



Hadronic Structure from Lattice QCD

James Zanotti
University of Edinburgh

Lattice 2008, College of William & Mary, July 14-19, 2008

A Mass of Work to Cover

Worst pun contender,
Lattice 2008!

- ♦ 32 parallel talks Lattice 2007: 26 talks, 1 poster
- ♦ 2 posters Lattice 2006: 19 talks 4 posters

Thanks to those who provided material

- ♦ >120 pages of material
- ♦ >200 figures!

Sorry if I've left out your favourite result

Jefferson Lab Research Program

<http://www.jlab.org/highlights/phys.html>

- Precision Test of the Standard Model
- Quark-Hadron Duality
- Strange Quarks in the Proton
- Generalized Parton Distributions
- Electric and Magnetic Proton Form Factors
- Neutron Charge Distribution
- Nucleon-Delta Transition
- Pion Form Factor
- Short-Range Correlations
- Deuteron Photodisintegration
- Spin Sum Rules
- Nucleon Spin Asymmetries
- Deuteron Form Factors
- Pentaquark Search
- Neutron Magnetic Structure

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- ◆ Nucleon Spin Asymmetries

Topics to Cover

- ◆ Form Factors (π , N, Delta, transition,...)
 - ◆ Provide information on size, shape and internal (charge) densities
 - ◆ eg. Neutron has charge zero, but charge density? +/-?
- ◆ Nucleon Axial Charge, g_A
 - ◆ Neutron beta decay, chiral symmetry breaking
- ◆ Generalised Parton Distributions
 - ◆ “Spin crisis”: quarks carry only 30% of proton’s spin
 - ◆ gluons? quark orbital angular momentum?
 - ◆ “3D” picture of nucleon
 - ◆ $\langle x \rangle$: Is this thing ever going to bend down?
- ◆ Distribution Amplitudes:
 - ◆ exclusive processes; hadron wave function
- ◆ Strange quark+ other disconnected contributions
 - ◆ Strangeness/gluonic content of nucleon

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- ◆ **Electric and Magnetic Proton Form Factors**
- ◆ **Neutron Charge Density**
- ◆ **Neutron Magnetic Structure**
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- ◆ Nucleon Spin Asymmetries
- ◆ Strange Quarks in the Proton

Form Factors

Nucleon

Q^2 Scaling

$$\langle p', s' | J^\mu(\vec{q}) | p, s \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2m} F_2(q^2) \right] u(p, s)$$

$$G^{BJ^\mu B}(t, \tau) = \sum_{s, s'} e^{-E_{p'}(t-\tau)} e^{-E_p\tau} \langle \Omega | \chi | p', s' \rangle \langle p', s' | J^\mu | p, s \rangle \langle p, s | \bar{\chi} | \Omega \rangle$$

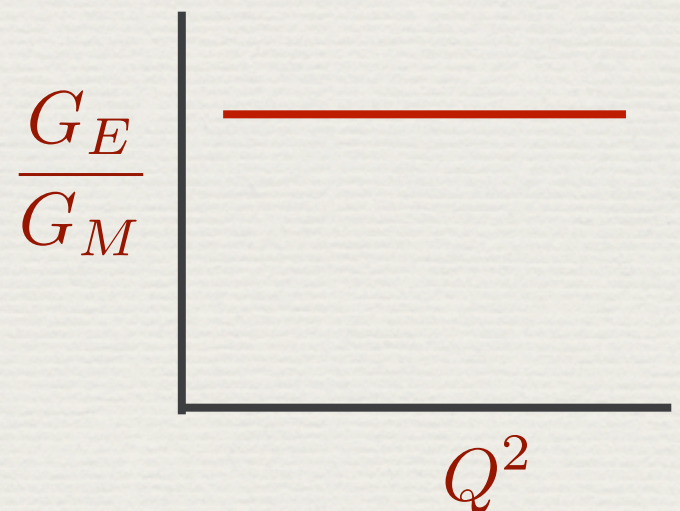
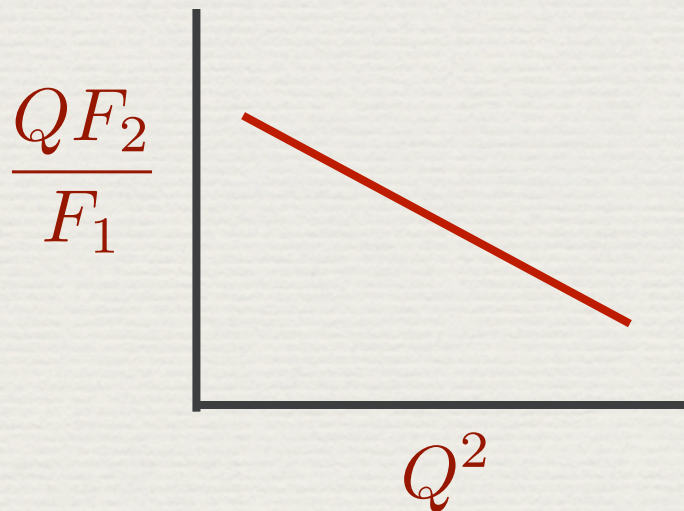
Naive expectation from dimensional counting

[Brodsky & Farrar, 1973]

$$F_1 \propto \frac{1}{Q^4} \quad (\text{dipole?})$$

$$F_2 \propto \frac{1}{Q^6} \quad (\text{tripole?})$$

$$\frac{F(0)}{(1 + Q^2/M^2)^p}$$



Form Factors

Nucleon

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Naive expectation from dimensional counting

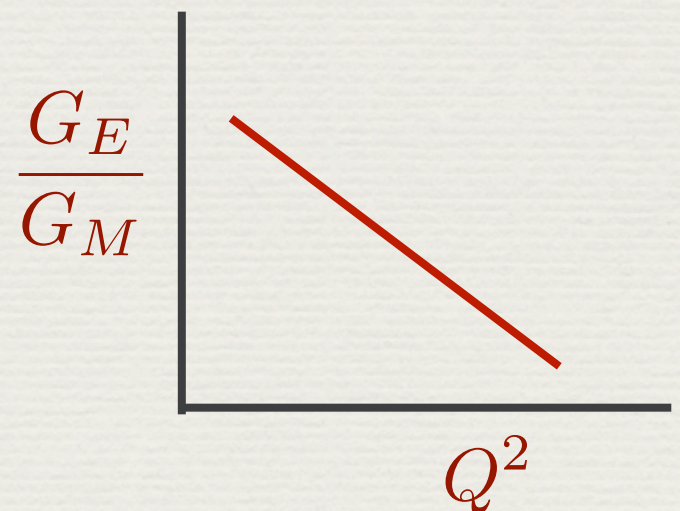
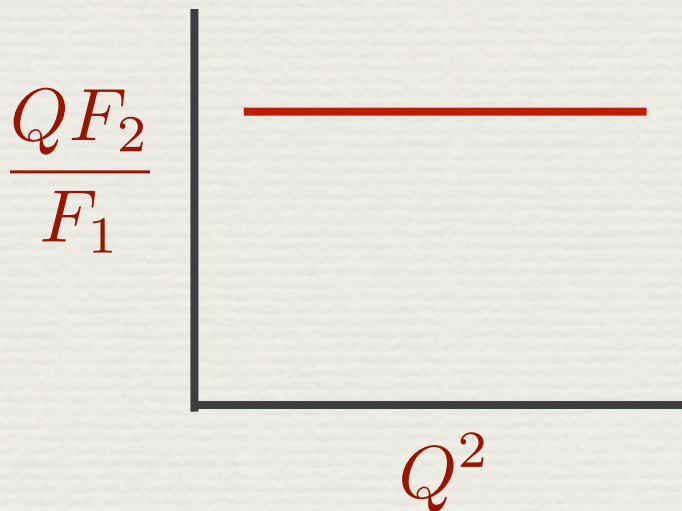
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Form Factors

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Naive expectation from dimensional counting

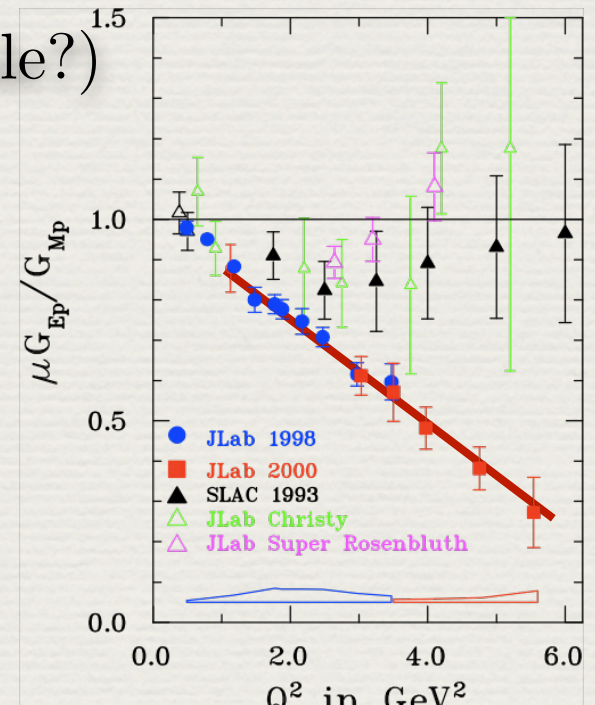
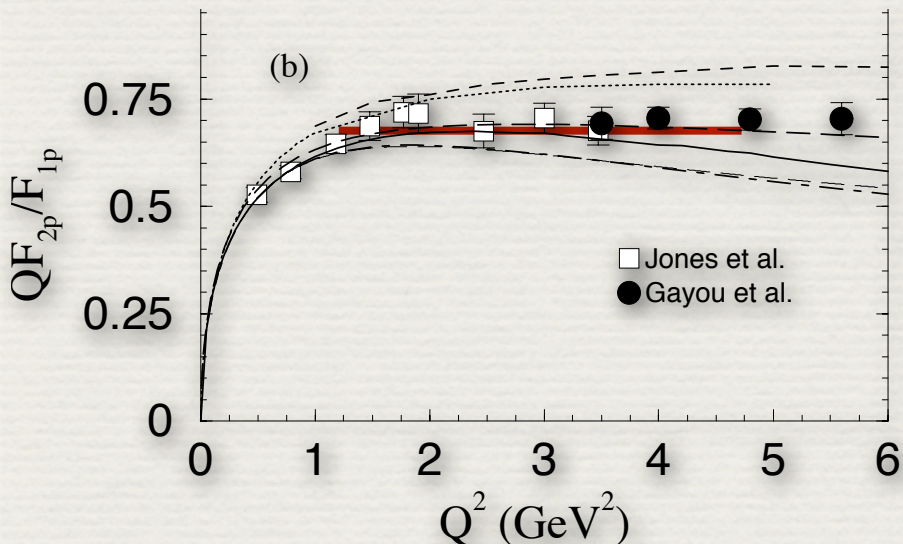
[Brodsky & Farrar, 1973]

$$F_1 \propto \frac{1}{Q^4} \quad (\text{dipole?})$$

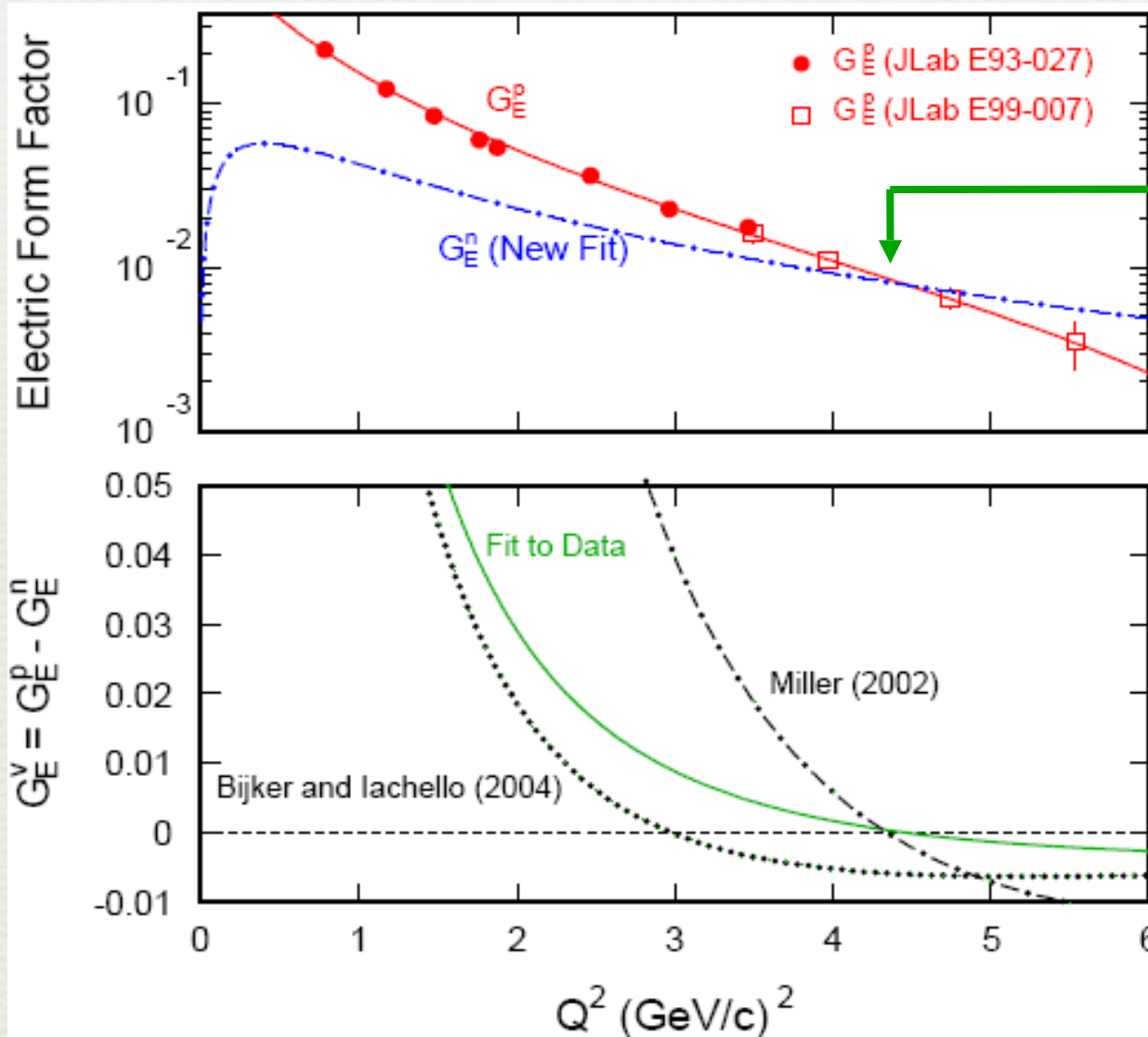
$$F_2 \propto \frac{1}{Q^6} \quad (\text{tripole?})$$

$$\frac{F(0)}{(1 + Q^2/M^2)^p}$$

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Zero Crossing?

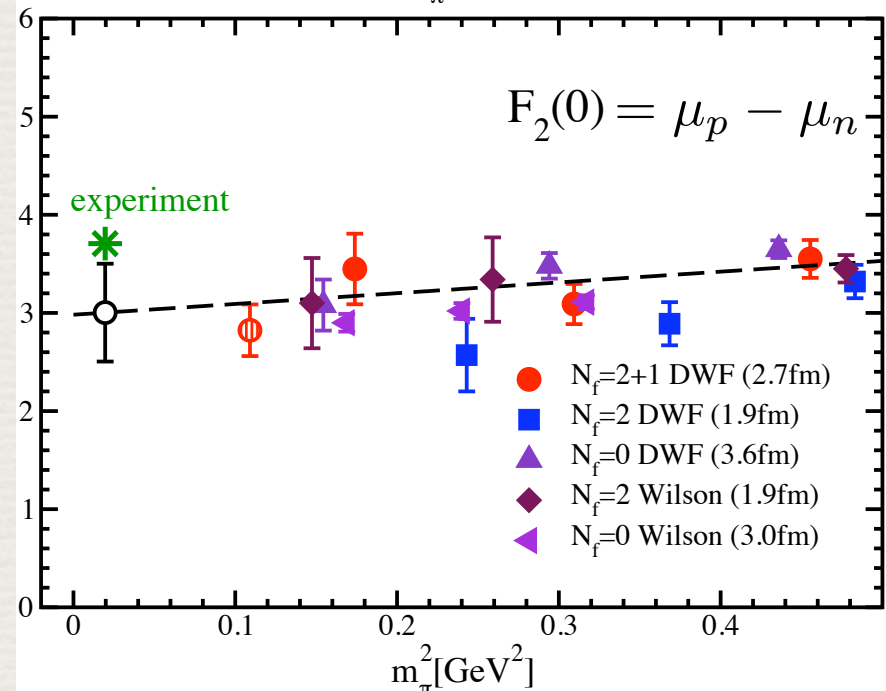
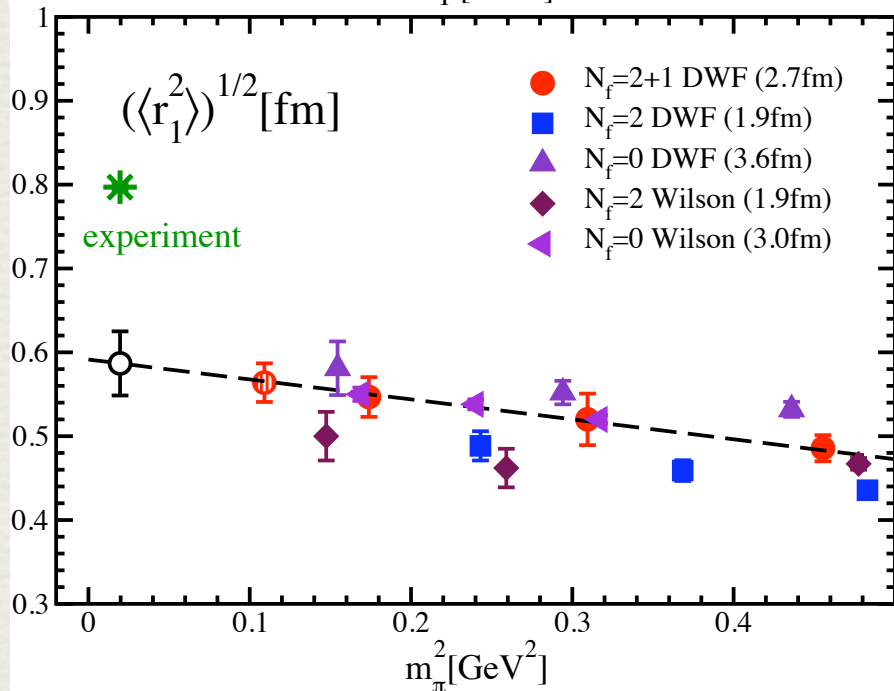
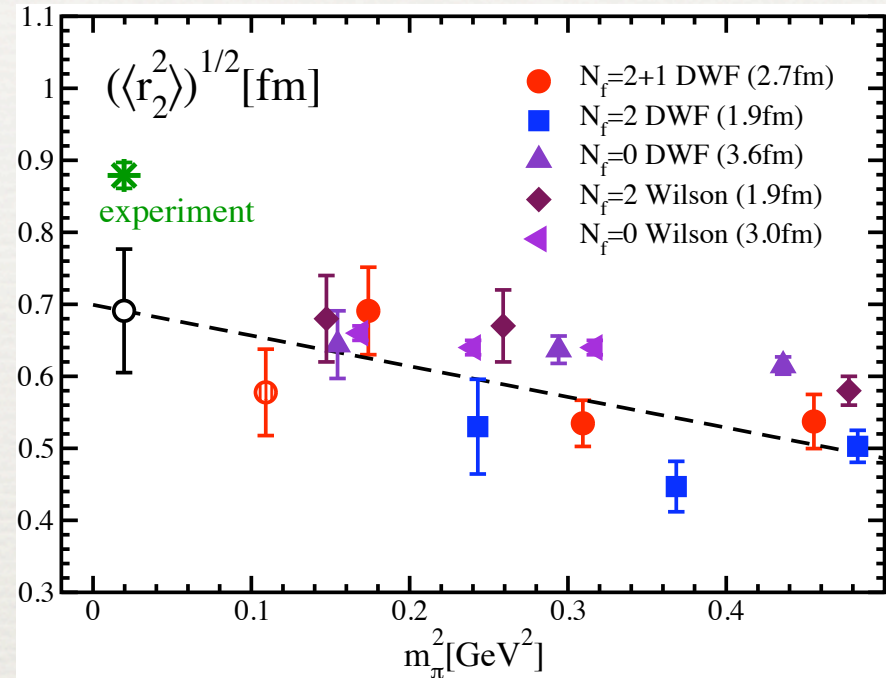
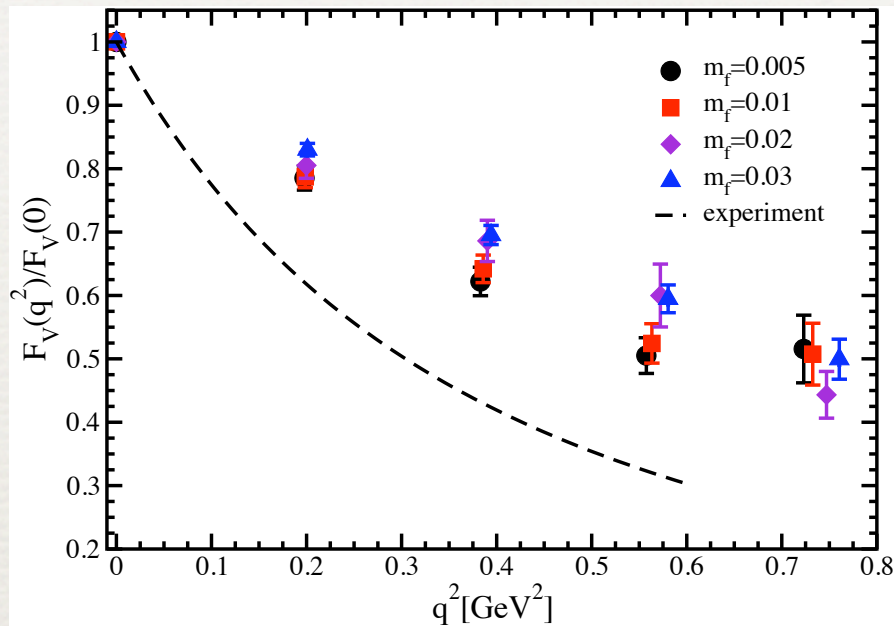


Possible zero crossing
in isovector electric
form factor G_E^V at
 $Q^2 \sim 4.5 (\text{GeV}/c)^2$

RBC/UKQCD [Takeshi Yamazaki] Monday 4:10

$N_f=2+1$ Domain Wall Fermions

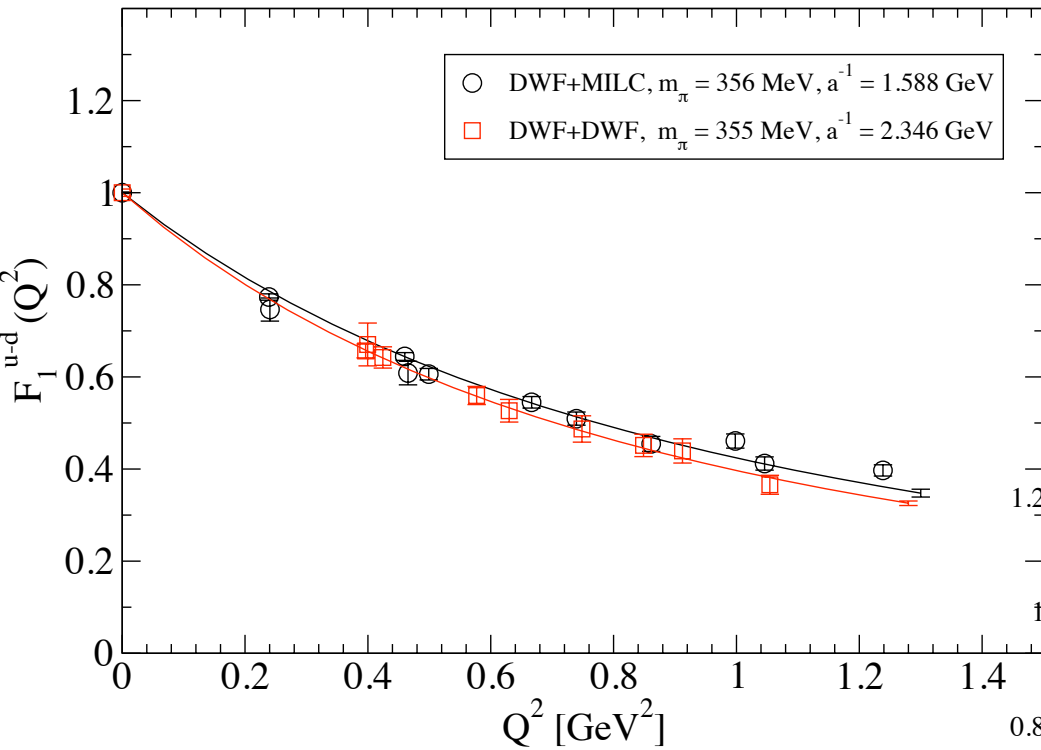
$24^3 \times 64 a^{-1} = 1.729 \text{ GeV}$



LHPC on RBC/UKQCD Configurations

$N_f=2+1$ Domain Wall Fermions

S. Syritsyn Friday 5:40



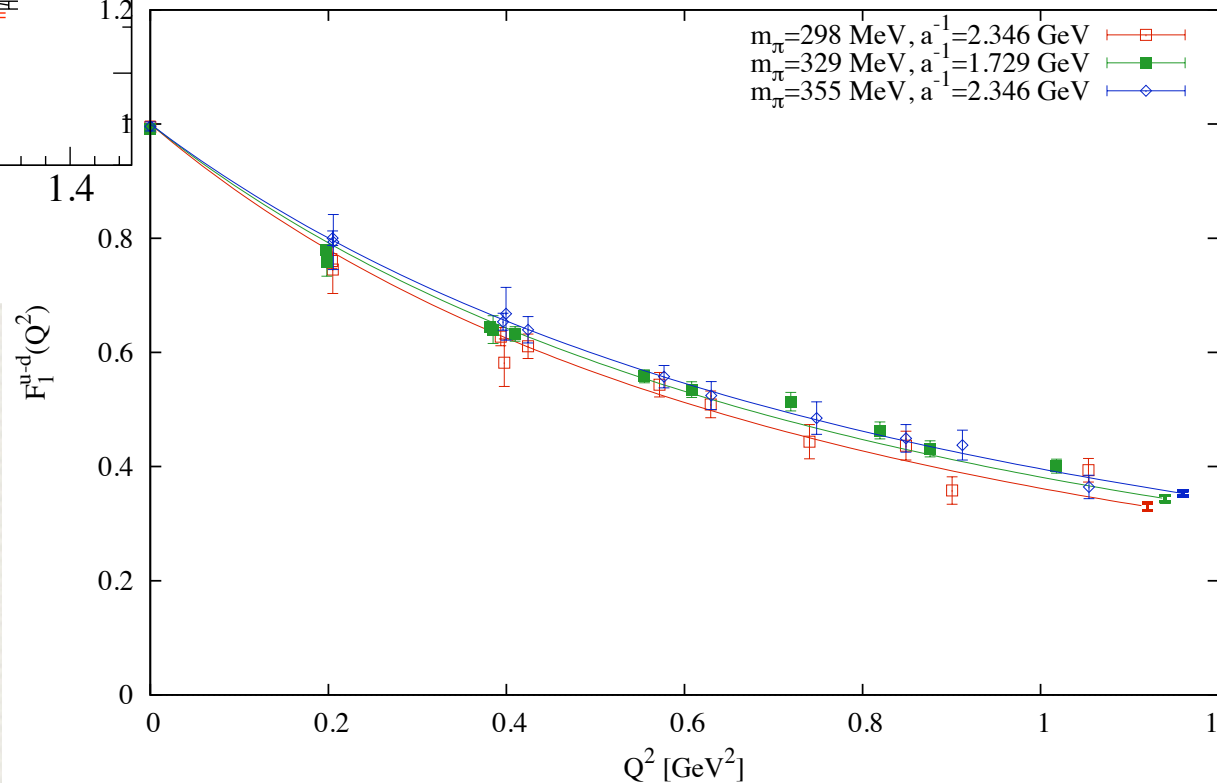
$24^3 \times 64$ $a^{-1} = 1.729$ GeV

$m_\pi \approx 330$ MeV

$32^3 \times 64$ $a^{-1} = 2.35$ GeV

$m_\pi \approx 355, 298$ MeV

DWF $n_f=3$ formfactor comparison



Also see Monday 3:30

M. Lin, DWF + asqtad

New results at $m_{\pi i}=293$ MeV

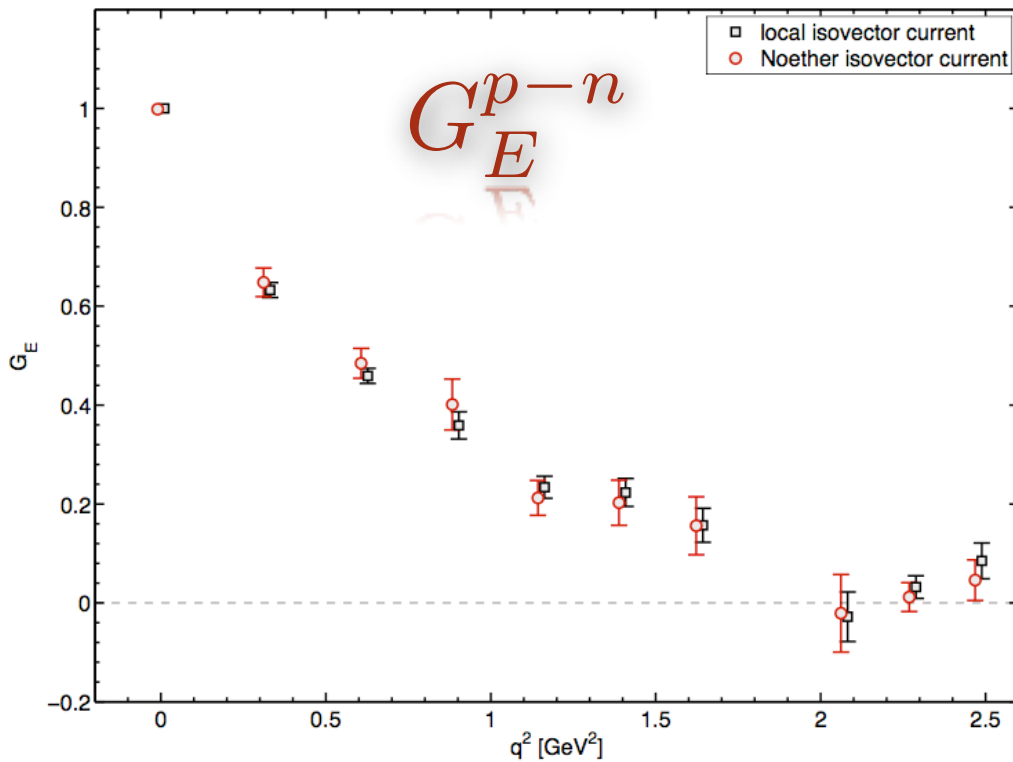
$N_f=2$ Twisted Mass Fermions

$$a = 0.089(1) \text{ fm}$$

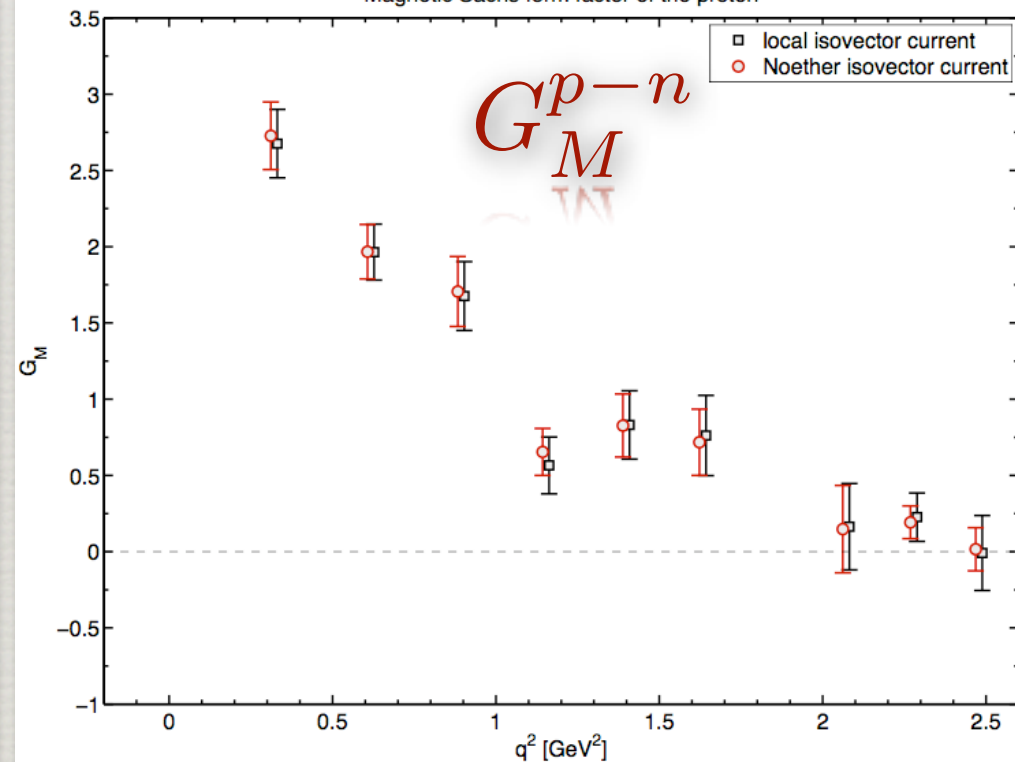
$$L = 2.13 \text{ fm}$$

$$m_\pi = 447, 313 \text{ MeV}$$

Electric Sachs form factor of the proton



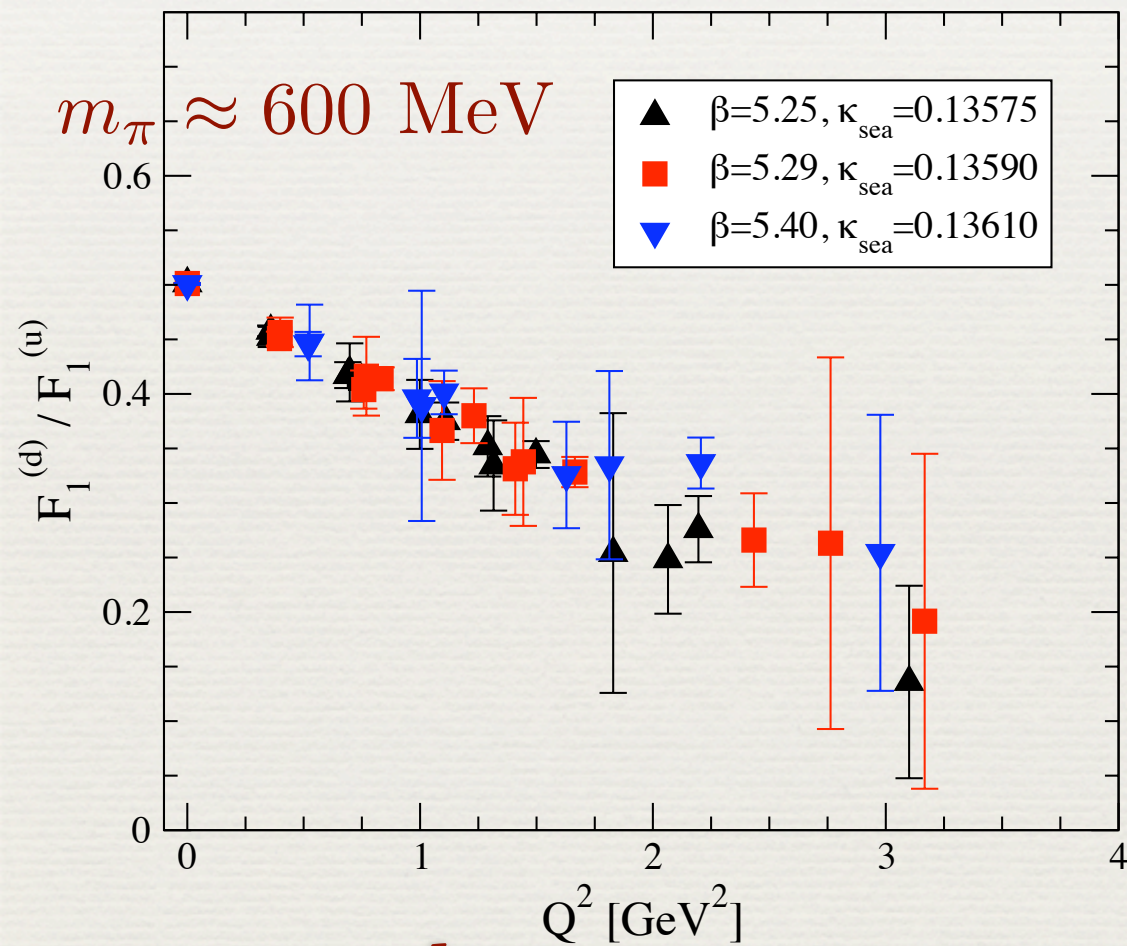
Magnetic Sachs form factor of the proton



Flavour Distribution

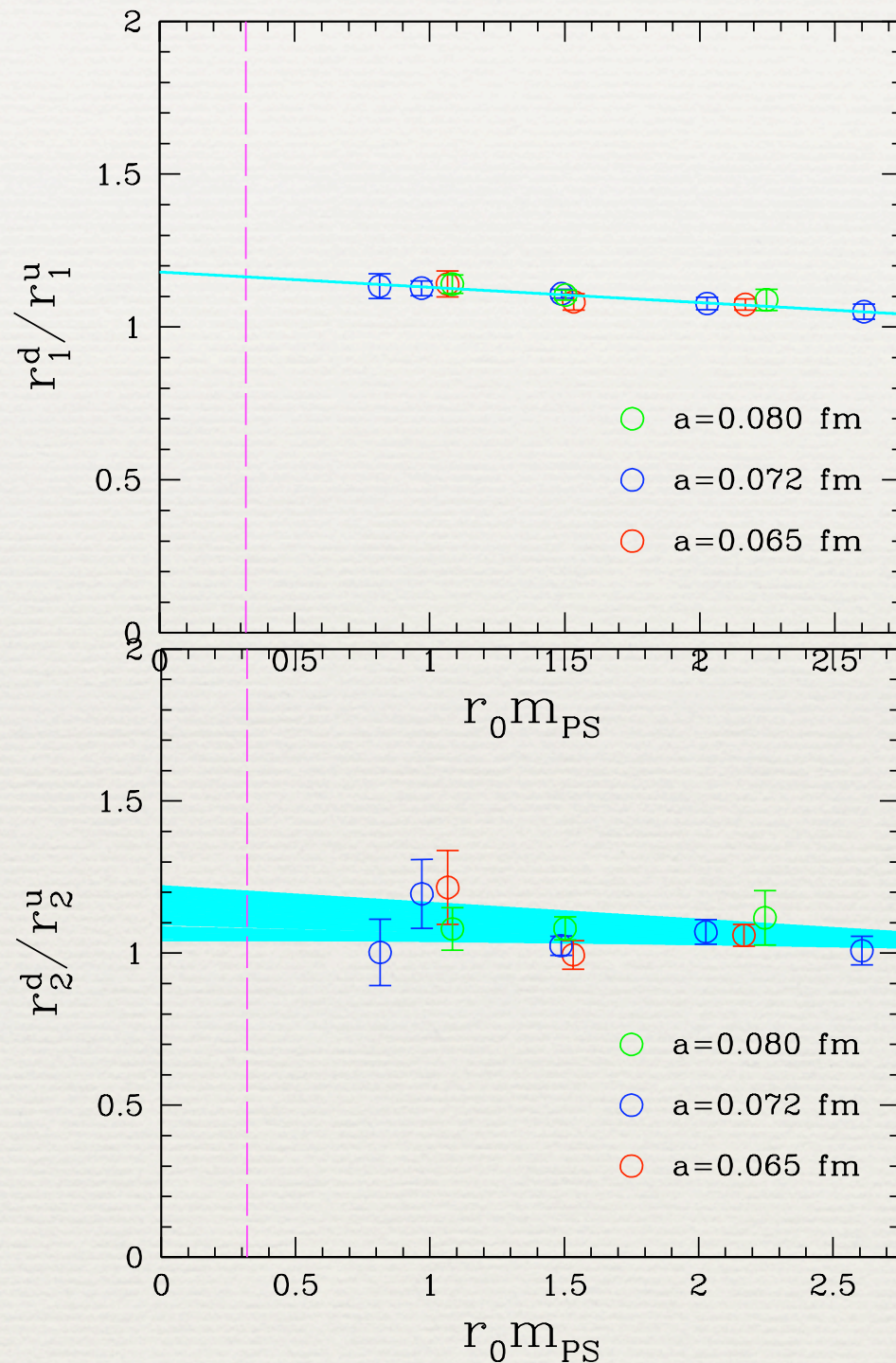
$$F^p = \frac{2}{3}F^u - \frac{1}{3}F^d$$

$$F^n = -\frac{1}{3}F^u + \frac{2}{3}F^d$$



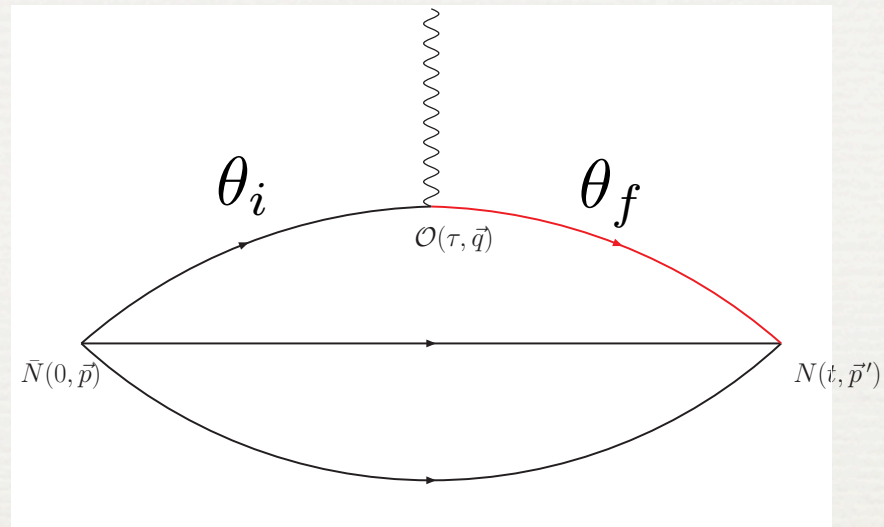
$$r_{1,2}^d > r_{1,2}^u$$

[QCDSF, $N_f=2$ Clover]



Accessing Small Q^2 : Partially Twisted Boundary Conditions

Bedaque hep-lat/0411033
Sachrajda & Villadoro hep-lat/0703005



- ✱ Modify boundary conditions on the valence quarks

$$\psi(x_k + L) = e^{i\theta_k} \psi(x_k), \quad (k = 1, 2, 3)$$

- ✱ allows to tune the momenta continuously $\vec{p}_{\text{FT}} + \vec{\theta}/L$

$$q^2 = (p_f - p_i)^2 = \left\{ [E_f(\vec{p}_f) - E_i(\vec{p}_i)]^2 - [(\vec{p}_{\text{FT},f} + \vec{\theta}_f/L) - (\vec{p}_{\text{FT},i} + \vec{\theta}_i/L)]^2 \right\}$$

- ✱ Introduces additional finite volume effect $\sim e^{-m_\pi L}$

Jiang & Tiburzi, Finite Volume effects for partially twisted b.c.
arXiv:0806.4371: small for pion form factor in the Breit frame

Nucleon form factors??

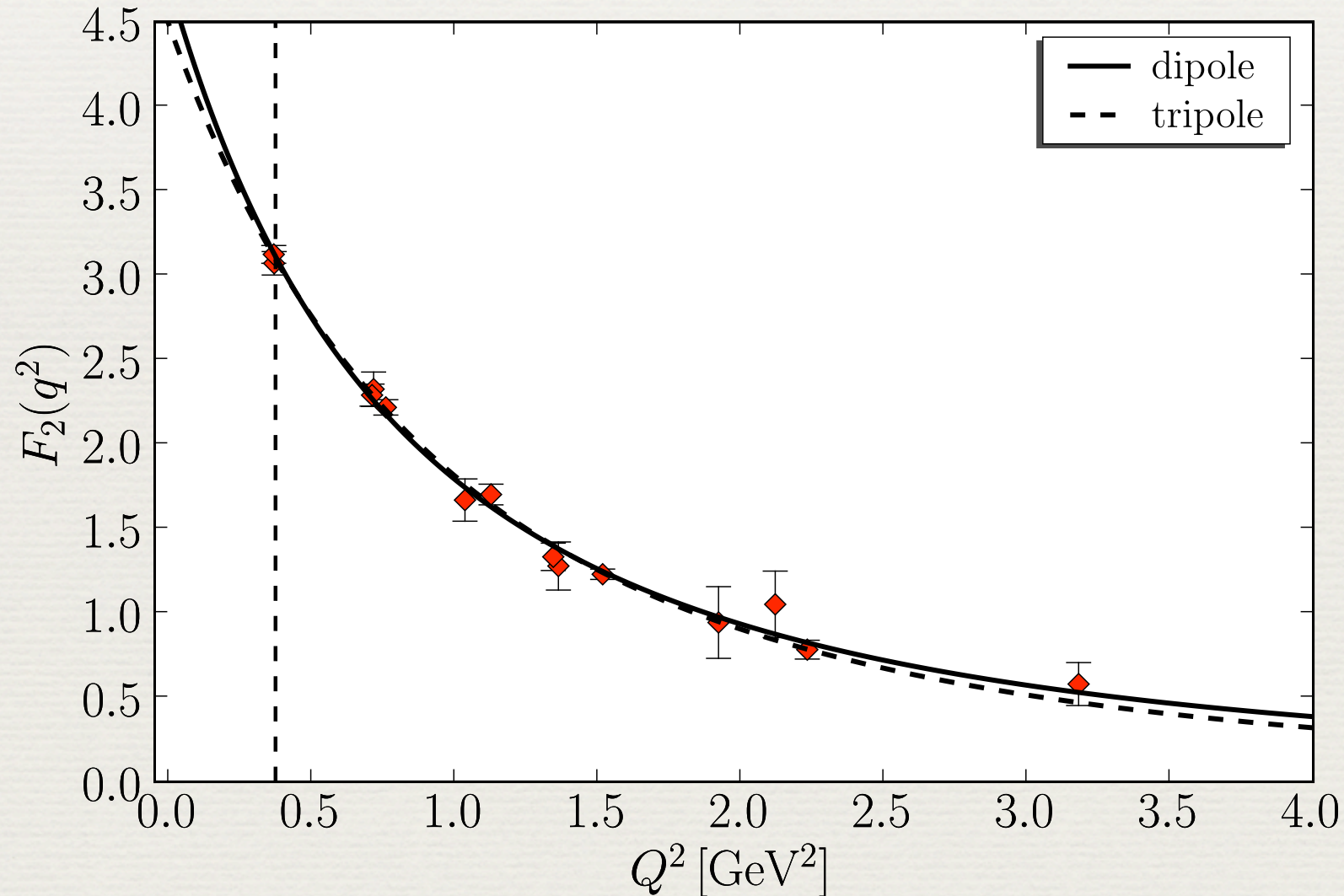
Accessing Small Q^2 : Partially Twisted Boundary Conditions

Ph. Hägler [QCDSF]: $N_f=2$ Clover Monday 2:30

We need to extrapolate $F_2(q^2)$ to $q^2=0$



Model dependence



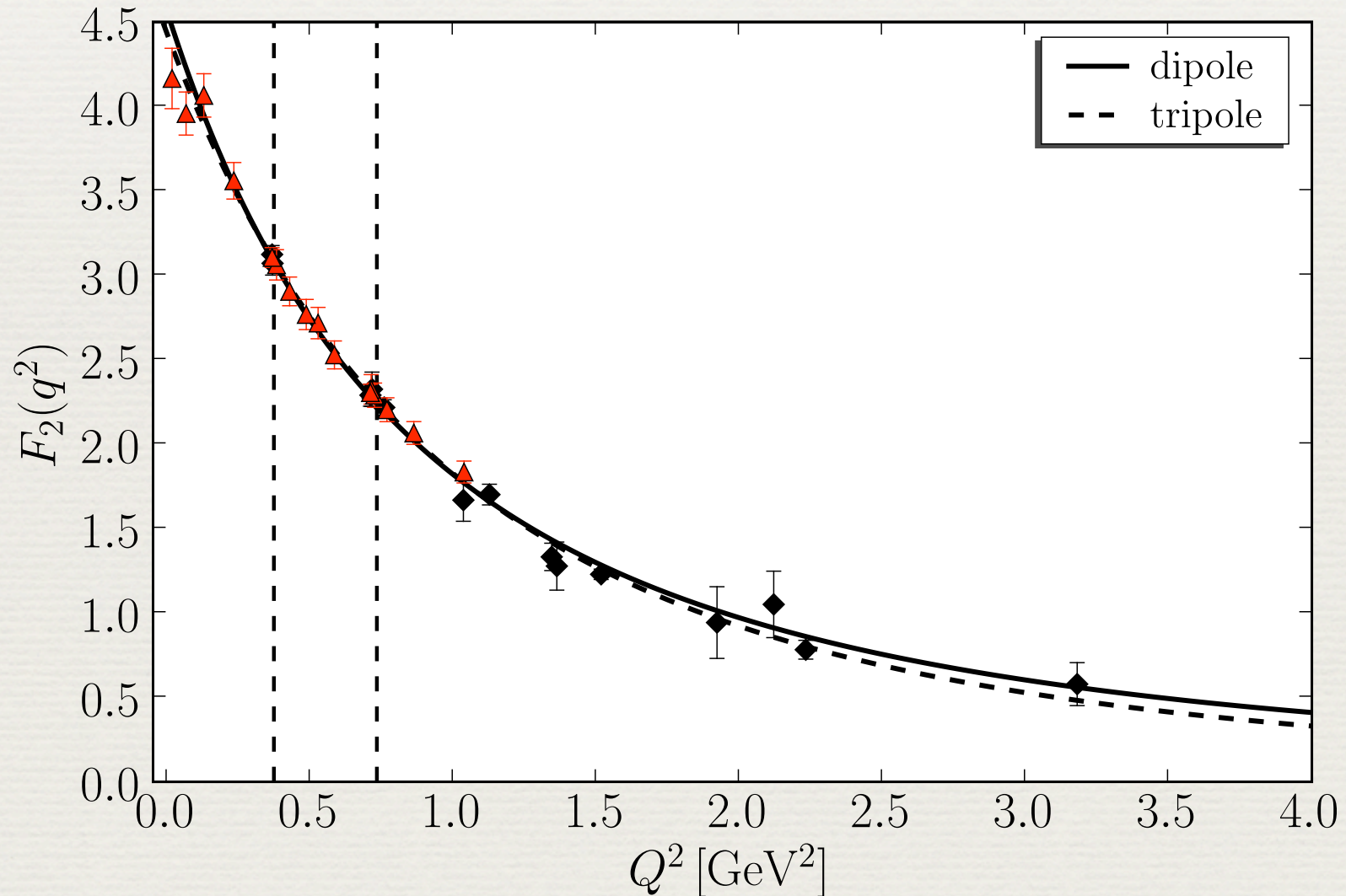
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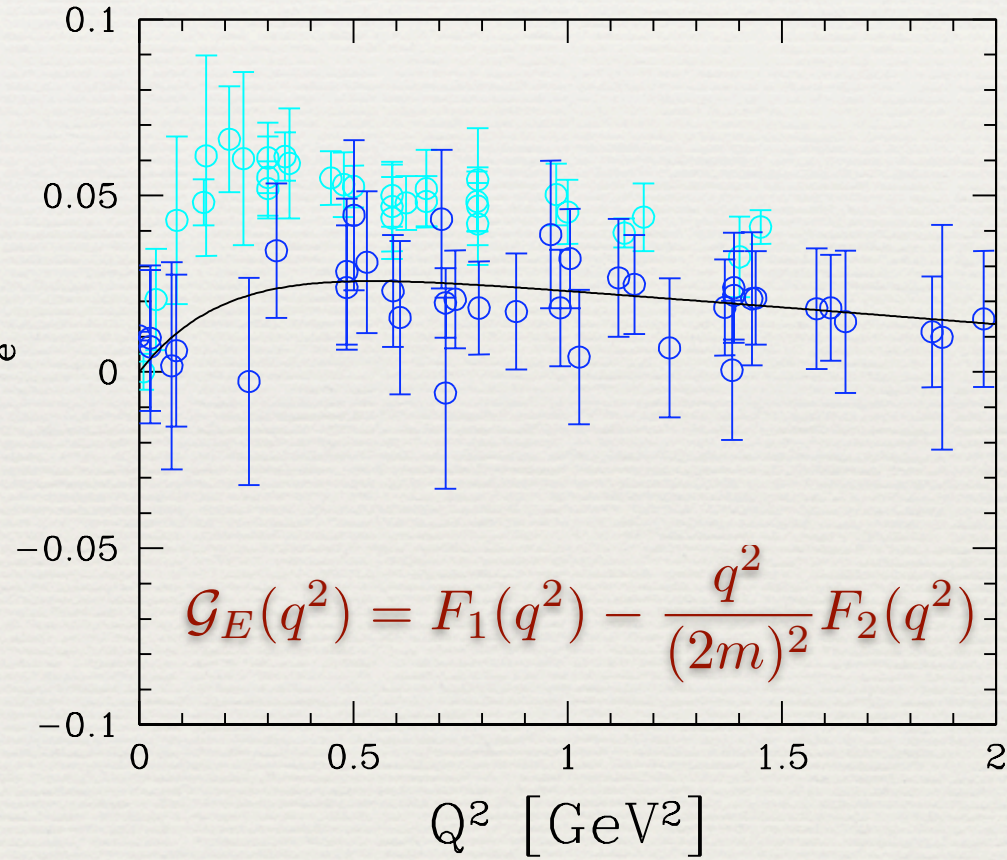
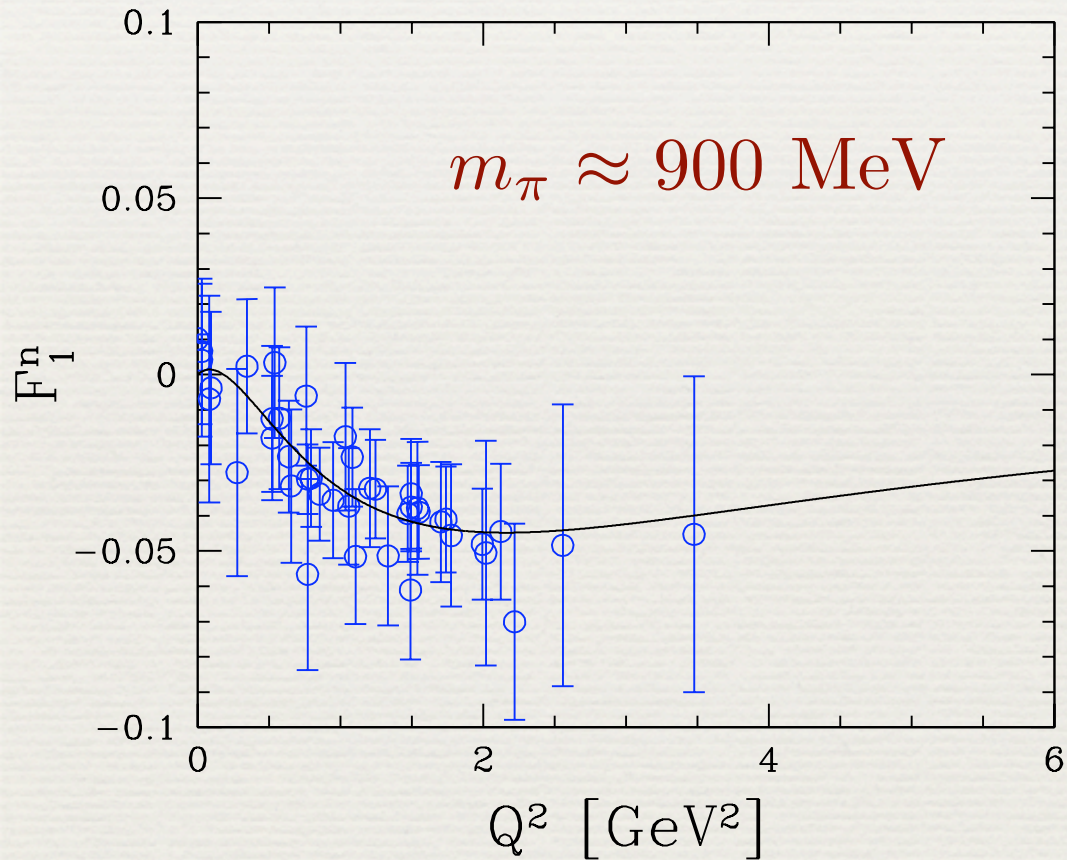


Neutron Form Factors

QCDSF: $N_f=2$ Clover

Phenomenological multipole fit (a la Kelly)

$$F(Q^2) = F(0)(1 + c_1 Q^2 + c_n Q^{2n})^{-1}$$



F_1 neutron negative at small Q^2

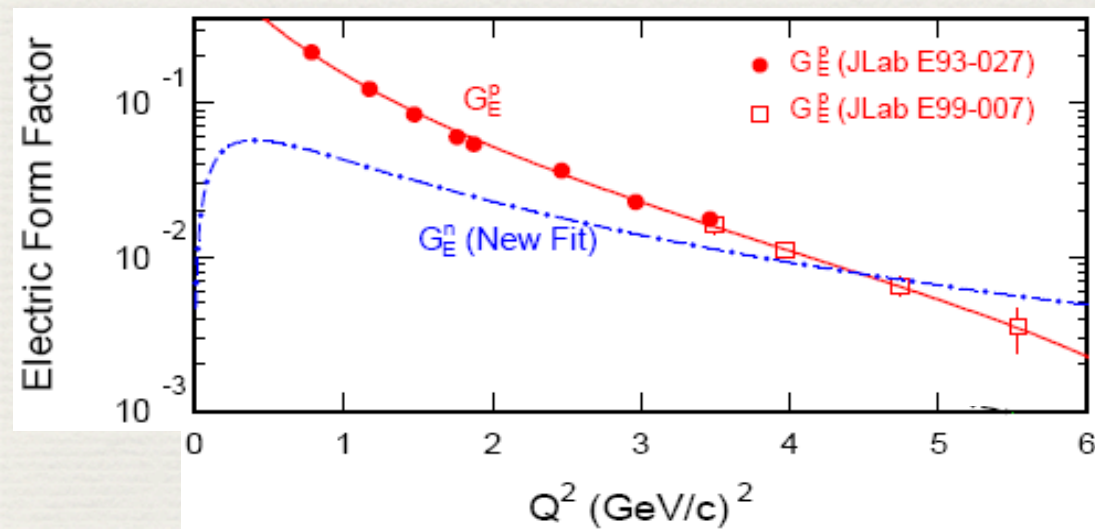
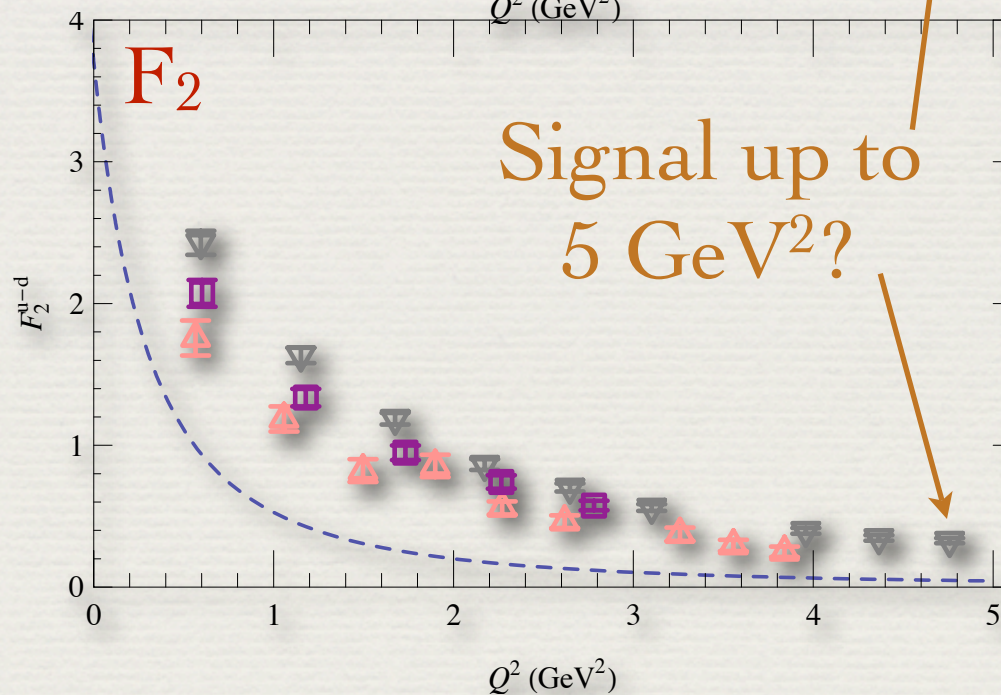
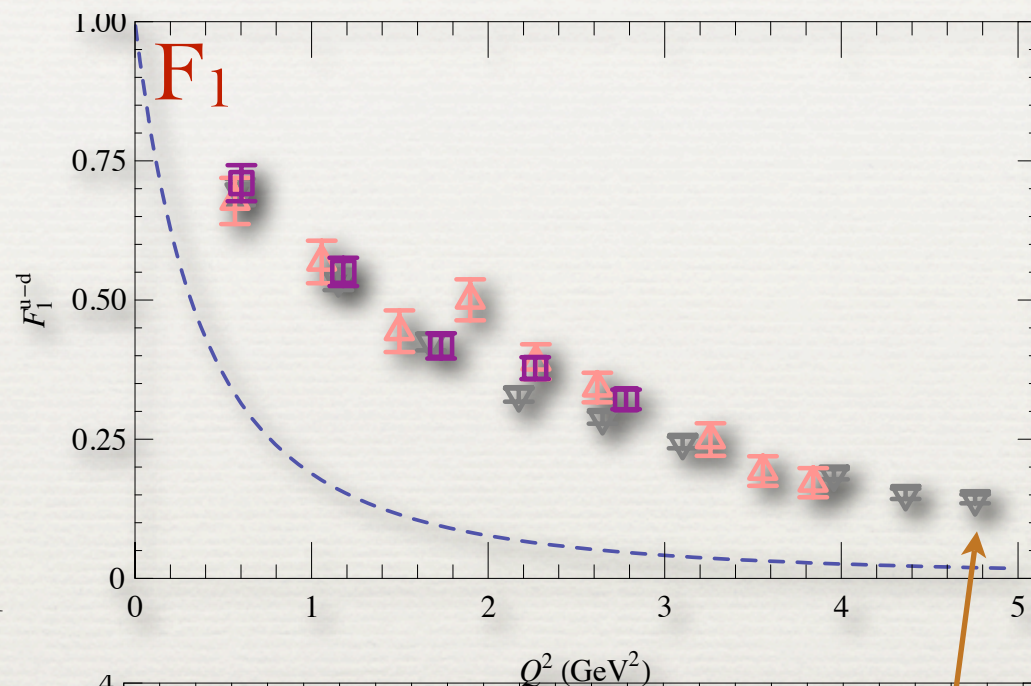
How does "hump" change with quark mass?

Large Q^2

H.-W. Lin, S. Cohen, R. Edwards, D. Richards Monday 3:10

$m_{\pi} = 480, 720, 1100 \text{ MeV}$

- ◆ Quenched
- ◆ Variational methods
- ◆ Three gaussian smearings
- ◆ Solve generalised eigenvalue problem to obtain Z 's and m 's
- ◆ Use Z 's and m 's in 3pt correlator and solve for form factors
- ◆ Smearing tuned for $p=0$ not ideal at large p



Nucleon-Roper Form Factors Monday 3:10

H.-W. Lin, S. Cohen, R. Edwards, D. Richards [arXiv:0803.3020]

Quenched study with variational methods at $m_{\text{pi}} = 480, 720, 1100 \text{ MeV}$

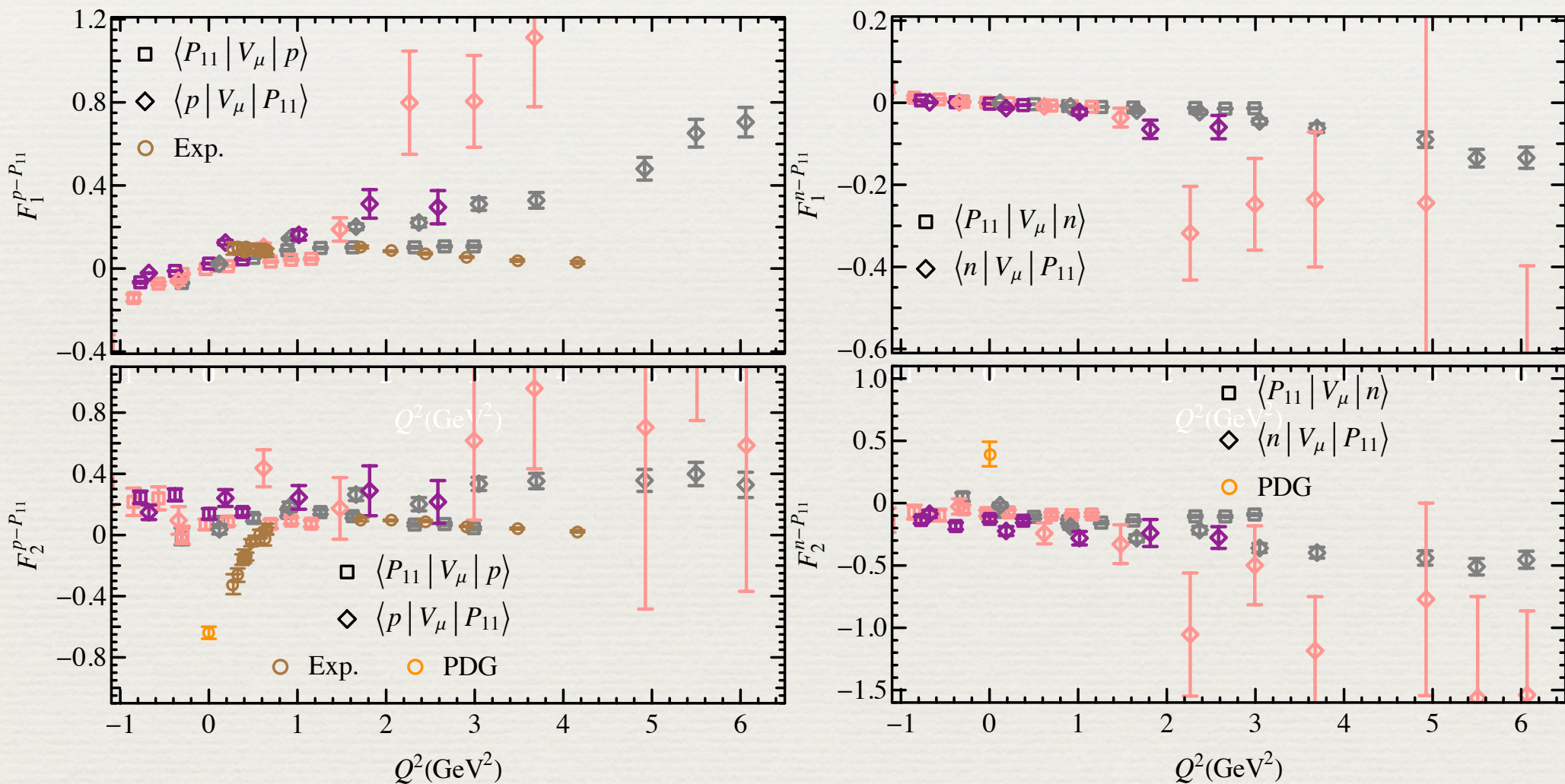
$$\langle N_2 | j_\mu(\vec{q}) | N_1 \rangle = \bar{u}_{N_2}(p') \left[F_1(q^2) \left(\gamma_\mu - \frac{q_\mu}{q^2} \not{q} \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p)$$

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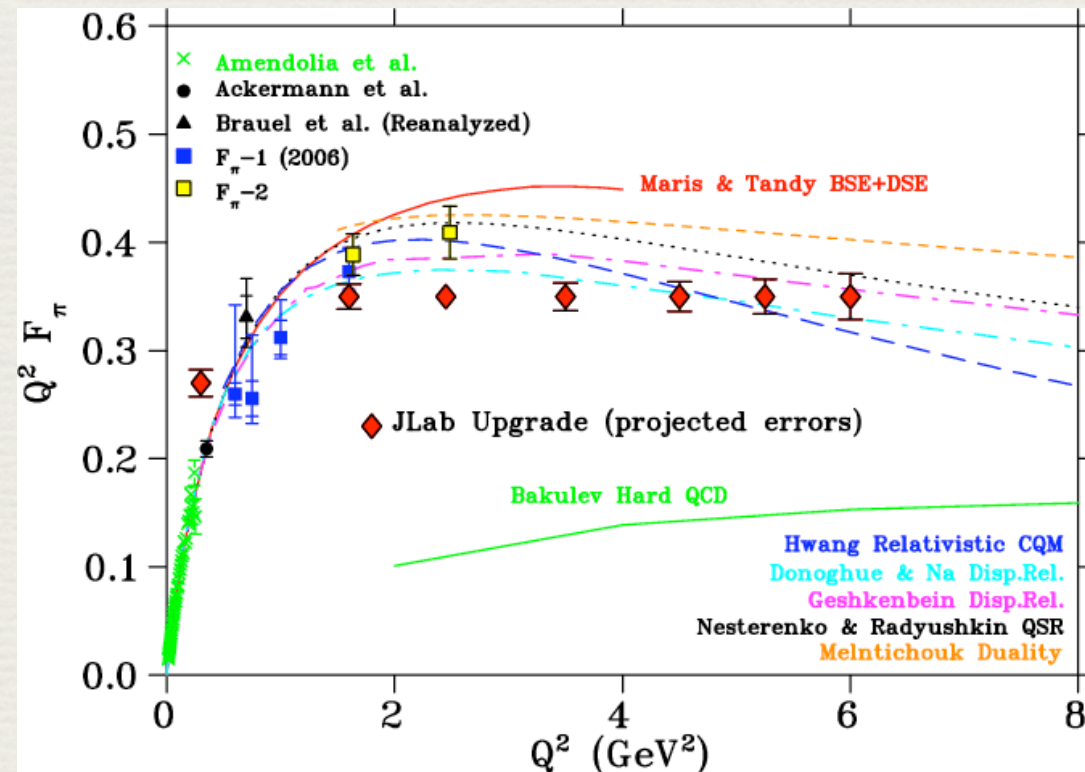
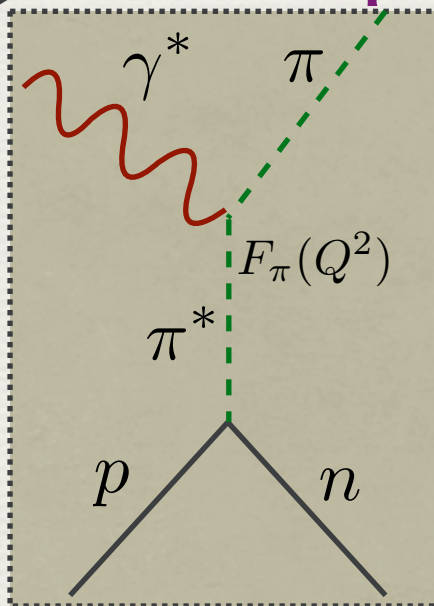
Pion Form Factor

- Asymptotic normalisation known from $\pi \rightarrow \mu + \nu$ decay

$$F_\pi(Q^2 \rightarrow \infty) = \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2}$$

- Allows to study the transition from the soft regime (quark-gluon correlations) to the hard regime (perturbative QCD)
- Low Q^2 : F_π measured directly by scattering high energy π^+ from atomic electrons [CERN] $r_\pi = 0.657 \pm 0.006$ fm
- High Q^2 : quasi-elastic scattering off virtual pions [DESY & JLab]

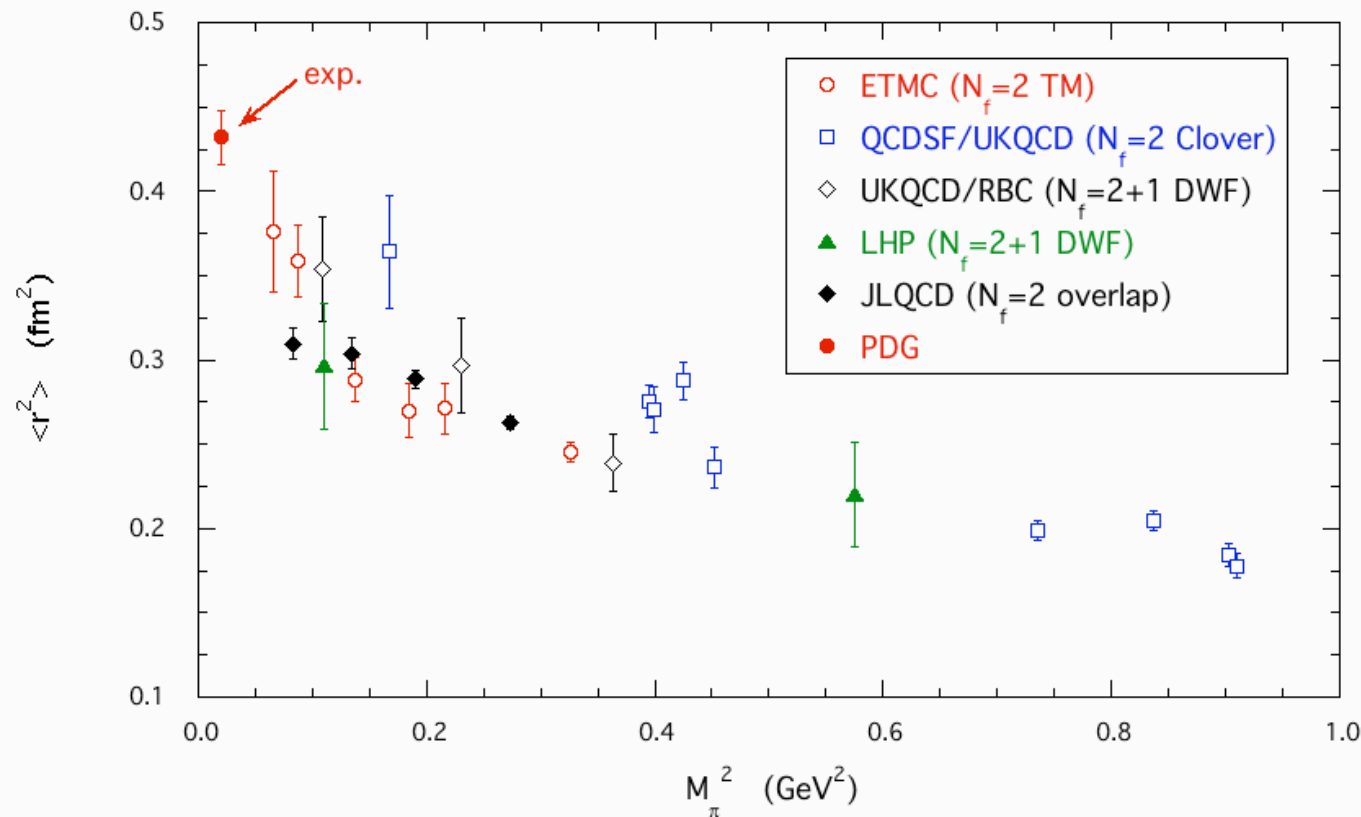
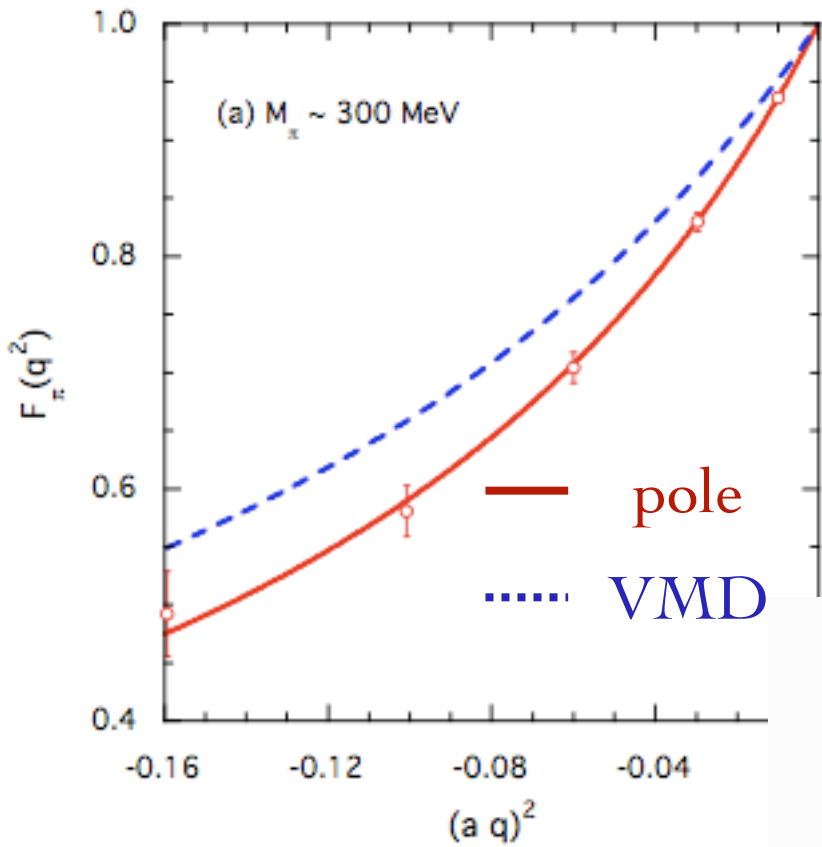
➔ **Model dependence**



Pion Charge Radius

S. Simula et al., [ETMC]

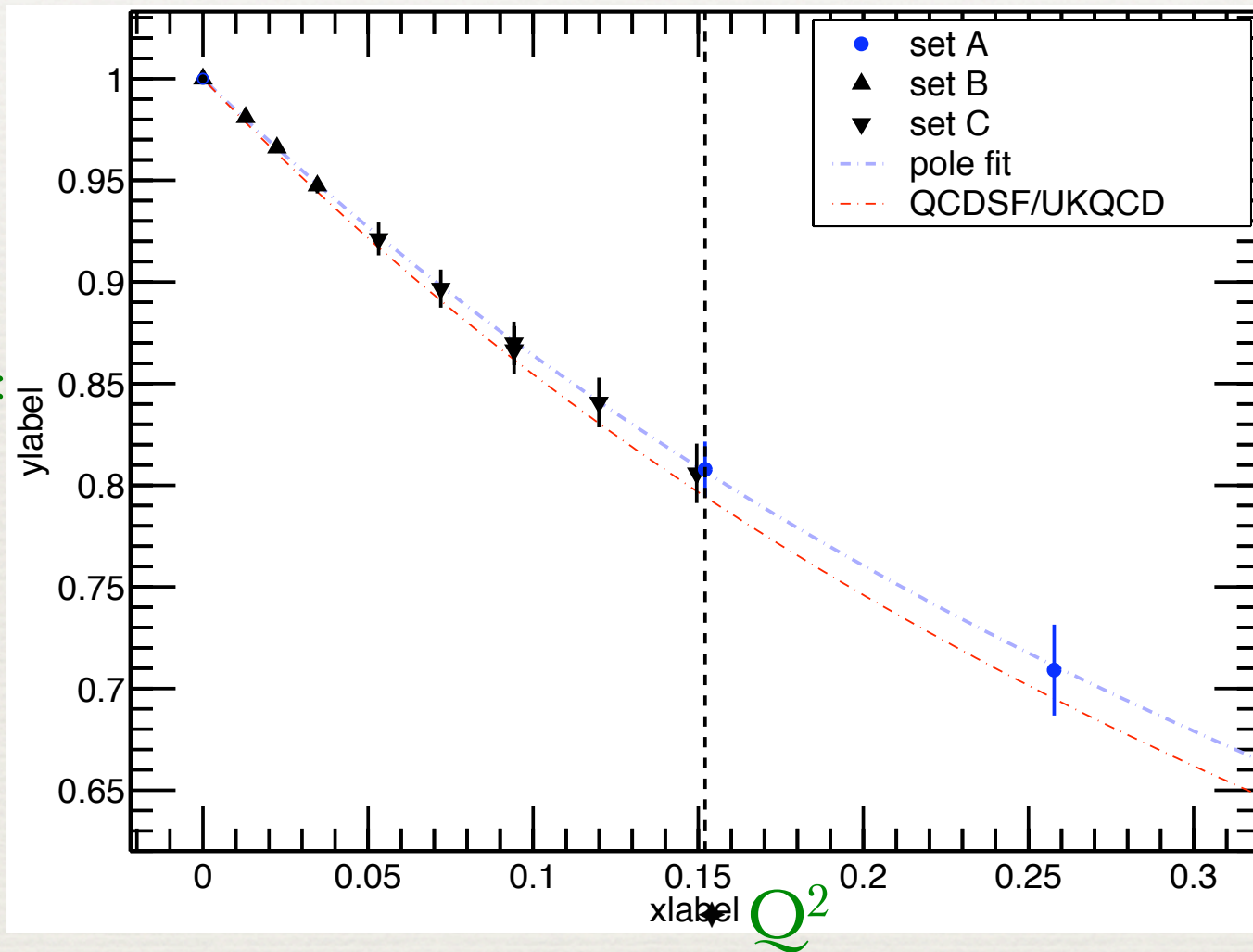
- ◆ $N_f=2$ Twisted Mass fermions
- ◆ Stochastic prop (one-end trick)
- ◆ Twisted boundary conditions



Finite Volume?
Discretisation?

Pion Charge Radius

[Boyle et al. [UKQCD/RBC], DWF $N_f=2+1$, arXiv: 0804.3971]



- Stochastic prop (one-end trick)
- Twisted boundary conditions

$$m_\pi \approx 330 \text{ MeV}$$

$$24^3 \times 64$$

$$a \approx 0.114 \text{ fm}$$

$$[0.418(28)]$$

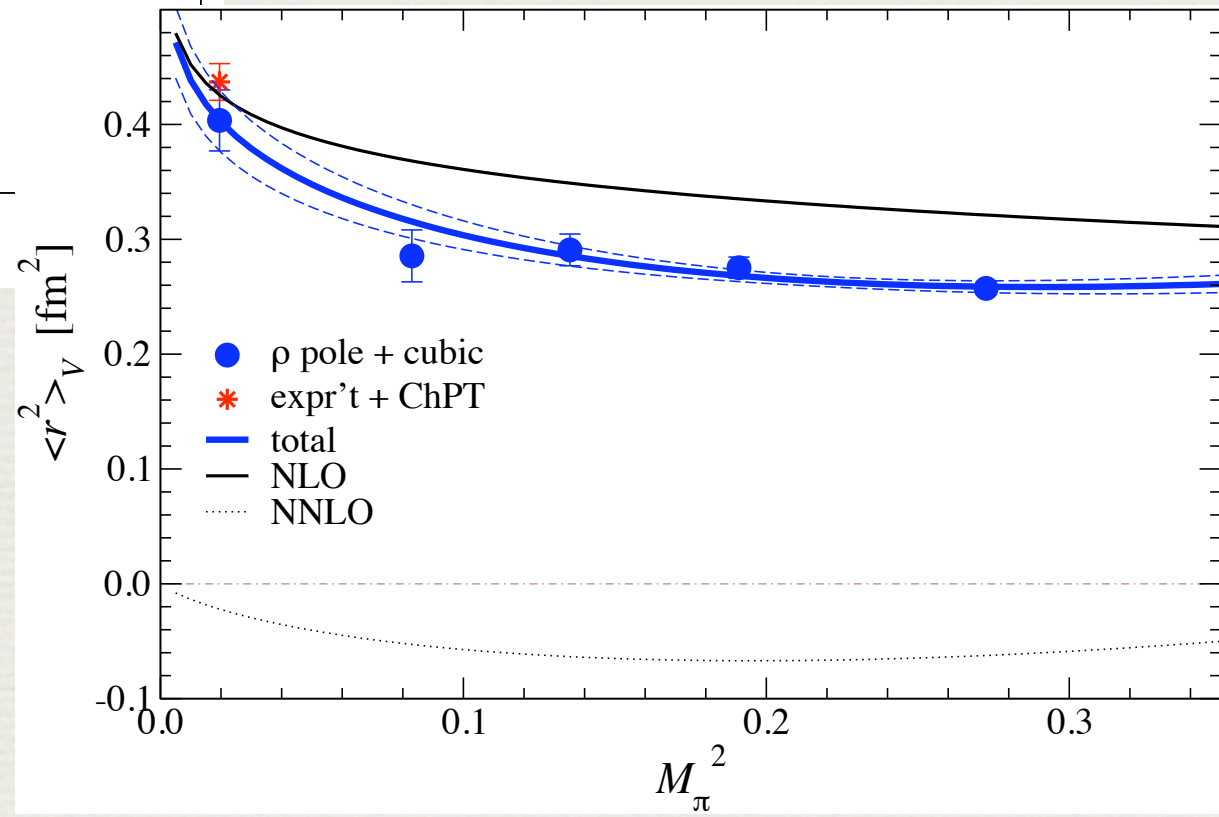
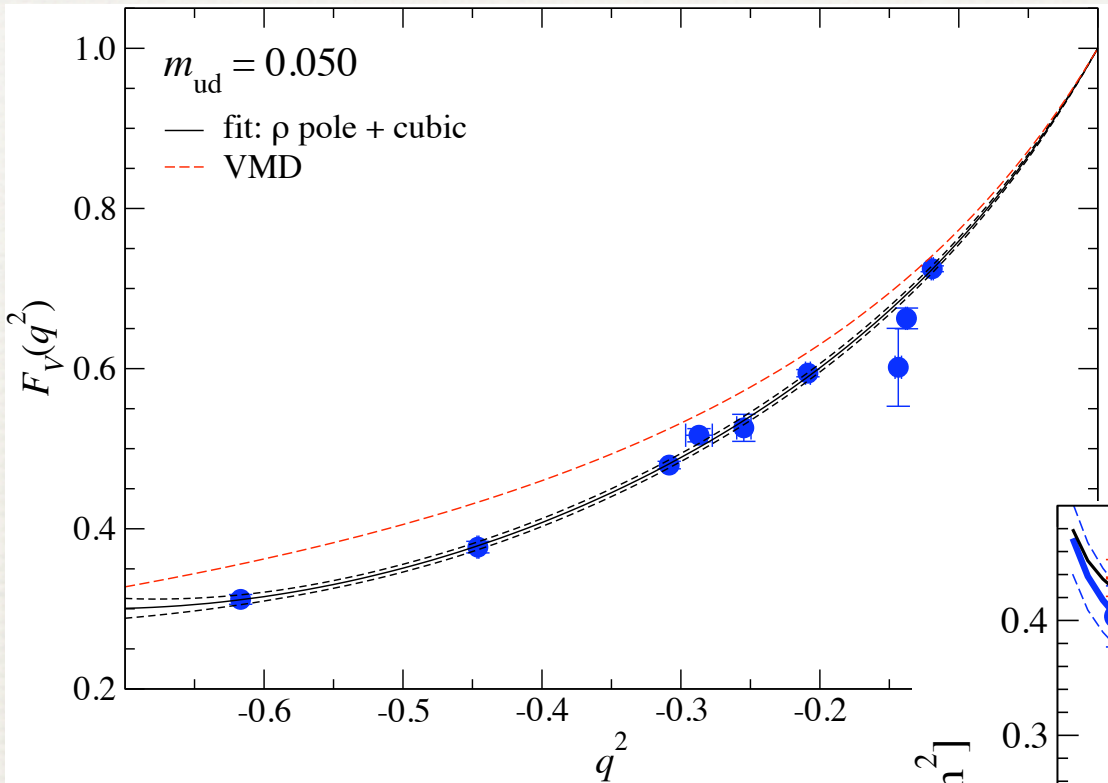
$$[0.452(11)]_{exp}$$

$$\langle r_\pi^2 \rangle_{(2), \text{NLO}} = -\frac{12l_6^r}{f^2} - \frac{1}{8\pi^2 f^2} \left(\log \frac{m_\pi^2}{\mu^2} + 1 \right)$$

Pion Form Factor

Thursday 9:10

T. Kaneko [JLQCD]: $N_f=2$ Overlap, all-to-all propagators



Pion Charge Radius

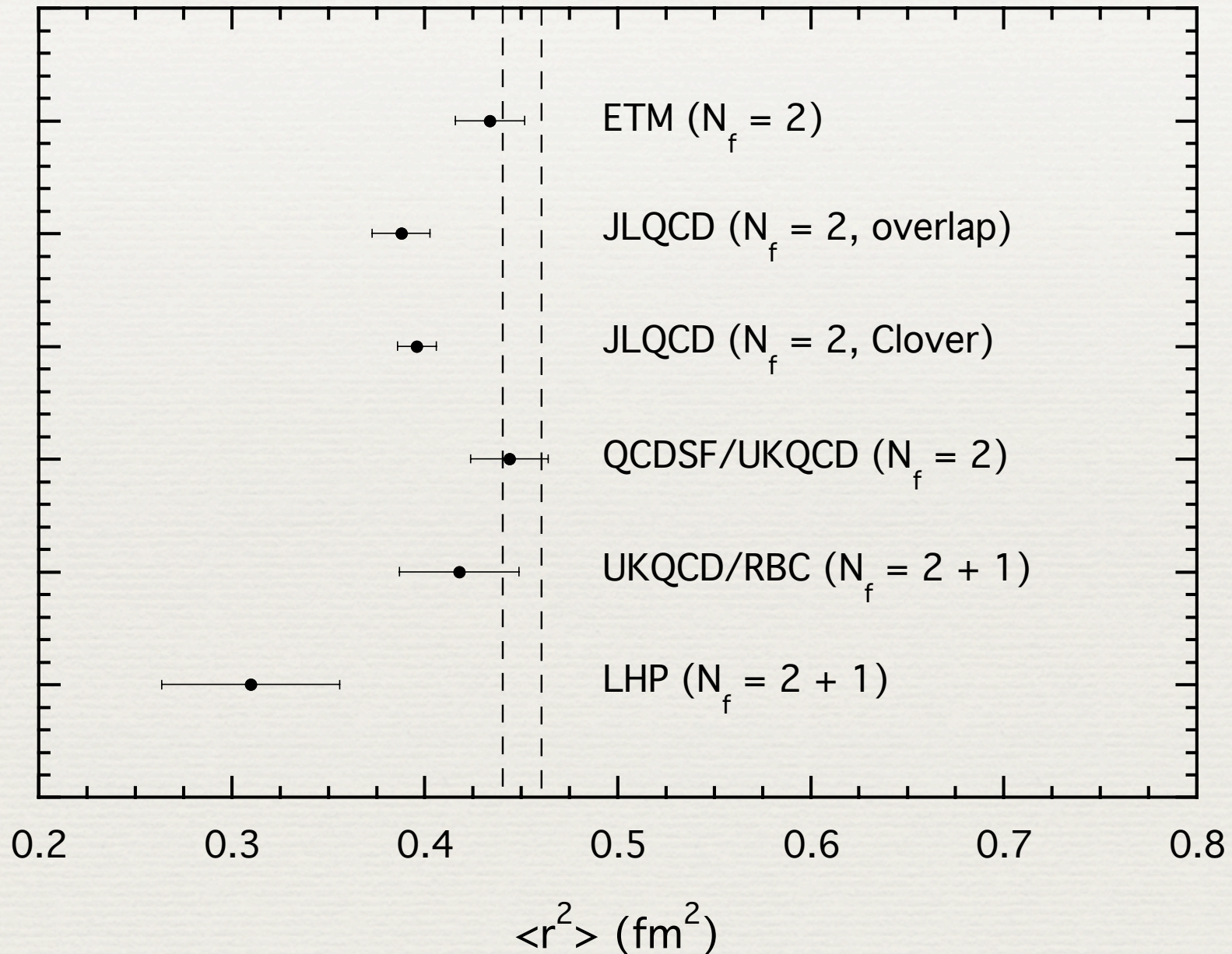


Figure by S. Simula

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Spin Sum Rule

- Spin decomposed in terms of quark and gluon angular momentum

$$\frac{1}{2} = \sum_q J_q(\mu^2) + J_g(\mu^2)$$

- Further decomposition into spin and orbital angular momentum

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta \Sigma_q + \sum_q L_q + \Delta G + L_g$$

- Also expressed in terms of moments of GPDs

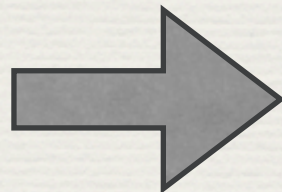
$$J_{q/g} = \frac{1}{2} [A_{20}^{q/g}(\Delta^2 = 0) + B_{20}^{q/g}(\Delta^2 = 0)]$$

- Matrix elements of the energy momentum tensor

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{U}(P') \left\{ \gamma^\mu \bar{P}^\nu A_{20}(\Delta^2) + \frac{i\sigma^{\mu\rho} \Delta_\rho \bar{P}^\nu}{2m_N} B_{20}(\Delta^2) + \frac{\Delta^\mu \Delta^\nu}{m_N} C_{20}(\Delta^2) \right\} U(P)$$

Momentum conservation:

$$\begin{aligned} 1 &= \sum_q A_{20}^q(0) + A_{20}^g(0) \\ &= \sum_q \langle x \rangle_q + \langle x \rangle_g \end{aligned}$$



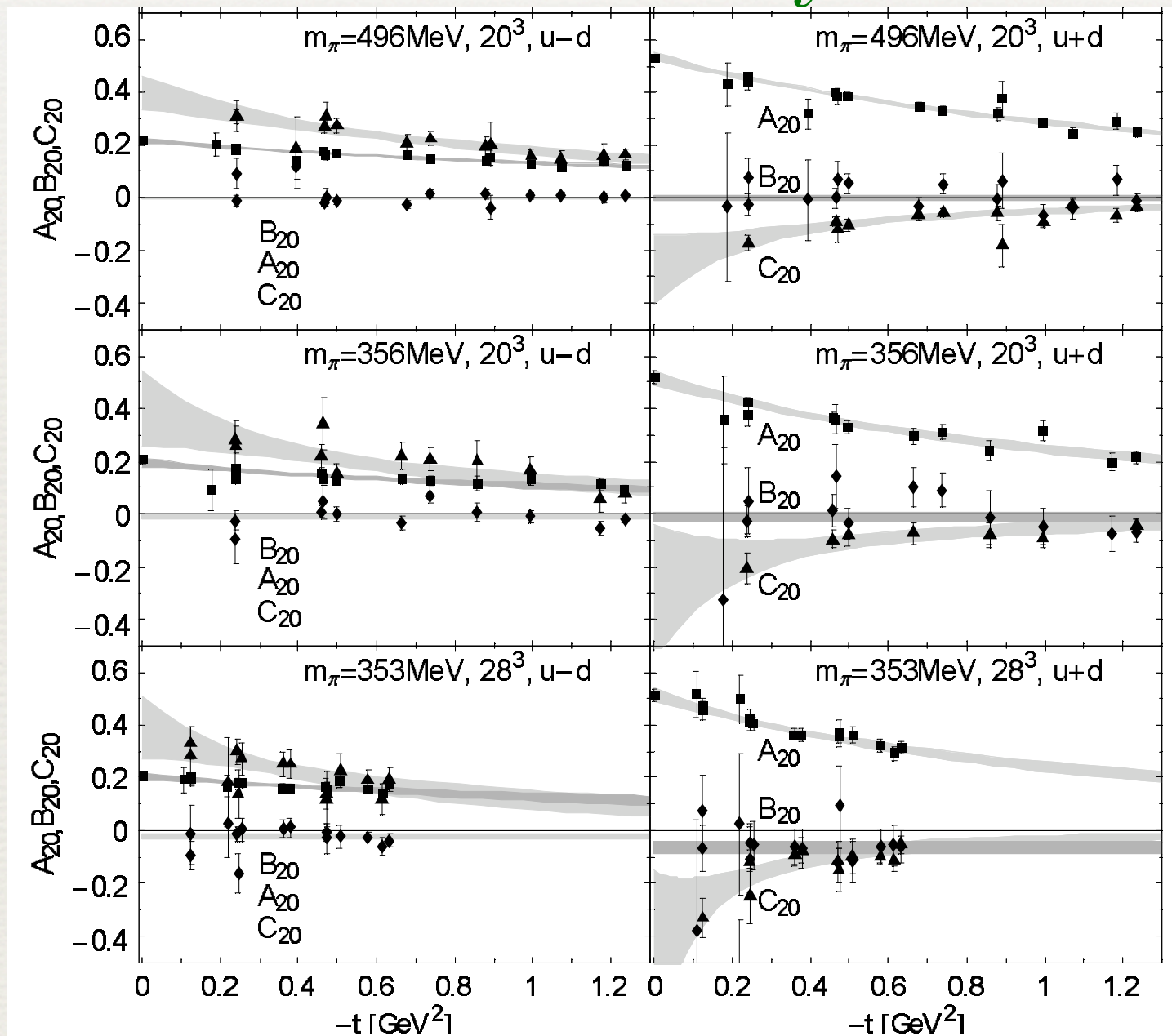
Anomalous gravitomagnetic moment

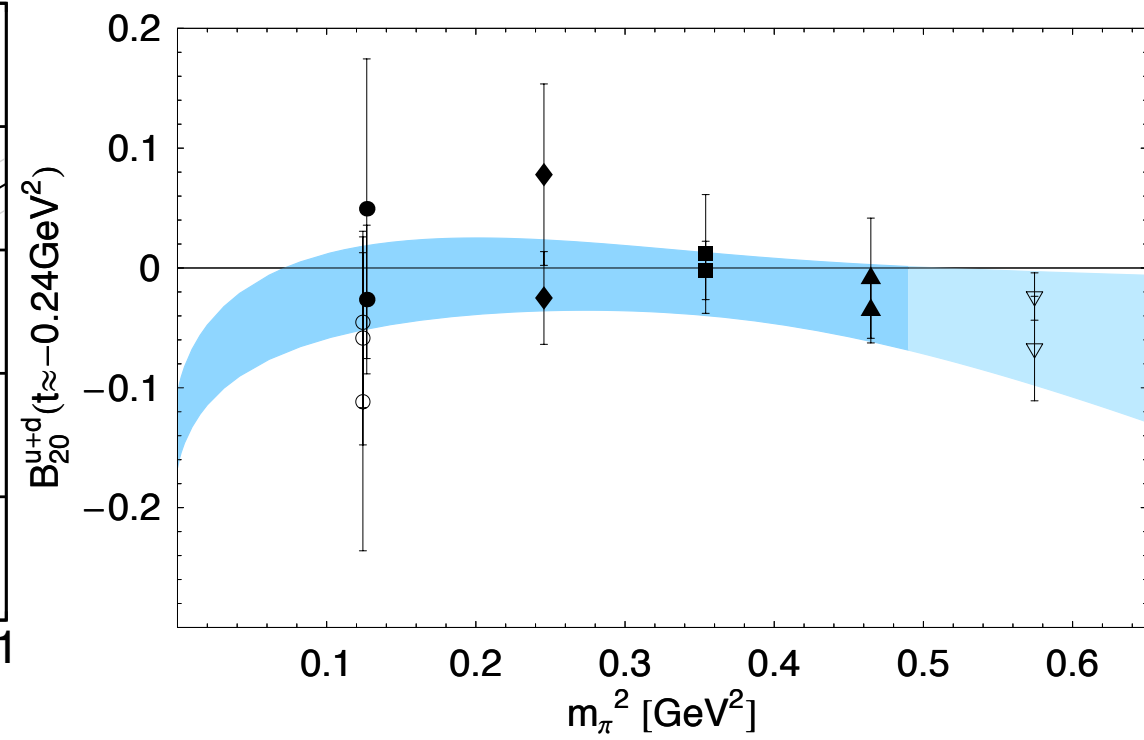
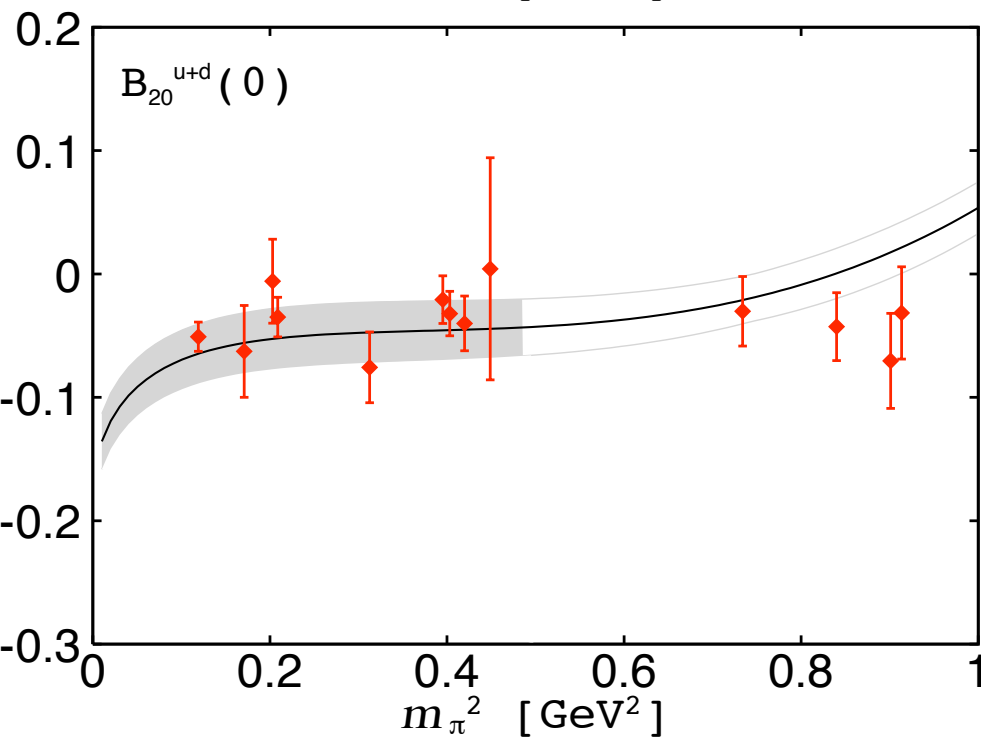
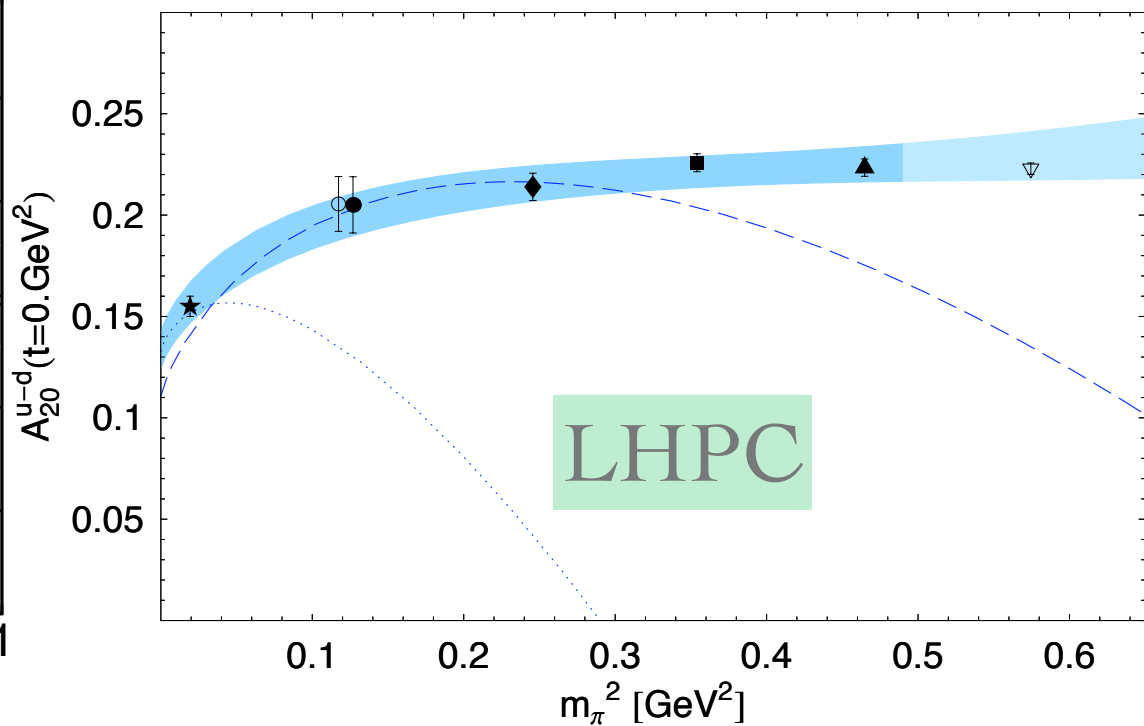
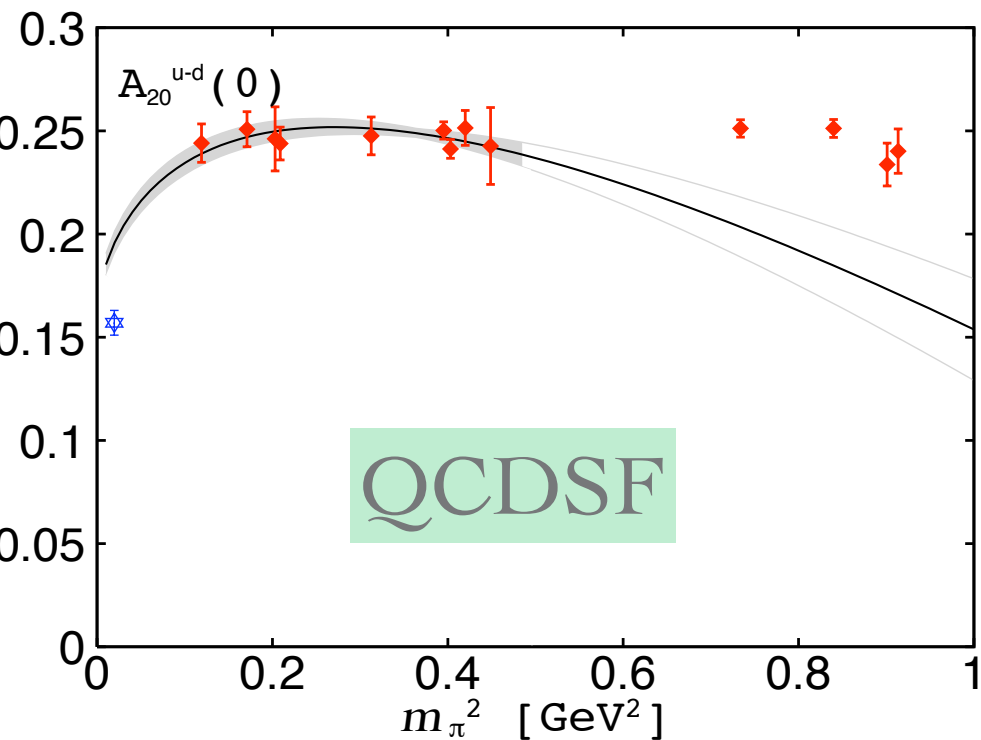
$$0 = \sum_q B_{20}^q(0) + B_{20}^g(0)$$

Generalised Form Factors A_2 , B_2 , C_2

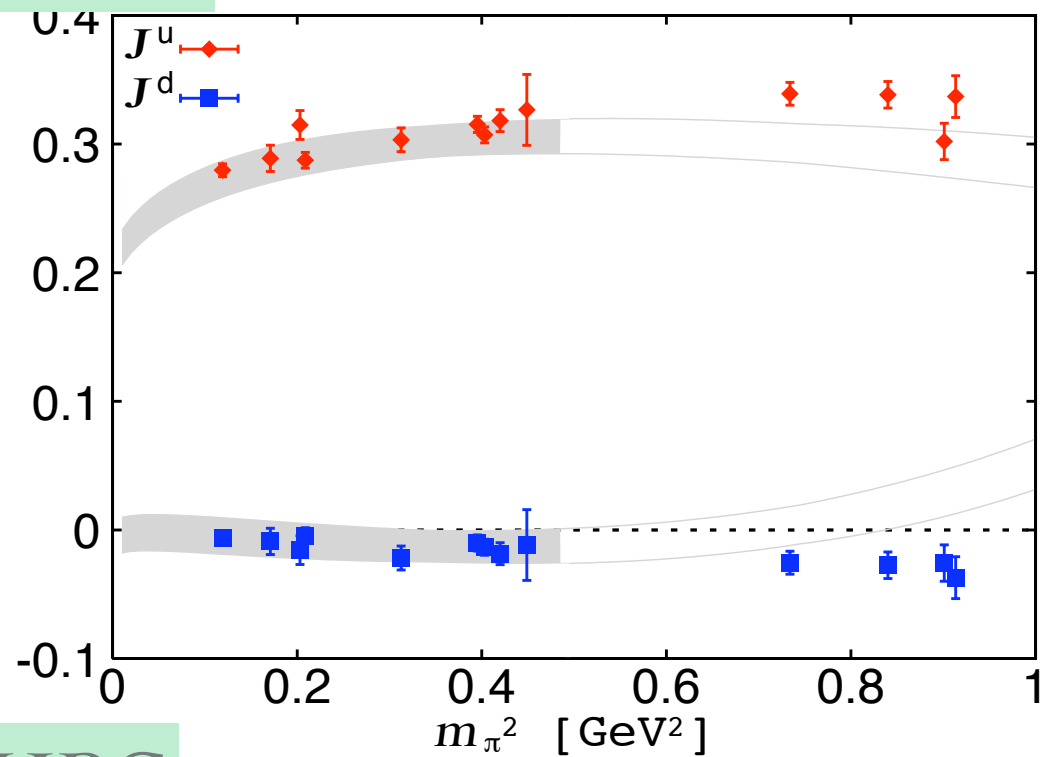
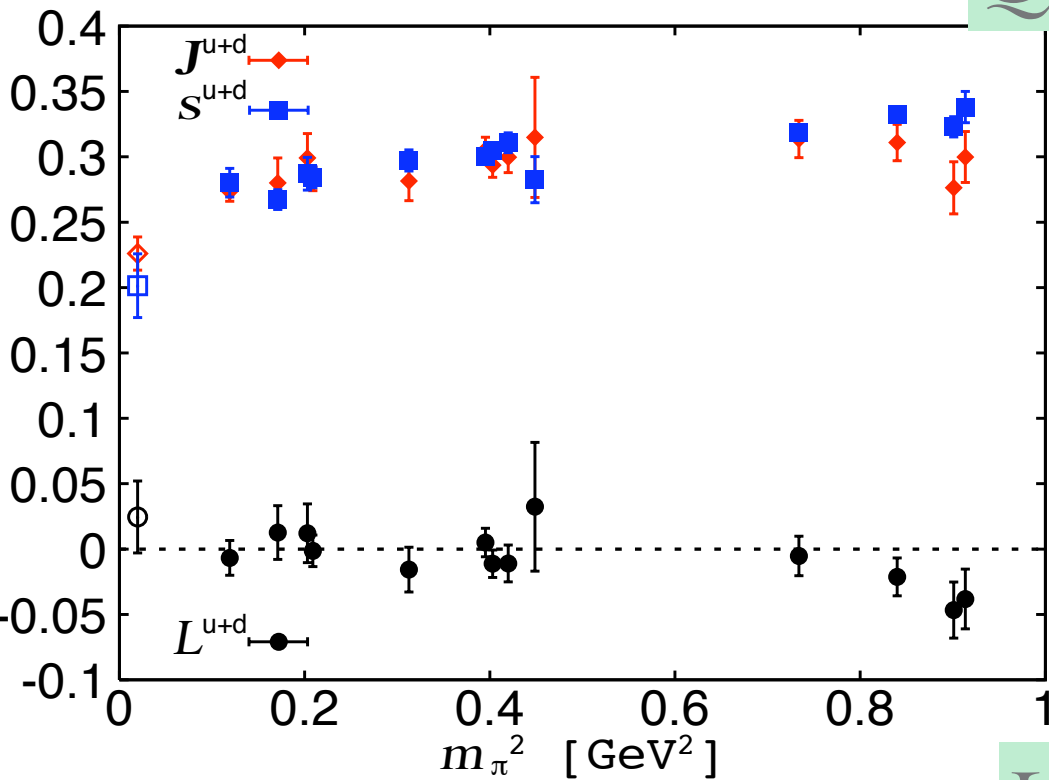
LHPC: PRD 77, 094502(2008), 0705.4295

J. Negele Friday 5:00

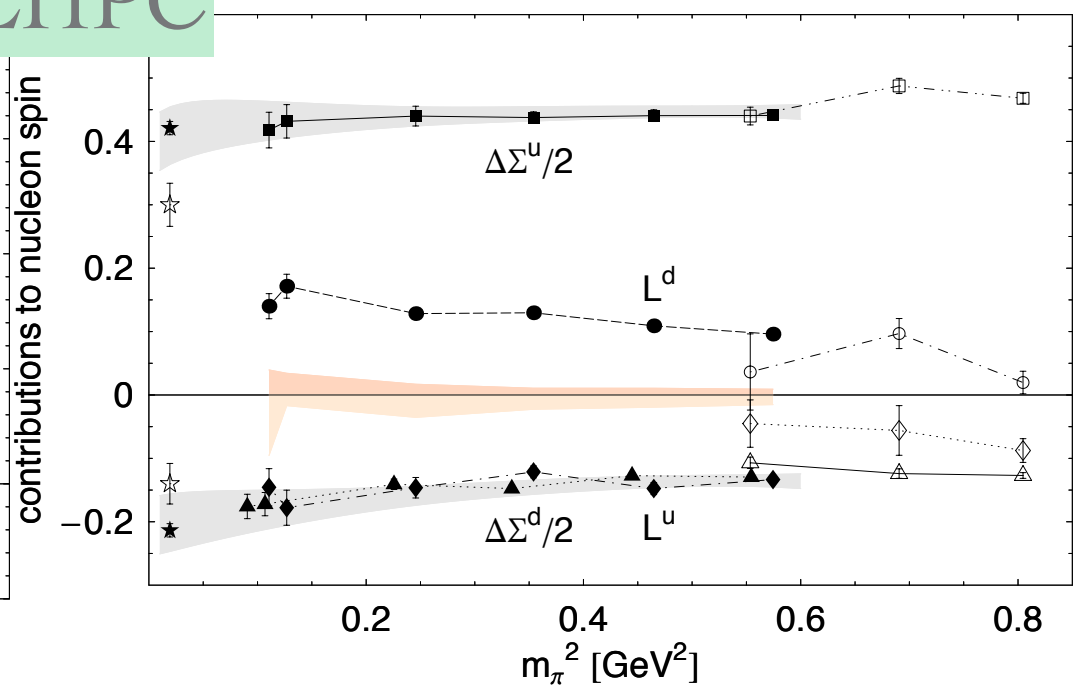
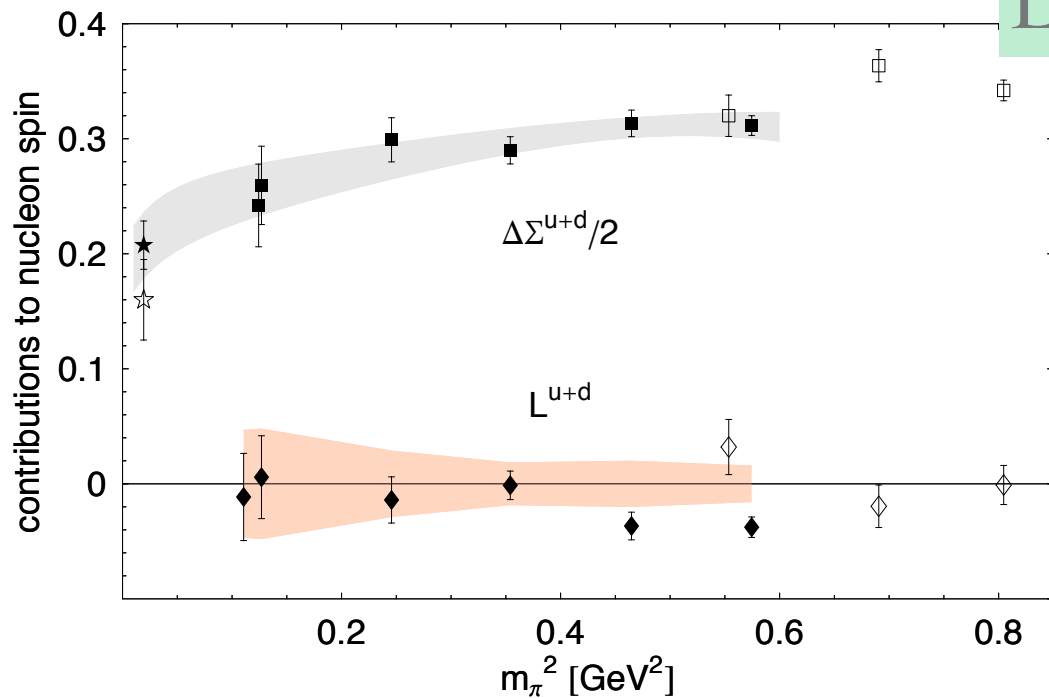


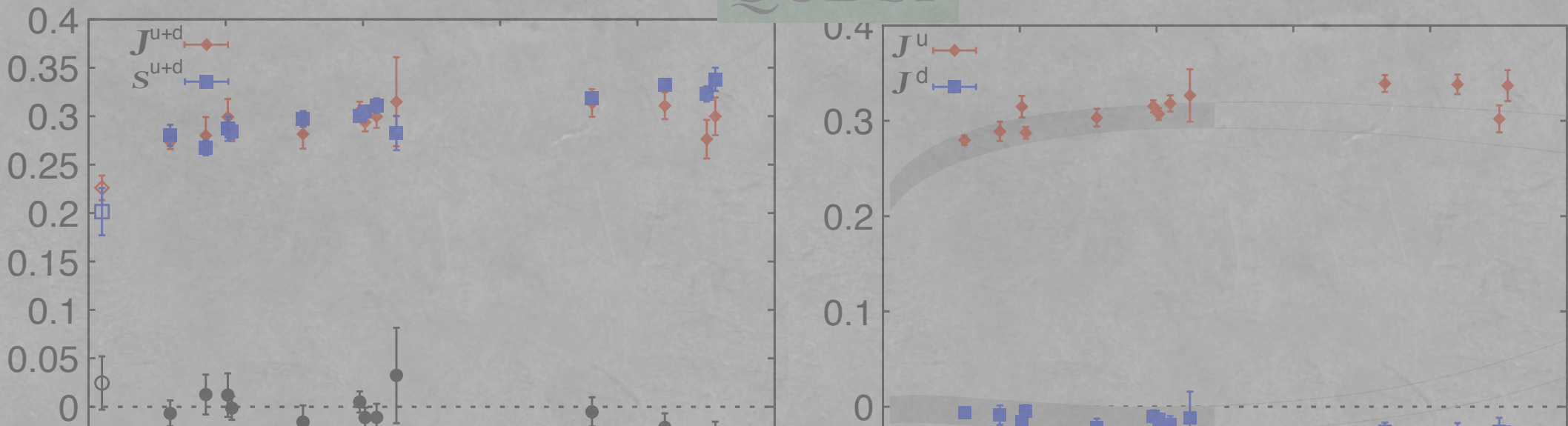


QCDSF

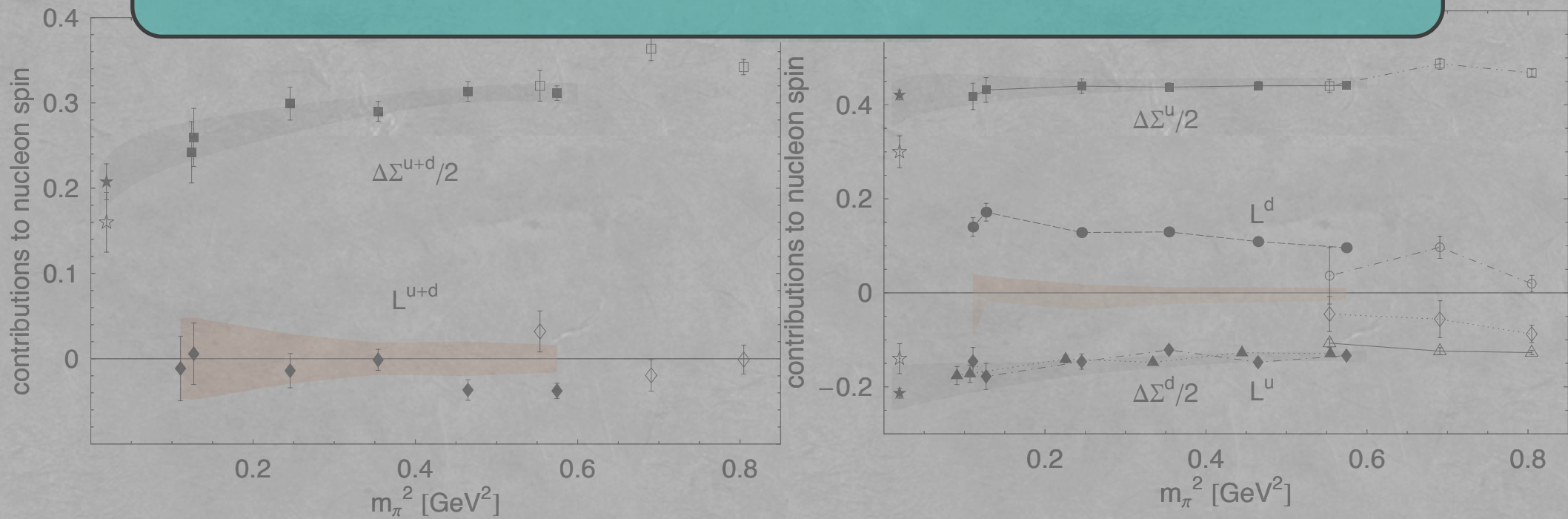


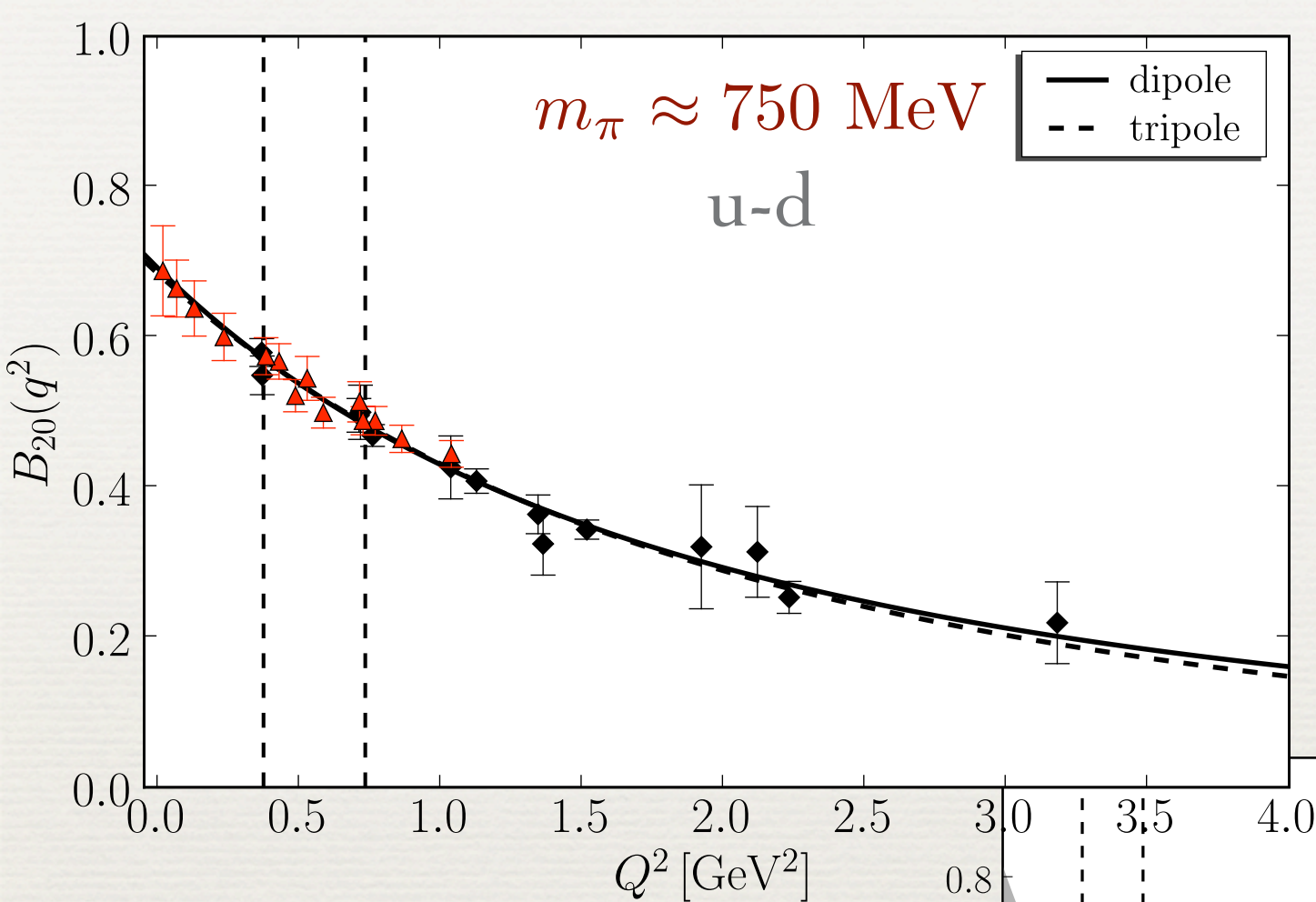
LHPC



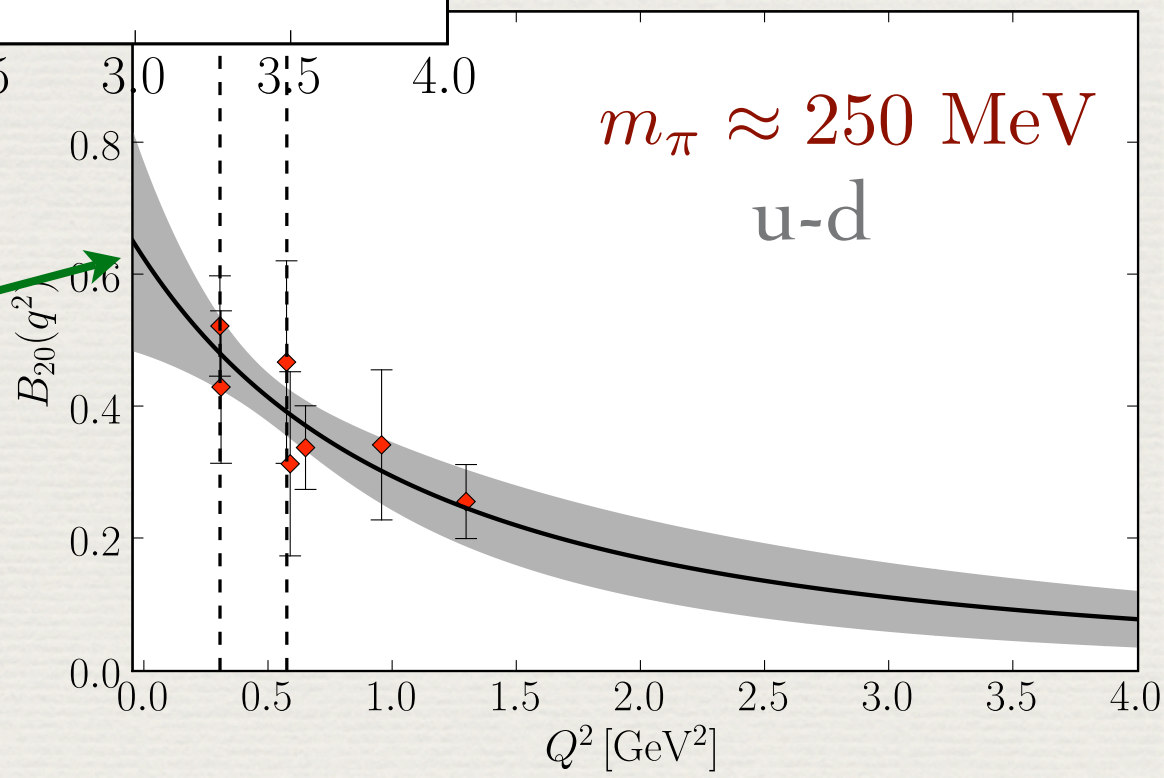


Caveat: No disconnected diagrams!





Accessing
Small Q^2 :
Partially
Twisted
Boundary
Conditions



TBC to help?

Ph. Hägler [QCDSF]:

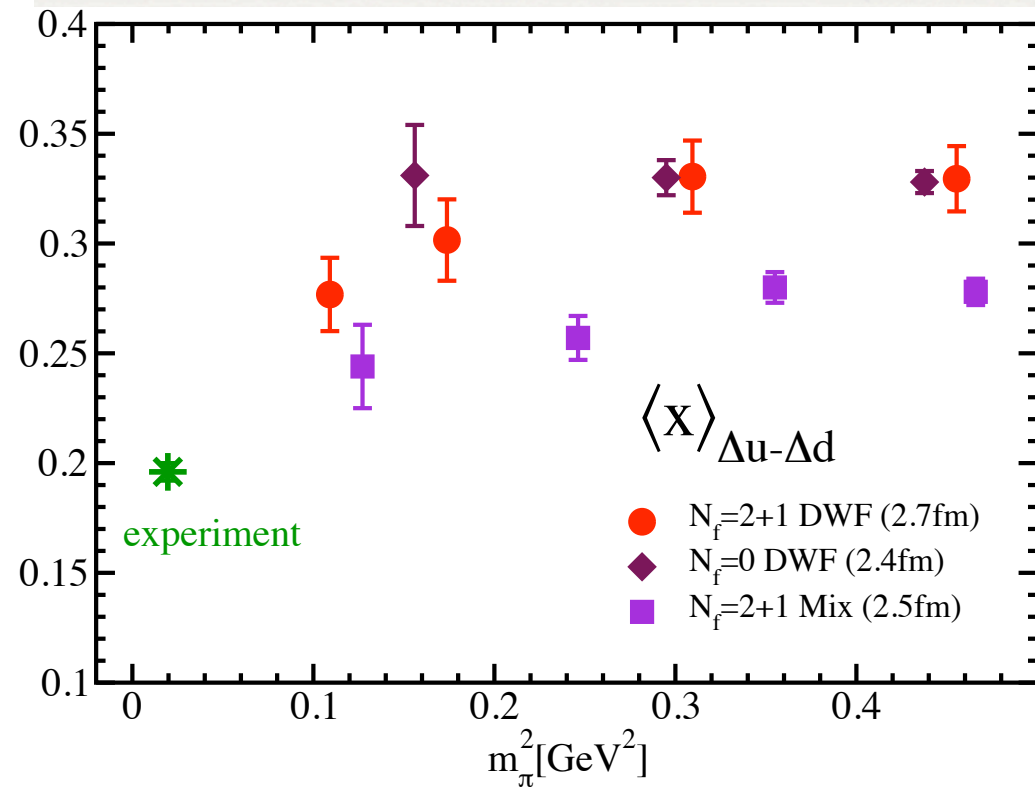
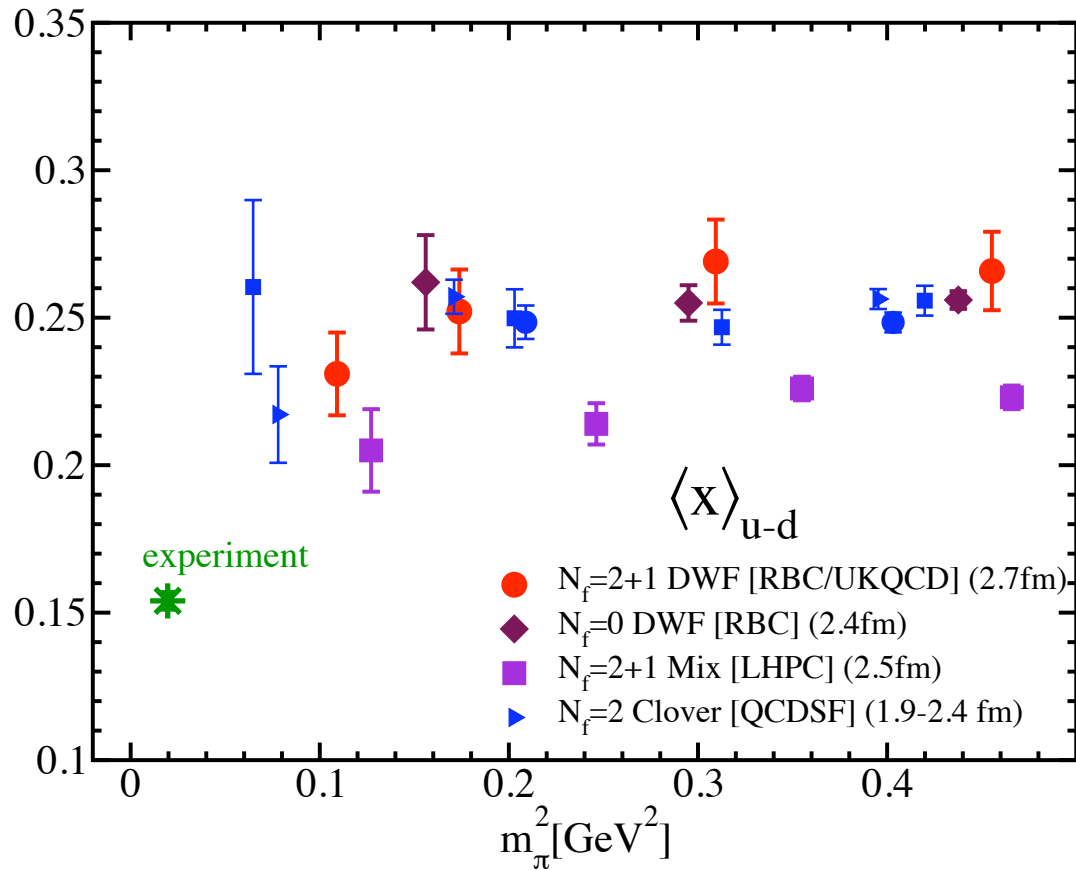
$N_f=2$ Clover

Monday 2:30

$$\langle x \rangle_{u-d}, \quad \langle x \rangle_{\Delta u-\Delta d}$$

S. Ohta [RBC/UKQCD]
2+1 DWF

Friday 5:20



Also see *D. Mankame [Kentucky] $N_f=2+1$ Clover (CP-PACS/JLQCD)*

Monday 3:50

Finite size

$$\langle x \rangle_{u-d}, \quad \langle x \rangle_{\Delta u-\Delta d}$$

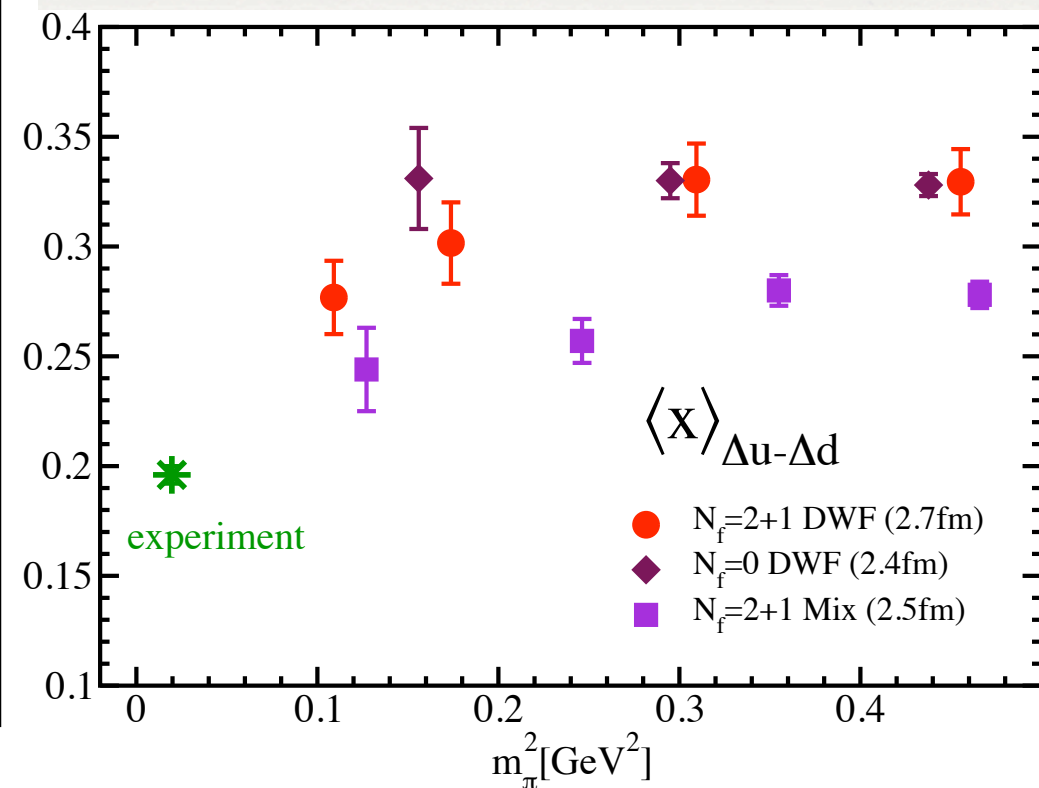
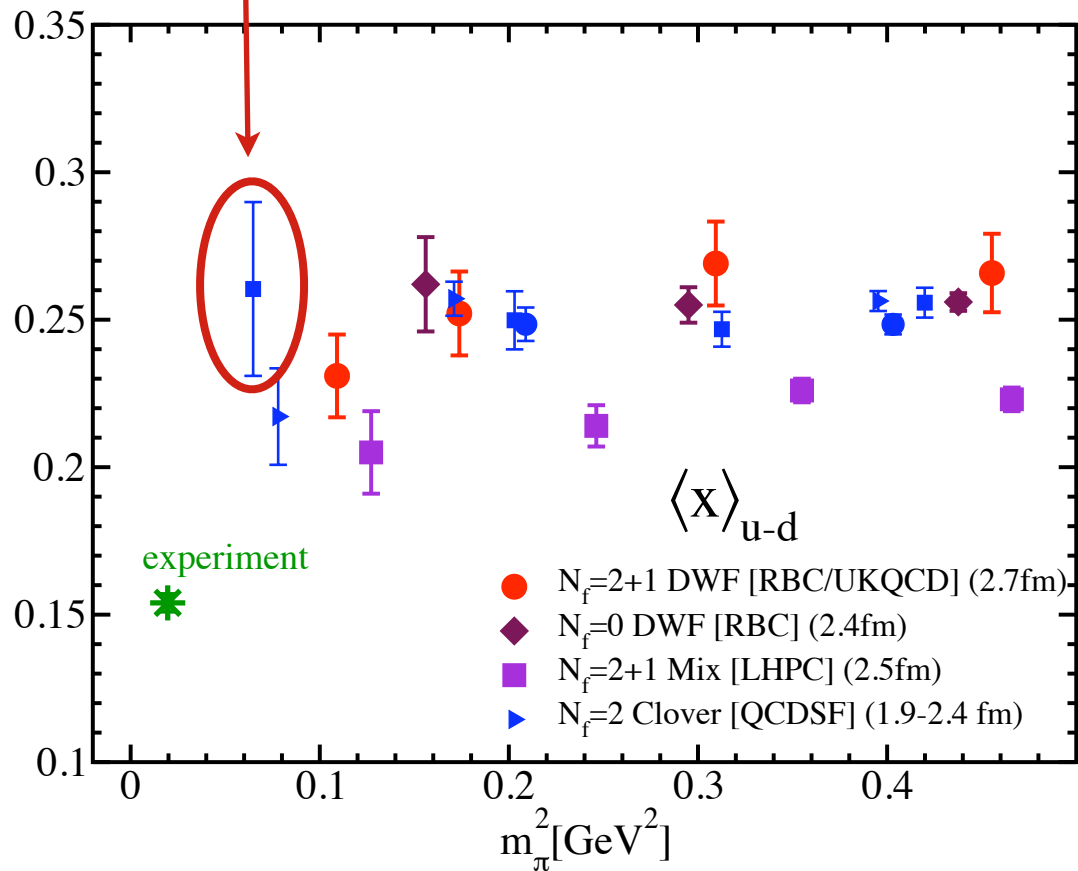
effect?

$(m_\pi L = 2.78)$

S. Ohta [RBC/UKQCD]

2+1 DWF

Friday 5:20



Also see *D. Mankame [Kentucky] $N_f=2+1$ Clover (CP-PACS/JLQCD)*

Monday 3:50

Nucleon Axial Charge g_A

[RBC/UKQCD] 2+1 DWF PRL 100, 171602 (2008)

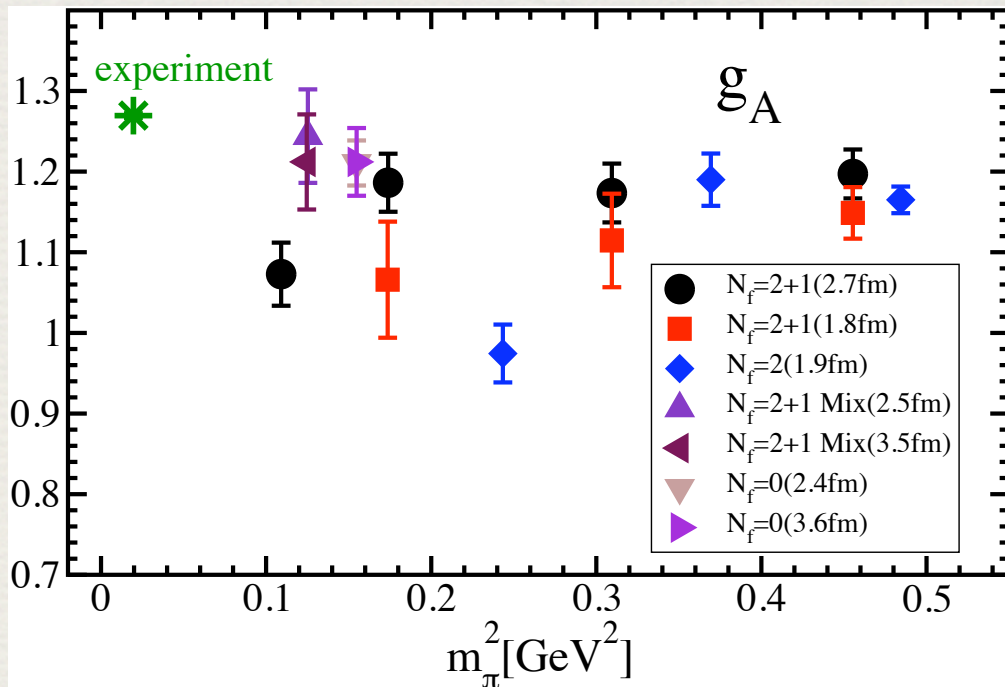
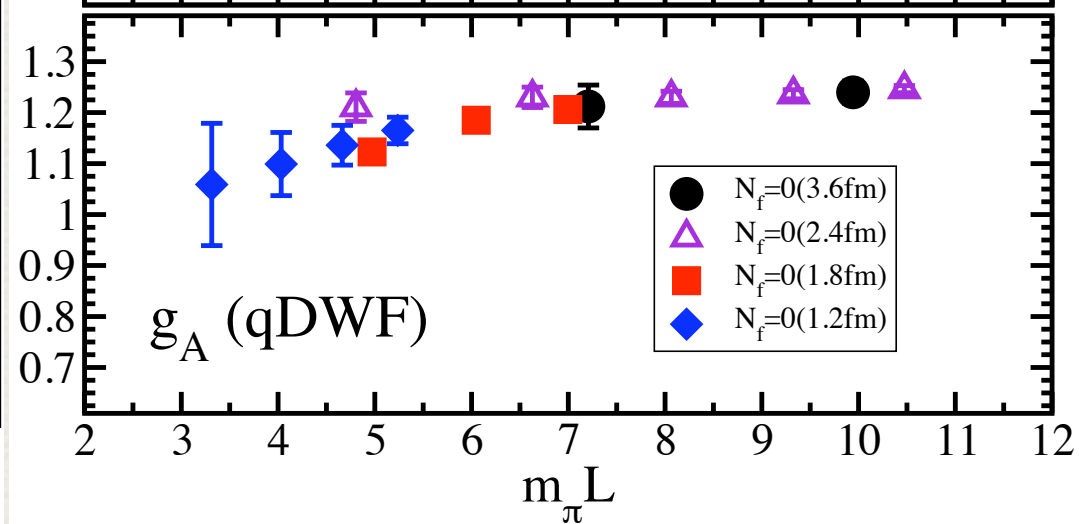
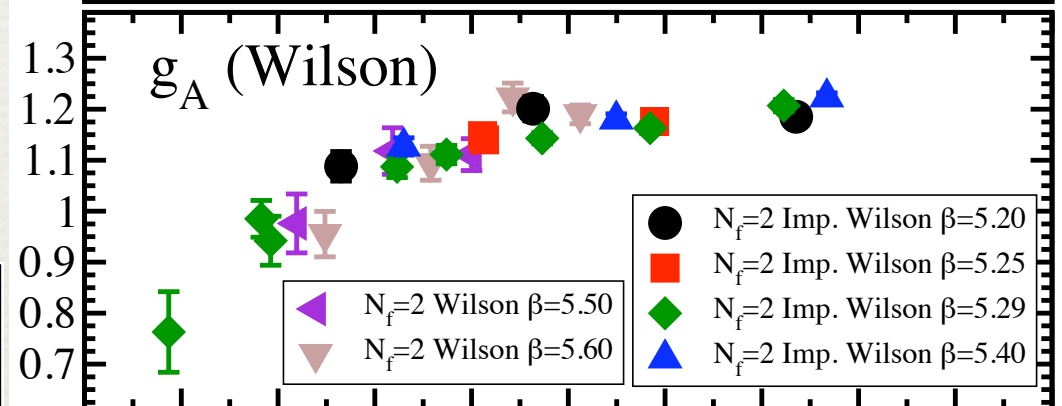
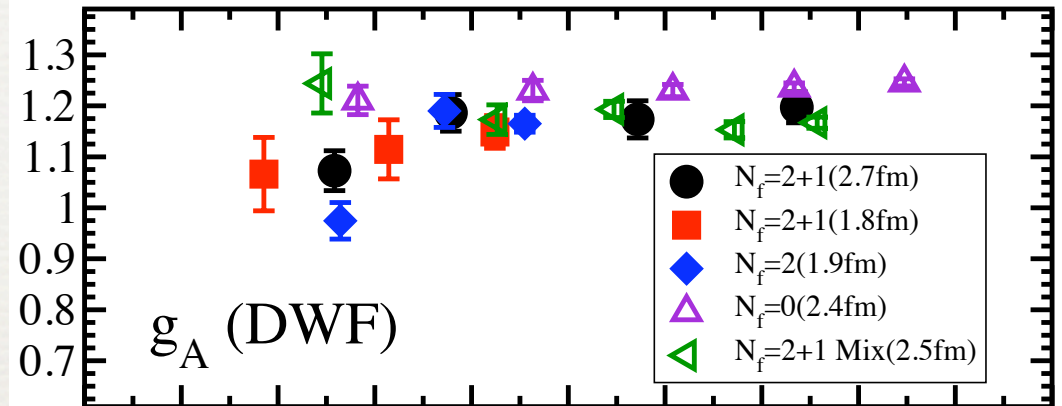
T.Yamazaki Monday 4:10

$$\langle p, s | A_{u-d}^\mu | p, s \rangle = 2g_A s^\mu$$

Also see

R. Baron $N_f=2$ Twisted Mass

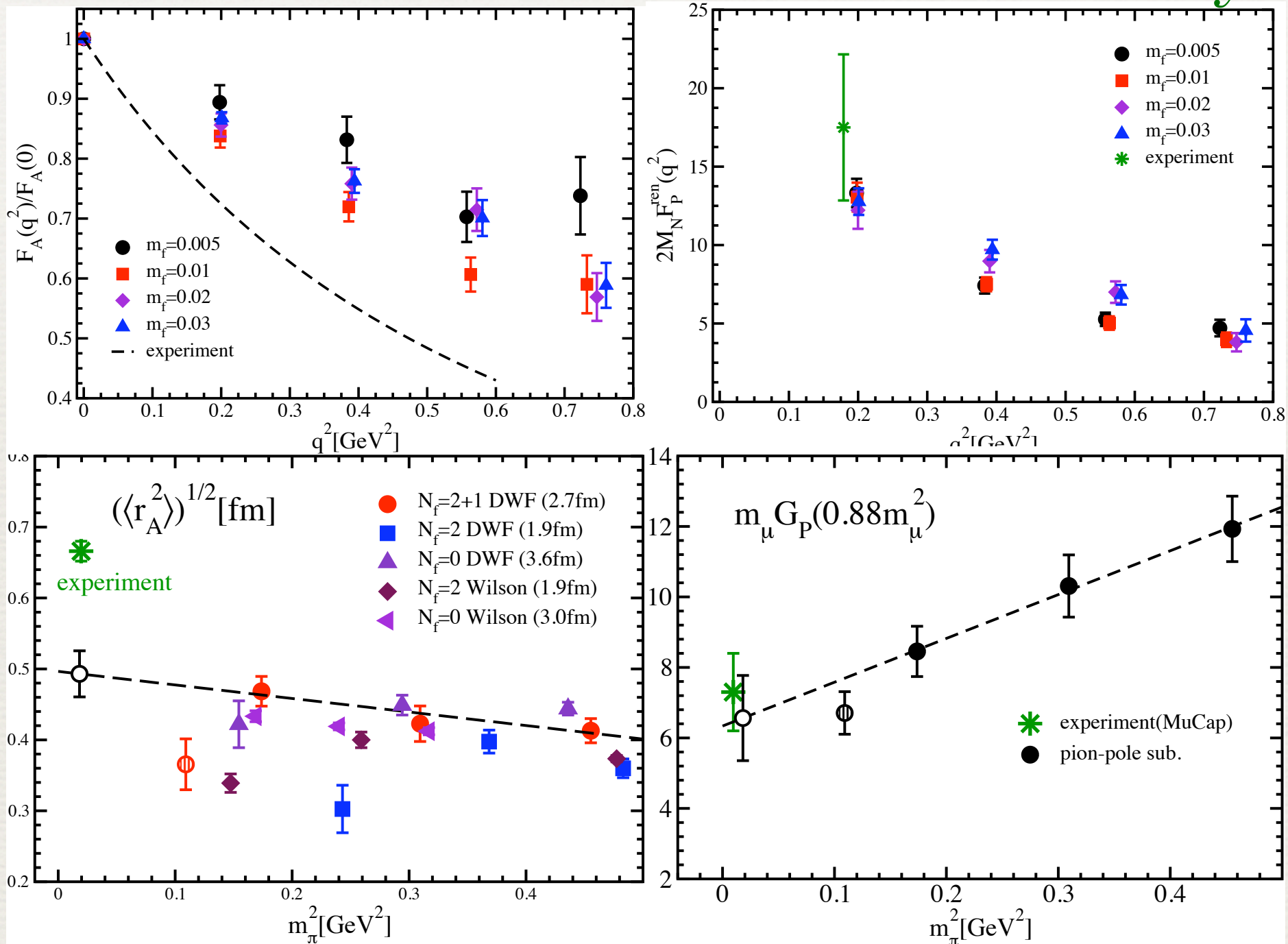
Thursday 11:20



Axial & Induced Pseudoscalar Form Factor

T.Yamazaki [RBC/UKQCD] 2+1 DWF

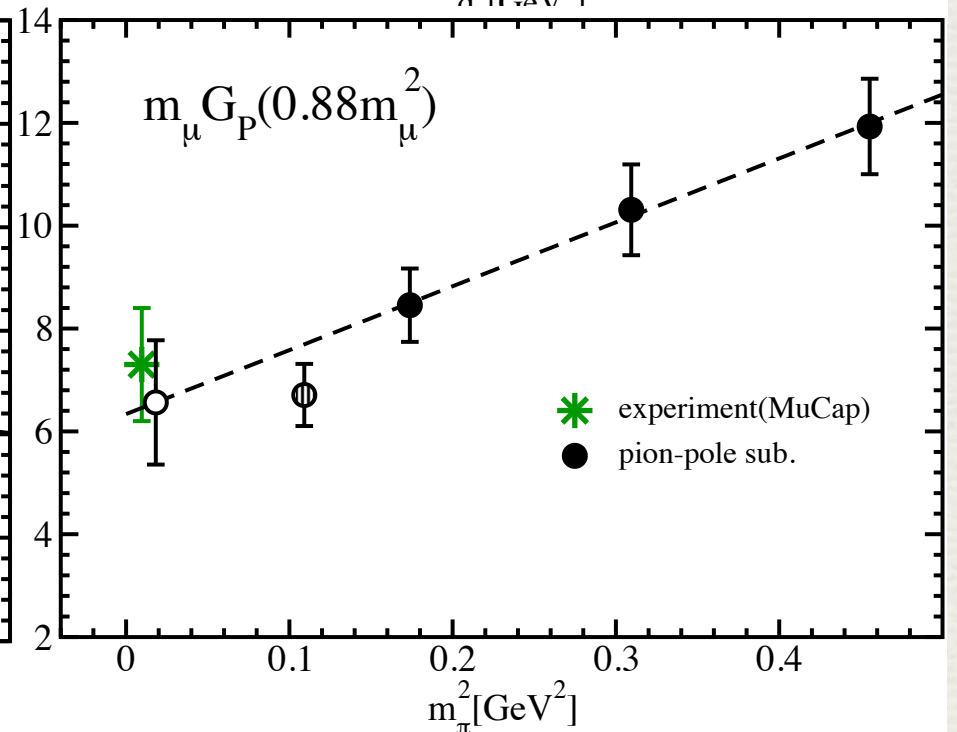
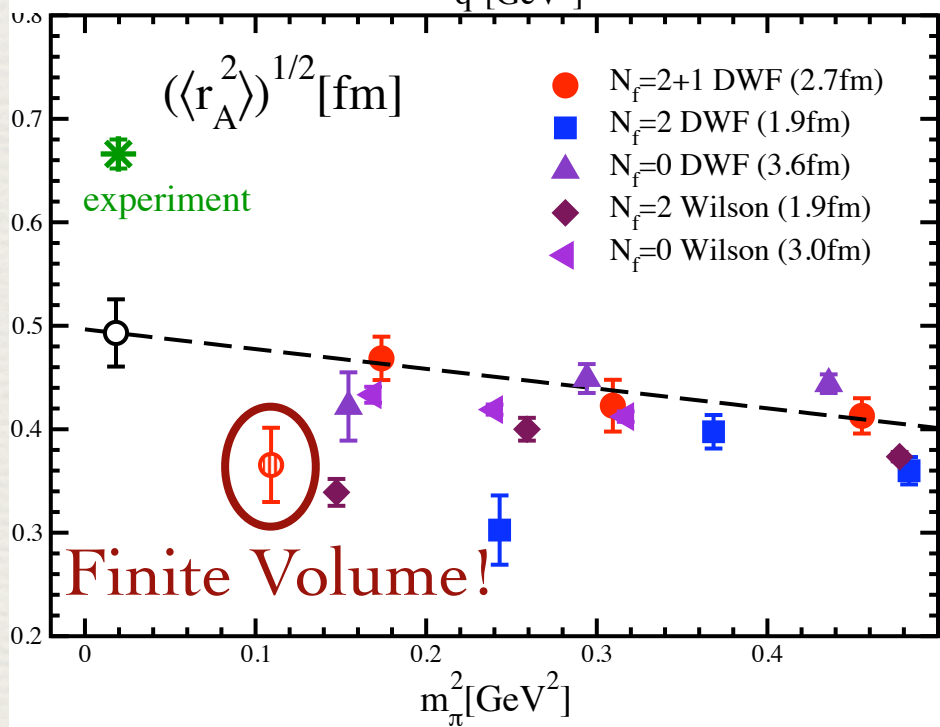
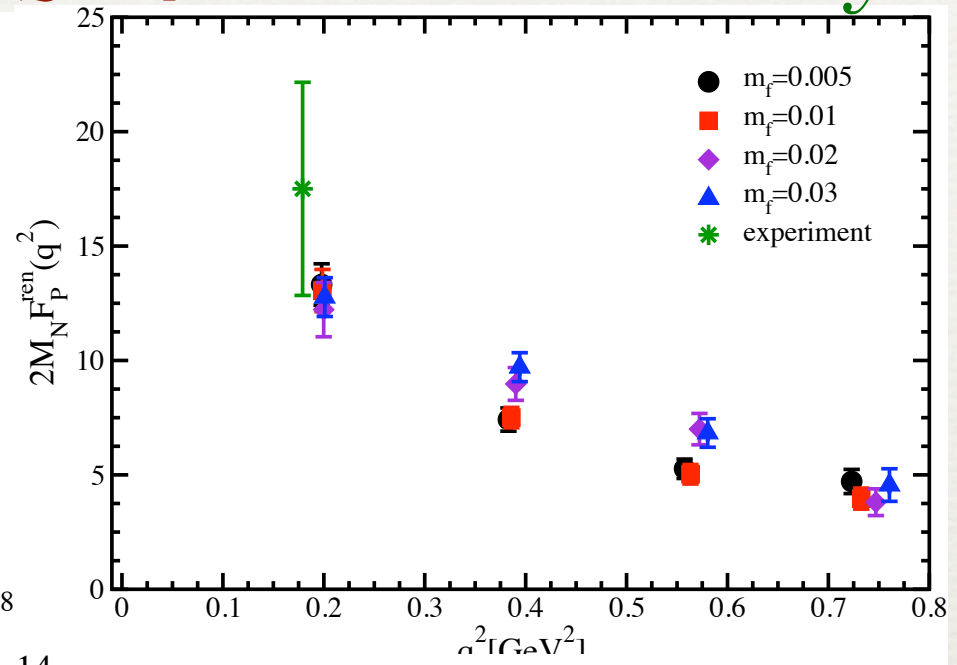
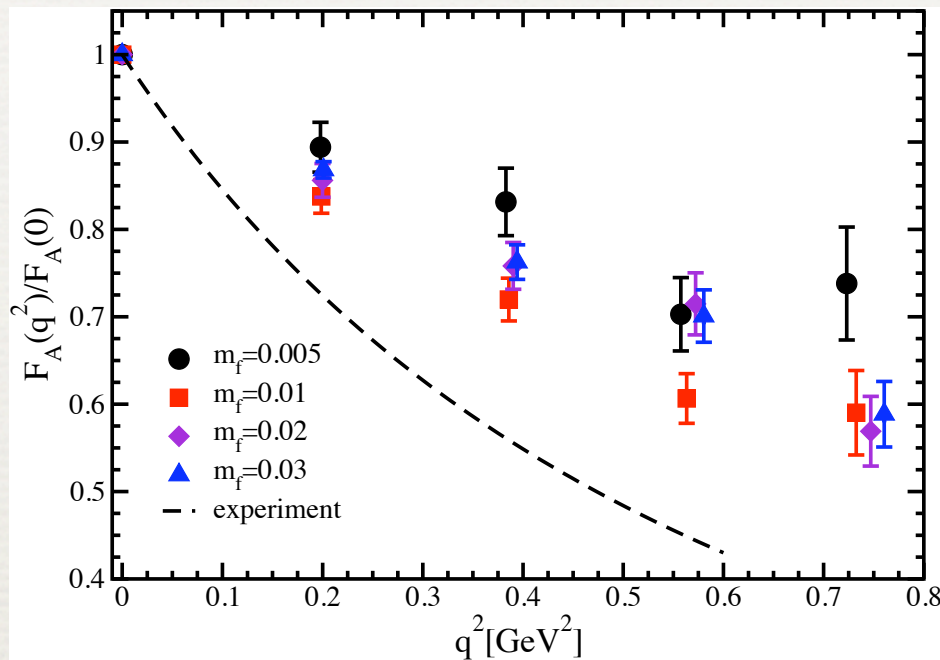
Monday 4:10



Axial & Induced Pseudoscalar Form Factor

T.Yamazaki [RBC/UKQCD] 2+1 DWF

Monday 4:10



Axial Coupling Constants $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

H.-W. Lin, K. Orginos [arXiv:0712.1214]

Mixed action (DWF+asqtad) at $m_{\text{pi}} = 350\text{-}750$ MeV

$$g_A = D + F + \sum_n C_N^{(n)} x^n$$

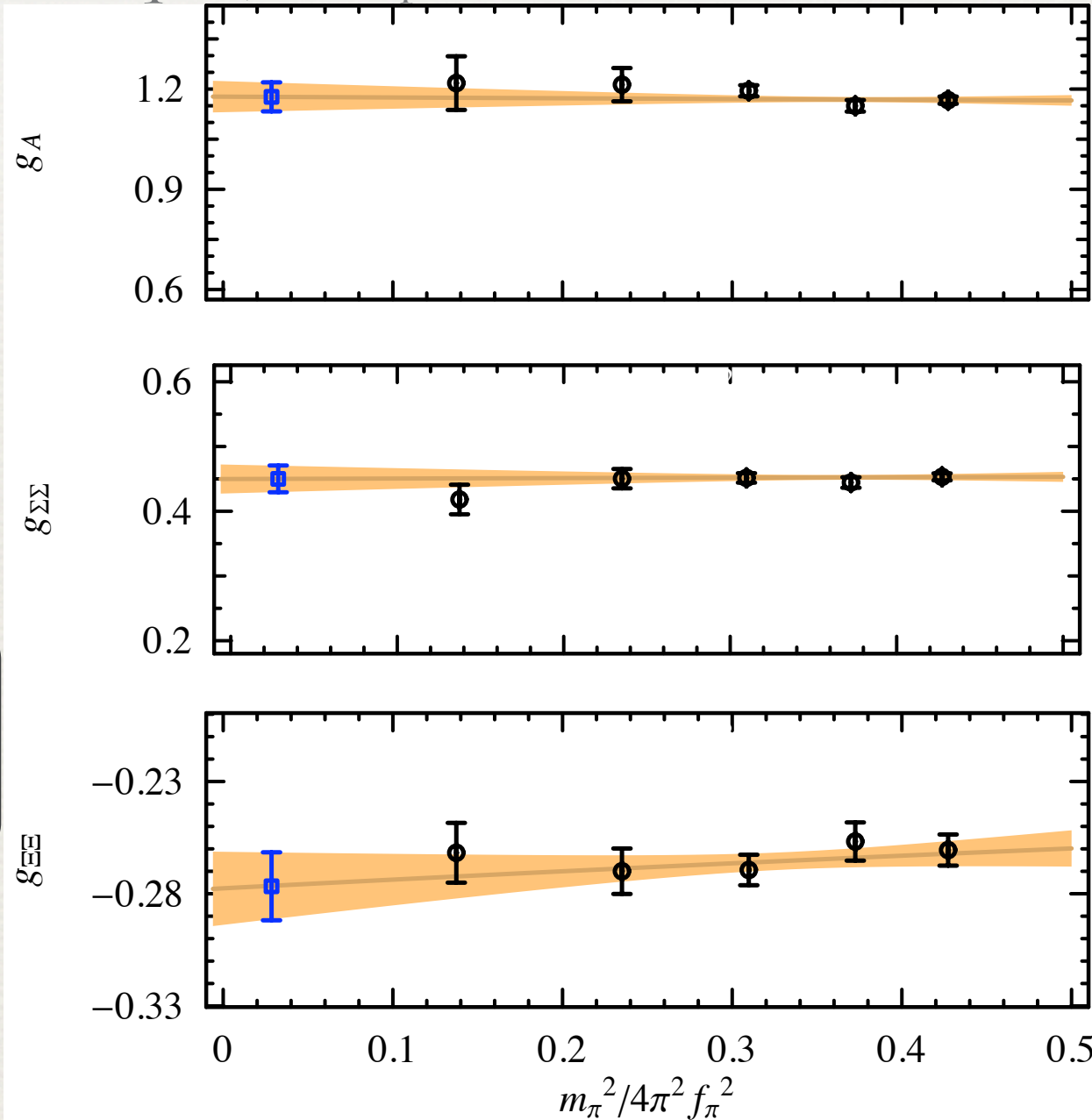
$$g_{\Xi\Xi} = F - D + \sum_n C_{\Xi}^{(n)} x^n$$

$$g_{\Sigma\Sigma} = F + \sum_n C_{\Sigma}^{(n)} x^n$$

$$x = (m_K^2 - m_\pi^2) / (4\pi f_\pi^2)$$

$g_A = 1.18(4)_{\text{stat}}(6)_{\text{sys}}$
 $g_{\Xi\Xi} = 0.450(21)_{\text{stat}}(27)_{\text{sys}}$
 $g_{\Sigma\Sigma} = -0.277(15)_{\text{stat}}(19)_{\text{sys}}$

$D=0.715(6)(29)$ $F=0.453(5)(19)$



N^* Axial Charges

T. Takahashi and T. Kunihiro, arXiv:0801.4707

♦ $N_f=2$ clover (CP-PACS)

♦ $16^3 \times 32$, $a=0.1555(17)$ fm

♦ $m_{ps}/m_v = 0.804(1), 0.752(1), 0.690(1)$

♦ Construct optimised sources/sinks from a combination of

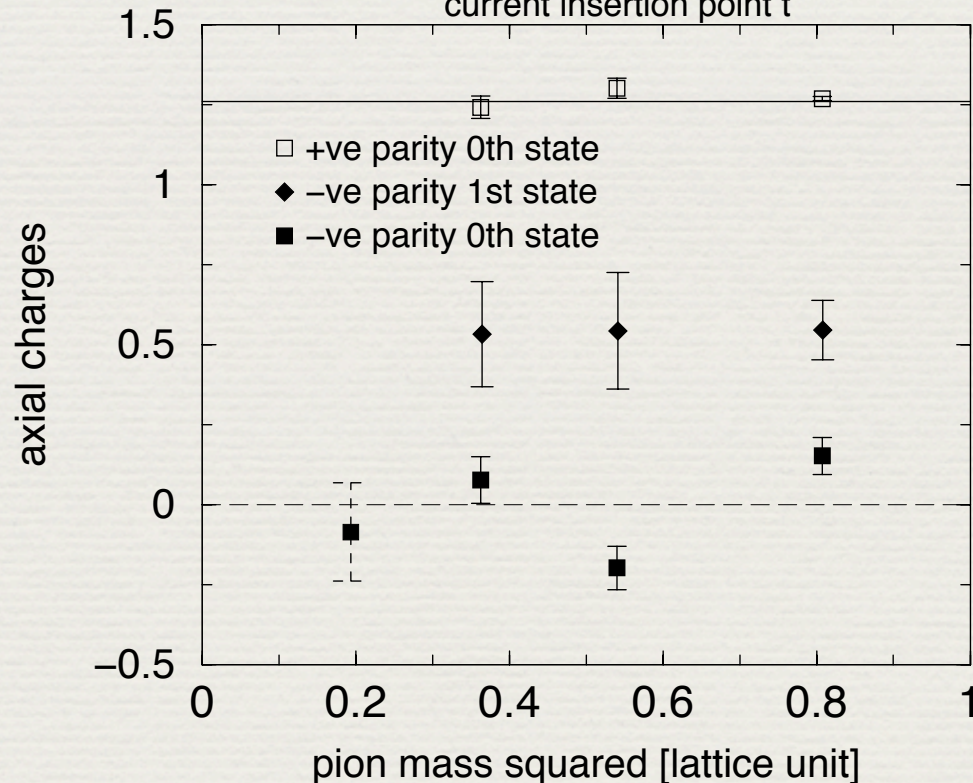
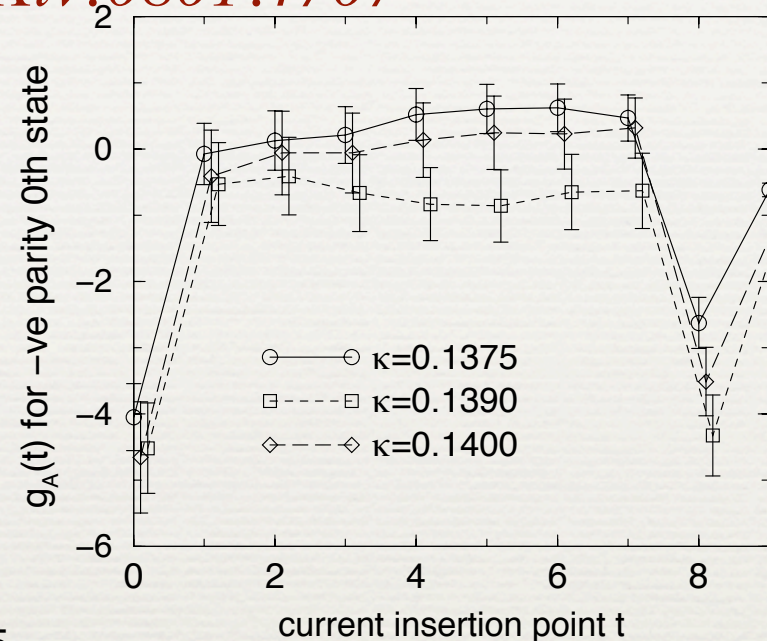
$$N_1(x) \equiv \epsilon_{abc} u^a(x) (u^b(x) C \gamma_5 d^c(x))$$

$$N_2(x) \equiv \epsilon_{abc} \gamma_5 u^a(x) (u^b(x) C d^c(x))$$

$$g_A^{0-} < 0.2$$

$$g_A^{1-} \approx 0.55$$

Consistent with NR quark model



Jefferson Lab Research Program

<http://www.jlab.org/highlights/phys.html>

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- ◆ Neutron Magnetic Structure
- ◆ **Nucleon-Delta Transition**
- ◆ Pion Form Factor
- ◆ Generalized Parton Distributions
- ◆ Spin Sum Rules
- ◆ Nucleon Spin Asymmetries
- ◆ Strange Quarks in the Proton



$$\langle \Delta(\vec{p}', s') | j_\mu | N(\vec{p}, s) \rangle = i \sqrt{\frac{2}{3}} \left(\frac{m_\Delta m_N}{E_\Delta(\vec{p}') E_N(\vec{p})} \right)^{1/2} \bar{u}^\sigma(\vec{p}', s') O_{\sigma\mu} u(\vec{p}, s)$$

$$O_{\sigma\mu} = G_{M1}(q^2) K_{\sigma\mu}^{M1} + G_{E2}(q^2) K_{\sigma\mu}^{E2} + G_{C2}(q^2) K_{\sigma\mu}^{C2}$$

Magnetic dipole:
dominant

Electric
quadrupole

Coulomb
quadrupole



$$\langle \Delta(\vec{p}', s') | j_\mu | N(\vec{p}, s) \rangle = i \sqrt{\frac{2}{3}} \left(\frac{m_\Delta m_N}{E_\Delta(\vec{p}') E_N(\vec{p})} \right)^{1/2} \bar{u}^\sigma(\vec{p}', s') O_{\sigma\mu} u(\vec{p}, s)$$

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N, Δ deformed?

Nonzero Quadrupole moments

Magnetic dipole:
dominant

Electric
quadrupole

Coulomb
quadrupole

$$N \longrightarrow \Delta \quad \langle \Delta(\vec{p}', s') | j_\mu | N(\vec{p}, s) \rangle = i \sqrt{\frac{2}{3}} \left(\frac{m_\Delta m_N}{E_\Delta(\vec{p}') E_N(\vec{p})} \right)^{1/2} \bar{u}^\sigma(\vec{p}', s') O_{\sigma\mu} u(\vec{p}, s)$$

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Electric
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Coulomb
quadrupole

Quantities measured in the lab frame of the Δ are the ratios

$$R_{EM}(EMR) = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}$$

$$R_{SM}(CMR) = -\frac{|\vec{q}|}{2m_\Delta} \frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$$

- Precise experimental data strongly suggest deformation of N/D
- First confirmation of non-zero EMR and CMR in full QCD



$$\langle \Delta(\vec{p}', s') | j_\mu | N(\vec{p}, s) \rangle = i \sqrt{\frac{2}{3}} \left(\frac{m_\Delta m_N}{E_\Delta(\vec{p}') E_N(\vec{p})} \right)^{1/2} \bar{u}^\sigma(\vec{p}', s') O_{\sigma\mu} u(\vec{p}, s)$$

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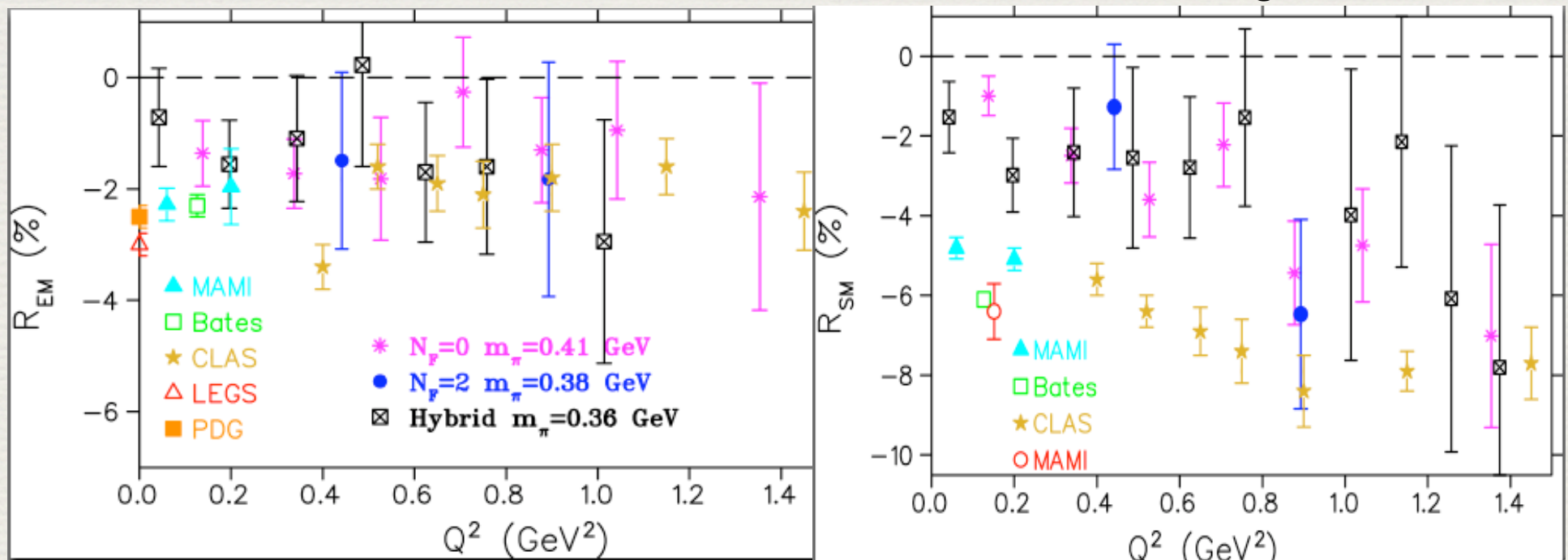
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Axial N to Δ

*C. Alexandrou, G. Koutsou, Th. Leontiou, J. W. Negele
and A. Tsapalis PRD76, 094511,2007.*

$$\langle \Delta(\vec{p}') | A_\mu | N(\vec{p}) \rangle = \bar{u}^\sigma \left(\frac{C_3^A}{m_N} \gamma^\nu + \frac{C_4^A}{m_N^2} p'^\nu \right) (g_{\sigma\mu} g_{\rho\nu} - g_{\sigma\rho} g_{\mu\nu}) q^\rho u + \bar{u}^\sigma C_5^A g_{\sigma\mu} u + \bar{u}^\sigma \frac{C_6^A}{m_N^2} q_\sigma q_\mu u$$

Axial vector $N \rightarrow \Delta$: four addition form factors $C_3^A(Q^2)$, $C_4^A(Q^2)$, $C_5^A(Q^2)$, $C_6^A(Q^2)$

Dominant axial form factors C_5 , C_6 correspond to the nucleon axial G_A , G_P

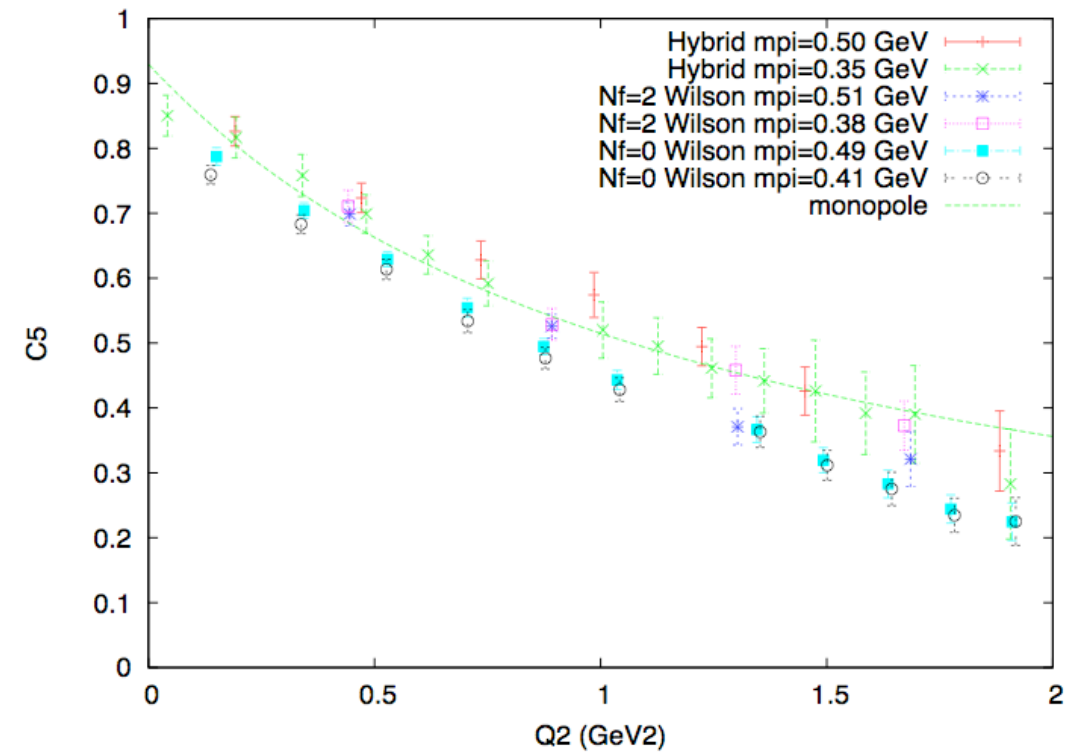
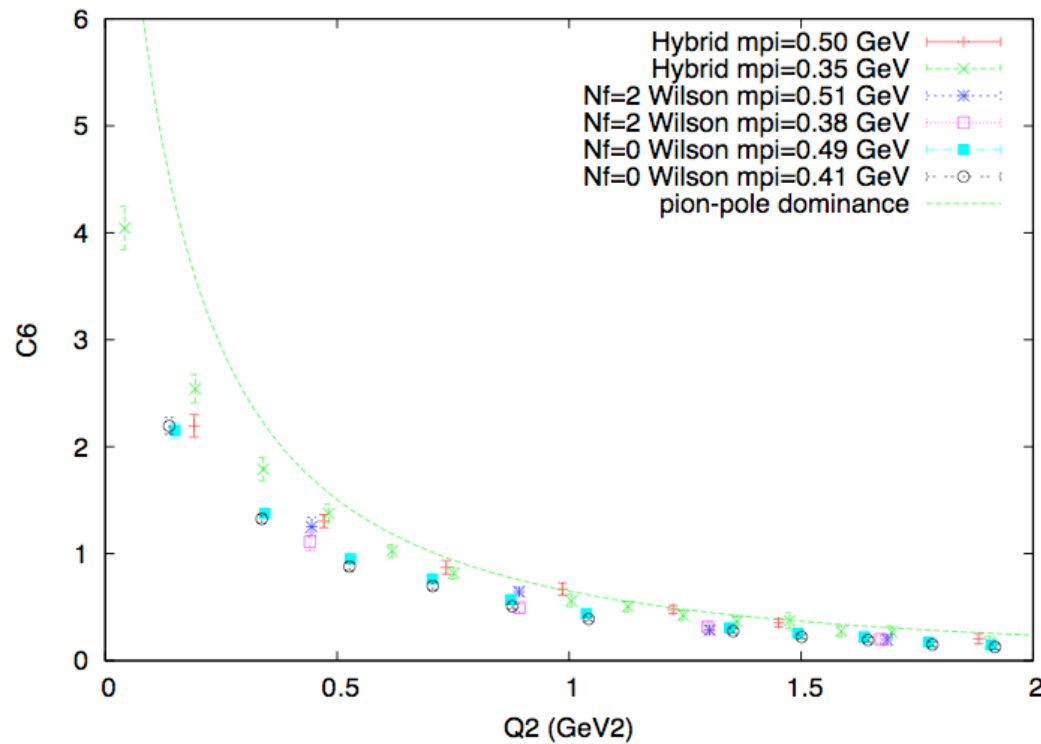
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Dominant axial form factors C_5 , C_6 correspond to the nucleon axial G_A , G_P



Unquenching effects at small Q^2 ??

Pion-pole dominance for C_6

$$\longrightarrow \frac{C_6^A}{m_N^2} = \frac{1}{m_\pi^2 + Q^2} C_5^A$$

Δ electromagnetic form factors *P. Moran et al. [Adelaide]*

$$\langle \Delta(\vec{p}', s') | j_\mu | \Delta(\vec{p}, s) \rangle = i \sqrt{\frac{m_\Delta^2}{E_\Delta(\vec{p}') E_\Delta(\vec{p})}} \bar{u}^\sigma(\vec{p}', s') O_{\sigma\mu\tau} u_\tau(\vec{p}, s)$$

$$O_{\sigma\mu\tau} = \delta^{\sigma\tau} \left[a_1(q^2) i\gamma^\mu + \frac{a_2(q^2)}{2m_\Delta} (p'^\mu + p^\mu) \right] - \frac{q^\sigma q^\tau}{2m_\Delta^2} \left[c_1(q^2) i\gamma^\mu + \frac{c_2(q^2)}{2m_\Delta} (p'^\mu + p^\mu) \right]$$

Four form factors: G_{E0} , G_{E2} , G_{M1} , G_{M3} given in terms of a_1 , a_2 , c_1 , c_2

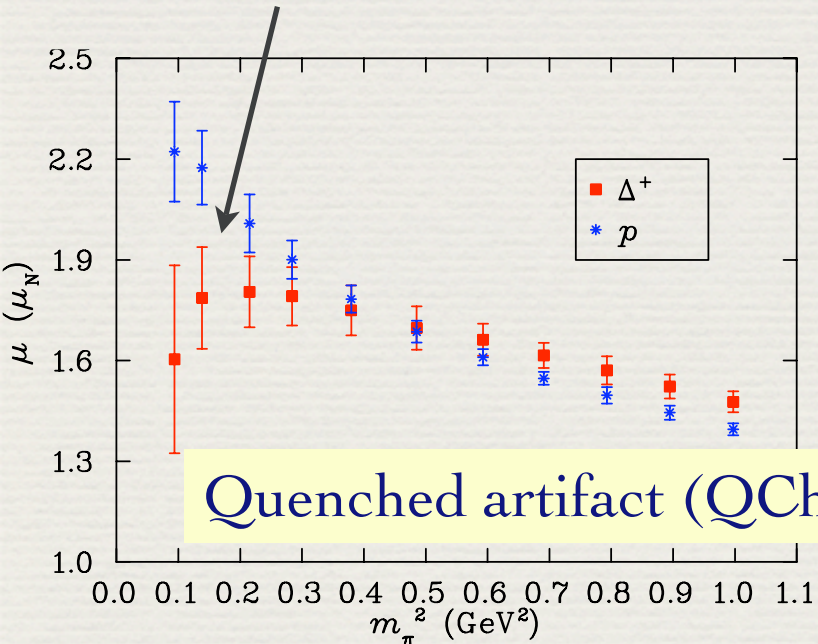
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Should go away with dynamical fermions

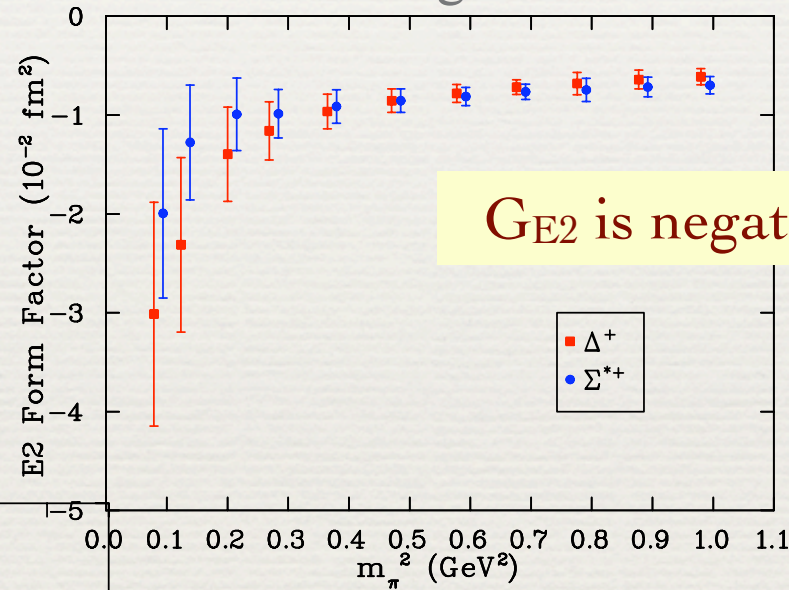


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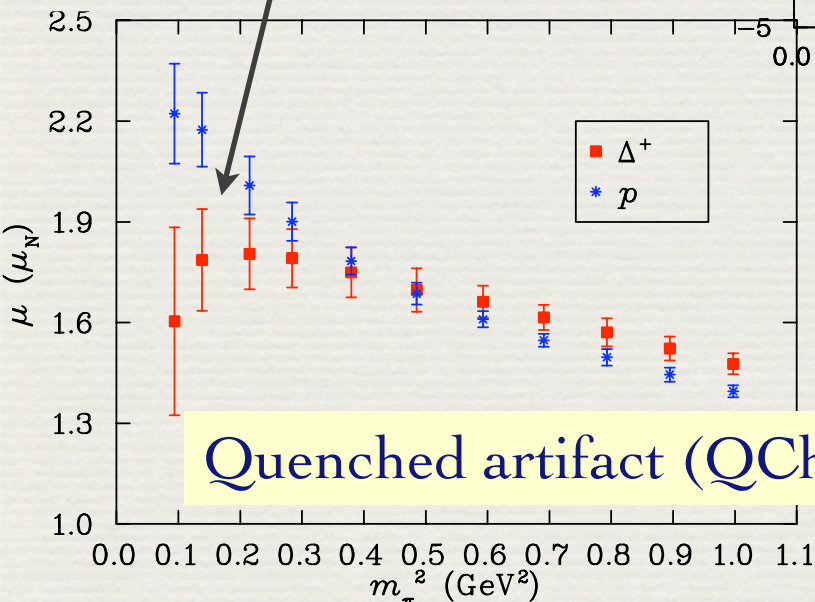
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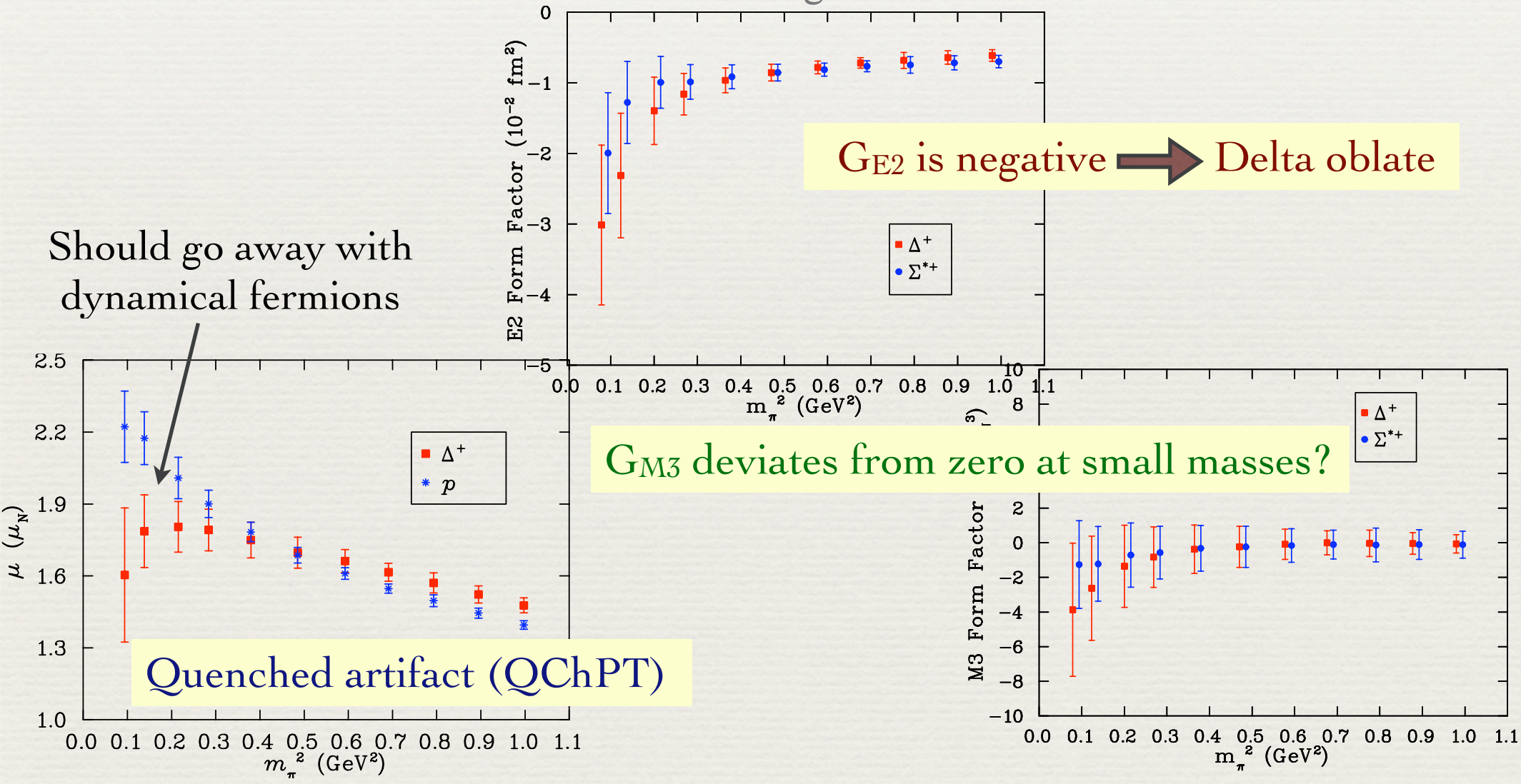


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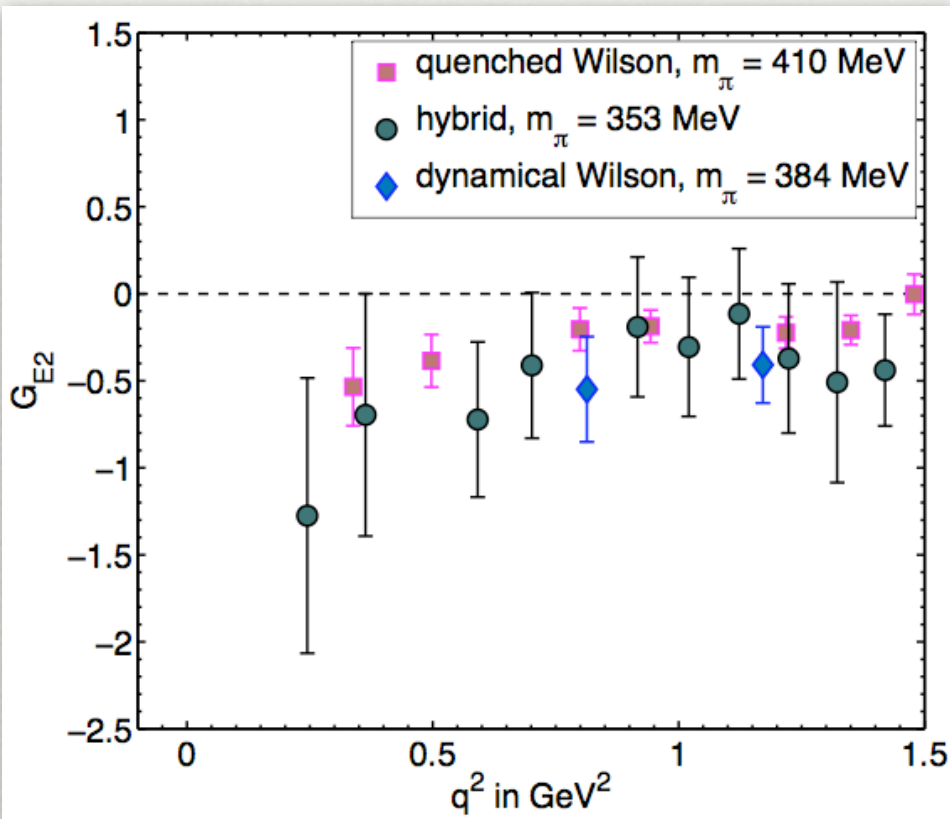
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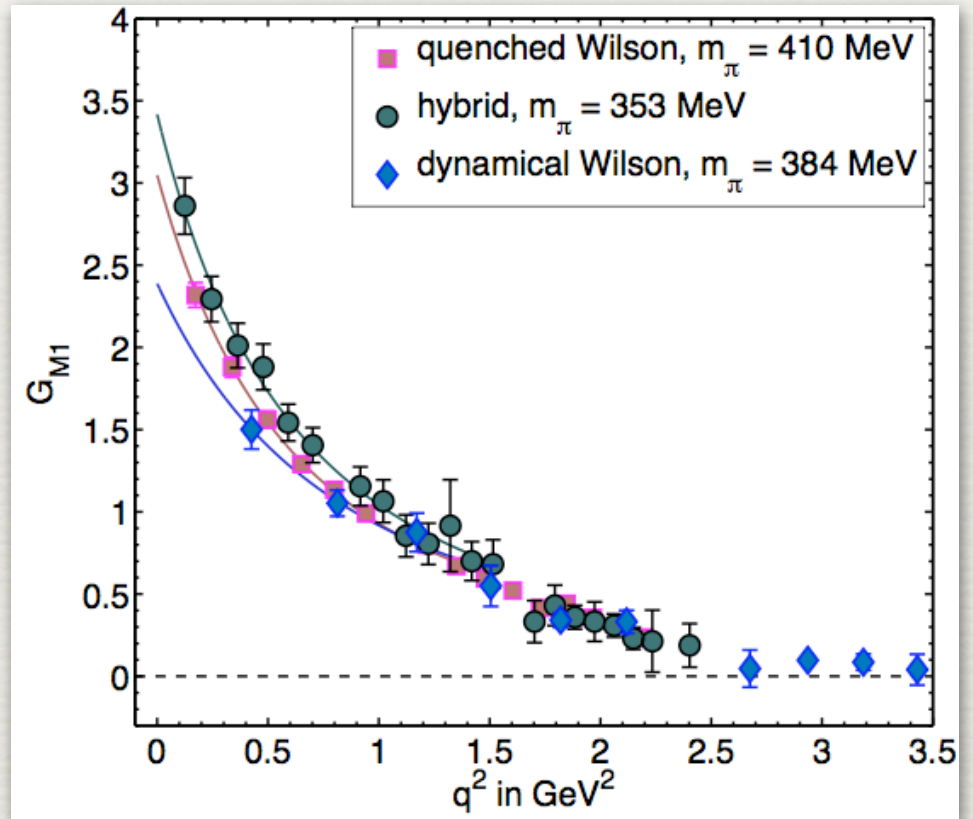
Δ electromagnetic form factors

[C. Alexandrou, Th. Korzec, Th. Leontiou, J. W. Negele, A. Tsapalis]

G_{E2} is negative \rightarrow Delta oblate



Entering area to look for
“bending down”



G_{M3} consistent with zero.
Deviate from zero at small masses?

Rho Form Factor

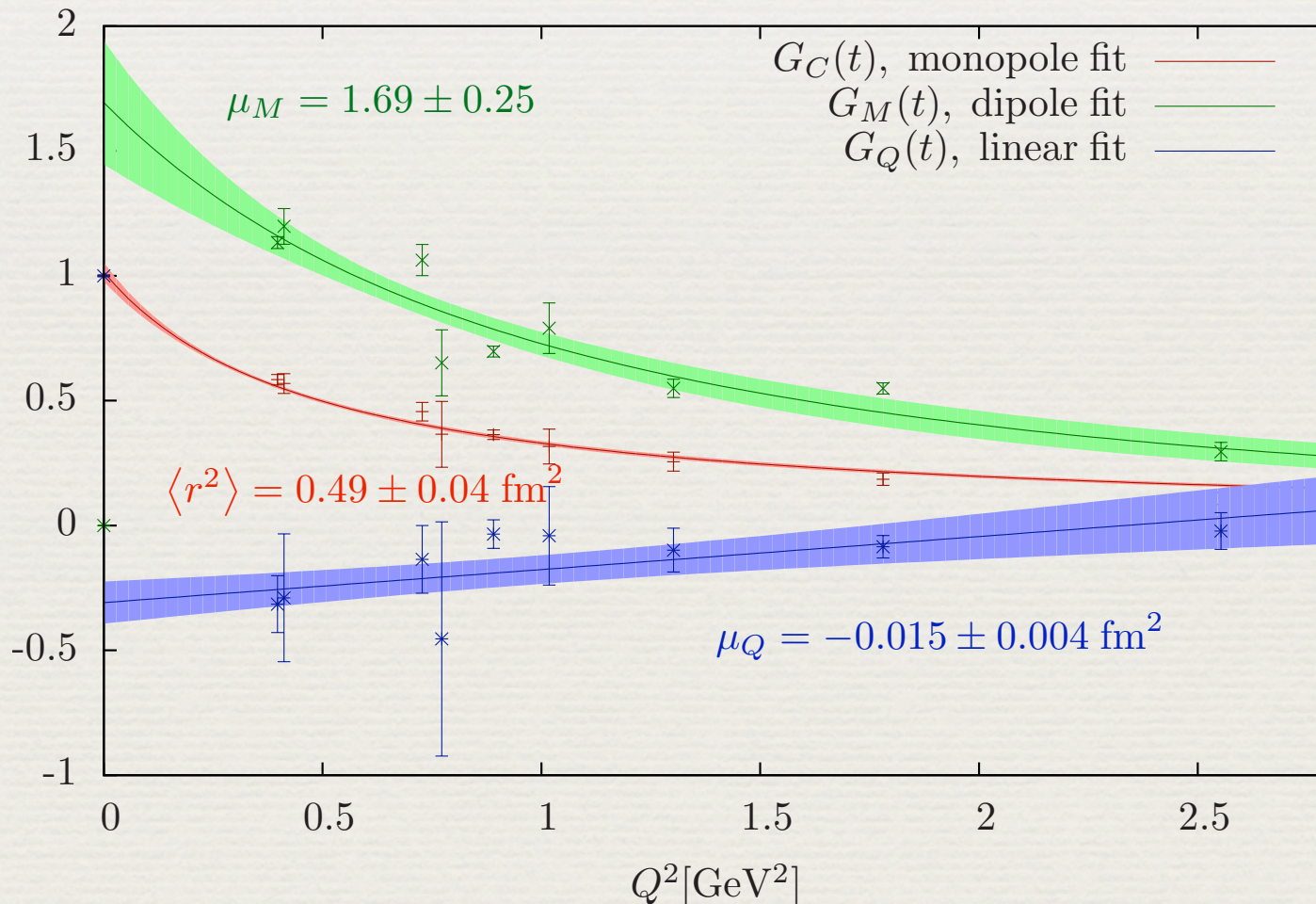
M. Gürtler [QCDSF] Thursday 9:40

$$\langle \rho(\vec{p}', s') | J^\alpha | \rho(\vec{p}, s) \rangle \propto G_E(Q^2), G_M(Q^2), G_Q(Q^2)$$

$G_Q(Q^2 = 0)$ \longrightarrow Quadrupole moment

$\neq 0$ \longrightarrow Spatial deformation

$$\beta = 5.29 \quad \kappa = .13620 \quad \text{Vol}24^348 \quad m_\pi = 406\text{MeV}$$



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- ◆ Strange Quarks in the Proton

Transverse Spin Structure of the Nucleon

Ph. Hägler (QCDSF) [PRL 98, 222001 (2007)]

Transverse densities:

$$\begin{aligned} \rho^n(b_\perp, s_\perp, S_\perp) &= \int_{-1}^1 dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) = \frac{1}{2} \left\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \Delta_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) \right. \\ &+ \left. \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \bar{B}'_{Tn0}(b_\perp^2) \right) + s_\perp^i (2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2) \right\} \end{aligned}$$

[Diehl & Haegler, 2005] [Burkardt, 2005]

$$F(b_\perp^2) = \int d^2 \Delta_\perp e^{-i\vec{b}_\perp \cdot \Delta_\perp} F(\Delta_\perp^2) = \int d^2 \Delta_\perp e^{-i\vec{b}_\perp \cdot \Delta_\perp} \frac{F(0)}{(1 - \Delta_\perp^2/M^2)^p}$$

Transverse Spin Structure of the ~~Nucleon~~ Pion

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Transverse Spin Structure of the Nucleon Pion

Transverse densities:

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^1 dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) = \frac{1}{2} \left\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \Delta_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) \right. \\ \left. + \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i \cancel{B'_{n0}}(b_\perp^2) + s_\perp^i \bar{B}'_{Tn0}(b_\perp^2) \right) + \cancel{s_\perp^i (2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j} \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2) \right\}$$

[Diehl & Haegler, 2005] [Burkardt, 2005]

$$F(b_\perp^2) = \int d^2 \Delta_\perp e^{-i\vec{b}_\perp \cdot \Delta_\perp} F(\Delta_\perp^2) = \int d^2 \Delta_\perp e^{-i\vec{b}_\perp \cdot \Delta_\perp} \frac{F(0)}{(1 - \Delta_\perp^2/M^2)^p}$$

Transverse Spin Structure of the ~~Nucleon~~ Pion

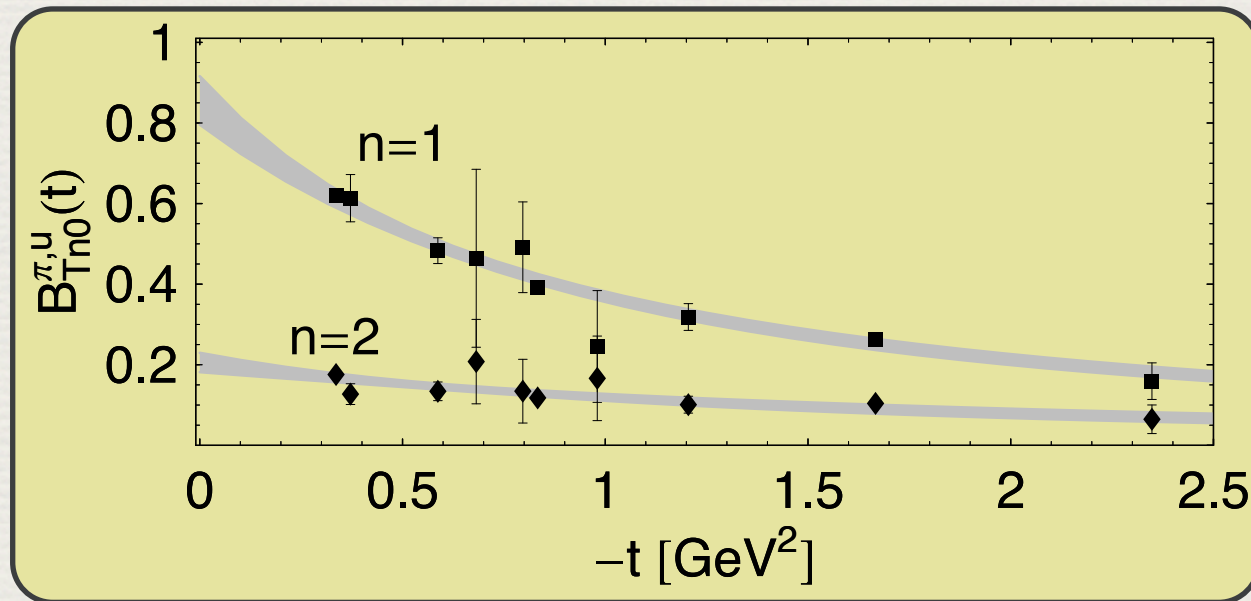
D. Brömmel (QCDSF) [arXiv:0708.2249]

Transverse densities:

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^1 dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) = \frac{1}{2} \left\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \Delta_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) \right. \\ \left. + \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i \cancel{B'_{n0}}(b_\perp^2) + s_\perp^i \bar{B}'_{Tn0}(b_\perp^2) \right) + \cancel{s_\perp^i (2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j} \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2) \right\}$$

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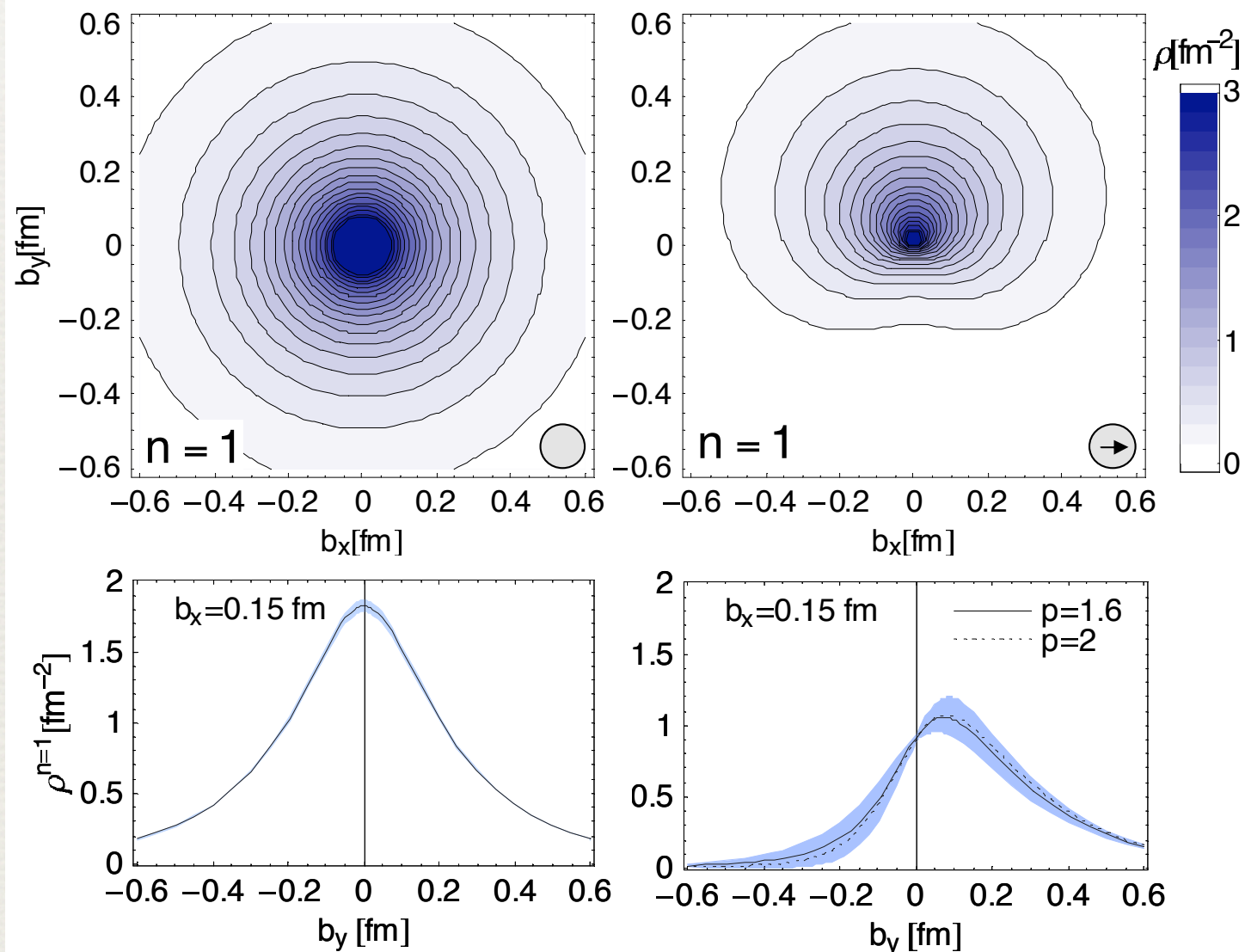
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Deformed Spin Densities

Pion

D. Brömmel (QCDSF) [arXiv:0708.2249]



Transverse Momentum Dependent PDFs

B. Musch [LHPC], *arXiv:0710.4423 (Lattice 2007)* [DWF + asqtad]

$f_1(x, k_\perp)$ Information on distribution of quarks with longitudinal momentum fraction, x , and transverse momentum, k_\perp


Factorisation in SIDIS $\langle P | \bar{q}(l) \Gamma \mathcal{U} q(0) | P \rangle$

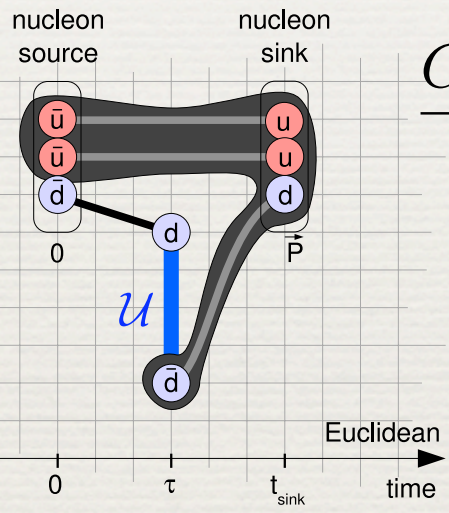


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Factorisation in SIDIS $\langle P | \bar{q}(l) \Gamma \mathcal{U} q(0) | P \rangle$  (Lattice)




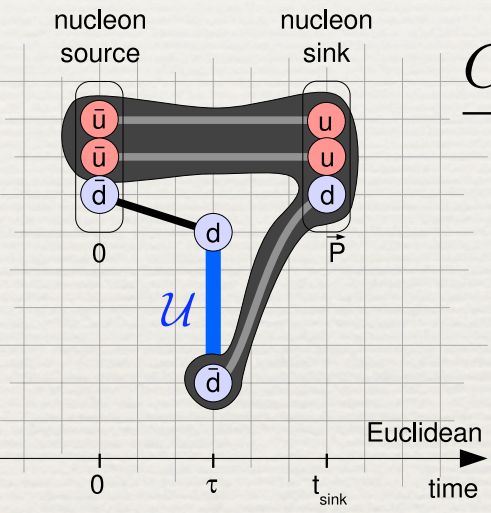
$$\frac{C_{3\text{pt}}(\tau, t_{\text{sink}}, P, \Gamma)}{C_{2\text{pt}}(t_{\text{sink}}, P)} \xrightarrow{0 \ll \tau \ll t_{\text{sink}}} \langle P | \bar{q}(l) \Gamma \mathcal{U} q(0) | P \rangle \propto \tilde{A}_i(l^2, l \cdot P)$$

Transverse Momentum Dependent PDFs

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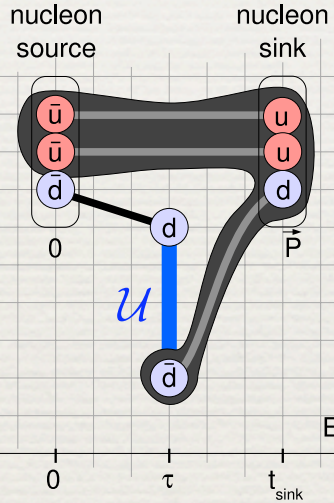
| | |
|-----------------------|---|
| Γ | \tilde{A}_i |
| γ_μ | \tilde{A}_2, \tilde{A}_3 |
| $\gamma_\mu \gamma_5$ | $\tilde{A}_6, \tilde{A}_7, \tilde{A}_8$ |

Transverse Momentum Dependent PDFs

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$$\Gamma \quad \tilde{A}_i$$

$$\gamma_\mu \quad \tilde{A}_2, \tilde{A}_3$$


$$\gamma_\mu \gamma_5 \quad \tilde{A}_6, \tilde{A}_7, \tilde{A}_8$$

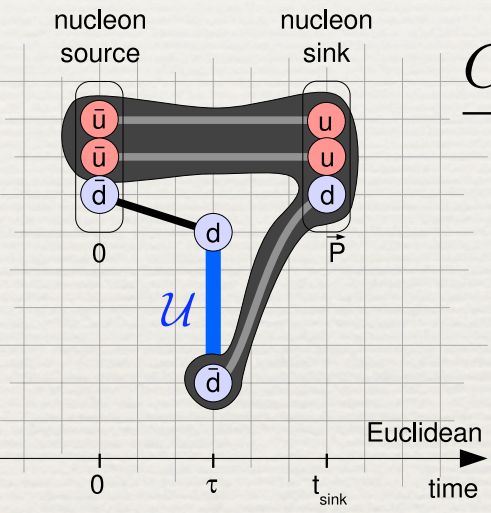
$$f_{1,\text{lat}}^{n=1}(\vec{k}_T) = \int dx f_1(x, \vec{k}_\perp) = \int \frac{d^2 \vec{\ell}_\perp}{(2\pi)^2} e^{i \vec{k}_\perp \cdot \vec{\ell}_\perp} 2 \tilde{A}_2(|\vec{\ell}_\perp|, 0)$$

Transverse Momentum Dependent PDFs

B. Musch [LHPC], arXiv:0710.4423 (Lattice 2007) [DWF + asqtad]

$f_1(x, k_\perp)$ Information on distribution of quarks with longitudinal momentum fraction, x , and transverse momentum, k_\perp

Factorisation in SIDIS $\langle P | \bar{q}(l) \Gamma \mathcal{U} q(0) | P \rangle$  (Lattice)



$$\frac{C_{3pt}(\tau, t_{\text{sink}}, P, \Gamma)}{C_{2pt}(t_{\text{sink}}, P)} \xrightarrow{0 \ll \tau \ll t_{\text{sink}}} \langle P | \bar{q}(l) \Gamma \mathcal{U} q(0) | P \rangle \propto \tilde{A}_i(l^2, l \cdot P)$$

| | |
|-----------------------|---|
| Γ | \tilde{A}_i |
| γ_μ | \tilde{A}_2, \tilde{A}_3 |
| $\gamma_\mu \gamma_5$ | $\tilde{A}_6, \tilde{A}_7, \tilde{A}_8$ |

$$f_{1,\text{lat}}^{n=1}(\vec{k}_T) = \int dx f_1(x, \vec{k}_\perp) = \int \frac{d^2 \vec{\ell}_\perp}{(2\pi)^2} e^{i \vec{k}_\perp \cdot \vec{\ell}_\perp} 2 \tilde{A}_2(|\vec{\ell}_\perp|, 0)$$

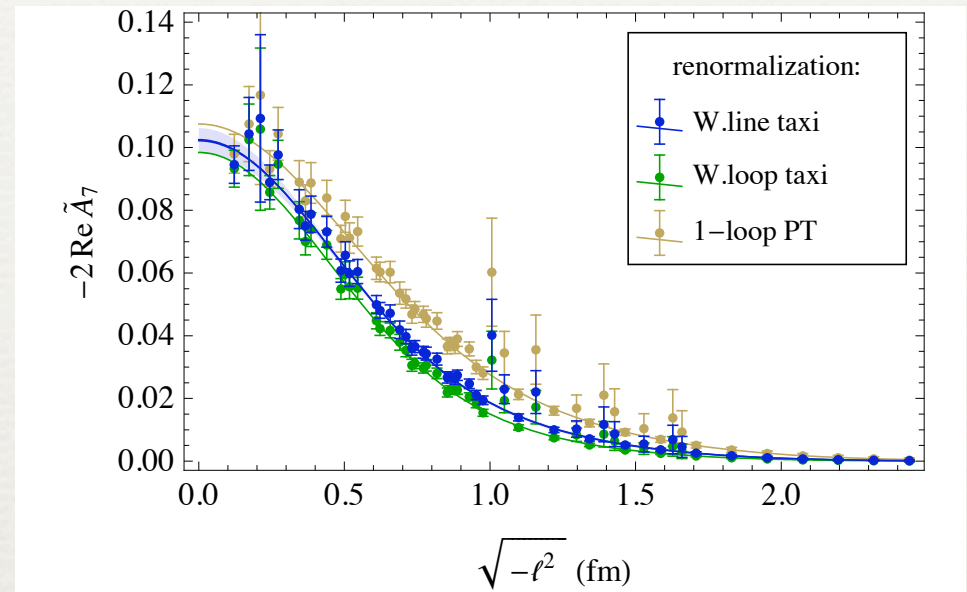
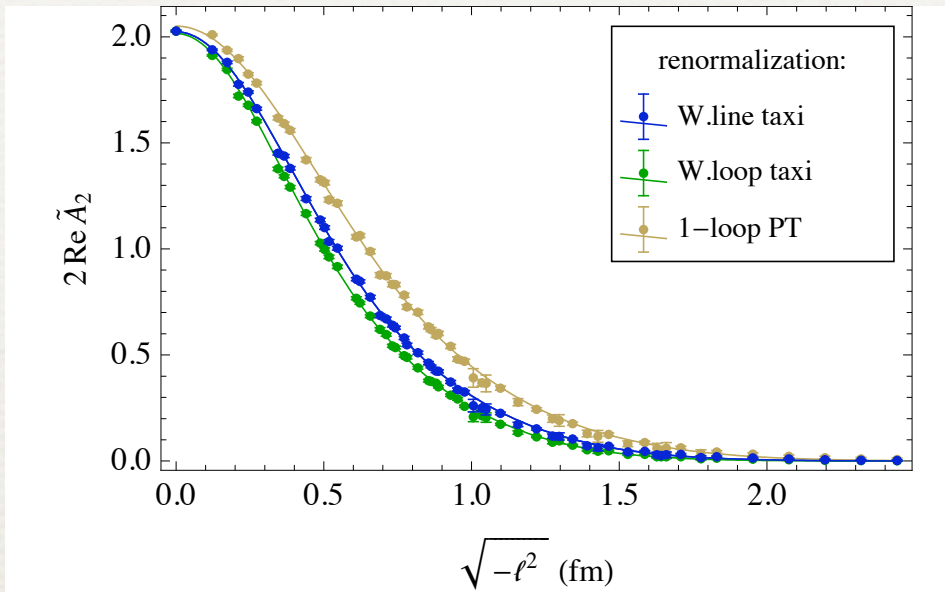
In a transversely polarised nucleon:

$$\frac{1}{2} \left(f_1^{(1)\text{lat}}(k_\perp) + \frac{k_\perp \cdot S_\perp}{m_N} g_{1T}^{(1)\text{lat}}(k_\perp) \right)$$

← from \tilde{A}_7

Transverse Momentum Dependent PDFs

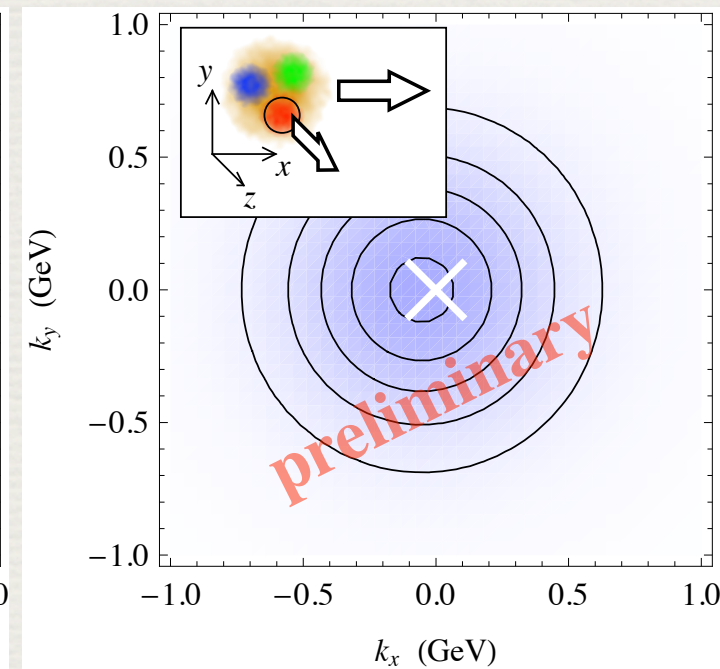
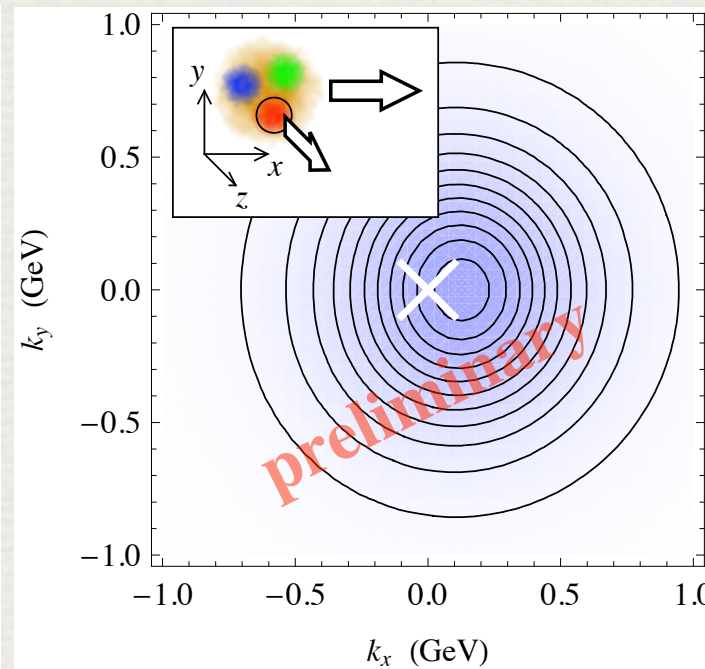
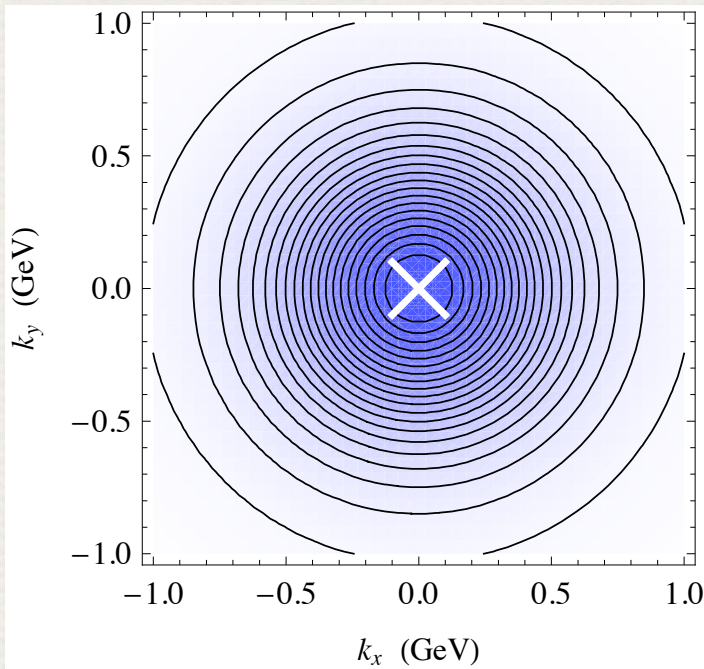
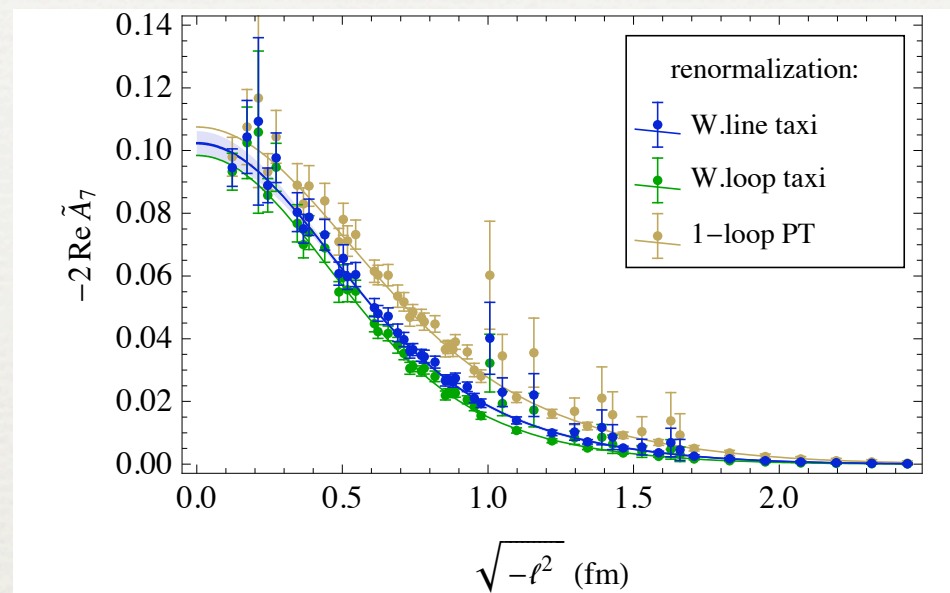
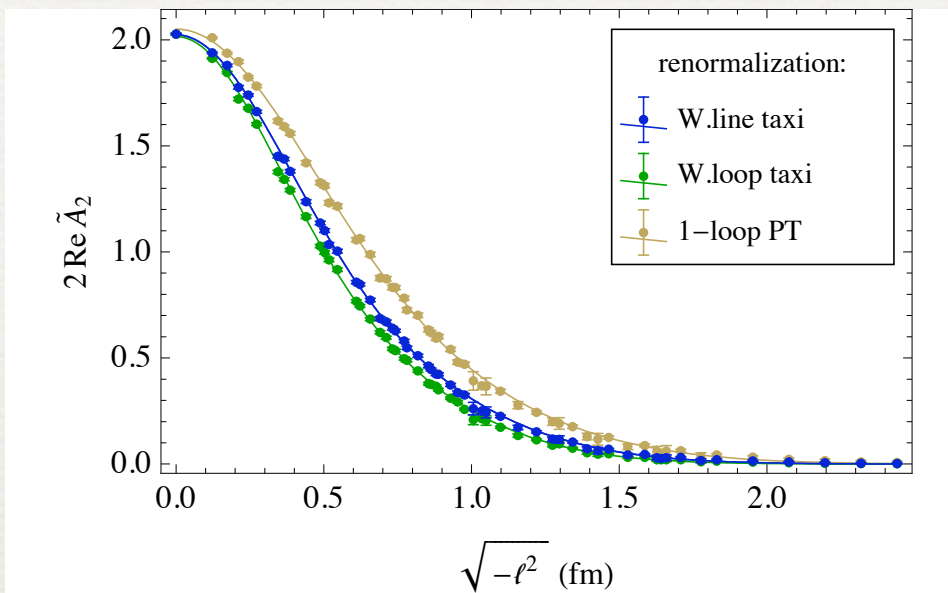
B. Musch [LHPC], arXiv:0710.4423 (Lattice 2007) Friday 3:30



Transverse Momentum Dependent PDFs

B. Musch [LHPC], arXiv:0710.4423 (Lattice 2007)

Friday 3:30



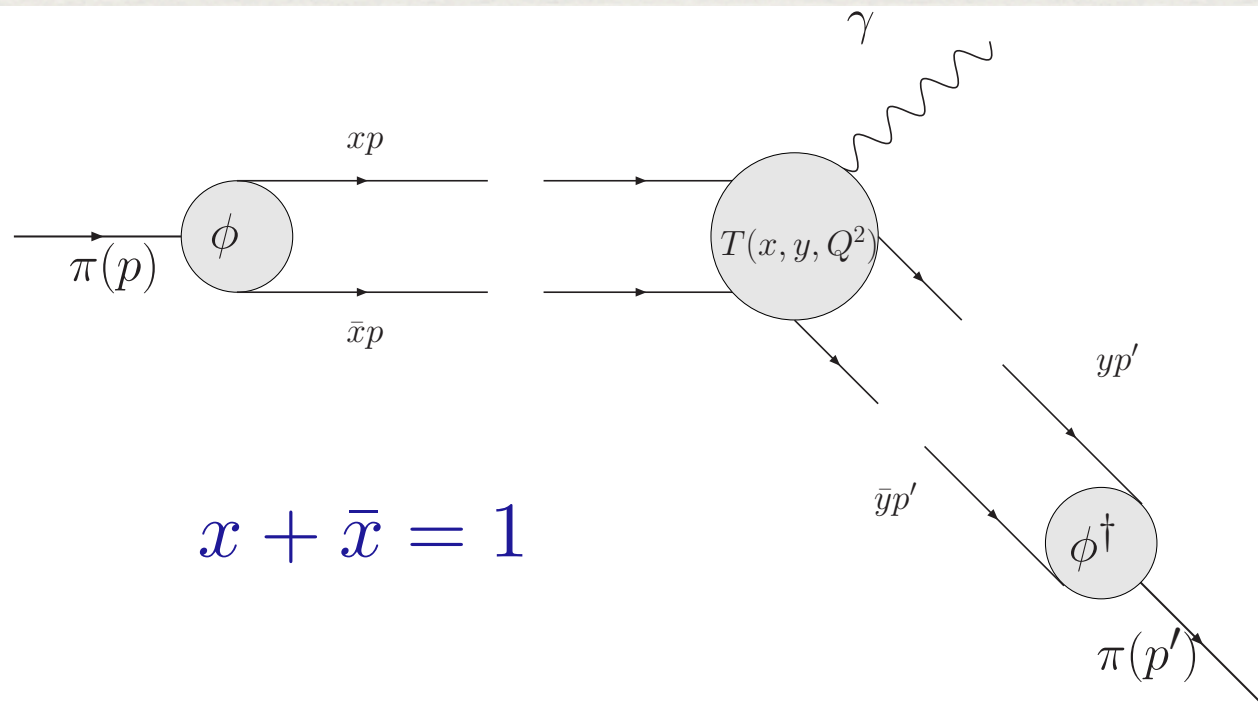
Distribution Amplitudes

Exclusive processes at large $Q^2 \rightarrow \infty$ can be factorised into:

- perturbative hard scattering amplitude (process dependent)
- nonperturbative wave functions describing the hadron's overlap with lowest Fock state (process independent)

$$F(Q^2) = \int_0^1 dx \int_0^1 dy \phi^\dagger(y, Q^2) T(x, y, Q^2) \phi(x, Q^2) [1 + \mathcal{O}(m^2/Q^2)]$$

$$\langle 0 | \bar{q}(-z) \gamma_\mu \gamma_5 [-z, z] u(z) | \Pi^+(p) \rangle = i f_\Pi p_\mu \int_{-1}^1 d\xi e^{-i\xi p \cdot z} \phi_\Pi(\xi, \mu^2)$$



$$\xi = x - \bar{x}$$

$$\int_{-1}^1 d\xi \phi_\Pi(\xi, \mu^2) = 1$$

- For spin 1 mesons:

$$\phi^\parallel(\xi), \phi^\perp(\xi)$$

Meson Distribution Amplitudes

D. Brömmel [UKQCD/RBC] 2+1 DWF Friday 3:10

Meson Distribution Amplitudes

D. Brömmel [UKQCD/RBC] 2+1 DWF Friday 3:10

π, K *QCDSF: PRD 74, 074501 (2006)*

UKQCD/RBC: PLB 641, 67 (2006)

arXiv:0710.0869 (Lattice 2007)

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ρ, K^*

QCDSF: PoS, Lattice 2007

UKQCD/RBC: in preparation

$$\langle 0 | \bar{q}(0) \gamma_{\{\rho} \overleftrightarrow{D}_{\mu\}} s(0) | K^*(p, \lambda) \rangle = m_{K^*} f_{K^*} p_{\{\rho} \epsilon_{\mu\}}^{(\lambda)} \langle \xi^1 \rangle_{K^*} \|$$

$$\langle 0 | \bar{q}(0) \gamma_{\{\rho} \overleftrightarrow{D}_{\mu} \overleftrightarrow{D}_{\nu\}} q(0) | \rho(p, \lambda) \rangle = -i m_{\rho} f_{\rho} p_{\{\rho} p_{\mu} \epsilon_{\nu\}}^{(\lambda)} \langle \xi^2 \rangle_{\rho} \|$$

Meson Distribution Amplitudes

D. Brömmel [UKQCD/RBC] 2+1 DWF Friday 3:10

π, K QCDSF: PRD 74, 074501 (2006)

UKQCD/RBC: PLB 641, 67 (2006)

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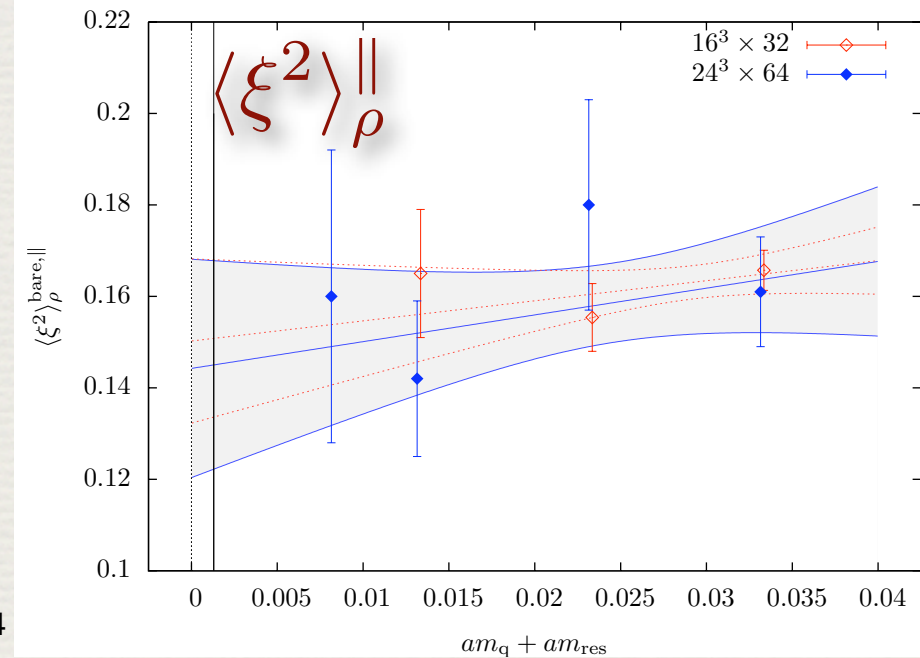
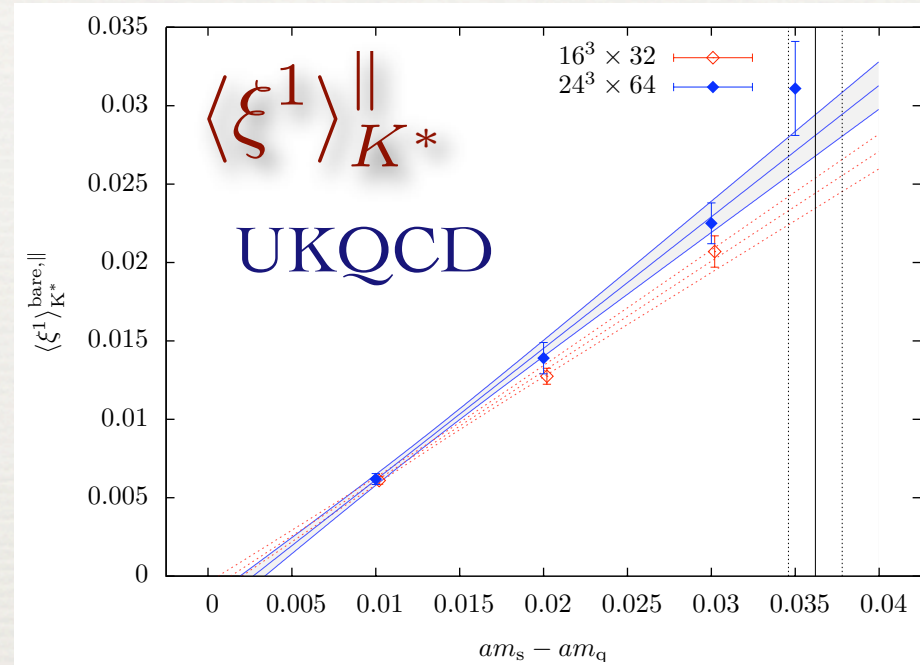
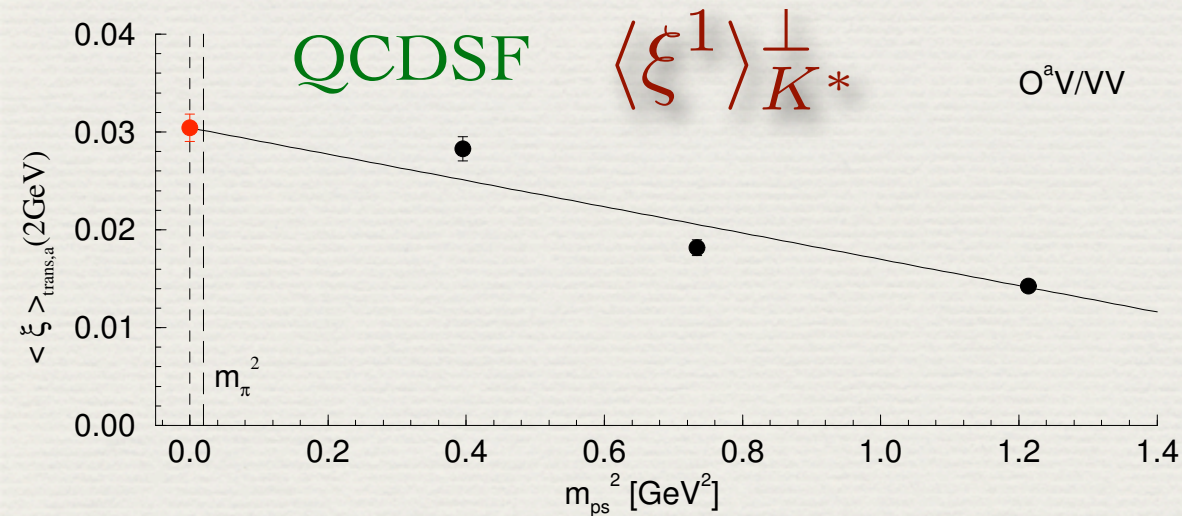
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QCDSF: PoS, Lattice 2007

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Vector Meson Distribution Amplitudes

$$\rho, K^* \quad \overline{\text{MS}}, \quad \mu^2 = 4 \text{ GeV}^2$$

QCDSF:

$$\langle \xi \rangle_{K^*}^{\parallel} \approx 0.036(3)$$

$$\langle \xi \rangle_{K^*}^{\perp} \approx 0.030(2)$$

UKQCD:

$$\langle \xi \rangle_{K^*}^{\parallel} \approx 0.0359(17)(22)$$

$$\langle \xi^2 \rangle_{\rho}^{\parallel} \approx 0.240(36)(12)$$

$$\langle \xi^2 \rangle_{K^*}^{\parallel} \approx 0.252(17)(12)$$

Sum Rules $\langle \xi \rangle_{K^*}^{\parallel} \sim 0.02(2)$

$$\langle \xi \rangle_{K^*}^{\perp} \sim 0.03(3)$$

$$\langle \xi^2 \rangle_{K^*, \rho}^{\parallel} \sim 0.24(2)$$

Asymptotic $\langle \xi \rangle_{K^*}^{\parallel} = 0$

$$\langle \xi \rangle_{K^*}^{\perp} = 0$$

$$\langle \xi^2 \rangle_{K^*, \rho}^{\parallel} = 0.2$$

Nucleon Distribution Amplitudes

N. Warkentin (QCDSF) [arXiv:0804.1877]

Wednesday 3:30

- Three distribution amplitudes for the proton: V, A, T

- Expansion in local matrix elements give **moments**

$$\begin{aligned} & \epsilon^{abc} \langle 0 | [i^l D^{\lambda_1} \dots D^{\lambda_l} u_\alpha^a(0)] (C \gamma^\rho)_{\alpha\beta} [i^m D^{\mu_1} \dots D^{\mu_m} u_\beta^b(0)] [i^n D^{\nu_1} \dots D^{\nu_n} (\gamma_5 d_\gamma^c(0))_\gamma] | p \rangle \\ & = -f_N V^{lmn} p^\rho p^{\lambda_1} \dots p^{\lambda_l} p^{\mu_1} \dots p^{\mu_m} p^{\nu_1} \dots p^{\nu_n} N_\gamma(p) \end{aligned}$$

Useful combination:

$$\phi^{lmn} = \frac{1}{3} (V^{lmn} - A^{lmn} + 2T^{lnm})$$

Momentum conservation: $\phi^{lmn} = \phi^{(l+1)mn} + \phi^{l(m+1)n} + \phi^{lm(n+1)}$

DA normalisation

$$\begin{aligned} \phi^{000} & \equiv 1 \\ 1 & = \phi^{100} + \phi^{010} + \phi^{001} \\ & = \phi^{200} + \phi^{020} + \phi^{002} + 2(\phi^{011} + \phi^{101} + \phi^{110}) \end{aligned}$$

Asymptotic behaviour $\varphi(x_i, Q^2 \rightarrow \infty) = 120x_1x_2x_3$

$$\phi^{100} = \phi^{010} = \phi^{001} = \frac{1}{3}$$

$$\phi^{200} = \phi^{020} = \phi^{002} = \frac{1}{7}$$

$$\phi^{110} = \phi^{101} = \phi^{011} = \frac{2}{21}$$

Asymmetries important

$$\phi^{100} - \phi^{010}$$

Constrained Analysis

N. Warkentin (QCDSF) [arXiv:0804.1877]

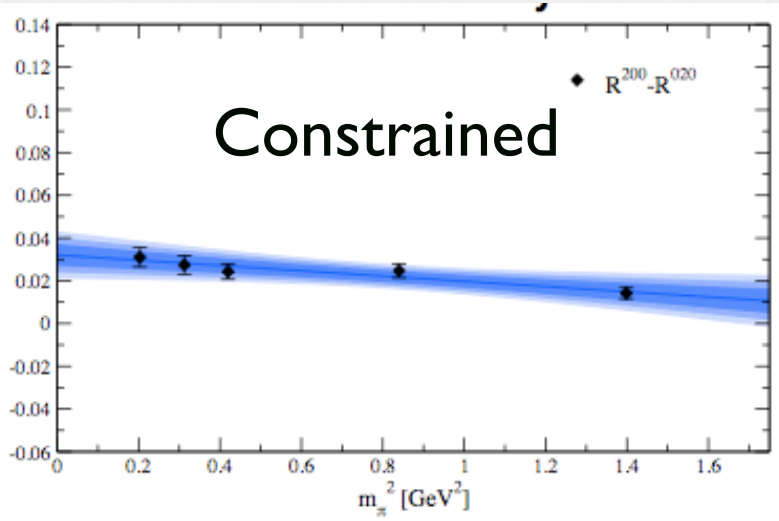
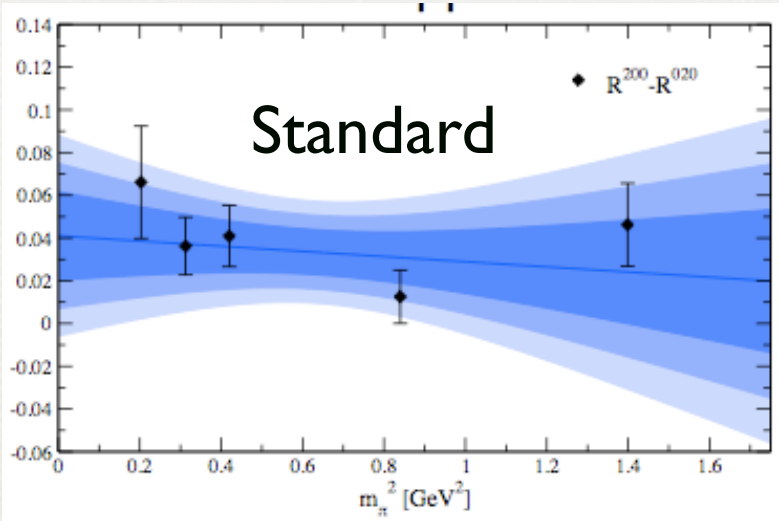
Wednesday 3:30

DA correlator ratios

$$R^{lmn} = \frac{\phi^{lmn}}{S_{(l+m+n)}}$$

$$S_1 = \phi^{100} + \phi^{010} + \phi^{001}$$

$$S_2 = \phi^{200} + \phi^{020} + \phi^{002} + 2(\phi^{110} + \phi^{101} + \phi^{011})$$

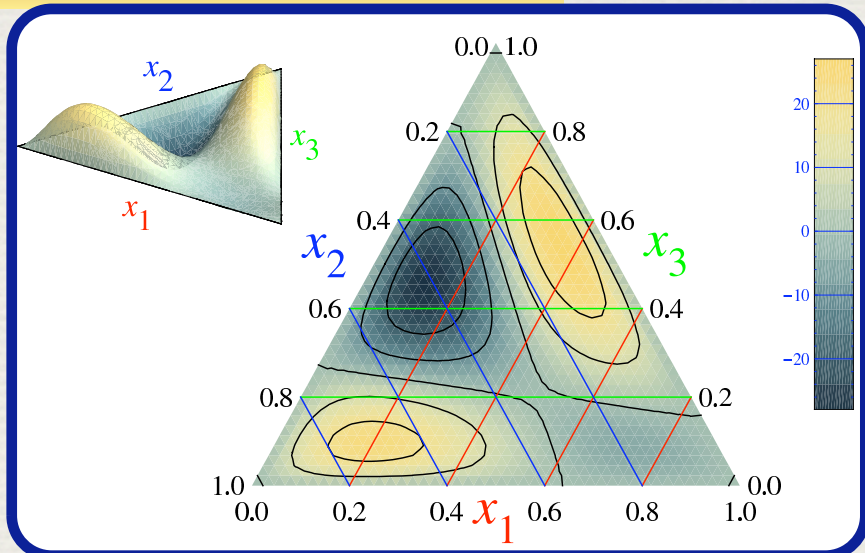
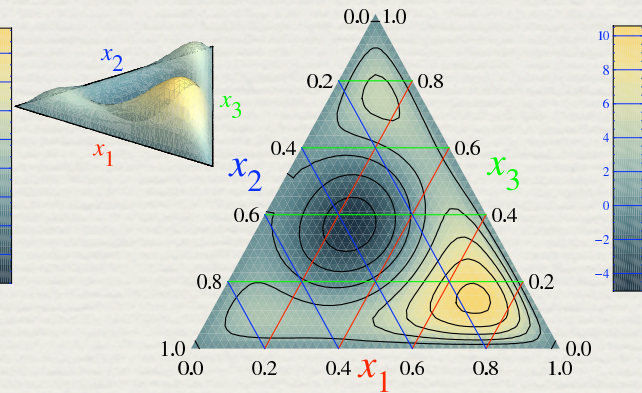
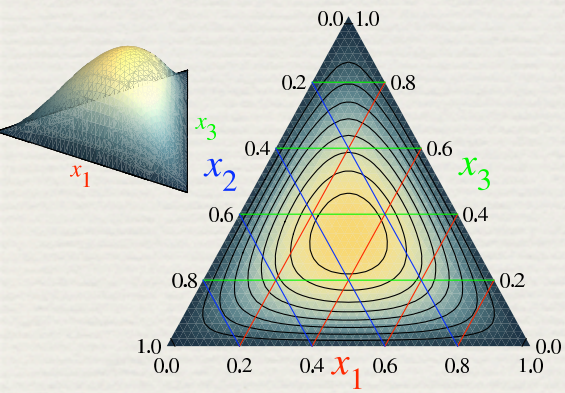


u-quark with spin aligned with proton spin has largest momentum fraction

Asymmetries less pronounced than for QCD-SR!

Asymptotic case:

Sum Rules



Jefferson Lab Research Program

<http://www.jlab.org/highlights/phys.html>

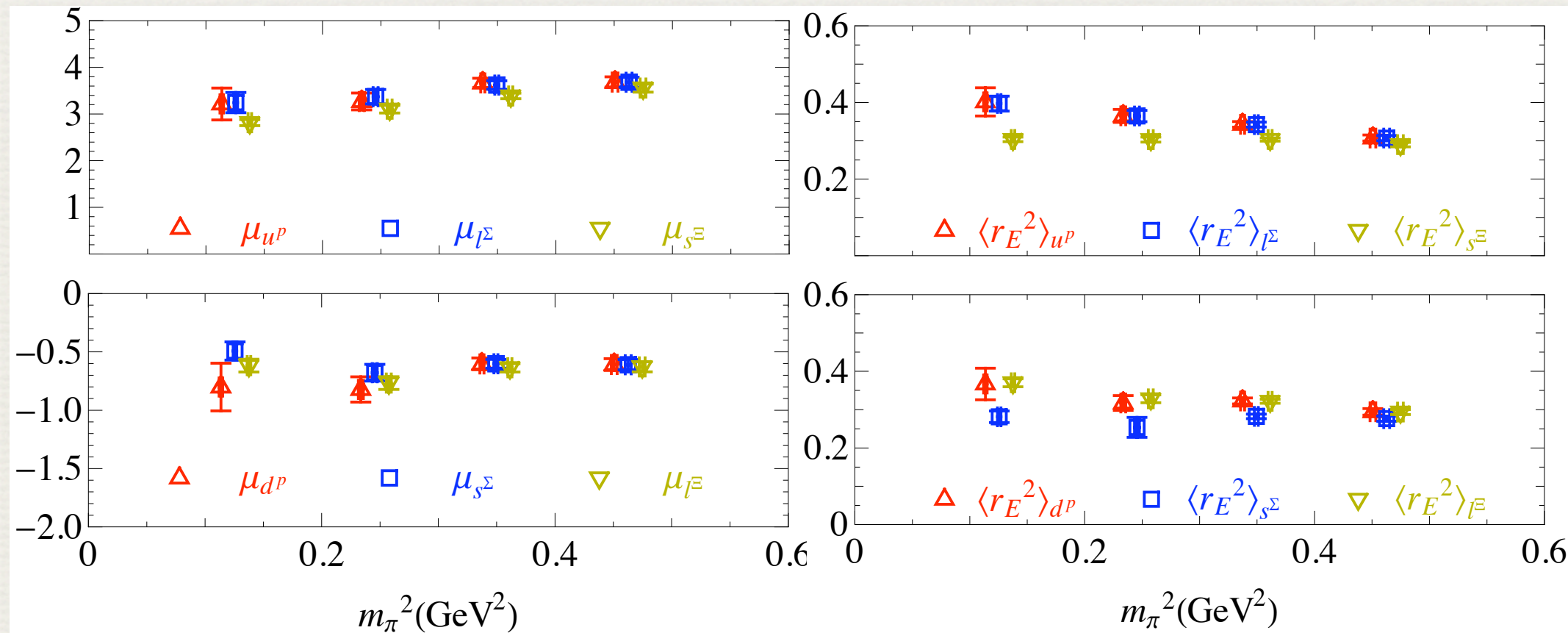
- ◆ Electric and Magnetic Proton Form Factors
- ◆ Neutron Charge Density
- ◆ Neutron Magnetic Structure
- ◆ Nucleon-Delta Transition
- ◆ Pion Form Factor
- ◆ Generalized Parton Distributions
- ◆ Spin Sum Rules
- ◆ Nucleon Spin Asymmetries
- ◆ **Strange Quarks in the Proton**

Octet Baryon Form Factors

H.-W. Lin, K. Orginos

Mixed action (DWF+asqtad) at $m_{\pi} = 350-750$ MeV

- Smaller strange contribution to charge radii
- Similar behaviour to quenched (Adelaide)



Strangeness

H.-W. Lin, K. Orginos

Mixed action (DWF+asqtad) at $m_{\text{pi}} = 350\text{-}750$ MeV

Indirect: Charge symmetry + chiral extrapolations

[D. Leinweber et al., PRL 94, 212001 (2005); 97, 022001(2006)]

$$G_M^s = \left(\frac{l R_d^s}{1 - l R_d^s} \right) \left[2p + n - \frac{u^p}{u^\Sigma} (\Sigma^+ - \Sigma^-) \right]$$

$$G_M^s = \left(\frac{l R_d^s}{1 - l R_d^s} \right) \left[p + 2n - \frac{u^n}{u^\Xi} (\Xi^0 - \Xi^-) \right]$$

ChPT

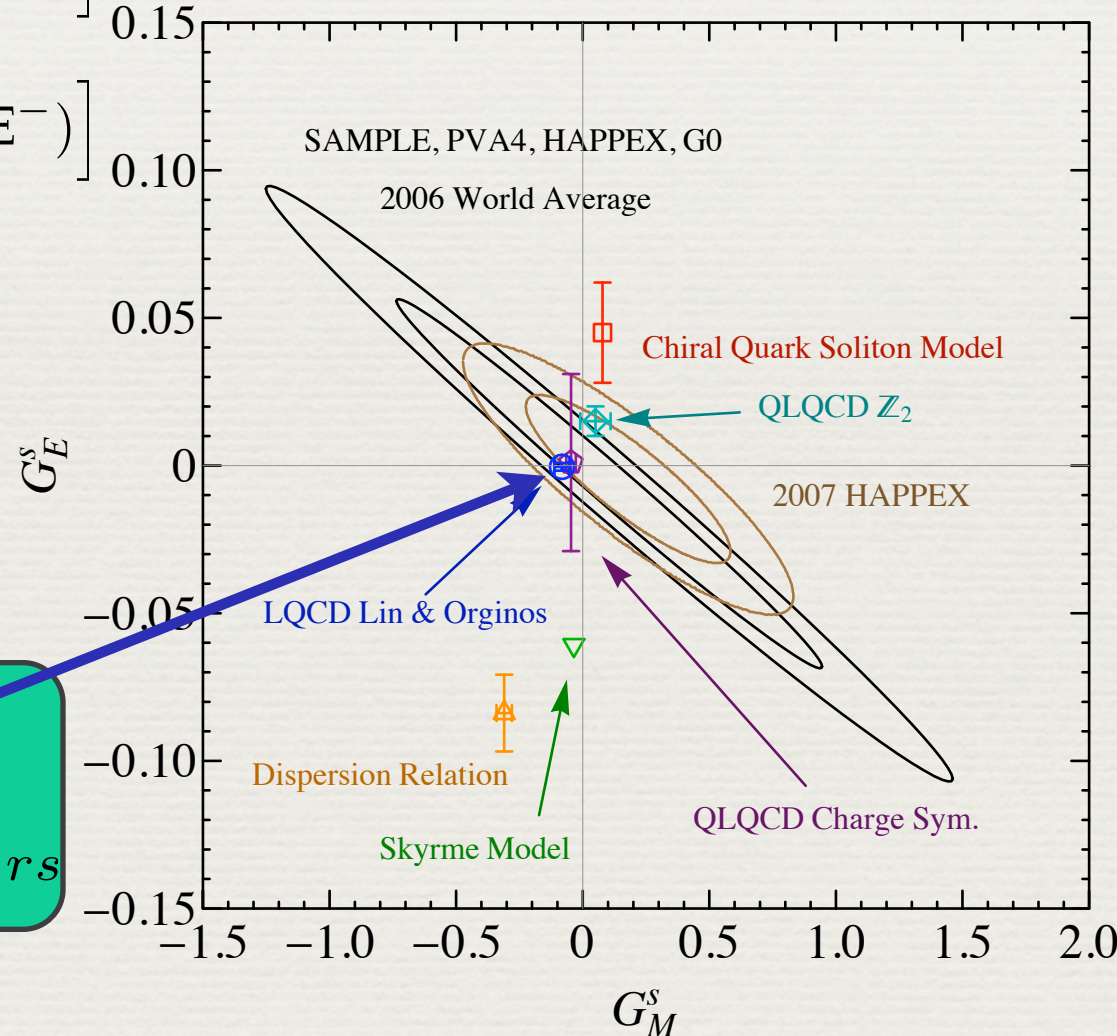
Exp

Lattice

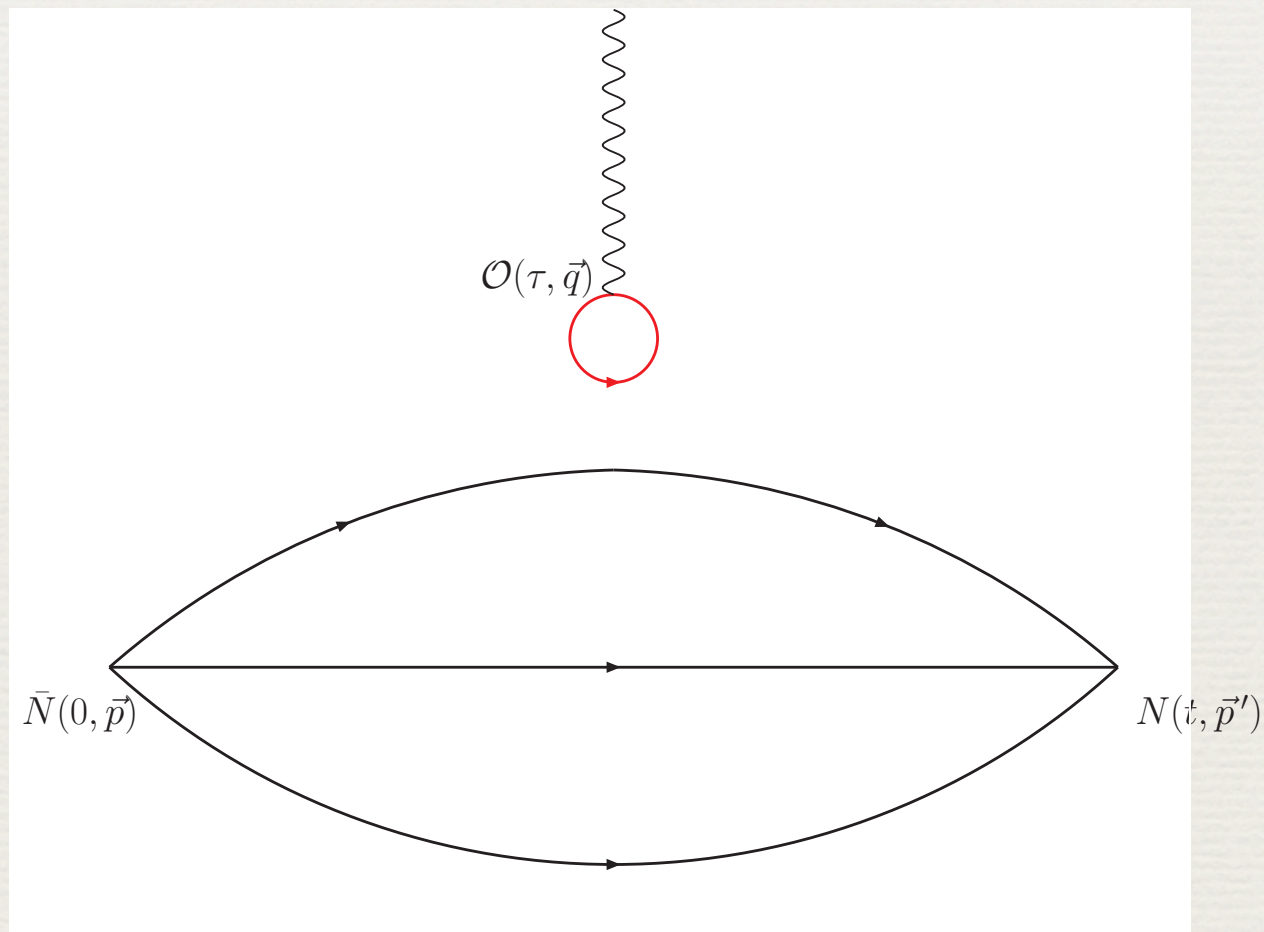
$$Q^2 \sim 0.1 \text{ GeV}^2$$

$$G_M^s = -0.082(8)_{\text{stat}}(25)_R$$

$$G_E^s = -0.00044(1)_{\text{stat}}(130)_{rs}$$

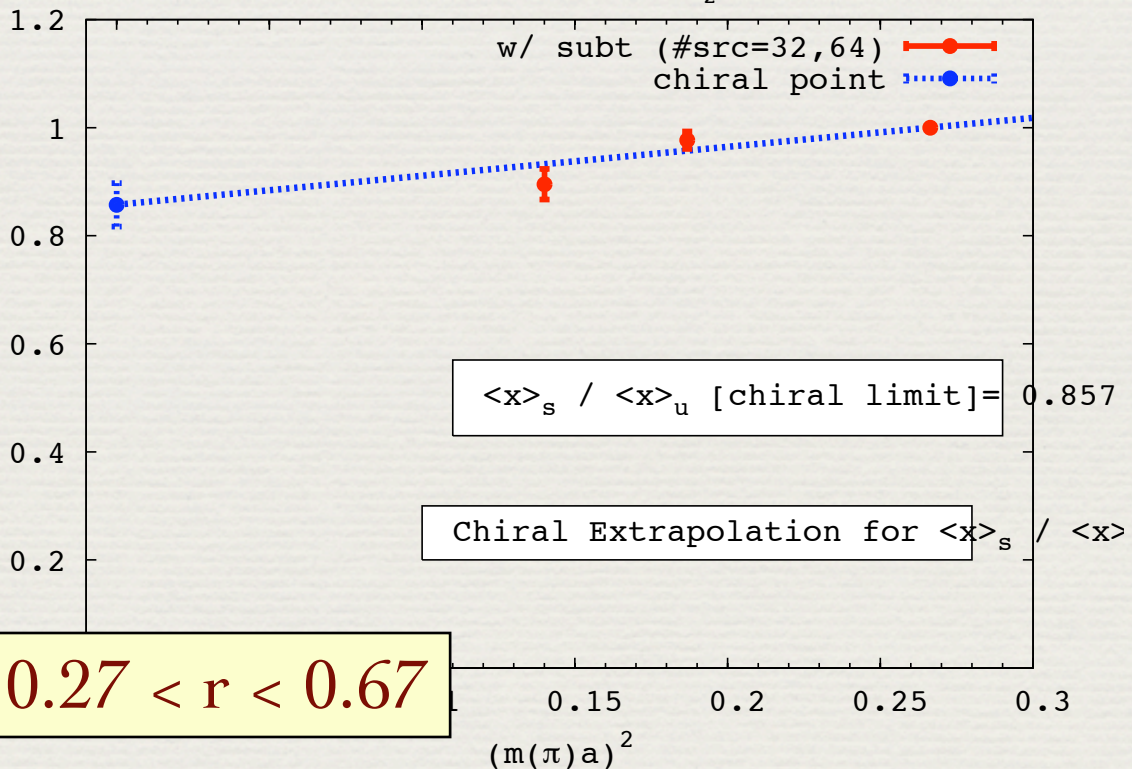
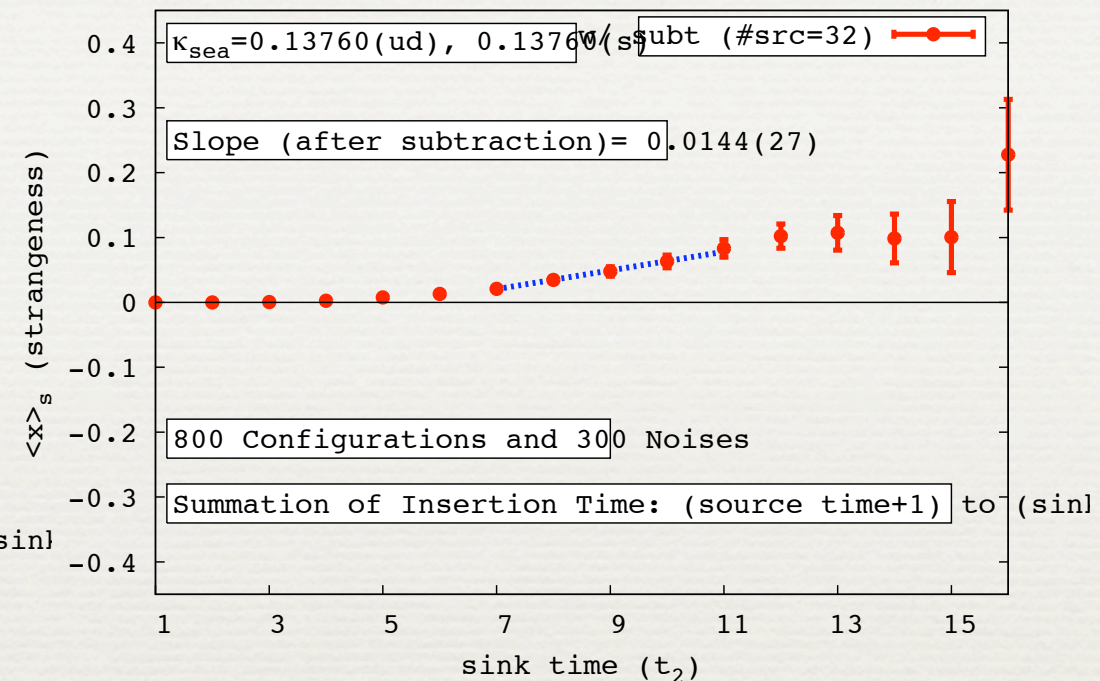
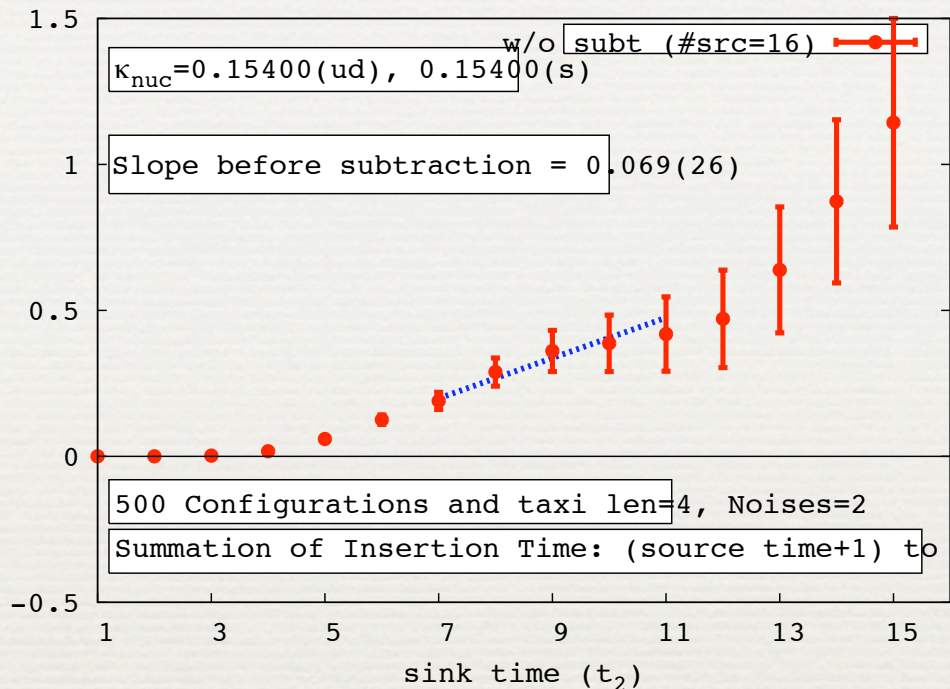


Disconnected



$$\langle x \rangle_g, \langle x \rangle_s$$

T. Doi [Kentucky] Thursday 11:40



- ◆ $N_f=2+1$ clover (CP-PACS/JLQCD)
- ◆ Z(4) noise sources (300)
- ◆ Multiple source locations
- ◆ Sum over operator insertion times
- ➔ fit to slope
- ◆ Overlap operator for $F_{\mu\nu}$

[PLB 659, 773 (2008)]

$$\frac{\langle x \rangle_{\bar{s}}}{\frac{1}{2}(\langle x \rangle_{\bar{u}} + \langle x \rangle_{\bar{d}})} \approx 85\%$$

CTEQ: $0.27 < r < 0.67$

$$G_S^s(q^2 = 0), G_A^s(q^2 = 0) = \Delta s \quad \text{Thursday 10:40}$$

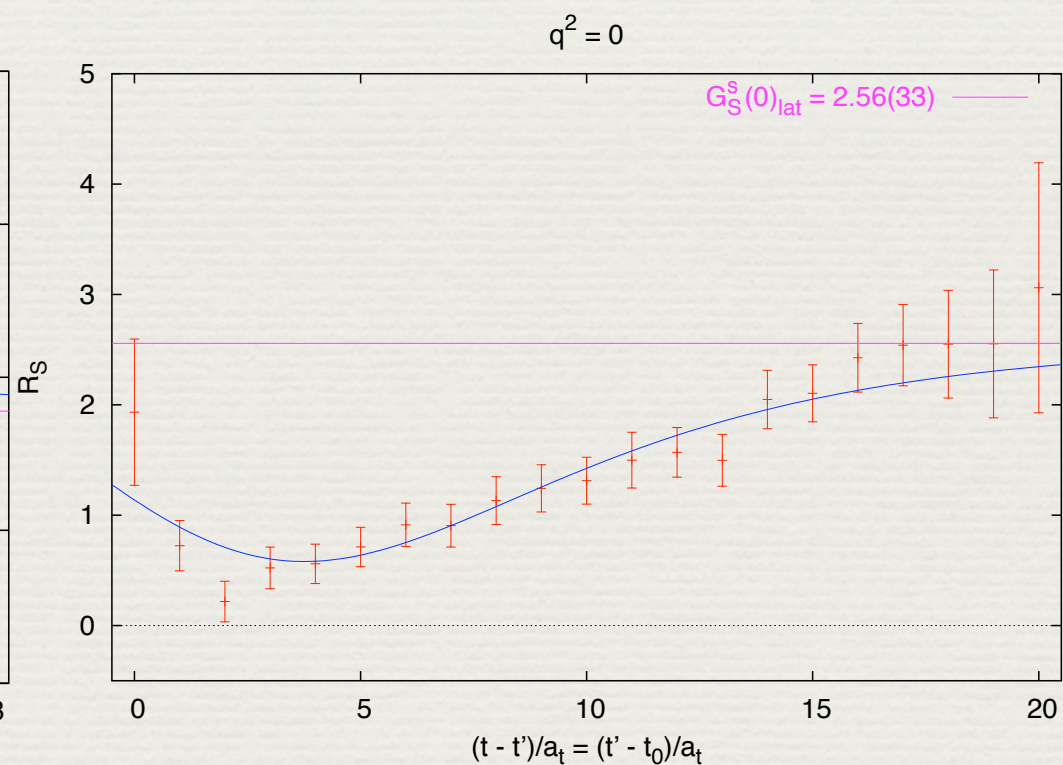
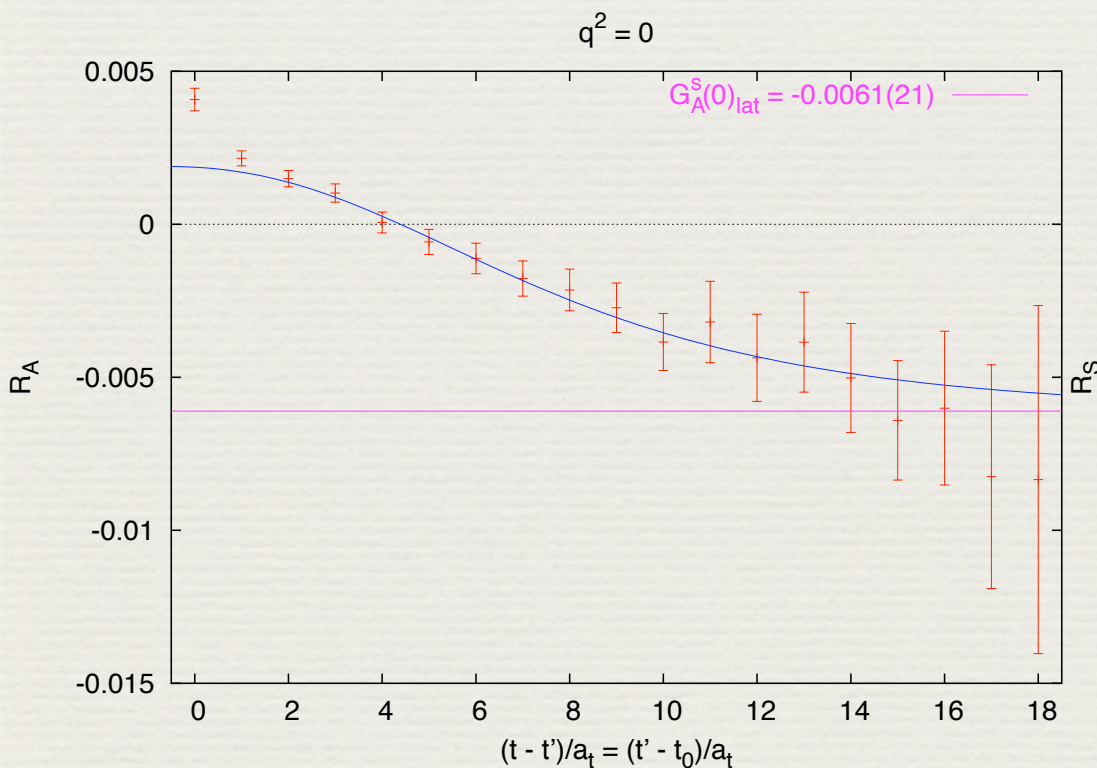
$24^3 \times 64$ anisotropic $N_f=2$ Wilson
 Stochastic sources with
 maximum dilution

R. Babich [Boston]

G_M^s Consistent with zero

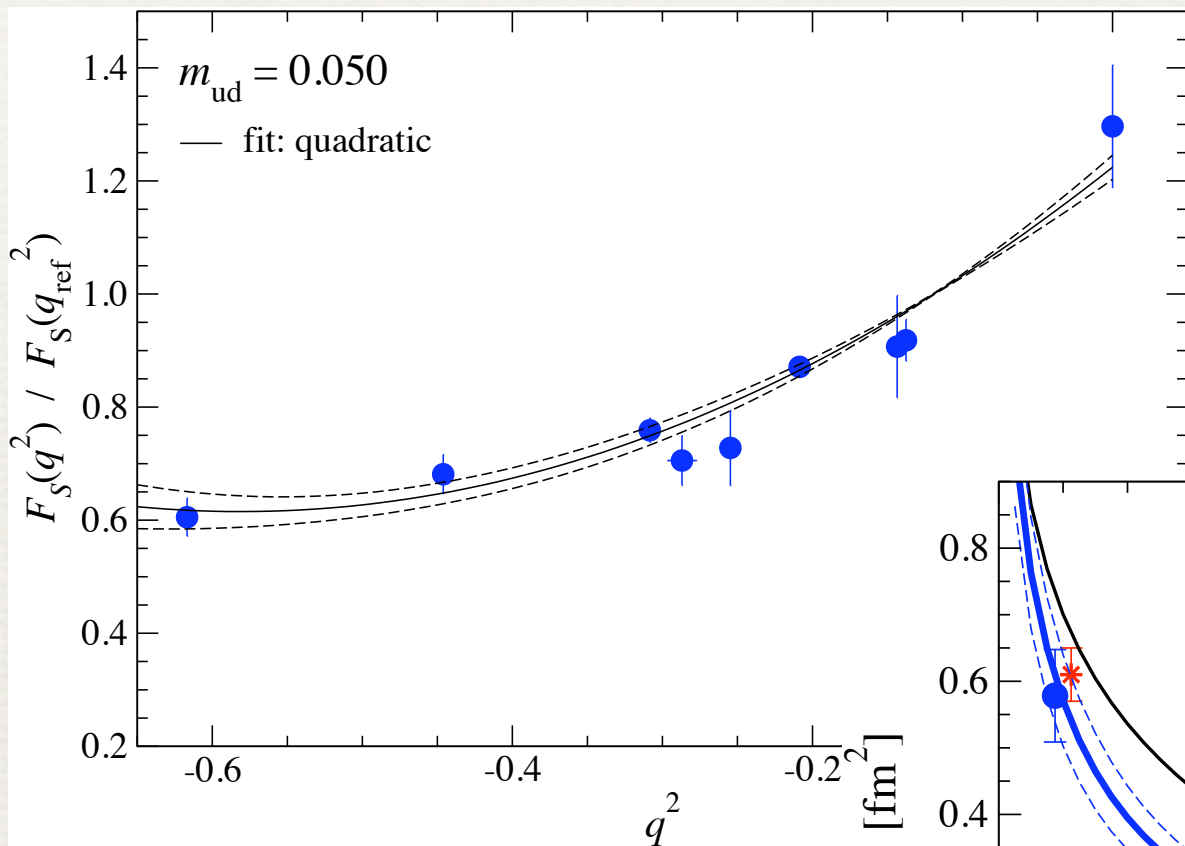
Δs can be obtained with
 30% errors

$$f_{T_s} = \frac{m_s \langle N | \bar{s} s | N \rangle}{M_N} = 0.48(7)(3)$$

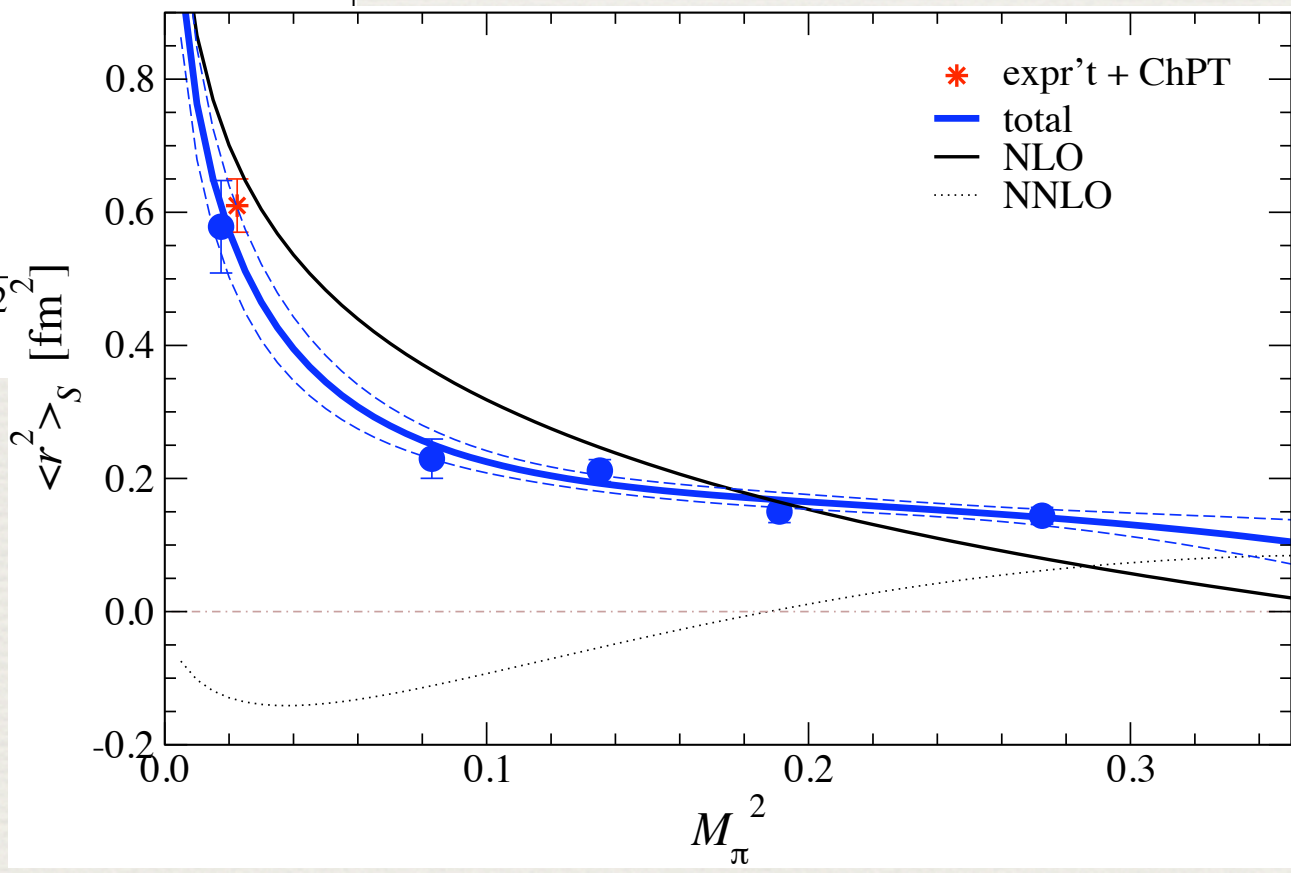


Scalar Form Factor

T. Kaneko [JLQCD]: $N_f=2$ Overlap, all-to-all propagators Thursday 9:10
Include disconnected contribution



$$\langle r^2 \rangle_\pi^S = 0.578(69)(46) \text{ fm}^2$$



Other Disconnected/ Strangness Talks

♦ Gunnar Bali

Thursday 11:00

♦ *“Hunting for the strangeness content of the nucleon”*

ΔS using variance-reduced all-to-all propagators

Background Field & Polarizabilities

- ◆ Andrei Alexandru Monday 5:20 α_E^n
 - ◆ *“The background field method on the lattice”*
- ◆ Christopher Aubin Monday 5:40 $\mu_{\Delta^+}, \mu_{\Delta^{++}}, \mu_{\Omega^-}$
 - ◆ *“Finite volume study of the Delta magnetic moments using dynamical clover fermions”*
- ◆ Brian Tiburzi Monday 6:00 α_E
 - ◆ *“Polarizabilities from Lattice QCD”* both neutral and charge hadrons (Clover on DWF)
- ◆ Scott Moerschbacher Monday 6:20 $\beta_M^{p,n}$
 - ◆ *“Magnetic polarizability of hadrons from dynamical configurations”* (CP-PACS configs)

Conclusion & Outlook

- ◆ Calculations of hadronic quantities becoming available at $m_\pi \approx 250$ MeV (beware finite size effects)
- ◆ q^2 scaling of hadronic form factors
 - ◆ Twisted b.c.s give access to small q^2 (charge radii, F_1^n negative)
 - ◆ Large $q^2 > 4$ GeV²? (JLab)
- ◆ Moments of Generalised Parton Distributions
 - ◆ Quark contribution to nucleon spin and angular momentum $J_u \approx 46\%$
 - ◆ Non-trivial transverse spin densities in pion and nucleon $J_d \approx 0$
 - $L_{u+d} \approx 0$
- ◆ Moments of Ordinary Parton Distribution Functions
 - ◆ Finite sized effects severe for g_A
 - ◆ Results for $\langle x \rangle_{u-d}$ appear to be “bending down”
- ◆ Moments of Distribution Amplitudes
 - ◆ Proton: Asymmetries are less pronounced as in QCD-SR

Conclusions and Outlook

Also important to develop new techniques/ideas

- ♦ Moments of Transverse Momentum Dependent PDFs $f_1^{(1)}(\vec{k}_\perp)$, $g_1^{(1)}(\vec{k}_\perp)$
 - ♦ **Densities of longitudinally polarised quarks in a transversely polarised nucleon are deformed**
- ♦ Disconnected contributions
 - ♦ **Strangeness/gluonic content of nucleon**
 - ♦ **Contribution to nucleon spin**
- ♦ Background field methods
 - ♦ **Magnetic moments**
 - ♦ **Polarisabilities**