Lattice Calculations of Hadronic Fluctuations with Staggered Fermions

Chuan Miao for the RBC-Bielefeld Collaboration
Brookhaven National Laboratory

Abstract

Starting from the Taylor expansion coefficients [1] of the pressure in terms of the quark chemical potentials, we study the baryon number, strangeness and electric charge fluctuations and their correlations. The results indicate a rapid change of the nature of the carriers of these quantum numbers when the temperature is increased.

Taylor Expansion of the Pressure

For a large homogeneous system, the pressure of QCD with u, d and s quarks can be expressed as

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_u, \mu_d, \mu_s) \,,$$

where the partition function Z is a function of the volume V, temperature T and chemical potentials of u, d, and s quarks. Direct lattice simulations are not feasible due to the sign problem. Alternatively, we perform a Taylor expansion of the pressure in terms of the chemical potentials

$$\frac{p}{T^4} = \sum_{i,j,k} c_{ijk}(T) \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k,$$

and calculate the coefficients at $\mu_{u,d,s} = 0$. The leading order term gives the pressure at zero chemical potentials, which can be calculated using integration method. The coefficients of the non-leading order terms are derivatives of the pressure with respect to chemical potentials, and therefore related to the quark number fluctuations (correlations) by the fluctuation-dissipation theorem. For example,

$$c_2^x \equiv \frac{1}{2VT^3} \frac{\partial^2 \ln Z}{\partial(\hat{\mu}_x)^2}\bigg|_{\mu_{u,d,s}=0} = \frac{1}{2VT^3} \left\langle (\delta N_x)^2 \right\rangle,$$

$$c_4^x \equiv \frac{1}{24VT^3} \frac{\partial^4 \ln Z}{\partial(\hat{\mu}_x)^4}\bigg|_{\mu=0} = \frac{1}{24VT^3} \left(\left\langle (\delta N_x)^4 \right\rangle - 3 \left\langle (\delta N_x)^2 \right\rangle^2 \right),$$

$$c_{11}^{xy} \equiv \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial(\hat{\mu}_x)\partial(\hat{\mu}_y)}\bigg|_{\mu=0} = \frac{1}{VT^3} \left(\langle N_x N_y \rangle - \langle N_x \rangle \langle N_y \rangle \right).$$

Notice that all odd order coefficients ($i+j+k$ is odd) vanish due to the CP symmetry.

Random Noise Estimators

Derivatives of the fermion matrix determinant appear in the coefficients formulae, e.g.

$$c_2^x = \frac{N_\tau}{2N_\sigma^3} \left(\frac{1}{4} \left\langle \frac{\partial^2 \ln \det M}{\partial \hat{\mu}_x^2} \right\rangle + \left(\frac{1}{4}\right)^2 \left\langle \left(\frac{\partial \ln \det M}{\partial \hat{\mu}_x} \right)^2 \right\rangle \right),$$

which lead to the appearances M^{-1} inside traces:

$$\frac{\partial \ln \det M}{\partial \mu} = \mathrm{Tr} \left(M^{-1} \frac{\partial M}{\partial \mu} \right).$$

To avoid full matrix inversions, we use the random noise method in estimating such traces.

$$\mathrm{Tr}\left(\mathcal{O}\, M^{-1}\right) \approx \frac{1}{N} \sum_{a=1}^{N} R^{(a)} \mathcal{O} M^{-1} R^{(a)},$$

where $R^{(a)}$ are random vectors. The errors arising from the random estimators should be reduced, by using large amount of random noise vectors, to be compatible with the statistical fluctuations. We find this is crucial especially for the low temperature phase.

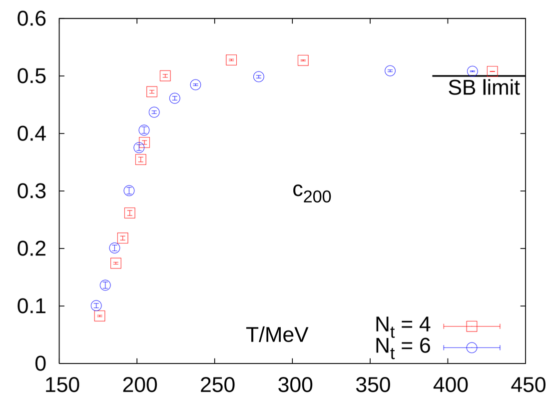
Simulation Details

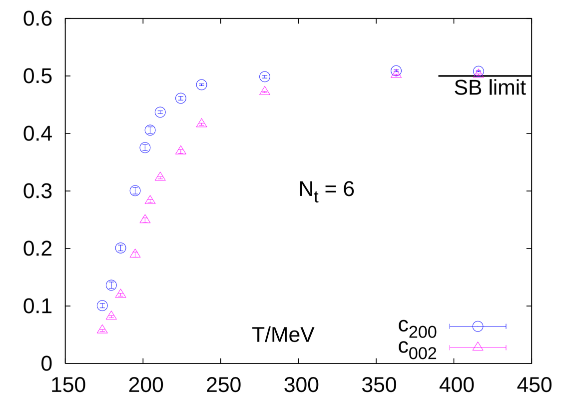
Our calculations are carried out on the p4-improved staggered ensembles from the thermodynamics study on a constant line of physics with pseudo scalar masses approximately 220 MeV for two sets of lattice cut-off $aT = 1/4$ and $1/6$. [2] Table below shows the configurations and RNV numbers used in this study.

$N_\tau = 4$			$N_\tau = 6$		
β	Conf.	RNV #	β	Conf.	RNV #
3.24	1013	480	3.41	800	400
3.28	1550	480	3.42	890	400
3.29	1550	480	3.43	850	400
3.30	875	384	3.445	920	400
3.315	475	384	3.455	700	350
3.32	475	384	3.46	600	200
3.335	264	384	3.47	570	150
3.351	365	384	3.49	560	150
3.41	199	192	3.51	670	100
3.46	302	96	3.57	540	50
3.61	618	48	3.69	350	50
			3.76	345	50

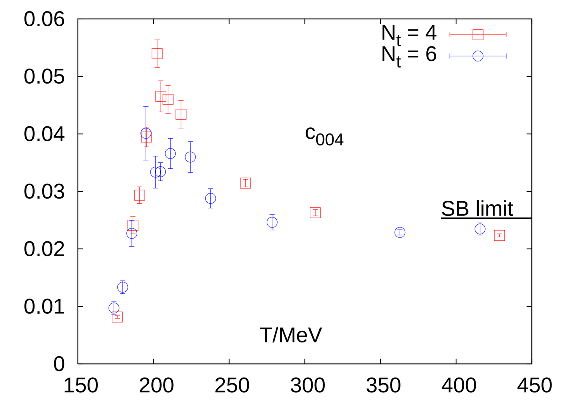
Measurements of the coefficients

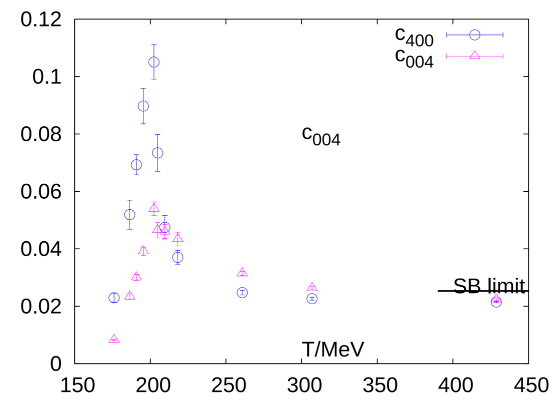
Second order diagonal coefficients, or the quark number fluctuations, increase rapidly through the phase transition region. On the left (below), we compare u(d) quark coefficients on $N_\tau = 4$ and 6 lattices. The lattice cut-off effects are evidently under control. On the right (below), we compare the u(d) quark and s quark coefficients.



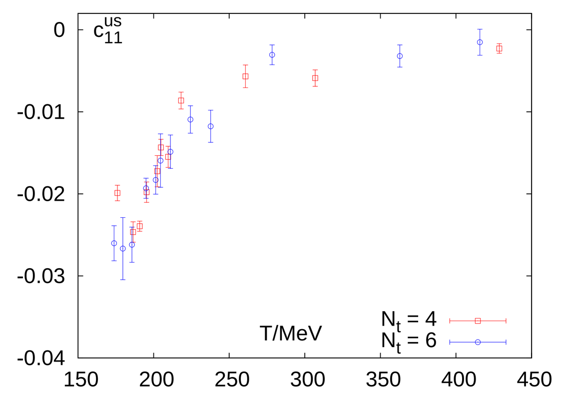


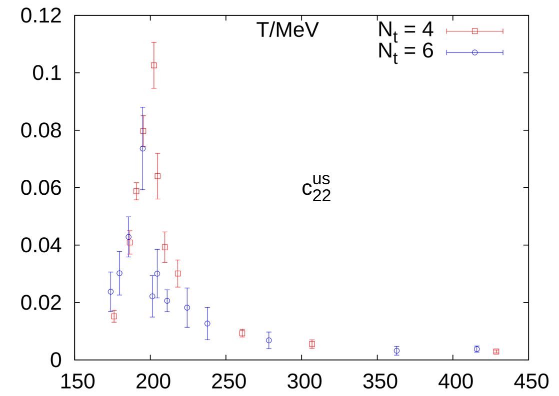
Fourth order diagonal coefficients, also known as quartic quark number fluctuations, show strong peaks, reflecting the deconfinement transition nature of QGP. The strange quark coefficients on $N_\tau = 4$ and 6 lattices are compared on left (next column top). In the right hand figure we compare the u and s quark coefficients; the peaks are more pronounced for lighter quarks.





We also show two typical off-diagonal coefficients below:





Taylor Expansion in $\mu_{B,Q,S}$

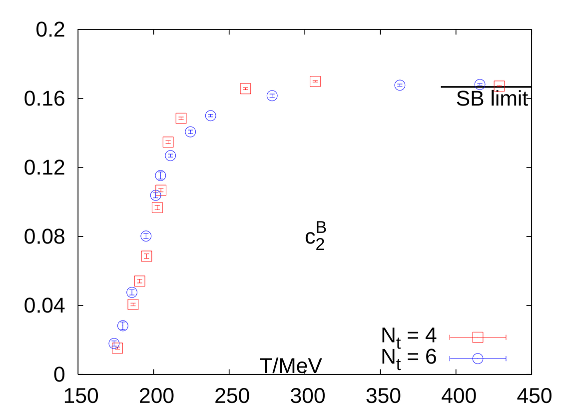
Baryon number B, strangeness S and electric charge Q are experimental observables, therefore we rearrange the Taylor expansion in $\mu_{B,Q,S}$:

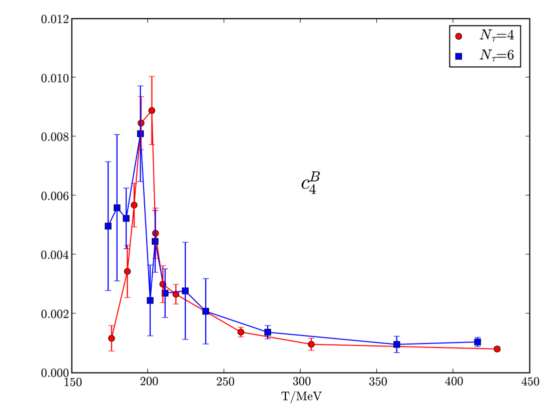
$$\frac{p}{T^4} = \sum_{i,j,k} c_{ijk}^{BSQ}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_S}{T}\right)^j \left(\frac{\mu_Q}{T}\right)^k,$$

with $\mu_B = \mu_u + \mu_d + \mu_s$, $\mu_Q = \mu_u - \mu_d$ and $\mu_S = \mu_s - \mu_d$. c_{ijk}^{BSQ} are straightforwardly related to c_{ijk}^{uds}, e.g.

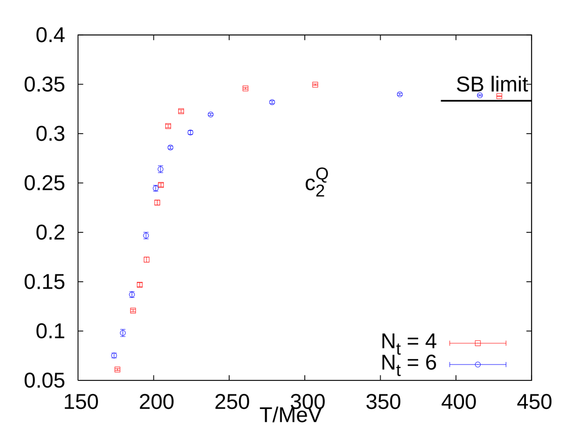
$$c_2^B \equiv c_{200}^{BSQ} = \frac{1}{9}\left(c_{200}^{uds} + c_{020}^{uds} + c_{002}^{uds} + c_{110}^{uds} + c_{011}^{uds} + c_{101}^{uds} \right)$$

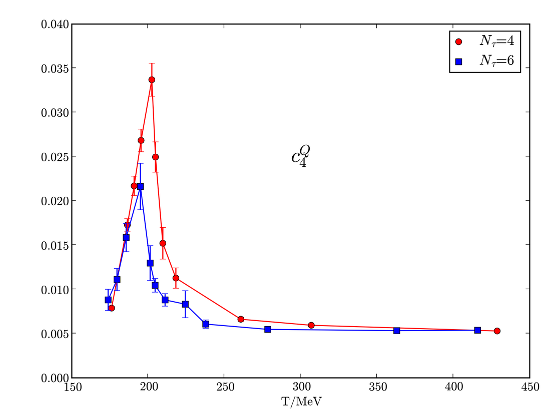
Below, we show c_2^B and $c_4^B \equiv c_{400}^{BSQ}$ from $N_\tau = 4$ and 6 lattices:



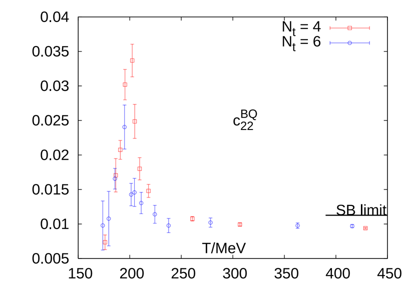


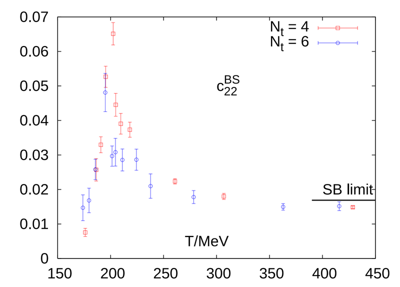
Also, c_2^Q and c_4^Q:





In addition, some off-diagonal coefficients:





Hadronic Fluctuations at Finite Baryon Number Density

With the measured coefficients, one can construct hadronic fluctuations at non-zero baryon number density. We show here fluctuation $\chi_{B,S,Q}$

$$\frac{\chi_B(\mu_B/T)}{T^2} = 2c_2^B + 12c_4^B \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^4\right],$$

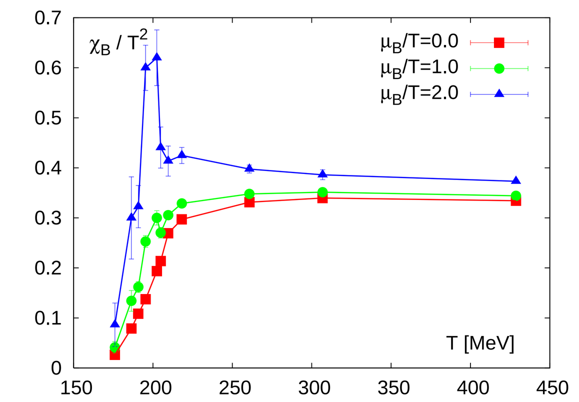
$$\frac{\chi_S(\mu_B/T)}{T^2} = 2c_2^S + 2c_{22}^{BS} \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^4\right],$$

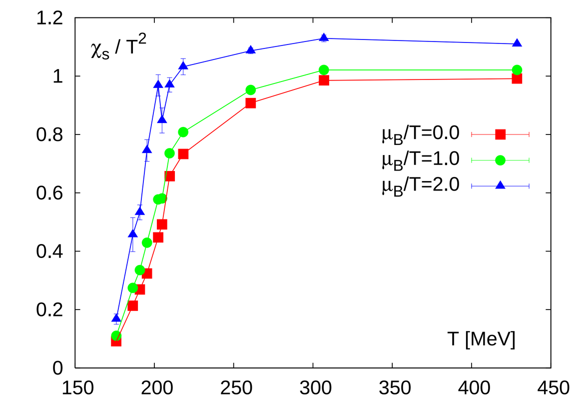
$$\frac{\chi_Q(\mu_B/T)}{T^2} = 2c_2^Q + 2c_{22}^{BQ} \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^4\right].$$

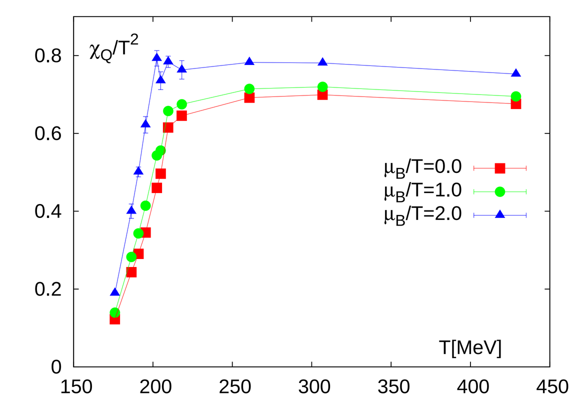
and correlation of baryon number and strangeness

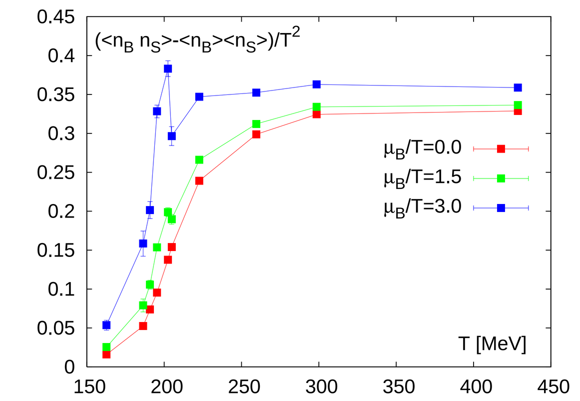
$$\frac{1}{T^2}\left(\langle n_B n_S \rangle - \langle n_B \rangle \langle n_S \rangle \right) = c_{11}^{BS} + 3c_{31}^{BS} \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^4\right]$$

up to fourth order in μ_B/T. Figures below are results from $N_\tau = 4$ lattices.









References

[1] R. V. Gavai and S. Gupta, *Phys. Rev. D* **64** (2001) 074506;
C. R. Allton *et al.*, *Phys. Rev. D* **68** (2003) 014507;
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Acknowledgments

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