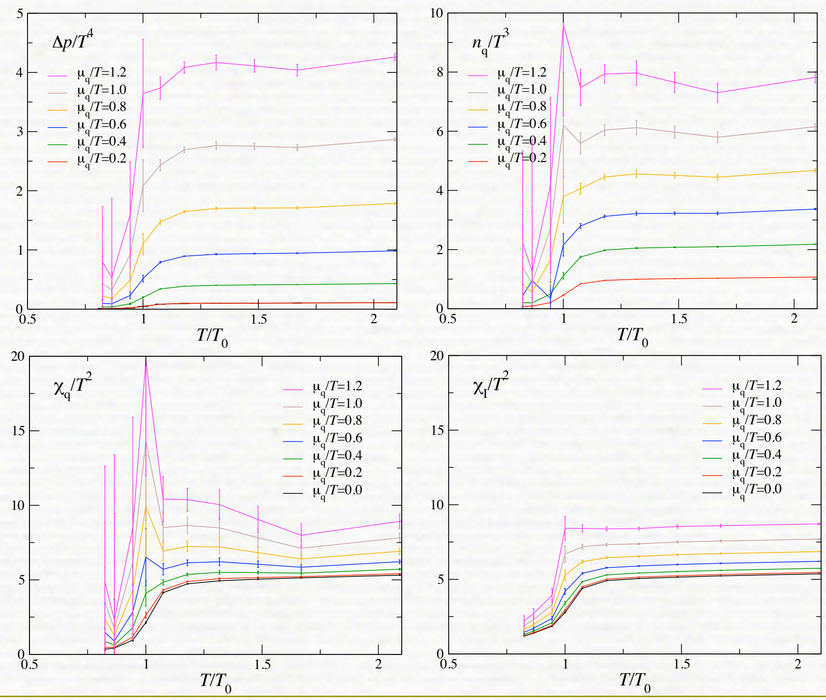
$$\mathcal{D}_2 = N_f\left[\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial \mu^2}\right) - \mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial \mu}M^{-1}\frac{\partial M}{\partial \mu}\right)\right],$$

etc.

where M is the quark matrix. Similar expansions can be written down for quark number density and its susceptibility.

We evaluate the traces with the random noise method. (i) Because the elements off-diagonal in color and spin indices are not suppressed by |x-y| with Wilson-type quarks, the number of the same-magnitude off-diagonal elements in the quark matrix is 11 times larger than the diagonal one. This is different from the case of staggered-type quarks, in which off-diagonal elements in spin indices are slightly suppressed by the spatial offset. Because a large number of noises is required to pick up a signal from data with S/N = 1/11, we decide not to apply the noise method for color and spin indices and generate noise vectors only for spatial indices, i.e. we repeat the inversion of M for each color and spin indices. (ii) We find that the dominant errors are from $\mathcal{D}_1$ which has much larger fluctuations than other traces. We adopt 10-40 times more noise vectors for $\mathcal{D}_1$. while we generate only 10 noise vectors for other traces.

Results for $\Delta p = p(\mu_q)-p(0)$, quark number density, and quark number as well as isospin susceptibilities at m_{PS}/m_V = 0.65 are as follows:



4. Improvement: A Hybrid Method

Evaluation of $\mathcal{D}_n$ with $n>4$ is compuationally demanding. Here, we note that $\mathcal{D}_n=0$ at $n>4$ for free quarks. Therefore, at high temperatures, we may approximate $\mathcal{D}_n=0$ for $n>4$ in the evaluation of c_n with $n>4$. This corresponds to a hybrid reweighting method in which the grand canonical potential is approximated by a truncated Taylor expansion

$$\omega(T,\mu_q) \approx \frac{1}{VT^3}\ln\mathcal{Z}(T,0) + \frac{1}{VT^3}\ln\left\langle \exp\left[\sum_{n=1}^{N_{\max}}\mathcal{D}_n(\mu_q a)^n\right]\right\rangle_{(\mu_q=0)},$$

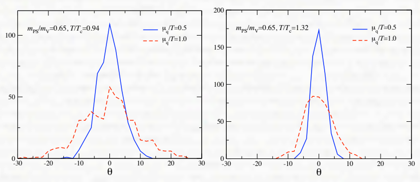
with N_{max} = 4.

We have $$\mathcal{Z}(T,\mu_q) = \mathcal{Z}(T,0)\left\langle e^F e^{i\theta}\right\rangle_{(\mu=0)}$$

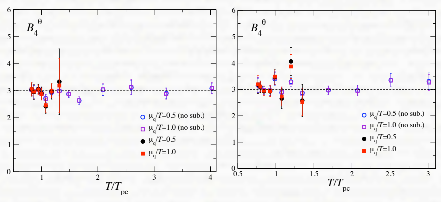
with $\theta(\mu) \equiv N_f \mathrm{Im}[\ln\det M(\mu)] \approx \sum_{n=0}^{1}\frac{1}{(2n+1)!}\mathrm{Im}\mathcal{D}_{2n+1}\mu^{2n+1}$ and $F(\mu) \equiv N_f \mathrm{Re}\left[\ln\left(\frac{\det M(\mu)}{\det M(0)}\right)\right] \approx \frac{1}{2!}\mathrm{Re}\mathcal{D}_2\mu^2$

In a previous study, SE noted that the θ-distribution is well described by a Gaussian form, and showed that this fact can be used to carry out the θ-averaging with small errors [Phys.Rev.D77,014508(2008)].

We find that our data are also well Gaussian:



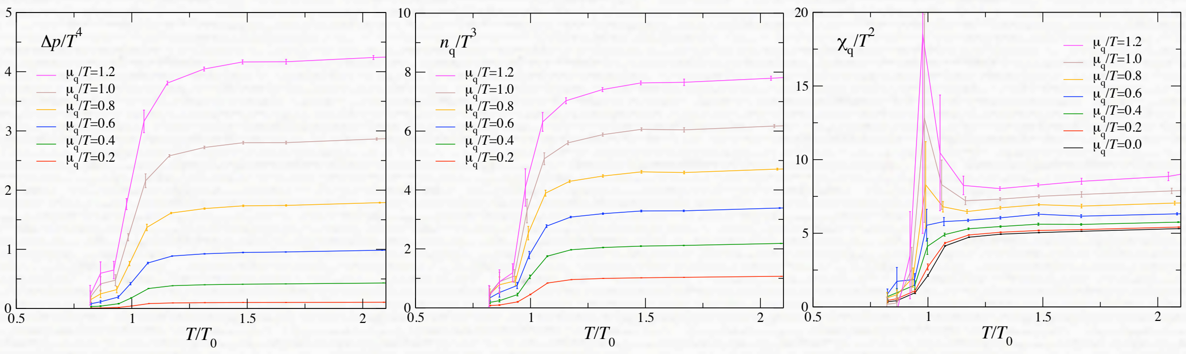
Test by the Binder cummurant: $$B_4^\theta \equiv \frac{\langle\theta^4\rangle}{\langle\theta^2\rangle^2} = 3 \text{ for Gaussian}$$



We thus have $$\mathcal{Z}(T,\mu) \approx \mathcal{Z}(T,0)\left\langle e^F e^{-\frac{1}{2}\langle\theta^2\rangle_F}\right\rangle_{(\mu=0)}.$$

The remaining F-averaging at finite μ_q can induce large statistical fluctuations due to the factor e^F. At small μ_q, the problem can be largely resolved by shifting β adopting a reweighting method because F is sensitively correlated with the gauge action. We calculate a optimal β by minimizing the fluctuation in $\left\langle e^F e^{-1/(4a_2)}e^{6N_{\rm site}(\beta-\beta_0)P}\right\rangle$ where $P = S_{gauge}/(6N_{\rm site}\beta)$ is the extended plaquette for our gauge action. The small shifts in β (which turn out to be less than about 0.03) are translated to slight shifts in T in the final plots.

With these improvements, we obtain at m_{PS}/m_V = 0.65

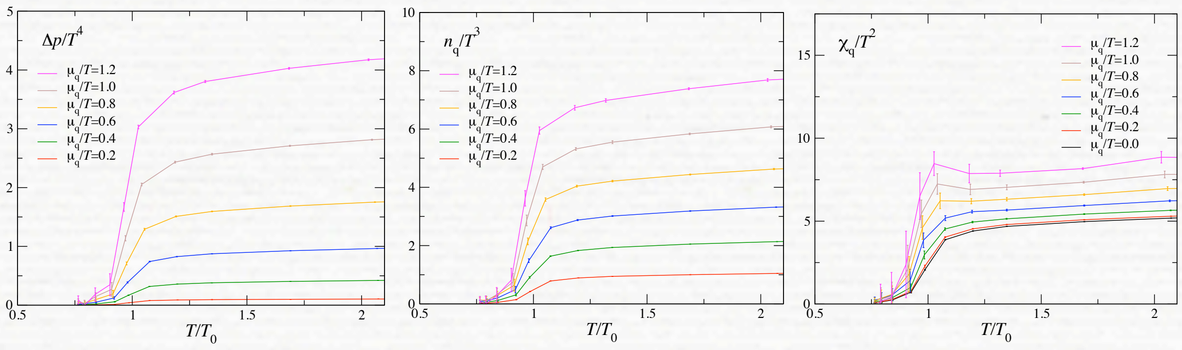


Here, we calculate the quark number density and its susceptibility by numerical differentiations of the grand canonical potential using the following thermodynamic formulae:

$$\frac{n_q}{T^3} = \frac{N_t^3}{N_s^3}\frac{\partial(\ln\mathcal{Z})}{\partial(\mu_q/T)}, \qquad \frac{\chi_q}{T^2} = \frac{N_t^3}{N_s^3}\frac{\partial^2(\ln\mathcal{Z})}{\partial(\mu_q/T)^2}$$

We find reduction of statistical fluctuations over the results of the previous section. Resulting T- and μ_q-dependences are smooth and in accordance with theoretical expectations. Therefore, we think that the assumptions introduced in the course of the improvement calculations are well under control.

Results at m_{PS}/m_V = 0.80 are



4. Conclusions

We have carried out the first calculation of the equation of state at non-zero densities with two flavors of improved Wilson quarks. Statistical fluctuations of physical observables at finite density are much severer with Wilson-type quarks than with staggered-type quarks. To tame the problem, we combined and developed several improvement techniques.

We find that the peak height of the quark number fluctuation at the pseudo-critical temperature increases as μ_q increases. In contrast, isospin susceptibilities show no sharp peaks at the pseudo-critical temperature. These results agree with previous observations by the Bielefeld-Swansea Collaboration using staggered-type quarks.

This suggests a critical point at finite μ_q, which is expected to locate at the end point of a first order transition line between confining and deconfining phases in the coupling parameter space of T and μ_q.