Hadronic Interactions and Nuclear Physics

Silas Beane

University of New Hampshire

US Lattice Quantum Chromodynamics

- Motivation
- Signal/Noise Estimates
- MM
- $MM \ldots M$
- MB
- BB
- The Future

Lattice QCD : So What??

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Particle and Nuclear physics perspectives . . .

The particle physics perspective

The particle physics perspective

QCD is "Background" for beyond-the-Standard-Model physics $(f_K/f_\pi\ ,\ f_{D_s}\ ,\ B_K\ ,\ \ldots)$

The nuclear physics perspective

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Intrinsically interesting QCD physics! $(YN\ ,\ K\pi\ ,\ h_{\pi NN}\ ,\ \ nnn, \ldots)$

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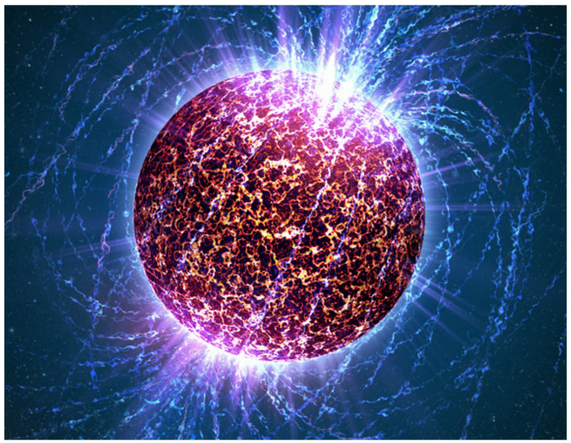
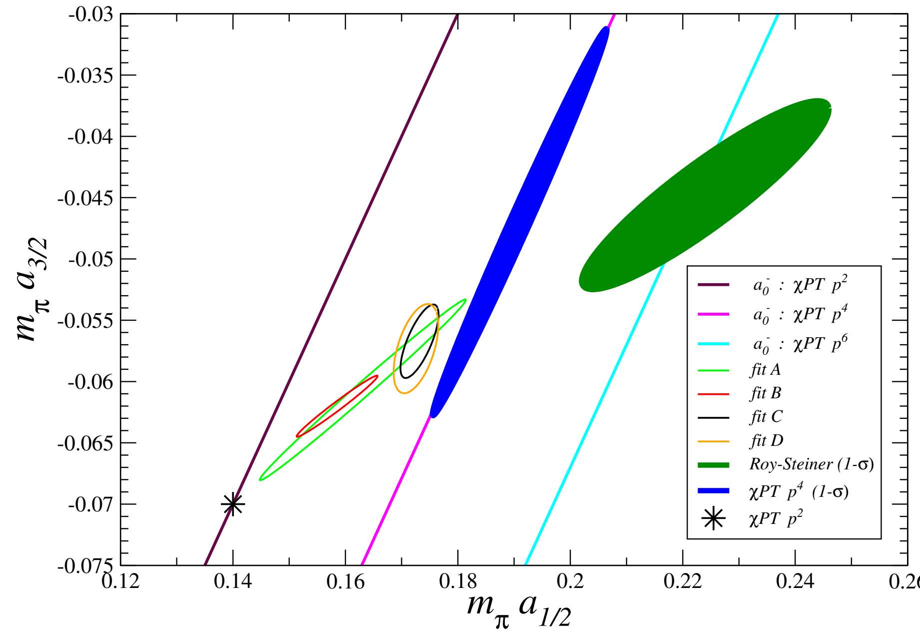
Intrinsically interesting QCD physics! $(YN\ ,\ K\pi\ ,\ h_{\pi NN}\ ,\ \ nnn\ ,\ \ldots)$

NEOS $p\,\Sigma^-$ *interactions*



$K^+\pi^-$ Atoms *Dirac collaboration*



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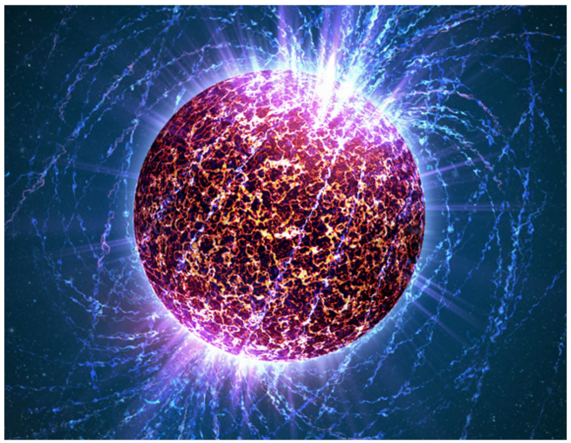
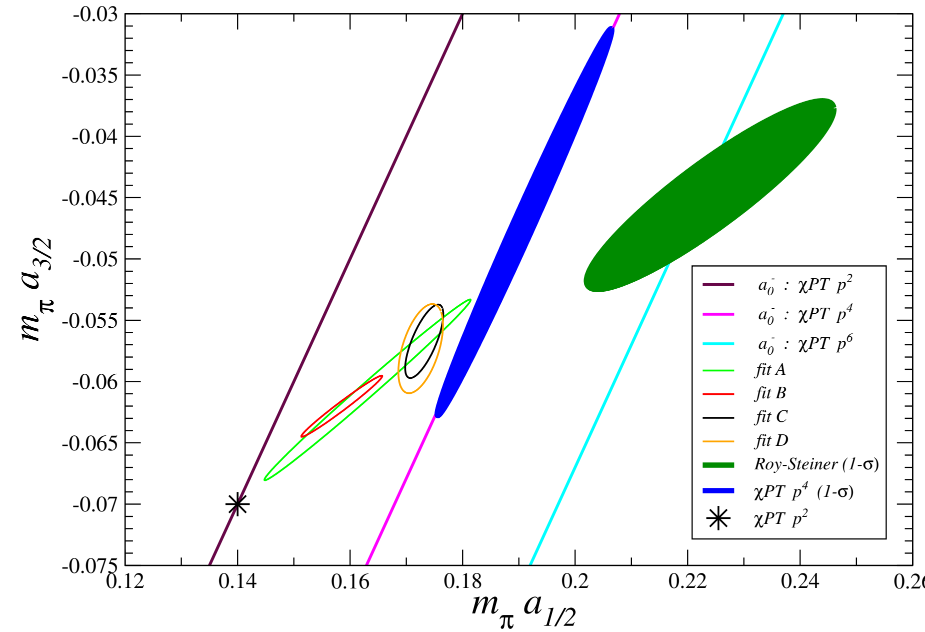
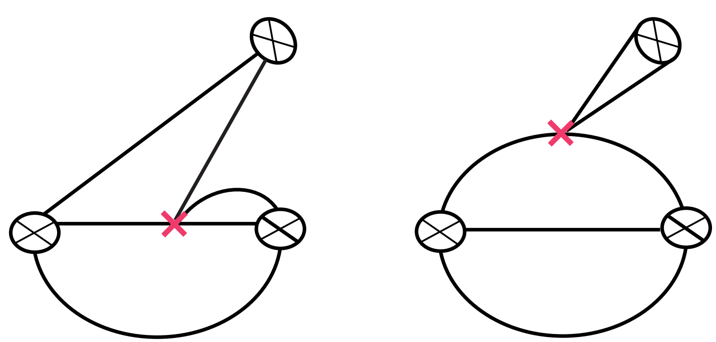




NPDgamma Experiment $h_{\pi NN} \sim \langle 0|\mathcal{O}_N(t)\ \mathcal{O}_\pi(\tau_2)\mathcal{O}_w^{\Delta I=1}(\tau_1)\bar{\mathcal{O}}_N(0)|0\rangle$



Signal/Noise Estimates Lepage (1989)

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$$\textbf{PIONS}: \quad \frac{\text{noise}}{\text{signal}} \quad \sim \quad \frac{\sigma(t)}{\langle\theta(t)\rangle} \sim \frac{\sqrt{\left(A_2 - A_0^2\right)\, e^{-n m_\pi t}}}{\sqrt{N} A_0 \; e^{-n m_\pi t}} \sim \frac{1}{\sqrt{N}}$$

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Noise independent of t!

$$\mathbf{BARYONS}: \quad \frac{\text{noise}}{\text{signal}} \sim \frac{\sigma(t)}{\langle\theta(t)\rangle} \sim \frac{1}{\sqrt{N}}\, e^{\left(m_p - \frac{3}{2} m_\pi\right) t}$$

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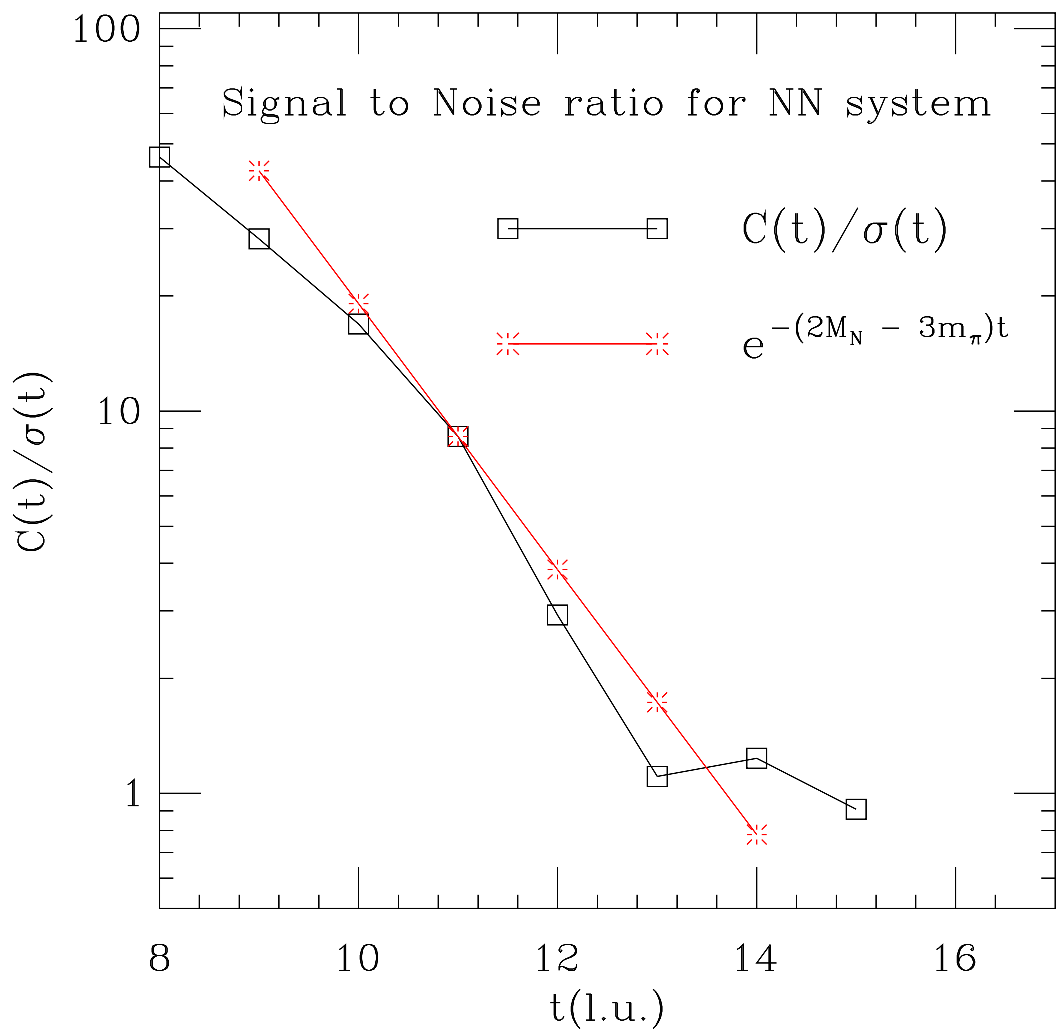
Exponential growth of noise!

np ($^1S_0$) NPLQCD MILC/2064f21b676m010m050



(Courtesy of Bedaque,Walker-Loud `arXiv:0708.0207`)

Currently the main obstacle to lattice QCD calculations of nucleon and nuclear quantities is the signal/noise problem.

Nuclear physics requires exponentially more resources than meson physics.

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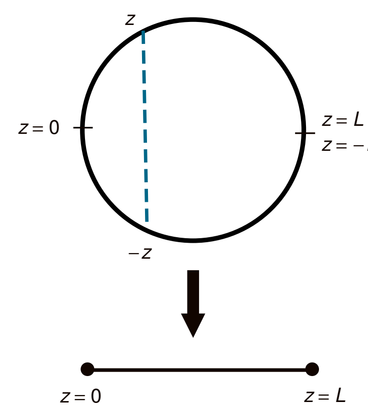
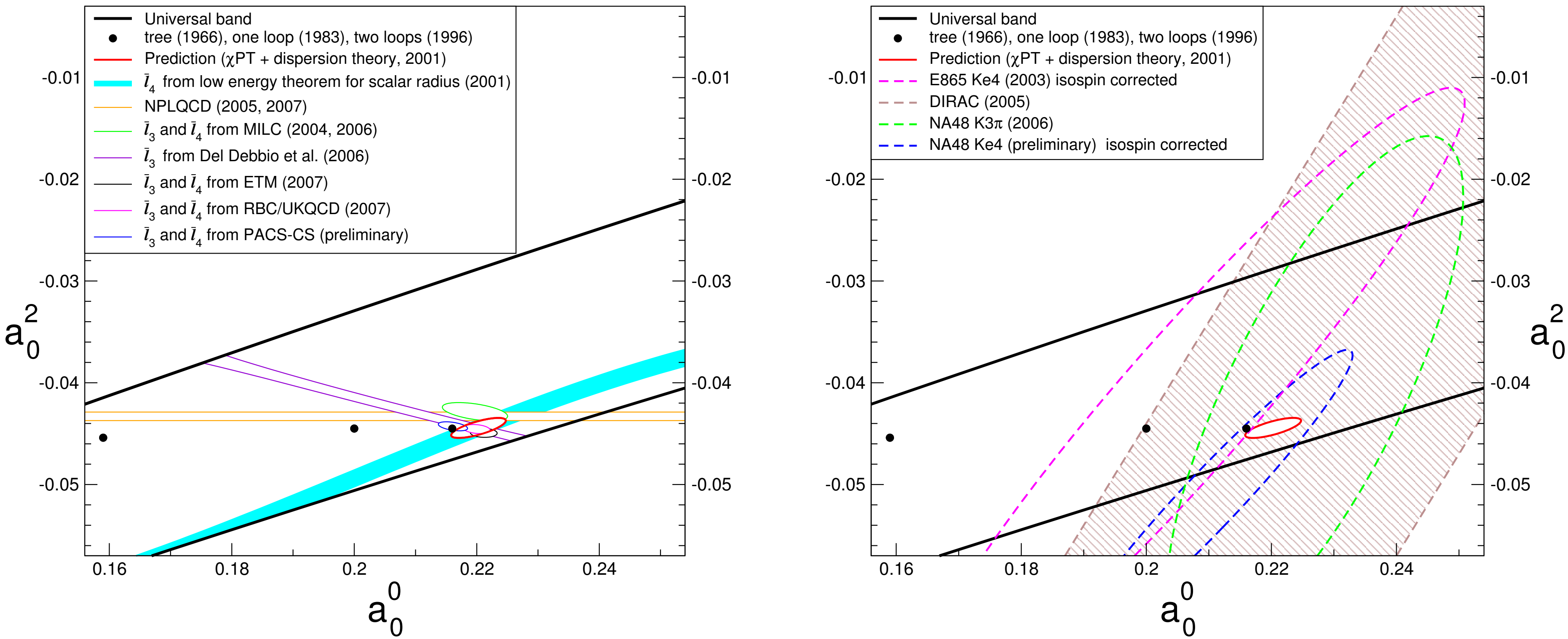
Theoretical fixes?

Orbifold boundary conditions



Bedaque,Walker-Loud/ arXiv:0708.0207

$\pi\pi$



(Courtesy of H. Leutwyler `arXiv:0804.3182`)

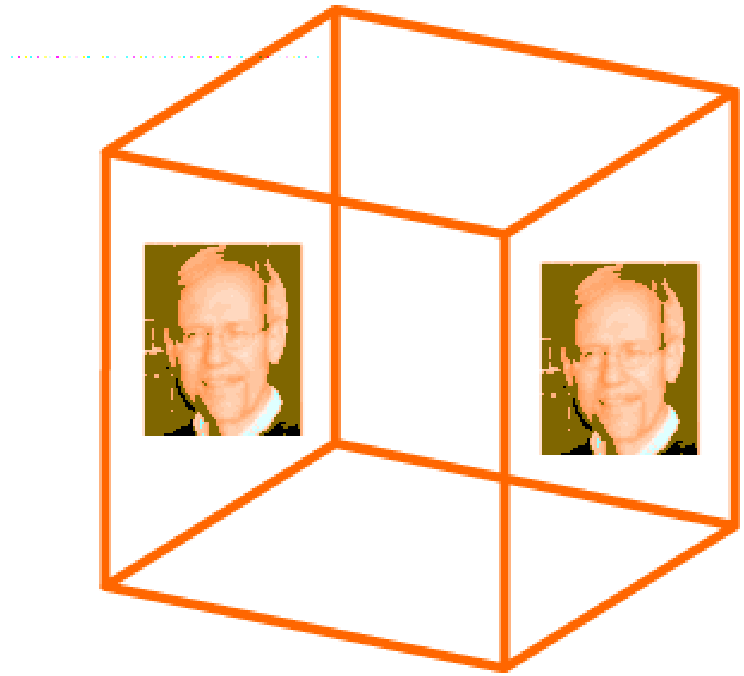
Hybrid of staggered sea quarks (MILC) and domain-wall valence quarks (LHPC+NPLQCD)

(2+1) dynamical flavors

Coarse lattices "chopped" from 64 to 32 with sources displaced in time and space

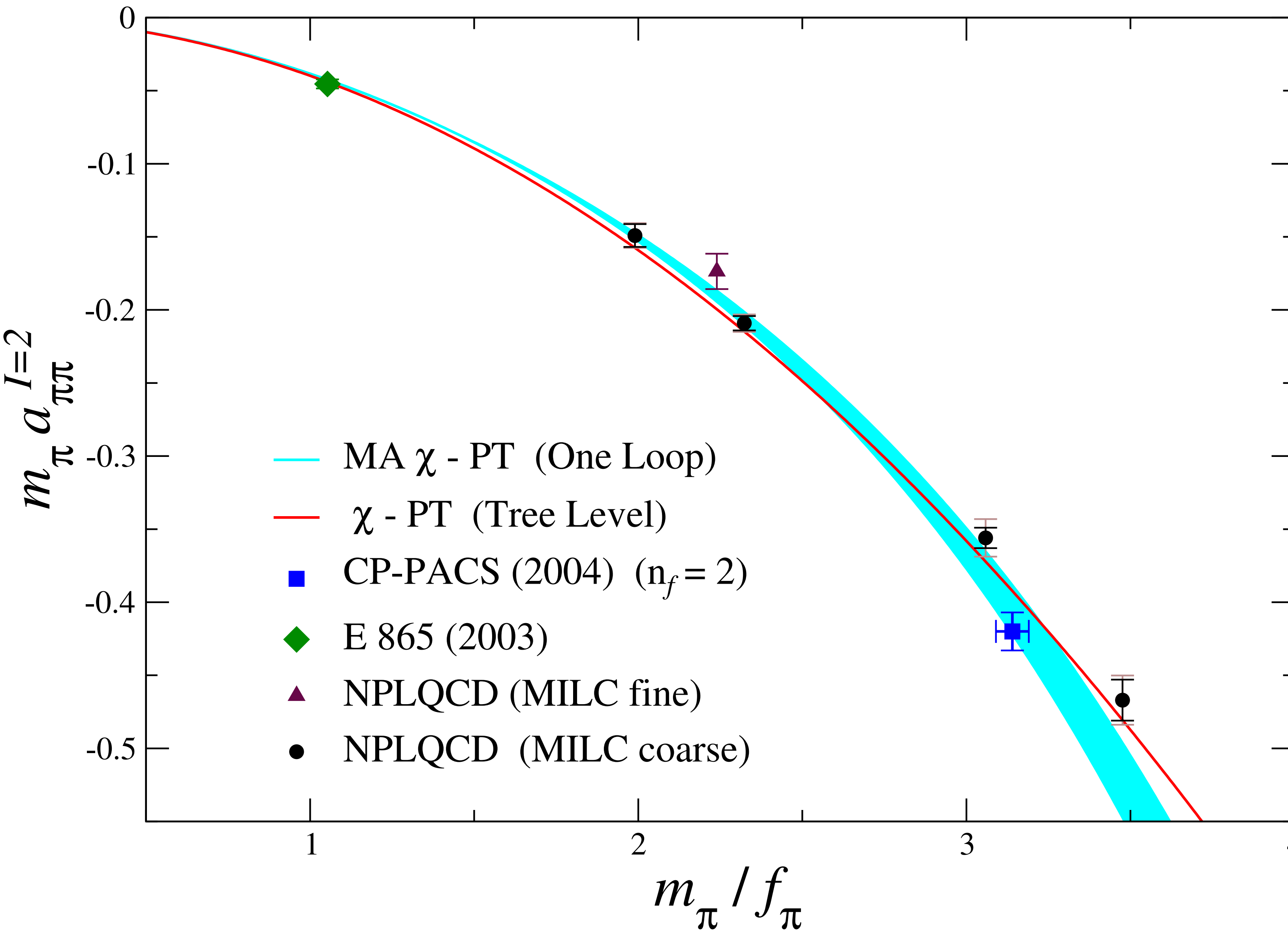
Config Set	Dimensions	m_π	# configs	# sources
2064f21b676m007m050	$20^3 \times 64$	291 MeV	1039	24
2064f21b676m010m050	$20^3 \times 64$	352 MeV	769	24
2064f21b679m020m050	$20^3 \times 64$	491 MeV	486	24
2064f21b681m030m050	$20^3 \times 64$	591 MeV	564	24
2896f21b709m0062m031	$28^3 \times 96$	318 MeV	1001	7

$b_{MILC}^{coarse} \sim 0.125$ fm $b_{MILC}^{fine} \sim 0.09$ fm $L \sim 2.5$ fm



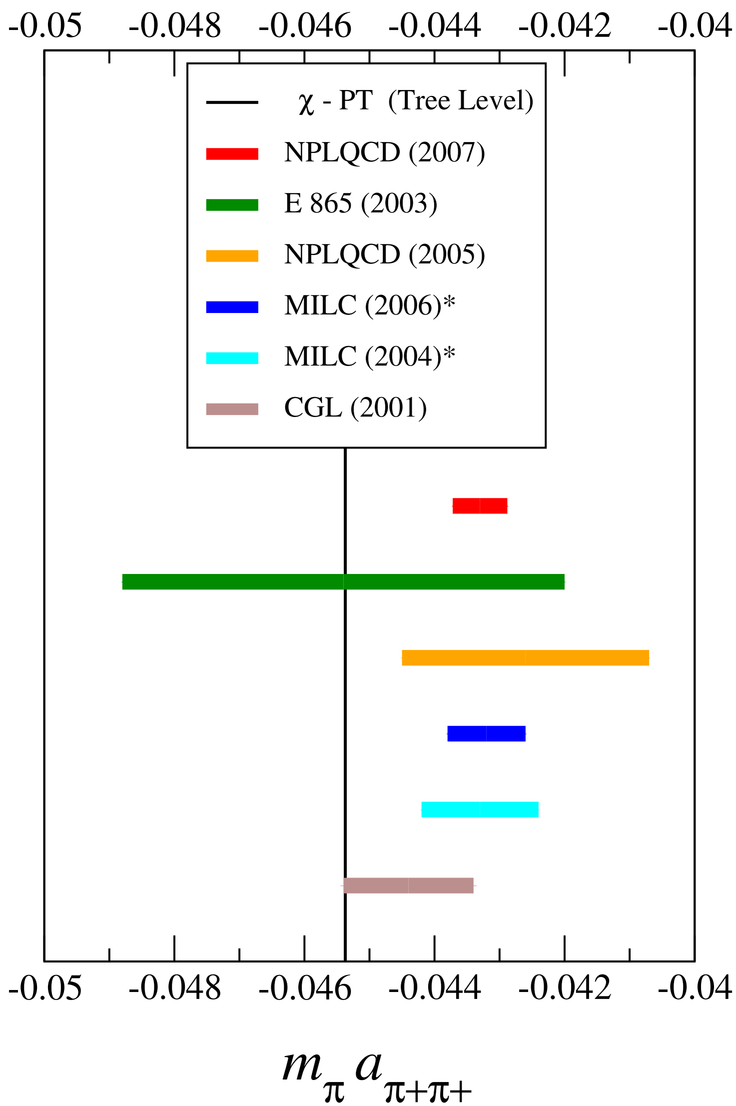
$$\Delta E_0(2,L) = \frac{4\pi\, a_{\pi\pi}}{m_\pi\, L^3}\left\{1 - \left(\frac{a_{\pi\pi}}{\pi\, L}\right)\mathcal{I} + \left(\frac{a_{\pi\pi}}{\pi\, L}\right)^2\left[\mathcal{I}^2 - \mathcal{J}\right] + \left(\frac{a_{\pi\pi}}{\pi\, L}\right)^3\left[-\mathcal{I}^3 + 3\mathcal{I}\mathcal{J} - \mathcal{K}\right]\right\} + \frac{8\pi^2 {a_{\pi\pi}}^3}{m_\pi\, L^6}\, r_{\pi\pi} + \mathcal{O}\left(L^{-7}\right)$$

$$\mathcal{I} = \lim_{\Lambda_j\to\infty}\sum_{\mathbf{i}\neq\mathbf{0}}^{|\mathbf{i}|\le\Lambda_j}\frac{1}{|\mathbf{i}|^2} - 4\pi\Lambda_j = -8.91363291781 \quad , \quad \mathcal{J} = \sum_{\mathbf{i}\neq\mathbf{0}}\frac{1}{|\mathbf{i}|^4} = 16.532315959 \quad , \quad \mathcal{K} = \sum_{\mathbf{i}\neq\mathbf{0}}\frac{1}{|\mathbf{i}|^6} = 8.401923974433$$



Savage/PAR-Mon-6pm

Status of $\pi\pi$



	$m_\pi\ a_{\pi\pi}^{I=2}$
χPT (Tree Level)	-0.04438
NPLQCD (2007)	-0.04330 ± 0.00042
E 865 (2003)	$-0.0454 \pm 0.0031 \pm 0.0010 \pm 0.0008$
NPLQCD (2005)	$-0.0426 \pm 0.0006 \pm 0.0003 \pm 0.0018$
MILC (2006)*	-0.0432 ± 0.0006
MILC (2004)*	-0.0433 ± 0.0009
CGL (2001)	-0.0444 ± 0.0010

- $\pi\pi$ in χ-PT for Wilson lattice actions

  Buchoff/ arXiv:0802.2931

  Aoki *et al.*/ arXiv:0806.4863/PS-B

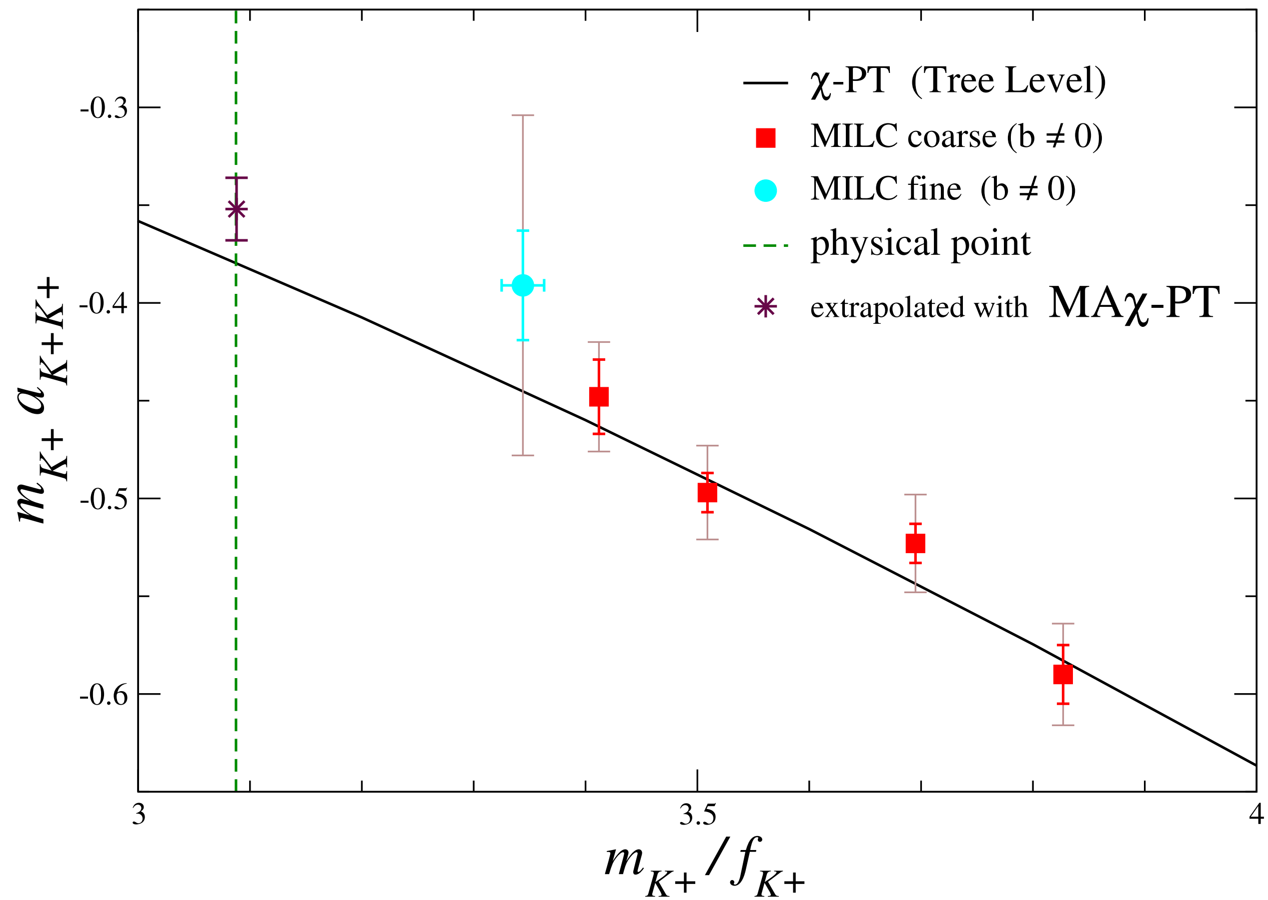
- Wavefunction method for $\pi\pi$ phase shift

  Sasaki,Ishizuka/ arXiv:0804.2941

- MM interactions from multi-M interactions

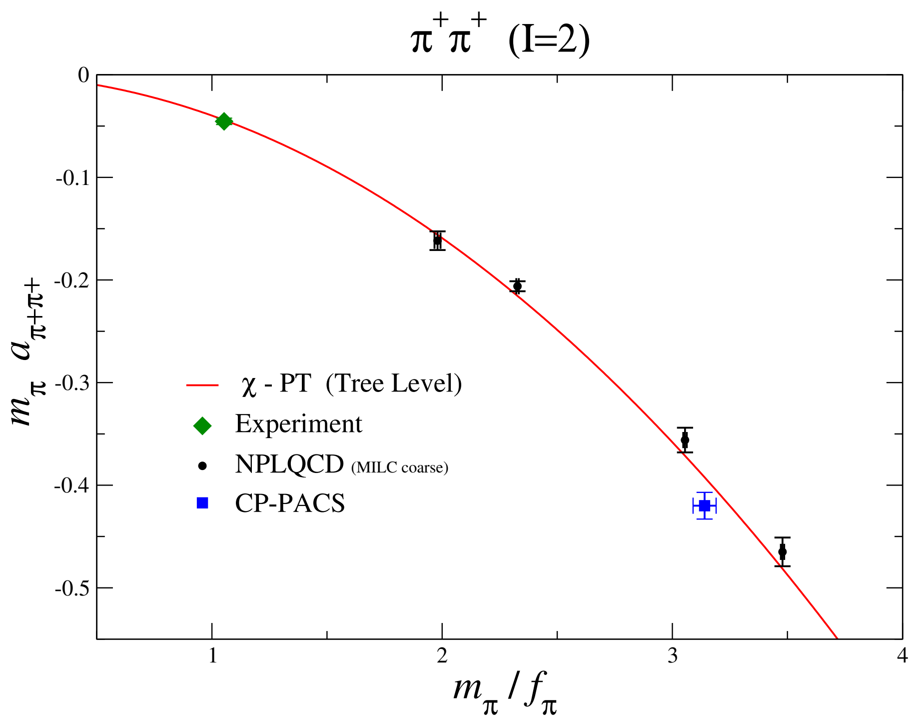
  NPLQCD/ arXiv:0710.1827/PAR-Tue-2:30pm

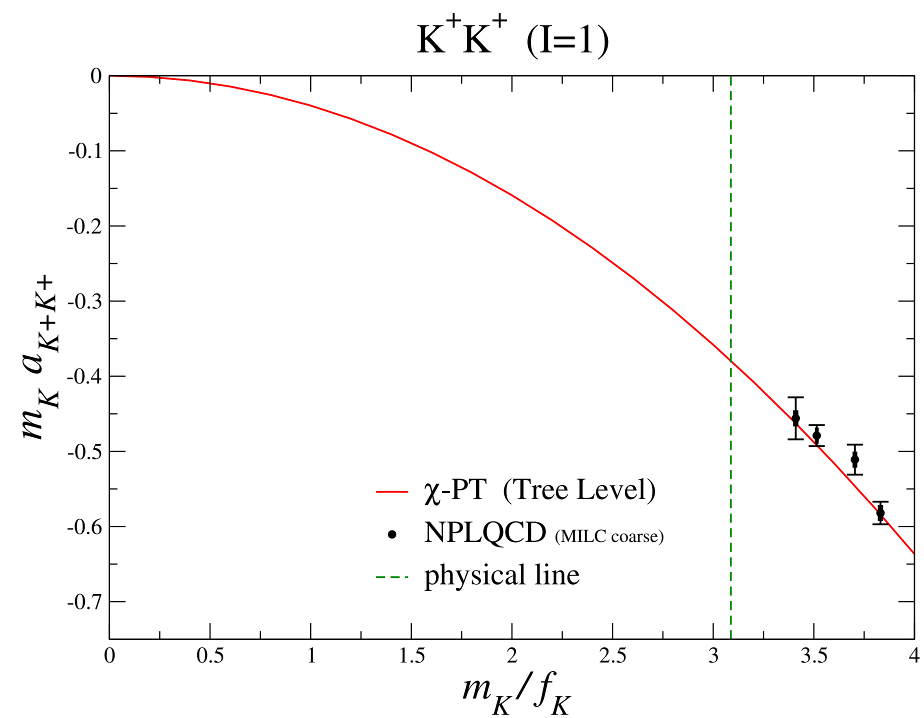
KK



$$m_{K^+}\ a_{K^+K^+} = -0.352 \pm 0.016$$ *NPLQCD*/arXiv : 0709.1169

PAR-Mon-6pm





Mysterious disappearance of higher-order effects!

$MM\ldots M$

n pions in a finite volume

Detmold *et al.*/ arXiv:0707.1670,0801.0763/PAR-Tue-2:30pm Tan/ arXiv:0709.2530

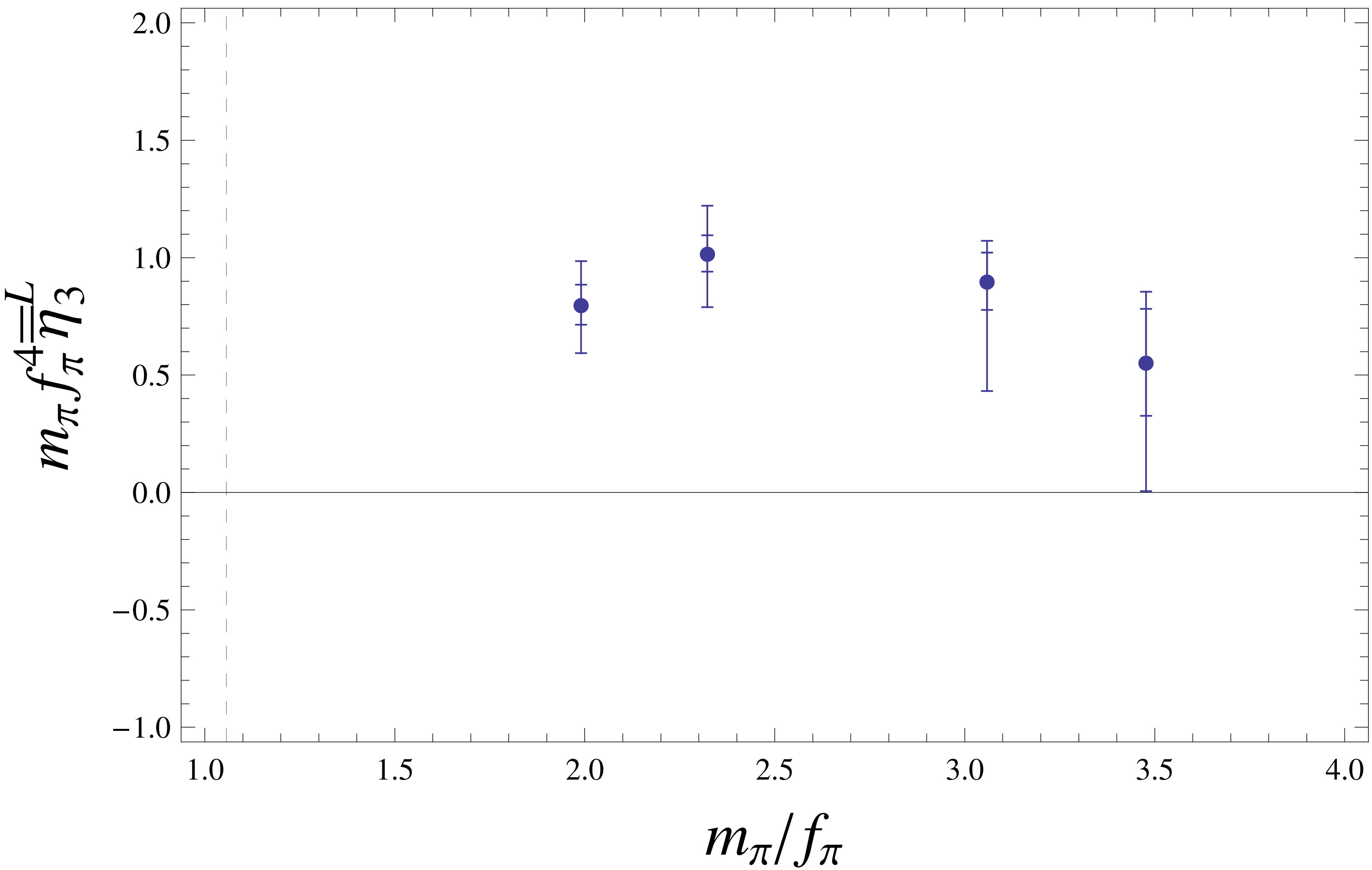
$$\Delta E_0(n,L) = \frac{4\pi\, a_{\pi\pi}}{m_\pi\, L^3}\binom{n}{2}\left\{1-\left(\frac{a_{\pi\pi}}{\pi\, L}\right)\mathcal{I}+\left(\frac{a_{\pi\pi}}{\pi\, L}\right)^2\left[\mathcal{I}^2+(2n-5)\mathcal{J}\right]\right.$$
$$\left.+\left(\frac{a_{\pi\pi}}{\pi\, L}\right)^3\left[-\mathcal{I}^3-(2n-7)\mathcal{I}\mathcal{J}-\left(5n^2-41n+63\right)\mathcal{K}\right]\right\}$$
$$+\binom{n}{2}\frac{8\pi^2 {a_{\pi\pi}}^3}{m_\pi\, L^6}\, r_{\pi\pi}+\binom{n}{3}\frac{\bar{\eta_3}(L)}{L^6}+\mathcal{O}\left(L^{-7}\right)$$

$$\bar{\eta_3}(L) = \eta_3(\mu) + \frac{64\pi\, {a_{\pi\pi}}^4}{m_\pi}(3\sqrt{3}-4\pi)\log(L\,\mu) - \frac{96\, {a_{\pi\pi}}^4}{m_\pi \pi^2}(2\mathcal{Q}+\mathcal{R})$$

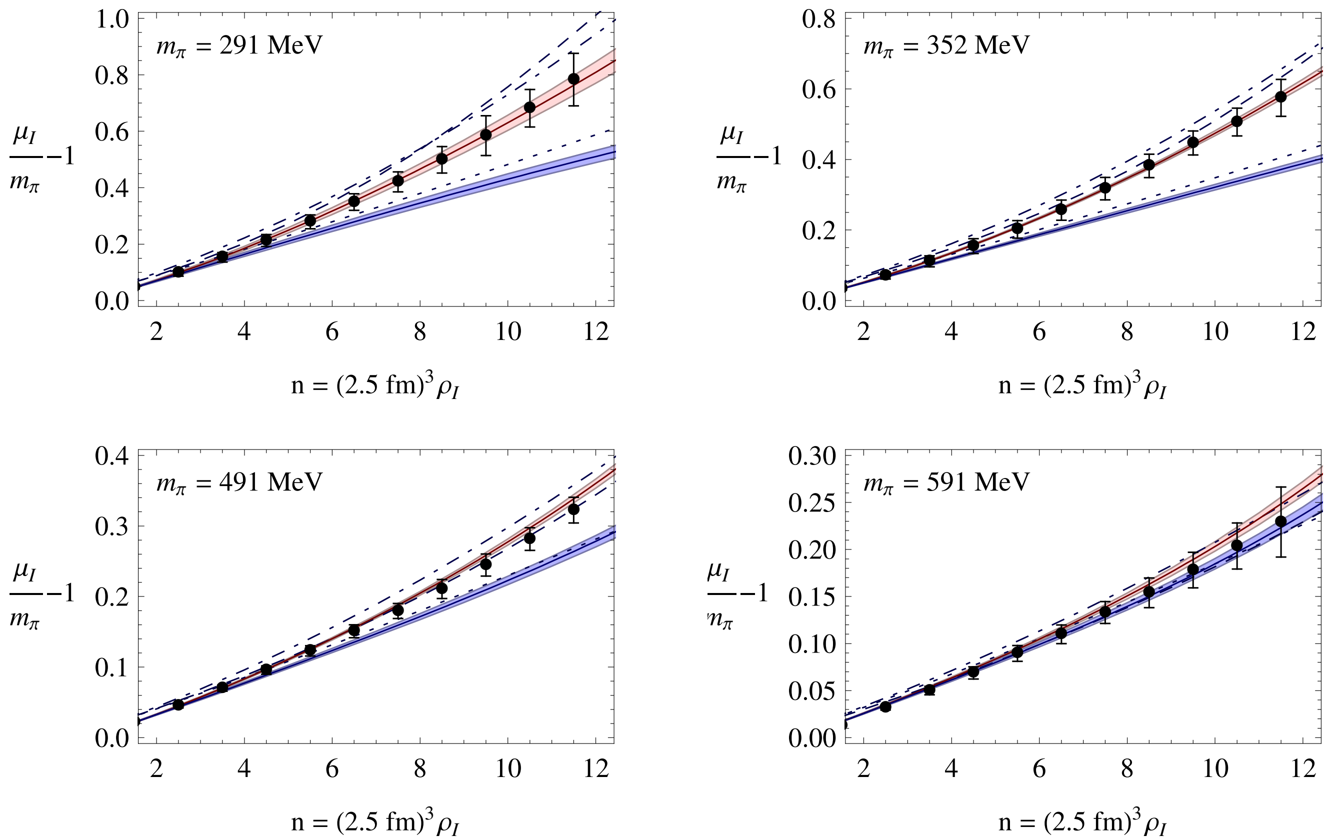
$$dim\ \ reg\ \ /w\ \ MS: \qquad \mathcal{Q}=-100.75569 \quad , \quad \mathcal{R}=19.186903$$

Three-body force

NPLQCD/arXiv:0710.1827,0803.2728

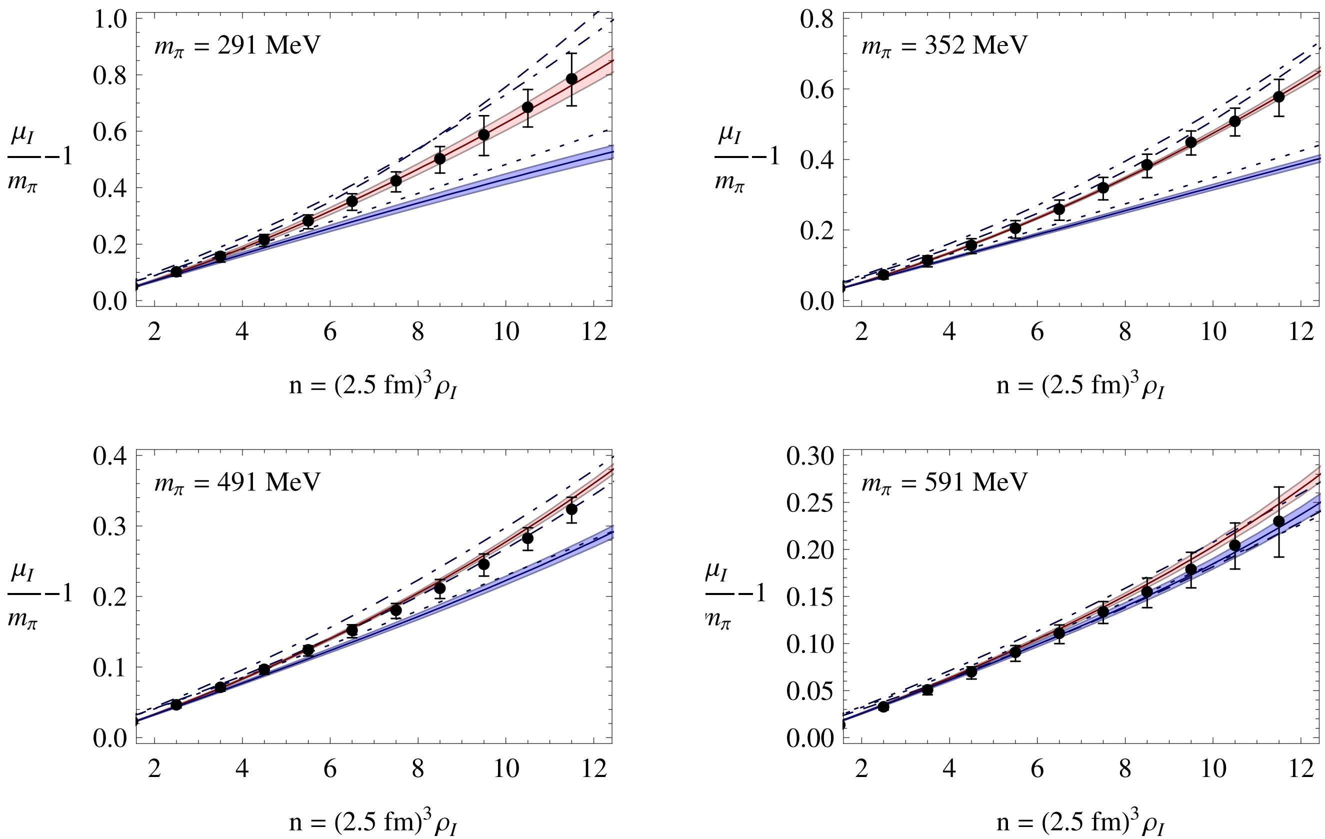


Pion condensate



$$\chi\text{−PT}: \qquad \rho_I = \frac{1}{2} f_\pi^2 \mu_I \left(1 - \frac{m_\pi^4}{\mu_I^4}\right)$$

Pion condensate



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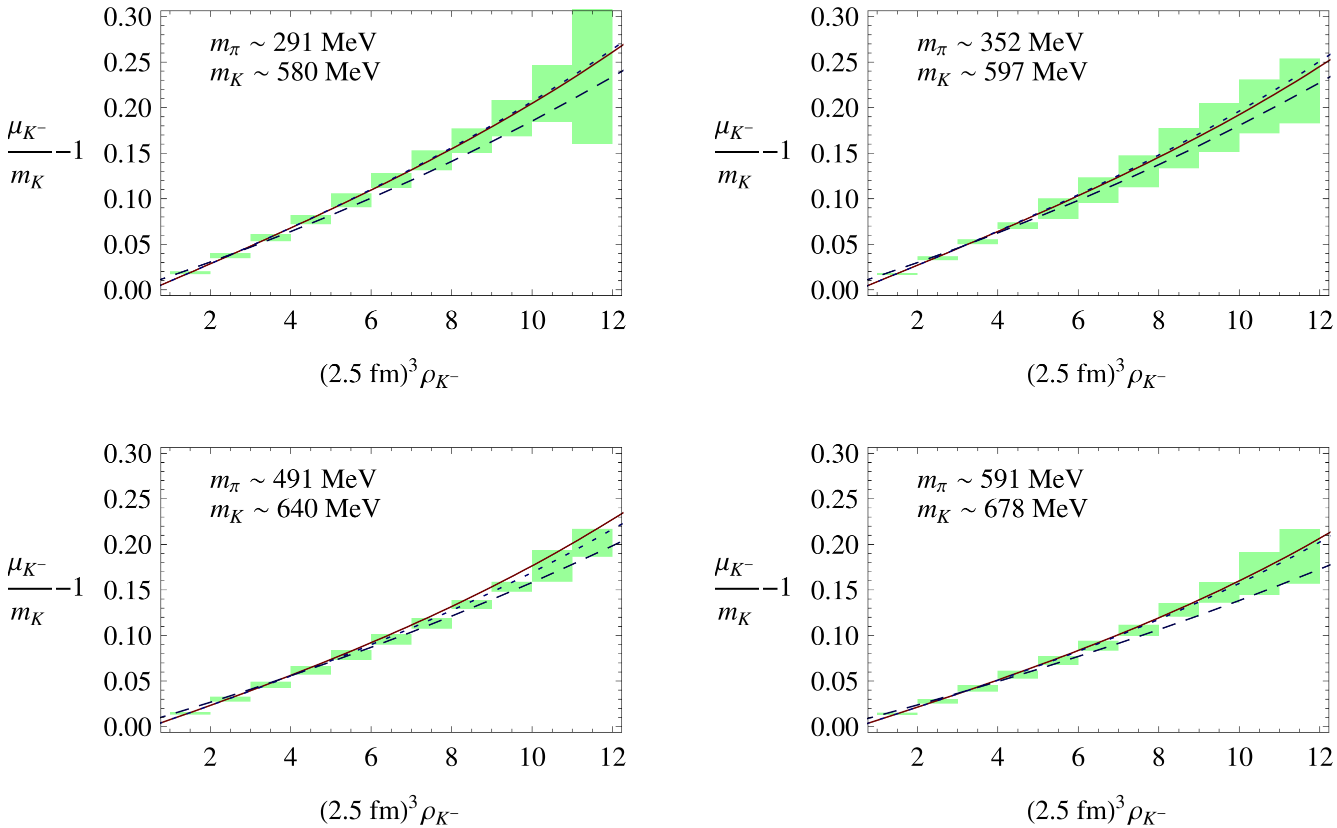
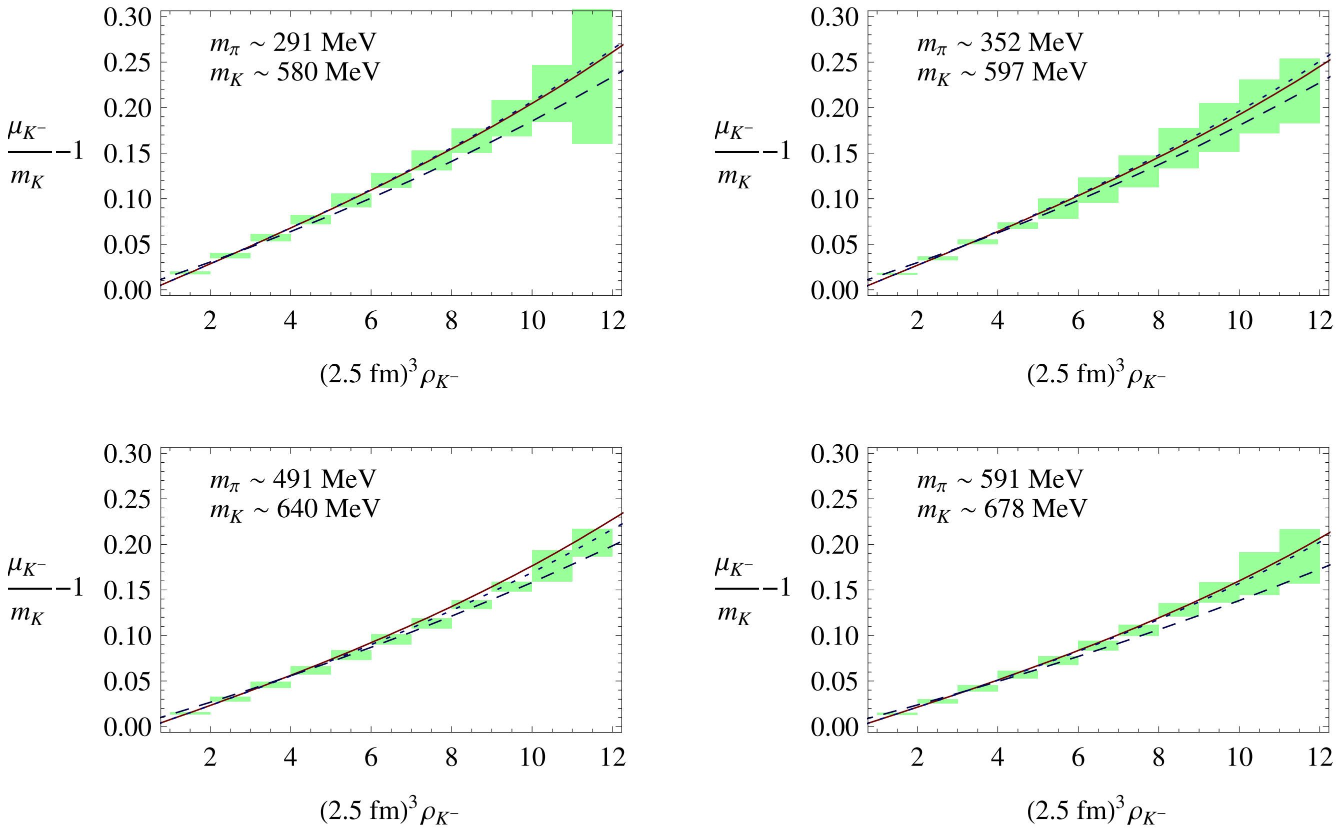
Three-body force is important!

Kaon condensate NPLQCD/arXiv:0807.1924



$$\chi\text{–PT}: \qquad \rho_K = \frac{1}{2} f_K^2 \left(\mu_K - \frac{m_K^4}{\mu_K^3} \right)$$

Kaon condensate NPLQCD/arXiv:0807.1924



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Why does χPT work so well??

Meson-Baryon scattering Torok/**PAR-Thu-8:50am**

- Interesting phenomenology
- Important for baryon spectroscopy Juge/PAR-Tue-3:30pm

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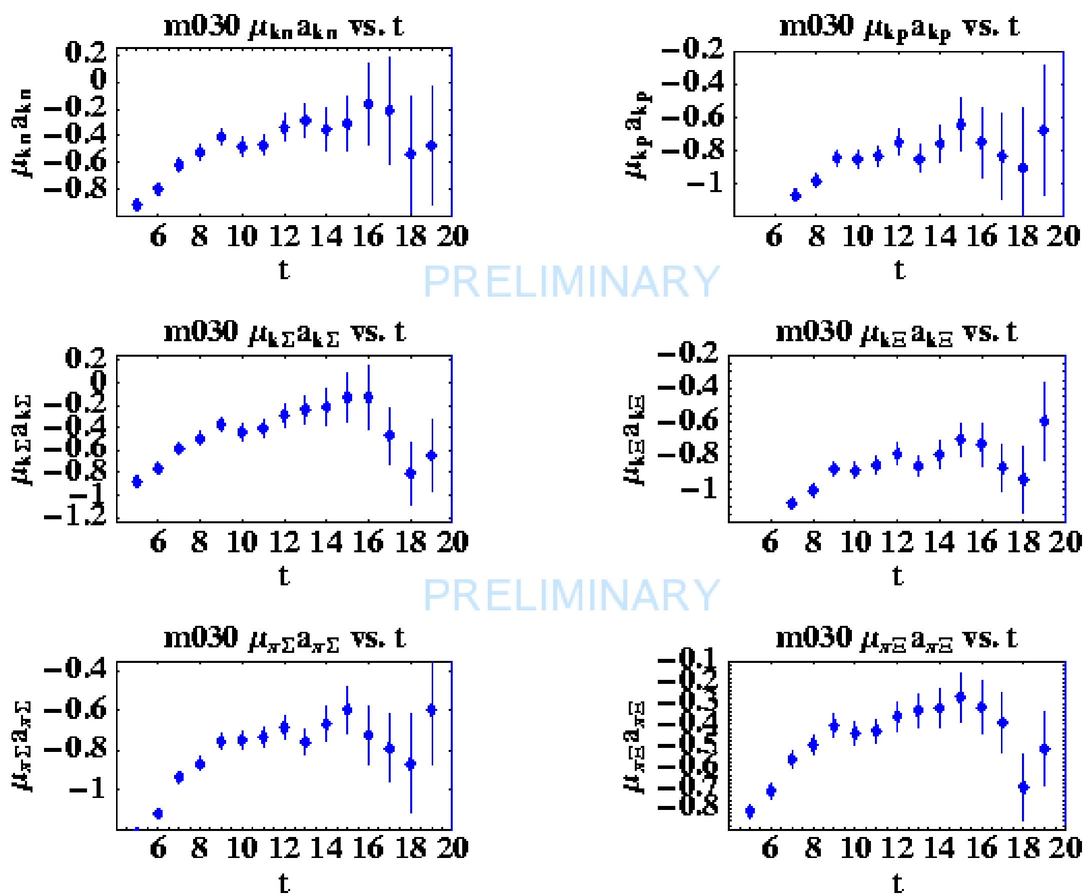

What should one calculate using lattice QCD ?

$V_{NN}(r)$ or $\delta_{NN}(E)$?

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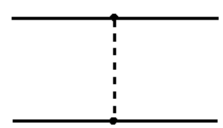
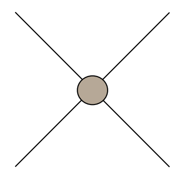
Historically nuclear physicists have used *energy-independent* potentials $V_{NN}(r)$ to fit NN phaseshifts with $\chi^2/d.o.f. \sim 1$ over a wide range of (low) energies.

$V_{NN}(r) \quad =$  $+$ 

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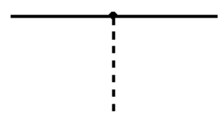
$V_{NN}(r)$ =



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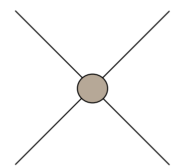
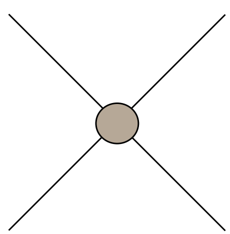
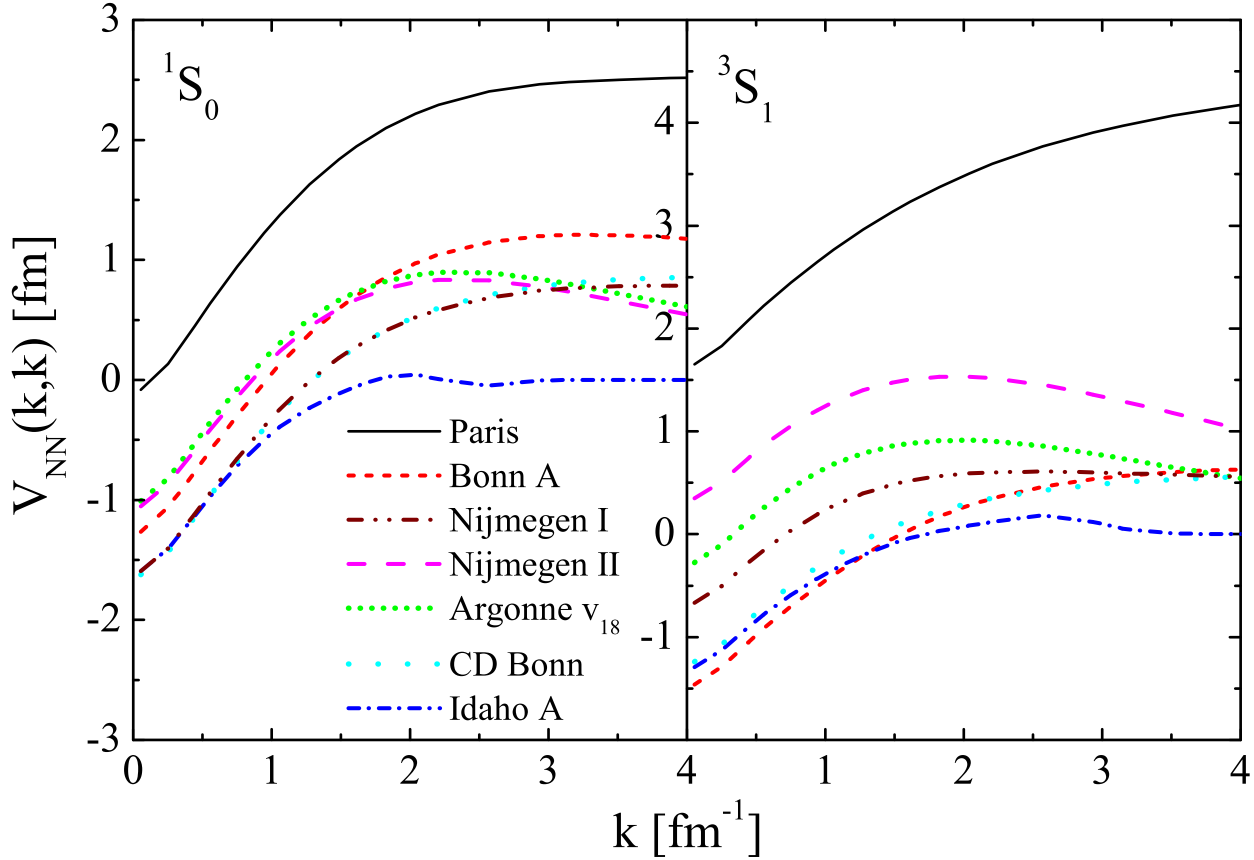




=

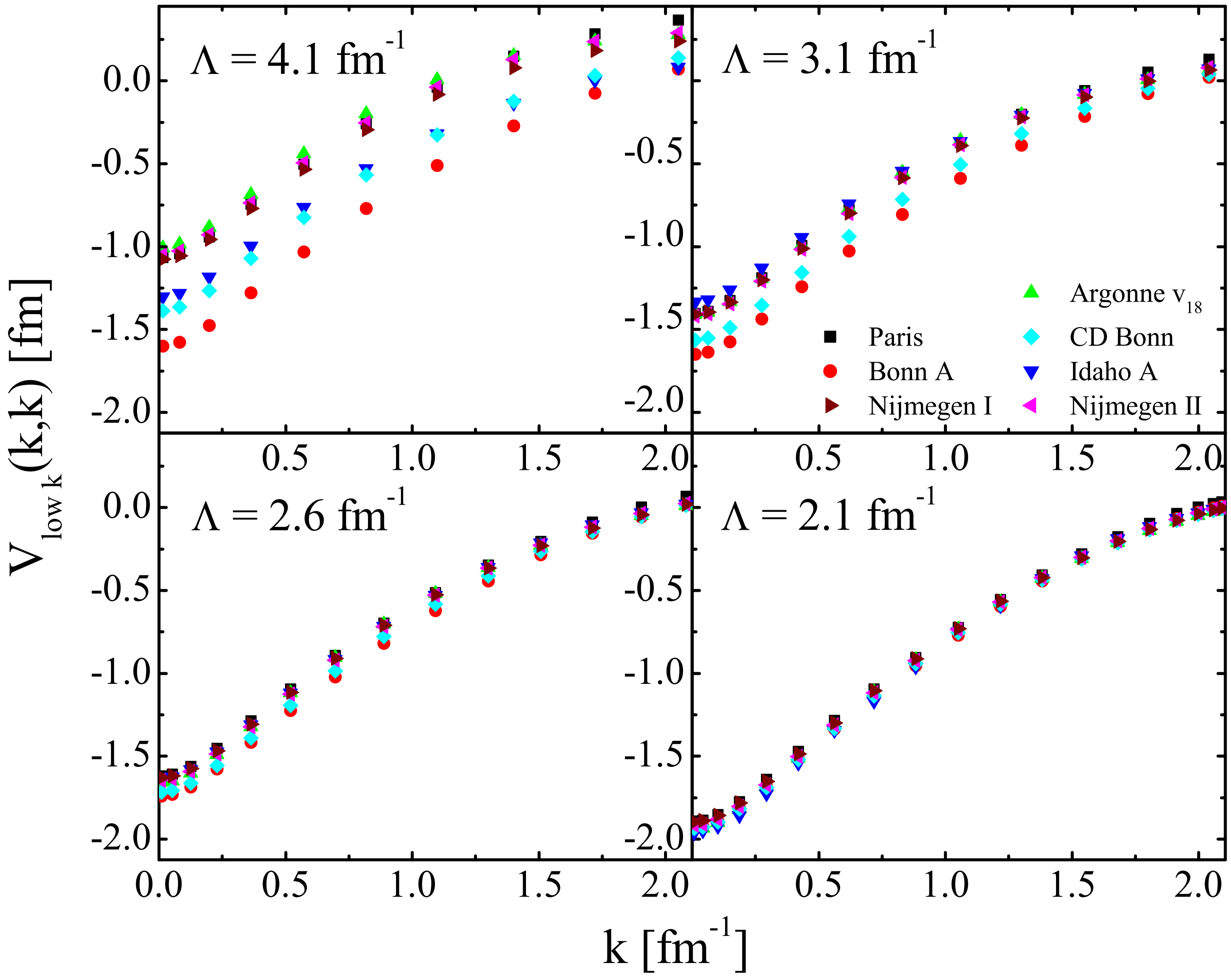
$(\chi^2/d.o.f. \sim 1)$

The modern viewpoint: Effective Field Theory



(Courtesy of A. Schwenk *et al.* `nucl-th/0305035`)

Use RG to integrate out



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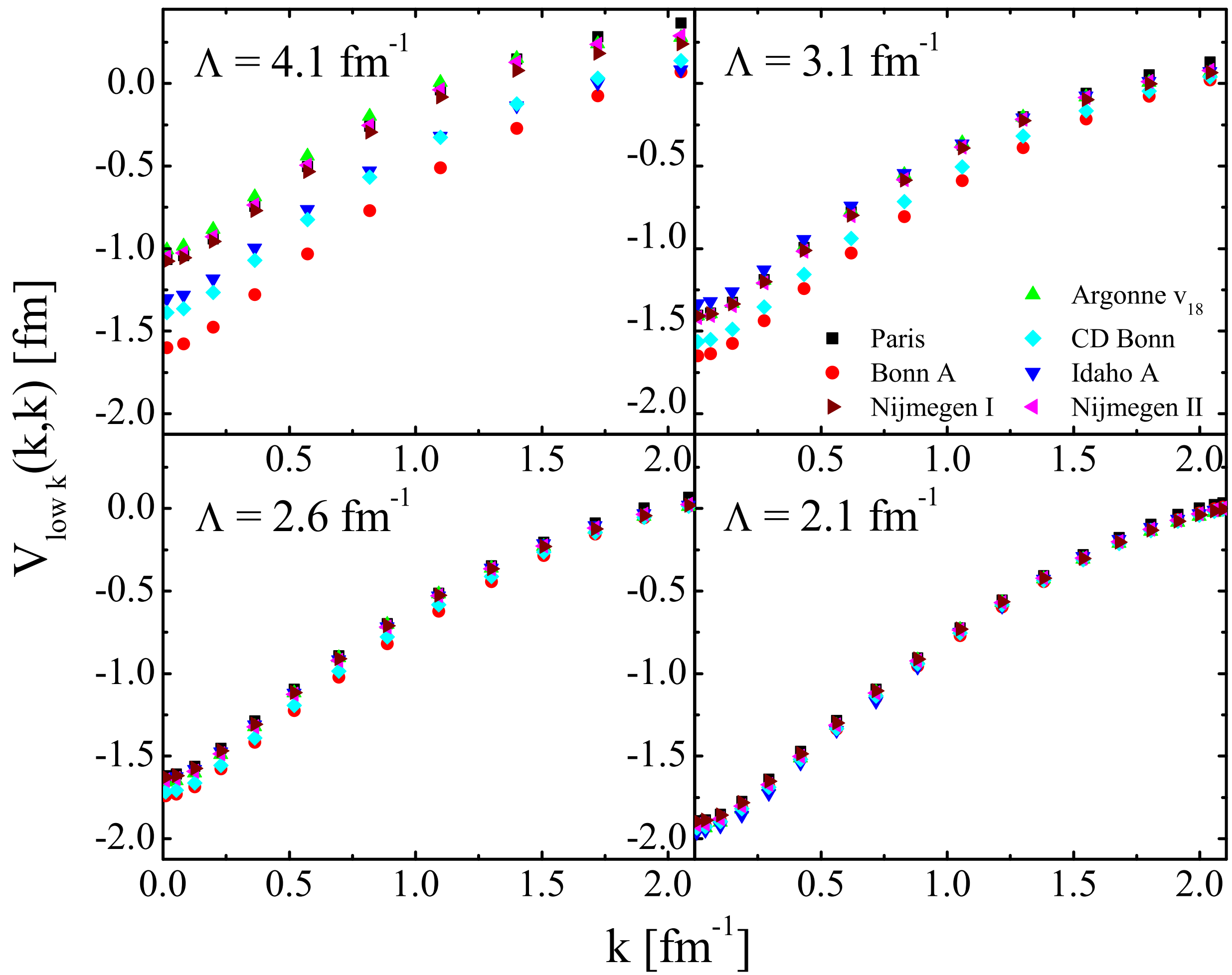

Bottom line: short distance part of $V_{NN}(r)$ not meaningful!

Can one *define* and *calculate* $V_{BB}^{LATT}(r)$?

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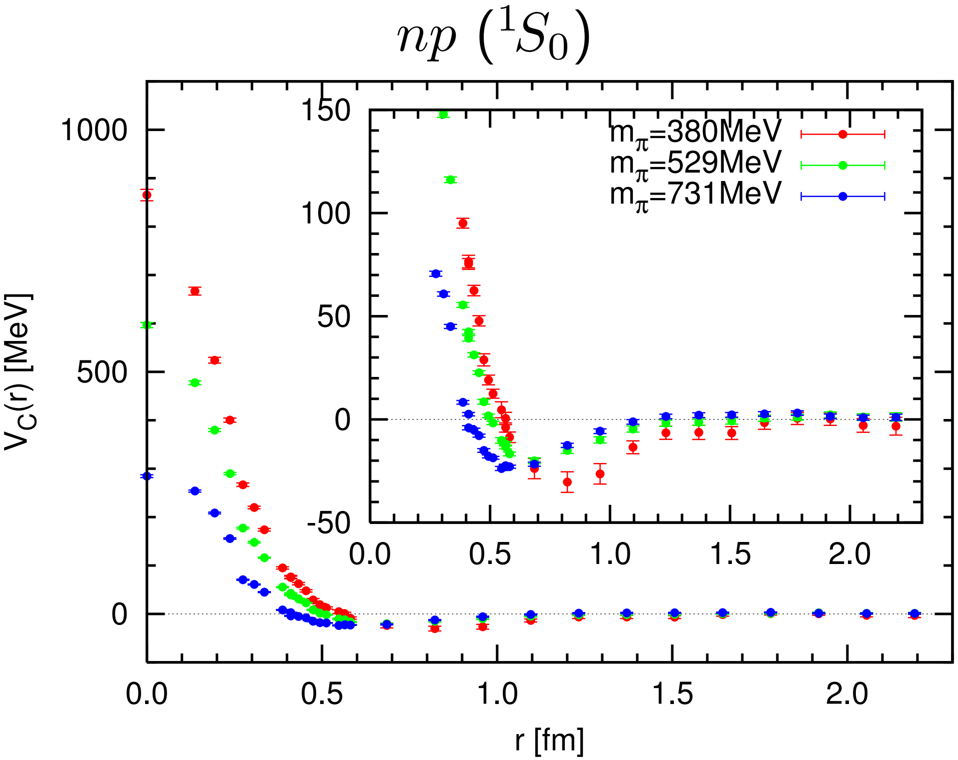
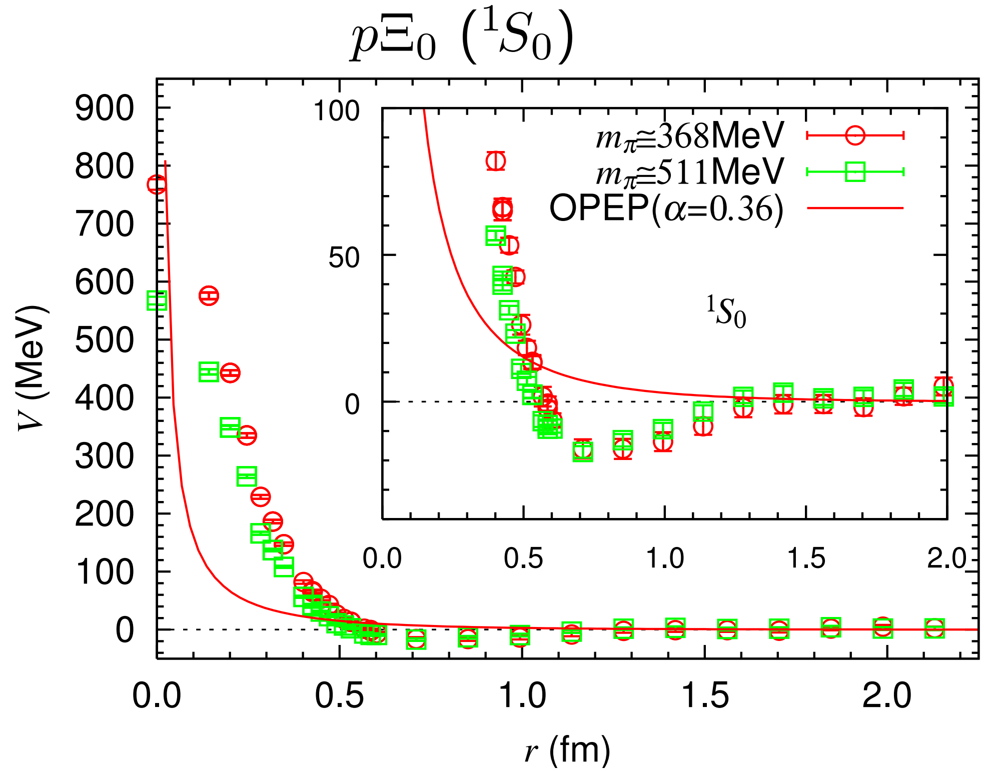
- NN: Aoki *et al.*/ `arXiv:0805.2462`/PAR-Wed-3:50pm
- YN: Nemura *et al.*/ `arXiv:0806.1094`/PAR-Wed-4:10pm

Bethe-Salpeter wave function $\Rightarrow$ V_{BB}^{LATT}

QQCD potentials





Wilson quark action $\beta = 5.7$ $b \sim 0.137\text{fm}$ $L \sim 4.4\text{fm}$

PROBLEMS

Detmold *et al.*/ arXiv:hep-lat/0703009v1

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- $V_{BB}^{LATT}(r, E)$ Aoki/PAR-Wed-2:50pm

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- $V_{BB}^{LATT}(r, E)$ Aoki/PAR-Wed-2:50pm

$$V_{BB}^{LATT}(r, E) \Rightarrow \delta_{BB}^{LATT}(E)$$

PROBLEMS

Detmold *et al.*/ arXiv:hep-lat/0703009v1

- $V_{BB}^{LATT}(r, E)$ Aoki/PAR-Wed-2:50pm

$$V_{BB}^{LATT}(r, E) \Rightarrow \delta_{BB}^{LATT}(E)$$

$$V_{BB}^{LATT}(r, E) \not\Rightarrow \delta_{BB}^{LATT}(E')\,!!$$

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- $V_{BB}^{LATT}(r, E, J)$ Aoki *et al.*/ *in preparation*

$$V_{BB}^{LATT}(r, E, J) \underset{r \to m_\pi^{-1}}{\longrightarrow} V_{BB}^{LATT}(r, E)??$$

PROBLEMS

Detmold *et al.*/ arXiv:hep-lat/0703009v1

- $V_{BB}^{LATT}(r, E)$ Aoki/PAR-Wed-2:50pm

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- $V_{BB}^{LATT}(r, E, J)$ Aoki *et al.*/*in preparation*

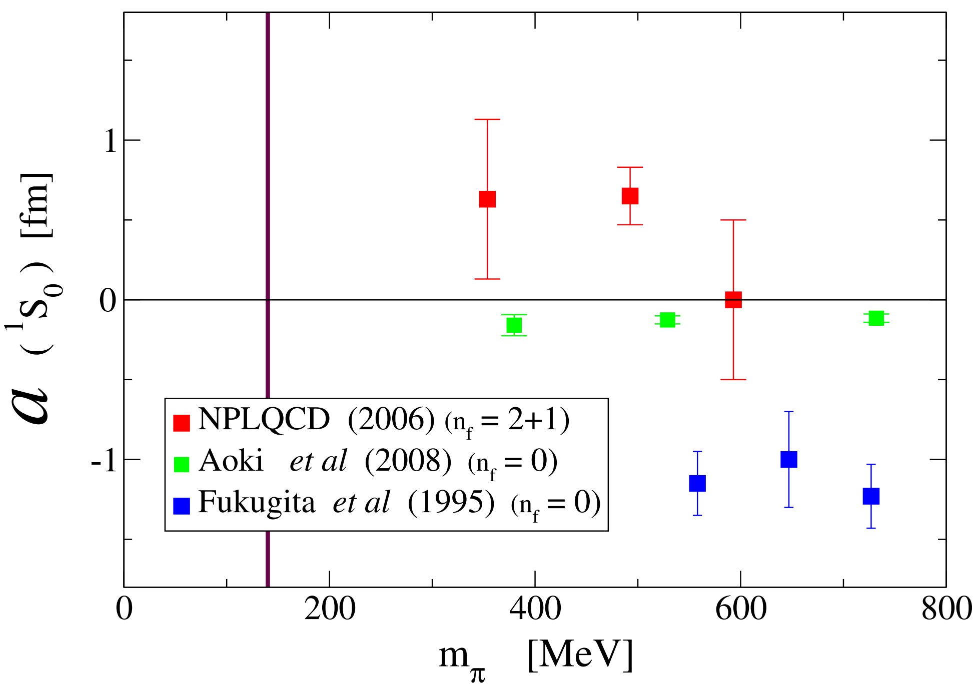
$$V_{BB}^{LATT}(r, E, J) \xrightarrow[r \to m_\pi^{-1}]{} V_{BB}^{LATT}(r, E)??$$

- QQCD : $V_{NN}^{LATT}(r) \to \left(M_0 - \alpha_\Phi m_\pi^2\right) e^{-m_\pi r}$

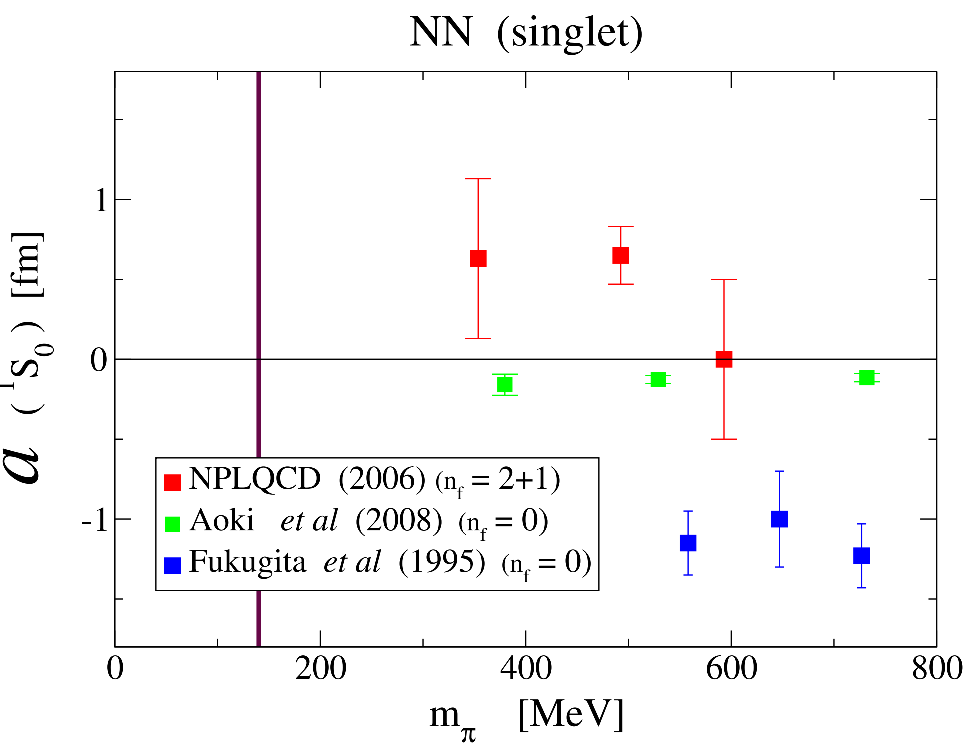
Dominates over Yukawa force!!

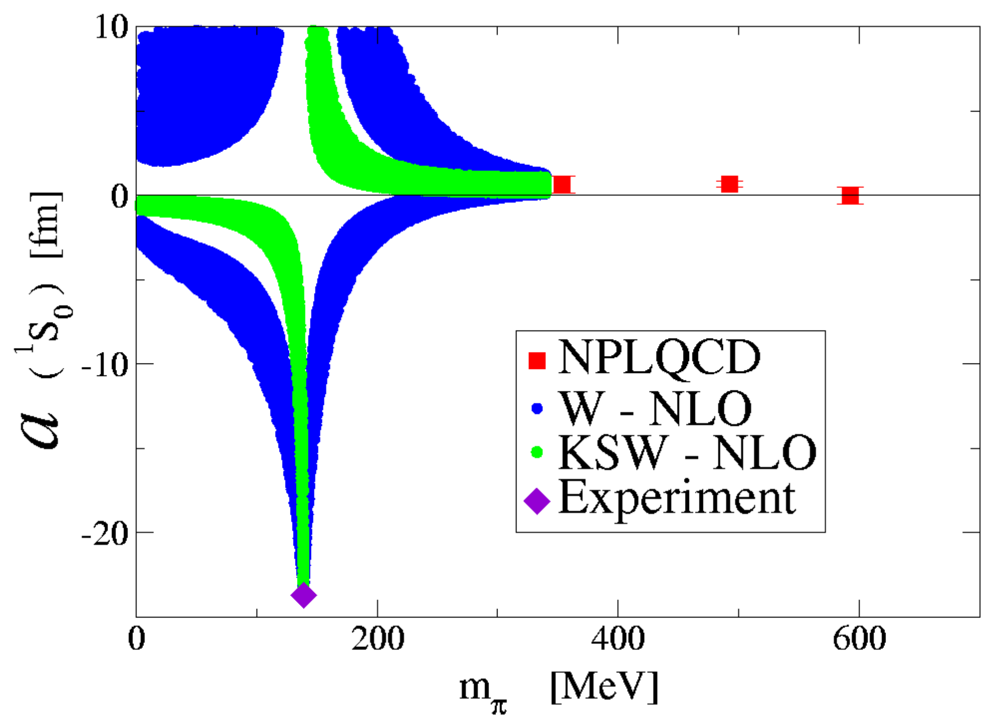
Results: NN

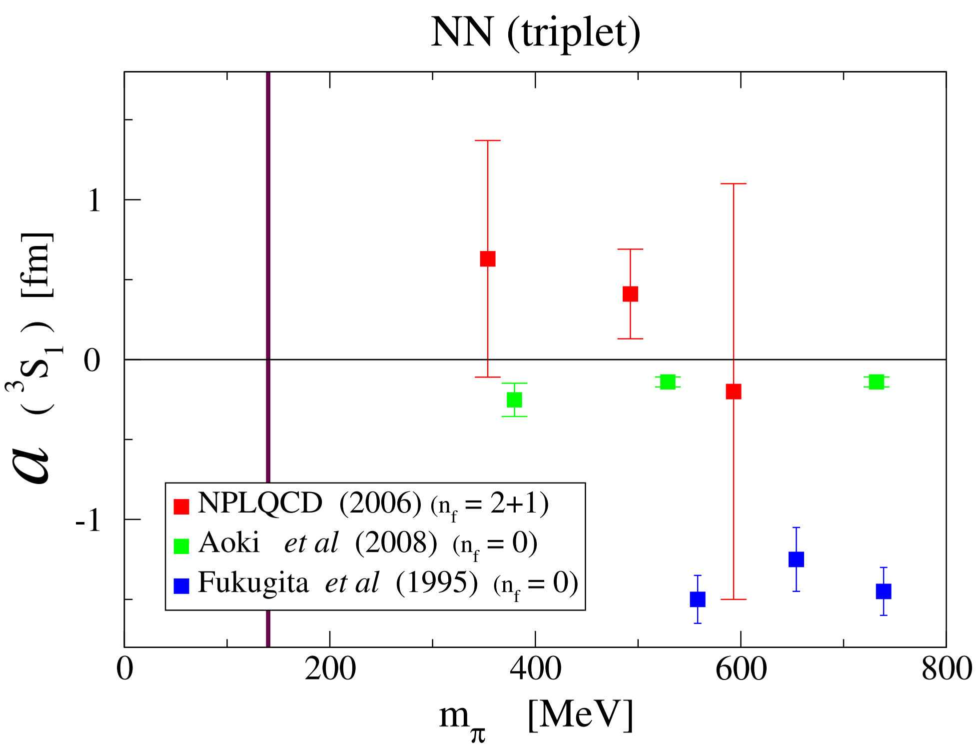
NN (singlet)



Results: NN


NN (singlet)
NPLQCD (2006) (n_f = 2+1)
Aoki et al (2008) (n_f = 0)
Fukugita et al (1995) (n_f = 0)
a (1S0) [fm]
m_π [MeV]
0
200
400
600
800
1
0
-1


NPLQCD
W - NLO
KSW - NLO
Experiment
a (1S0) [fm]
m_π [MeV]
10
0
-10
-20
0
200
400
600


NN (triplet)
1
0
-1
0
200
400
600
800
a ($^3S_1$) [fm]
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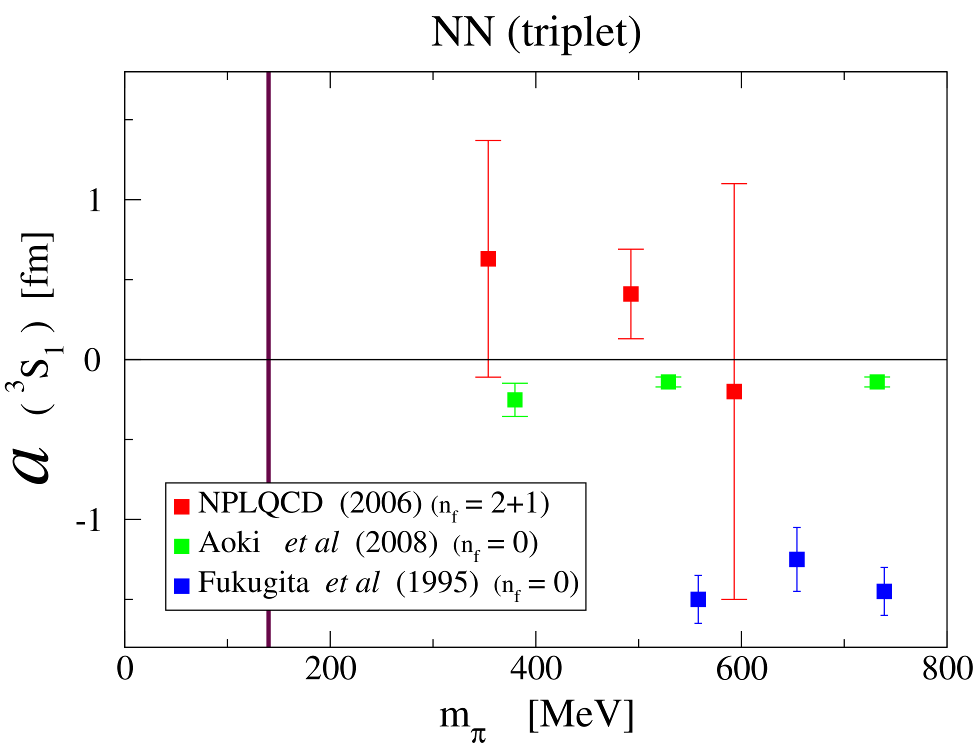
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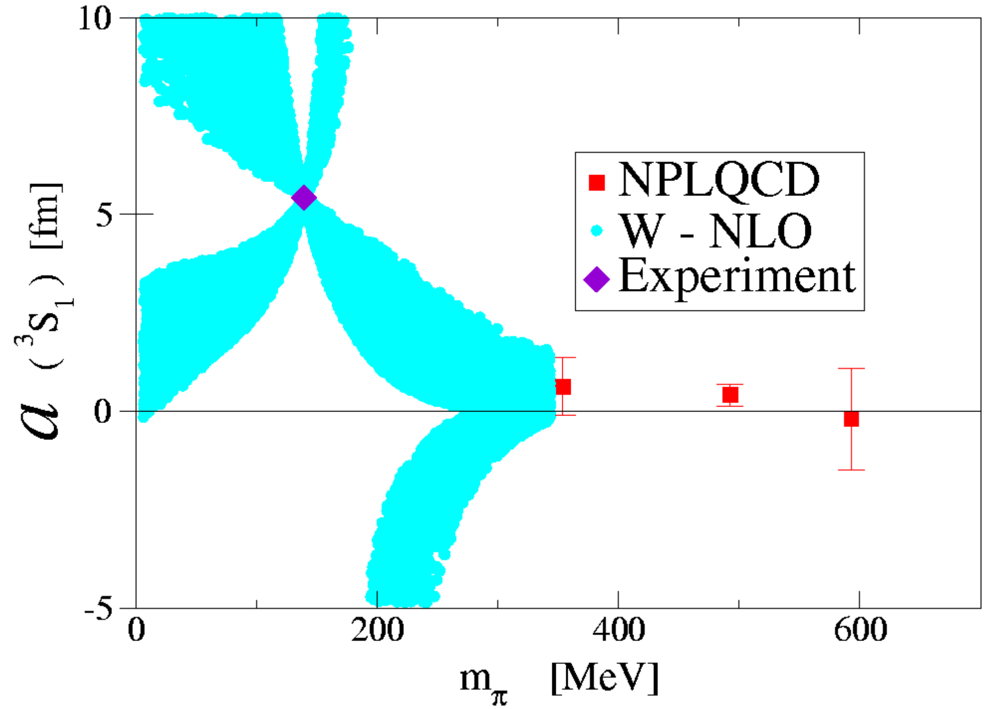
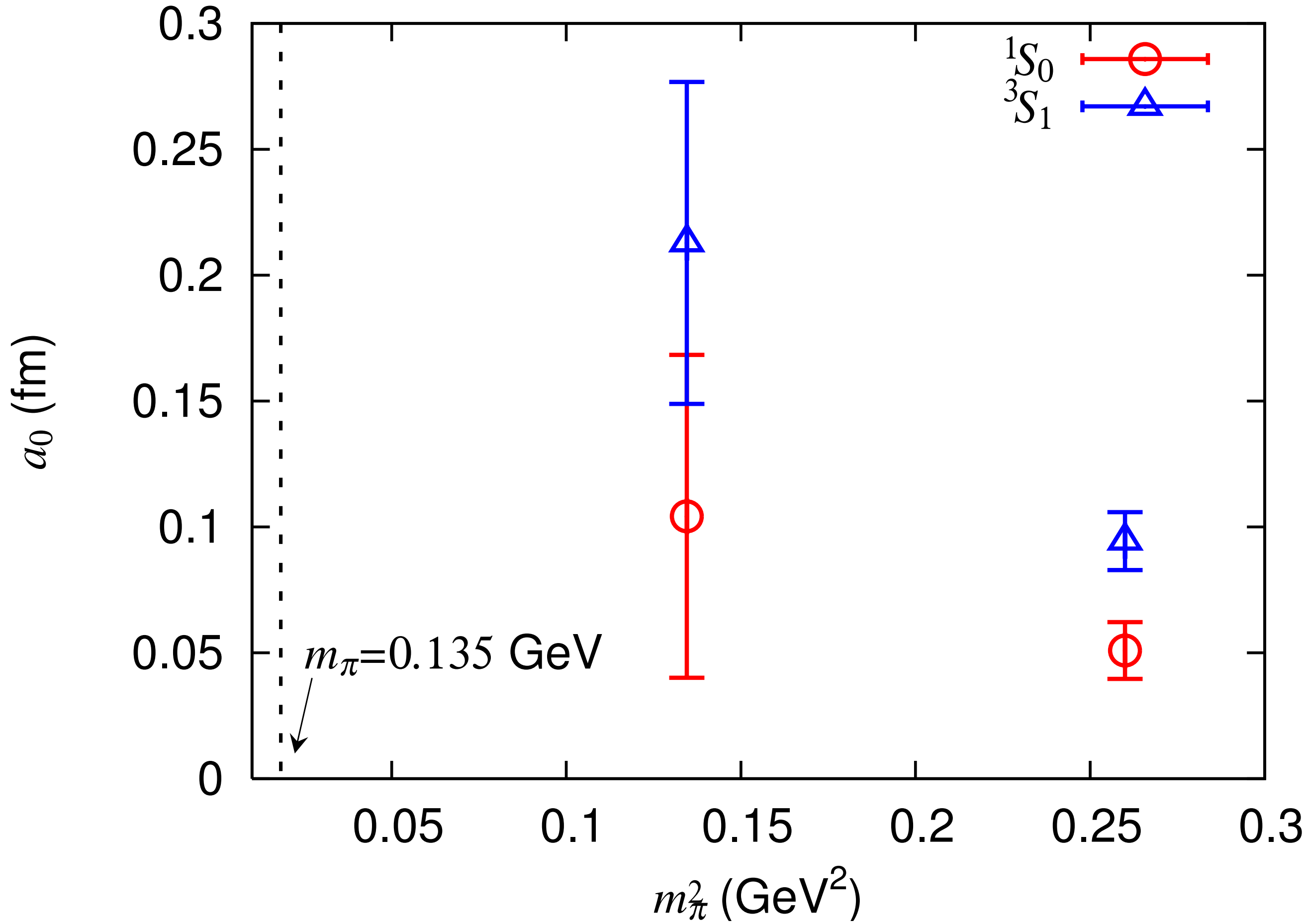
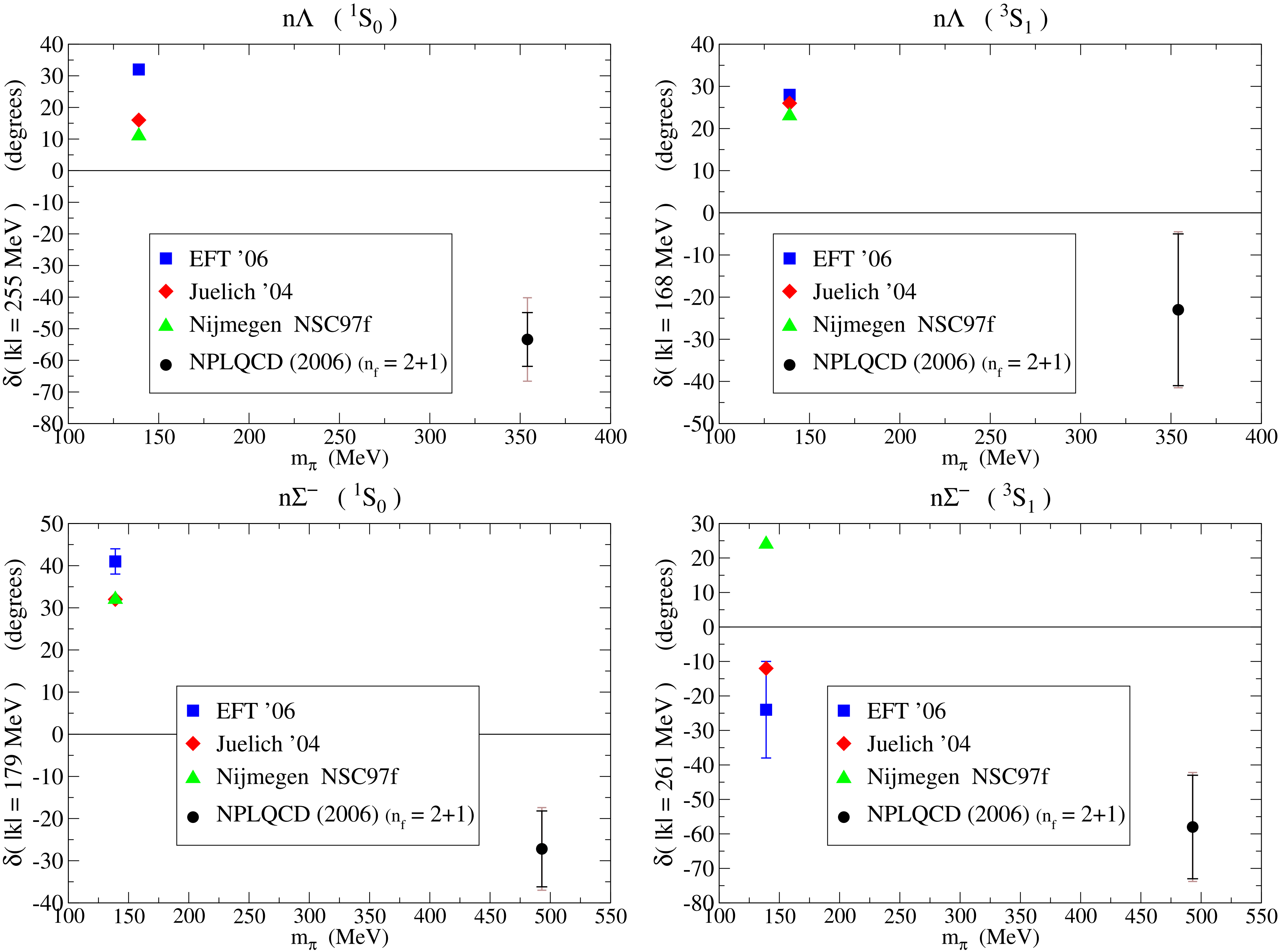
Aoki et al (2008) (n_f = 0)
Fukugita et al (1995) (n_f = 0)

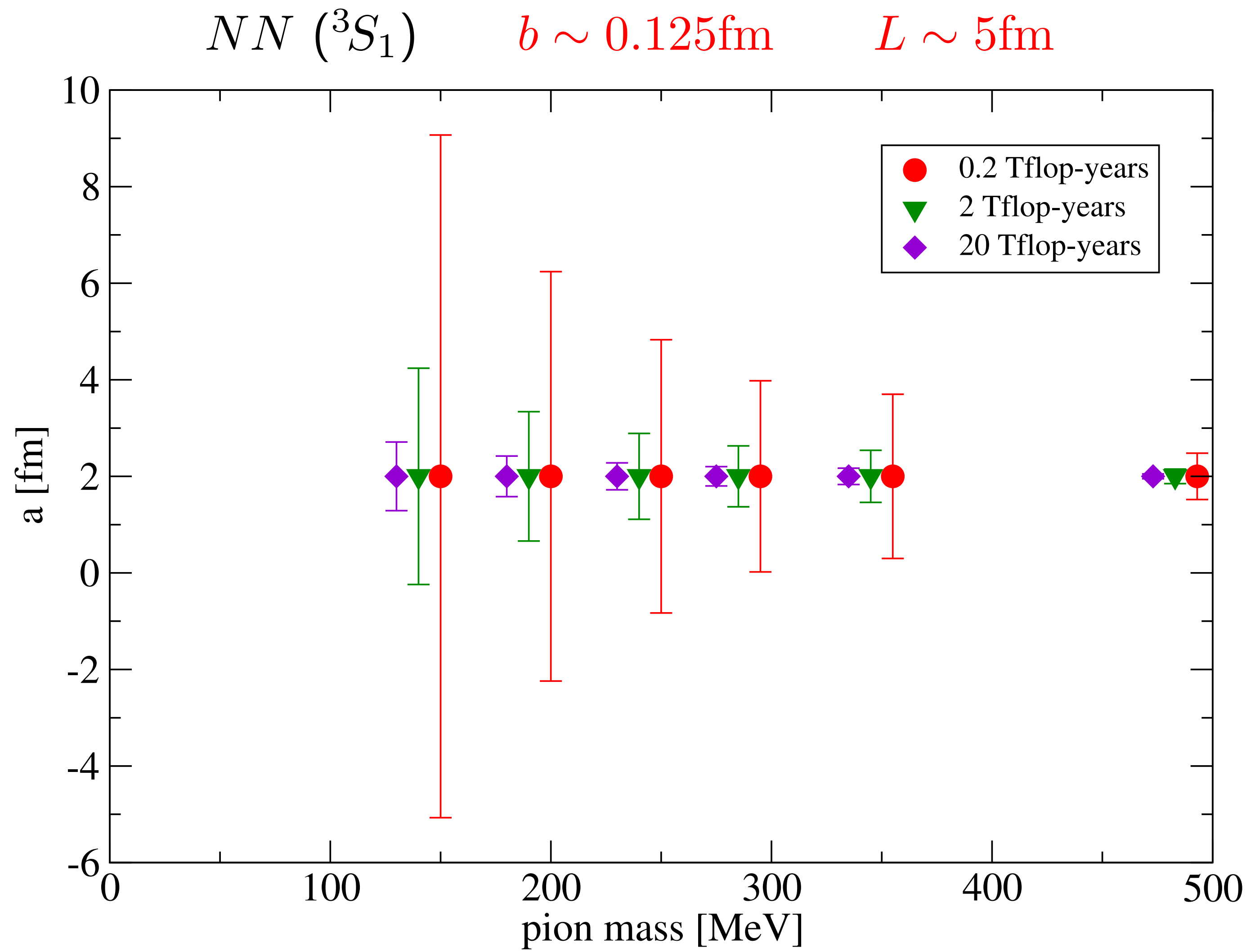

10
5
0
-5
a ($^3S_1$) [fm]
0
200
400
600
m_π [MeV]
NPLQCD
W - NLO
Experiment

$p\Xi_0$ (QQCD)



Nemura *et al.*/ arXiv:0806.1094/PAR-Wed-4:10pm


nΛ ($^1S_0$)
δ(|k| = 255 MeV) (degrees)
nΛ ($^3S_1$)
δ(|k| = 168 MeV) (degrees)
nΣ⁻ ($^1S_0$)
δ(|k| = 179 MeV) (degrees)
nΣ⁻ ($^3S_1$)
δ(|k| = 261 MeV) (degrees)
m_π (MeV)
EFT '06
Juelich '04
Nijmegen NSC97f
NPLQCD (2006) (n_f = 2+1)


$NN\ (^3S_1)$
$b \sim 0.125\text{fm}$
$L \sim 5\text{fm}$
0.2 Tflop-years
2 Tflop-years
20 Tflop-years
a [fm]
pion mass [MeV]
10
8
6
4
2
0
-2
-4
-6
0
100
200
300
400
500

Currently the main obstacle to lattice QCD calculations of nucleon and nuclear quantities is the signal/noise problem.

Nuclear physics requires exponentially more resources than meson physics.

The advent of petascale computing will overcome this obstacle and allow the calculation of nuclear properties and interactions!

The future is bright!