Kaon physics: a lattice perspective

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Lattice 2008 @ William & Mary

18 July 2008

Motivations: phenomenology

Test the standard model (SM), determine some of its parameters and constrain new physics (NP)

- High precision
  - unitarity of first row of CKM matrix and quark-lepton universality $\rightarrow f_+^{K^0\pi^-}(0)$, F_K/F_π, etc.
  - appearance of right-handed currents $\rightarrow f_0(M_K^2 - M_\pi^2) = F_K/F_\pi + O(m_{ud}/\Lambda_\chi)$ (Callan-Treiman point), etc.
- Precision
  - CP violation in neutral kaon mixing in the SM and beyond $\rightarrow B_K$ and BSM matrix elements
  - light-quark masses
- Exploratory
  - $\Delta I = 1/2$ rule
  - Direct CP violation in $K \rightarrow \pi\pi$ and $\mathrm{Re}(\epsilon'/\epsilon)$

Motivations: interface with ChPT

ChPT $\longrightarrow$ low energy dymamics of the pseudo-Golsdstone bosons of chiral symmetry breaking and related infrared singularities
$\longrightarrow$ widely used and successful in many phenomenological applications

$\Rightarrow$ very useful tool for understanding **dependence of lattice results** on **light-quark masses** and **volume**

Recent $N_f = 2$ and $2+1$ simulations w/ $M_\pi \lesssim 350\mathrm{MeV}$ allow to begin returning the favor:

- To what extent does ChPT apply to the strange quark?
  $\longrightarrow$ $SU(3)$ vs $SU(2)$ ChPT
- What are the couplings (i.e. LECs) of ChPT?

Here, concentrate on regime where $M_\pi L,\ F_\pi L \gg 1$ (p-regime of ChPT)
$\longrightarrow$ unfortunately no time to cover very interesting finite-volume regimes, where $M_\pi L \lesssim 1$

Extrapolation/interpolation to the physical mass point

In practice today $(\phi \leftrightarrow$ physical$)$

- calculate w/ $M_\pi \sim 200 \div 700\,\mathrm{MeV}$ and $M_K^\chi \simeq M_K^2 - M_\pi^2/2 \simeq M_K^{\chi\phi} \simeq 485\,\mathrm{MeV}$
- extrapolate/interpolate to $M_\pi = M_\pi^\phi \simeq 135\,\mathrm{MeV}$ and to $M_K^\chi = M_K^{\chi\phi}$

Three distinct sets of questions

(1) What is the best way to interpolate to $M_K^\chi = M_K^{\chi\phi}$?

(2) What is the best way to extrapolate to $M_\pi = M_\pi^\phi$?

(3) Do $SU(2)$ and/or $SU(3)$ chiral forms fit the lattice results? To what order? Are the parameters obtained the true LECs of QCD?

Simple answer to (1)

- Typically two values of M_K^2 around $M_K^{\phi,2}$ with total spread of $\sim 10\%$
- *Flavor* expand in $M_K^{\chi,2}$ about non-singular point, $M_K^{\chi\phi,2}$

$\Rightarrow$ expansion parameter: $\delta_K^2 \equiv (M_K^{\chi,2} - M_K^{\chi\phi,2})/M_{QCD}^2 \leq 0.012$, w/ $M_{QCD} \sim 1\,\mathrm{GeV}$

$\Rightarrow$ generically $\leq 1.2\%$ systematic error w/ 2 strange masses and $< 0.015\%$ w/ 3

$\Rightarrow$ no information about $SU(3)$ ChPT

Extrapolation to $M_\pi = M_\pi^\phi$

Much more difficult: need simulations w/ many different $M_\pi < M_\pi^{cut} \simeq 450\,\mathrm{MeV}$ extending preferably below $200\,\mathrm{MeV}$

Flavor expansion approach, in its simplest form

- expansion around $\bar{M}_\pi^2 = (M_\pi^{\phi,2} + M_\pi^{cut,2})/2 \simeq (330\,\mathrm{MeV})^2$
- expansion parameter: $\delta_\pi^2 \equiv (M_\pi^2 - \bar{M}_\pi^2)/M_{QCD}^2 \lesssim 0.1$, w/ $M_{QCD} \sim 1\,\mathrm{GeV}$

$\Rightarrow$ w/ simple 3 parameter quadratic form in δ_π^2 should reach $\lesssim 1\%$ accuracy in extrapolation (can fit a chiral log that gives a correction $\gtrsim -30\%$ for $M_\pi = M_\pi^\phi \to M_\pi^{cut}$ to less than $\sim 0.5\%$)

- minimal assumptions, but no symmetry constraints
- no information about $SU(2)$ or $SU(3)$ ChPT

ChPT approach

- expansions about the singular point $(M_\pi^2 \to 0,\ M_K^{\chi,2} = M_K^{\chi\phi,2})$ for $SU(2)$ and $M_{\pi,K}^2 \to 0$ for $SU(3)$
- $SU(2)$ or $SU(3)$?
- $SU(2)$ relies on flavor expansion approach for interpolation to $M_K^{\chi,2} = M_K^{\chi\phi,2}$

$SU(3)$ vs $SU(2)$ ChPT: what's the difference?

	$SU(3)$	$SU(2)$
dofs	π, K, η i.e. K and η are treated as PGB	π i.e. K and η are integrated out
expansion in	$\left(\frac{M_{\pi,K,\eta}}{\Lambda_\chi}\right)^2$ w/ $\Lambda_\chi \sim 4\pi F_\pi$	$\left(\frac{M_\pi}{\sqrt{2}M_K}\right)^2, \left(\frac{M_\pi}{\Lambda_\chi}\right)^2$
LECs	$f(m_c, m_b, m_t, \Lambda_{\rm QCD})$	$f(m_s, m_c, m_b, m_t, \Lambda_{\rm QCD})$
resums	$\left(\frac{M_\pi}{\sqrt{2}M_K}\right)^2$ at each order in $\left(\frac{M_\pi}{\Lambda_\chi}\right)^2$	$\left(\frac{M_{K,\eta}}{\Lambda_\chi}\right)^2$ at each order in $\left(\frac{M_\pi}{\sqrt{2}M_K}\right)^2$
NLO accuracy at physical masses	$\left(\frac{M_\eta}{4\pi F_\pi}\right)^4 \sim 5\%$	$\left(\frac{M_\pi}{\sqrt{2}M_K}\right)^4 \sim 0.4\%$
NLO matching	For $M_K^{\chi,2} \gg M_\pi^2$,	$\mathrm{LEC}_{SU(3)}\left[1+O\left(\frac{M_K^{\chi,2}}{\Lambda_\chi^2}\ln\frac{M_K^{\chi,2}}{\Lambda_\chi^2}, \frac{M_K^{\chi,2}}{\Lambda_\chi^2}\right)\right] = \mathrm{LEC}_{SU(2)}$

$SU(3)$ vs $SU(2)$ ChPT: an example

$SU(3)$ NLO ($\chi_n(M^2) = M^{2n}\ln(M^2/\mu^2)$) (Gasser & Leutwyler '85)

$$F_\pi = F_3\left\{1 - \frac{1}{(4\pi F_3)^2}\left[\chi_1(M_\pi^2) + \frac{1}{2}\chi_1(M_K^2)\right] + 4(L_5 + L_4)(\mu)\frac{M_\pi^2}{F_3^2} + 8L_4(\mu)\frac{M_K^2}{F_3^2}\right\}$$

$$F_K = F_3\left\{1 - \frac{1}{(4\pi F_3)^2}\left[\frac{3}{8}\chi_1(M_\pi^2) + \frac{3}{4}\chi_1(M_K^2) + \frac{3}{8}\chi_1(M_\eta^2)\right] + 4(L_5 + 2L_4)(\mu)\frac{M_K^2}{F_3^2} + 4L_4(\mu)\frac{M_\pi^2}{F_3^2}\right\}$$

i.e. 3 parameters: $F_3,\ L_4,\ L_5$

$SU(2)$ NLO (Gasser & Leutwyler '84, RBC/UKQCD '08)

$$F_\pi = F_2(1 + \alpha_F\delta_K^2)\left\{1 - \frac{1}{(4\pi F_2)^2}\left[\chi_1(M_\pi^2) - \ell_4(\mu)M_\pi^2\right]\right\} + O\left(M_\pi^2\delta_K^2\right)$$

$$F_K = F_2^K(1 + \alpha_F^K\delta_K^2)\left\{1 - \frac{1}{(4\pi F_2)^2}\left[\frac{3}{8}\chi_1(M_\pi^2) - \ell_4^K(\mu)M_\pi^2\right]\right\} + O\left(M_\pi^2\delta_K^2\right)$$

i.e. at least 6 parameters ($F_2,\ \ell_4,\ \alpha_F,\ F_2^K,\ \ell_4^K,\ \alpha_F^K$), 8 with $O\left(M_\pi^2\delta_K^2\right)$ terms

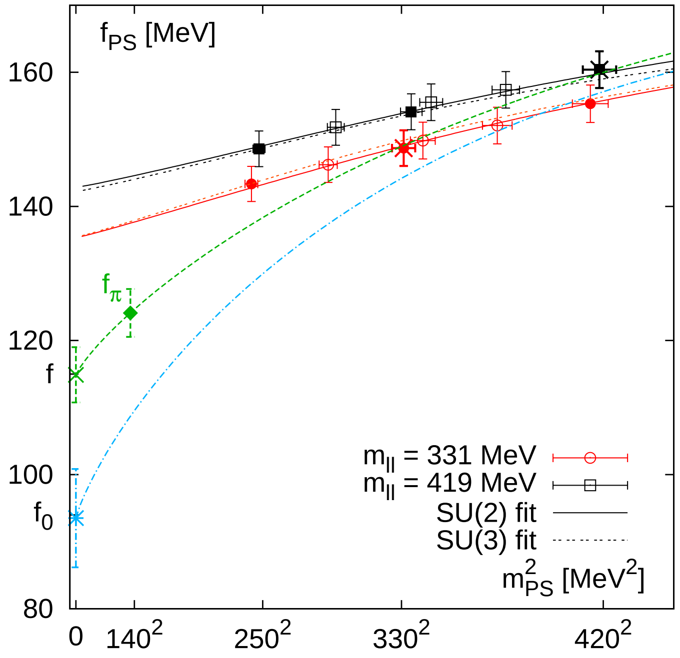
Flavor expansion has 8 parameters to $O(\delta_\pi^4, \delta_K^2)$

$SU(3)$ vs $SU(2)$ ChPT: so what *is* the difference?

- As M_π is lowered below $\sqrt{2}M_K^\chi$:
  $SU(3)$ ChPT $\longrightarrow$ $SU(2)$ ChPT, but with many constraints amongst LECs
- Constraints are released by addition of NNLO and higher order $SU(3)$ terms
  $\Rightarrow$ recover $SU(2)$ form
- If $M_K^{\chi,2}$ expansion in $SU(3)$ appears to "converge", fitted LECs may be the true $SU(3)$ LECs of QCD
- If $M_K^{\chi,2}$ expansion in $SU(3)$ "converges" poorly, a fit may be obtained by addition of higher order terms, but fitted LECs will most likely not be the true $SU(3)$ LECs of QCD
- M_π^2 terms in $SU(3)$ expansion can be better behaved
  $\Rightarrow$ an $SU(2)$ fit ought to work and should give the true $SU(2)$ LECs of QCD
- If goal is to obtain LECs of QCD, one should simulate closer to the chiral limit than the physical point, especially in the case of $SU(3)$

NLO $SU(3)$ vs $SU(2)$ fit examples

Details of simulations below



(RBC/UKQCD '08) (Scholz)
$(f_\pi = \sqrt{2} F_\pi)$

- $SU(3)$ expansion parameters at $M_\pi^{max} \simeq 419$ MeV: $(M_\pi^{max}/4\pi F_\pi^\phi)^2 \simeq 0.1$ and $(M_K/4\pi F_\pi^\phi)^2 \simeq 0.4$

- $SU(2)$ expansion parameter at M_π^{max}: $(M_\pi^{max}/\sqrt{2} M_K^{\chi\phi})^2 \simeq 0.4$

$\Rightarrow$ not clear which is better at top of M_π range

- $SU(2)$ improves as M_π decreases while $(M_K/4\pi F_\pi^\phi)^2$ of $SU(3)$ remains $\sim$ constant
- Very large NLO $SU(3)$ corrections $\sim 70\%$ at lightest unitary $M_\pi \simeq 331$ MeV
- Also find that NLO $SU(3)$ does not fit results with $M_\pi \to M_K^\phi$

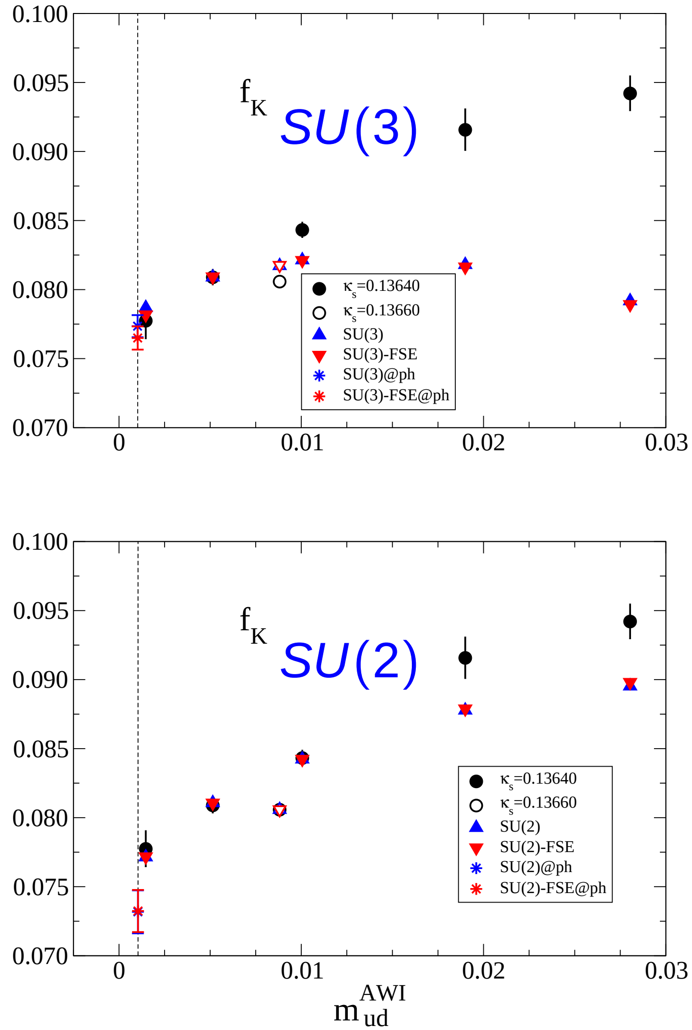
$\rightarrow$ conclude $SU(3)$ fails while $SU(2)$ is OK

- Relies heavily on partial quenching
- Fits are uncorrelated $\rightarrow$ no meaningful measure of quality of fit
- Does not account for possible distortions of mass behavior by discretization erorrs ($a \simeq 0.11$ fm)

NLO $SU(3)$ vs $SU(2)$ fit examples: cont'd

- All points are unitary, i.e. no partial quenching
- Fits restricted to $M_\pi \lesssim 410\,\mathrm{MeV}$
- NLO $SU(3)$ fits fail to reproduce M_π^2 dependence around $400\,\mathrm{MeV}$ and M_K^χ dependence around $M_K^{\chi\phi}$ for $M_\pi \simeq 400\,\mathrm{MeV}$
- NLO $SU(2)$ fits work well up to $410\,\mathrm{MeV}$ and above, failing by $\sim 5\%$ at $M_\pi \simeq 570\,\mathrm{MeV}$
- $LM_\pi \sim 2.3$ at $156\,\mathrm{MeV} \Rightarrow$ difficult to control FV effects at low M_π end
- Fits are uncorrelated $\to$ no meaningful measure of quality of fit
- Discretization errors are not accounted for ($a \simeq 0.09\,\mathrm{fm}$)

Together w/ RBC/UKQCD results $\to$ evidence that $SU(3)$ ChPT may fail at m_s^ϕ



$M_\pi\,[\mathrm{MeV}] = 156, 296, 385, 411, 570, 702$

(PACS-CS '08) (Kuramashi)

$(f_{\pi,K} = \sqrt{2} F_{\pi,K})$

$|V_{us}|$ from experiment and the lattice

Precision tests of CKM unitarity/quark-lepton universality and constraints on new physics (NP) from

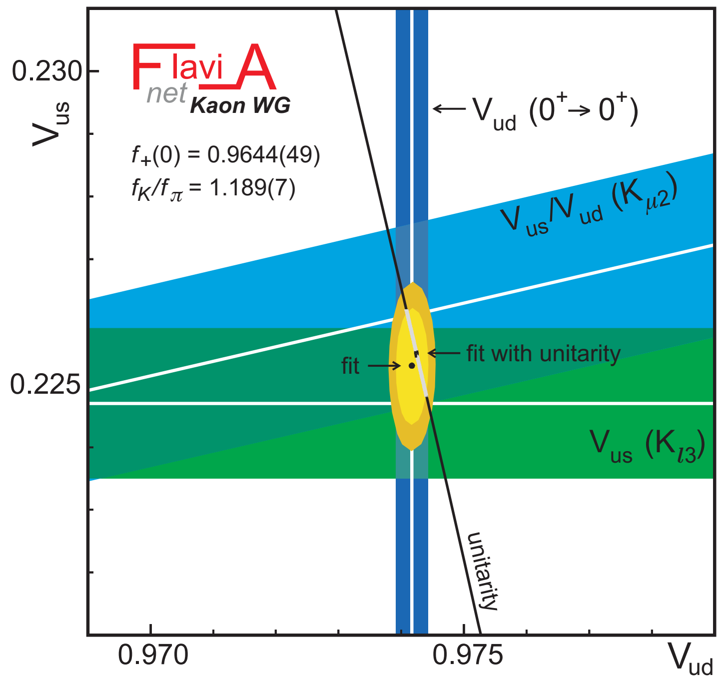
$$G_\mu^2\left[|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2\right]=G_\mu^2\left[1+O\left(\frac{M_W^2}{\Lambda_{NP}^2}\right)\right]$$

Large amounts of new data: BNL-E865, KLOE, KTEV, ISTRA+, NA48

Currently

- $|V_{ud}|=0.97418(26)$ [0.03%] from nuclear β decays (Hardy & Towner '07)
- $|V_{us}|=0.2246(12)$ [0.5%] from K_{l3} (Flavianet '07)
- $|V_{us}/V_{ud}|=0.2321(15)$ [0.6%] from K_{l2} (Flavianet '07)
- $|V_{ub}|=3.86(9)(47)\cdot 10^{-3}$ (HFAG '07, CKMfitter '07)

$|V_{us}|$ from experiment and the lattice



Combined fit (Flavianet '07)

- $|V_{ud}| = 0.97417(26)$ [0.03%]
  $\Rightarrow \delta|V_{ud}|^2 = 5.1 \cdot 10^{-4}$
- $|V_{us}| = 0.2253(9)$ [0.4%]
  $\Rightarrow \delta|V_{us}|^2 = 4.1 \cdot 10^{-4}$
- and $|V_{ub}|^2 \simeq 1.5 \cdot 10^{-5}$

$\Rightarrow$ error from $|V_{us}|$ is no longer dominant uncertainty!

Find

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9998(6) \quad [0.6\%]$$

$\Rightarrow$ scale of new physics: $\Lambda_{NP} \gtrsim 3 \div 1\,\mathrm{TeV}$ @ $1 \div 3\sigma$

$|V_{us}|$ from $K \to \mu\bar{\nu}$

Marciano '04: window of opportunity

$$\frac{\Gamma(K\to\mu\bar{\nu}(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}(\gamma))} \longrightarrow \frac{|V_{us}|}{|V_{ud}|}\frac{F_K}{F_\pi} = 0.2760(6)\ [0.22\%]$$

Need:

- F_K/F_π to 0.5% to match $K\to\pi\ell\nu$ determination (assuming that systematics in that determination are controlled to that level)
- F_K/F_π to 0.22% to match experimental error in $K\to\mu\bar{\nu}(\gamma)/\pi\to\mu\bar{\nu}(\gamma)$

Also

- $F_K/F_\pi = 1 + O\left(\frac{M_K^2-M_\pi^2}{M_{QCD}^2}\right)$
- On lattice, get F_K from e.g.

$$C_{A_0P}(t) \equiv \frac{1}{(L/a)^3}\sum_{\vec{x}}\langle[\bar{s}\gamma_5\gamma_0 u](x)[\bar{u}\gamma_5 s](0)\rangle \overset{0\ll t\ll T}{\longrightarrow} \frac{\langle 0|\bar{s}\gamma_5\gamma_0 u|K^+(\vec{0})\rangle\langle K^+(\vec{0})|\bar{u}\gamma_5 d|0\rangle}{2M_K}e^{-M_K t}$$

and

$$\langle 0|\bar{s}\gamma_5\gamma_0 u|K^+(\vec{0})\rangle = \sqrt{2}M_K F_K$$

F_K/F_π from the lattice: unquenched calculations

ref.	N_f	action	a[fm]	LM_π	M_π[MeV]	F_K/F_π
PDG '06						1.223(15)
ETM '08 (Tarantino)	2	tmQCD	$\gtrsim 0.07[F_\pi]$	3.2	$\gtrsim 300$	1.196(13)(7)(8)
NPLQCD '06	2+1	$\mathrm{KS_{MILC}}$ /DWF	$0.13[r_0]$	3.7	$\gtrsim 290$	$1.218(2)^{+11}_{-24}$
MILC '04-'07	2+1	$\mathrm{KS^{AsqTad}_{MILC}}$	$\gtrsim 0.06[F_\pi]$	4	$\gtrsim 240$	$1.197(3)^{+6}_{-13}$
HPQCD/ UKQCD '07	2+1	$\mathrm{KS^{HISQ}_{MILC}}$	$\gtrsim 0.09[\Upsilon]$	3.8	$\gtrsim 250$	1.189(2)(7)
RBC/ UKQCD '08 (Scholz)	2+1	DWF	$0.11[\Omega]$	4.6	$\gtrsim 330$	1.205(18)(62)
PACS-CS '08 (Kuramashi)	2+1	NP-SW	$0.09[\Omega]$	2.3	$\gtrsim 160$	1.189(20)
BMW '08 (Dürr)	2+1	SW	$\gtrsim 0.065[\Xi]$	$\gtrsim 4$	$\gtrsim 190$	1.19(1)(1)

A parte on color coding of lattice simulations

In the process of being put together by the FLAVIAnet Lattice Averaging Group (FLAG) (personalized version here)

- **publication status**
  - published
  - preprint
  - proceedings, talk
- **flavors, action and algorithm**
  - $N_f = 2 + 1$ w/ an exact algorithm and an action whose universality class is QCD
  - $N_f = 2$ or use of an action whose universality class is not know to be QCD
  - $N_f = 0$
- **renormalization**
  - non-perturbative w/ non-perturbative running
  - non-perturbative w/ perturbative running or $\geq$ two-loops
  - one-loop perturbative

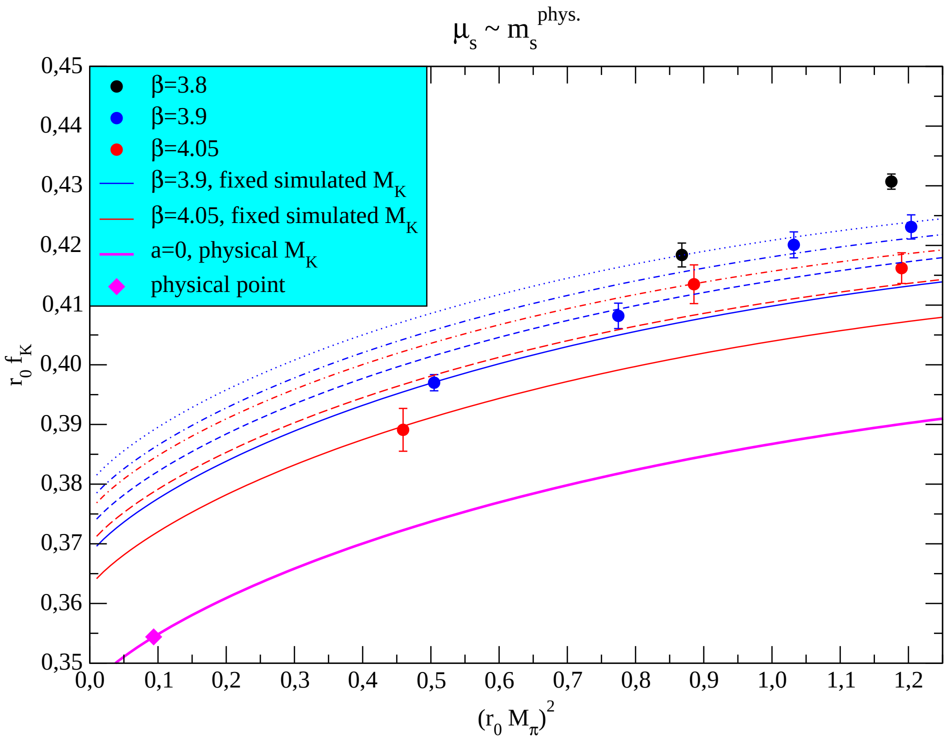
A parte on color coding of lattice simulations

- **extrapolation/interpolation to physical mass point**
  - (green) minimum unitary $M_\pi \leq 250\,\mathrm{MeV}$ and NLO or better ChPT or any other demonstrably controlled functional mass dependence
  - (orange) minimum unitary $M_\pi \leq 350\,\mathrm{MeV}$ and reliable estimate of extrapolation error
  - (red) minimum unitary $M_\pi > 350\,\mathrm{MeV}$
- **continuum extrapolation**
  - (green) ≥ 3 lattice spacings with at least one $a < 0.08\,\mathrm{fm}$ and controlled scaling
  - (orange) 2 lattice spacings with one $a \lesssim 0.1\,\mathrm{fm}$
  - (red) a single lattice spacing or all $a > 0.1\,\mathrm{fm}$
- **finite volume**
  - (green) $LM_\pi \geq 4$ and numerical volume scaling study (with ChPT)
  - (orange) $3 < LM_\pi \leq 4$ and ChPT corrections
  - (red) $LM_\pi \leq 3$ and/or no study of finite volume effects

F_K/F_π: consumer report

ref.	publication	N_f, action, etc	mass extrap	$a \to 0$	finite volume
ETM '08	🔴	🟠	🟠	🟢	🟠
NPLQCD '06	🟢	🟠	🟠	🔴	🔴
MILC '04-07	🟢	🟠	🟢	🟢	🟢
HPQCD/UKQCD '07	🟢	🟠	🟢	🟢	🟢
RBC/UKQCD '08	🟠	🟢	🟠	🔴	🟠
PACS-CS '08	🟠	🟢	🟢	🔴	🔴
BMW '08	🔴	🟢	🟢	🟢	🟢

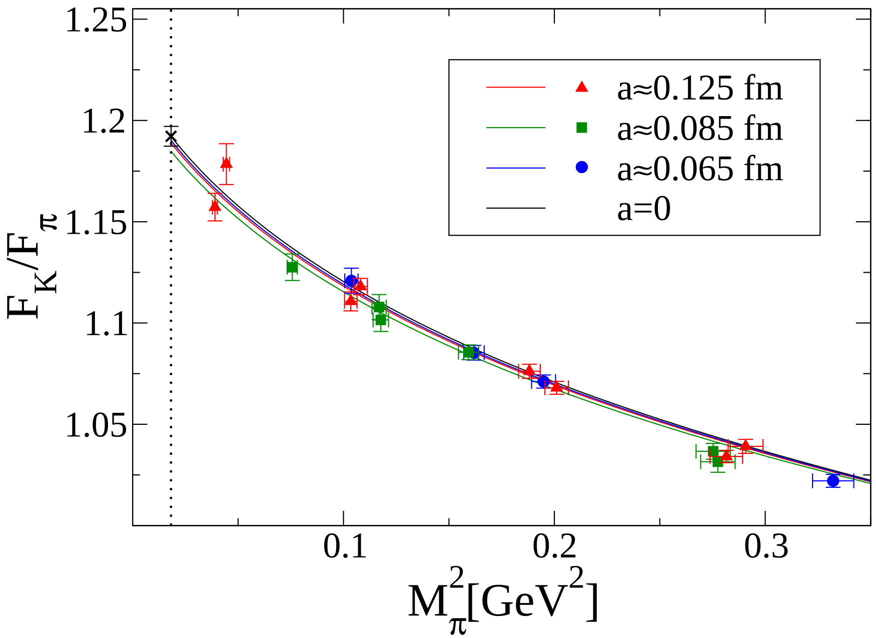
F_K/F_π: chiral extrapolations



(ETM '08) (Tarantino)

- $N_f = 2$, tmQCD, partially quenched
- $a \simeq 0.07,\ 0.09,\ 0.10\,\mathrm{fm}$ ($0.10\,\mathrm{fm}$ not included in fit)
- $M_\pi : 300 \longrightarrow 480\,\mathrm{MeV}$, $LM_\pi \gtrsim 3.2$
- NLO $SU(2)$ analysis w/ $O(a^2)$ term included and 1-loop FV corrections
- $\sim 10\%$ extrapolation to physical point

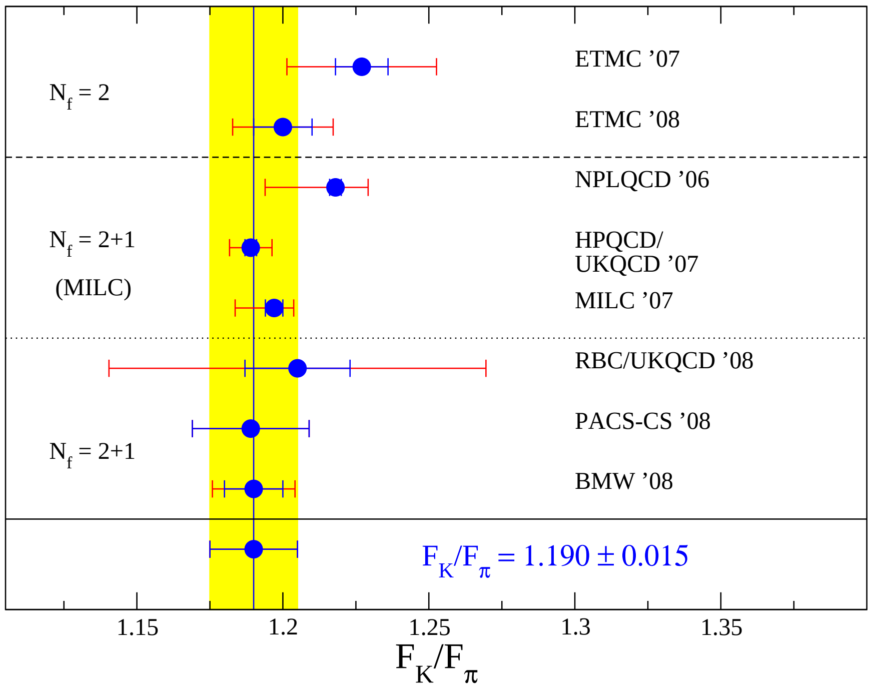
F_K/F_π: chiral extrapolations



(BMW '08) (Dürr)
(Fit with $M_\pi^{cut} = 470\,\mathrm{MeV}$)

- $N_f=2+1$ & $a \simeq 0.065,\ 0.085\,,\ 0.125\,\mathrm{fm}$
- M_π : $190 \to 570\,\mathrm{MeV}$, $LM_\pi \gtrsim 4$
- Large variety of $SU(2)$ and $SU(3)$ fits w/ $600\,\mathrm{MeV}$, $470\,\mathrm{MeV}$ and $420\,\mathrm{MeV}$ cuts on M_π, a^2 or a terms included, 2-loop FV corrections (Colangelo et al '05), many fit times, etc.
- Analyses done w/ 2000 boostrap samples
- Create distributions for central value and stat. error from different procedures weighed by fit CL
- Median of central value and stat. error distributions $\longrightarrow$ final value and stat. error
- Central 68% $\longrightarrow$ systematic error
- $\lesssim 2\%$ extrapolation to physical point

F_K/F_π from the lattice: summary



- $\delta(F_K/F_\pi)^{lat} = 1.3\% \Leftrightarrow \delta(F_K/F_\pi - 1)^{lat} \simeq 8\%$

$\Rightarrow$ relative accuracy on calculated $SU(3)$ breaking effect much better than for $f_+^{K^0\pi^-}(0)$

$\Rightarrow$ still leads to larger theory error on $|V_{us}|$ (1.3% vs 0.5%)

- F_K/F_π straightforward to calculate

$\Rightarrow$ will be able to reach the $\delta(F_K/F_\pi - 1)^{lat} = 3\%$ required for $\delta^{th}|V_{us}| = 0.5\%$ w/ results closer to the physical point

- more difficult to match the 0.22% experimental accuracy

$|V_{us}|$ from $K \to \pi\ell\nu$

Measurement of $|V_{us}|$ requires theoretical determination of $f_+(q^2)$:

$$\langle\pi^+(p')|\bar{u}\gamma_\mu s|\bar{K}^0(p)\rangle \longrightarrow f_+(q^2),\ f_0(q^2) \qquad q = p - p'$$

$\Rightarrow$ form factor shape measured in experiment and extract (Flavianet '07)

$$|V_{us}| \times f_+(0) = 0.21664(48)\ [0.22\%]$$

- Same error as in $K\ell_2/\pi\ell_2$
- Need $f_+(0)$ to 0.22% to fully exploit new experimental results!

Theoretical framework: ChPT (Leutwyler & Roos '84, Gasser & Leutwyler '85)

$$f_+(0) = 1 + f_2 + f_4 + \cdots$$

- Ademollo-Gatto thm and χPT: $f_2 = O\left(\frac{(M_K^2 - M_\pi^2)^2}{M_K^2\Lambda_\chi^2}\right) = -0.023$

  $\to$ no contributions from the $O(p^4)$ L_i's
  $\to$ NLO chiral logs fully determined in terms of M_K, M_π and F_π!

$|V_{us}|$ from $K \to \pi \ell \nu$

$\Rightarrow$ need a precise calculation of

$$\Delta f \equiv f_+(0) - 1 - f_2 = O\left(\frac{(M_K^2 - M_\pi^2)^2}{\Lambda_\chi^4}\right) \sim 3\%$$

$\Rightarrow$ Δf is comparable to f_2

$\Rightarrow$ "Only" need Δf to $\sim$ 7% to match experiment

- f_4:
  - NNLO logs computed (Post & Schilcher '02, Bijnens & Talavera '03)
  - requires $O(p^6)$ LECs; estimates in Bijnens & Talavera '03, Jamin et al '04, Cirigliano et al '05, Portoles '07
  - $O(p^6)$ LECs can be determined from slope and curvature of $f_+(q^2)$ (Bijnens & Talavera '03)
  - Reference result (Leutwyler & Roos '85): $\Delta f = -0.016(8)$

$K \to \pi\ell\nu$: unquenched calculations

Ref.	N_f	action	a[fm]	L[fm]	M_π[MeV]	$f_+(0)$
JLQCD '05	2	NP SW	0.09	1.8	$\gtrsim$ 550	0.967(6)
RBC '06	2	DWF	0.12	2.5	$\gtrsim$ 490	0.968(9)(6)
FNAL/MILC '04	2+1	KS+Wil				0.962(6)(9)
RBC/UKQCD '08	2+1	DWF	0.11	1.8, 2.8	$\gtrsim$ 330	0.9644(33)(34)(14)

ref.	publication	N_f, action, etc	mass extrap	$a \to 0$	finite volume
JLQCD '05	🔴	🟠	🔴	🔴	🔴
RBC '06	🟢	🟠	🔴	🔴	🔴
FNAL/MILC '04	🔴	🟠	🔴	🔴	🔴
RBC/UKQCD '08	🟢	🟢	🟠	🔴	🟠

$K \to \pi\ell\nu$: lattice methodology

Becirevic et al '04: $f_+(0) - 1$ using double ratio of 3-pt fns

1. $$f_0(q^2_{max}) = \frac{2\sqrt{M_K M_\pi}}{M_K + M_\pi} \frac{\langle\pi|V_0|K\rangle\langle K|V_0|\pi\rangle}{\langle\pi|V_0|\pi\rangle\langle K|V_0|K\rangle}$$

   $\to$ statistical error $\lesssim 0.1\%$!

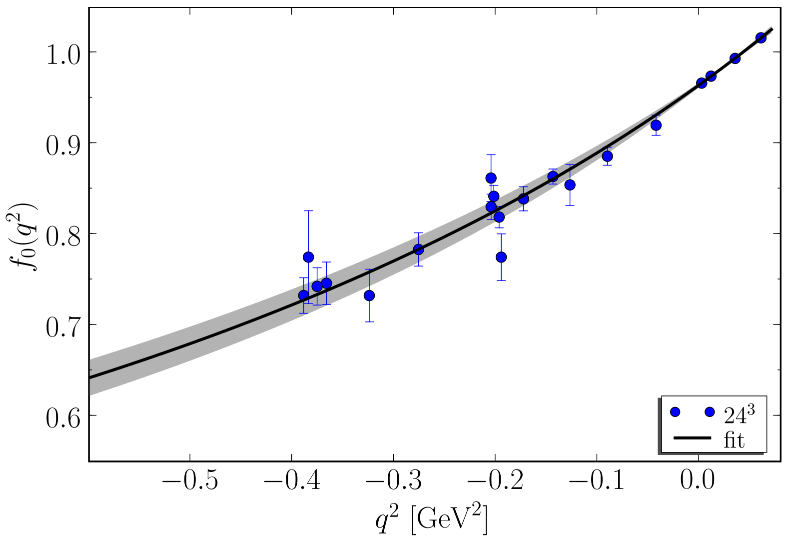
2. Compute $f_0(q^2)$ at various q^2 and interpolate to get $f_+(0) = f_0(0)$ using ansatz
3. RBC/UKQCD '08 perform q^2 interpolation and chiral extrapolation together, e.g.

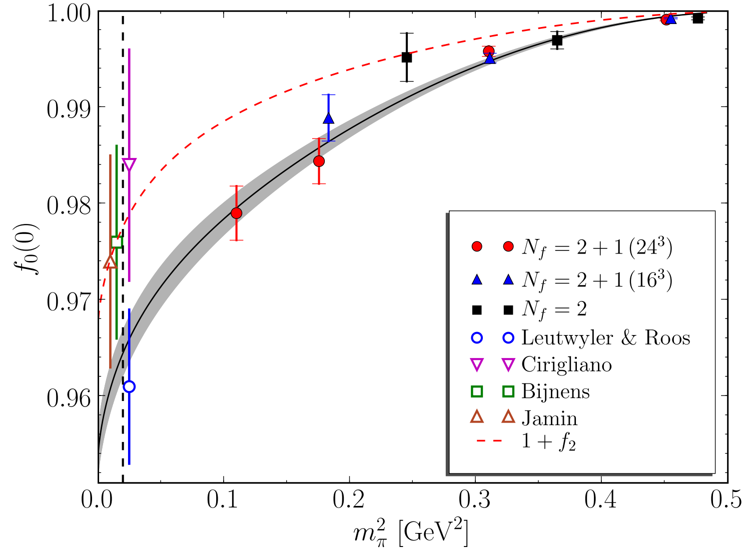
$$f_0(q^2; M_K, M_\pi) = \frac{1 + f_2 + (M_K^2 - M_\pi^2)^2(A_0 + A_1(M_K^2 + M_\pi^2))}{1 - q^2/(M_0 + M_1(M_K^2 + M_\pi^2))^2}$$

w/ A_0, A_1, M_0, M_1 parameters

$K \to \pi\ell\nu$: q^2 and chiral fits

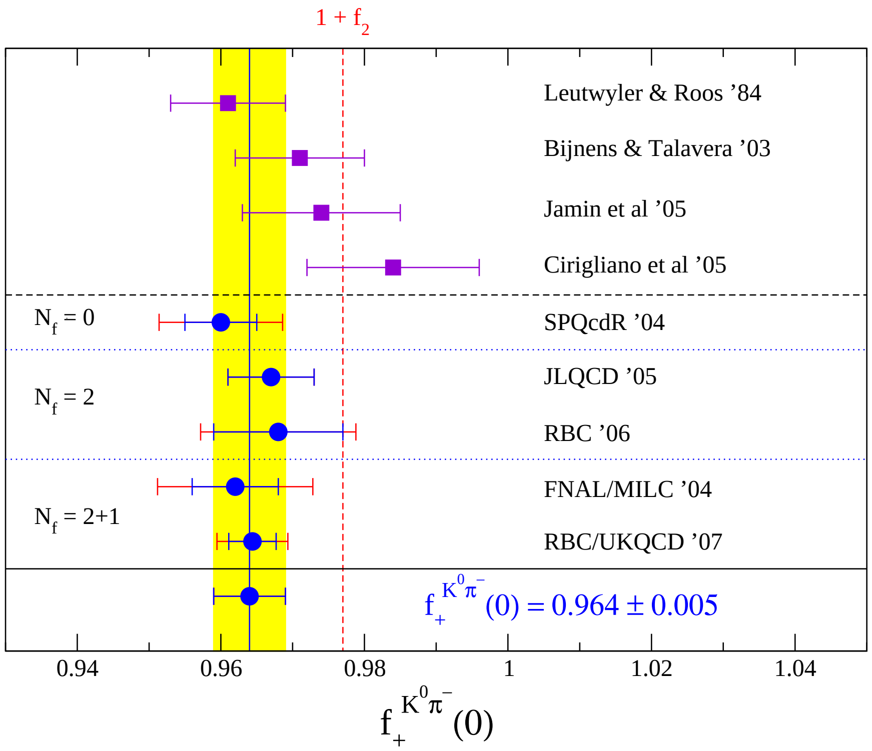
Combined q^2 and chiral fit by RBC/UKQCD '08





- Results fit $1 + f_2(M_K, M_\pi) + \mathrm{NNLO}$ "well" (but fits are uncorrelated)
  $\Rightarrow$ claim of being able to determine NNLO effects is justified
- Extrapolated result is only 2σ below result for lightest point and claimed error on $f_+(0) - 1$ is 14%
- m_s approx. 15% too high
- Single rather coarse lattice w/ spacing $a = 0.114(2)\,\mathrm{fm}$
  $\Rightarrow$ discretization systematics can only be guessed
- Nevertheless, first realistic lattice calculation

$f_+(0)$ from the lattice: summary



- $\delta f_+(0)^{lat} = 0.5\%$

$\Rightarrow$ still gives best accuracy for $|V_{us}|$

- $\delta(f_+(0)-1)^{\rm lat} \simeq 15\%$ will be reduced
  $\longrightarrow$ by use of stochastic sources for the propagators (e.g. ETM '07, RBC/UKQCD '08)
  $\longrightarrow$ by use of partially twisted boundary conditions discussed in (Bedaque '04, Sachrajda et al '05) and applied to form factors in (Guadagnoli et al '06, RBC/UKQCD '07-08, ETM '07) w/ the possibility of obtaining the $f_+(q^2)$ directly at $q^2 = 0$ (UKQCD '07)
  $\longrightarrow$ simulations closer to the physical QCD point

- critical to check $a \longrightarrow 0$, as $a^2(m_s - m_{ud})^2$ effects in $f_+(0) - 1$ may not be so small compared to the desired Δf

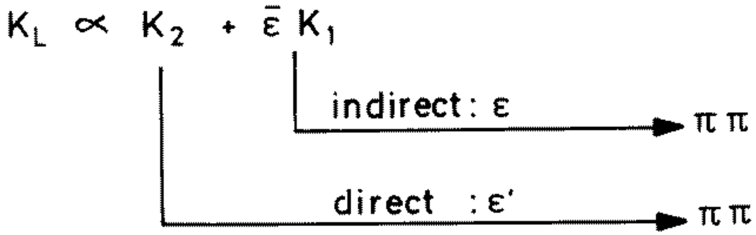
$K \to \pi\pi$ decays: phenomenology

$$-iT[K^0 \to \pi^+\pi^-] = \sqrt{\tfrac{1}{3}}A_0 e^{i\delta_0} + \sqrt{\tfrac{1}{6}}A_2 e^{i\delta_2} \qquad -iT[K^+ \to \pi^+\pi^0] = \tfrac{\sqrt{3}}{2}A_2 e^{i\delta_2}$$

$$-iT[K^0 \to \pi^0\pi^0] = -\sqrt{\tfrac{1}{3}}A_0 e^{i\delta_0} + \sqrt{\tfrac{2}{3}}A_2 e^{i\delta_2}$$

CP violation implies $A_I^* \neq A_I$

$$\Delta M_K = M_{K_L} - M_{K_S} \simeq 2\,\mathrm{Re}M_{12}$$



$$\epsilon \equiv \frac{T[K_L \to (\pi\pi)_{I=0}]}{T[K_S \to (\pi\pi)_{I=0}]} \simeq \frac{1}{\sqrt{2}} e^{i\pi/4} \frac{\mathrm{Im}M_{12}}{\Delta M_K}$$

$$\epsilon' \simeq \frac{1}{\sqrt{2}} e^{i\pi/4}\, \mathrm{Im}\left(\frac{A_2}{A_0}\right)$$

Experimentally:
(PDG '06)

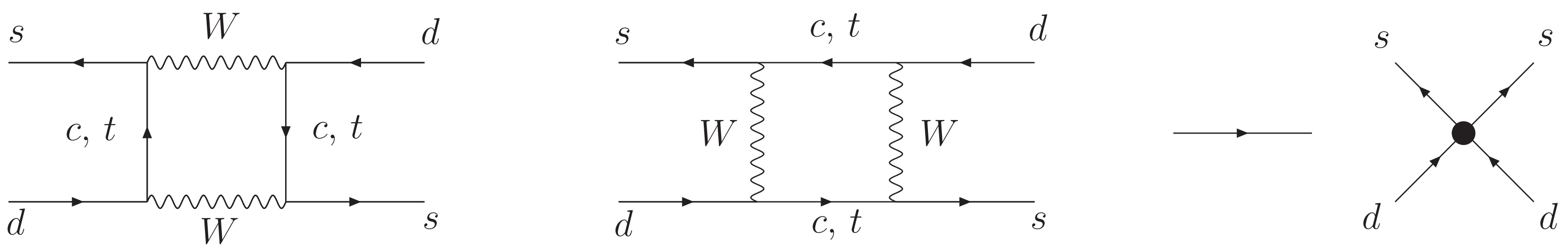
$\Delta M_K = (3.483 \pm 0.006) \times 10^{-12}\,\mathrm{MeV}$ [0.2%]

$|A_0/A_2| \simeq 22.2$ ($\Delta I = 1/2$ rule)

$|\epsilon| = (2.232 \pm 0.007) \cdot 10^{-3}$ [0.3%]

$\mathrm{Re}(\epsilon'/\epsilon) = (1.66 \pm 0.26) \cdot 10^{-3}$ [16%]

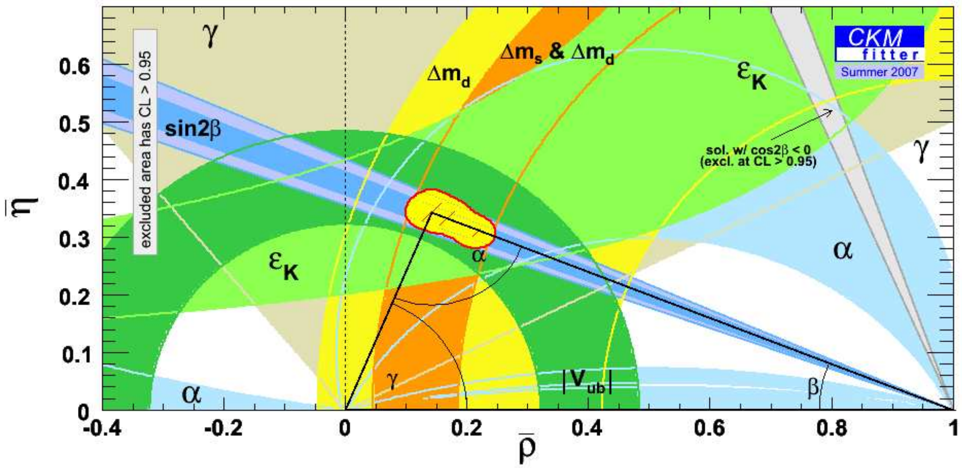
K^0-$\bar{K}^0$ mixing in the SM: B_K



$$2M_K M_{12}^* = \langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle = C_1^{\rm SM}(\mu) \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle$$

$$O_1 = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \qquad \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \frac{16}{3} M_K^2 F_K^2 B_K(\mu)$$

Constraint $\mathrm{Im}\lambda_t^2$, $\mathrm{Im}\lambda_c^2$ and $\mathrm{Im}\lambda_t\lambda_c$, with $\lambda_q = V_{qs}^* V_{qd}$



Constraint from ϵ on UT summit is rather weak

$\longrightarrow$ why?

ϵ from global CKM fit

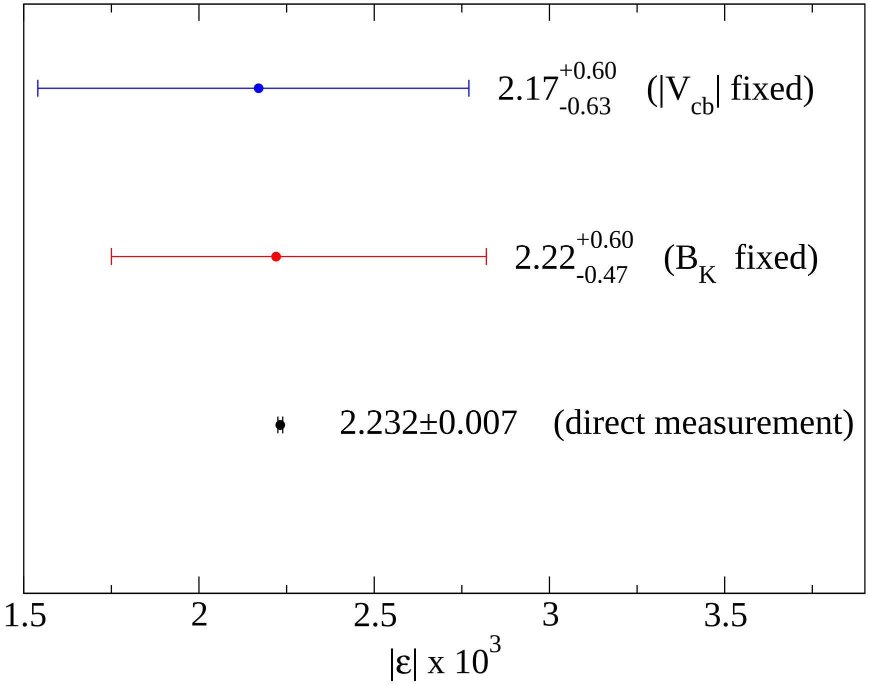
In the standard analysis of the SM (e.g. Buras '98)

$$|\epsilon| = C_\epsilon \hat{B}_K \lambda^2 \bar{\eta}^2 |V_{cb}|^2 \left[|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c \right]$$

with $\hat{B}_K = C_1^{\rm SM}(\mu) B_K(\mu)$

From global CKM fit w/ $\hat{B}_K = 0.78(2)(8)$ [11%] and $|V_{cb}| = 0.0415(9)$ [2.2%]

(CKMfitter '08)



Contribution from δB_K large . . .

. . . but so is contribution from $\delta|V_{cb}|^4$

$\Rightarrow$ important to improve determination of B_K, but must also reduce error on $|V_{cb}|$, etc.

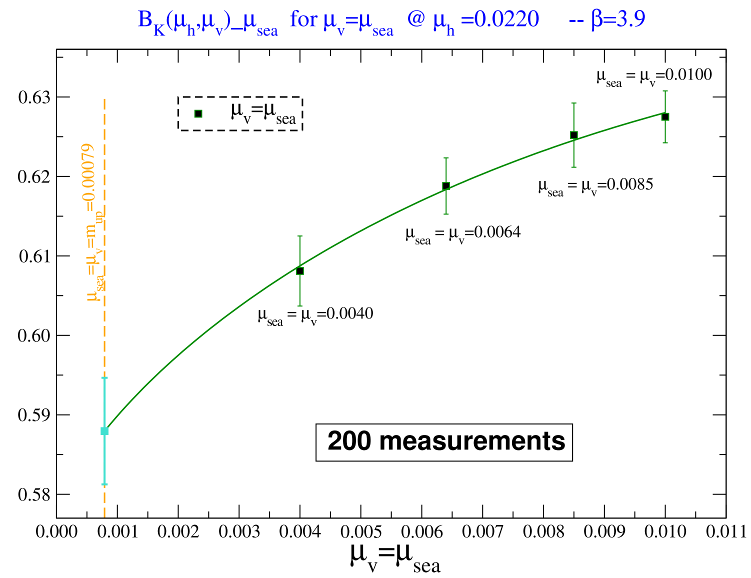
B_K from the lattice: unquenched simulations

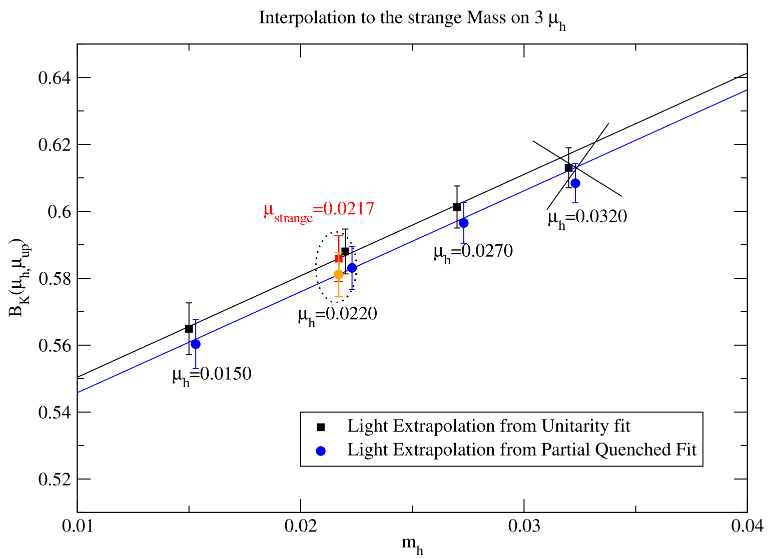
ref.	N_f	action	a[fm]	LM_π	M_π[MeV]	$\hat{B}_K$
JLQCD '08 (Hashimoto)	2	Overlap	0.12	2.7	$\gtrsim 290$	0.734(5)(50)
ETM '08 (Vladikas)	2	OS/tmQCD	0.07,0.09	3.1	$\gtrsim 300$	0.785(10)(16)
HPQCD/ UKQCD '06	2+1	$\mathrm{KS}^{\mathrm{HYP}}_{\mathrm{MILC}}$	0.125	4.5	$\gtrsim 360$	0.85(2)(18)
RBC/ UKQCD '07-08 (Scholz)	2+1	DWF	0.11	4.6	$\gtrsim 330$	0.717(14)(39)
Bae et al '08 (Lee)	2+1	$\mathrm{KS}^{\mathrm{HYP}}_{\mathrm{MILC}}$	$\gtrsim 0.06$	4	$\gtrsim 240$	$\delta B_K \longrightarrow 3\%$

ref.	publication	N_f, action, etc	mass extrap	$a \to 0$	finite volume	renorm
JLQCD '08	🟢	🟠	🟠	🔴	🔴	🟠
ETM '08	🔴	🟠	🟠	🟠	🟠	🟠
HPQCD/UKQCD '06	🟢	🟠	🔴	🔴	🟠	🔴
RBC/UKQCD '07-08	🟢	🟢	🟠	🔴	🟠	🟠

B_K: extraplation/interpolation to physical point

(ETM '08) (Vladikas)



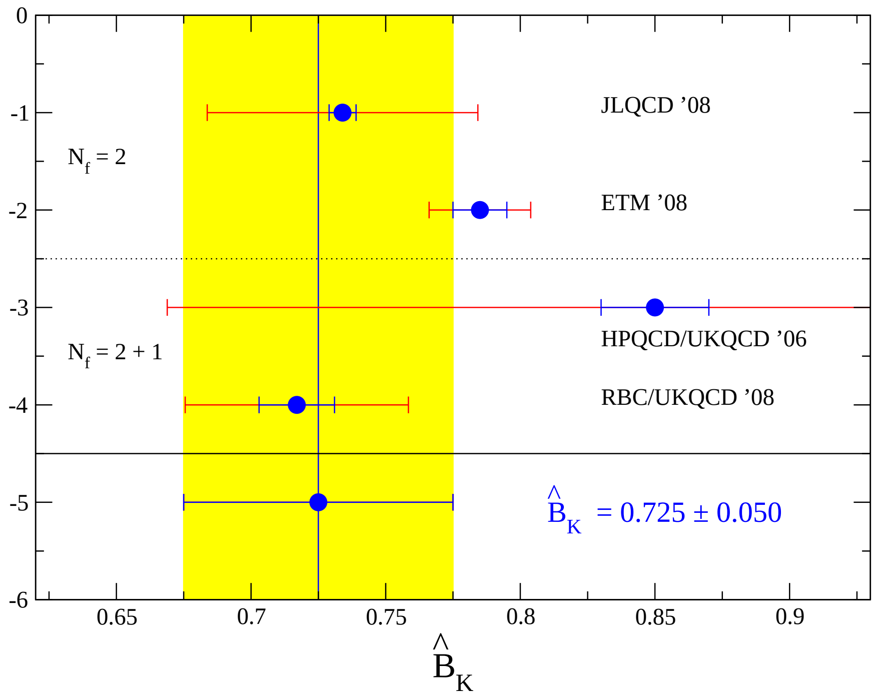


- Osterwalder-Seiler valence on twisted sea to ensure automatic improvement and multiplicative renormalization

$\Rightarrow$ $O(a^2)$ unitarity violations which must be controlled

- $a \simeq 0.07,\ 0.09,\ 0.10\,\mathrm{fm}$
- $M_\pi\ :\ 300 \to 480\,\mathrm{MeV}$, $LM_\pi \gtrsim 3.2$
- NLO $SU(2)$ analysis at single a for the moment
- continuum limit not yet taken
- FV corrections not yet accounted for
- $\sim 3\%$ extrapolation to physical light-quark mass

B_K from the lattice: summary



- $B_K|^{N_f=2+1}_{RBC} \simeq 0.83 \cdot B_K|^{N_f=0}_{JLQCD}$ (ca. '97)
- $\delta B_K^{lat} = 7\%$, i.e. comparable to other uncertainties in SM expression for ϵ

$\Rightarrow$ to improve constraint on UT must also improve $\delta|V_{cb}|$

- need to investigate continuum scaling of B_K for $N_f \geq 2$
- BSM contributions to $K^0 - \bar{K}^0$-mixing currently investigated by RBC/UKQCD w/ $N_f = 2+1$ DWF (Wennekers)

$\Rightarrow$ Will observation that ratios of non-SM to SM matrix elements are roughly twice as large in Babich et al '06 as in Donini et al '99 be confirmed?

Is there tension between ϵ and $\sin 2\beta_{\psi K_s}$?

Actually

Lunghi & Soni '08
Buras & Guadagnoli '08

$$\epsilon = e^{i\phi}\sin\phi\left[\frac{\mathrm{Im}M_{12}}{\Delta M_K}+\xi\right]$$

with $\phi = 43.5(5)^o$ (PDG '06) instead of 45^o and $\xi/(\sqrt{2}|\epsilon|) \simeq -0.06(2)$ (Buras & Guadagnoli '08)

Keeping only tt contribution and assuming no NP in CP conserving part of $B_{d(s)}-\bar{B}_{d(s)}$

$$|\epsilon| \sim \kappa\, F_K^2 \hat{B}_K |V_{cb}|^4 \left[f_{B_s}^2 B_{B_s}/f_{B_d}^2 B_{B_d}\right] \sin 2\beta$$

w/ $\kappa \simeq \sqrt{2}\sin\phi(1+\xi/(\sqrt{2}|\epsilon|)) = 0.92(2)$ a suppression factor

Combined with a lower $\hat{B}_K$, measured $|\epsilon|$ favors $\sin 2\beta$ larger than $\sin 2\beta_{\psi K_s}$
$\Rightarrow$ possible NP

Global CKM fit gives (w/ κ and B_K scaled down by 8%, gaussian errors and thanks to J. Charles)

$$\sin 2\beta = 0.787^{+47}_{-52} \qquad \text{vs} \qquad \sin 2\beta_{\psi K_S} = 0.681(25) \qquad (1.8\,\sigma)$$

$$|\epsilon| = 1.79^{+31}_{-29}\cdot 10^{-3} \qquad \text{vs} \qquad |\epsilon|_{direct} = 2.232(7)\cdot 10^{-3} \qquad (1.4\,\sigma)$$

i.e. errors on B_K, $|V_{ub}|$, etc. must come down to answer question

Conclusion

- Lattice QCD simulations have made tremendous progress in the last few years
- It is now possible to perform $2+1$ flavor lattice calculations with $M_\pi \sim 190\,\mathrm{MeV}$, $L \sim 4\,\mathrm{fm}$ and $a \to 0.065\,\mathrm{fm}$
  $\Rightarrow$ extrapolations to the physical QCD point ($M_\pi = 135\,\mathrm{MeV}$, $a \to 0$, $L \to \infty$) can be performed in a controlled manner (e.g. spectrum talk by Hoelbling (BMW))
- Quantities such as F_K/F_π and $f_+^{K^0\pi^-}(0)$ are already being computed with % or better accuracy and are having an important impact on SM and BSM tests
- Quantities such B_K are reaching the sub 10% accuracy leval, have errors which match those from other sources and may already be pointing to NP
- ϵ'/ϵ still have 100% despite the impressive $N_f = 2+1$ RBC/UKQCD effort, but not for long ... (talk by Christ)
- NLO $SU(3)$ ChPT appears to be having trouble at physical strange mass while $SU(2)$ ChPT performs better $\Rightarrow$ needs further investigation, w/ variety of a's
- Concerning extrapolations to the physical mass point, if you have the data, keep an open mind regarding functional forms
- Most quantities are still missing controlled continuum extrapolations
- The age of precision non-perturbative QCD calculations is finally dawning

Apologies to

- Norman Christ and RBC/UKQCD who presented a heroic $N_f = 2+1$ DWF calculation of ϵ'/ϵ that I was hoping to have time to cover
- Claude Bernard and MILC who sent very interesting preliminary results for their simulations with a light strange
- Carsten Urbach and EMT for not having the time to cover the results on $SU(2)$ LECs which they sent
- Derek Leinweber et al who sent information about the electromagnetic form factors of K and K^* mesons (CSSM Lattice Collaboration '07)
- Shoji Hashimoto, Jun Noaki and JLQCD's for not mentioning their $N_f = 2\,\&\,2+1$ overlap results
- All the other colleagues whose work I have not had the time to cover

And many thanks to my colleagues of the BMW collaboration (Christian Hoelbling, Zoltan Fodor, Stephan Dürr, etc.), to Jérôme Charles, Steve Sharpe, Claude Bernard, Chris Sachrajda, Amarjit Soni, . . . for helping in the preparation of this talk