Investigation of the η'-η_c-mixing with improved stochastic estimators

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A reliable lattice calculation of charmonium S-wave fine splittings is still a very challenging spectroscopy problem. The situation is complicated by the flavour-singlet nature of charmonia. Possible effects from $c\bar{c}$ annihilation diagrams have been studied previously, with inconclusive results. Here we extend the investigation to also include mixing effects with other pseudoscalar flavour singlet states, in particular with the η' meson, in unprecedented accuracy. We employ improved stochastic all-to-all propagator techniques (including new methods) to calculate the diagrams that appear within the mixing matrix. The runs are initially performed on $N_f = 2$ 16^3x32 configurations with the non-perturbatively improved Clover-Wilson action, both for valence and sea quarks.

Introduction

1S Hyperfine splitting for charmonia consistently underestimated by lattice calculations

Possible reasons:

- missing continuum limit
- too high see quark masses
- disconnected contributions [a]
- glueball mixing
- mixing with lighter mesons, especially with the η'

[a] P. de Forcrand *et al.* JHEP 0408:004, C. McNeile and C. Michael Phys.Rev.D70:034506

Simulation details

- valence & sea quark action: Clover-Wilson
- gluon action: plaquette
- QCDSF configuration in use:

β	κ	volume	m_π[GeV]	a[fm]	L[fm]
5.20	0.13420	$16^3 \times 32$	1.007(2)	0.1145	1.8

A. Ali Khan *et al.*, Phys.Lett.B564:235-240

- charm quark mass parameter set by tuning $\frac{1}{4}m_{\eta_c} + \frac{3}{4}m_{J/\Psi}$
- runs performed on 16 node partitions on the local QCDOC using the Chroma software library (see arXiv:hep-lat/0409003; Nucl. Phys B1 40 p832, 2005; http://www.ph.ed.ac.uk/ paboyle/bagel/Bagel.html, 2005)

All-to-All Propagators

- create N random noise vectors

$$\eta^i_{\alpha,a,x} = \frac{1}{\sqrt{2}}(v + iw) \qquad v, w \in \{\pm 1\}, i = 1, \ldots, N$$

- define the *random contraction*

$$\frac{1}{N}\sum_i \eta^i_{\alpha,a,x}\eta^{i*}_{\beta,b,y} = \delta_{x,y}\delta_{a,b}\delta_{\alpha,\beta} + O(\frac{1}{\sqrt{N}})$$

- by inverting on these sources we obtain N solution vectors

$$s^i = M^{-1}\eta^i, \quad i = 1, \ldots, N.$$

- naive estimate for A2AP

$$\sum_i s^i\eta^{i\dagger} = \sum_i M^{-1}\eta^i\eta^{i\dagger} = M^{-1}\left(1 + O(\frac{1}{\sqrt{N}})\right)$$

- improvements: SSD, HPA, RNS, TSM

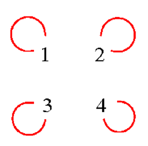
Staggered Spin Dilution (SSD)

- for heavy quarks coupling between upper and lower spin components is weak
- exploit fact that typically $L \bmod 4 = 0$

Scheme / **Nearest neighbor coupling**

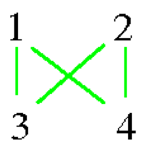
Standard

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
...							



SSD

1	3	2	4	1	3	2	4
3	2	4	1	3	2	4	1
2	4	1	3	2	4	1	3
...							

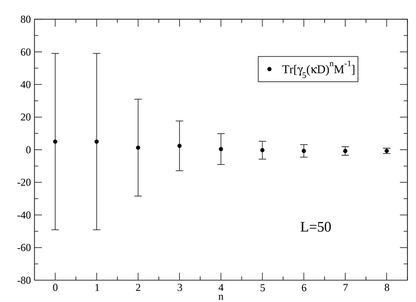


Hopping Parameter Acceleration (HPA)

Consider HPE of the Greens Function of a Wilson-like Dirac Operator $M = \mathbb{1} - \kappa D$:

$$M^{-1} = (\mathbb{1} - \kappa D)^{-1} = \mathbb{1} + \kappa D + \ldots + (\kappa D)^{n-1} + \sum_{i=n}^{\infty}(\kappa D)^i$$
$$= 1 + \kappa D + \ldots + \kappa D^{n-1} + (\kappa D)^n M^{-1}$$

$$\Rightarrow (\kappa D)^n M^{-1} = M^{-1} - (1 + \kappa D + \ldots + \kappa D^{n-1})$$

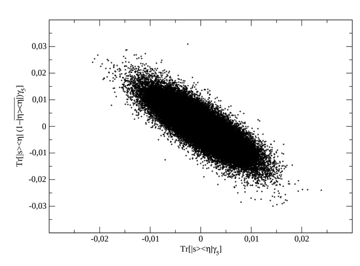


Recursive Noise Subtraction (RNS)

Idea: Calculate noise terms by hand and subtract them

Notation: $\overline{|s\rangle\langle\eta|} \equiv \sum_i |s^i\rangle\langle\eta^i|$

$$M^{-1}\overline{|\eta\rangle\langle\eta|} = \overline{|s\rangle\langle\eta|}$$
$$M^{-1} = \overline{|s\rangle\langle\eta|} + M^{-1}(1 - \overline{|\eta\rangle\langle\eta|})$$
$$M^{-1} \approx \overline{|s\rangle\langle\eta|} + \overline{|s\rangle\langle\eta|}(1 - \overline{|\eta\rangle\langle\eta|})$$
$$= \overline{|s\rangle\langle\eta|}\,(2 - \overline{|\eta\rangle\langle\eta|})$$



Improvements - An Overview

- Consider pseudoscalar disconnected correlator: $\langle\, \mathrm{Tr}(M^{-1}\gamma_5)\, \mathrm{Tr}(M^{-1}\gamma_5)\, \rangle$

- Effective gain (extra computational cost already divided out):

m	no	spin	color	color + spin	SSD	SSD + color	RNS
0	1	1.43	1.80	2.52	1.97	3.63	1.87
2	2.89	6.32	5.06	10.24	7.16	11.80	5.44

Variational Method

- Choose basis of operators O_i, $i = 1, \ldots, N$ and construct a cross-correlator matrix:

$$C_{ij}(t) = \langle O_i(t)O_j^\dagger(0)\rangle = \sum_n v_i^n v_j^{n*} e^{-tE_n}$$

- Solve symmetric generalized eigenvalue problem:

$$C(t_0)^{-1/2}\, C(t)\, C(t_0)^{-1/2}\, \psi^\alpha = \lambda^\alpha(t, t_0)\, \psi^\alpha$$

- Eigenvalues behave like:[a]

$$\lambda^\alpha(t, t_0) \propto e^{-(t-t_0)E_\alpha}[1 + \mathcal{O}(e^{-(t-t_0)\Delta E_\alpha})]$$

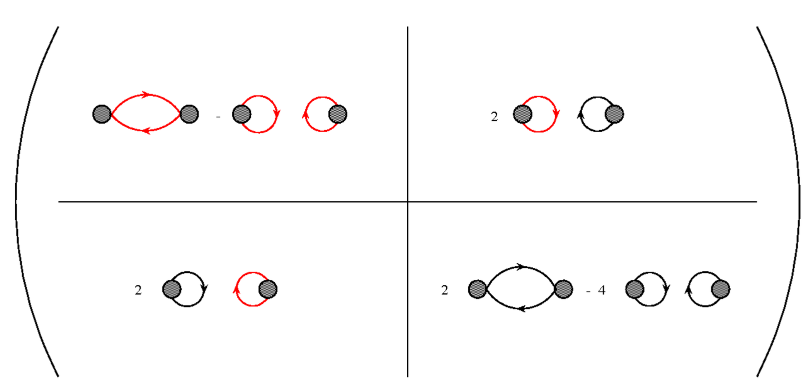
- Eigenvectors behave like:

$$\psi^\alpha(t, t_0) \approx v^n + \sum_{l>N} e^{-(E_l - E_n)t}\, F(n, l, N; t, t_0)$$

[a] C. Michael, NPB259, 58; M. Lüscher and U. Wolff, NPB339, 222

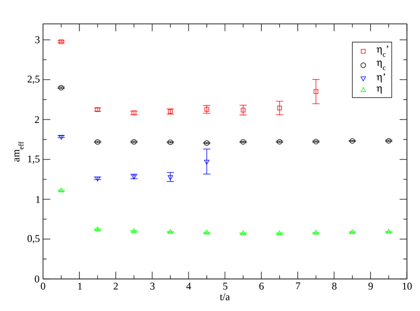
The Mixing Matrix

- simulations performed on $N_f = 2$ lattice without valence strange quarks
- so we are actually in the flavor SU(2) $\rightarrow$ $\eta_c - \eta$ - mixing
- eigenvectors will give information about magnitude of mixing
- "unperturbed" states (subscripts indicate the number of Jacobi smearing steps): $(c\bar{c})_0$, $(c\bar{c})_{10}$, $(c\bar{c})_{80}$, $(u\bar{u})_0$, $(u\bar{u})_5$, $(u\bar{u})_{40}$

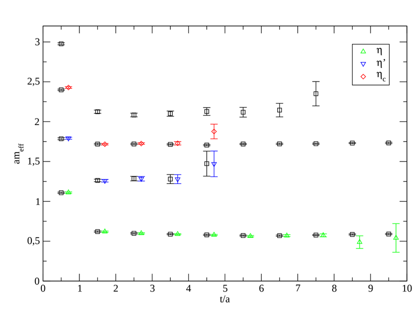


Effective Masses

- Submatrices Spectrum

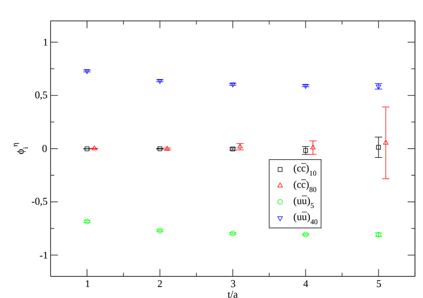


- Full Matrix Spectrum

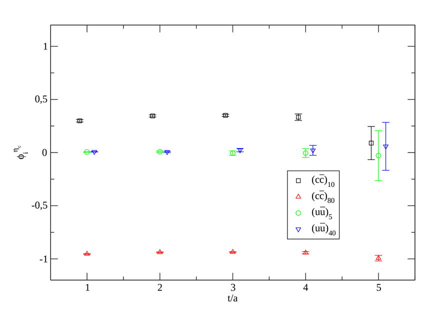


Eigenvectors

- η' Eigenvectors



- η_c Eigenvectors



Summary and Outlook

- sophisticated estimator improvements bring a net noise reduction factor of 10-15 at the charm quark mass
- how does the efficiency depend on the quark mass?
- further estimator improvements, e.g. RNS in subspace ($\approx$ partitioning)
- no significant $\eta_c - \eta'$ - mixing visible
- same analysis on $24^3 \times 48$ lattices with $m_\pi \approx 400$ MeV in progress
- how does the magnitude of mixing depend on the light quark mass?
- similar study for η_c-glueball-mixing desirable