Heavy-light hadrons and their excitations

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We study the excitations of hadrons containing a single heavy quark. We present meson and baryon mass splittings and ratios of meson decay constants resulting from quenched and dynamical two-flavor configurations. Light quarks are simulated using the chirally-improved (CI) lattice Dirac operator. The heavy quark is approximated by a static propagator, appropriate for the b quark on our lattices ($1/a \sim 1-2$ GeV). We also include some preliminary calculations of the heavy-quark kinetic corrections to the states.

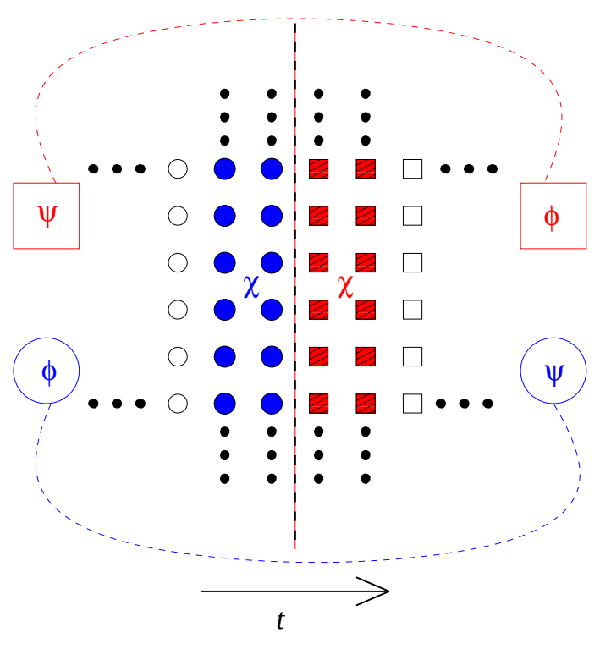
SFB
TR55

Domain-Decomposition Improvement

Propagator (P) between regions 1 and 2 estimated using N random sources (χ):

$$\begin{aligned} P_{12} &= -M_{11}^{-1}M_{12}P_{22} \\ &\approx -M_{11}^{-1}M_{12}\frac{1}{N}\chi_2^n\chi_2^{n\dagger}P \\ &\approx -\frac{1}{N}\left(M_{11}^{-1}M_{12}\chi_2^n\right)\left(\gamma_5P\gamma_5\chi_2^n\right)^\dagger \\ &\approx -\frac{1}{N}\psi_1^n\phi_2^{n\dagger}\ . \end{aligned}$$

Note: No sources needed in region 1 and those in region 2 should reach region 1 with one application of M.
Random sources surrounding one boundary:



[1] T. Burch & C. Hagen, Comput Phys Commun 176, 137
[2] C. Michael & J. Peisa, Phys Rev D58, 034506

Static-Light Correlators

Using different "wavefunctions" for the light-quark source and sink, we construct the following correlators:

$$\begin{aligned} C_{ij}(t) &= \left\langle 0\left|(\bar{Q}\,O_j\,q)_t\ (\bar{q}\,\bar{O}_i\,Q)_0\right|0\right\rangle \\ &= \left\langle \sum_x \mathrm{Tr}\left[\frac{1+\gamma_4}{2}\prod_{i=0}^{t-1}U_4^\dagger(x+i\hat{4})\,O_jP_{x+t\hat{4},x}\bar{O}_i\right]\right\rangle\ , \end{aligned}$$

where x is in one domain and $x+t\hat{4}$ is in the other.

We then solve the generalized eigenvalue problem:

$$C(t)\ \vec{\psi}^{(\alpha)} = \lambda^{(\alpha)}(t,t_0)C(t_0)\ \vec{\psi}^{(\alpha)}\ . \qquad (1)$$

For sufficiently large t, the eigenvalues are

$$\lambda^{(\alpha)}(t,t_0) = c^{(\alpha)}e^{-(t-t_0)M^{(\alpha)}}\left[1+O(e^{-(t-t_0)\Delta^{(\alpha)}})\right]\ ,$$

where $\Delta^{(\alpha)}$ is the energy difference to the closest state.

[3] C. Michael, Nucl Phys B259, 58
[4] M. Lüscher & U. Wolff, Nucl Phys B339, 222
[5] T. Burch, C. Gattringer, L. Y. Glozman, C. Hagen, & C. B. Lang, Phys Rev D73, 017502

Interpolators / Lattices

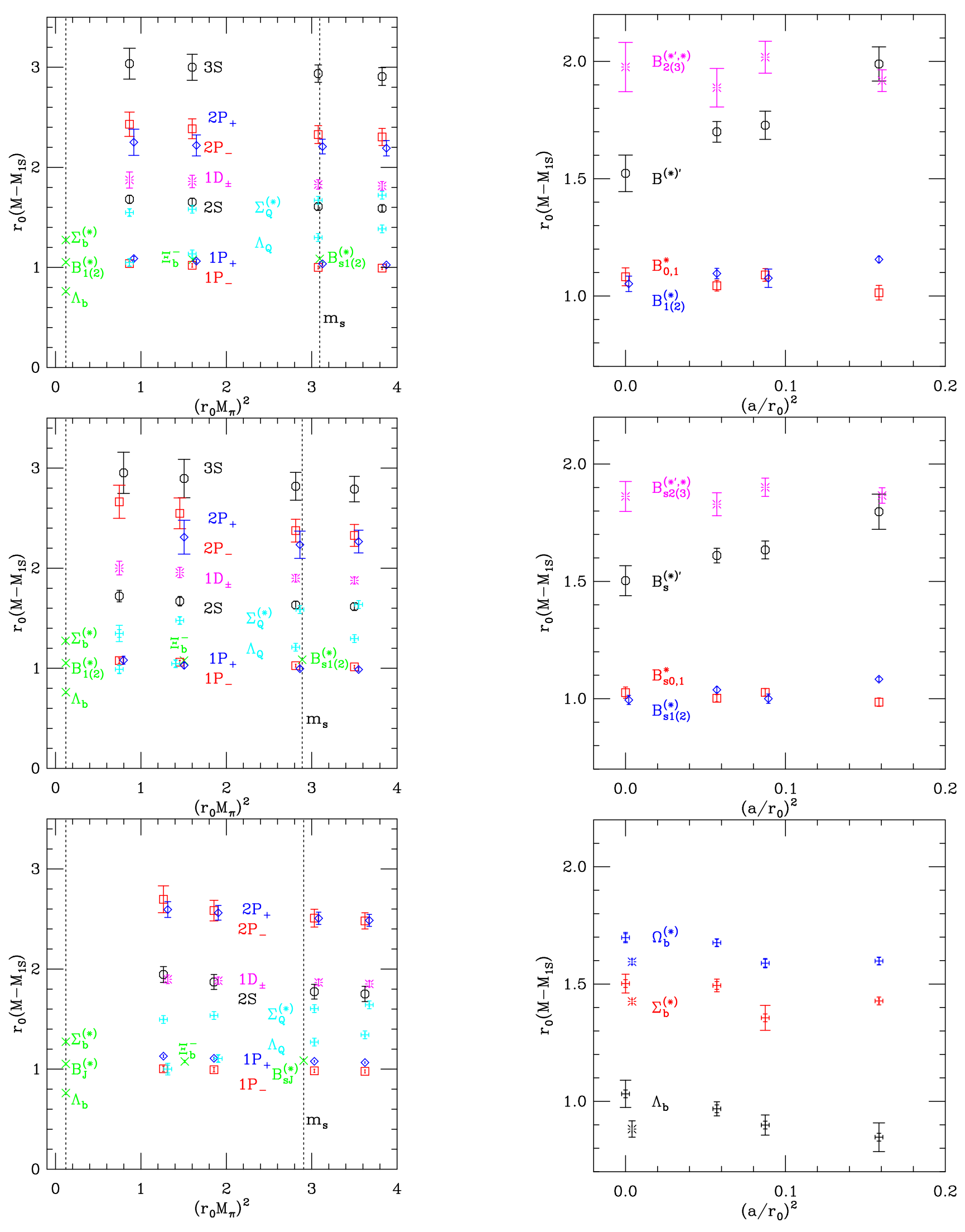
label	j	J^P	states	operator
S	$\frac{1}{2}$	$(0,1)^-$	$B^{(*)}$ $(B_s^{(*)})$	$\bar{Q}_a\,\gamma_{5,i}\,q_a$
P_-	$\frac{1}{2}$	$(0,1)^+$	$B_{0,1}^*$ $(B_{s0,1}^*)$	$\bar{Q}_a\,\gamma_i(D_i)_{ab}\,q_b$
P_+	$\frac{3}{2}$	$(1,2)^+$	$B_{1(2)}^{(*)}$ $(B_{s1(2)}^{(*)})$	$\bar{Q}_a\,(\gamma_1D_1-\gamma_2D_2)_{ab}\,q_b$
$D_\pm$	$(\frac{3}{2},\frac{5}{2})$	$(1,2,3)^-$	$B_{2(3)}^{(*',*)}$ $(B_{s2(3)}^{(*',*)})$	$\bar{Q}_a\,\gamma_5(D_1^2-D_2^2)_{ab}\,q_b$
Λ_Q	0	$\frac{1}{2}^+$	Λ_b (-)	$\epsilon_{abc}\,Q_a\,q_b\,C\gamma_5\,q_c$ $i\epsilon_{abc}\,Q_a\,q_b\,C\gamma_4\gamma_5\,q_c$
$\Sigma_Q^{(*)}$	1	$(\frac{1}{2},\frac{3}{2})^+$	$\Sigma_b^{(*)}$ $(\Omega_b^{(*)})$	$\epsilon_{abc}\,Q_a\,q_b\,C\gamma_i\,q_c$ $i\epsilon_{abc}\,Q_a\,q_b\,C\gamma_4\gamma_i\,q_c$

We present results from three sets of (Lüscher-Weisz gauge) configurations; three quenched (Hyp-blocked links), two $N_f=2$ dynamical (Stout links):

$L^3\times T$	a [fm]	$M_{\pi,sea}$	link smear	N_{conf}	$\left(\begin{smallmatrix} l_1 & l_2 \\ l_3 & l_4\end{smallmatrix},\ \begin{smallmatrix} N_{sm,1} & N_{sm,2} \\ N_{sm,3} & N_{sm,4}\end{smallmatrix},\ \kappa\right)$
$12^3\times 24$	0.20	∞	HYP	200	$\left(\begin{smallmatrix} 0 & 0 \\ 1 & 2\end{smallmatrix},\ \begin{smallmatrix} 0 & 8 \\ 12 & 16\end{smallmatrix},\ 0.20\right)$
$16^3\times 32$	0.15	∞	HYP	100	$\left(\begin{smallmatrix} 0 & 0 \\ 1 & 2\end{smallmatrix},\ \begin{smallmatrix} 0 & 12 \\ 18 & 24\end{smallmatrix},\ 0.20\right)$
$20^3\times 40$	0.12	∞	HYP	100	$\left(\begin{smallmatrix} 0 & 0 \\ 1 & 2\end{smallmatrix},\ \begin{smallmatrix} 0 & 16 \\ 24 & 32\end{smallmatrix},\ 0.20\right)$
$12^3\times 24$	0.115	500 MeV	Stout	74	$\left(\begin{smallmatrix} 0 & 0 \\ 1 & 2\end{smallmatrix},\ \begin{smallmatrix} 0 & 8 \\ 12 & 16\end{smallmatrix},\ 0.20\right)$
$16^3\times 32$	0.16	450 MeV	Stout	100	$\left(\begin{smallmatrix} 0 & 0 \\ 1 & 2\end{smallmatrix},\ \begin{smallmatrix} 0 & 12 \\ 18 & 24\end{smallmatrix},\ 0.20\right)$

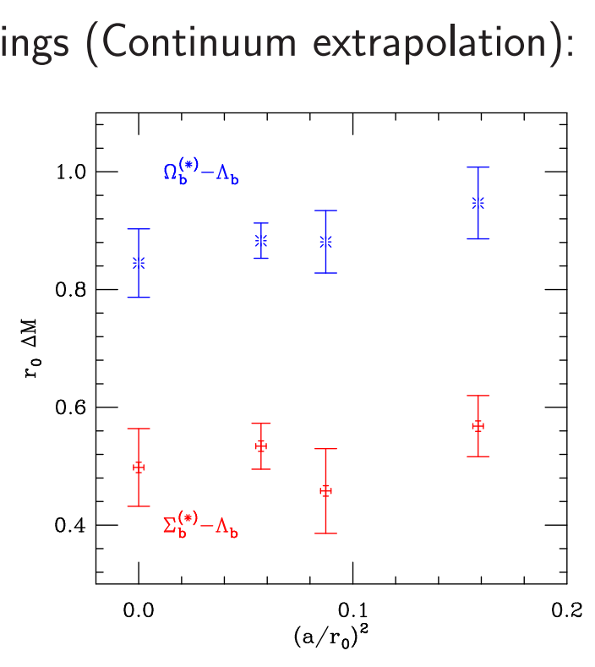
[6] C. B. Lang, P. Majumdar, & W. Ortner, Phys Rev D73, 034507

Quenched results/Continuum extrapolation

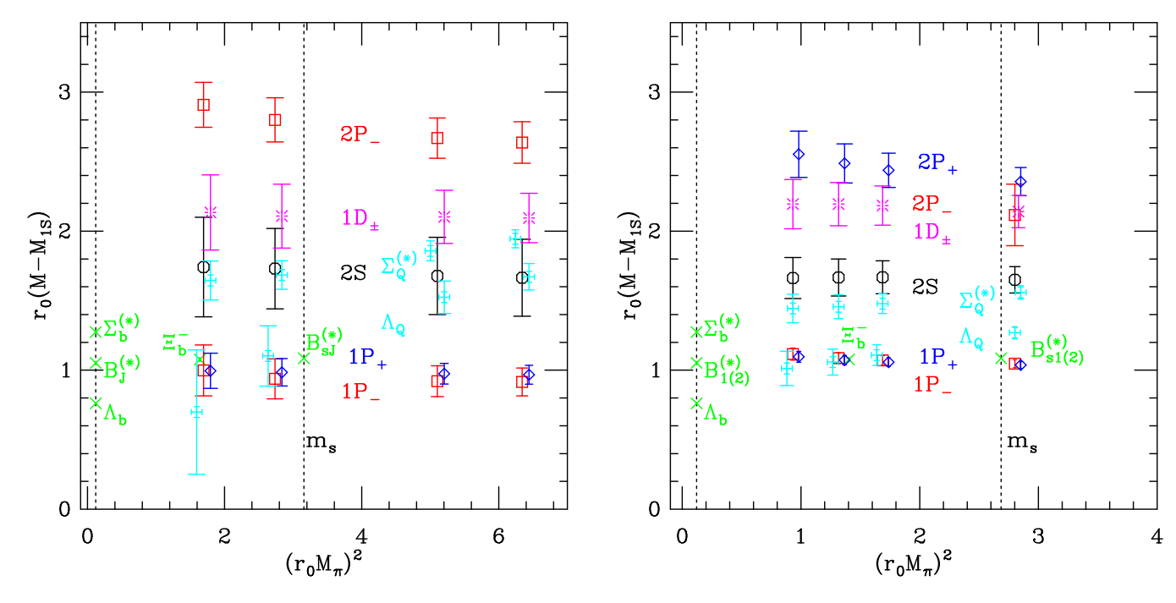


Quenched baryon splittings/Dynamical results

Quenched baryon splittings (Continuum extrapolation):



Dynamical results:



Experimental numbers (green crosses):

[7] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **99**, 172001 (2007).
[8] T. Aaltonen *et al.* [CDF Collaboration], arXiv:0710.4199 [hep-ex].
V. M. Abazov *et al.* [D0 Collaboration],Phys. Rev. Lett. **100**, 082002 (2008).
[9] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **99**, 202001 (2007).
[10] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **99**, 052001 (2007).
T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **99**, 052002 (2007).
[11] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006) and 2007 partial update for 2008.

Mass splittings

Summary of mass differences (in MeV) and comparison to experimental results:

difference	$N_f=0$ $a\to 0$	$N_f=2$ $a=0.156(3)$ fm	experiment
$B_{1(2)}^{(*)}-B^{(*)}$	423(13)(9)	446(17)(9)	423(4)
$B_{s1(2)}^{(*)}-B_s^{(*)}$	400(8)(8)	417(10)(9)	436(1)
$\Lambda_b-B^{(*)}$	415(23)(8)	358(55)(7)	306(2)
$\Sigma_b^{(*)}-B^{(*)}$	604(16)(12)	555(47)(11)	512(4)
$\Xi_b-B^{(*)}$	466(17)(10)	426(37)(9)	476(5)
$\Sigma_b^{(*)}-\Lambda_b$	200(27)(4)	195(72)(4)	206(4)
$\Xi_b-\Lambda_b$	95(17)(2)	111(37)(2)	170(5)

Summary of mass differences (in MeV) where no experimental results are known:

difference	$N_f=0$ $a\to 0$	$N_f=2$ $a=0.156(3)$ fm
$B^{(*)'}-B^{(*)}$	612(31)(13)	674(66)(14)
$B_s^{(*)'}-B_s^{(*)}$	604(26)(12)	664(39)(13)
$B_{0,1}^*-B^{(*)}$	435(15)(9)	454(19)(9)
$B_{s0,1}^*-B_s^{(*)}$	412(10)(8)	421(12)(9)
$\Omega_b^{(*)}-B_s^{(*)}$	683(9)(14)	624(21)(13)
$\Omega_b^{(*)}-\Lambda_b$	340(23)(7)	342(55)(7)
$\Xi_b^{(',*)}-\Lambda_b$	272(23)(6)	269(55)(5)
$\Xi_b^{(',*)}-\Xi_b$	173(20)(4)	158(50)(3)

The strange quark mass m_s is set via the splitting $M_{1S_s}-M_{1S_{ud}}=76.9$ MeV. The later value is obtained by the $1/M_{H^{(*)}}\to 0$ linear extrapolation of the experimental values $M_{B_s^{(*)}}-M_{B^{(*)}}=86.8$ MeV and $M_{D_s^{(*)}}-M_{D^{(*)}}=103.5$ MeV. The second error estimate on quantities is due to the $\approx 2\%$ error we use for $r_0=0.49(1)$ fm.

Decay constants/Kinetic corrections

Couplings

are obtained by considering

$$\begin{aligned} \sum_{j=1}^{r}C(t)_{ij}\psi_j^{(\alpha)} &= \sum_{n=1}^{\infty}\sum_{j=1}^{r}v_i^{(n)}v_j^{(n)*}\psi_j^{(\alpha)}\,e^{-t\,E^{(n)}} \\ &\overset{(1)}{=}\sum_{n=1}^{\infty}\sum_{j=1}^{r}v_i^{(n)}\left\{A^{(\alpha)}\,\delta^{(n,\alpha)}+\mathcal{O}(e^{-t\Delta^{(\alpha)}})\right\}\,e^{-t\,E^{(n)}} \\ &\overset{t\gg 1}{\to}v_i^{(\alpha)}A^{(\alpha)}\,e^{-t\,E^{(\alpha)}}\ . \end{aligned} \qquad (2)$$

Analogously with two eigenvectors on obtains

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\psi_i^{(\alpha)*}C(t)_{ij}\psi_j^{(\alpha)} \approx A^{(\alpha)*}A^{(\alpha)}\,e^{-t\,E^{(\alpha)}}. \qquad (3)$$

We may then construct and fit the following ratio to determine the couplings:

$$R(t)_i^{(\alpha)} = \frac{\left|\text{Eq. (2)}\right|^2}{\text{Eq. (3)}} \approx v_i^{(\alpha)}v_i^{(\alpha)*}\,e^{-tE^{(\alpha)}}\ . \qquad (4)$$

Ratios of different couplings to the same mass eigenstate are even easier:

$$\frac{v_i^{(\alpha)}}{v_k^{(\alpha)}} \approx \frac{\sum_j C(t)_{ij}\psi_j^{(\alpha)}}{\sum_l C(t)_{kl}\psi_l^{(\alpha)}}\ . \qquad (5)$$

These ratios are related to the decay constants:

$$f_P^{(\alpha)} = \sqrt{\frac{2}{M^{(\alpha)}}}\left(v_i^{(\alpha)}+\mathcal{O}(k^{(\alpha)2})/m_Q\right)\ .$$

The ratio of meson decay constants, for example $f_{B_s'}/f_{B_s}$, may be extracted from the $m_q=m_s$ point of

$$\frac{f_P^{(2)}}{f_P^{(1)}} = \frac{v_i^{(2)}}{v_i^{(1)}}\sqrt{\frac{M_{B_s^{(*)}}}{M_{B_s^{(*)}}+(E^{(2)}-E^{(1)})}}\ .$$

Kinetic corrections

to the mass and matrix elements are given by

$$\begin{aligned} \delta M^{(n)} &= \frac{1}{2m_Q}\varepsilon^{(n,n)} \\ \delta v_i^{(n)} &= \frac{1}{2m_Q}\sum_{n'\neq n}\frac{\varepsilon^{(n',n)}}{E^{(n)}-E^{(n')}}\,v_i^{(n')}, \end{aligned}$$

with

$$\varepsilon^{(n,n')} = \langle\,H_Q^{(n)}\,|\,\bar{Q}\vec{D}^2Q\,|\,H_Q^{(n')}\,\rangle,$$

For the calculation we consider the 3-point function

$$T(t,t')_{ij} = \langle\,0\,|\,\bar{q}O_iQ(t)\ \bar{Q}\vec{D}^2Q(t')\ \bar{Q}O_j^\dagger q(0)\,|\,0\,\rangle$$

and two 2-point functions

$$\begin{aligned} C(t-t')_{ij} &= \langle\,0\,|\,\bar{q}O_iQ(t)\ \bar{Q}O_j^\dagger q(t')\,|\,0\,\rangle \\ C(t')_{ij} &= \langle\,0\,|\,\bar{q}O_iQ(t')\ \bar{Q}O_j^\dagger q(0)\,|\,0\,\rangle. \end{aligned}$$

The variational method yields

$$\begin{aligned} \mathbf{C}(t-t')\,\vec{\psi}^{(\alpha)} &= \lambda(t-t',t_0'-t')^{(\alpha)}\,\mathbf{C}(t_0'-t')\,\vec{\psi}^{(\alpha)} \\ \mathbf{C}(t')\,\vec{\phi}^{(\beta)} &= \lambda(t',t_0)^{(\beta)}\,\mathbf{C}(t_0)\,\vec{\phi}^{(\beta)}. \end{aligned}$$

with eigenvectors $\vec{\psi}^{(\alpha)}$ and $\vec{\phi}^{(\beta)}$ which are applied to the 3-point function

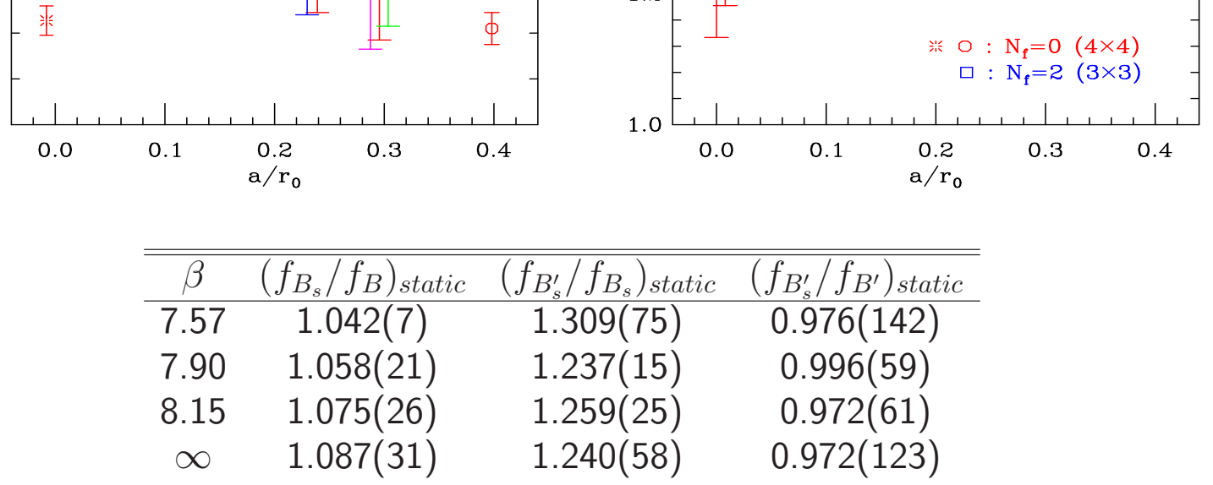
$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r}\psi_i^{(\alpha)*}T(t,t')_{ij}\phi_j^{(\beta)} \\ &\approx A^{(\alpha)*}\varepsilon^{(\alpha,\beta)}A^{(\beta)}\,e^{-(t-t')E^{(\alpha)}}e^{-t'E^{(\beta)}} \end{aligned}$$

The corrections can then be extracted from the ratio

$$\begin{aligned} R_{kl}^{(\alpha,\beta)} &= \frac{\sum_i\sum_j\psi_i^{(\alpha)*}T(t,t')_{ij}\phi_j^{(\beta)}}{\sum_a\psi_a^{(\alpha)*}C(t-t')_{ak}\cdot\sum_b C(t')_{lb}\phi_b^{(\beta)}} \\ &\approx \frac{\varepsilon^{(\alpha,\beta)}}{v_k^{(\alpha)*}v_l^{(\beta)}} \end{aligned} \qquad (6)$$

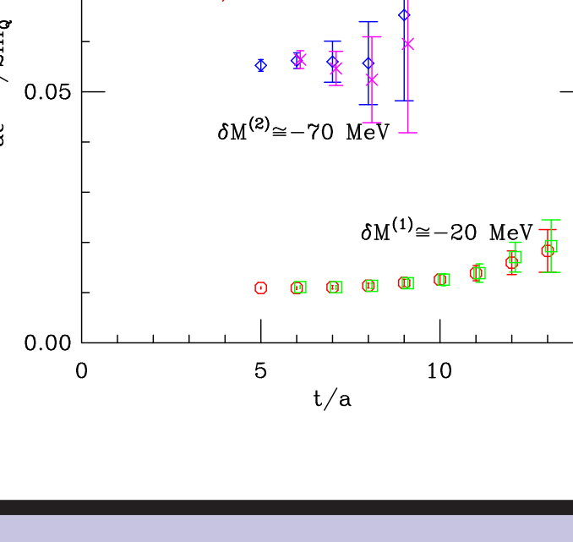
Results for decay constants/kinetic corrections

Ratios of decay constants:



β	$(f_{B_s}/f_B)_{static}$	$(f_{B_s'}/f_{B_s})_{static}$	$(f_{B_s'}/f_{B'})_{static}$
7.57	1.042(7)	1.309(75)	0.976(142)
7.90	1.058(21)	1.237(15)	0.996(59)
8.15	1.075(26)	1.259(25)	0.972(61)
∞	1.087(31)	1.240(58)	0.972(123)
4.65	1.108(29)	1.356(142)	1.089(259)
5.20	1.089(41)	1.453(168)	1.026(128)

From the $\epsilon^{(1,1)}$ and $\epsilon^{(2,2)}$,
preliminary $\mathcal{O}(1/m_b)$ corrections to $2S-1S$ masses:



Conclusions / Outlook

Conclusion:

- Successfully isolated excited static-light mesons via variational method on a large number of lattices.
- Several excited states found: $2S$, $3S$, $1P$, $2P$, $1D$.
- Good agreement with experiment for static-light mesons.
- Unexpected increase of $1P_+-1S$ splittings towards chiral limit (due to static approximation?)
- Extraction of a number of baryon ground states.
- Large discrepancies for Λ_b most likely due to the static approximation ⟶ Have to include kinetic corrections.
- Successful extraction of ratios of decay constants for ground and excited states.
- Include kinetic corrections as current insertion (preliminary results presented here)

Outlook:

- Splittings between Glueballino and R-Meson
- Heavy-light 4-quark operator (⟶ Bag parameter)
- 3-point functions in the light sector