Improving B physics simulations

Eric B. Gregory & the HPQCD Collaboration
Department of Physics & Astronomy, University of Glasgow, Glasgow, UK, G12-8QQ
e.gregory@physics.gla.ac.uk

UNIVERSITY of GLASGOW

Abstract

We give improved results for B meson masses using NRQCD b quarks and HISQ light valence quarks for a range of lattice spacings and sea quark masses enabling controlled extrapolation to the physical point.

Introduction

The precise calculation of the charmonium and bottomium spectra is an important goal of lattice QCD for several reasons:

- There are many 'gold-plated' states: narrow, stable, and experimentally accessible.
- The splittings have particularly good properties for determining the lattice scale.
- It is an important test of the actions used for b and c, which can then be used to calculate decay constants which in turn are crucial for determining CKM matrix elements.
- Control of systematic errors are well developed in calculations of the B and D spectrum on the lattice due to relative insensitivity to heavy-quark polarisation effects and to light quark masses.

The Highly Improved Staggered Quark (HISQ) action [1], allows unprecedented control of discretization errors in numerical lattice calculations. We use HISQ s and c valence quarks with NRQCD b quarks on MILC lattices with $N_f = 2+1$ flavors of ASQTAD sea quarks to calculate the masses of the B_s and B_c mesons.

Heavy-light 2-point functions

For increased statistics, we use random sources $\eta(x, t_0)$, defined as a three-component random complex unit-vector defined on each point in the source time slice t_0. These are the sources for the inversion of the HISQ strange and charmed valence quark propagators.

We convolute the noise source with a collection of smearing functions (e.g., Gaussian), of varying radii chosen to allow resolution of different ground and excited B_S and B_c states. We then initialize N_{smear} NRQCD b quark propagators by setting

$$G_i(x, t=0) = \sum_{x'} S(|x - x'|; r_i)\eta(x', t_0))$$

From this, dynamical HISQ propagators for the s and c quarks are generated. We evolve NRQCD b quark propagators from the same noise source:

$$G_i(x, t+1) = \left(1 - \frac{\delta H}{2}\right)\left(1 - \frac{H_0}{2n}\right)^n U_t^\dagger(x)\left(1 - \frac{H_0}{2n}\right)^n\left(1 - \frac{\delta H}{2}\right) G_i(x, t)$$

We use an improved lattice NRQCD Hamiltonian [2]:

$$H_0 = -\frac{\Delta^{(2)}}{2M^0}$$

$$\delta H = -c_1\frac{(\Delta^{(2)})^2}{8(M^0)^3} + c_2\frac{ig}{8(M^0)^3}(\mathbf{\Delta}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{\Delta}) - c_3\frac{ig}{8(M^0)^3}\sigma\cdot(\mathbf{\Delta}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\mathbf{\Delta}) - c_4\frac{g}{2M^0}\sigma\cdot\tilde{\mathbf{B}} + c_5\frac{a^2\Delta^{(4)}}{24M^0} - c_6\frac{a(\delta^{(2)})^2}{16n(M^0)^2}$$

Finally, at each time-slice, we combine the N_{smear} NRQCD propagators and the valence s or c quark propagator with the same smearing functions at the sink end, giving us $N_{\text{smear}} \times N_{\text{smear}}$ B_s and B_c meson correlators.

2-point function effective masses and noise

The expression

$$\left[\langle G_{B_s}(i,j;t-t_0)G_{B_s}(i,j;t-t_0)\rangle - \langle G_{B_s}(i,j;t-t_0)\rangle^2\right]$$

contains in it propagators for $\bar{b}\bar{s}bs$ four-quark states. The lightest combination on the lattice is $\eta_b + \eta_s$. Therefore the error on the propagator falls like $e^{\frac{1}{2}(M_{\eta_b}+M_{\eta_s})}$ as can be seen in Figure 1. This figure also illustrates that effective masses of propagators $G_{B_s}(i,i;t-t_0)$ with Gaussian smearing with radius 2 and 4 ($G2G2$ and $G4G4$) approach the M_{B_s} plateau rapidly compared to the local source-sink combination ($L0L0$).

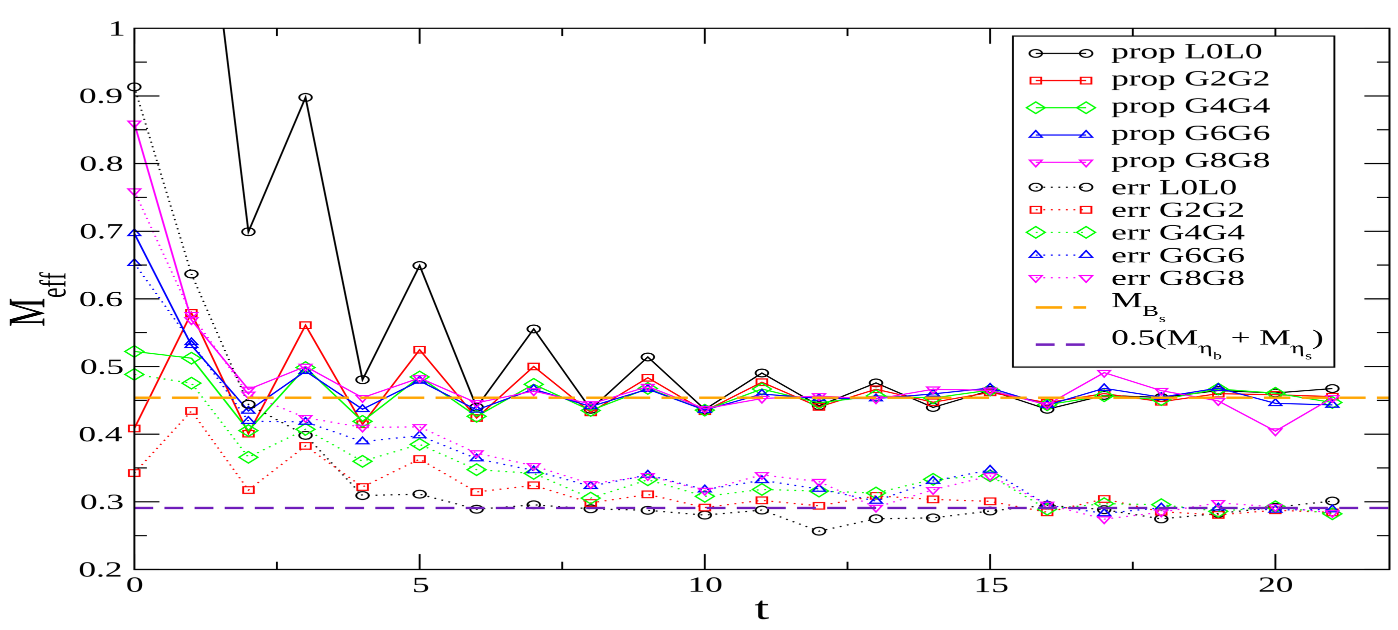

Figure 1: Plot of effective masses of (local-local) B_s correlator and B_s correlator error. While effective mass of the correlator matches the experimental M_{B_s} (corrected for the energy shift), the effective mass of the correlator error gives $\frac{1}{2}(M_{\eta_b} + M_{\eta_s})$. Several source-sink smearing combinations are shown.

Extracting M_{B_s} and M_{B_c}

We fit the measured B_s and B_c correlators to the form

$$G_{\text{meson}}(i,j;t-t_0) = \sum_{k=1}^{N_{\text{exp}}} a_{i,k}a^*_{j,k}e^{-E_k(t-t_0)} + \sum_{k'=1}^{N_{\text{exp}}-1} b_{i,k'}b^*_{j,k'}(-1)^{(t-t_0)}e^{-E'_{k'}(t-t_0)},$$

where i and j respectively index the source and sink smearing functions. The second term is an oscillating parity partner state. We perform simultaneous Bayesian fits of the $G_{\text{meson}}(i,j;t-t_0)$, looking for stability with respect to fit range and N_{exp}.

Since the NRQCD Hamiltonian does not include a mass term, there is a shift in the energy of the $p=0$ states relative to the continuum mass. To correct for the energy shift in M_{B_s} and M_{B_c} we use the relationship:

$$M_{B_{s/c}} = \left(E_{B_{s/c}} - \frac{1}{2}E_\Upsilon\right)_{\text{latt}} + \frac{1}{2}M_\Upsilon, \qquad \text{(I)}$$

where E_{B_s} is the ground-state E_0, and E_Υ is calculated with two NRQCD b quark propagators on the same configurations.[3] The Ms on the right hand side are experimental values. For M_{B_c} we also explore two other methods for cancelling the energy shift:

$$M_{B_c} = \left(E_{B_c} - \frac{1}{2}(E_\Upsilon + E_{\eta_c})\right)_{\text{latt}} + \frac{1}{2}\left(M_\Upsilon + M_{\eta_c}\right) \qquad \text{(II)}$$

$$M_{B_c} = \left(E_{B_c} - (E_{B_s} + E_{D_s})\right)_{\text{latt}} + \left(M_{B_s} + M_{D_s}\right) \qquad \text{(III)}$$

We show results for M_{B_s} and M_{B_c} on several ensembles at three lattice spacings and different light sea quark masses in Figure 2. The statistical error (shown in figure) is dominated by the uncertainty in a^{-1}. Expression (II) minimizes the statistical uncertainty by using $(E_{B_c} - \frac{1}{2}(E_\Upsilon + E_{\eta_c}))$, which is small relative to the $()_{\text{latt}}$ quantities in the other expressions.

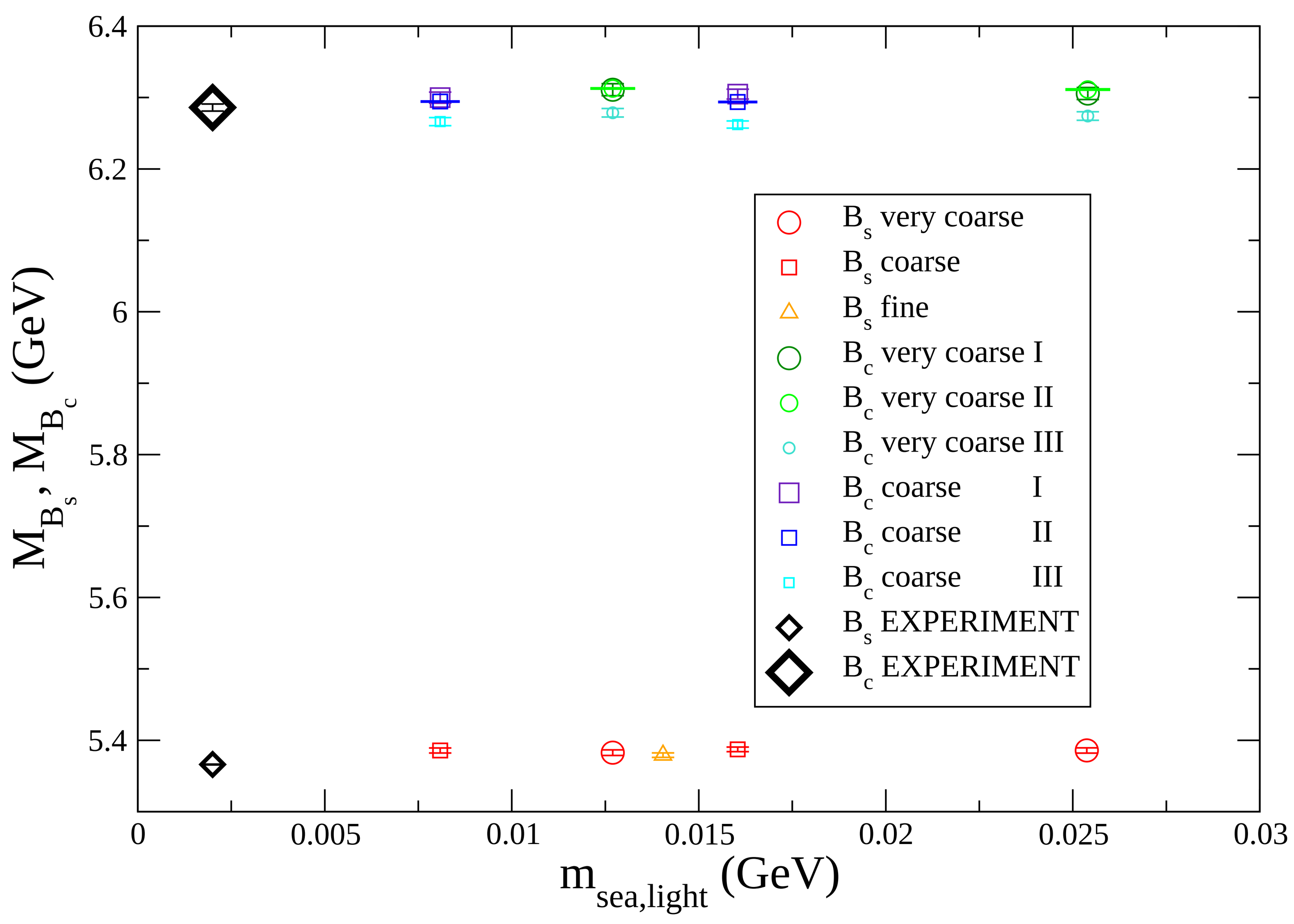

Figure 2: Lattice calculations and experiment for M_{B_s} and M_{B_c} on "very coarse" $16^3 \times 48$ $a \approx 1.3$ GeV and "coarse" $20^3 \times 64$ and $24^3 \times 64$ $a \approx 1.6$ GeV configurations. We extracted M_{B_c} separately with expressions with relations I, II, and III. All M_{B_c} values come from I. Only statistical uncertainties are shown. The smaller error bars from II are enhanced in width for visibility.

Conclusions

While preliminary, these results show the potential for precise calculations of M_{B_s} and M_{B_c} using HISQ valence quarks and NRQCD b quarks on a $2+1$ flavor ASQTAD sea. The lattice results show little sensitivity to sea quark mass and only a small dependence on the discretization scale. The ~ 20MeV discrepancy in M_{B_s} and the ~ 30 MeV discrepancy between III and the other methods for getting M_{B_c} (also noted in [4]) indicate systematic error that will hopefully be understood with further work. Relativistic corrections and electromagnetic effects are two possible sources of systematic error. Future efforts will improve statistics and include finer lattices.

	Ensemble	N_{cfg}	M_{B_s}	M_{B_c} (I)	M_{B_c} (II)	M_{B_c} (III)
$16^3 \times 48$	$\beta = 6.572$ $am = 0.0097, 0.0484$	628	5.3826(39)	6.3109(84)	6.3127(9)	6.2787(59)
$16^3 \times 48$	$\beta = 6.586$ $am = 0.0194, 0.0484$	628	5.3857(37)	6.3056(84)	6.3112(9)	6.2741(60)
$20^3 \times 64$	$\beta = 6.76$ $am = 0.010, 0.050$	592	5.3873(32)	6.3048(69)	6.2938(8)	6.2621(50)
$24^3 \times 64$	$\beta = 6.76$ $am = 0.005/0.050$	191	5.3857(81)	6.2999(78)	6.2945(8)	6.2663(57)
$28^3 \times 96$	$\beta = 7.09$ $am = 0.0062, 0.031$	149	5.3792(31)	—	—	—
	EXPERIMENT		5.3661(6)	6.286(5)	6.286(5)	6.286(5)

Table 1: All errors are statistical only. All masses are in GeV.

References

[1] E. Follana *et al.* [HPQCD Collaboration], Phys. Rev. D **75**, 054502 (2007).

[2] A. Gray, I. Allison, C. T. H. Davies, E. Dalgic, G. P. Lepage, J. Shigemitsu and M. Wingate, Phys. Rev. D **72**, 094507 (2005).

[3] see I, Kendall, parallel talk, this conference.

[4] I. F. Allison, C. T. H. Davies, A. Gray, A. S. Kronfeld, P. B. Mackenzie and J. N. Simone [HPQCD Collaboration and Fermilab Lattice Collaboration and UKQCD Colla], Phys. Rev. Lett. **94**, 172001 (2005).