SIMULATION OF $4d\ \mathcal{N}=1$ SUSY YANG-MILLS THEORY WITH LIGHT WILSON GLUINOS

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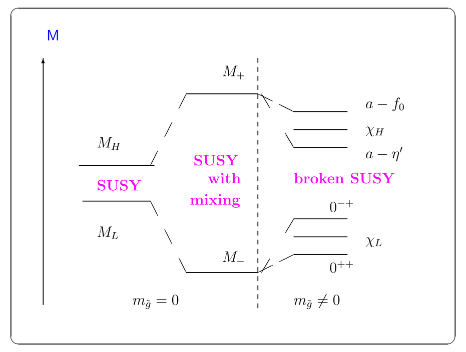
Introduction

We present Monte Carlo lattice calculations of the $\mathcal{N}=1$ SYM theory spectrum. The theory possesses one flavour (effectively $N_f=\frac{1}{2}$) of Majorana fermions in the adjoint representation of the colour gauge group SU(2). The interesting feature is the SUSY restoration in the continuum limit. The masses of the low lying bound states are expected to form two chiral supermultiplets as described by the effective actions [1, 2] (see Figure).



- SUSY is softly broken by a non-zero gluino mass
- $U(1)_\lambda$ chiral symmetry is broken by anomaly to Z_{N_c}
- Z_{N_c} is spontaneously broken to Z_2 by the gluino condensate $\langle\bar\lambda\lambda\rangle$
- Two vacua coexist: first order phase transition
- Confinement is realized by colorless bound states

Simulation of SYM on the lattice

Curci-Veneziano Lattice action

- We use the Curci-Veneziano action to simulate the Wilson gluinos (with eventually improved gauge action). The effective action for the gauge links reads

$$S_{CV}=\beta\left(c_0\sum_{pl}\left(1-\tfrac{1}{N_c}\mathrm{ReTr}U_{pl}\right)+c_1\sum_{rect}\left(1-\tfrac{1}{N_c}\mathrm{ReTr}U_{rect}\right)\right)-\tfrac{1}{2}\mathrm{logdet}Q[U].$$

- Q : Dirac-Wilson fermion matrix

$$Q^{ab\alpha\beta}_{xy}[U]=\delta^{ab}\delta_{xy}\delta_{\alpha\beta}-\kappa\sum_{\mu=1}^{4}\left(\delta_{y,x+\hat\mu}(1+\gamma^{\alpha\beta}_{\mu})V^{ab}_{\mu}(x)+\delta_{y+\hat\mu,x}(1-\gamma^{\alpha\beta}_{\mu})V^{ab^T}_{\mu}(y)\right),$$

where V are the real adjoint links $V^{ab}_{\mu}(x)\equiv 2\mathrm{Tr}[U^{\dagger}_{\mu}(x)T^aU_\mu T^b]=[V^{ab}_{\mu}(x)]^*=[V^{ab^T}_{\mu}(x)]^{-1}$

- ADVANTAGES: low computational cost, theoretically well-defined
- PRICE TO PAY: *Pfaffian* sign problem + fine-tuning towards the SUSY ($\sim$ chiral) point
- SUSY and chiral symmetry are expected to be recovered in the continuum limit at $m_{\tilde g}=0$

Algorithms

- The fractional power of the fermion determinant is represented by the pseudo-fermion integral with a polynomial approximation

$$|\det(Q[U])|^{N_f}\propto\int\mathcal{D}[\Phi^\dagger,\Phi]\exp\left\{-\sum_{xy}\Phi^\dagger_y P_{n_1}(Q^\dagger Q)_{yx}\Phi_x\right\}$$

- Keep n_1 small and correct the first polynomial approximation by a global correction step: Two-Step Multi-Boson algorithm [3]: local updating + *noisy correction step*

$$|\det(Q[U])|^{N_f}\simeq\frac{1}{\det\left(P_{n_1}(\tilde Q^2)P_{n_2}(\tilde Q^2)\right)}$$

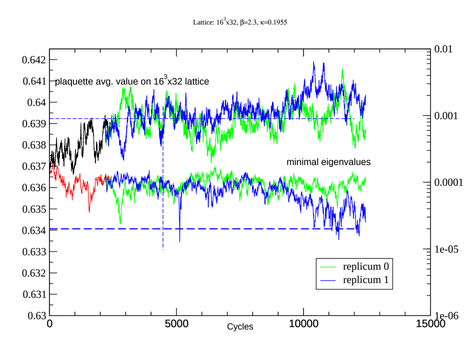
- Recently, we implemented a two-step Polynomial-Hybrid-Monte-Carlo algorithm [4]: sequence of PHMC trajectories + noisy correction step
- Algorithm improvements: determinant break-up, *even-odd* preconditioning

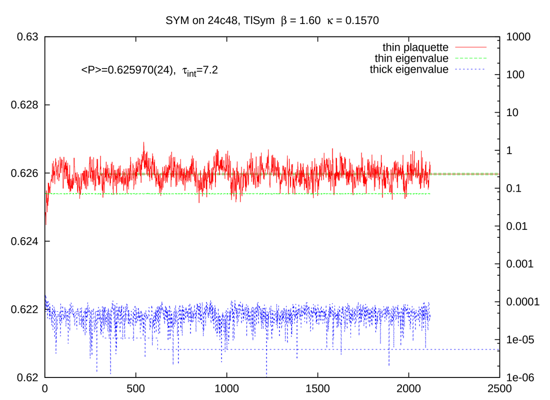
Improved lattice actions

- Besides Wilson (plaquette) gauge action in TSMB runs ($c_0=1, c_1=0$), we implemented Tree-Level Symanzik improved action to lower lattice artifacts in the new PHMC runs

$$c_0=\tfrac{5}{3}\ \text{and}\ c_1=-\tfrac{1}{12}$$

- In the fermionic part of the action we used one-step **Stout** smearing ($\rho=0.15$) which is useful for reducing the fluctuation of the smallest eigenvalues





Simulation details & Analysis

SETUP of TSMB runs

Run	$L^3.T$	β	κ	# Sweep	A_{nc} %	τ^{plaq}	ϵ	λ	n_1	n_2	offset
(a)	$16^3.32$	2.3	0.1955	12500	50-80	167.6	$2.0\cdot10^{-5}$	4.0	40	800	5
(b)	$16^3.32$	2.3	0.1960	23500	50-80	181.1	$4.0\cdot10^{-6}$	4.0	40	1800	5
(c)	$16^3.32$	2.3	0.1965	18000	50-62	254.2	$4.0\cdot10^{-6}$	4.0	40	1800	10

SETUP of PHMC Runs

Run	$L^3.T$	β	κ	# Traj.	A_{nc} %	τ^{plaq}
A	$16^3.32$	1.6	0.1800	2500	95.6	7.5
B	$16^3.32$	1.6	0.1900	2700	96.4	3.08
C1a	$16^3.32$	1.6	0.2000	1973	82.9	4.91
C1b	$16^3.32$	1.6	0.2000	8874	88.3	27.6
C2	$24^3.48$	1.6	0.2000	6465	88.6	7.4
D	$16^3.32$	1.6	0.2020	6947	88.5	45.7
C_{stout}	$24^3.48$	1.6	0.1570	2110	92.4	7.2

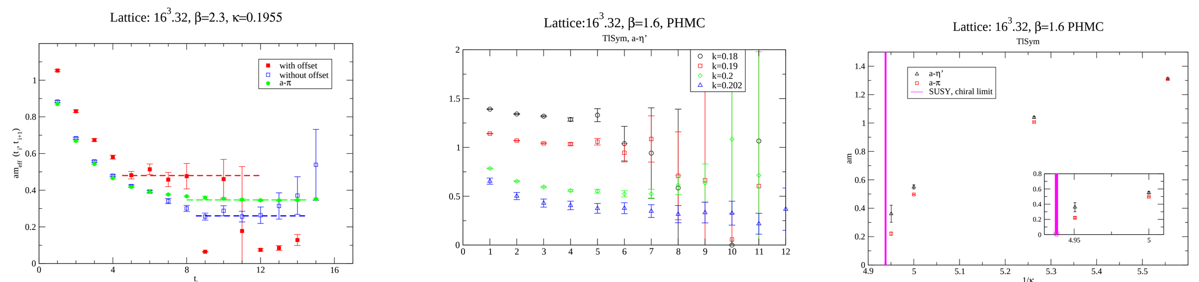
- The MD equations are implemented by Sexton-Weingarten integrator with multiple-time scale
- Pfaffian sign is computed with ARPACK and checked with *spectral flow* method
- The Pfaffian sign and the correction factors C are included by the *reweighting* in the analysis

$$\langle A\rangle=\frac{\langle sign[U]C[U]A[U]\rangle_g}{\langle sign[U]C[U]\rangle_g}$$

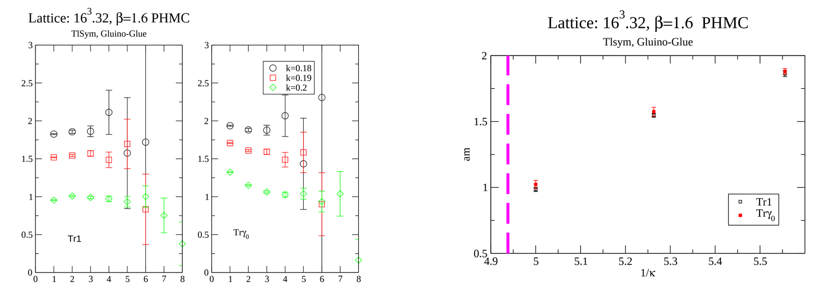
- One key for the reliable analysis of the SUSY spectrum is the physical volume
- On TSMB runs we have $\frac{r_0}{a}\sim 8$ $\rightarrow$ small volume $L^3\sim(1\text{fm})^3$ on $16^3\cdot32$ lattices
- On the new produced PHMC ensembles $\frac{r_0}{a}\sim 4$, the spatial volume is $L^3\sim(2.2\text{fm})^3$
- Larger lattices are being simulated, on $24^3\cdot48$ lattices $L\sim$ 3fm
- We found few configurations ($\sim$ 15 out of 5160) with negative sign of the Pfaffian at $\kappa=0.202$

Determination of masses

- The particle masses in lattice units are calculated from the zero-momentum correlation function of the corresponding interpolating operator $\mathcal{O}$
- Adjoint mesons: in analogy with the flavour singlet QCD mesons the correlator consists of a connected and a disconnected part
- We use the Stochastic Estimators Technique (SET) with spin dilution, and the Improved Volume Source Technique (IVST) [8] to compute the *all-to-all* propagators
- $a-\eta'$: $\mathcal{O}=\bar\lambda\gamma_5\lambda$ dominated by disconnected part for $\kappa\rightarrow\kappa_{cr}$



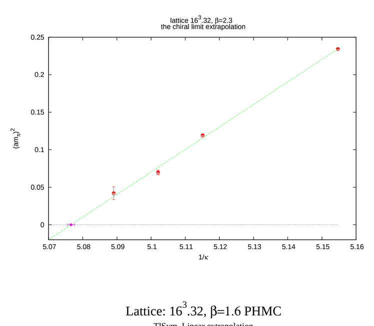
- $a-f_0$: $\mathcal{O}=\bar\lambda\lambda$. As in QCD (and in experiments), the mass of the scalar meson is difficult to determine, higher statistics are needed
- Glueballs: $\mathcal{O}_{0^{++}}=\mathrm{Tr}[U_{12}+U_{23}+U_{31}]$, $\mathcal{O}_{0^{-+}}=\sum_{\mathcal{R}\in O_h}(\mathrm{Tr}[W(C_{\mathcal{R}})]-\mathrm{Tr}[W(\mathcal{R}C_{\mathcal{R}})])$
- APE smearing combined with variational method
- χ_l/χ_h Gluino-glueball: $\mathcal{O}^\alpha=\sum_{i,j}\sigma^{\alpha\beta}_{ij}\mathrm{Tr}[P_{ij}\lambda^\beta]$ To optimize the overlap with the physical state we used Jacobi smearing for gluinos and APE smearing for links

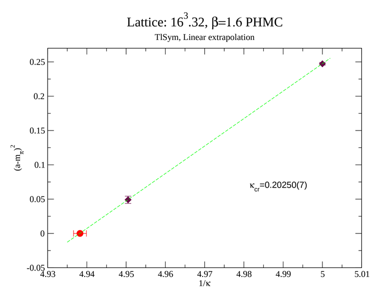


Chiral (SUSY) limit

OZI Arguments

- The connected part of the $a-\eta'$ correlator refers to the adjoint pseudoscalar $a-\pi$ which is not a physical particle in SYM
- The vanishing of the $a-\pi$ mass can be used to determine the chiral limit $m_{\tilde g}\rightarrow 0$, while $a-\eta'$ is expected to remain massive
- Within the OZI picture, and when approaching the chiral limit, the mass square of $a-\pi$ behaves like $(am_\pi)^2=A(\frac{1}{\kappa}-\frac{1}{\kappa_{cr}})$



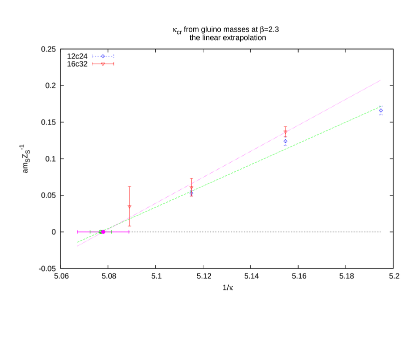


SUSY Ward-Takahashi identities

- The *gluino mass* $m_{\tilde g}$ is defined through the SUSY WI's [7]; $m_{\tilde g}\rightarrow 0$ in the chiral (SUSY) limit
- The gluino mass behaves linearly in $1/\kappa$; up to renormalization constants:

$$am_{\tilde g}\sim\tfrac{1}{2}(\tfrac{1}{\kappa}-\tfrac{1}{\kappa_{cr}})$$

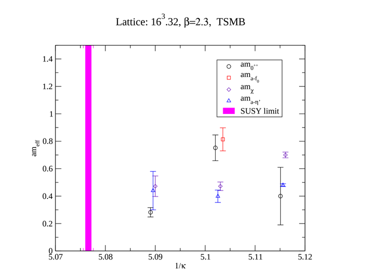
- On TSMB ensembles, from the linear extrapolation of $(am_\pi)^2$ we find $\kappa^{OZI}_{cr}(\beta=2.3)=0.1969$, which is consistent with the estimate using the WI's
- On PHMC ensembles (no Stout), we determine $\kappa^{OZI}_{cr}(\beta=1.6)=0.2025$

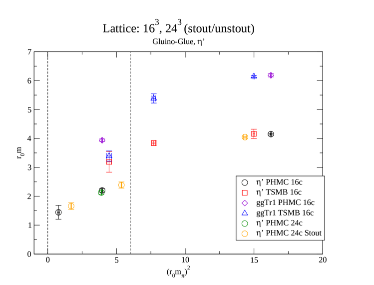


Conclusion & References

Conclusion & Outlook

- We presented new analysis of the SYM spectrum for lighter gluino masses as a continuation of the early DESY-Münster-collaboration investigations [6, 5]
- In this work, a first simulation of SYM with PHMC+NC algorithm [4] using improved action and Stout smearing has been performed
- Analysis of mass spectrum, WI's and chiral condensate on large volume and stout configurations is ongoing
- In the next steps, investigation of chiral transition is planned
- Better variance reduction methods and acceleration algorithms are being tested to optimize the disconnected correlator analysis (e. g. hybrid method, spectral decomposition and deflation)





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