PACS-CS Results for 2+1 Flavor Lattice QCD Simulation on and off the Physical Point

Y.Kuramashi for the PACS-CS Collaboration
(Univ. of Tsukuba)

July 18, 2008

Plan of talk

§1. The PACS-CS project

§2. Simulation details

- Algorithms $\Rightarrow$ Ishikawa's talk(Wed)
- Simulation parameters

§3. SU(3) and SU(2) ChPT analyses

- PS sector (Low energy constants, FSE)
- Baryon

§4. Results of the physical point simulation

- Comparison with the extrapolated results

§5. Summary

§1. The PACS-CS project

Parallel Array Computer System for Computational Sciences

operation started on 1 July 2006 at CCS in U.Tsukuba

collaboration members

physicists:

S.Aoki, N.Ishii, N.Ishizuka, D.Kadoh, K.Kanaya,	Tsukuba
Y.Kuramashi, K.Sasaki, Y.Taniguchi, A.Ukawa,	
N.Ukita, T.Yoshié	
K-I.Ishikawa, M.Okawa	Hiroshima
T.Izubuchi	Kanazawa

computer scientists:

T.Boku, M.Sato, D.Takahashi, O.Tatebe	Tsukuba
T.Sakurai, H.Tadano	

Physics plan

aim: 2+1 flavor QCD simulation at the physical point

strategy: reduce $m_{\rm ud}$ keeping $m_{\rm s}$ around the physical value

by-product: investigate viability of extrapolation method with ChPT

	PACS-CS	CP-PACS/JLQCD
gauge action	Iwasaki	Iwasaki
quark action	clover with $c_{\rm SW}^{\rm NP}$	clover with $c_{\rm SW}^{\rm NP}$
a[fm]	0.07,0.1,0.122	0.07,0.1,0.122
volume	$\gtrsim$ (3fm)^3	$\sim$ (2fm)^3
$m_{\rm ud}^{\rm AWI}$	physical point	64MeV
algorithm for ud	DDHMC with improvements	HMC
algorithm for s	UV-filtered exact PHMC	exact PHMC

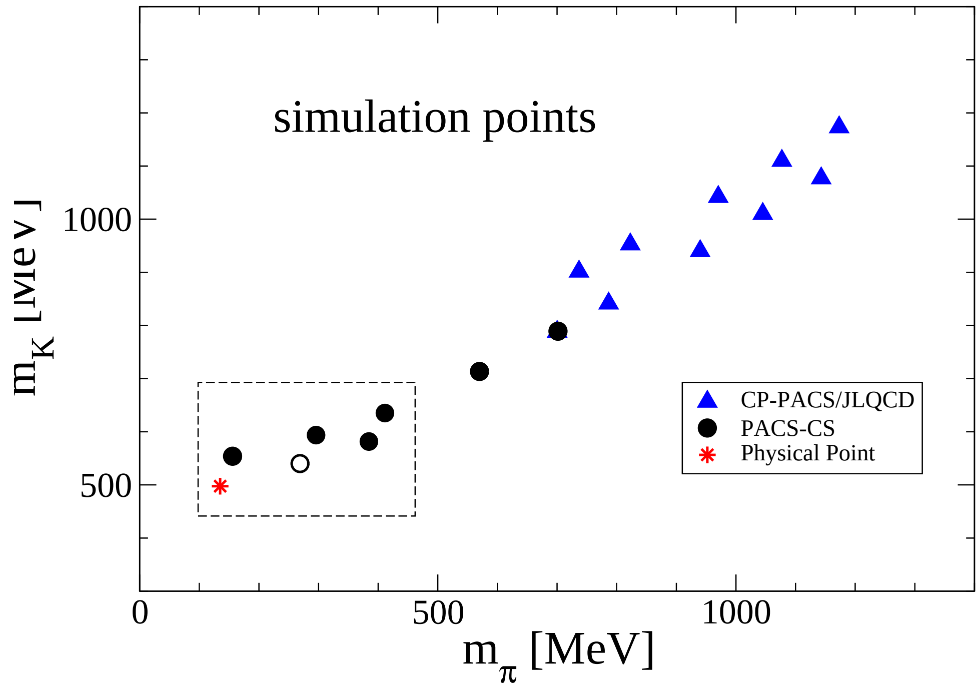
§2. Simulation details

parameters

κ_{ud}	κ_{s}	ud algorithm	$m_\pi L$	τ	MD time
0.13700	0.13640	DDHMC	10.3	0.5	2500
0.13727	0.13640	DDHMC	8.4	0.5	2000
0.13754	0.13640	DDHMC	6.0	0.5	2250
0.13754	0.13660	DDHMC	5.7	0.5	2000
0.13770	0.13640	DDHMC	4.3	0.25	2000
0.13770	0.13660	MP2DDHMC	–	0.25	–
0.13781	0.13640	MPDDHMC	2.3	0.25	990

$m_{\text{PS}} = 156\text{MeV}$ ($m_{\text{ud}}^{\text{AWI}}$=3MeV) is reached

$\kappa_{\text{ud}} \geq 0.13754$ for ChPT fits



unitary points with $m_\pi \lesssim 400$MeV

Further improvements on DDHMC for $m_\pi \lesssim 200$MeV

$\Rightarrow$ Ishikawa's talk(Wed)

- Mass-preconditioned DDHMC(MPDDHMC,MP2DDHMC)
  $\kappa'_{\mathrm{ud}} = \rho\kappa_{\mathrm{ud}}$ for the preconditioner of IR part

$$\det R = \det R' \cdot \det\left(\frac{R}{R'}\right)$$

  tame the fluctuation of $||F_{\mathrm{IR}}||$

- Chronological inverter for IR part
  strict tolerance: $|Dx - b|/|b| < 10^{-14}$ both for force and H

- solver improvement
  incorporate deflation technique

§3. SU(3) and SU(2) ChPT analyses

NLO SU(3) ChPT formula Gasser-Leutwyler, 85

$$
\begin{aligned}
m_\pi^2 &= 2m_{\mathrm{ud}}B_0\left\{1+\mu_\pi-\frac{1}{3}\mu_\eta+2m_{\mathrm{ud}}K_3+K_4\right\}\\
m_K^2 &= (m_{\mathrm{ud}}+m_{\mathrm{s}})B_0\left\{1+\frac{2}{3}\mu_\eta+(m_{\mathrm{ud}}+m_{\mathrm{s}})K_3+K_4\right\}\\
f_\pi &= f_0\left\{1-2\mu_\pi-\mu_K+2m_{\mathrm{ud}}K_6+K_7\right\}\\
f_K &= f_0\left\{1-\frac{3}{4}\mu_\pi-\frac{3}{2}\mu_K-\frac{3}{4}\mu_\eta+(m_{\mathrm{ud}}+m_{\mathrm{s}})K_6+K_7\right\}
\end{aligned}
$$

$$\mu_P=\frac{1}{16\pi^2}\frac{m_P^2}{f_0^2}\ln\left(\frac{m_P^2}{\mu^2}\right)$$

unknown parameters: $B_0, f_0, K_{3,4,6,7}(L_{4,5,6,8})$

should use the formula of WChPT?

NLO WChPT formula in terms of m_q^{AWI} Aoki-Bär-Takeda-Ishikawa

$$
\begin{aligned}
m_\pi^2 &= 2m_{\mathrm{ud}}B_0\left\{1+\mu_\pi-\frac{1}{3}\mu_\eta+2m_{\mathrm{ud}}K_3+K_4-\frac{2H''}{f_0^2}\right\}\\
m_K^2 &= (m_{\mathrm{ud}}+m_s)B_0\left\{1+\frac{2}{3}\mu_\eta+(m_{\mathrm{ud}}+m_s)K_3+K_4-\frac{2H''}{f_0^2}\right\}\\
f_\pi &= f_0\left\{1-2\mu_\pi-\mu_K+2m_{\mathrm{ud}}K_6+K_7-\frac{2H'}{f_0^2}\right\}\\
f_K &= f_0\left\{1-\frac{3}{4}\mu_\pi-\frac{3}{2}\mu_K-\frac{3}{4}\mu_\eta+(m_{\mathrm{ud}}+m_{\mathrm{s}})K_6+K_7-\frac{2H'}{f_0^2}\right\}
\end{aligned}
$$

NLO WChPT formula in terms of m_q^{AWI} Aoki-Bär-Takeda-Ishikawa

$$
\begin{aligned}
m_\pi^2 &= 2m_{\mathrm{ud}}B_0\left\{1+\mu_\pi-\frac{1}{3}\mu_\eta+2m_{\mathrm{ud}}K_3+K_4-\frac{2H''}{f_0^2}\right\}\\
m_K^2 &= (m_{\mathrm{ud}}+m_s)B_0\left\{1+\frac{2}{3}\mu_\eta+(m_{\mathrm{ud}}+m_s)K_3+K_4-\frac{2H''}{f_0^2}\right\}\\
f_\pi &= f_0\left\{1-2\mu_\pi-\mu_K+2m_{\mathrm{ud}}K_6+K_7-\frac{2H'}{f_0^2}\right\}\\
f_K &= f_0\left\{1-\frac{3}{4}\mu_\pi-\frac{3}{2}\mu_K-\frac{3}{4}\mu_\eta+(m_{\mathrm{ud}}+m_{\mathrm{s}})K_6+K_7-\frac{2H'}{f_0^2}\right\}
\end{aligned}
$$

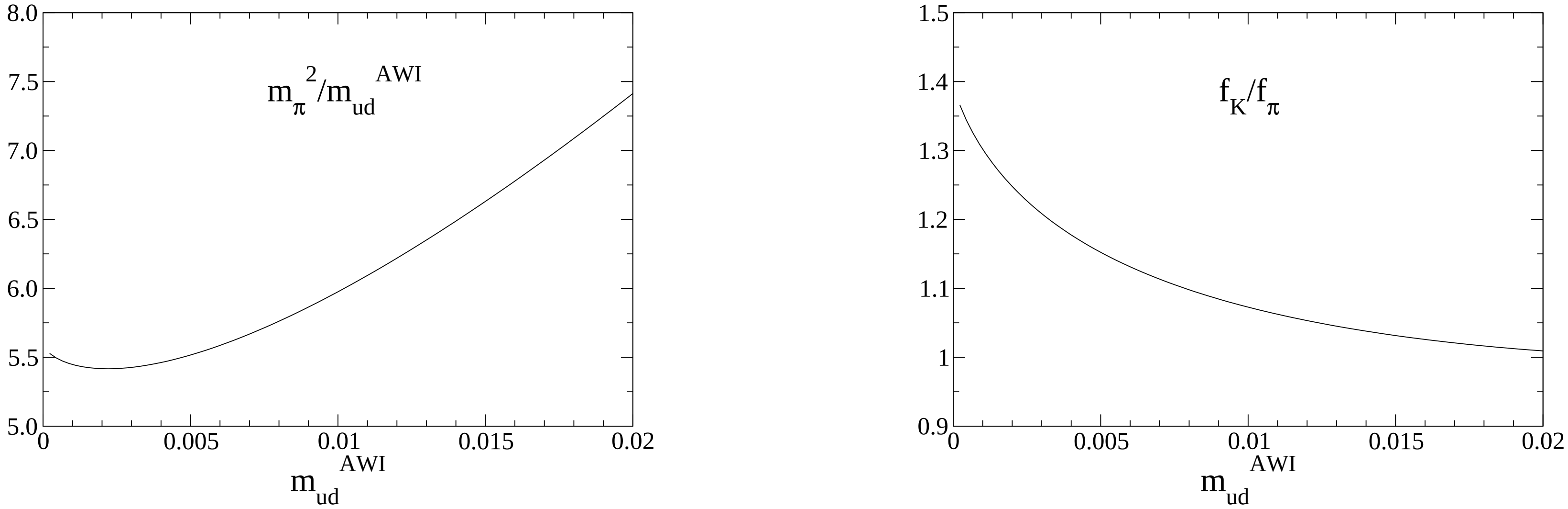
redefinition: $B_0' = B_0\left\{1-\frac{2H''}{f_0^2}\right\},\ \ f_0' = f_0\left\{1-\frac{2H'}{f_0^2}\right\}$

$O(a^2)$ terms can be absorbed in B_0' and f_0'

expected chiral behavior

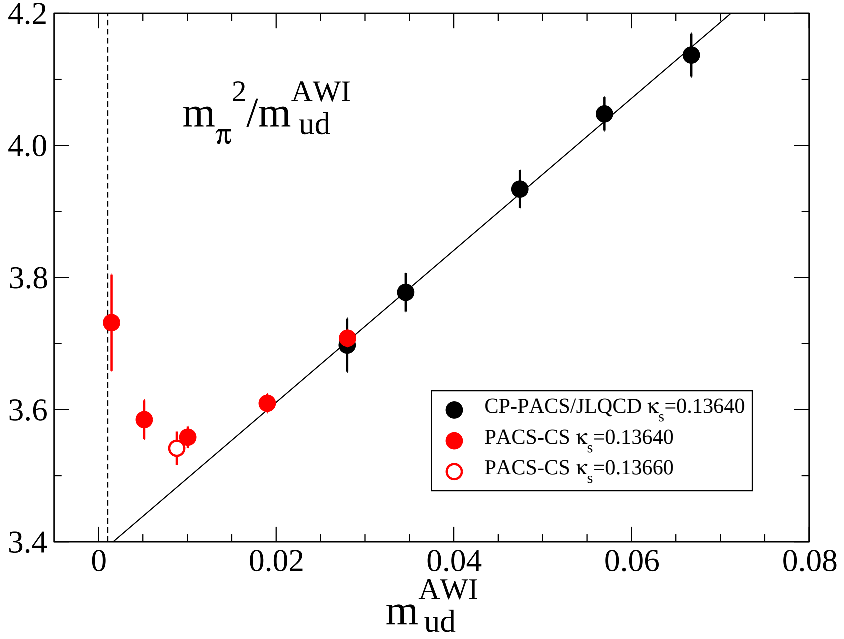
parameters are determined by experimental inputs

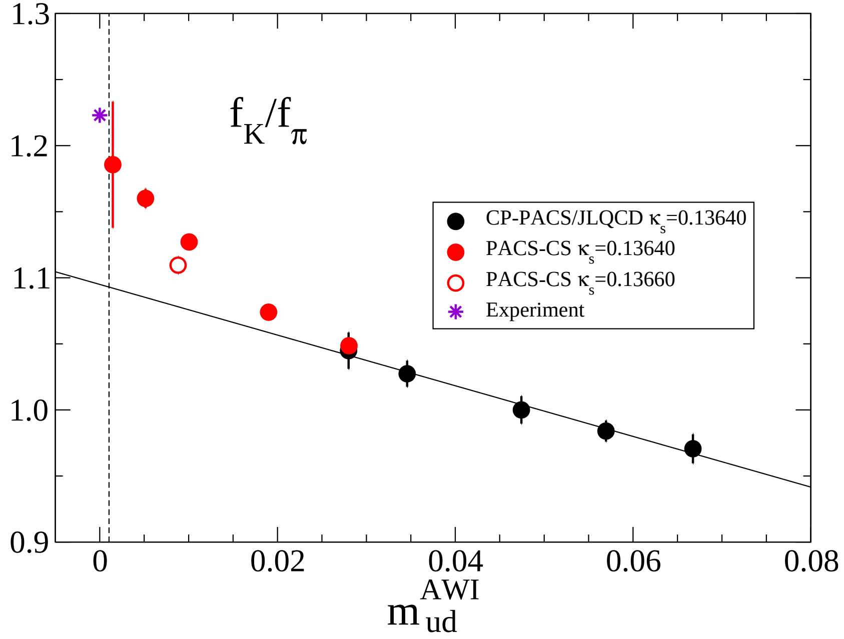
Amorós-Bijnens-Talavera, 01



curvature due to logarithmic function

$\Rightarrow$ good test ground for light quark mass simulations chiral behavior





curvature is clearly observed

$\Rightarrow$ try SU(3) ChPT fit

simultaneous fit for $m_\pi^2/m_{\mathrm{ud}}^{\mathrm{AWI}}$,$m_K^2/(m_{\mathrm{ud}}^{\mathrm{AWI}}+m_{\mathrm{s}}^{\mathrm{AWI}})$,$f_\pi$,$f_K$

	PACS-CS	phenom*	RBC/UKQCD08	MILC07
f_0[GeV]	0.1160(88)	0.115	0.0935(73)	
f_π/f_0	1.159(57)	1.139	1.33(7)	$1.21(5)(^{+13}_{-3})$
L_4	−0.04(10)	0.00(80)	0.139(80)	$0.1(3)(^{+3}_{-1})$
L_5	1.43(7)	1.46(10)	0.872(99)	$1.4(2)(^{+2}_{-1})$
$(2L_6-L_4)$	0.10(2)	0.0(1.0)	-0.001(42)	$0.3(1)(^{+2}_{-3})$
$(2L_8-L_5)$	−0.21(3)	0.54(43)	0.243(45)	0.3(1)(1)
χ^2/dof	4.2(2.7)		0.7	

L_i are defined at $\mu = m_\rho$ in units of 10^{-3}
* taken from Colangelo-Dürr-Haefeli, hep-lat/0503014

consistent with other groups?

conversion to $\bar{l}_3$ and $\bar{l}_4$ in SU(2) ChPT

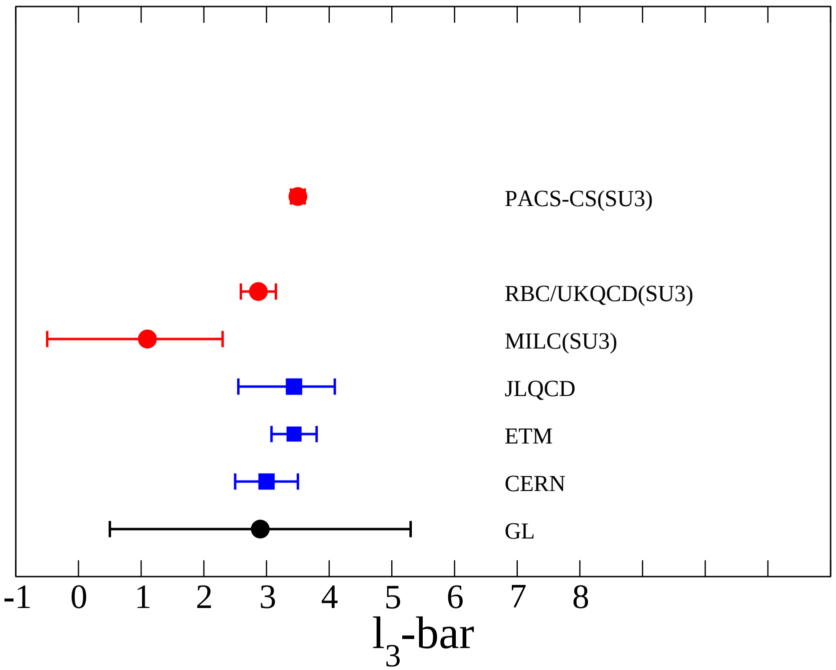
Gasser-Leutwyler, 84

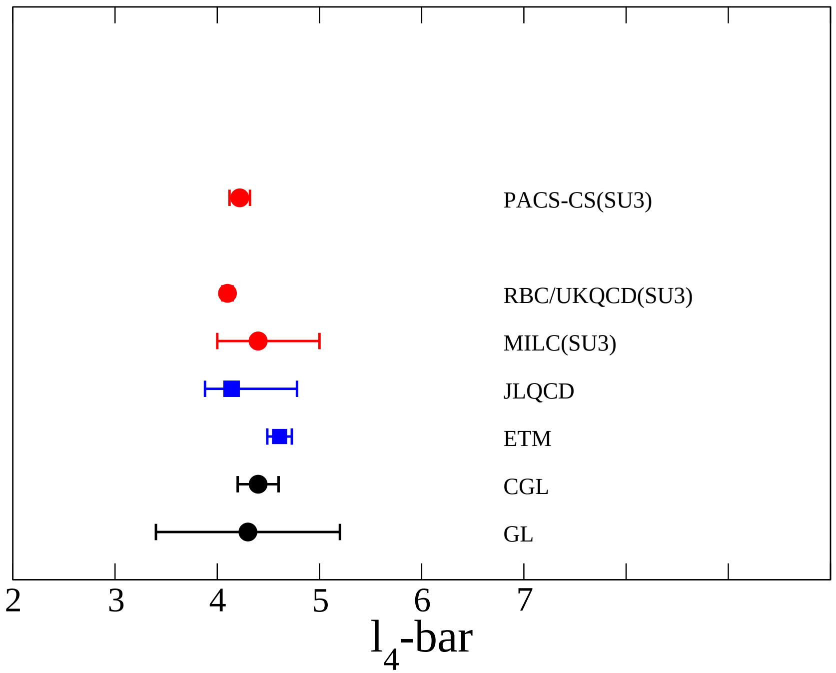
$$m_\pi^2 = 2m_{\mathrm{ud}}B\left\{1 + \frac{2m_{\mathrm{ud}}B}{16\pi^2 f^2}\bar{l}_3\right\}$$
$$f_\pi = f\left\{1 - \frac{2m_{\mathrm{ud}}B}{8\pi^2 f^2}\bar{l}_4\right\}$$

$\{B_0, f_0, L_{4,5,6,8}\}$ are converted to $\{B, f, \bar{l}_{3,4}\}$

Gasser-Leutwyler, 85

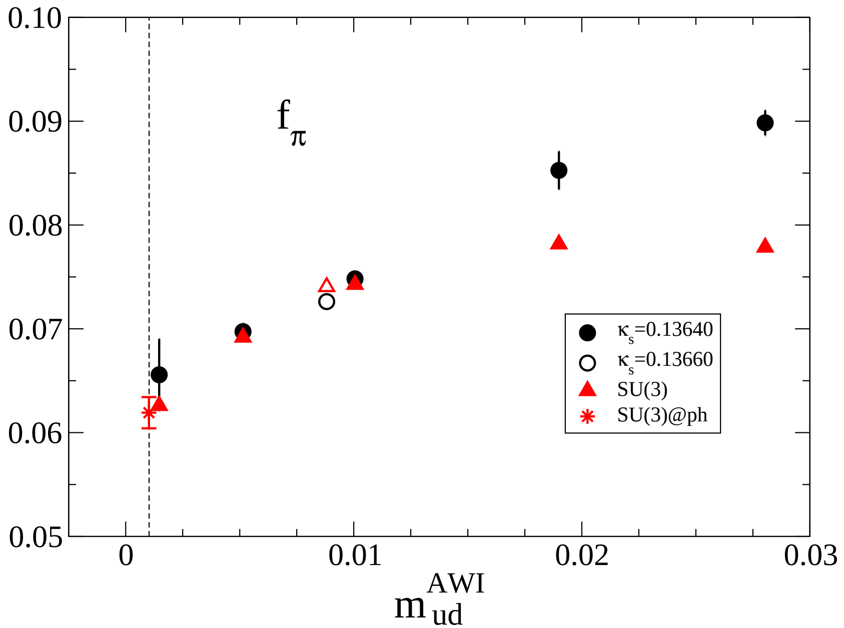
$\bar{l}_3$ and $\bar{l}_4$ in comparison with other groups

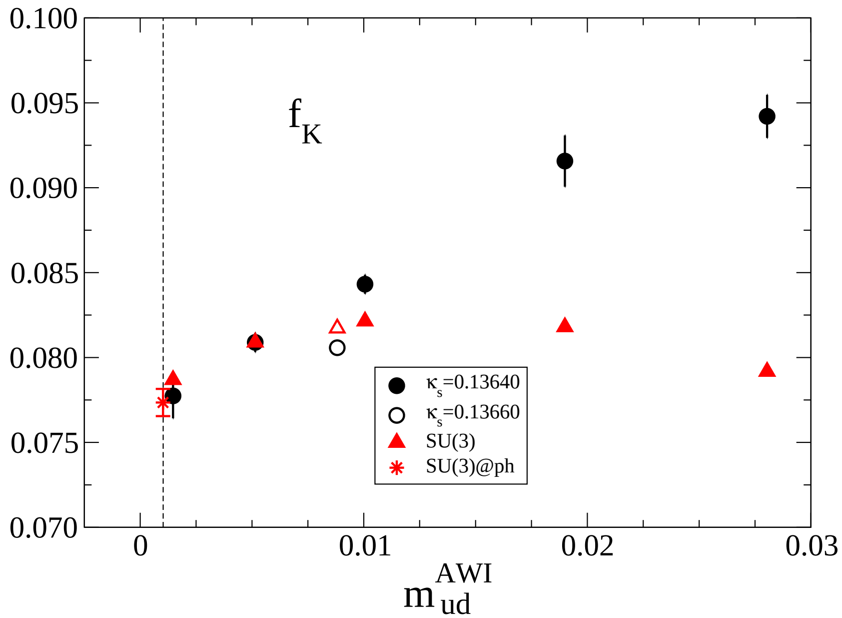




MILC result for $\bar{l}_3$ is exceptionally small

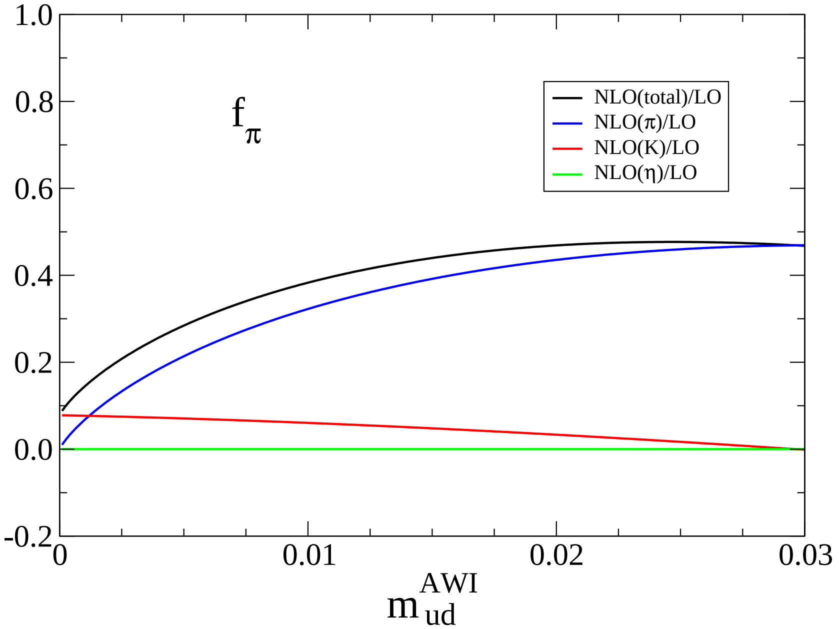
what is responsible for χ^2/dof=4.2(2.7)?

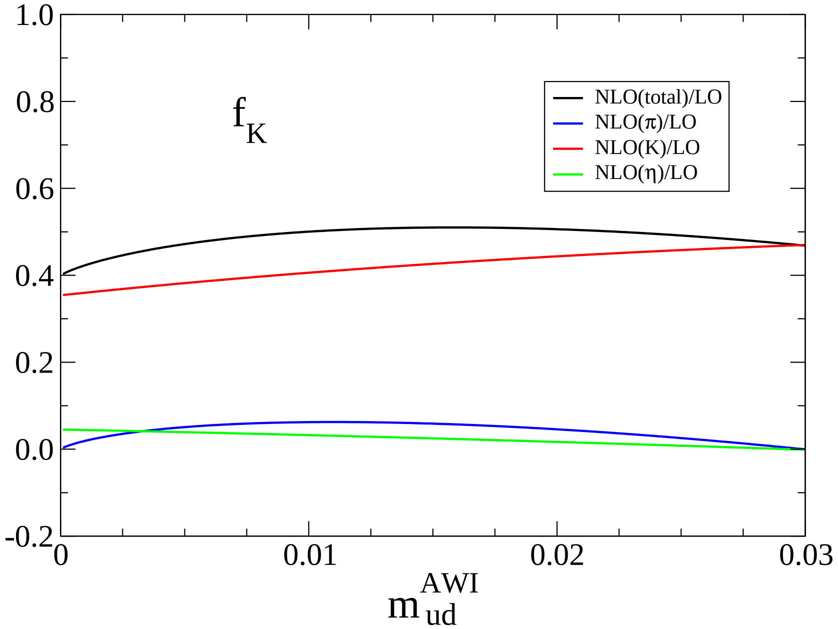




m_s^{AWI} dependence is not well reproduced

what about NLO/LO ratio?





NLO/LO ratio is 40∼50%

⇒ NNLO contributions would be considerable

alternative choice

Roessl, 99

NLO SU(2) ChPT + analytical expansion around the physical m_{s}

$$
\begin{aligned}
m_\pi^2 &= 2m_{\mathsf{ud}}B\left\{1+\frac{2m_{\mathsf{ud}}B}{16\pi^2f^2}\bar{l}_3\right\}\\
f_\pi &= f\left\{1-\frac{2m_{\mathsf{ud}}B}{8\pi^2f^2}\bar{l}_4\right\}\\
m_K^2 &= \alpha+\beta\cdot m_{\mathsf{ud}}+\gamma\cdot m_{\mathsf{s}}\\
f_K &= (\bar{f}_{\mathsf{s}}^{(0)}+m_{\mathsf{s}}\bar{f}_{\mathsf{s}}^{(1)})\left\{1+\beta'\cdot m_{\mathsf{ud}}-\frac{3}{4}\left(\frac{2m_{\mathsf{ud}}B}{16\pi^2f^2}\right)\ln\left(\frac{2m_{\mathsf{ud}}B}{\mu^2}\right)\right\}
\end{aligned}
$$

fit results

simultaneous fit to m_π, f_π, f_K

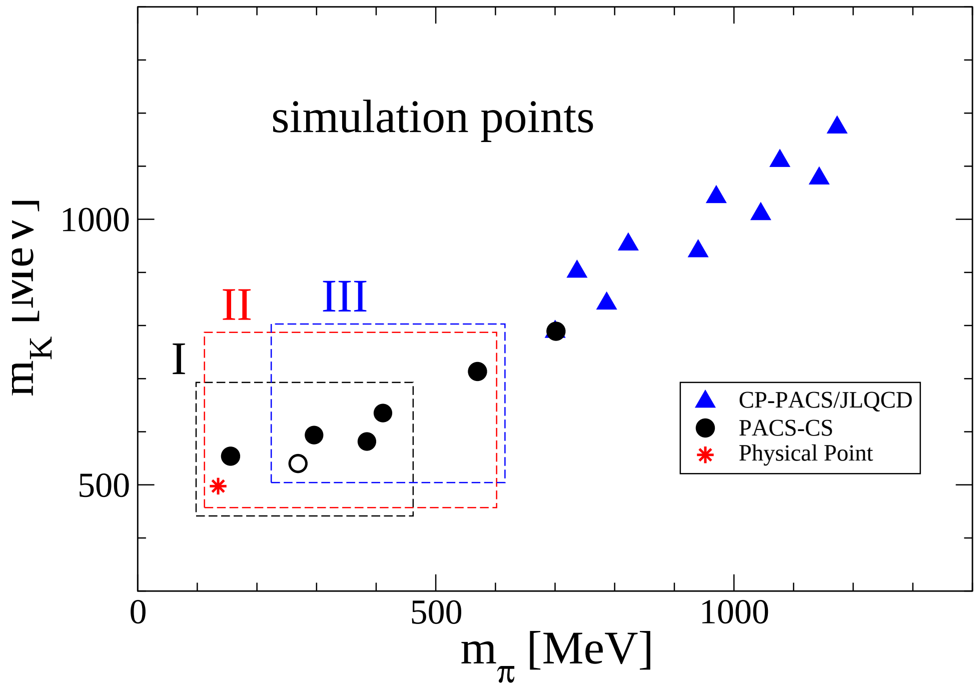
3 choices of fit ranges

	PACS-CS Range I	Range II	Range III	phenom
f[GeV]	0.1248(51)	0.1181(30)	0.1158(28)	0.1219(7)
f_π/f	1.063(8)	1.074(5)	1.078(5)	1.072(7)
$\bar{l}_3$	3.23(21)	3.32(10)	3.31(10)	2.9(2.4)
$\bar{l}_4$	4.10(20)	4.32(9)	4.36(9)	4.4(2)
χ^2/dof	0.33(68)	2.0(1.0)	2.8(1.8)	

results are stable against choice of fit range

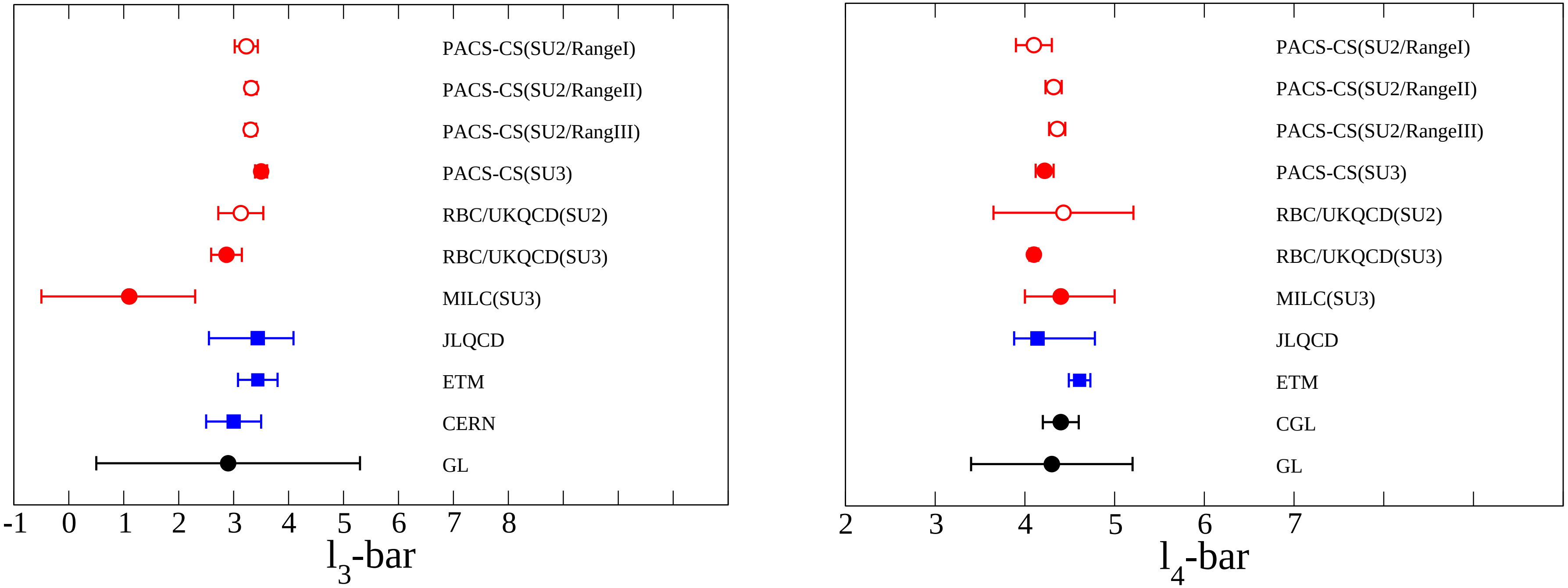
χ^2/dof is increased by inclusion of heavier pion

fit ranges



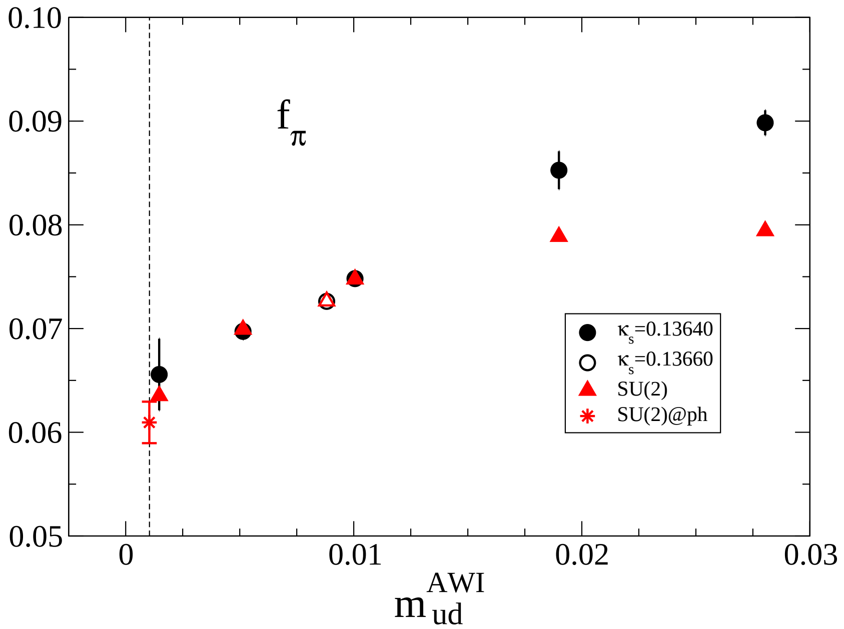
simulation points
I
II
III
CP-PACS/JLQCD
PACS-CS
Physical Point
1000
500
0
500
1000
m_K [MeV]
m_π [MeV]

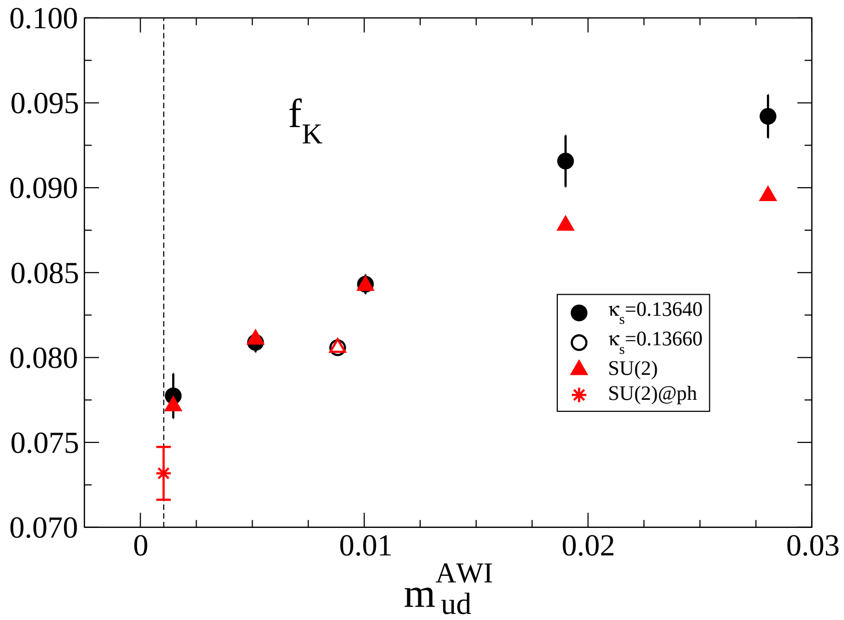
$\bar{l}_3$ and $\bar{l}_4$ in comparison with other groups



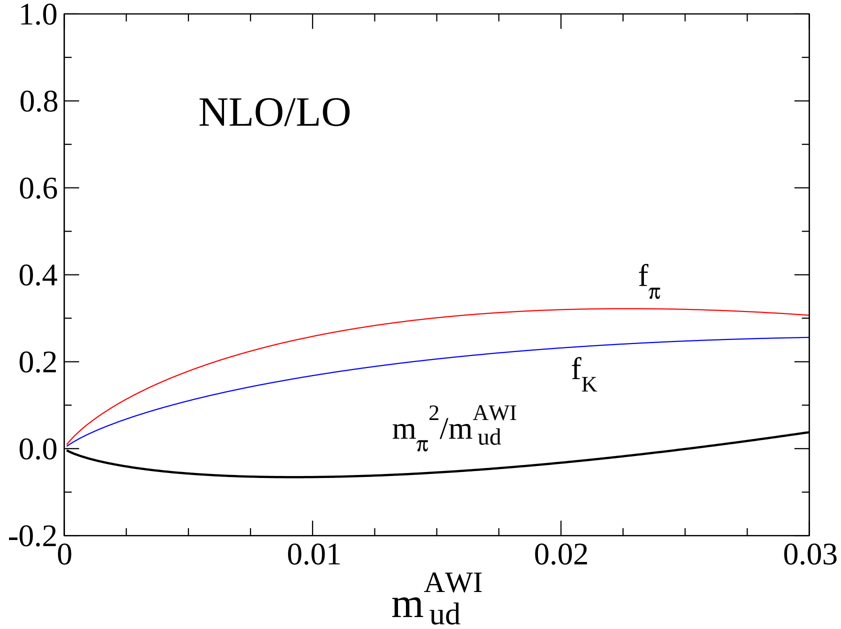
consistent with other groups

how well described by NLO SU(2) ChPT?





m_s^{AWI} dependence is well reproduced convergence



NLO/LO ratio is $\lesssim$20% in the fit range $m_{\mathrm{ud}}^{\mathrm{AWI}} \lesssim 0.01$

NNLO contributions could be 5%

FSE based on NLO SU(2) ChPT

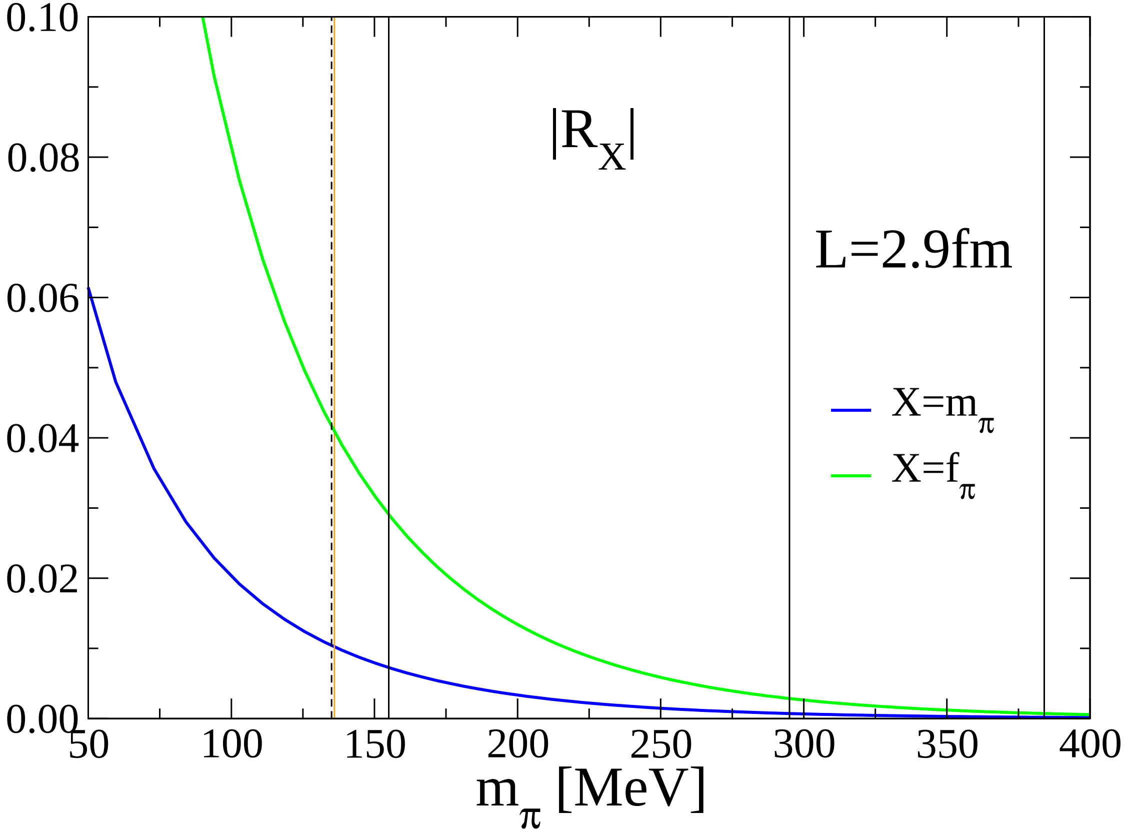
$R_X = (X(L) - X(\infty))/X(\infty)$ for $X = m_\pi, f_\pi$

Colangelo-Dür-Haefeli, 04

$$
\begin{aligned}
R_{m_\pi} &= \frac{1}{4}\xi_\pi \tilde{g}_1(m_\pi L), \\
R_{f_\pi} &= -\xi_\pi \tilde{g}_1(m_\pi L)
\end{aligned}
$$

$$
\xi_\pi \equiv \frac{m_\pi^2}{(4\pi f_\pi)^2}, \quad \tilde{g}_1(x) = \sum_{n=1}^{\infty} \frac{4m(n)}{\sqrt{n}x} K_1(\sqrt{n}x)
$$

valid for $m_\pi L > 2$ expected by the authors



$R_{m_{\rm PS}} > 0$ and $R_{f_{\rm PS}} < 0$

at most a few % even at the physical point

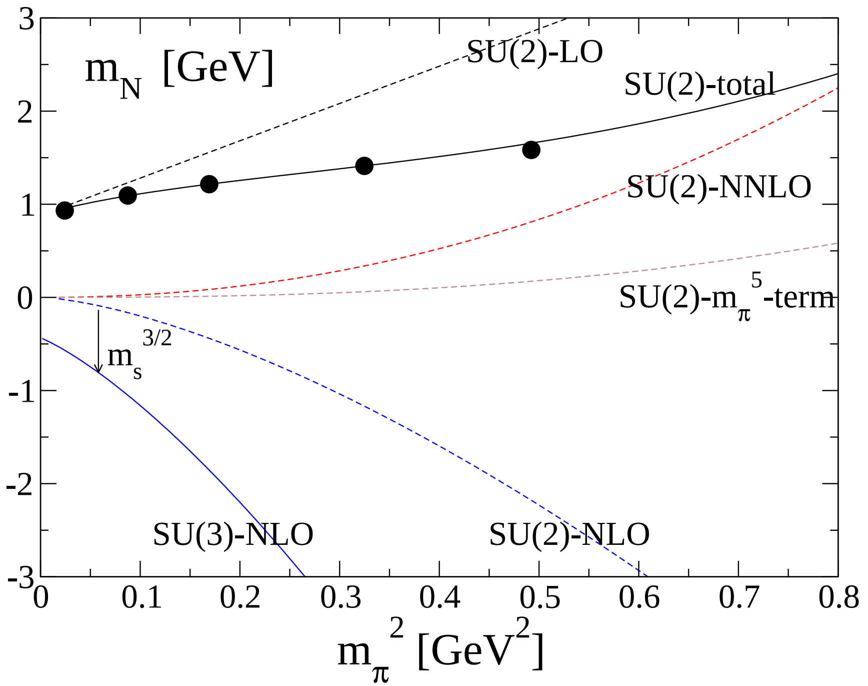
ChPT for nucleon

NNLO SU(2) formula

$$
\begin{aligned}
m_N \;=\; & m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f^2} m_\pi^3 + \left[e_1(\mu) - \frac{3}{32\pi^2 f^2}\left(\frac{g_A^2}{m_0} - \frac{c_2}{2}\right) \right. \\
& \left. - \frac{3}{16\pi^2 f^2}\left(\frac{g_A^2}{m_0} - 8c_1 + c_2 + 4c_3\right) \ln\left(\frac{m_\pi}{\mu}\right)\right] m_\pi^4 \\
& + \frac{3g_A^2}{128\pi f^2 m_0^2} m_\pi^5 + O(m_\pi^6)
\end{aligned}
$$

fit procedures

- g_A, c_2, c_3 given, m_0, c_1 $e_1(\mu)$ to be determined by fit
- apply to $\kappa_s = 0.13640$ series



drastic cancellations between different orders

strange contributions in SU(3) ChPT are significant

Are we satisfied at NLO SU(2) ChPT on the physical m_s?

Are we satisfied at NLO SU(2) ChPT on the physical m_s?

absolutely NO

Are we satisfied at NLO SU(2) ChPT on the physical $m_{\rm s}$?

absolutely NO

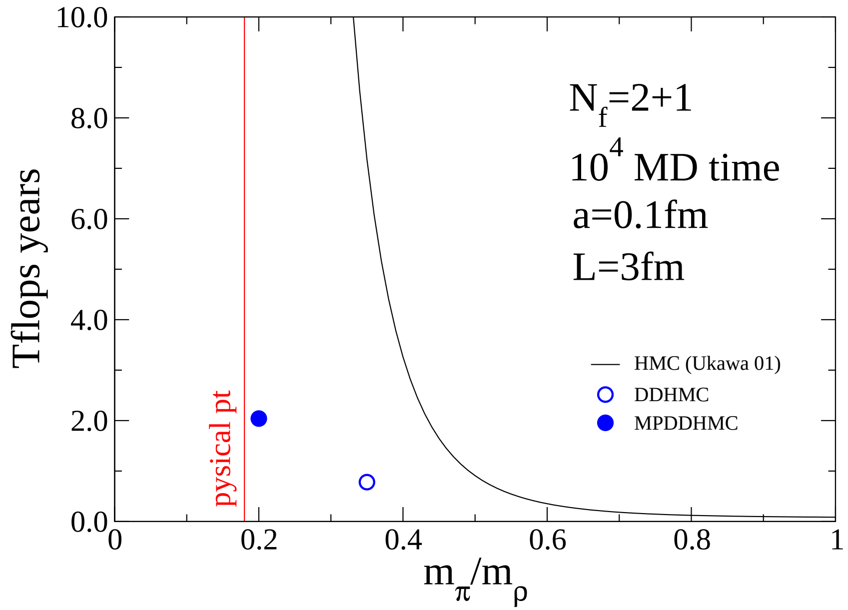
<u>reasons</u>

- inappropriate for precision measurement
  NNLO contributions could be 5%
- discouraging convergence behaviors for baryons

direct simulation on the physical point is inevitable to control the systematic error

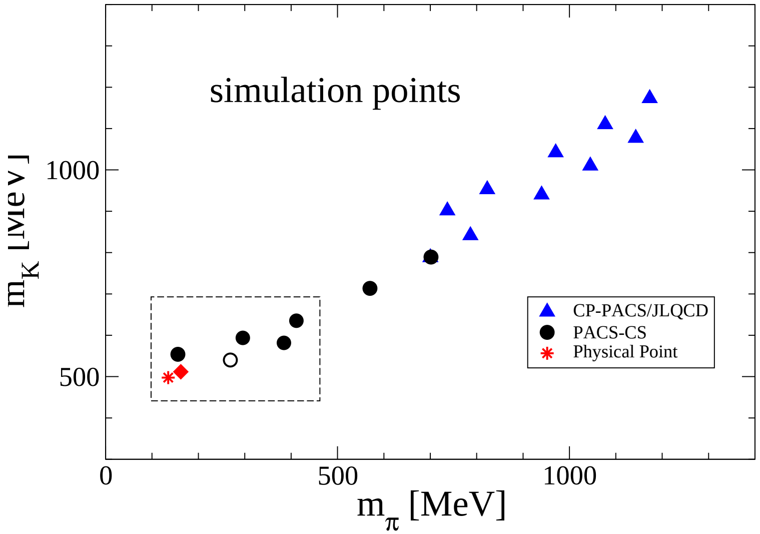
§4. Results of the physical point simulation

practical feasibility



physical point is within our reach

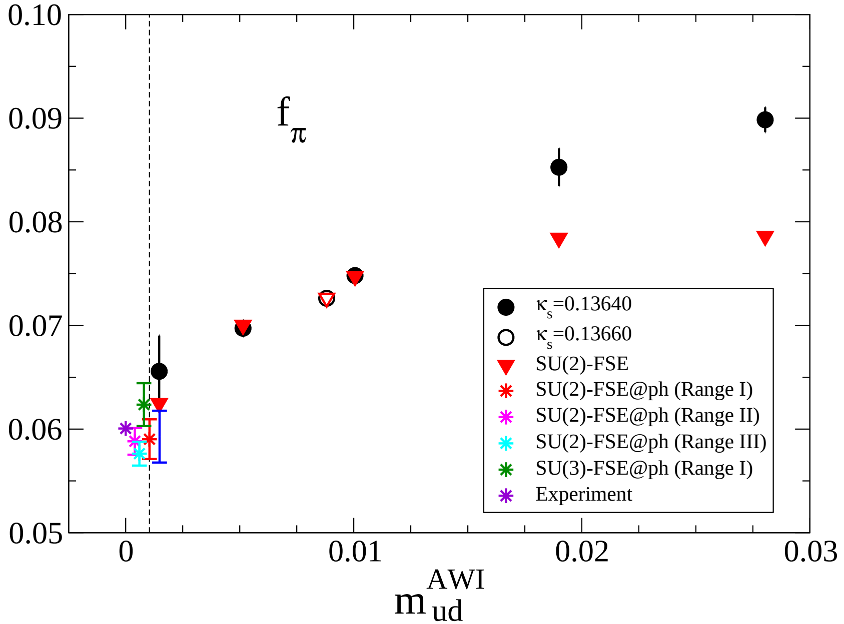
$\kappa_{\rm ud}$	$\kappa_{\rm s}$	ud algorithm	$m_\pi L$	τ	MD time
0.137785	0.13660	MP2DDHMC	2.3	0.25	850

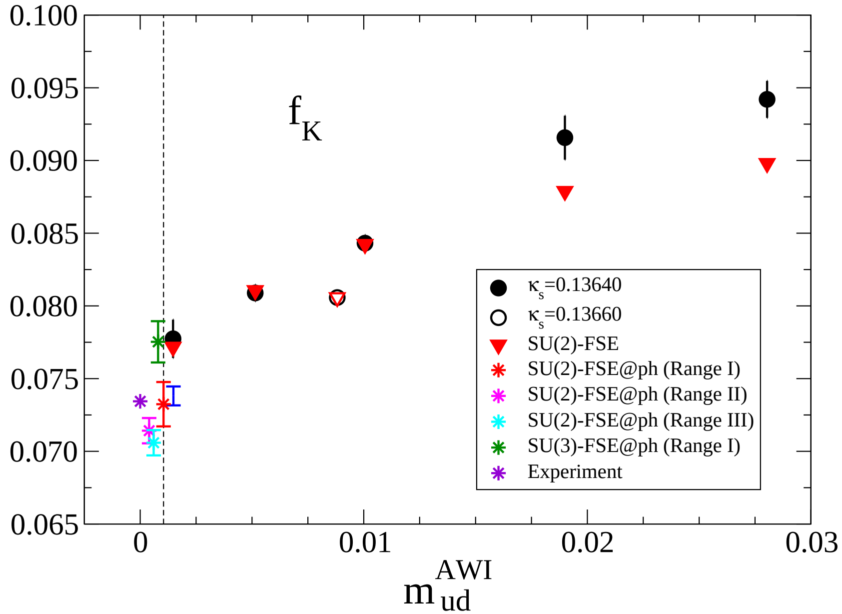


slightly off the physical point at the present statistics

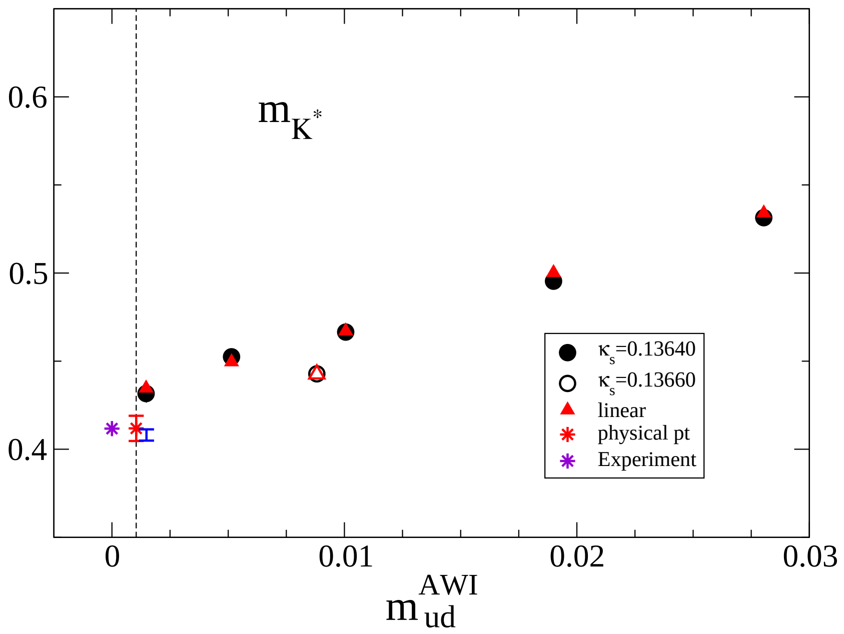
interesting to compare the results with the extrapolated values

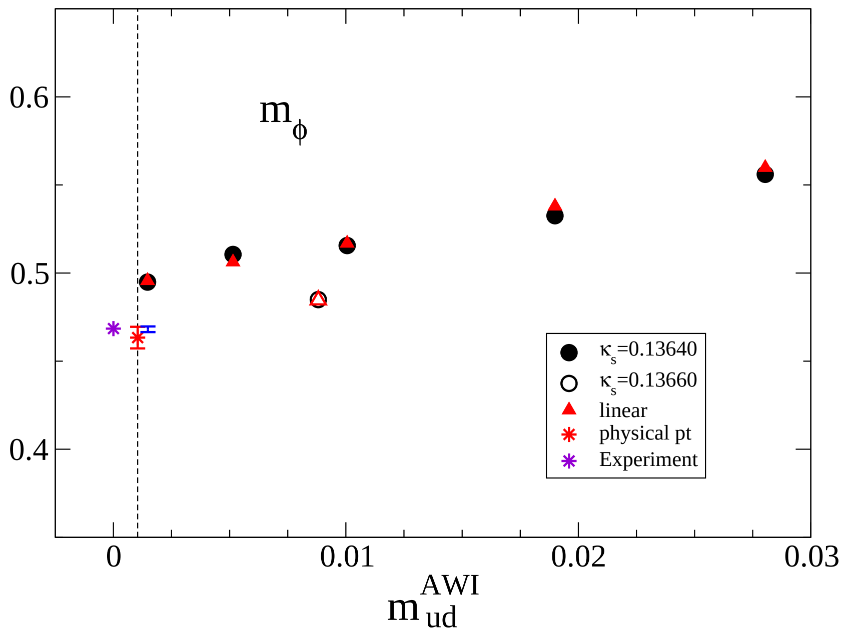
decay constants





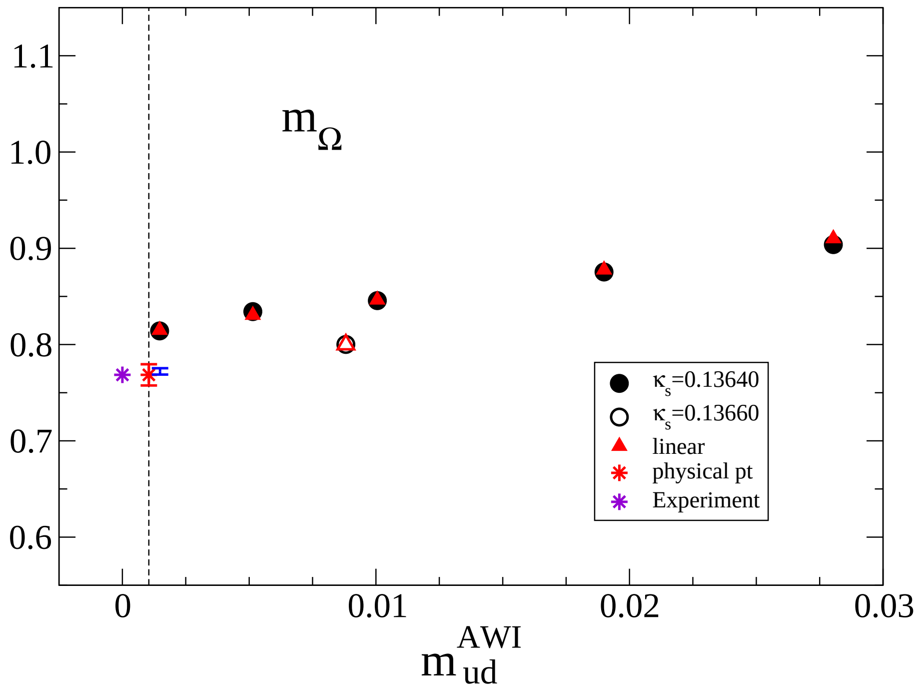
extrapolation ambiguity doesn't matter at the level of 5% error vector meson masses





refined with smaller error bars

Ω baryon as physical input



more precise scale determination at the level of $< 1\%$

§4. Summary

what we learn

– large NLO contributions in SU(3) ChPT due to m_s

– NLO SU(2) ChPT is not enough for precision measurement

– necessary to simulate the physical point directly

next step

– increasing the statistics on $32^3 \times 64$ at the physical point

– T2K-Tsukuba (95TF peak) started operation in U.Tsukuba on 2 June

– check FSE $\Rightarrow$ 64^4 lattice at the physical point

– extension to another lattice spacings