Physics results from dynamical overlap fermion simulations

Shoji Hashimoto (KEK)

@ Lattice 2008 at Williamsburg, Virginia, July 16, 2008.

JLQCD+TWQCD collaborations

- JLQCD
  - SH, T. Kaneko, H. Matsufuru, J. Noaki, E. Shintani, N. Yamada (KEK)
  - H. Ohki, T. Onogi (Kyoto)
  - S. Aoki, N. Ishizuka, K. Kanaya, Y. Kuramashi, Y. Taniguchi, A. Ukawa, T. Yoshie (Tsukuba)
  - K. Ishikawa, M. Okawa (Hiroshima)
  - H. Fukaya (Niels Bohr Inst)
- TWQCD
  - T.W. Chiu, T.H. Hsieh, K. Ogawa (National Taiwan Univ)
- Machines at KEK (since 2006)
  - SR11000 (2.15 Tflops)
  - BlueGene/L (10 racks, 57.3 Tflops)

Project: dynamical overlap fermions

- Theoretically clean =
  - Respect *the* symmetry
  - Slow to develop…

Project: dynamical overlap fermions

First large scale simulation with *exact* chiral symmetry

Theoretical interest	Phenomenological interest
• Dirac operator spectrum: Banks-Casher relation, chiral RMT • Chiral symmetry breaking: chiral condensate and related • Topology: θ-vacuum, topological susceptibility	• Controlled chiral extrapolation with the *continuum* ChPT • Physics applications: B_K, form factors, etc. • Sum rules, OPE • Flavor-singlet physics

Publications from the project

Not including conference proceedings

1. Fukaya et al. "*Lattice gauge action suppressing near-zero modes of H_W*," Phys. Rev. D, 094505 (2006).
2. Fukaya et al. "*Two-flavor QCD simulation in the ε-regime…*," Phys. Rev. Lett 98, 172001 (2007).
3. Fukaya et al. "*Two-flavor lattice QCD in the ε-regime…*," Phys. Rev. D76, 054503 (2007).
4. Aoki, Fukaya, SH, Onogi, "*Finite volume QCD at fixed topological charge*," Phys. Rev. D76, 054508 (2007).
5. Aoki et al., "*Topological susceptibility in two-flavor QCD…*," Phys. Lett. B665, 294 (2008).
6. Fukaya et al., "*Lattice study of meson correlators in the ε-regime…*," Phys. Rev. D77, 074503 (2008).
7. Aoki et al. "*B_K with two flavors of dynamical overlap fermions*," Phys. Rev. D77, 094503 (2008).
8. Aoki et al. "*Two-flavor QCD simulation with exact chiral symmetry*," arXiv:0803.3197 [hep-lat], to appear in PRD.
9. Noaki et al. "*Convergence of the chiral expansion…*," arXiv:0806.0894 [hep-lat].
10. Shintani et al. "*S-parameter and pseudo NG boson mass…*," arXiv:0806.4222 [hep-lat].
11. Ohki et al., "*Nucleon sigma term and strange quark content…*," arXiv:0806.4744 [hep-lat] .
12. Shintani et al., "*Lattice calculation of strong coupling constant…*," arXiv:0807.0556 [hep-lat].

Plan

1. Simulation status
   - Overlap implementation, runs, …
2. Topology issues
   - Physics from fixed topology
   - Topological susceptibility
3. Physics applications
   - Chiral condensate
   - Convergence of the chiral expansion (m_π, f_π)
   - Pion form factor, B_K
   - VV-AA, αs
   - Nucleon sigma-term, strange content

1. Simulation status

See also, Matsufuru, poster session

Overlap fermion

- Neuberger-Narayanan (1998)

$$D = \frac{1}{a}\left[1 + \frac{X}{\sqrt{X^\dagger X}}\right],\ X = aD_W - 1$$

$$= \frac{1}{a}\left[1 + \gamma_5 \,\mathrm{sgn}(aH_W)\right],\ aH_W = \gamma_5(aD_W - 1)$$

- Exact chiral symmetry through the Ginsparg-Wilson relation.

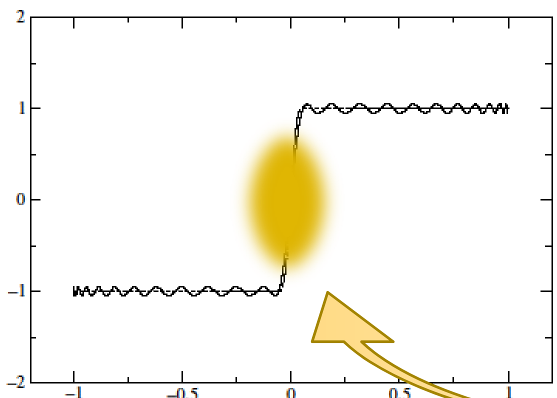
$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

  - Continuum-like Ward-Takahashi identities hold
  - Index theorem (relation to topology) satisfied

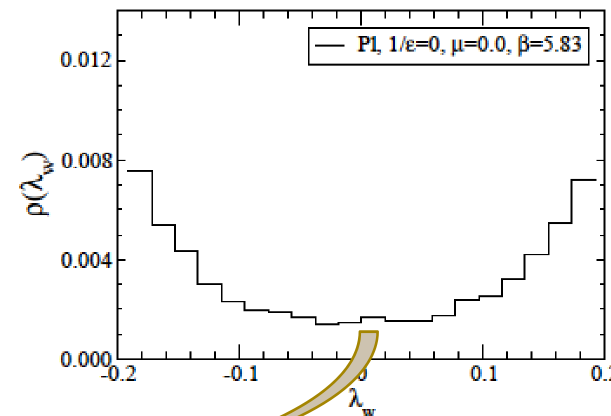
Sign function

- Rational approximation (Zolotarev)

$$\varepsilon[x] = x\left(p_0 + \sum_{l=1}^{N_{pole}} \frac{p_l}{x^2 + q_l} \right)$$



- Problem of near-zero modes of H_W.
  - Their density is non-zero at any finite β (Edwards, Heller, Narayanan, 1998)



Need to subtract before approximate
= potentially $O(V^2)$

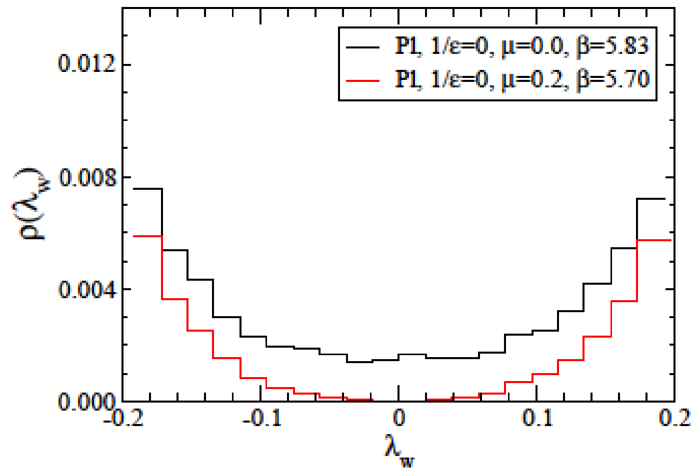
Near-zero mode suppression

- Near-zero modes are unphysical (associated with a local lump or dislocation = lattice artifact)

- lattice action to suppress them
  - Introduce unphysical (heavy negative mass) Wilson fermions (Vranas, JLQCD, 2006)

$$\det\left[\frac{H_W(-m_0)^2}{H_W(-m_0)^2+\mu^2}\right]$$

Plaquette gauge, β=5.83, μ=0; β=5.70, μ=0.2



Completely wash-out the near-zero modes.

Topological freezing

- Suppress the dislocations, e.g. by adding extra Wilson fermions

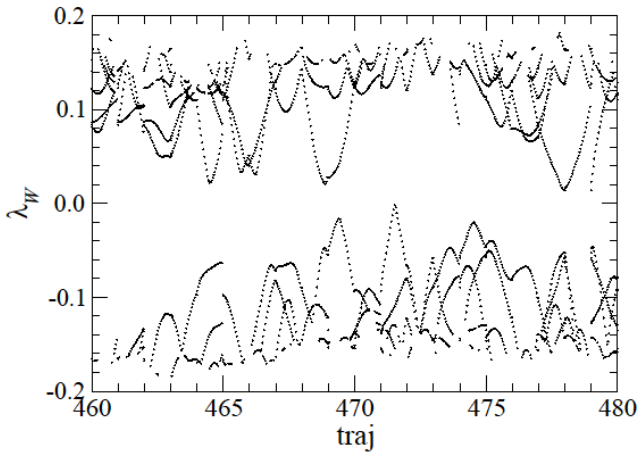
$$\det\left[\frac{H_W(-m_0)^2}{H_W(-m_0)^2+\mu^2}\right]$$

- Zero probability to have an exact zero-mode

⇕

- No chance to tunnel between different topological sectors.

- If the MD-type algorithm is used, the global topology never changes.
  - Provided that the step size is small enough.



Property of the continuum QCD: common for all lattice formulations as the continuum limit is approached.

Dynamical overlap

Recent attempts:

- Fodor-Katz-Szabo (2003)
  - Reflection/refraction trick
- Cundy et al. (2004)
  - Many algorithmic improvements
- DeGrand-Schaefer (2005)
  - Fat-link
  - Some physics results

Our project:

Aoki et al., arXiv:0803.3197 [hep-lat]

- Fixed topology
- Large scale simulation with $L \approx 2$ fm, $m_q \sim m_s/6$.
- 2-flavor and 2+1-flavor runs

Broad physics program:

- Pion/kaon physics
- ε-regime
- Nucleon sigma term etc.

Parameters

N_f = 2 runs

many physics analysis completed/on-going.

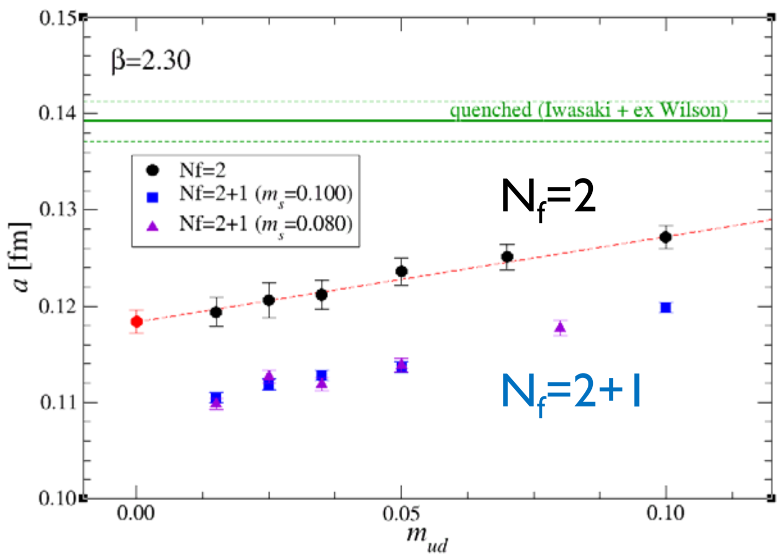
- β=2.30 (Iwasaki), a=0.12 fm, 16^3x32
- 6 sea quark masses covering $m_s/6 \sim m_s$
- 10,000 HMC traj.
- Q=0 sector only, except Q=–2, –4 runs at m_q=0.050

N_f = 2+1 runs

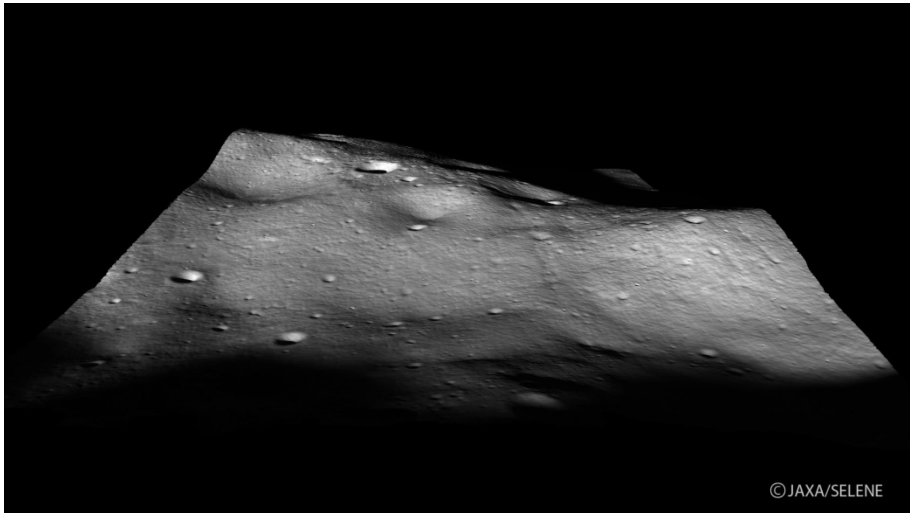
some physics analysis begun.

- β=2.30 (Iwasaki), a=0.11 fm, 16^3x48
- 5 ud quark masses, covering $m_s/6 \sim m_s$
- x 2 s quark masses
- 2,500 HMC traj.
  - Using 5D solver
- Q=0 sector only

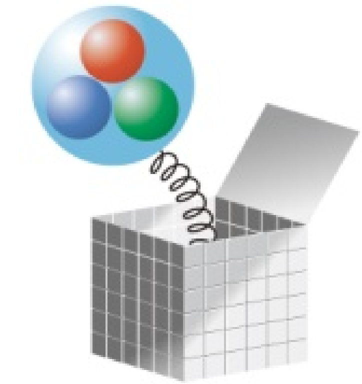
Lattice spacing



- β fixed (= 2.30) with varying m_q
  - Overlap fermion: close to the mass independent renormalization = no $O(am_q)$ term.
- Sommer scale r_0, from the static quark potential
  - $N_f = 2$
    $a = 0.118(2)$ fm
  - $N_f = 2+1$
    $a = 0.108(2)$ fm



SELENE (KAGUYA)



2. Topology issues

Instanton behind the moon…?

Topology is fixed. Any problem?

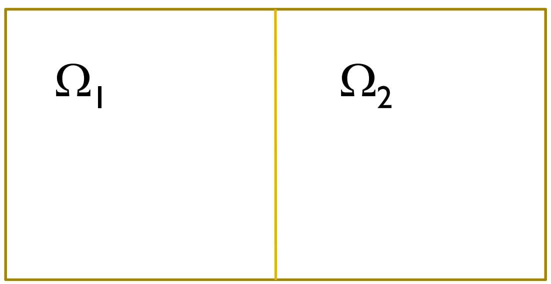
- Yes = the real QCD vacuum is the θ-vacuum, a superposition of different topological sectors.
- A serious problem for *everyone*
  - Topological tunneling occurs through rough gauge configs. If you observe frequent topology change, you are far apart from the continuum.
- A solution: *accept* it and reconstruct the θ-vacuum physics

Aoki, Fukaya, SH, Onogi, PRD76, 054508 (2007)

  - Finite volume effect of O(1/V)
  - Topological susceptibility calculable on a fixed topology configs.

Cluster decomposition

- In QCD, the real vacuum has a certain distribution of the topological charge = the θ vacuum.
  - Required to satisfy the cluster decomposition property: topology distribution must satisfy



$$f(Q_1+Q_2)=f(Q_1)\,f(Q_2)$$

$$\Rightarrow \quad f(Q)=e^{i\theta Q}$$

- Can one reproduce the physics of the θ vacuum from the fixed topology simulations?
  - Sum-up the topology! Or, not?

Sum-up the topology!

- Partition function of the vacuum $Z(\theta)=\exp\left[-VE(\theta)\right]$
  - Vacuum energy density E(θ) $E(\theta)=\frac{\chi_t}{2}\theta^2+\frac{c_4}{12}\theta^4+\ldots$
  - Partition function for a fixed Q

$$Z_Q=\frac{1}{2\pi}\int_{-\pi}^{+\pi}d\theta Z(\theta)e^{i\theta Q}=\frac{1}{2\pi}\int_{-\pi}^{+\pi}d\theta\exp\left[-VE(\theta)+i\theta Q\right]$$

  - Using a saddle point expansion around ($\theta_c=iQ/V$), one can evaluate the θ integral to obtain

$$Z_Q=\frac{1}{\sqrt{2\pi\chi_t V}}\exp\left[-\frac{Q^2}{2\chi_t V}\right]\left[1-\frac{c_4}{8V\chi_t}+O\left(\frac{1}{(\chi_t V)^2},\frac{Q^2}{(\chi_t V)^2}\right)\right]$$

  - Then, the original partition function can be recovered, if one knows χ_t, c_4, etc., as $Z(\theta)=\sum_Q Z_Q e^{-i\theta Q}$

Or, not?

- Fixing topology = Finite volume effect

Brower et al., PLB560, 64 (2003); Aoki, Fukaya, SH, Onogi, PRD76, 054508 (2007)

  - When the volume is large enough, the global topology is irrelevant.
  - Topological charge fluctuate locally, according to χ_t, topological susceptibility.
  - Physics of the θ-vacuum can be recovered by a similar saddle-point analysis, e.g. Some Green's function:

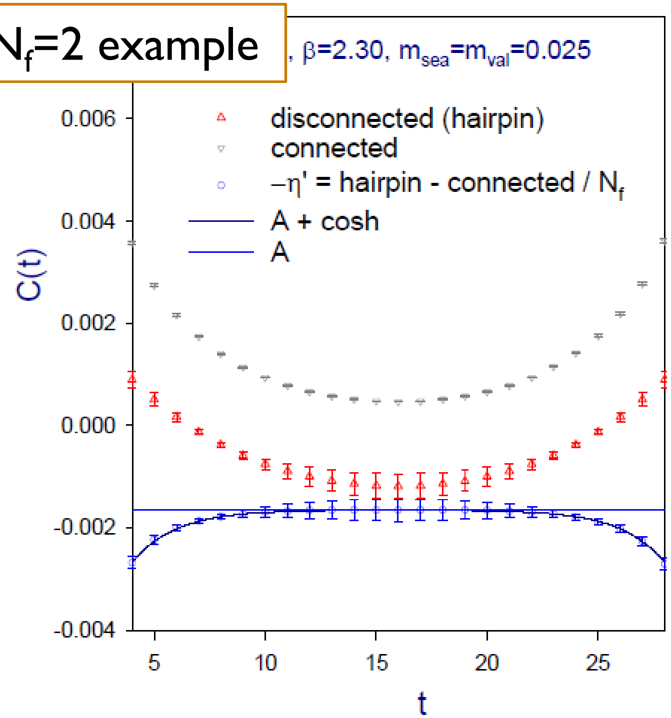
$$G_Q^{\mathrm{CPeven}} = G(0) + G^{(2)}(0)\frac{1}{2\chi_t V}\left[1 - \frac{Q^2}{\chi_t V} - \frac{c_4}{2\chi_t^2 V}\right] + G^{(4)}(0)\frac{1}{8\chi_t^2 V^2} + \ldots$$

Topological susceptibility $\chi_t = \langle Q^2 \rangle / V$

- Applying the same formula for the flavor-singlet PS density, χ_t can be extracted.

$$\lim_{x\to\infty}\langle mP(x)mP(0)\rangle_Q = -\frac{1}{V}\left(\chi_t - \frac{Q^2}{V} + \ldots\right) + O(e^{-m_{\eta'}x})$$

N_f=2 example

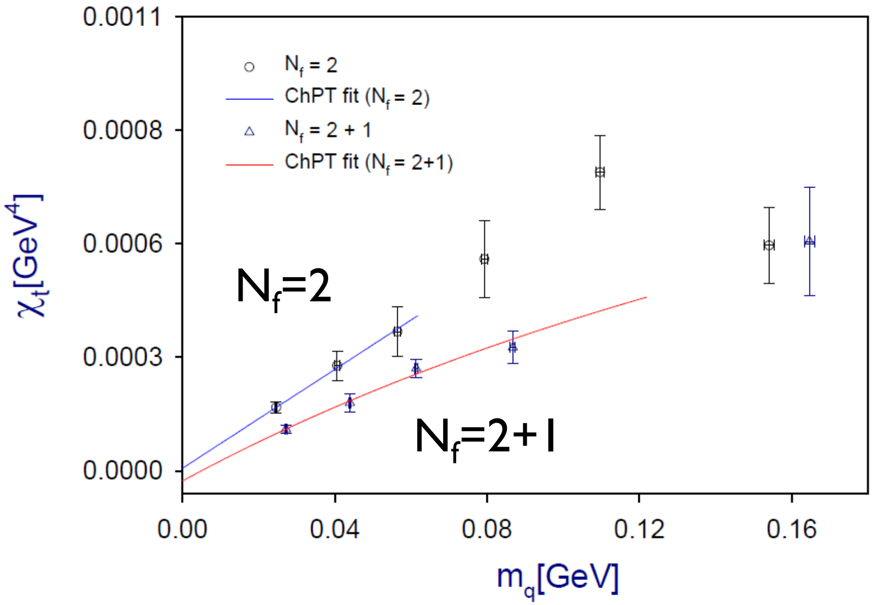


arXiv:0710.1130 [hep-lat]

Look at a (negative) constant correlation of the local topological charges.

- Found a clear plateau.
- Results from other topological sectors are consistent.

Sea quark mass dependence



Leutwyler-Smilga (1992)

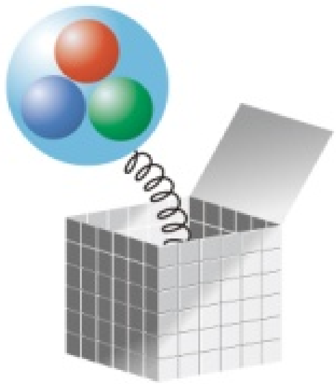
$$\chi_t = m\Sigma / N_f, \quad \text{or}$$

$$\chi_t = \frac{\Sigma}{m_u^{-1} + m_d^{-1} + m_s^{-1}}$$

- N_f=2 done; 2+1 on-going
  - Disconnected loops constructed from low modes (saturation confirmed)

Clear evidence of the sea quark effects: 2 and 2+1.

- Fit with ChPT expectation
  - N_f=2:
    Σ = [242(5)(10) MeV]3
  - N_f=2+1:
    Σ = [240(5)(2) MeV]3



3. Physics applications

Chiral condensate

Chiral condensate

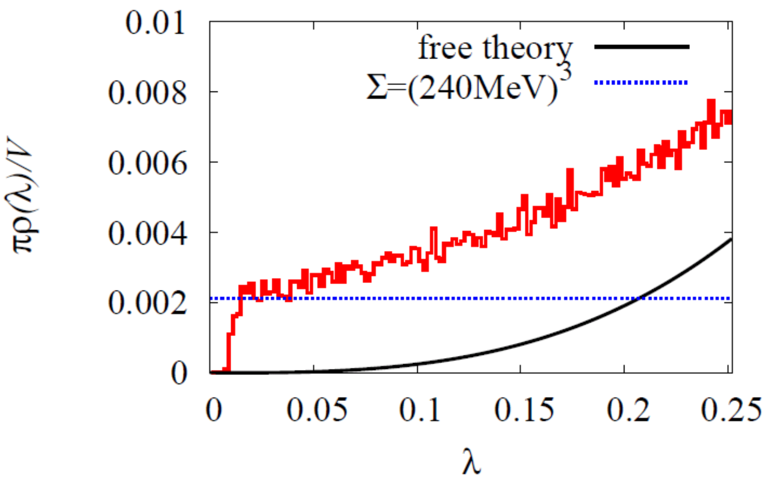
- Thanks to the exact chiral symmetry, additive renormalization is prohibited.

$$(\bar{\psi}\psi)^{cont} = Z_S (\bar{\psi}\psi)^{lat} + Z_{mix} \frac{1}{a^3}(1)^{lat}$$

- Many ways to extract
  - Banks-Casher relation
  - Low-lying eigenmodes (ChRMT)
  - ε-regime correlator
  - Topological susceptibility
  - GMOR relation

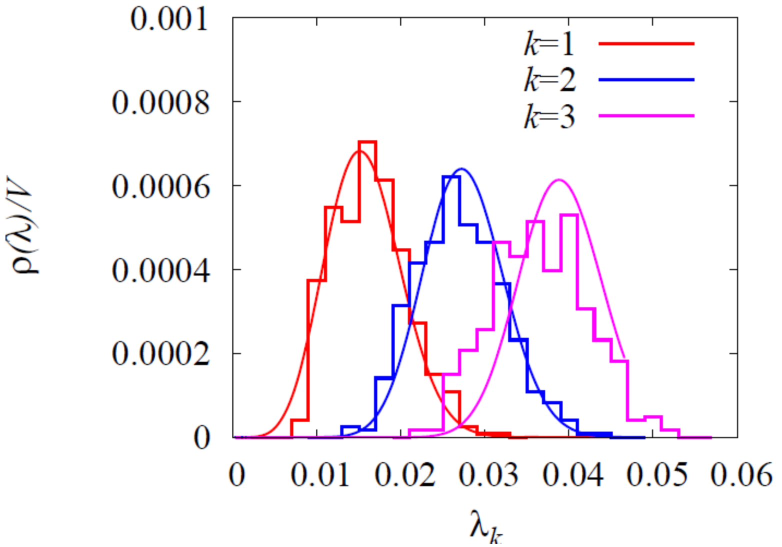
$$m_\pi^2 = (m_u + m_d)\frac{\Sigma}{f^2}[1+\ldots]$$

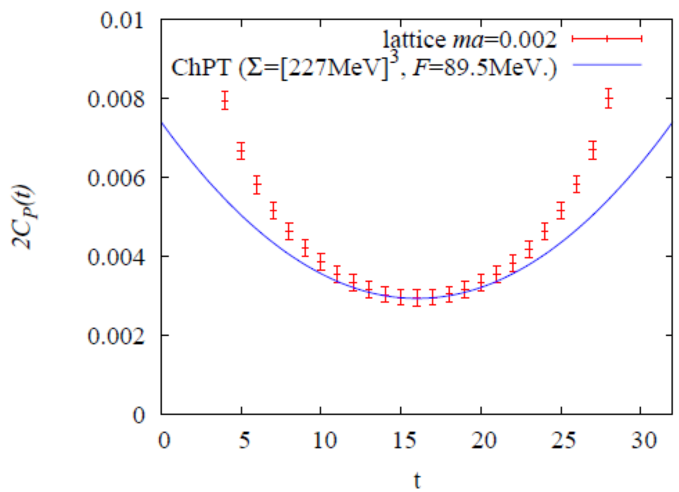
Banks-Casher relation (N_f=2)



ε-regime

- Matching the low-lying eigenvalue distribution with Chiral Random Matrix Theory (ChRMT)
- Matching the PS and A correlators with ε-regime ChPT





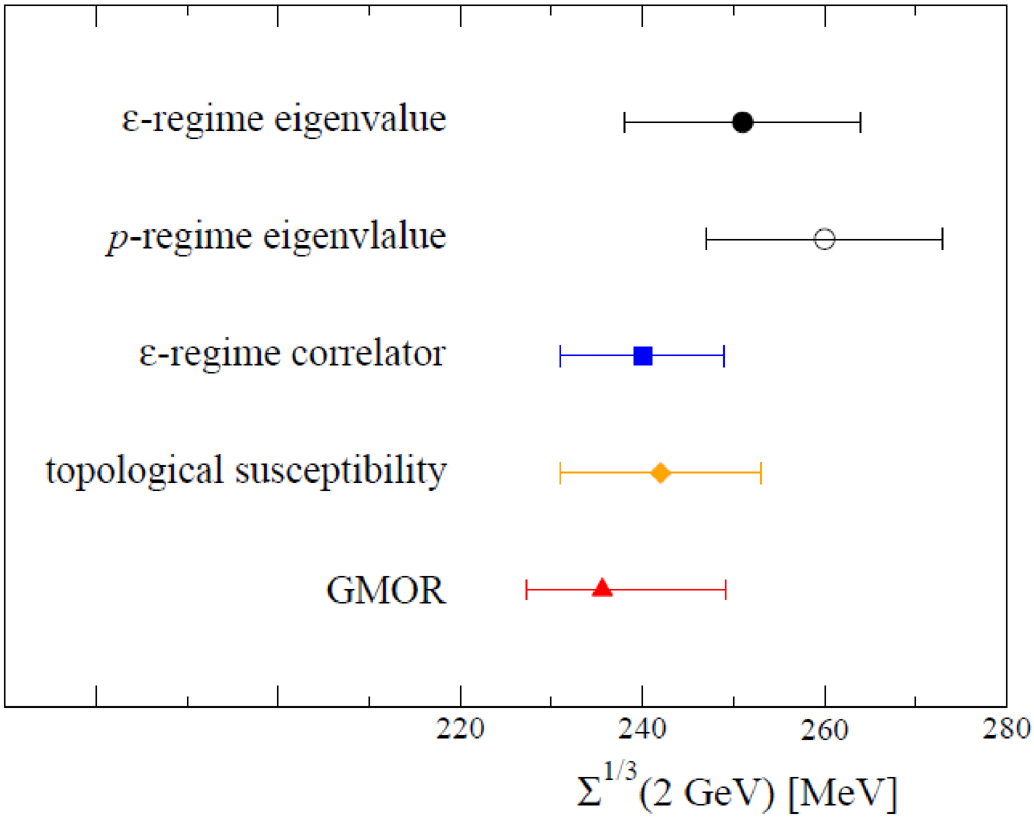
Σ = [251(7)(11) MeV]^3

F = 87(6)(8) MeV

Σ = [240(4)(7) MeV]^3

Two-flavor results

$\Sigma^{1/3}(2\,\mathrm{GeV})$



Phys. Rev. Lett 98, 172001 (2007)

Phys. Rev. D76, 054503 (2007)

Phys. Rev. D77, 074503 (2008)

arXiv:0710.1130 [hep-lat]

arXiv:0806.0894 [hep-lat]

In good agreement

m_π and f_π

Talk by Noaki, Tue 6:20, "Auditorium"

Convergence of chiral expansion

arXiv:0806.0894 [hep-lat]

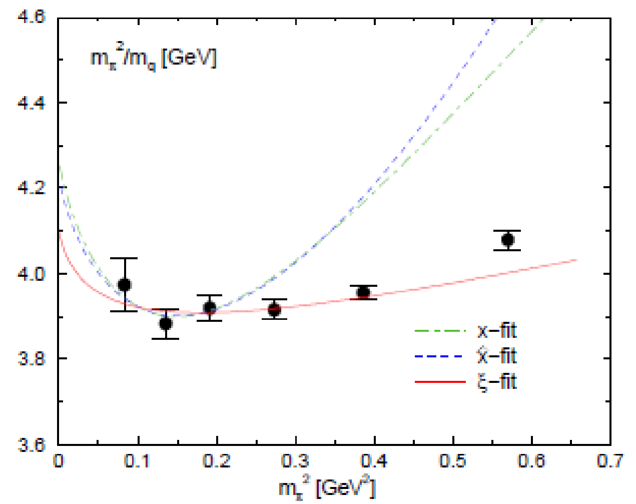
- Chiral expansion
  - The region of convergence is not known a priori.
  - Test with lattice QCD; conceptually clear with exact chiral symmetry.

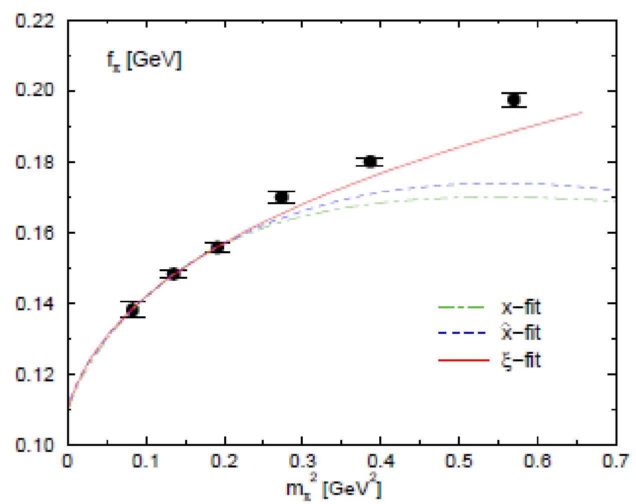
$$\frac{m_\pi^2}{m_q} = 2B\left[1 + x\ln x + c_3 x + O(x^2)\right],$$

$$f_\pi = f\left[1 - 2x\ln x + c_4 x + O(x^2)\right].$$

  - Expand in either

$$x \equiv \frac{m^2}{(4\pi f)^2},\ \hat{x} \equiv \frac{m_\pi^2}{(4\pi f)^2},\ \xi \equiv \frac{m_\pi^2}{(4\pi f_\pi)^2}$$





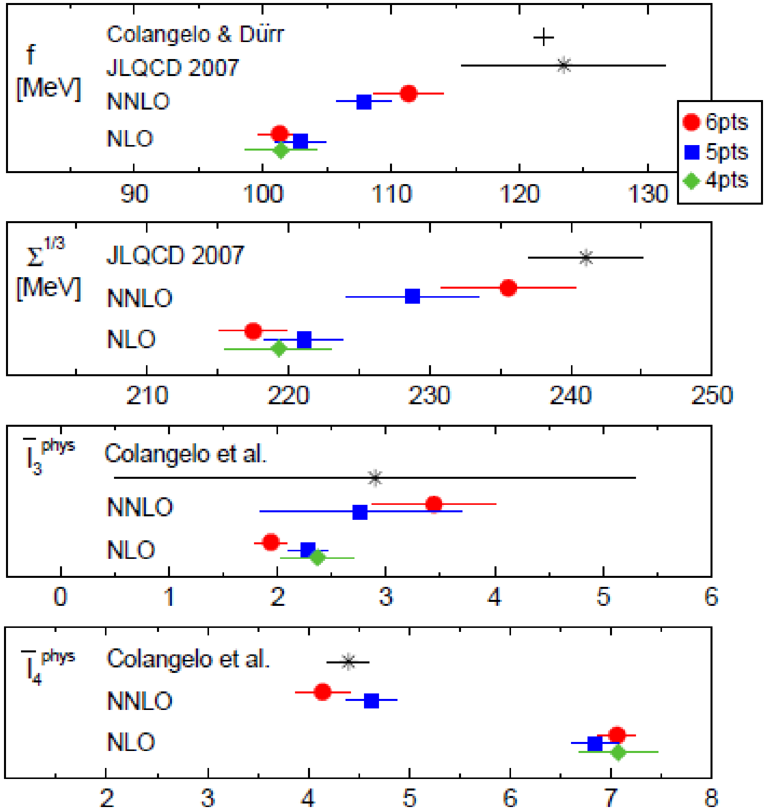
ξ extends the region significantly.

Two-loop analysis

- Analysis including NNLO
  - With the ξ-expansion

$$m_\pi^2/m_q = 2B\left[1 + \xi\ln\xi + \tfrac{7}{2}(\xi\ln\xi)^2 + \left(\tfrac{c_4}{2f} - \tfrac{4}{3}(\tilde{l}^{\,\mathrm{phys}} + 16)\right)\xi^2\ln\xi\right] + c_3\,\xi(1 - 9\xi\ln\xi) + \alpha\,\xi^2,$$

$$f_\pi = f\left[1 - 2\xi\ln\xi + 5(\xi\ln\xi)^2 + \tfrac{3}{2}(\tilde{l}^{\,\mathrm{phys}} + \tfrac{53}{2})\xi^2\ln\xi\right] + c_4\,\xi(1 - 10\xi\ln\xi) + \beta\,\xi^2.$$



  - For reliable extraction of the low energy constants, the NNLO terms are mandatory.

Talk by Noaki, Tue 6:20, "Auditorium"

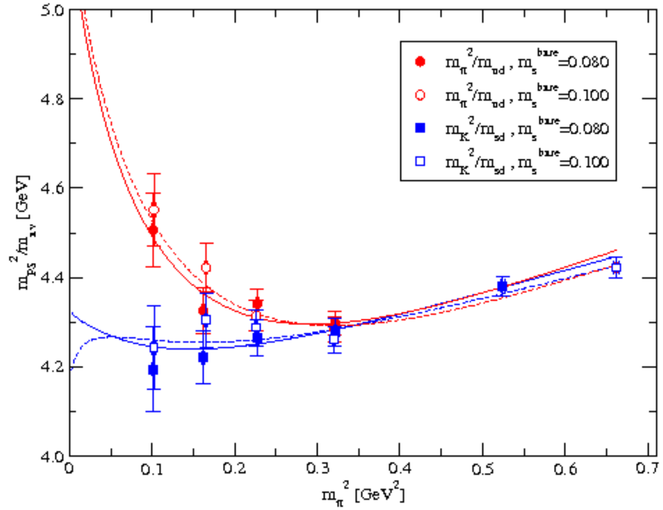
2+1 flavors

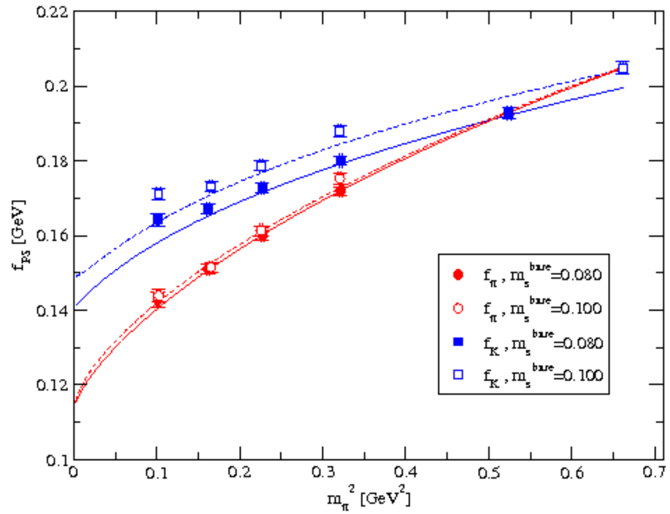
- Similar analysis including NNLO is on-going for 2+1-flavor data. Preliminary results.

$$\overline{m}_{ud}(2\text{GeV})=3.76(45)\,\text{MeV},$$

$$\overline{m}_{s}(2\,\text{GeV})=116(12)\,\text{MeV},$$

$$\frac{f_K}{f_\pi}=1.201(30).$$





Pion form factors

Talk by Kaneko, Thu 9:10, "Chesapeake A"

Pion form factors

- Another testing ground of ChPT
  - Vector and scalar

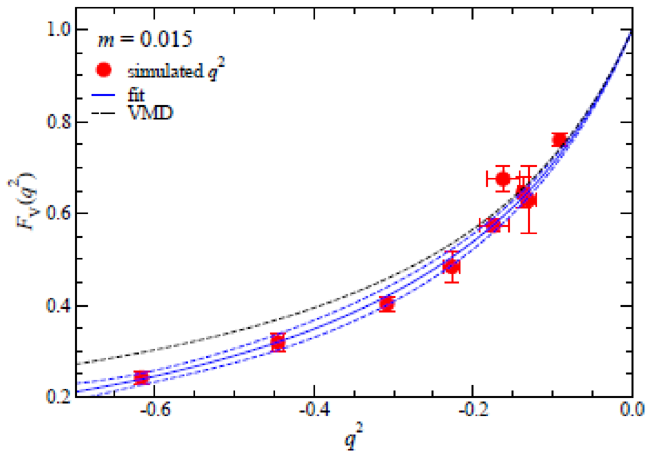
$$\langle \pi(p')|V_\mu|\pi(p)\rangle = i(p_\mu + p_\mu')F_V(q^2),$$
$$\langle \pi(p')|S|\pi(p)\rangle = F_S(q^2),\ q_\mu \equiv p_\mu' - p_\mu$$

  - Charge and scalar radius

$$F_V(q^2) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi q^2 + O(q^4),$$
$$F_S(q^2) = F_S(0)\left[1 + \frac{1}{6}\langle r^2\rangle_S^\pi q^2 + O(q^4)\right],$$

  - Calculation using the all-to-all technique.

Vector form factor

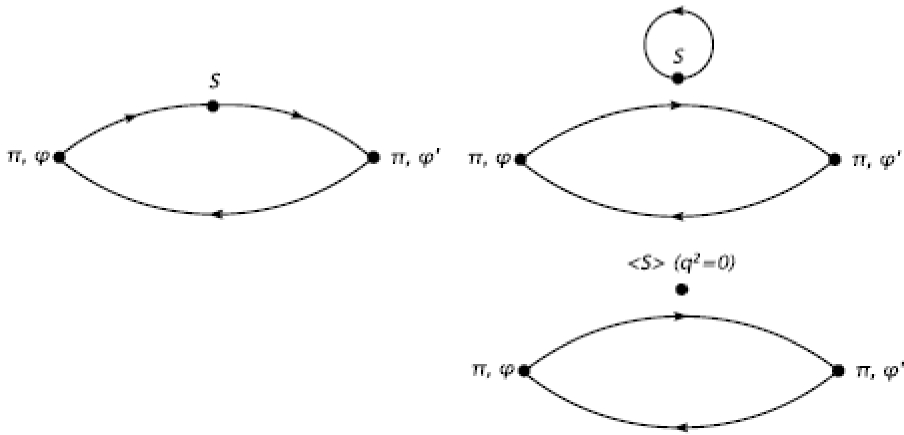


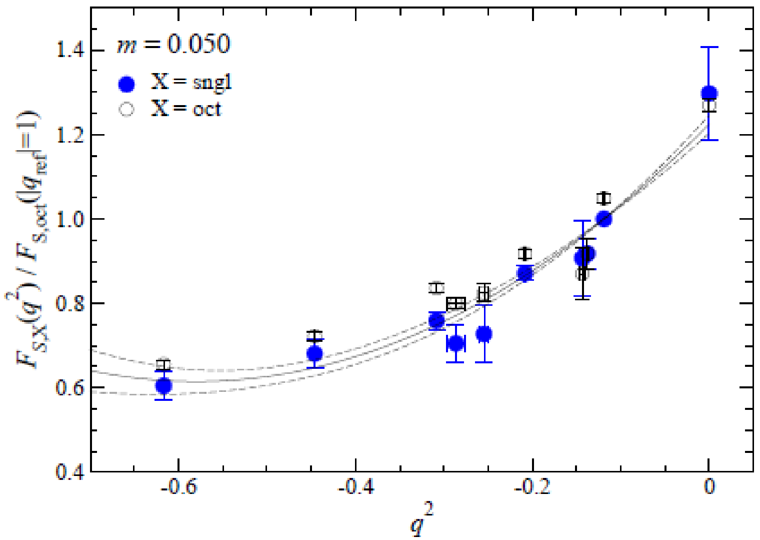
q^2 dependence well described by a vector meson pole + corrections.

$$F_\pi(q^2) = \frac{1}{1 - q^2/m_V^2} + c_1 q^2 + \ldots$$

All-to-all

- Disconnected diagram
  - Relevant for the scalar form factor.
  - Calculated using the all-to-all technique.
  - Lowmodes are averaged over space-time.

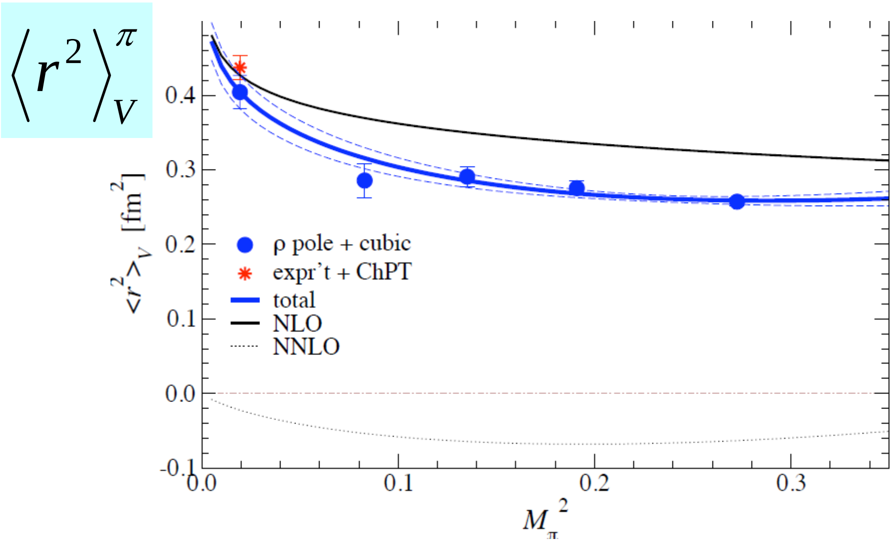


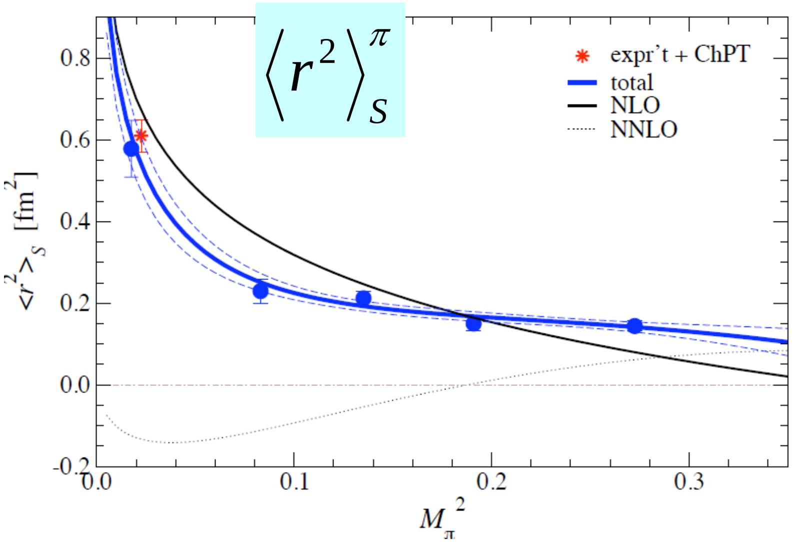


Disconnected contribution is visible.

Chiral extrapolation

- Fit with NNLO ChPT
  - Data do not show clear evidence of the chiral log. But, it is expected to show up even smaller pion masses.
  - NNLO contribution is significant; necessary to reproduce the phenomenological values.





Vacuum polarization functions

Vacuum polarization functions

- Vector and axial correlators in the momentum space.

$$\langle J_\mu J_\nu \rangle = \left(g_{\mu\nu} q^2 - q_\mu q_\nu\right) \Pi_J^{(1)}(Q^2) - q_\mu q_\nu \Pi_J^{(0)}(Q^2)$$

$$= \int_0^\infty \frac{ds}{s - q^2 + i\varepsilon} \left[\left(g_{\mu\nu} s^2 - s_\mu s_\nu\right) \mathrm{Im}\,\Pi_J^{(1)}(s) - s_\mu s_\nu\, \mathrm{Im}\,\Pi_J^{(0)}(s) \right]$$

  - Directly calculable on the lattice for space-like momenta
- Weinberg sum rules:

$$f_\pi^2 = -\lim_{Q^2 \to 0} Q^2 \left[\Pi_V^{(1+0)}(Q^2) - \Pi_A^{(1+0)}(Q^2) \right],$$

$$S = -\lim_{Q^2 \to 0} \frac{\partial}{\partial Q^2} Q^2 \left[\Pi_V^{(1+0)}(Q^2) - \Pi_A^{(1+0)}(Q^2) \right] \quad \text{or } L_{10}$$

  - Another probe of the chiral symmetry breaking.
  - S is relevant for the precision EW test of new strong dynamics.

Pion electromagnetic mass splitting

- Das-Guralnik-Mathur-Low-Young sum rule (1967)

$$\Delta m_\pi^2 = -\frac{3\alpha_{\rm EM}}{4\pi f_\pi^2}\int_0^\infty dQ^2\; Q^2\left[\Pi_V^{(1+0)}(Q^2)-\Pi_A^{(1+0)}(Q^2)\right]$$

  - Valid in the chiral limit (soft pion theorem)
  - Gives dominant contribution to the $\pi^\pm$-π^0 splitting.
  - Related to the pseudo-NG boson mass in the context of new strong dynamics.
- Exact chiral symmetry is essential.
  - The quantity of interest is obtained after huge cancellation between V and A.

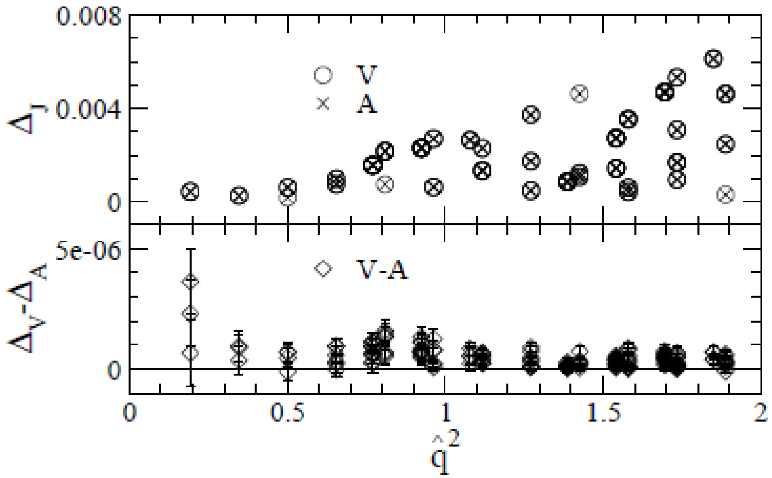
Lattice artifact

- Lorentz violation + currents not conserving

$$\Pi_{J\mu\nu}(q) = \int d^4x e^{iq\cdot x} \langle 0 | T\left[J_\mu(x) J_\nu(y) \right] | 0 \rangle = \sum_{n=0}^{\infty} B_J^{(n)} q_\mu^{2n} + \sum_{m,n=1}^{\infty} C_J^{(m,n)} q_\mu^{2m-1} q_\nu^{2n-1}$$

  - J = V or A.
  - Only $B_J^{(0)}$ and $C_J^{(1,1)}$ are physical.
  - Thanks to the exact chiral symmetry, B_J and C_J are common (up to m_q) between V and A, thus cancel in V-A.

$$\Delta_J = \sum_{\mu,\nu} \hat{q}_\mu \hat{q}_\nu \left(\frac{1}{\hat{q}^2} - \frac{\hat{q}_\nu}{\sum_\lambda (\hat{q}_\lambda)^3} \right) \Pi_{J\mu\nu}$$



Lattice results

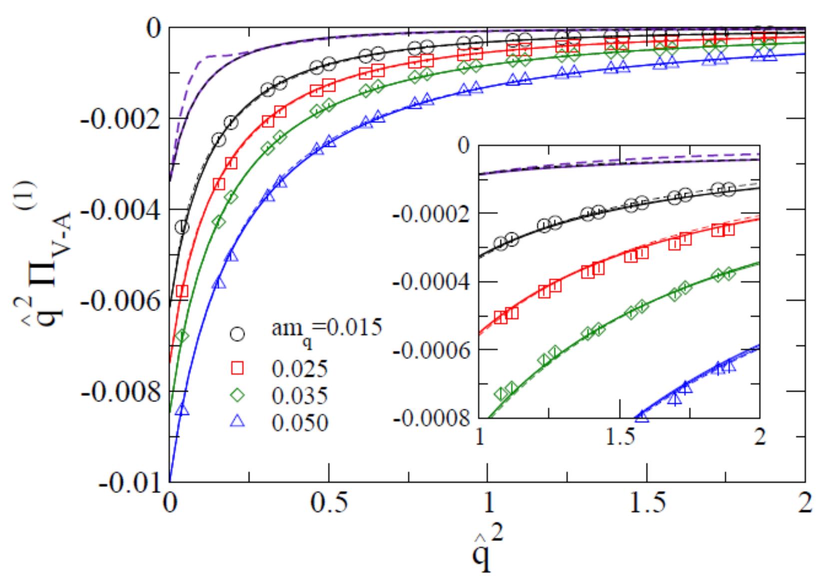
- Can be fitted with
  - ChPT in the low q^2 region

$$\Pi^{(1)}_{V-A}(q^2) = -\frac{f_\pi^2}{q^2} - 8L^r_{10}(\mu)$$
$$-\frac{1}{24\pi^2}\left[\ln\frac{m_\pi^2}{\mu^2} + \frac{1}{3} - H(x)\right]$$

L_{10} is extracted.

$$L^r_{10}(m_\rho) = -5.2(2)(^{+0}_{-3})(^{+5}_{-0})\times 10^{-3}$$

arXiv:0806.4222 [hep-lat].



  - OPE in the high q^2 region. In the massless limit, $1/Q^6$ is the leading.
  - Summing up the two regions, Δm_π^2 is obtained.

$$\Delta m_\pi^2 = 993(12)(^{+\ \ 0}_{-135})(149)\ \mathrm{MeV}^2$$

Exp: $\Delta m_\pi^2 = 1261.2\ \mathrm{MeV}^2$

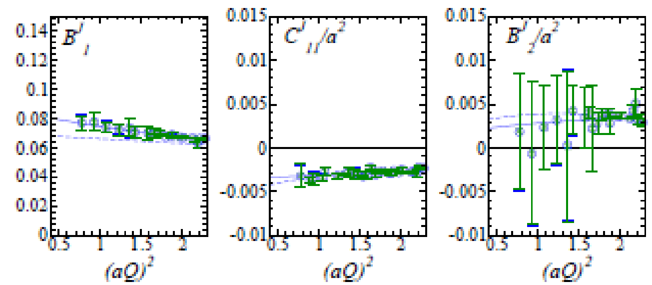
Strong coupling constant

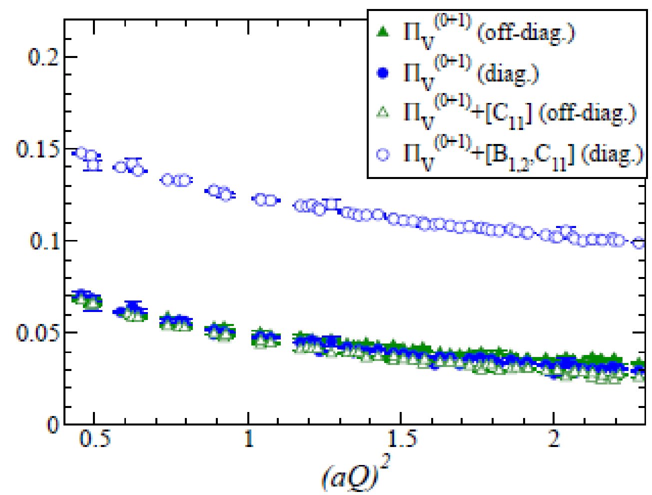
- Matching of $\Pi_J^{(0+1)}(Q^2)$ with its perturbative expansion
  - Adler function

$$D_J(Q^2) \equiv -Q^2 \frac{d\Pi_J(Q^2)}{dQ^2}$$

is finite, renormalization scheme independent.

- Lattice artifacts are non-perturbatively subtracted.





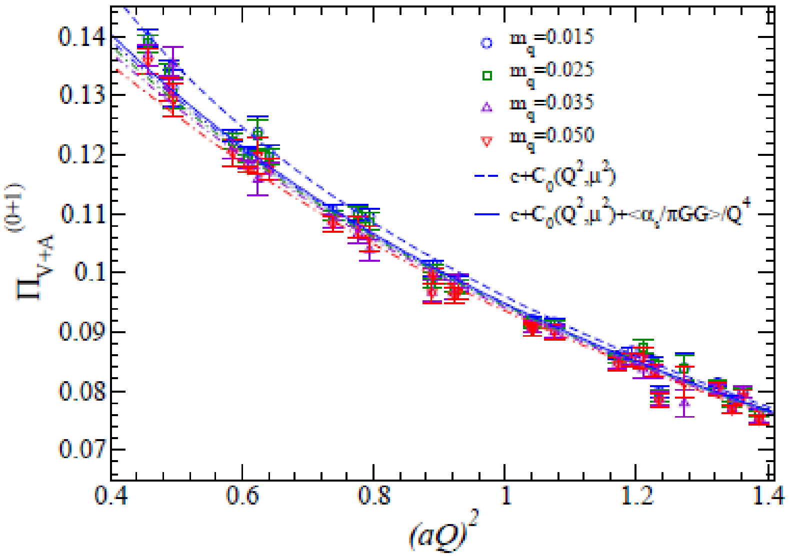
$$\langle J_\mu J_\nu \rangle^{\rm lat}(Q) = \Pi_J^{(1)}(Q)Q^2\delta_{\mu\nu} - \Pi_J^{(0+1)}(Q)Q_\mu Q_\nu$$
$$- \sum_{n=0} B_n^J(Q) Q_\mu^{2n}\delta_{\mu\nu} - \sum_{m,n=1} C_{mn}^J(Q)\{Q_\mu^{2m+1}Q_\nu^{2n-1} + Q_\nu^{2m+1}Q_\mu^{2n-1}\}$$

Talk by Shintani, Fri 3:10, "Chesapeake A"

Strong coupling constant

- High Q^2 region described by OPE.

$$\Pi_J^{(0+1)}(Q^2) = c + C_0(Q^2,\mu^2) + \frac{C_m^J(Q^2)}{Q^2} + C_{qq}^J(Q^2)\frac{\langle m\bar{q}q\rangle}{Q^4} + C_{GG}(Q^2)\frac{\langle \frac{\alpha_s}{\pi} GG\rangle}{Q^4} + \ldots$$



  - Perturbative expansions known to α_s^2.
  - Chiral condensate is an input.

$$\Lambda_{\overline{MS}}^{(2)} = 234(9)\left(^{+16}_{-\ 0}\right)\,\mathrm{MeV}$$

cf.) ALPHA: 250(16)(16) MeV
QCDSF: 249(16)(25) MeV

Nucleon structure

Nucleon sigma term

- Finite quark mass effect on the nucleon mass

$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

  - Contains both connected and disconected contrib.

- Strange quark content

$$y \equiv \frac{2\langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle}$$

  - Disconnect contrib only.

- Feynman-Hellman theorem:
  - Relates them to derivatives of nucleon mass in terms of m_{val} and m_{sea}.

$$\frac{\partial M_N}{\partial m_{\mathrm{val}}} = \langle N | \bar{u}u + \bar{d}d | N \rangle_{\mathrm{conn}},$$

$$\frac{\partial M_N}{\partial m_{\mathrm{sea}}} = \langle N | \bar{u}u + \bar{d}d | N \rangle_{\mathrm{disc}}$$

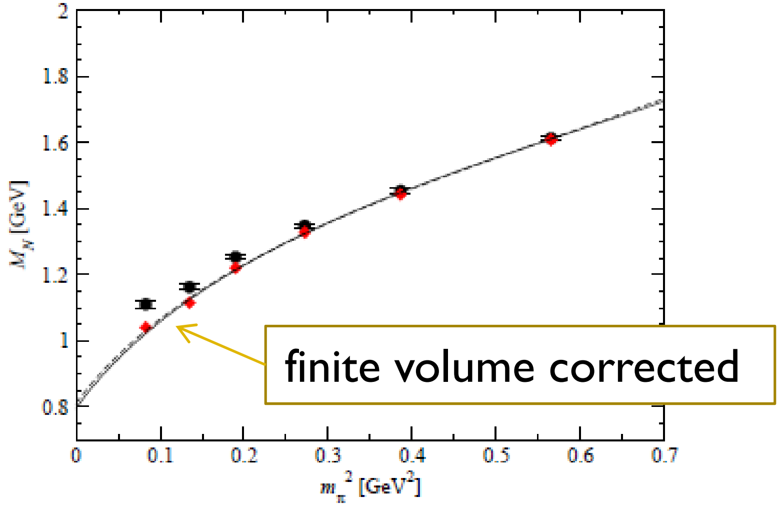
$$\left(= 2\langle N | \bar{s}s | N \rangle\right)$$

  - Analysis with partially quenched data set ($m_{val} \neq m_{sea}$)
  - Use PQChPT

Talk by Ohki, Thu 9:50, "Auditorium"

Fit with HBChPT

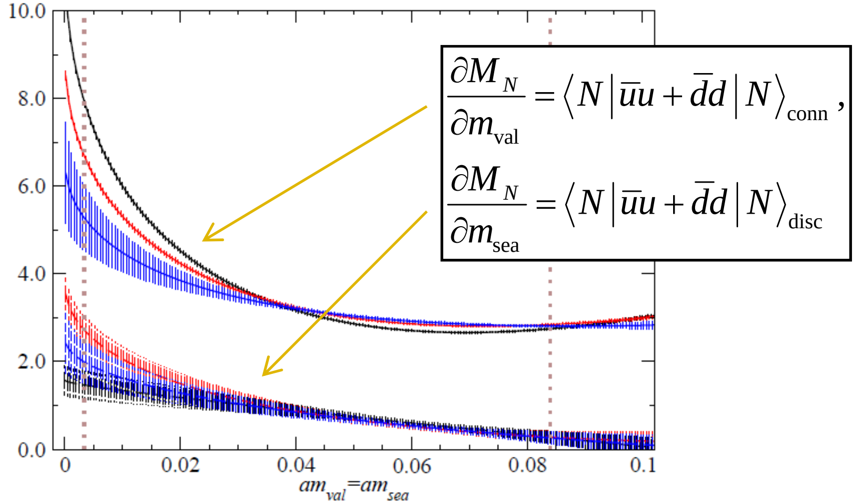
- Nucleon mass



- Finite volume effect significant.
- Downward shift observed

$$\sigma_{\pi N} = 52(2)(^{+20}_{-\ 2})(^{+5}_{-0})\ \text{MeV}$$

- Valence and Sea derivatives



- Disconnected contribution relatively small.

$$y = 0.030(16)(^{+6}_{-8})(^{+1}_{-2})$$

Talk by Ohki, Thu 9:50, "Auditorium"

So what?

- Previous lattice results:
  - Fukugita et al. (1995)
    $y = 0.66(15)$, quenched
  - Dong-Lagae-Liu (1996)
    $y = 0.36(3)$, quenched
  - SESAM (1999)
    $y = 0.59(13)$, $N_f=2$
  - UKQCD (2001)
    $y = -0.28(33)$, $N_f=2$
  - JLQCD (2008)
    $y = 0.030(18)$, $N_f=2$

- Problem and solution
  - Was difficult to calculate due to m_{sea} dependence of m_{cr}.= easily spoils the physical effect.
  - Problem persists in the quenched calculation.
  - If subtracted, too large error.
  - Exact chiral symmetry is the key.

Conclusion

Dynamical overlap fermion

= clean approach producing interesting physics.

- Feasible with O(10 Tflops) machines
  - 16^3x48 → 24^3x48: test runs started
- Frozen topology = the property of continuum QCD
  - New strategy successful, e.g. topological susceptibility
- Physics applications (so far)
  - Chiral condensates
  - Test of *continuum* ChPT
  - Sum rules, OPE
  - Nucleon structure, flavor-singlet physics
  - More to come…

Backup slides

Locality

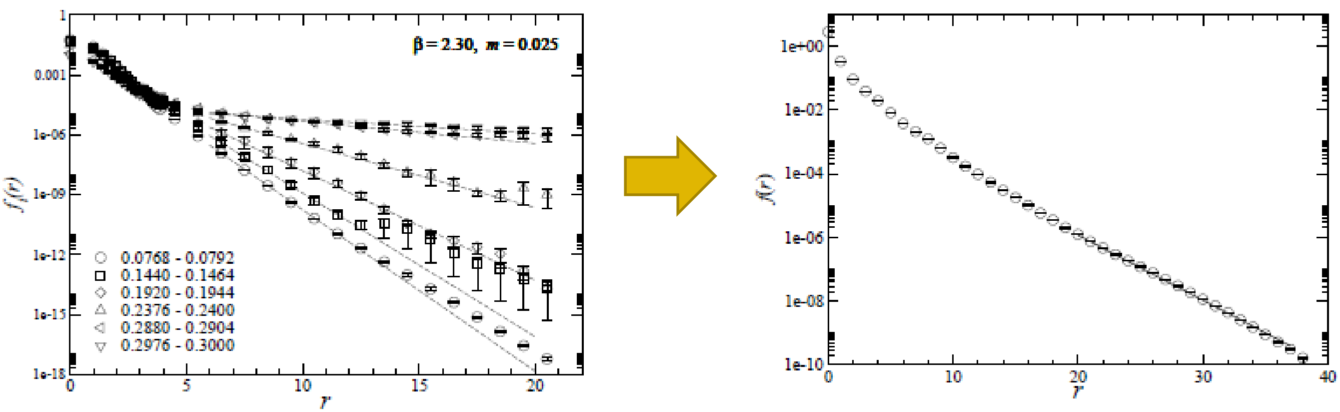
- Lattice Dirac operator must be local in order that a local theory is obtained in the continuum limit.
  - Locality is not obvious for the overlap operator due to 1/√.

$$D = \frac{1}{a}\left[1 + \frac{X}{\sqrt{X^\dagger X}}\right],\ X = aD_W - 1$$

  - Locality in the sense that |D|<exp(-μx), with μ a number of order 1/*a*, may be satisfied.
  - "Proof" is known for smooth enough gauge canfigurations (Hernandez, Jansen, Luscher (1999)).
  - No mathematical proof in more realistic situations where there is non-zero density of the near-zero modes.
    ⇒ Okay if near-zero modes are always localized.

Locality

- Maybe analyzed by looking at individual eigenmodes of H_W.
  - Near-zero modes are more localized. Higher modes are extended. There is a critical value above which the modes are extended = "mobility edge" (Golterman, Shamir (2003)).



  - An important lessen: do not use the overlap fermion in the Aoki phase (where the near-zero modes are extended).

FAQ on topology fixing

1. Extra Wilson fermions:

Don't they spoil the continuum limit or the $O(a^2)$ scaling?

- No. They are heavy: $m\sim 1/a$. Low-lying modes of H_W are local and irrelevant in the continuum limit.

2. Ergordicity:

Is the ergordicity maintained?

- No. HMC visits only the fixed topological sector. It restricts the path integral to a given Q. But the same physics can be obtained as explained above.
- Probably yes, within a given Q. A fixed Q manifold is connected in the continuum theory. No proof on the lattice; no counter example, either. Lattice aims at approaching the continuum, anyway.

Measurement techniques

Measurements at every 20 traj ⇒
  500 conf / m_{sea}

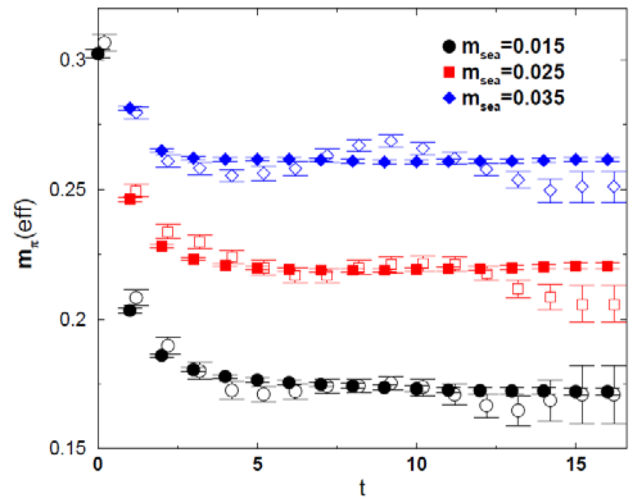
- Improved measurements
  - 50 pairs of low modes calculated and stored.
  - Used for low mode preconditioning (deflation)

    ⇒ (multi-mass) solver is then x8 faster
  - Low mode averaging

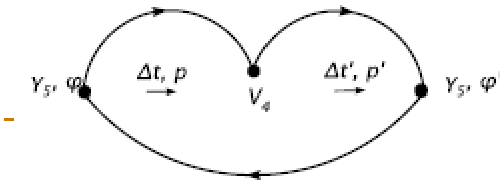
    (and all-to-all)

$$D_m^{-1}(x,y) = \sum_{k=1}^{N} \frac{u_k(x)u_k^\dagger(y)}{\lambda_k + m} + D_m^{(h)-1}(x,y)$$

$$C(x,y) = C^{ll}(x,y) + C^{hh}(x,y) + C^{hl}(x,y) + C^{lh}(x,y)$$



All-to-all



To improve the signal

- Usually, the quark propagator is calculated with a fixed initial point (one-to-all)
- Average over initial point (or momentum config) will improve statistics; possible with all-to-all

$$D^{-1}(x,y) = \sum_{k=1}^{N_{ev}} \frac{1}{\lambda^{(k)}} u^{(k)}(x) u^{(k)\dagger}(y) + \sum_{d=1}^{N_d} \left[D_{high}^{-1} \eta^{(d)} \right](x) \eta^{(d)}(y)$$

Low mode contribution

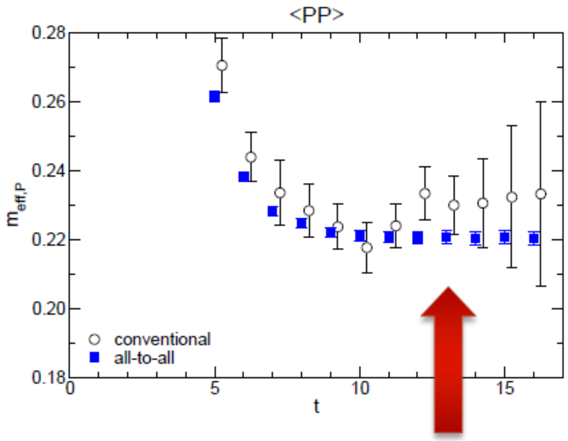
High mode propagation From the random noice

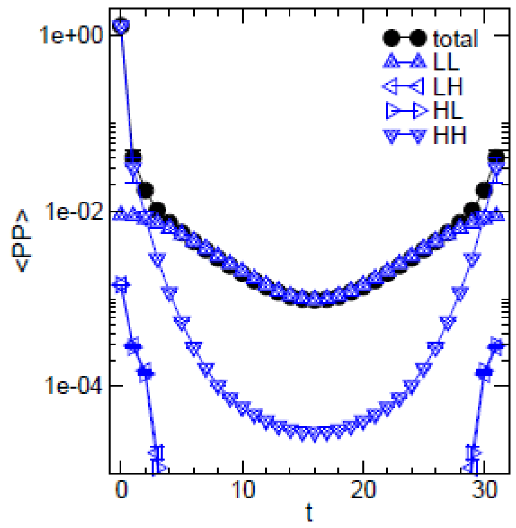
Random noise

An example: two-point func

Dramatic improvement of the signal, thanks to the averaging over source points

- Similar to the low mode averaging; but all-to-all can be used for any n-point func.
- PP correlator is dominated by the low-modes



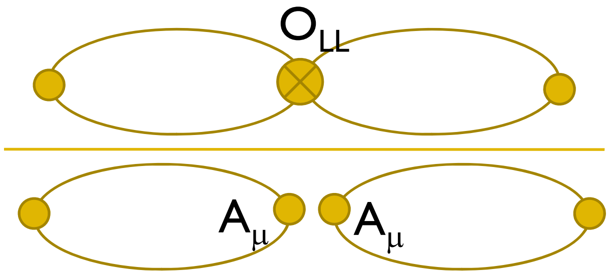


B_K

First (unquenched) lattice calculation with exact chiral symmetry:

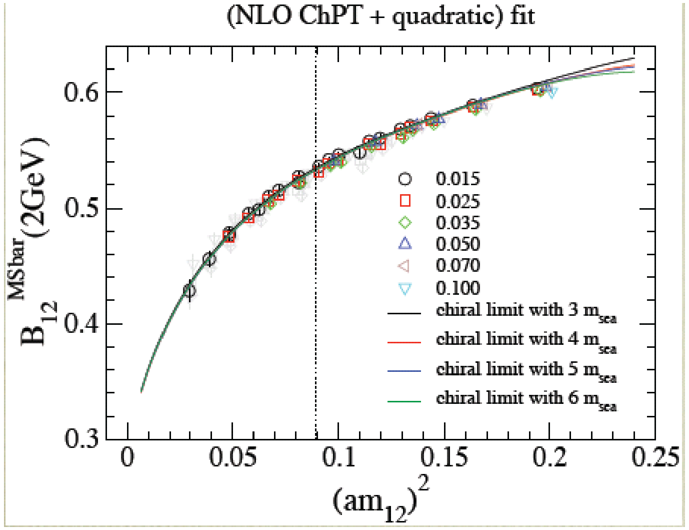
JLQCD collab, arXiv:0801.4186 [hep-lat].

$$\langle \bar{K}^0 | O_{LL}(\mu) | K^0 \rangle = \frac{8}{3} B_K(\mu) f_K^2 m_K^2$$



- No problem of operator mixing; otherwise, mixes with O_{LR}, for instance. Enhanced by its wrong chiral behavior.
- Another test of chiral log. Here the data follows the NLO ChPT.

$$B_P = B_P^\chi \left[1 - \frac{6m_P^2}{(4\pi f)^2} \ln \frac{m_P^2}{\mu^2} + b m_P^2 + O(m_P^4) \right]$$



$$B_K(2\text{GeV}) = 0.534(5)(30)$$