Tests of Electric Polarizability on the Lattice

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I. Introduction:

Electric and magnetic polarizabilities characterize the rigidity of both charged and uncharged hadrons in external fields and are important fundamental properties of particles. In particular, the electric polarizability of a hadron characterizes the reaction of quarks to a weak external electric field and can be measured by experiment via Compton scattering. These sorts of quantities can be measured on the lattice**[1]** using background fields for neutral particles. We report here on the ongoing work at Jefferson Lab by the "polar" collaboration to measure particle polarizabilities**[2,3,4]**.

It turns out there are many different ways of putting an electric field on the lattice. In this poster, we discuss and present the results of various tests we have done in formulating a lattice quantum chromodynamics (QCD) calculation of neutral hadron electric polarizabilities. We shall consider the effects of periodic and fixed spatial boundary conditions, various types of quark sources and the field linearization postulate. We also consider two ways of formulating the classical vector potential for a uniform electric field from Maxwell's equations along with the other possibilities.

Classically, hadron polarizability is measured as a change in mass from an uniform, external electric field:

$$\delta m = -\tfrac{1}{2}\,\alpha\,\mathbf{E}^2,$$

where α is the electric polarizability coefficient. On the lattice, one looks for a plateau in the effective mass shift plots vs. lattice time, which we show below.

II. Results:

We have used a small number of configurations (18-20) to test out various sources, boundary conditions and formulations. We are doing this in the context of the dynamical 2+1 CP-PACS clover configurations**[5]**. We use a single mass, κ_{ud} = 0.13580 with κ_s = 0.13640 at β =1.9 on 20^3x40 lattices (a =0.098+_ .002 fm). We include the electric field as a phase on the links which affect the Wilson term but not the clover loops. We actually simulated with two different nonzero electric fields, but found that the results were exactly proportional to $\mathbf{E}^2$, and will only show the results at the smallest field value.

In many of the tests below we used a volume or spatial plane quark source to study neutral mesons and baryons in an electric field. Surprisingly, we found that this gave a good signal for the mesons but a poor or nonexistent one for the baryons. It turns out that when using an extended interpolation field for a hadron, the electric field phases will cancel on the sources for mesons but not for baryons. We believe this is the reason we obtained reasonable statistical signals only for the mesons with extended sources in an electric field. Therefore, with the exception of point interpolation results in Figs. 1 and 2, which did not suffer from this problem, we will only show the π^0 results since the statistical signal for other mesons are not as good with our small statistics. Please note that we have not attempted to include any disconnected diagrams.

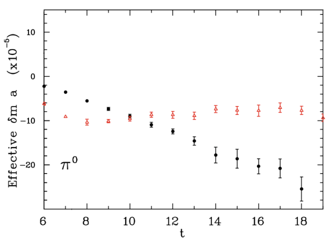

Figure 1: Point source results for effective mass shift for our smallest electric field for the π^0. Results are from time dependent spatial link (black symbols) or spatially dependent time link (red symbols).

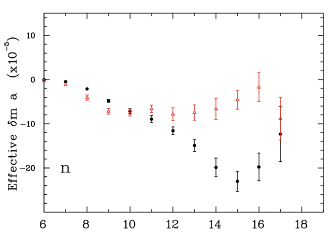

Figure 2: Point Source effective mass shifts for the neutron. Results are from time dependent spatial link (black symbols) or spatially dependent time link (red symbols).

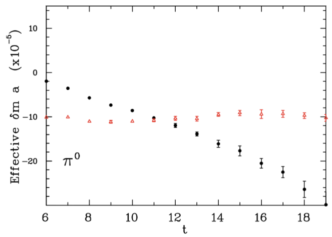

Figure 3: Volume source effective mass shifts for the π^0. Results are from time dependent spatial link (black symbols) or spatially dependent time links (red symbols).

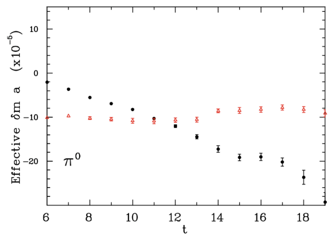

Figure 4: Plane source effective mass shifts for the π^0. Results are from time dependent spatial link (black symbols) or spatially dependent time links (red symbols).

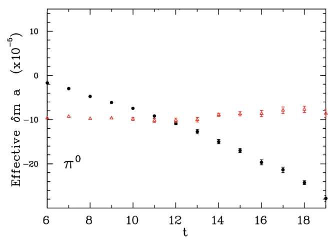

Figure 5: Non-periodic volume source effective mass shifts for the π^0. Results are from time dependent spatial link (black symbols) or spatially dependent time link (red symbols).

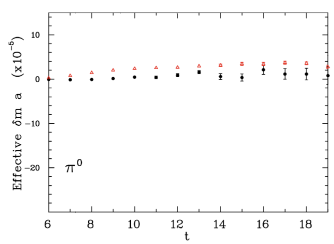

Figure 6: Volume source with exponential E-field effective mass shifts for the π^0. Results are from time dependent spatial link (black symbols) or spatially dependent time links (red symbols).

Figures 1 and 2 show the results from our "usual" way of calculating electric polarizabilities for the π^0 and neutron. We are using point interpolation fields and periodic spatial boundary conditions. The electric field is in the z-direction and the point sources are started in the "middle" of the lattice in order to avoid quark wrap around effects as much as possible. The black symbol results are consistent with the types of results we have previously obtained. The π^0 seems not to have a plateau but the neutron has possible plateaus at larger times. Here as in the other figures, all mass shifts obtained are negative. As mentioned in the Introduction, we also looked at the effect of simulating an electric field with a linear z-dependent time link. The red symbols give these results. As one can see, the results do not agree with one another. The black points drop nearly linearly. On the other hand the red points seem to show a plateau in the effective mass from the very first time step! (The source is at time step 6.)

Fig. 3 shows the results of using a quark volume source rather than a point source as the interpolation field. Again, the lattice is kept periodic in spatial directions. The results are very similar to the point interpolation field results, except one has smaller statistical error bars, which would be expected. The disagreement between the two classical ways of formulating an electric field persists.

Fig. 4 shows the result of using a spatial plane quark source. The z=10 plane is chosen to be at the approximate mid-point of the lattice in the z-direction to minimize wrap around effects. We see again the disagreement between the two classical ways of putting in the electric field. The statistical error bars are comparable to those in Fig. 3.

One can also avoid the quark wrap around effects by explicitly removing the coupling between the two sides of the lattice in the z-direction. Such nonperiodic boundary conditions make a linear "box" in the z-direction. Now the quarks do not feel the electric field discontinuity at the spatial edge of the lattice. (Of course, this is also avoided by using time dependent z-direction links, which "hides" the discontinuity in the time direction, which is always kept nonperiodic.) **Fig. 5** gives the result of such a change in boundary conditions. Remarkably, the results look very similar to those in Figs. 3 and 4, and seem to not show any effects of the decoupling of the two sides of the lattice. Again, the black and red points show the same behavior as for Figs. 3, 4, and 5.

Finally, we examine the effect of the field linearization postulate in **Fig. 6**. By this we mean the replacement of the phase $U_z = e^{i\eta t}$ by $(1 + i\,\eta t)$ in the space link case and $U_t = e^{i\eta' z}$ by $(1 + i\,\eta' z)$ in the time link case, as has been done in all of our simulations. Only by linearizing the electric field phase in the quark action does one couple the external field with the correct conserved lattice current. Failure to linearize will result in spurious non-linear photon interaction terms. We re-examine the mass shifts using the full exponential in Fig. 6. Even though the linear terms were kept quite small (we used the same η values as in Ref. 1), one can see that the results are affected by the presence of the extra local terms; we verified that these shifts are also proportional to $\mathbf{E}^2$. We regard this as a warning that failure to linearize is a serious error.

III. Conclusions:

Many of the results here were a surprise to us. The disagreement of the two classical ways of formulating the electric field is especially puzzling. On the other hand, the amazingly robust mass shift plateau we found for the alternate formulation of the classical electric field is encouraging. We will continue the process of studying various field boundary conditions/formulations on lattice fermions. In particular, it will be very interesting to re-examine the boundary condition tests for point baryon fields to assess their affect.

IV. Acknowledgements:

This work is supported in part by U.S. Department of Energy under grant DE-FG02-95ER40907. Our calculations were done with QDP++ and Chroma software at Jefferson Lab.

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