Nuclear Effective Field Theory on the Lattice

Hermann Krebs
FZ Jülich & Universität Bonn

July 19, 2008, Lattice 2008 Williamsburg

In collaboration with B. Borasoy, E. Epelbaum, D. Lee, U. Meißner

Outline

- EFT and Lattice QCD
- Nuclear forces in EFT
- Lattice EFT (Formalism)
- Lattice EFT to LO
- Nucleon-Nucleon scattering within Lattice EFT up to NLO
- Dilute neutron matter up to NLO
- First results for NNLO
- Conclusion & Outlook

ChPT and low energy QCD

Spontaneous + explicit (by small quark masses) breaking of chiral symmetry in QCD

⇩

Existence of light weakly interacting Goldstone bosons

⇩

Chiral Perturbation theory (ChPT)
Expansion in small momenta and masses of Goldstone bosons

⇩

Systematic description of QCD by ChPT in low energy sector
(low momenta $q \ll \Lambda_\chi \simeq 1\,\mathrm{GeV}$)

EFT and Lattice QCD

- Free parameters of QCD

α_s, quark masses

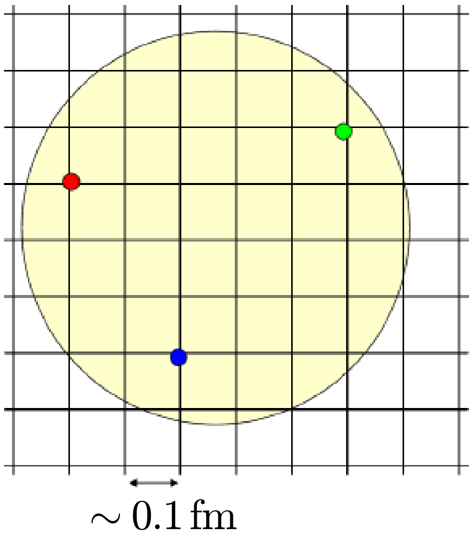
- Free parameters of ChPT

$F_\pi, g_{\pi N}, c_i \ldots, d_i \ldots, C_S, C_T \ldots$

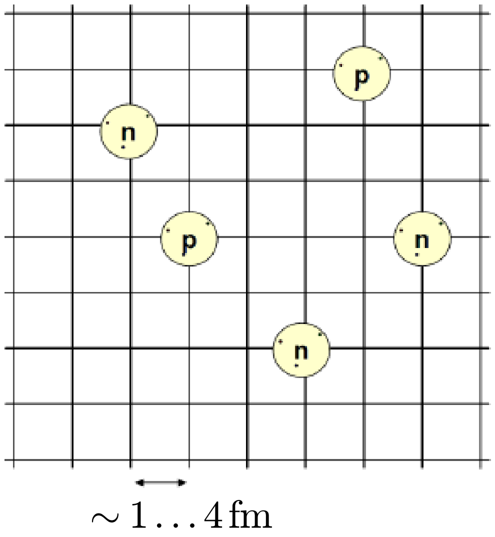
Computable by LQCD

Nucleon in LQCD

Nucleons as point particles on the lattice



Difference in computational effort



EFT on the Lattice so far

For review see: Lee, arXiv: 0804.3501 [nucl-th]

First Lattice study of nuclear matter: Brockmann, Frank, Phys. Rev. Lett. 68: 1830 (1992)
momentum Lattice & hadrodynamics model: Walecka, Annals Phys. 83: 491 (1974)

First Lattice EFT simul.: Müller, Koonin, Seki, van Kolck, Phys. Rev. C61: 044320 (2000)
infinite nuclear and neutron matter at nonzero density and temperature

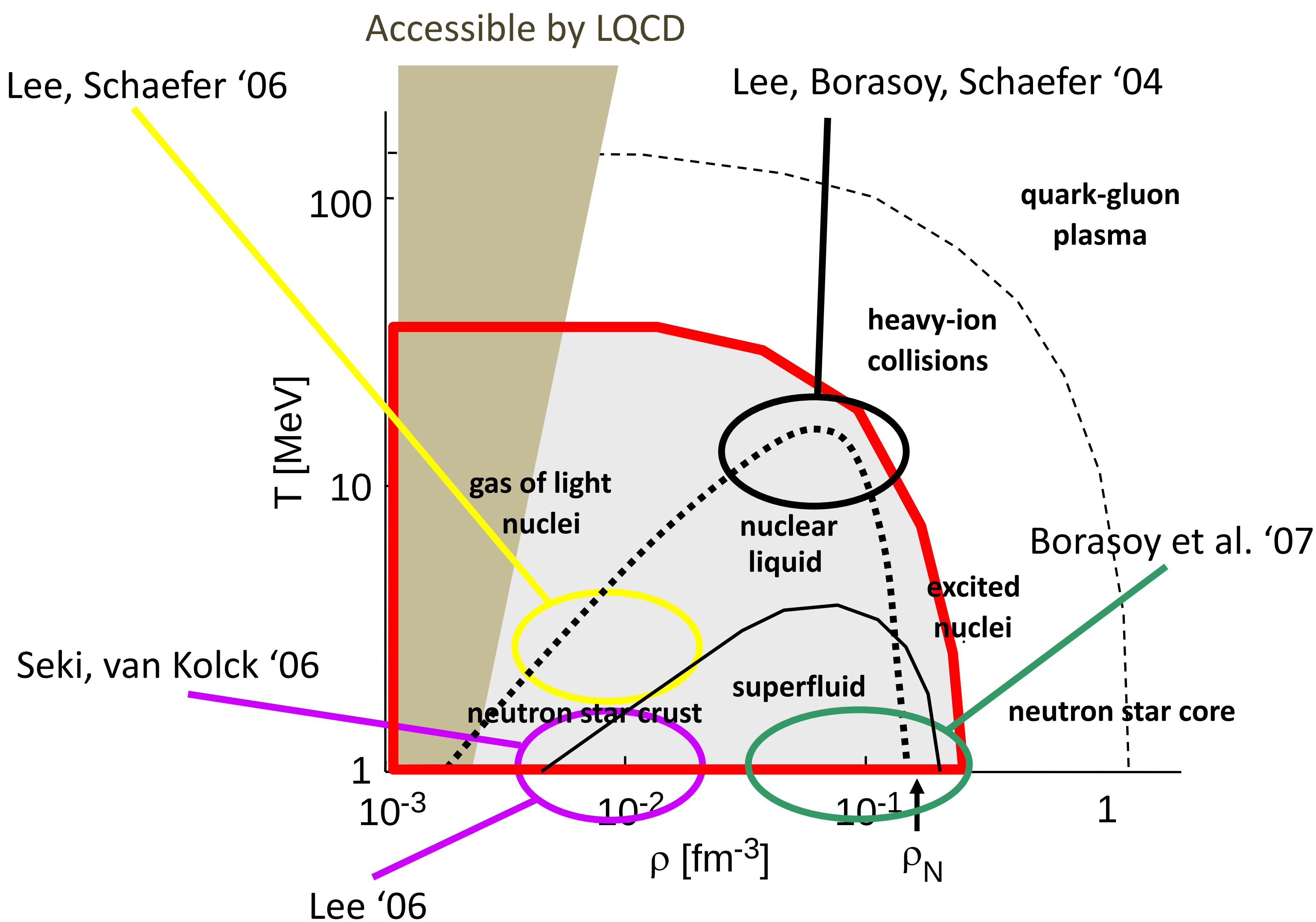
Absence of sign oscillation for nonzero chemical potential in Hubbard model:
Chen, Kaplan, Phys. Rev. Lett. 92: 257002 (2004)

Non-linear realization of chiral symmetry with static nucleons on the lattice:
Chandrasekharan, Pepe, Steffen, Wiese, Nucl. Phys. Proc. Suppl. 129: 507 (2004)

ChPT with lattice regularization: Shushpanov, Smilga, Phys. Rev. D59: 054013 (1999),
Lewis, Ouimet, Phys. Rev. D64: 034005 (2001), Borasoy, Lewis, Ouimet, hep-lat/0310054

Condense matter considerations (Talk by Uwe-Jens Wiese)

Phase region accessible by EFT

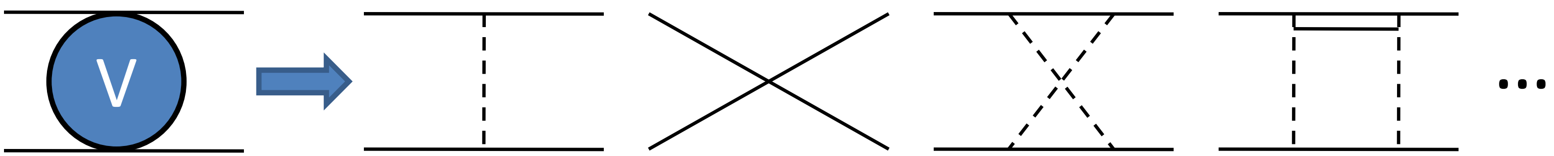


Weinberg's scheme for NN

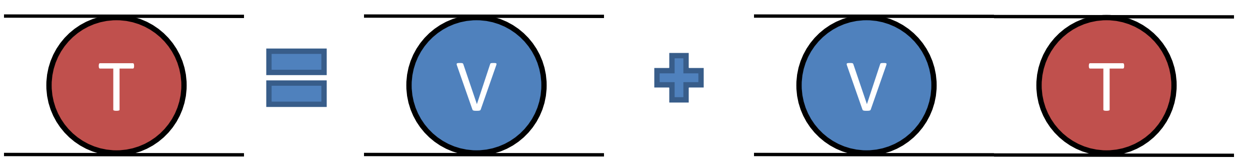
Weinberg, Nucl. Phys. B 363: 3 (1991)

- No perturbative description for bound states

- Construct effective potential perturbatively



- Solve Lippmann-Schwinger equation nonperturbatively

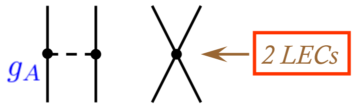


Nucleon-nucleon force up to N^{3}LO

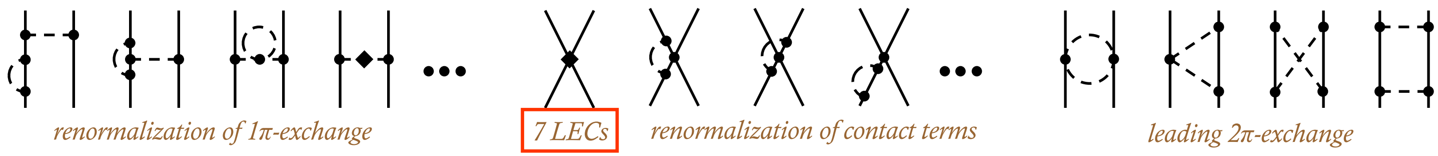
Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; …

Chiral expansion for the 2N force: $V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$

- **LO:**

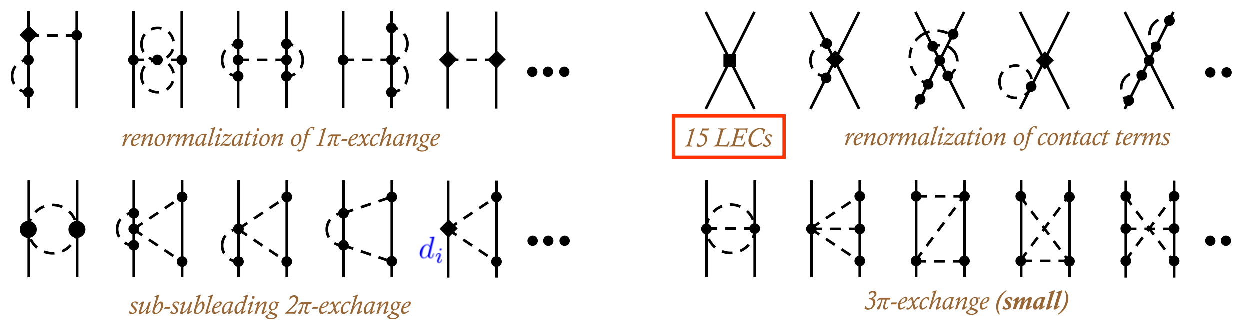


- **NLO:**



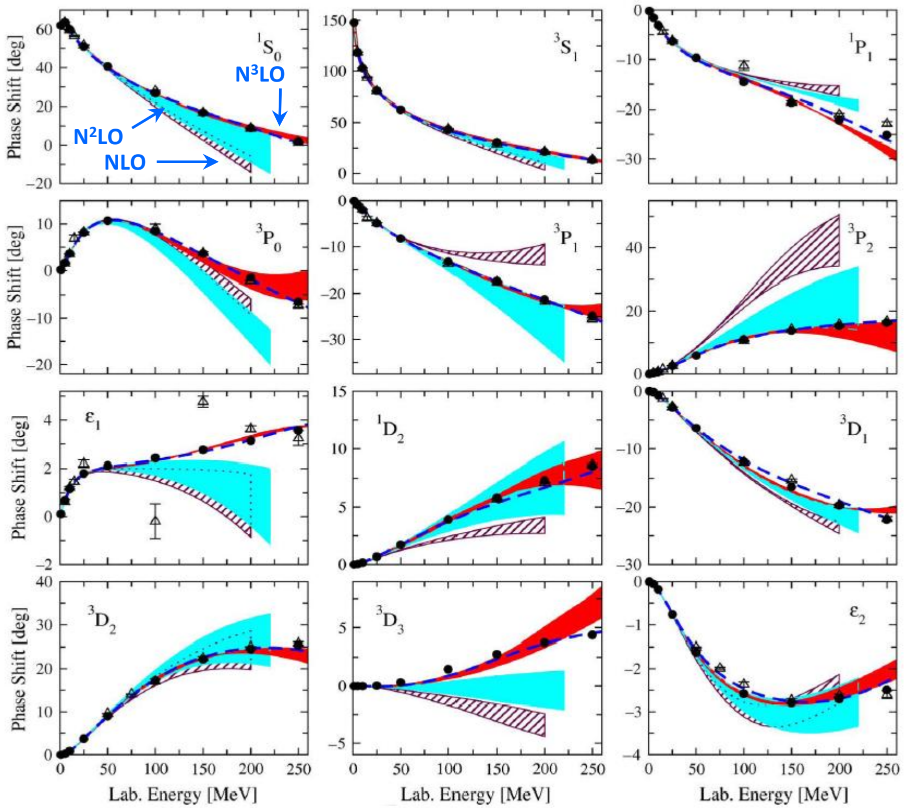
- **N^{2}LO:**

- **N^{3}LO:**

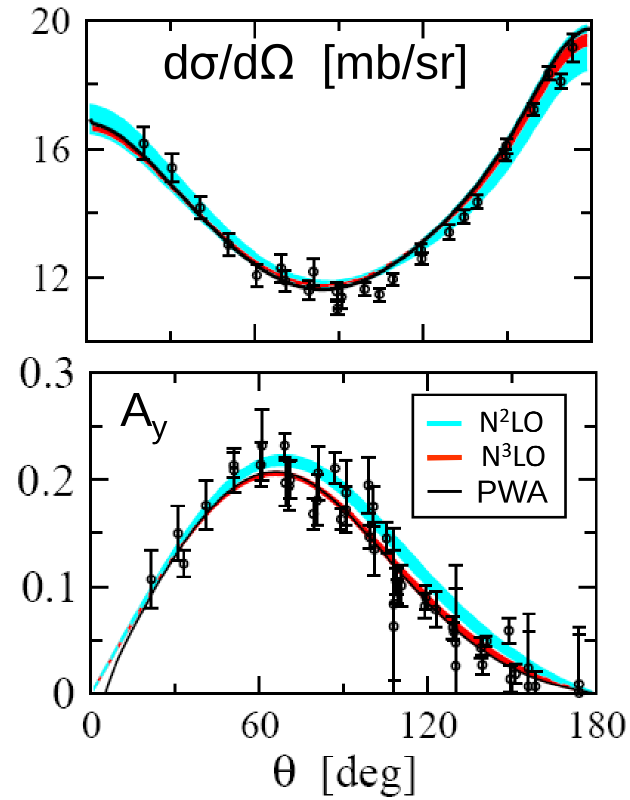


+ 1/m and isospin-breaking corrections…

Neutron-proton phase shifts up to N^{3}LO



np scattering at 50 MeV



Deuteron binding energy & asymptotic normalizations A_s and η_d

	NLO	N^{2}LO	N^{3}LO	Exp
E_d [MeV]	$-2.171\ldots-2.186$	$-2.189\ldots-2.202$	$-2.216\ldots-2.223$	$-2.224575(9)$
A_S [fm$^{-1/2}$]	$0.868\ldots0.873$	$0.874\ldots0.879$	$0.882\ldots0.883$	$0.8846(9)$
η_d	$0.0256\ldots0.0257$	$0.0255\ldots0.0256$	$0.0254\ldots0.0255$	$0.0256(4)$

Entem & Machleidt '03; Epelbaum, Glöckle & Meißner '05

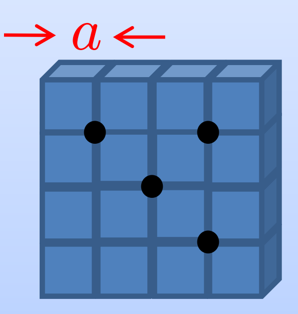
Nuclear lattice simulations

Borasoy, H.K., Lee, Meißner, Nucl.Phys. A768: 179 (2006)

- Low-energy constants fixed from experimental data with $A \leq 3$
- All nuclear spectra for $A \geq 4$ pure prediction
- Explicit methods for $A \geq 4$ too complicated ⇒ use EFT on the Lattice

- Nucleons are represented as point-like Grassmann fields
- Point-like instantaneous pions to reproduce effective potential
- Typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300\,\mathrm{MeV}(a \simeq 2\,\mathrm{fm})$$
$$L \simeq 20\,\mathrm{fm}$$



- Strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry
- Method: hybrid Monte Carlo & transfer matrix (similar to LQCD)

Transfer matrix method

- Correlator function for A Nucleons $Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$

Slater Determinants for A free Nucleons

- Ground state energy by time derivative of the correlator

$$E(t) = -\frac{d}{dt} \ln Z_A(t)$$

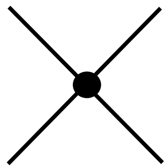
At large time only ground states survive $\Rightarrow$ $E_A^0 = \lim_{t\to\infty} E_A(t)$

- Expectation value of normal ordered operator $\mathcal{O}$:

$$Z_A^{\mathcal{O}}(t) = \langle \Psi_A | \exp(-tH/2) \mathcal{O} \exp(-tH/2) | \Psi_A \rangle$$

$$\lim_{t\to\infty} \frac{Z_A^{\mathcal{O}}(t)}{Z_A(t)} = \langle \Psi_A^0 | \mathcal{O} | \Psi_A^0 \rangle$$

Monte Carlo with auxiliary fields

- Contact interactions  can be represented by auxiliary fields

$$\exp(\rho^2/2) \sim \int_{-\infty}^{\infty} ds \exp(-s^2/2 - s\,\rho)$$

Hubbard-Stratonovich field

- Correlation function as path-integral over pions and auxiliary fields

$$Z_A(t) \propto \int_{-\infty}^{\infty} \mathcal{D}s \prod_{I=1,2,3} \mathcal{D}s_I \mathcal{D}\pi_I \exp(-S_{\pi\pi} - S_{ss}) \det \mathcal{M}(\pi_I, s, s_I)$$

Slater-determinant of single nucleon matrix elements

- Single-nucleon matrix elements

L_t-th temporal lattice step

$$\mathcal{M}_{i,j}(\pi_I, s, s_I) = \langle \psi_{i,X} | M_X^{(L_t-1)} \cdots M_X^{(0)} | \psi_{j,X} \rangle$$

Free nucleons and pions

- Positive definite free action: $\alpha_t = a_t/a$

$$S_{ss}(s, s_I) = \frac{1}{2}\sum_{\vec{n}} s(\vec{n})^2 + \frac{1}{2}\sum_{I=1,2,3}\sum_{\vec{n}} s_I(\vec{n})^2$$

Auxiliary field contributions

$$S_{\pi\pi}(\pi_I) = \frac{\alpha_t}{2}\sum_{I=1,2,3}\sum_{\vec{n}} \pi_I(\vec{n})(-\Delta + M_\pi^2)\pi_I(\vec{n})$$

Free instantaneous pions

- $O(a^4)$-improved free nucleon lattice Hamiltonian $f_{0,1,2,3} = \frac{49}{2}, -\frac{3}{4}, \frac{3}{40}, -\frac{1}{180}$

$$H_{\text{free}} = \frac{1}{m}\sum_{k=0}^{3}\sum_{\vec{n}_s, l_s, i, j} f_k \left[a_{i,j}^\dagger(\vec{n}_s)\left[a_{i,j}(\vec{n}_s + k\,\hat{l}_s) + a_{i,j}(\vec{n}_s - k\,\hat{l}_s)\right]\right]$$

- Nucleon density operators with different spin-isospin polarizations

$$\rho^{a^\dagger,a}(\vec{n}_s) = \sum_{i,j} a_{i,j}^\dagger(\vec{n}_s)a_{i,j}(\vec{n}_s) \qquad \rho_I^{a^\dagger,a}(\vec{n}_s) = \sum_{i,j,j'} a_{i,j}^\dagger(\vec{n}_s)[\tau_I]_{j,j'}a_{i,j'}(\vec{n}_s)$$

$$\rho_{S,I}^{a^\dagger,a}(\vec{n}_s) = \sum_{i,i',j,j'} a_{i,j}^\dagger(\vec{n}_s)[\sigma_S]_{i,i'}[\tau_I]_{j,j'}a_{i',j'}(\vec{n}_s)$$

Leading order interactions

- Transfer matrix from n_t - th step in temporal direction $C < 0, C_I > 0$

Small sign oscillation from pion-nucleon vertex

$$M^{(n_t)} = : \exp \left[-H_{\text{free}}\alpha_t - \frac{g_A \alpha_t}{2F_\pi} \sum_{S,I} \nabla_S \pi_I(\vec{n}_s, n_t) \rho^{a^\dagger,a}_{S,I}(\vec{n}_s) \right.$$

$$\left. + \sqrt{-C\alpha_t} \sum_{\vec{n}_s} [s(\vec{n}_s, n_t) \rho^{a^\dagger,a}(\vec{n}_s) + i\sqrt{C_I \alpha_t} \sum_I \sum_{\vec{n}_s} s_I(\vec{n}_s, n_t) \rho^{a^\dagger,a}_I(\vec{n}_s) \right] :$$

No sign oscillation for contact interactions if the number of protons and neutrons are equal

- Real determinant in the pion-less case $\tau_2 \mathcal{M} \tau_2 = \mathcal{M}^* \Longrightarrow \det \mathcal{M}^* = \det \mathcal{M}$

  Lee, Phys. Rev. C70: 064002 (2004)

Antisymmetry of $\tau_2 \Longrightarrow$ real eigenvalues of $\mathcal{M}$ are doubly degenerate

$$\Longleftarrow \det \mathcal{M} \geq 0$$

Approximate SU(4) symmetry

- Wigner spin-isospin SU(4) symmetry transformation:

Wigner, Phys. Rev. 51: 106 (1937)

$$\delta N = \alpha_{\mu\nu}\sigma^{\mu}\tau^{\nu}N \quad \text{with} \quad \sigma^{\mu} = (1,\vec{\sigma}), \tau^{\mu} = (1,\vec{\tau})$$

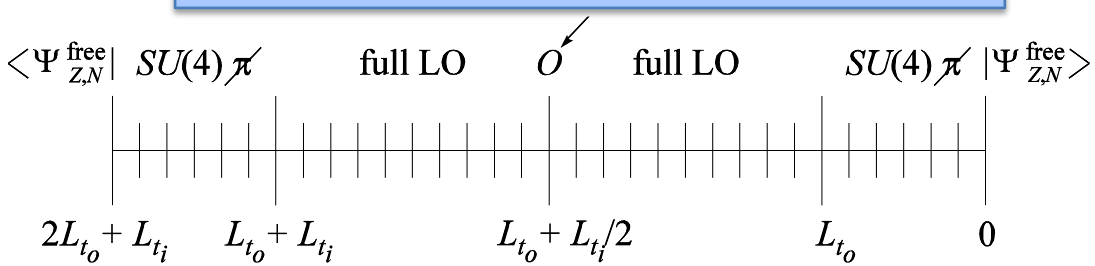
SU(4) invariance in the limit of infinite $a(^1S_0)$ and $a(^3S_1)$ scattering length
SU(4)-breaking terms $\sim 1/a(^3S_1) - 1/a(^1S_0), q/\Lambda_{\chi}$

Mehen, Stewart, Wise, Phys. Rev. Lett. 83: 931 (1999)

- Large NN scattering length ⟹ approximate SU(4) symmetry

$$M^{(n_t)}_{\mathrm{SU(4)}} =: \exp\left[-H_{\mathrm{free}}\alpha_t + \sqrt{-C\alpha_t}\sum_{\vec{n}_s}[s(\vec{n}_s,n_t)\rho^{a^{\dagger},a}(\vec{n}_s)]\right]:$$

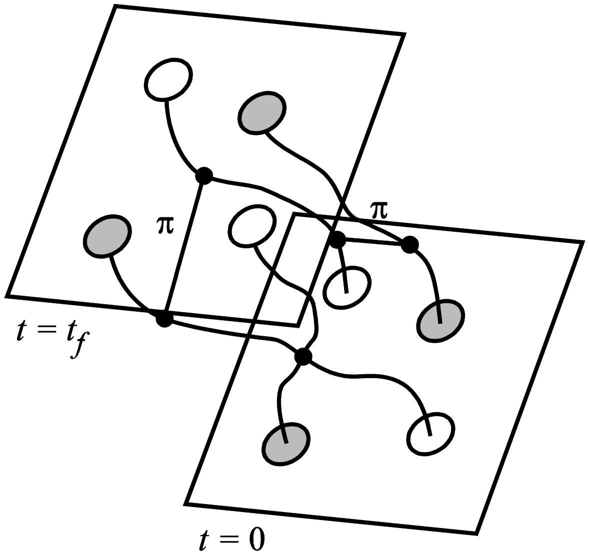
Operator insertion for expectation value



- Pionless SU(4)-symmetric simulations are cheaper
- Decrease computational effort by using SU(4)-filter

Hybrid Monte Carlo

- Introduce conjugate fields p_{π_I}, p_s, p_{s_I}



$$H_{\mathrm{HMC}} = \frac{1}{2} \sum_{I,\vec{n}} (p^2_{\pi_I}(\vec{n}\,) + p^2_s(\vec{n}\,) + p^2_{s_I}(\vec{n}\,)) + V(\pi_I, s, s_I)$$

$$V(\pi_I, s, s_I) = S_{\pi\pi} + S_{ss} - \log\{|\det \mathcal{M}|\}$$

Generate new configurations for $p_{\pi_I}, p_s, p_{s_I}, \pi_I, s, s_I$ by molecular dynamics trajectories

Apply Metropolis accept or reject step for the new configuration according to probability $\exp(-H_{\mathrm{HMC}})$

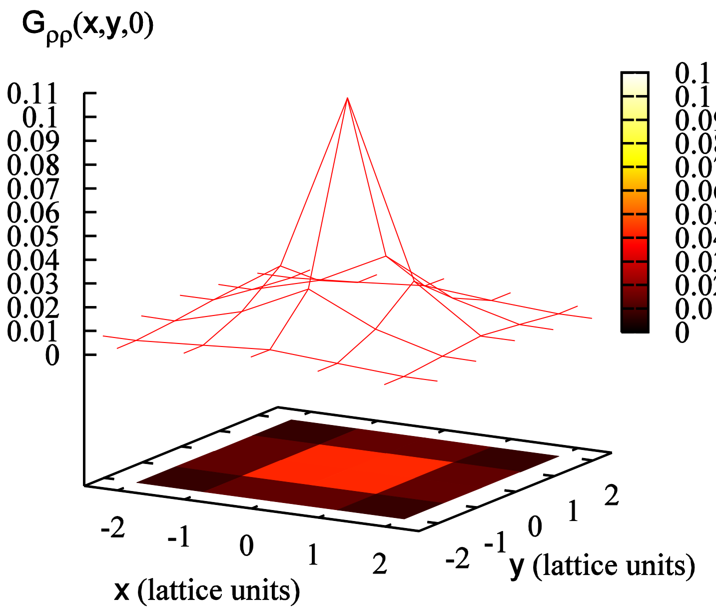
Repeat the steps many times

Leading-order considerations

Borasoy, Epelbaum, H.K., Lee, Meißner, Eur.Phys.J.A31:105 (2007)

- Promising results for $A = 2, 3, 4$

	Simulation	Experiment
r_d[fm]	$1.989(1)$	$1.9671(6)$
Q_d[fm^2]	$0.278(1)$	$0.2859(3)$
E_{3H}[MeV]	$-8.9(2)$	-8.482
r_{3H}[fm]	$2.27(7)$	$1.755(9)$
E_{4He}[MeV]	$-21.5(9)$	-28.296
r_{4He}[fm]	$1.50(14)$	$1.673(1)$

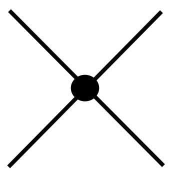


Nucleon density correlation in ^{3}H in the x-y plane

- CPU time appears to scale linear with A ($A \leq 10$)

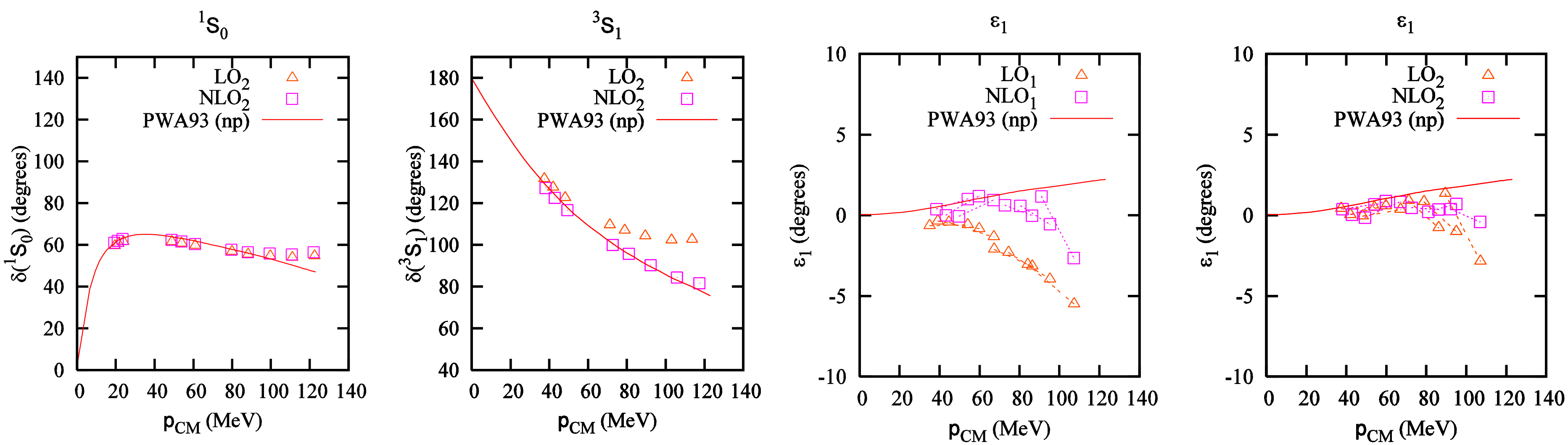
Chiral EFT at NLO: Scattering results

Borasoy, Epelbaum, H.K., Lee, Meißner, Eur. Phys. J. A35: 343 (2008)



9 NLO LECs fitted to the various S(D) and P(F)-wave phase-shifts
+ Quadrupole moment of the deuteron

- NLO results with different actions ⟹ estimate errors



- Fairly accurate description for momenta $\leq M_\pi$
- Deviations appear consistent with higher-order effects
- Small systematic errors at NLO

Dilute neutron matter

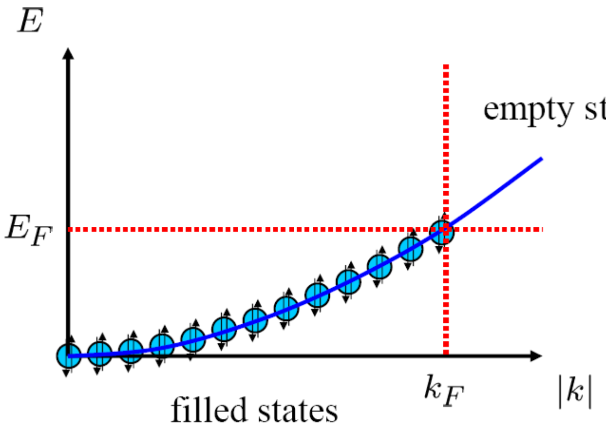
- Neutron-Neutron scattering matrix: $f_0(k) = \dfrac{1}{k\cot(\delta_0) - i\,k}$

Effective range expansion → $k\cot(\delta_0) = -a_{\rm scatt}^{-1} + \frac{1}{2}k^2 r_{\rm eff} + \ldots$

- Unitary limit: $a_{\rm scatt} \to \infty,\ r_{\rm eff} \to 0$ → $f_0(k) \to \dfrac{i}{k}$

Scattering amplitude is as strong as possible

- Free fermion ground states



Energy per particle free fermion case:

$$E_0^{\rm free}/A = \frac{3}{5}E_{\rm F},\ E_F = \frac{k_F^2}{2m}$$

In the unitary limit:

$$E_0/A = \xi E_0^{\rm free}/A = \xi\frac{3}{5}E_{\rm F}$$

ξ :dimensionless measurable number

Dilute neutron matter

- Neutron matter at $k_F \sim 80\,\mathrm{MeV}$ is close to the unitary limit

Scattering length at lower density → Important corrections ← Effective range and other effects at higher density

- Experiment: ultracold $^6\mathrm{Li}$ and $^{40}\mathrm{K}$ using a magnetic-field Feshbach resonance

$\xi = 0.46^{+12}_{-05}$	Stewart et al., Phys. Rev. Lett. 97, 220406 (2006)
$\xi = 0.32^{+10}_{-13}$	Kinast et al., Science 307, 1296 (2005)
$\xi = 0.51(4)$	Bartenstein et al., Phys. Rev. Lett. 92, 120401 (2004)

Larger values in earlier experiments → Further work to be needed

- Numerous calculations of ξ :

$$\xi \sim 0.2 - 0.6$$

For review see: Furnstahl, Rupak, Schäfer, arXiv: 0801.0729 [nucl-th]

Dilute neutron matter

Borasoy, Epelbaum, H.K., Lee, Meißner, Eur. Phys. J. A35: 357 (2008)

Put $N = 8, 12$ neutrons in a box with

$$L = 10, 12, 14\,\text{fm}, \quad k_F = \frac{1}{L}(3\pi^2 N)^{1/3}$$

2 – 8 % of normal nuclear matter density

Energy per particle at NLO

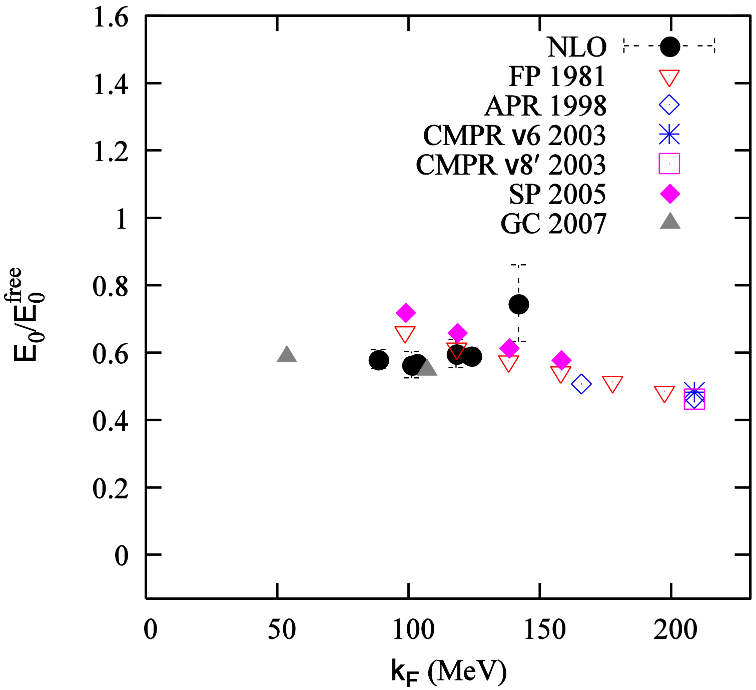
NLO lattice data in comparíson to earlier results

- Slope ?
- Close to unitary limit

$$\frac{E_0}{E_0^{\text{free}}} = \xi - \frac{\xi_1}{k_F a_{\text{scatt}}} + 0.15 k_F r_{\text{eff}}$$

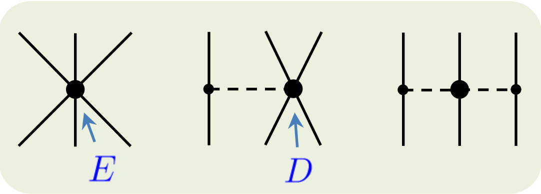
- Results from the fit

$$\xi \simeq 0.25$$
$$\xi_1 \simeq 1.0$$



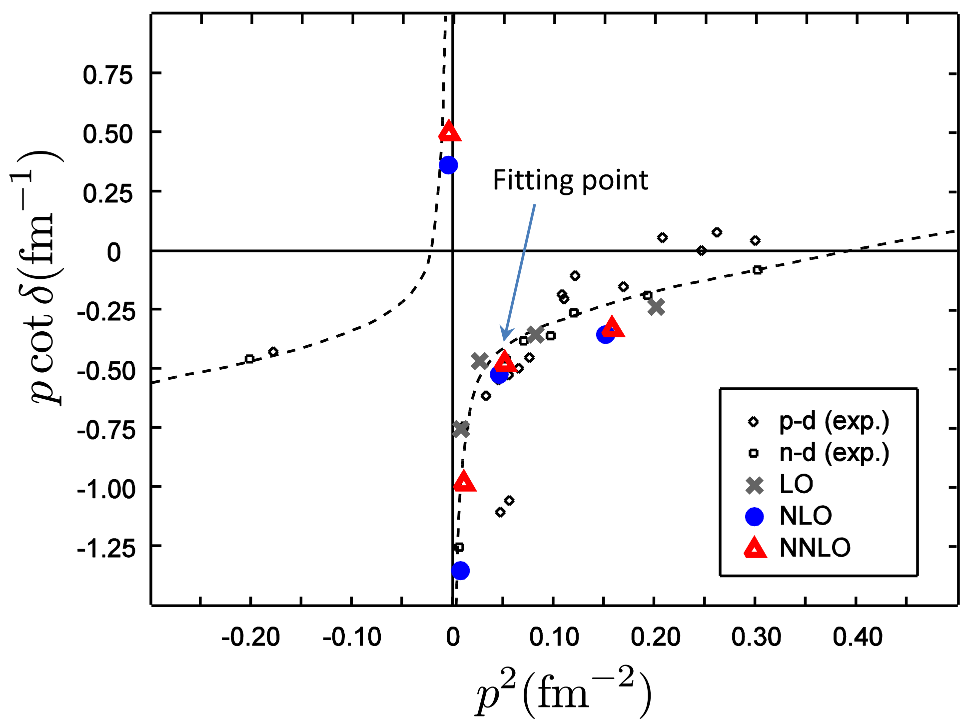
NNLO three-body forces

- Three-body forces at NNLO →

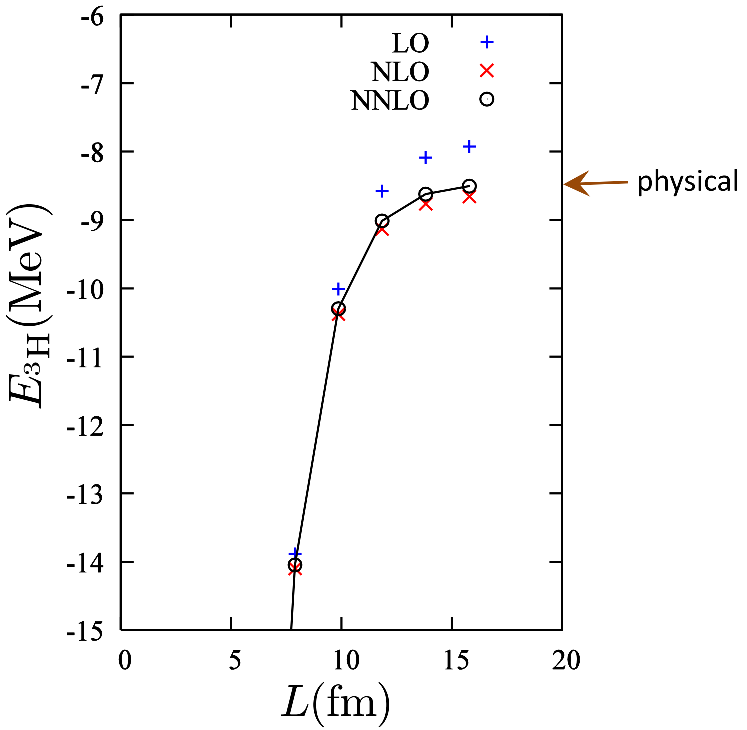


- Fit D and E to neutron-deuteron scattering data + $^3\mathrm{H}$ binding energy

Spin-1/2 doublet channel

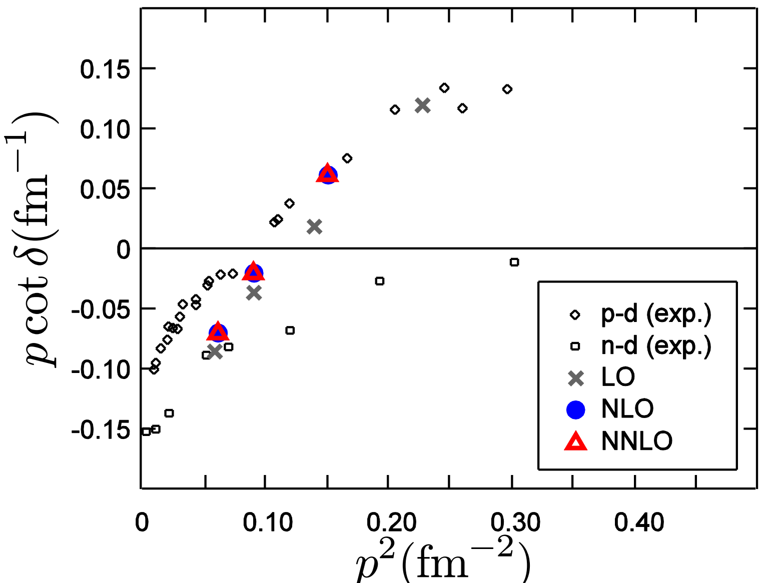


$^3\mathrm{H}$ binding energy



NNLO first applications

- Spin-3/2 quartet channel in neutron-deuteron scattering (Lanczos method)



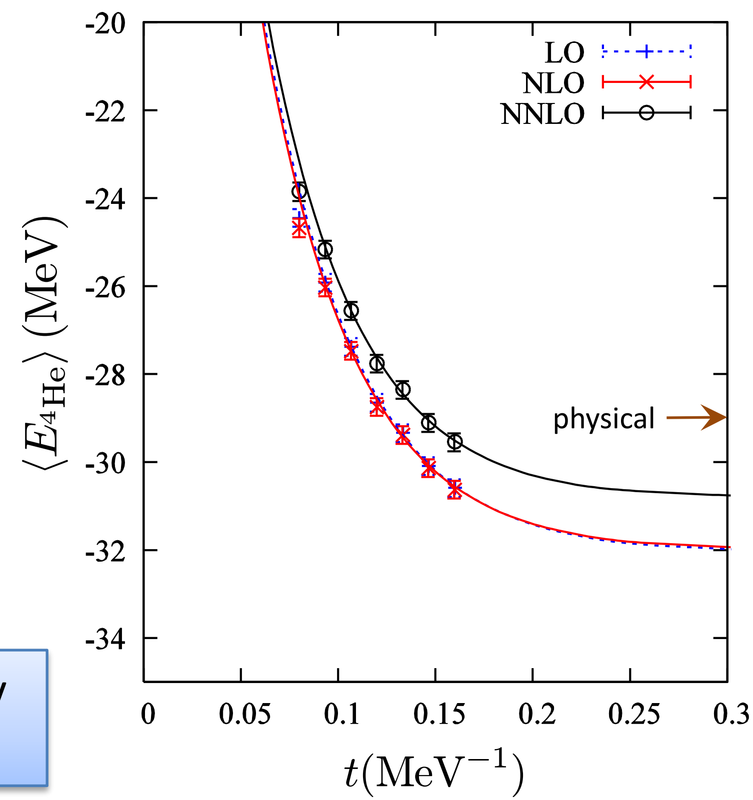
- Predictions for quartet phase-shifts are between p-d and n-d exp. data
- Small effects from NNLO corrections

- $^4\mathrm{He}$ binding energy versus Euclidean time

Monte-Carlo simulation with $L = 16\,\mathrm{fm}$

$$\langle E_{^4\mathrm{He}} \rangle = \frac{\langle \Psi_4 | \exp(-tH/2) H \exp(-tH/2) | \Psi_4 \rangle}{\langle \Psi_4 | \exp(-tH) | \Psi_4 \rangle}$$

5% overestimation of physical binding energy with subtracted Coulomb-effects



Conclusions

- Lattice EFT is a promising tool for a quantitative description of light nuclei
- At LO promising results for light nuclei up to $^4\mathrm{He}$
- At NLO 9 LECs are fitted to the phase-shifts by spherical wall method
- NLO Lattice EFT simulation of dilute neutron matter close to unitary limit with N=8,12 neutrons in a box
- NNLO three-body forces fitted to neutron-deuteron scattering data and triton binding energy
- Satisfactory NNLO description of quartet channel in neutron-deuteron scattering and $^4\mathrm{He}$ binding energy

Outlook

- NNLO simulations for light nuclei
- NNLO simulations of neutron matter with larger number of neutrons in a box

Scattering from finite volume

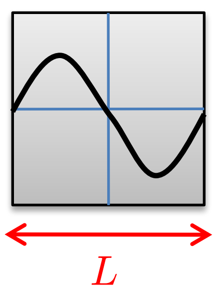
- Rotation group $SO(3) \longrightarrow SO(3,Z)$

Representation	J_z	Example
A_1	$0 \bmod 4$	$Y_{0,0}$
T_1	$0,1,3 \bmod 4$	$\{Y_{1,0}, Y_{1,1}, Y_{1,-1}\}$
E	$0,2 \bmod 4$	$\left\{Y_{2,0}, \frac{Y_{2,-2}+Y_{2,2}}{\sqrt{2}}\right\}$
T_2	$1,2,3 \bmod 4$	$\left\{Y_{2,1}, \frac{Y_{2,-2}-Y_{2,2}}{\sqrt{2}}, Y_{2,-1}\right\}$
A_2	$2 \bmod 4$	$\frac{Y_{3,2}-Y_{3,-2}}{\sqrt{2}}$

Every irreducible repr. includes definite $J \bmod 4$ quantum numbers

$$Z_{0,0}(s,q^2) = \sqrt{1/4\pi} \sum_{n\in Z^3} \frac{1}{(n^2-q^2)^s}$$

- Scattering from finite volume



Large L $\longrightarrow$

$$\exp(2\,i\,\delta_0) = \frac{Z_{0,0}(1;q^2) + i\,\pi^{3/2} q}{Z_{0,0}(1;q^2) - i\,\pi^{3/2} q}$$

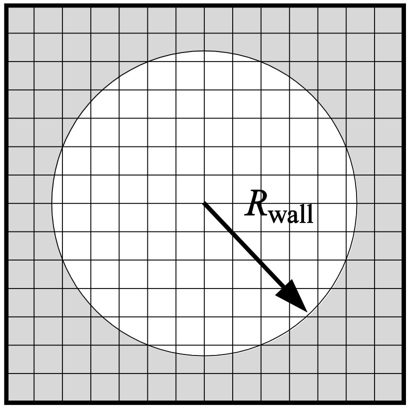
Lüscher formula for phase-shifts

- Note: No extension to mixing angles available

Spherical wall method

Borasoy, Epelbaum, H.K.,Lee, Meißner, Eur.Phys.J.A34:185 (2007)

- Spherical wall imposed in the center-of-mass frame



$$\Psi(\vec{r}) = [\cos\delta_L j_L(kr) - \sin\delta_L y_L(kr)] Y_{L,m}(\theta,\phi)$$

$$\Psi(\vec{R}_{\mathrm{Wall}}) = 0$$

$$\tan\delta_L = \frac{j_L(kR_{\mathrm{Wall}})}{y_L(kR_{\mathrm{Wall}})}$$

Similar for Spin-triplet case

- Spherical wall removes copies of interactions due to periodic boundaries
- Energy spectrum by solving Schrödinger Eq. on the lattice (Lanczos method)
- Illustration for a toy-model: $C = -2\,\mathrm{MeV}, R_0 = 2\times 10^{-2}\,\mathrm{MeV}^{-1}$

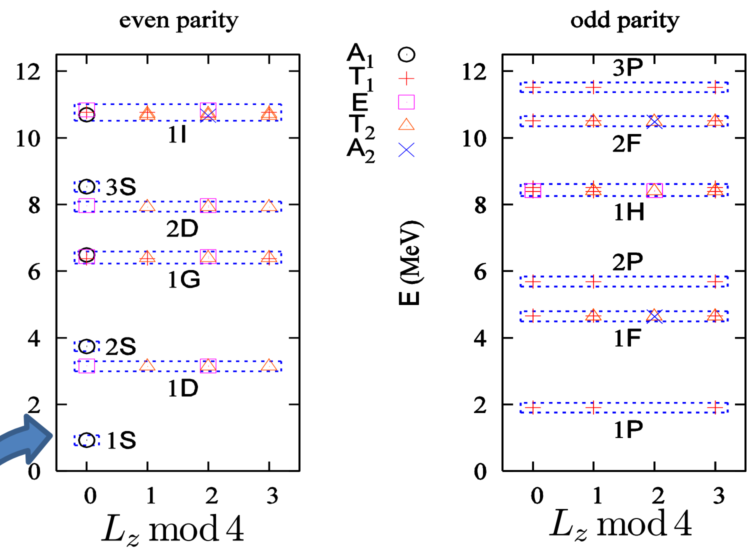
$$V(\vec{r}) = C\left\{1 + \frac{r^2}{R_0^2}\left[3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2\right]\right\}\exp\left(-\frac{1}{2}\frac{r^2}{R_0^2}\right)$$

Very shallow bound state in the ${}^3S(D)_1$ channel with energy $-0.155\,\mathrm{MeV}$

Illustration for the toy model

Borasoy, Epelbaum, H.K.,Lee, Meißner, Eur.Phys.J.A34:185 (2007)

Free particle spectrum for $R_{\mathrm{Wall}} = 10 + \epsilon$

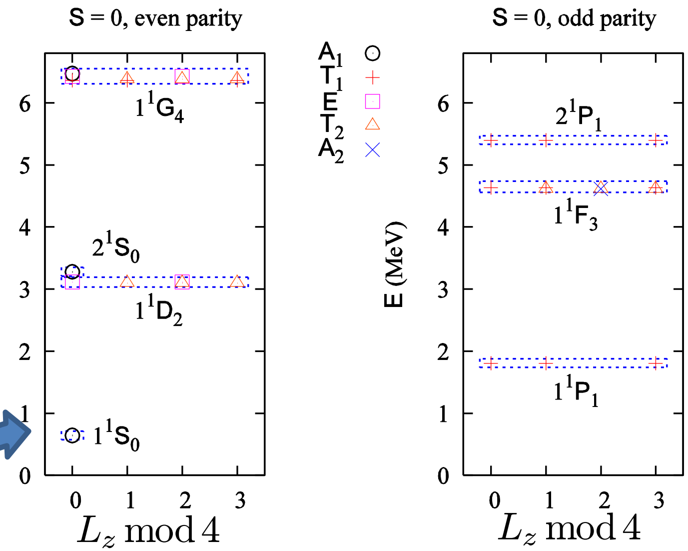


1^1S_0 energy $= 0.9280\,\mathrm{MeV}$

$k_{\mathrm{free}} = 29.52\,\mathrm{MeV},\ j_0(k_{\mathrm{free}} R_{\mathrm{Wall}}) = 0$

$$R_{\mathrm{Wall}} = \frac{\pi}{k_{\mathrm{free}}} = 0.1064\,\mathrm{MeV}^{-1}$$

Interacting spectrum for $S = 0$



1^1S_0 energy $= 0.6445\,\mathrm{MeV}$

$k = 24.60\,\mathrm{MeV}$

$$\delta({}^1S_0) = \tan^{-1}\left[\frac{j_0(kR_{\mathrm{Wall}})}{y_0(kR_{\mathrm{Wall}})}\right] = 30.0^o$$

Systematic error estimation

- Check for sensitivity on the cut off scale Λ
  - Deviations for cut offs Λ_1 & Λ_2 should be $\leq$ higher order corr.

    Fixed cut off in present work: check of cut off sensitivity in future

- Error estimation with fixed cut off
  - At LO different actions

$$V_{\mathrm{LO}_1} = V_1^{(0)} + V_{\mathrm{OPE}} + V_1^{Q^2/\Lambda^2}$$
$$V_{\mathrm{LO}_2} = V_2^{(0)} + V_{\mathrm{OPE}} + V_2^{Q^2/\Lambda^2}$$

Quasi-local operators with at least two powers of momenta

Physical observables should agree up to NLO corrections

  - At NLO different actions

$$V_{\mathrm{NLO}_1} = V_{\mathrm{LO}_1} + \Delta V_1^0 + V_1^{(2)} + V_1^{Q^4/\Lambda^4}$$
$$V_{\mathrm{NLO}_2} = V_{\mathrm{LO}_2} + \Delta V_2^0 + V_2^{(2)} + V_2^{Q^4/\Lambda^4}$$

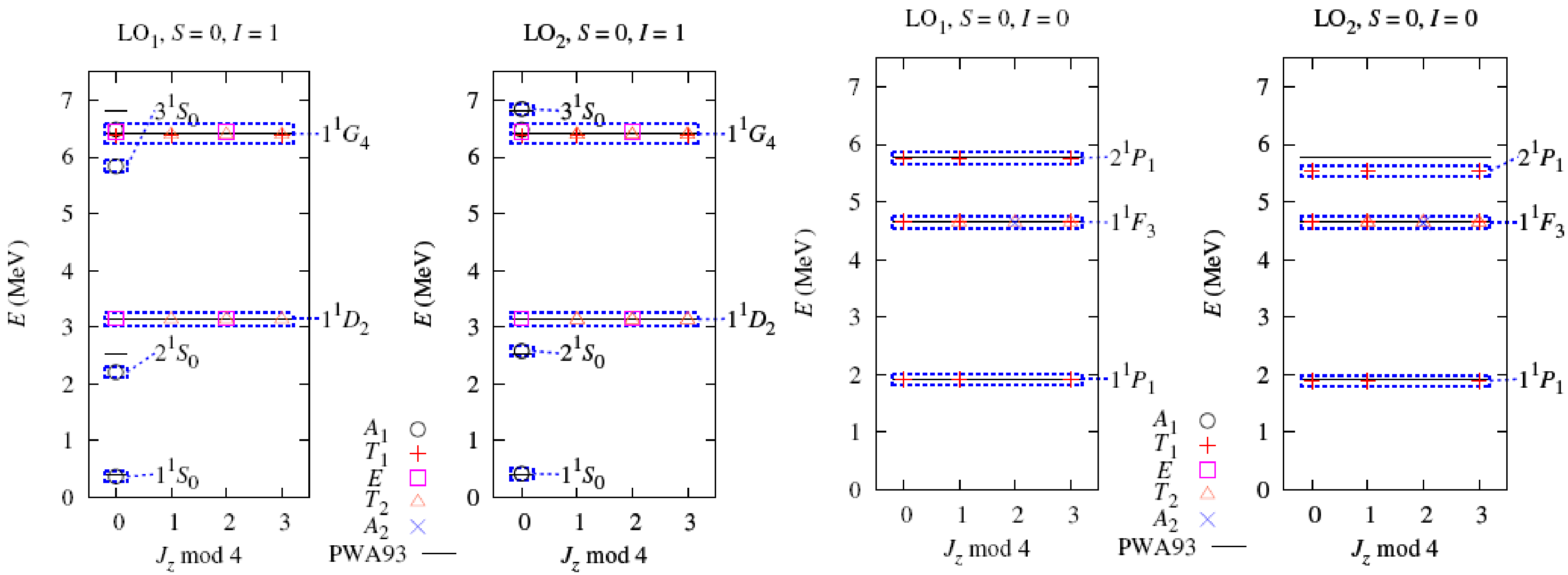
Expected agreement up to NNLO corr.

LO energy levels

Two different LO actions in momentum space

$$\mathcal{L}_C^{\mathrm{LO}_{1,2}} \sim \sum_{\vec{q}} f_{1,2}(\vec{q}) \left[C N^\dagger(\vec{q}) N(\vec{q}) N^\dagger(-\vec{q}) N(-\vec{q}) + C_I N^\dagger(\vec{q}) \vec{\tau} N(\vec{q}) \cdot N^\dagger(-\vec{q}) \vec{\tau} N(-\vec{q}) \right]$$

- $f_1(\vec{q}) = 1$ ➡ original repr.
- $f_2(\vec{q}) \sim \exp(-b \sum_{l=1,2,3} (1 - \cos q_l))$ ➡ Gaussian smearing

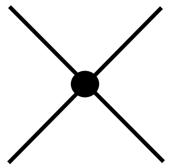


- LO_1 : 10-15% too low for ${}^1S_0$
- LO_2: correct to within few % for ${}^1S_0$
- Higher partial waves within 1%

- LO_1 : correct within 1% for ${}^1P_1$
- LO_2 : 5% too low for ${}^1P_1$
- Deviations consistent with NLO corr.

Chiral EFT at NLO: Mixing angles

Borasoy, Epelbaum, H.K., Lee, Meißner, arXiv:0712.2990 [nucl-th]



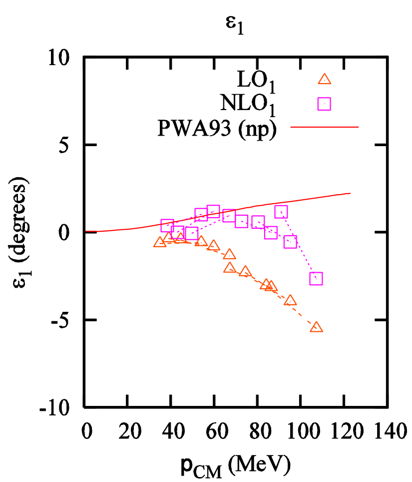
9 NLO LECs fitted to the various S(D) and P(F)-wave phase-shifts
+ Quadrupole moment of the deuteron

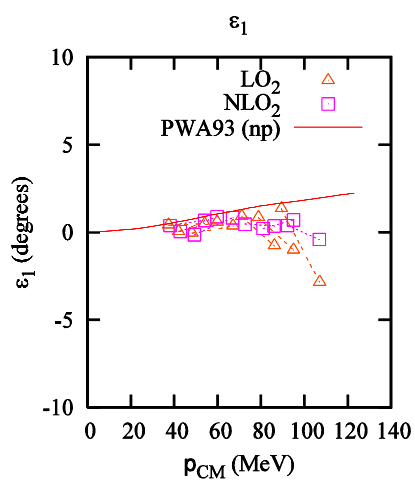
- NLO results with different actions ⟹ estimate errors

Possible LO contact interactions in momentum space

$$\mathcal{L}_C^{\mathrm{LO}_{1,2}} \sim \sum_{\vec{q}} f_{1,2}(\vec{q}) \left[C N^\dagger(\vec{q}) N(\vec{q}) N^\dagger(-\vec{q}) N(-\vec{q}) + C_I N^\dagger(\vec{q}) \vec{\tau} N(\vec{q}) \cdot N^\dagger(-\vec{q}) \vec{\tau} N(-\vec{q}) \right]$$

- $f_1(\vec{q}) = 1$ ⟹ original repr.
- $f_2(\vec{q}) \sim \exp(-b \sum_{l=1,2,3} (1 - \cos q_l))$ ⟹ Gaussian smearing

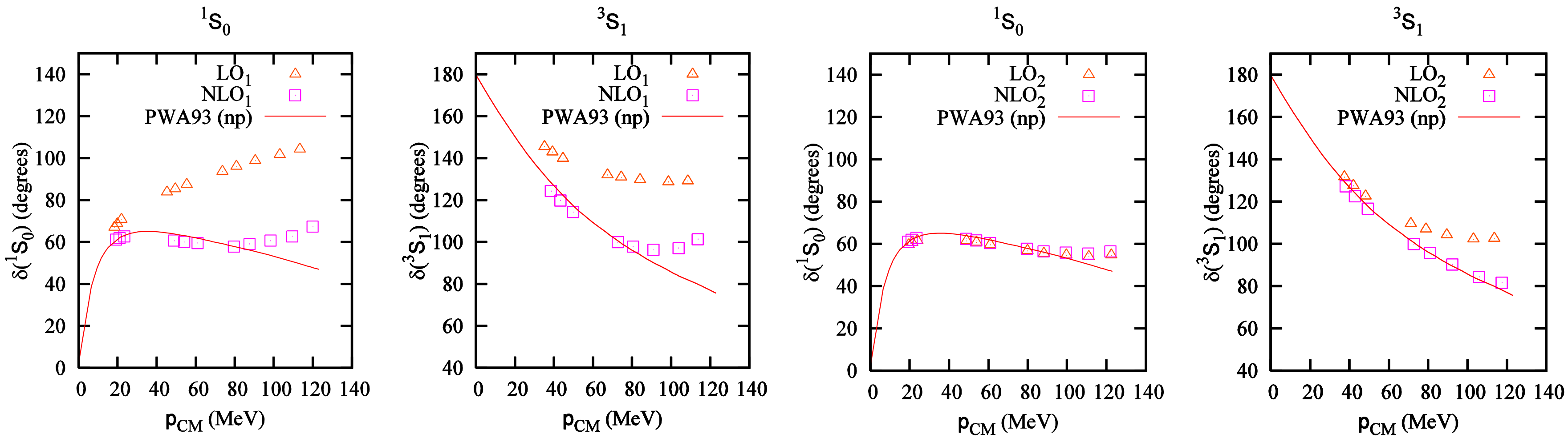




- Fairly accurate description for momenta $\leq M_\pi$
- Mixing angle changes sign by LO ⟹ NLO
- Deviations appear consistent with higher-order effects

S-wave phase shifts

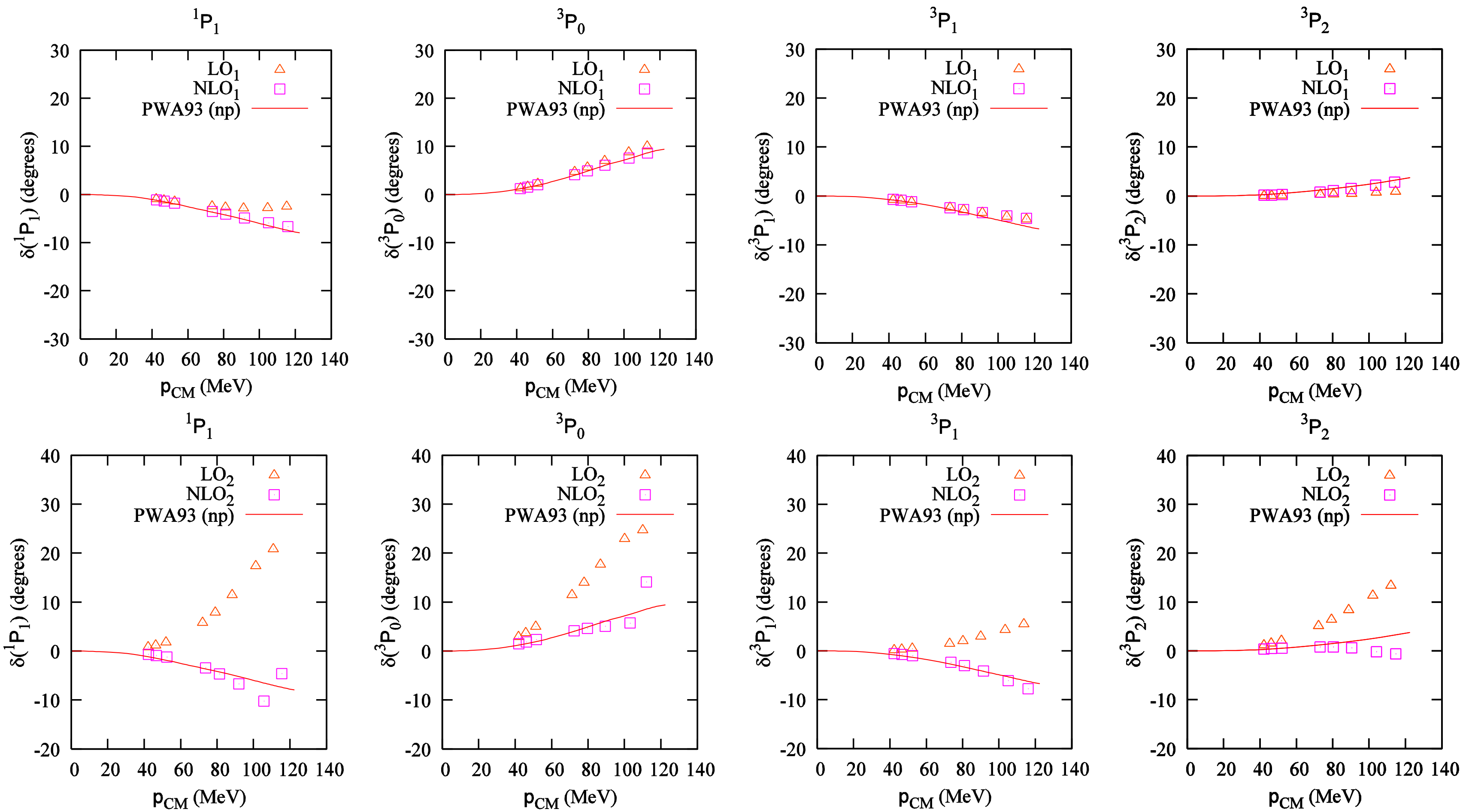
- S-wave phase shifts with different actions



- Accurate NLO-description in both cases for momenta $\leq 80\,\mathrm{MeV}$
- Better convergence with smeared action

P-wave phase shifts

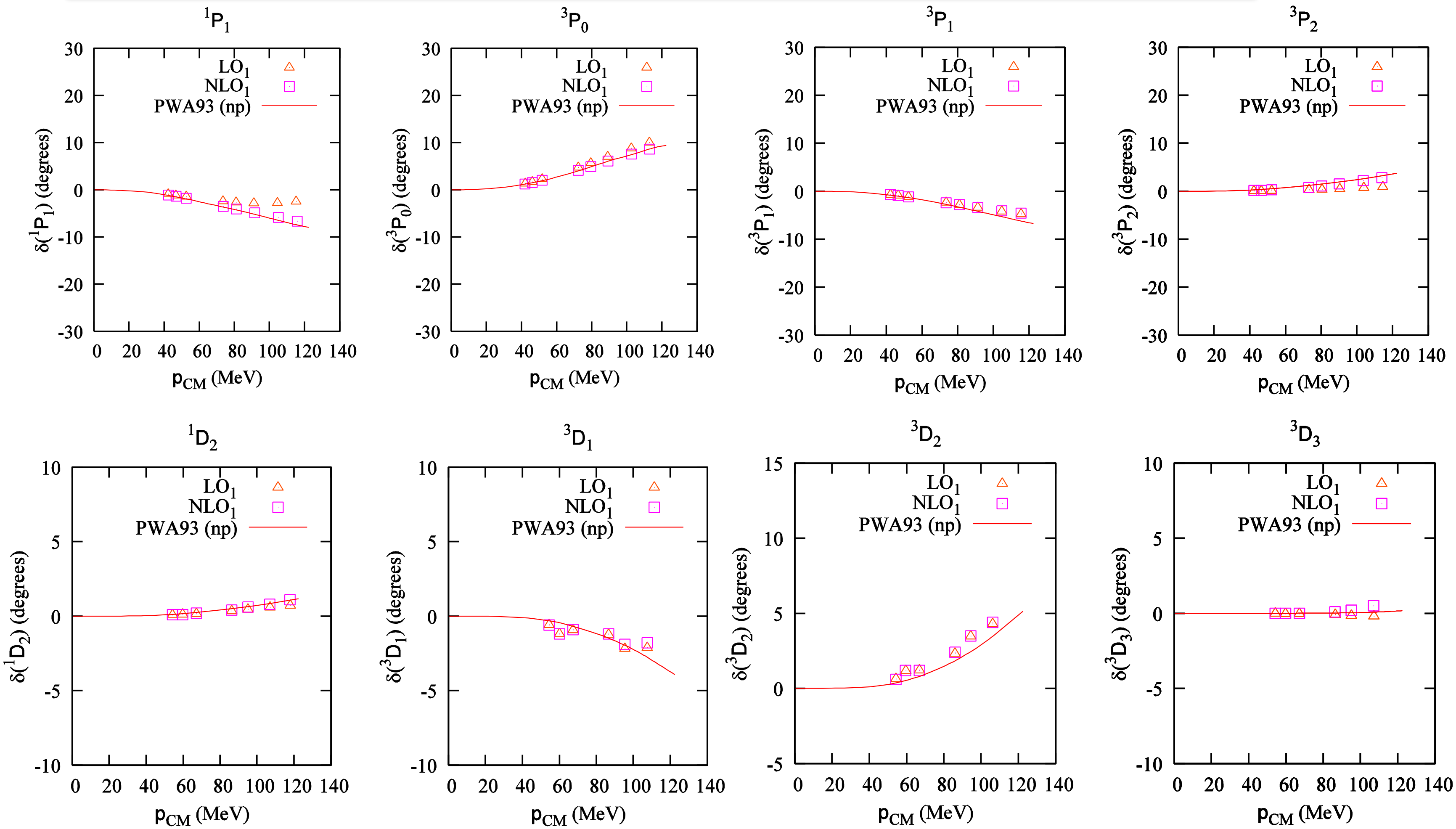
- P-wave phase shifts with different actions



- Better convergence with non-smeared action

Higher partial waves

- Accurate NLO description of higher partial waves
- Non of the D-wave data was used in the fitting of LECs



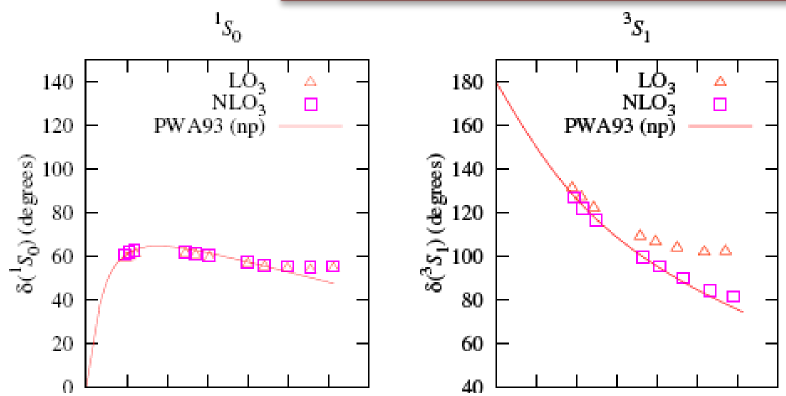
Improving convergence

- Project the smearing out of the P-waves

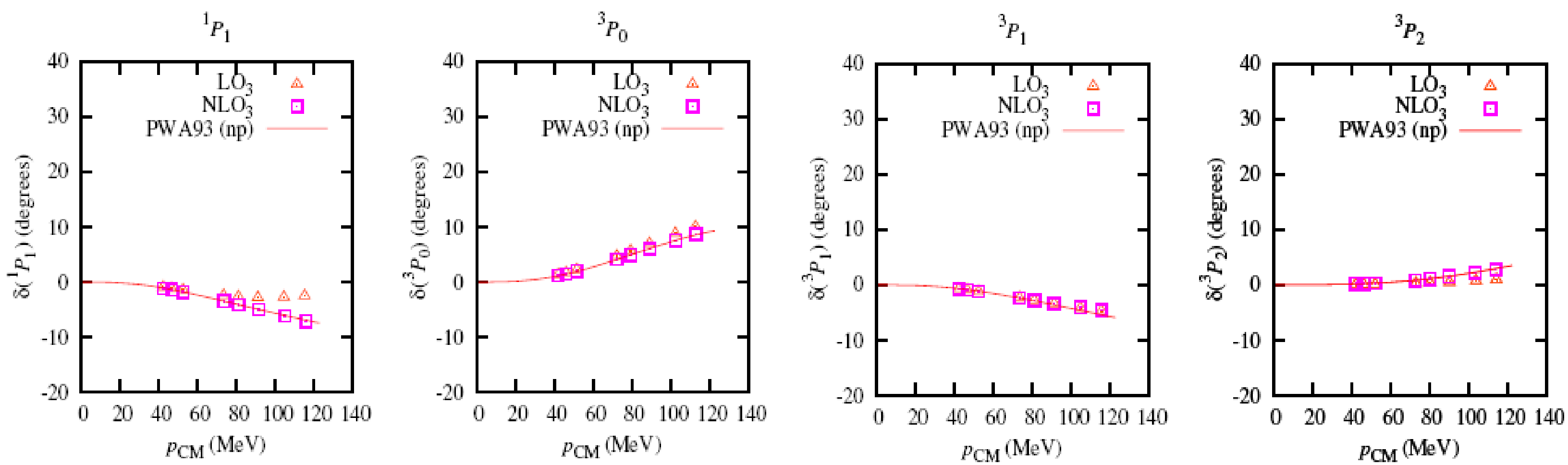
$$V_{\mathrm{LO}_3} = C_{^1S_0} f(\vec{q}) \left(\frac{1}{4} - \frac{1}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)\left(\frac{3}{4} + \frac{1}{4}\vec{\tau}_1 \cdot \vec{\tau}_2\right) + C_{^3S_1} f(\vec{q}) \left(\frac{3}{4} + \frac{1}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)\left(\frac{1}{4} - \frac{1}{4}\vec{\tau}_1 \cdot \vec{\tau}_2\right) + V_{\mathrm{OPE}}$$

Spin 0/Isospin 1 projector

Spin 1/Isospin 0 projector



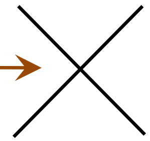
- Fast convergence in all partial waves
  - NLO can be treated perturbatively
- Small systematic errors at NLO



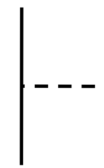
Instantaneous pions

- LO effective potential in continuum

$$V_C = C + C_I \vec{\tau}_1 \cdot \vec{\tau}_2 \rightarrow$$



$$V_{\mathrm{OPE}} = -\left(\frac{g_A}{2F_\pi}\right)^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \rightarrow$$



- Instantaneous pion to reproduce iteration of V_{OPE}

$$\frac{1}{2}\left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - M_\pi^2 \vec{\pi}^2\right] \Longrightarrow -\frac{1}{2}\left[\sum_{i=1}^{3} \partial_i \vec{\pi} \cdot \partial_i \vec{\pi} + M_\pi^2 \vec{\pi}^2\right]$$

No time-derivative

- Two-pion exchange potential

$$V_{\mathrm{TPEP}} \sim L(q) = \frac{1}{2q}\sqrt{4M_\pi^2 + q^2} \ln \frac{\sqrt{4M_\pi^2 + q^2} + q}{\sqrt{4M_\pi^2 + q^2} - q} = 1 + \frac{q^2}{12M_\pi^2} + \ldots$$

Approximation by series of contact interactions. Valid for q<<Λ