The Landau gauge lattice ghost propagator in stochastic perturbation theory

F. Di Renzo[1], E.-M. Ilgenfritz[2], H. Perlt[3], A. Schiller[3] and C. Torrero[4]

UNIVERSITÄT LEIPZIG

[1]University of Parma, [2]Humboldt-Universität zu Berlin, [3]Universität Leipzig, [4]Universität Regensburg

LATTICE 2008, Williamsburg, VA, July 14 - July 19, 2008

Abstract: We present one- and two-loop results for the ghost propagator in Landau gauge calculated in numerical stochastic perturbation theory (NSPT). The one-loop results are compared with available standard lattice perturbation theory in the infinite volume limit. We discuss in detail how to perform the different necessary limits in the NSPT approach and discuss a recipe to treat logarithmic terms by introducing "finite lattice logs". We find agreement with the one-loop result from standard lattice perturbation theory and estimate, from the non-logarithmic part of the ghost propagator in two-loop order, the unknown constant contribution to the ghost self-energy in the RI'-MOM scheme in Landau gauge. That constant vanishes within our numerical accuracy.

Supported by DFG under contract FOR 465

NSPT and Langevin equation

Numerical stochastic perturbation theory (NSPT) (e.g. Review by Scorzato/Di Renzo JHEP 2004) is a powerful tool to study higher loop contributions in lattice perturbation theory (LPT).
Applications: Wilson loops, Z-factor for bilinear quark operators,...

New application: The two-loop ghost propagator

Use the lattice Langevin equation with time t

$$\frac{\partial}{\partial t} U_{x,\mu}(t;\eta) = \mathrm{i}\left(\nabla_{x,\mu} S_G[U] + \eta_{x,\mu}(t)\right)\, U_{x,\mu}(t;\eta)$$

η – Gaussian random noise,
$\nabla_{x,\mu}$ – left Lie derivative within the gauge group.

For $t \to \infty$ the gauge fields are distributed according to the measure $\exp(-S_G[U])$.
Here we use the Wilson gauge action S_G.

Discretize $t = n\epsilon$ and get the Langevin equation solved within the Euler scheme:

$$U_{x,\mu}(n+1;\eta) = \exp(F_{x,\mu}[U,\eta])\, U_{x,\mu}(n;\eta)$$

$$F_{x,\mu}[U,\eta] = \mathrm{i}(\epsilon \nabla_{x,\mu} S_G[U] + \sqrt{\epsilon}\, \eta_{x,\mu})$$

Implementing the Langevin equation

Rescale $\varepsilon = \beta\epsilon$ and use the expansion ($g \propto \beta^{-1/2}$):

$$U_{x,\mu}(t;\eta) \to 1 + \sum_{l>0} \beta^{-l/2} U^{(l)}_{x,\mu}(t;\eta)$$

The Langevin equation transforms to a system of updates $U \to U'$, one for each perturbative $U^{(l)}$:

$$\begin{aligned} {U^{(1)}}' &= U^{(1)} - F^{(1)} \\ {U^{(2)}}' &= U^{(2)} - F^{(2)} + \frac{1}{2}(F^{(1)})^2 - F^{(1)} U^{(1)} \\ &\dots \end{aligned}$$

The random noise η is fed in only through $F^{(1)}$, higher orders become stochastic by propagation of noise through fields of lower order.

Keep also the gauge field variables (in the algebra), $A = \log U$, enforcing unitarity in all orders in g:

$$A_{x,\mu}(t;\eta) \to \sum_{l>0} \beta^{-l/2} A^{(l)}_{x,\mu}(t;\eta)\,, \qquad A^{(l)}_{x,\mu} = T^a A^{a,(l)}_{x,\mu}$$

$$A^{(l)\dagger} = -A^{(l)}\,, \qquad \mathrm{Tr} A^{(l)} = 0$$

The Landau gauge is reached by iterative gauge trafo's (perturbative Fourier acceleration, Davies et al.).

Ghost propagator in NSPT

The continuum ghost propagator in momentum space is defined as $G^{ab}(q) = \delta^{ab} G(q^2)$.

On the lattice we define

$$G(\hat{q}(k)) = \frac{1}{N_c^2 - 1} G^{aa}(\hat{q}(k)) = \frac{1}{N_c^2 - 1} \left\langle \mathrm{Tr} M^{-1}(k) \right\rangle_U$$

- $M = -\nabla \cdot D(U)$ - the Faddeev-Popov operator,
- with $D(U)$ - the lattice covariant derivative,
- $M^{-1}(k)$ - Fourier transform of the inverse FP op.,
- Lattice momenta: $\hat{q}_\mu(k_\mu) = \frac{2}{a}\sin\left(\frac{\pi k_\mu}{L_\mu}\right) = \frac{2}{a}\sin\left(\frac{a q_\mu}{2}\right)$.

Perturbative expansion based on the mapping
$A^{(n)}_{x,\mu} \to M^{(n)} \to \left[M^{-1}\right]^{(n)}$:
instead of inversion, recursive evaluation is possible:

$$\begin{aligned} \left[M^{-1}\right]^{(0)} &= \left[M^{(0)}\right]^{-1} \\ \left[M^{-1}\right]^{(n)} &= -\left[M^{0}\right]^{-1} \sum_{j=0}^{n-1} M^{(n-j)} \left[M^{-1}\right]^{(j)} \end{aligned}$$

Momentum space ghost propagator in NSPT from

$$\left[M^{-1}\right]^{(n)} \to G^{(n)}(\hat{q}(k)) = \langle k | \left[M^{-1}\right]^{(n)} | k \rangle$$

Ghost propagator in standard LPT

Discuss two forms of the dressing function:

$$J^{(l)}(aq) = (aq)^2\, G^{(l)}\,, \quad \hat{J}^{(l)}(\hat{q}) = \hat{q}^2\, G^{(l)}$$

Renormalization in the RI'-MOM scheme:

$$J^{\mathrm{RI}'}(q,\mu,\alpha_{\mathrm{RI}'}) = \frac{J(a,q,\alpha_{\mathrm{RI}'})}{Z_{\mathrm{gh}}(a,\mu,\alpha_{\mathrm{RI}'})}$$

$$J^{\mathrm{RI}'}(q,\mu,\alpha_{\mathrm{RI}'})|_{q^2=\mu^2} = 1\,.$$

Restricting to two loop order, we have $e.g.$

$$J(a,q,\alpha_{\mathrm{RI}'}) = 1 + \sum_{i=1}^{2} \alpha^i_{\mathrm{RI}'} \sum_{k=0}^{i} z^{\mathrm{gh},\mathrm{RI}'}_{i,k} \left(\frac{1}{2}\log(a^2 q^2)\right)^k$$

$z^{\mathrm{gh},\mathrm{RI}'}_{i,i}$: known from continuum PT
$z^{\mathrm{gh},\mathrm{RI}'}_{i,k}\Big|_{i>k>0}$: partly known from continuum PT

$$z^{\mathrm{gh},\mathrm{RI}'}_{1,1} = -\frac{3}{2} N_c\,, \quad z^{\mathrm{gh},\mathrm{RI}'}_{2,2} = -\frac{35}{8} N_c^2\,, \quad z^{\mathrm{gh},\mathrm{RI}'}_{2,1} = \left(-\frac{271}{24} + \frac{35}{6} z^{\mathrm{gh},\mathrm{RI}'}_{1,0}\right) N_c$$

$z^{\mathrm{gh},\mathrm{RI}'}_{1,0} = 13.8257$, is known from one-loop LPT
$z^{\mathrm{gh},\mathrm{RI}'}_{2,0}$ is unknown

Results: data and statement of problem

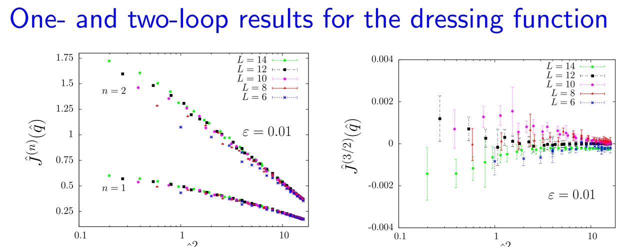
From the relation
$\alpha_{\mathrm{RI}'} = \alpha_0 + \left(-\frac{22}{3}\log(a\mu) + 73.9355\right)\, \alpha_0^2 + \dots$,
with bare $\alpha_0 = N_c/(8\pi^2\beta)$, we get for the measured dressing function:

$$J^{2-\mathrm{loop}}(a,q,\beta) = 1 + \frac{1}{\beta}\left(J_{1,1}\, \log(aq)^2 + J_{1,0}\right) + \frac{1}{\beta^2}\left(J_{2,2}\, \log^2(aq)^2 + J_{2,1}\, \log(aq)^2 + J_{2,0}\right)$$

Aim of this first investigation:

confirmation of known $J_{1,0}$, determination of unknown $J_{2,0}$

One- and two-loop results for the dressing function

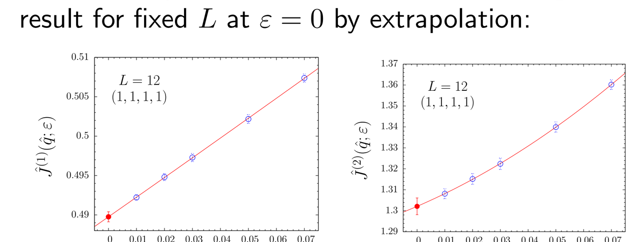


Measured ghost dressing function $\hat{J}(\hat{q})$ vs. $\hat{q}^2$ for all inequivalent lattice momentum 4-tuples near diagonal for $L = 6, 8, 10, 12, 14$ and $\varepsilon = 0.01$. Left: The one-loop (β^{-1}) and two-loop (β^{-2}) contributions, right: the vanishing ($\propto \beta^{-3/2}$) contribution.

Results: the limits to be taken

- Limit $\varepsilon \to 0$

Different step sizes: $\varepsilon =$ 0.07,..., 0.01 $\to$ Langevin result for fixed L at $\varepsilon = 0$ by extrapolation:



Linear plus quadratic correction extrapolation to $\varepsilon = 0$ of the one-loop (left) and two-loop (right) ghost dressing function for lattice size 12^4 at momentum tuple $(1,1,1,1)$.

- Limits $L \to \infty$ and $a \to 0$

In order to make contact with standard LPT both limits have to be performed.

Problem : How to represent – on finite lattices – the logs that appear in the $L \to \infty$ regime ?

Proposal: Replace divergent lattice integrals, that give the logarithms, by finite lattice sums and use these expressions in the fits at fixed L.

Handling the lattice logs encountered

Example: typical one-loop divergent integral

$$A(aq) = (4\pi)^2 \int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4} \frac{1}{\hat{k}^2 \widehat{(k+q)}^2}\,.$$

In the limit $aq \to 0$ (Lüscher, Weisz):

$$A(aq) = -\log(aq)^2 + a_1\,,\ a_1 = 2 + F_0 - \gamma_E = 5.79201$$

On a lattice with finite L we calculate lattice sums like:

$$A(i^q, L) = \frac{1}{L^4} \sum_{i_1,i_2,i_3,i_4} \frac{1}{\left[\sum_{\mu=1}^4 \sin^2\left(\frac{\pi}{L} i_\mu\right)\right]\left[\sum_{\nu=1}^4 \sin^2\left(\frac{\pi}{L}(i_\nu - i^q_\nu)\right)\right]}$$

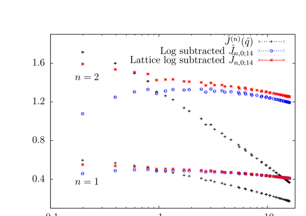
$$a k_\mu = \frac{2\pi i_\mu}{L}\,, \quad a q_\mu = \frac{2\pi i^q_\mu}{L}\,, \quad \{i_\mu, i^q_\mu\} \in \left(-\frac{L}{2}, \frac{L}{2}\right]$$

This leads – for each L – to the replacement:

$$J_{1,0} \log(aq)^2 \to -2J_{1,0}\,(A(i^q,L) - a_1)$$

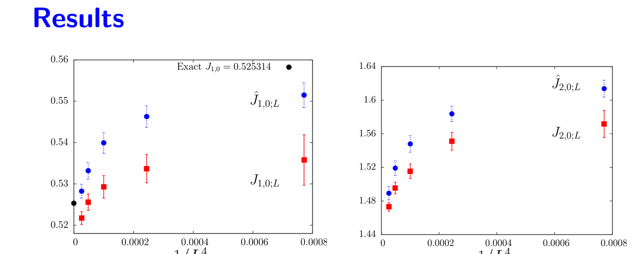
Result: Better flattening for the log-subtracted data

Original and remaining 'non-logarithmic' contributions to $\hat{J}$ using logarithms and lattice logarithms in one-loop and two-loop as function of $\hat{q}^2$, here for a lattice 14^4.



Results and Summary

Results



The $V \to \infty$ limit: volume dependence of the constants $\hat{J}_{k,0;L}$ and $J_{k,0;L}$

Linear extrapolation in $1/L^4$ for $L = 10, 12, 14$:

$$\hat{J}^{\mathrm{Fit}}_{1,0} = 0.5246(22)\,, \quad \hat{J}^{\mathrm{Fit}}_{2,0} = 1.4737(118)$$

$$\longrightarrow z^{\mathrm{gh},\mathrm{RI}'}_{2,0} = -1.42(8.18)$$

Summary

- First two-loop calculation of the lattice ghost propagator.
- One-loop constant $J_{1,0}$ agrees with known $V \to \infty$ result.
- Two-loop constant $J_{2,0}$ determined for the first time.
- Detailed analysis of all neccessary limits is performed.
- Proposal how to mimic logarithmic terms on finite lattices.
- Detailed comparison with Monte Carlo data next to be done.