

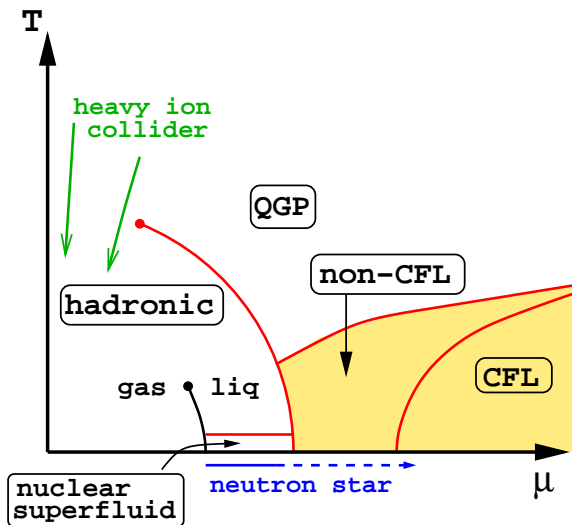
Energy-momentum tensor correlators and viscosity

Harvey Meyer

The XXVI International Symposium on Lattice Field Theory
College of William & Mary, 18 July 2008

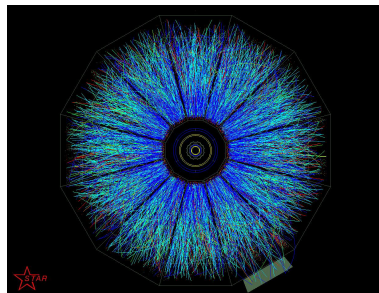
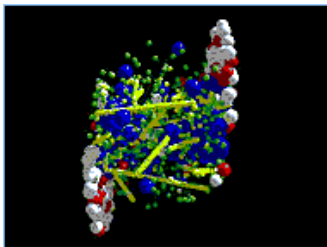
Phys.Rev.D76:101701,2007; Phys.Rev.Lett.100:162001,2008
arXiv:0805.4567; arXiv:0806.3914

Phase diagram of QCD

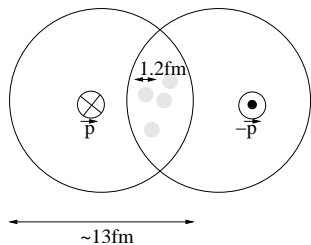


Alford, Schmitt, Rajagopal, Schäfer 0709.4635

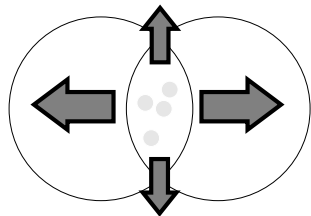
Heavy Ion collisions at RHIC



Heavy ion collisions and elliptic flow

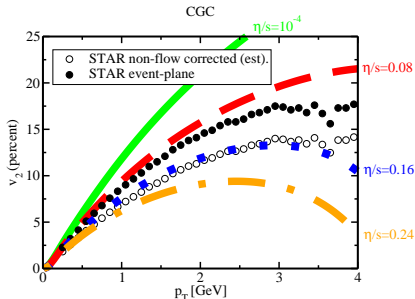
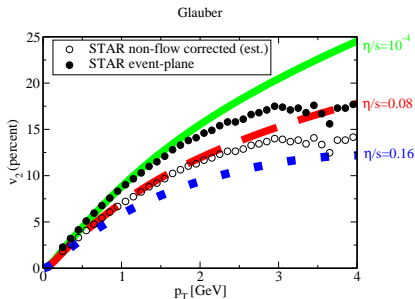


- transv. section of the collision of two gold nuclei
- impact vector \vec{b} determined experimentally by ϕ distribution of particles



- pressure gradient is greater in the (\vec{b}, \hat{z}) plane
- \Rightarrow excess of particles produced in that plane

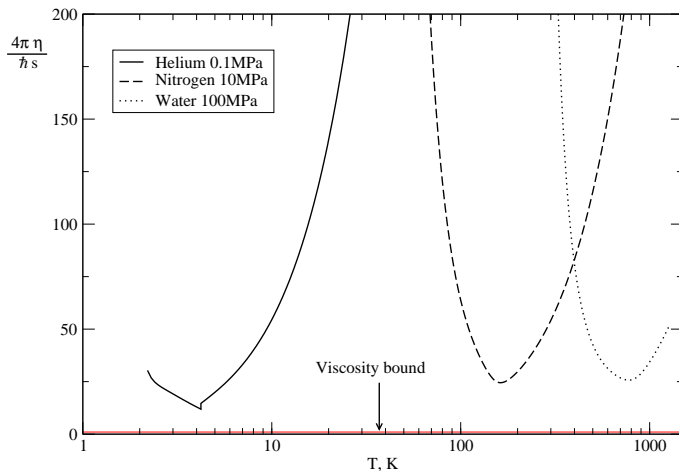
Hydrodynamics, ideal and viscous



Teaney '03; P. & U. Romatschke, '07; Luzum, Romatschke, '08;
Heinz, Song '07; Dusling, Teaney '07

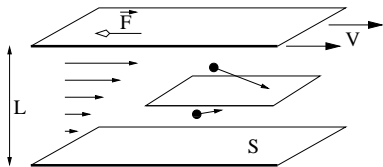
anisotropic flow incompatible with $\eta/s \gtrsim 0.2$

Shear viscosity of ordinary substances



from Kovtun, Son, Starinets PRL 94:111601,2005

Shear viscosity of a dilute gas

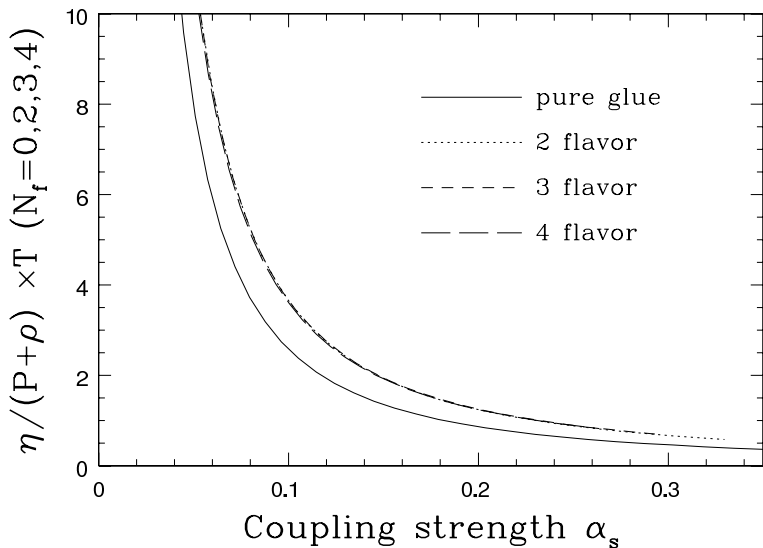


- $F/S = \eta V/L$
- the molecules flying through a horizontal slice have a longitudinal momentum characteristic of their surface of last scattering
- those moving \downarrow have greater longit. momentum than those moving \uparrow
- \Rightarrow transfer of longit. momentum from top to bottom

$$\eta = \frac{1}{3} \bar{p} \sigma, \quad \sigma = \text{cross-section}$$

- η is independent of the density for a dilute gas
- bulk viscosity: $\zeta \propto \text{density}$ (requires 3-body collisions).

Pure gauge theory vs. full QCD in LO PT



G. Moore, SEWM 04

Modes of linearized hydrodynamics

Macroscopic form of the energy-momentum tensor:

$$T^{\mu\nu} = -Pg^{\mu\nu} + (\epsilon + P)u^\mu u^\nu + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = \eta(\Delta^\mu u^\nu + \Delta^\nu u^\mu) + \left(\frac{2}{3}\eta - \zeta\right)H^{\mu\nu}\partial_\rho u^\rho$$

- u^μ is the velocity of energy-transport
- $\eta, \zeta =$ shear and bulk viscosities respectively
- $H^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu}$, $\Delta_\mu = \partial_\mu - u_\mu u^\beta \partial_\beta$

Kubo formula for the shear viscosity

- couple $T_{0i}(x)$ to a small external velocity field $v_i(x)$
- switch off the external field \Rightarrow the system relaxes to equilibrium
- the Kubo formula is a matching condition between the linearized hydro. description and the linear response formalism of the QFT:

$$\eta = -\lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} D_{12,12}^{\text{R}}(\omega, \mathbf{0}), \quad D^{\text{R}} \text{ the retarded correlator of } T_{12}$$

Dispersion relation and analytic continuation: (Karsch, Wyld, '86)

- $D_{12,12}^{\text{R}}(k) = -\int d\omega \frac{\rho(\omega)}{\omega - k_0 - i\epsilon}, \quad \text{Im} D_{12,12}^{\text{R}}(k) = -\pi \rho(k)$
- $\mathcal{D}_{12,12}(\omega_n, \mathbf{k}) = \int d\omega \frac{\rho(\omega, \mathbf{k})}{\omega + i\omega_n}$ (the Euclidean correlator)

$$\mathcal{D}_{12,12}(\tau, \mathbf{k}) = \int_0^\infty d\omega \frac{\cosh \omega(L_0/2 - \tau)}{\sinh \omega L_0/2} \rho(\omega, \mathbf{k}), \quad \eta = \pi \lim_{\omega \rightarrow 0} \rho(\omega, \mathbf{0})/\omega$$

The QCD energy-momentum tensor

Separating the traceless part $\theta_{\mu\nu}$ from the trace part S for gluons, denoted 'g', and quarks, denoted 'f',

$$T_{\mu\nu} \equiv \theta_{\mu\nu}^g + \theta_{\mu\nu}^f + \frac{1}{4}\delta_{\mu\nu}(\theta^g + \theta^f),$$

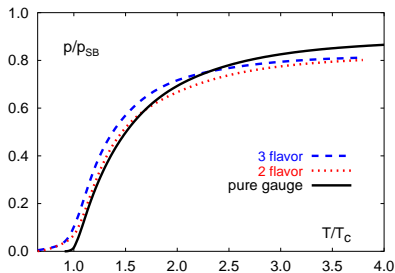
$$\theta_{\mu\nu}^g = \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}^a F_{\rho\sigma}^a - F_{\mu\alpha}^a F_{\nu\alpha}^a,$$

$$\theta_{\mu\nu}^f = \frac{1}{4}\sum_f \bar{\psi}_f \overleftrightarrow{D}_\mu \gamma_\nu \psi_f + \bar{\psi}_f \overleftrightarrow{D}_\nu \gamma_\mu \psi_f - \frac{1}{2}\delta_{\mu\nu} \bar{\psi}_f \overleftrightarrow{D}_\rho \gamma_\rho \psi_f,$$

$$\theta^g = \beta(g)/(2g) F_{\rho\sigma}^a F_{\rho\sigma}^a, \quad \theta^f = [1 + \gamma_m(g)] \sum_f \bar{\psi}_f m \psi_f$$

- $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$
- $\beta(g)$ is the beta-function
- $\gamma_m(g)$ is the anomalous dimension of the mass operator
- all expressions are written in Euclidean space.

Thermodynamics

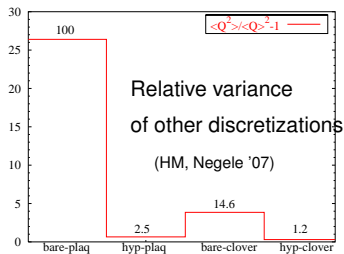
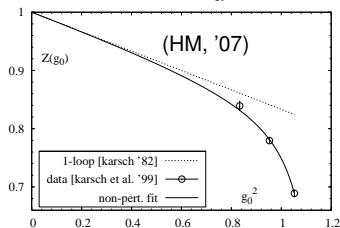


Karsch, *Hard Probes* '06

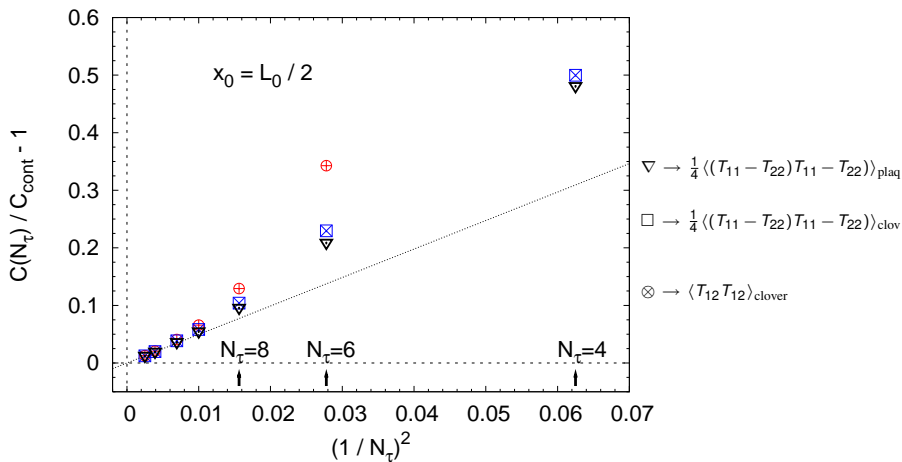
$$T_{00} = \theta_{00} + \frac{1}{4}\theta,$$

$$\begin{cases} \epsilon - 3P = \langle \theta \rangle_T - \langle \theta \rangle_0, \\ \epsilon + P = \frac{4}{3} \langle \theta_{00} \rangle_T \end{cases}$$

Normalization of the traceless energy-momentum tensor

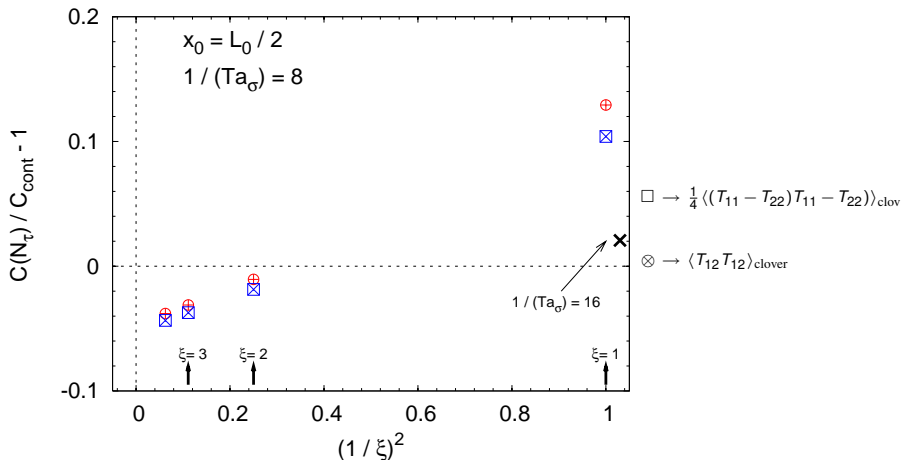


Cutoff effects in lattice perturbation theory



- clover & plaquette discretizations have comparable cutoff effects
- $N_\tau \geq 8$ is mandatory
- use these results to remove treelevel cutoff effects on non-pert. correlators.

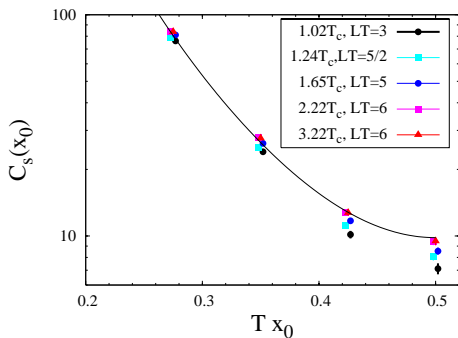
Perturbative cutoff effects: anisotropic lattice



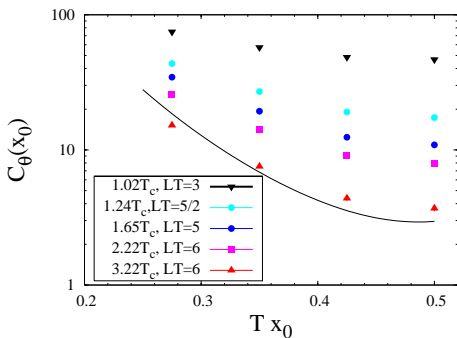
- for fixed L_0/a_σ , $\xi = 2$ or 3 is optimal
- $(N_\tau = 16, \xi = 2)$ is as good as $(N_\tau = 16, \xi = 1)$ and saves a factor 8 in the spatial volume.

The finite T Euclidean correlators ($N_\tau = 8$, $\xi = 1$)

$$\langle T_{12} T_{12} \rangle_c$$



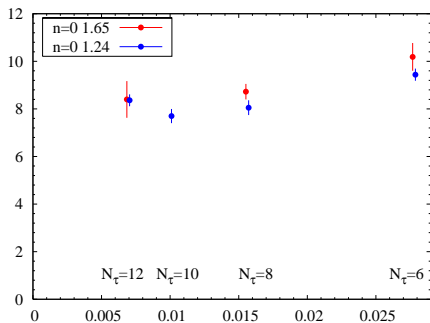
$$\langle \theta \theta \rangle_c$$



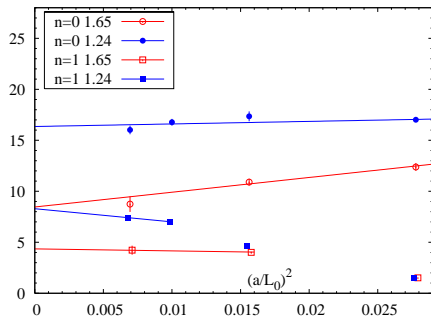
- near-conformal behaviour in one channel, large deviations in the other
- small errors thanks to multi-level algorithm (HM '03)

Taking the continuum limit

$\langle T_{12} T_{12} \rangle_c$



$\langle \theta \theta \rangle_c$



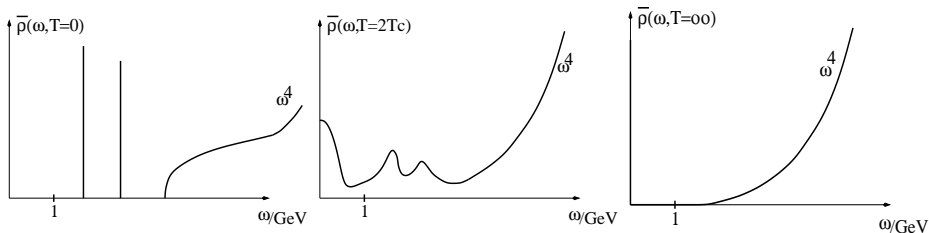
$$\langle \omega^{2n} \rangle \equiv \int_0^\infty d\omega \omega^{2n} \frac{\rho(\omega)}{\sinh \omega/2T} = T^5 \left. \frac{d^{2n} C}{dx_0^{2n}} \right|_{x_0=L_0/2}$$

• ... tree-level improvement works well.

Reconstructing the dinosaur



Expected form of the spectral functions



Free theory:

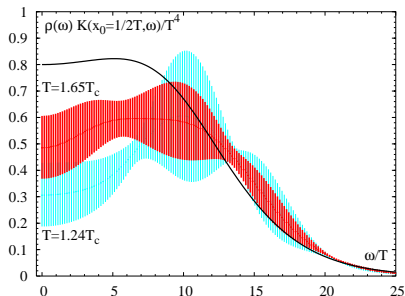
$$\rho_{\mu\nu, \rho\sigma}(\omega) \leftrightarrow \langle T_{\mu\nu} T_{\rho\sigma} \rangle$$

- $\rho_{12,12}(\omega, T=0) = \frac{d_A}{10(4\pi)^2} \omega^4$
- $\rho_{\theta,\theta}(\omega, T=0) = \left(\frac{11\alpha_s N_c}{6\pi}\right)^2 \frac{d_A}{4(4\pi)^2} \omega^4$
- $\rho_{12,12}(\omega, T) = \frac{d_A}{10(4\pi)^2} \frac{\omega^4}{\tanh \frac{\omega}{4T}} + \left(\frac{2\pi}{15}\right)^2 d_A T^4 \omega \delta(\omega)$
- $\rho_{\theta,\theta}(\omega, T) = \frac{d_A}{4(4\pi)^2} \left(\frac{11\alpha_s N_c}{6\pi}\right)^2 \frac{\omega^4}{\tanh \frac{\omega}{4T}}$

The reconstructed spectral functions (2007)

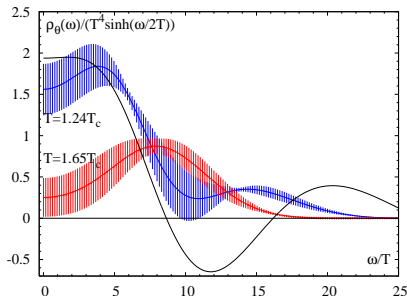
$$\hat{\rho}(\omega) = m(\omega) [1 + \sum_{\ell=1}^N c_{\ell} u_{\ell}(\omega)], \quad m(\omega \gg T) \text{ as predicted by PT}$$

$\langle T_{12} T_{12} \rangle_c$ ($N_{\tau} = 8, \xi = 1$)

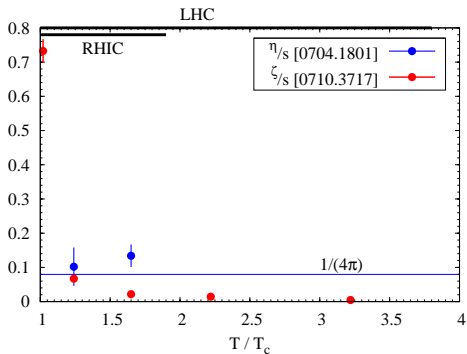


- $\eta/T^3 = \frac{\pi}{2} \times \text{intercept}$
- black curve = normalized $\mathcal{N} = 4$ SYM spectral function.

$\langle \theta \theta \rangle_c$ ($N_{\tau} = 12, \xi = 1$)



- $\zeta/T^3 = (\frac{\pi}{18} \times \text{intercept})$
increasing for $T \rightarrow T_c$
- black curve = $\hat{\delta}(0, \omega)$



Perturbative and AdS/CFT calculations:

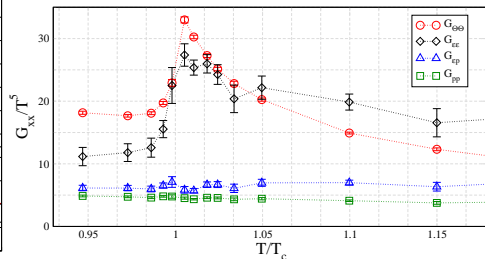
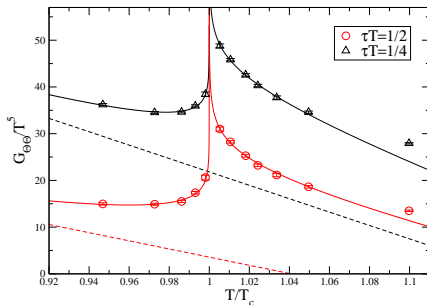
$$\eta/s, \zeta/s = \begin{cases} \frac{0.484}{\pi^2 \alpha_s^2 \log(0.608/\alpha_s)}, & \frac{1.25\alpha_s^2}{\pi^2 \log(4.06/\alpha_s)} & N_f = 0 \text{ PT} \\ 1/(4\pi), & 0 & \mathcal{N} = 4 \text{ SYM.} \end{cases}$$

Arnold, Moore, Yaffe '03; Arnold, Dogan, Moore '06;

Policastro, Son, Starinets '01; Kovtun, Son, Starinets '04

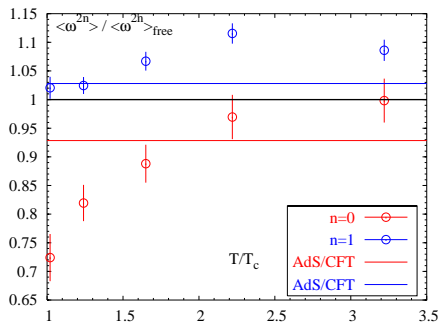
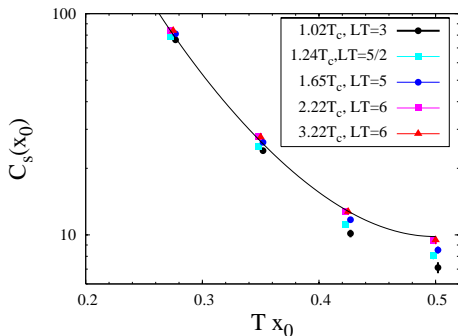
The $\langle\theta\theta\rangle$ correlator at $T = T_c(1 \pm \epsilon)$ in SU(2)

Hübner, Karsch, Pica; see C. Pica's talk



- beautiful confirmation of the 3d Ising class critical behavior
- what happens to ζ at the phase transition is still an open question.

Coming back to the $\langle T_{12} T_{12} \rangle$ correlator



$$\langle \omega^{2n} \rangle \equiv \int_0^\infty d\omega \omega^{2n} \frac{\rho(\omega)}{\sinh \omega/2T} = T^5 \left. \frac{d^{2n} C}{dx_0^{2n}} \right|_{x_0=L_0/2}$$

- due to the large UV contribution, the Euclidean correlator is not very sensitive to interactions...

Universal properties of spectral functions

Ward Identities: if $\mathbf{q} = (0, 0, q)$, $\partial_\mu T_{\mu\nu} = 0 \Rightarrow$

$$\rho_{00,00}(\omega, \mathbf{q}) = \frac{q^4}{\omega^4} \rho_{33,33}(\omega, \mathbf{q}) \Rightarrow \rho_{00,00}(\omega, \mathbf{q}) \stackrel{\omega \rightarrow \infty}{\sim} q^4$$

$$\rho_{01,01}(\omega, \mathbf{q}) = \frac{q^2}{\omega^2} \rho_{13,13}(\omega, \mathbf{q}) \Rightarrow \rho_{01,01}(\omega, \mathbf{q}) \stackrel{\omega \rightarrow \infty}{\sim} q^2 \omega^2.$$

Hydrodynamics prediction at small (ω, \mathbf{q}) :

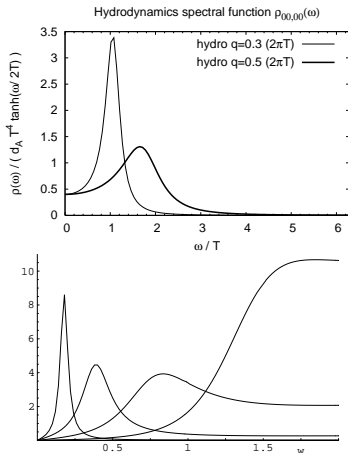
shear channel:
$$\frac{\rho_{01,01}(\omega, \mathbf{q})}{\omega} \stackrel{\omega, q \rightarrow 0}{\sim} \frac{\eta}{\pi} \frac{q^2}{\omega^2 + (\eta q^2 / (\epsilon + P))^2},$$

sound channel:
$$\frac{\rho_{00,00}(\omega, \mathbf{q})}{\omega} \stackrel{\omega, q \rightarrow 0}{\sim} \frac{\Gamma_s}{\pi} \frac{(\epsilon + P) q^4}{(\omega^2 - v_s^2 q^2)^2 + (\Gamma_s \omega q^2)^2},$$

$$\Gamma_s = \frac{\frac{4}{3}\eta + \zeta}{\epsilon + P} = \frac{\frac{4}{3}\eta + \zeta}{T_s}$$

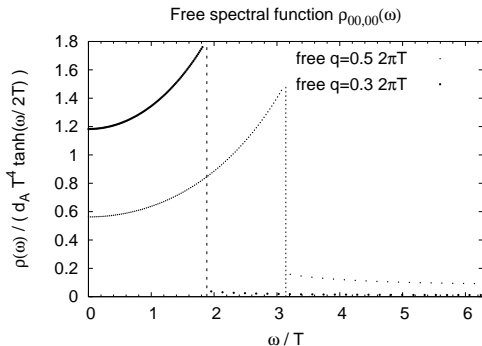
for a derivation see D. Teaney, 2006

Sound channel spectral function $\rho_{00,00}(\omega, \mathbf{q})$



← Hydro. spectral function:

$$v_s^2 = \frac{1}{3}, \quad s = \frac{3}{4} s_{SB}, \quad T\Gamma_s = \frac{1}{3\pi}$$



strongly coupled $\mathcal{N} = 4$ SYM

$$\frac{2\rho}{\pi d_A T^4} \text{ for } q/(2\pi T) = 0.3, 0.6, 1.0, 1.5$$

(Kovtun, Starinets 2006)

$SU(N_c)$ gauge theory (HM, 2008)

Asymptotic behavior of $\rho_{00,00}(\omega)$

asymptotic behavior

small ω :
$$\frac{\frac{2}{\pi} \tanh\left(\frac{\omega}{2T}\right) \Gamma_s (\epsilon + P) q^4}{(\omega^2 - v_s^2 q^2)^2 + (\Gamma_s \omega q^2)^2}$$

large ω :
$$\frac{2d_A q^4}{15(4\pi)^2} \tanh \frac{\omega}{2T}$$

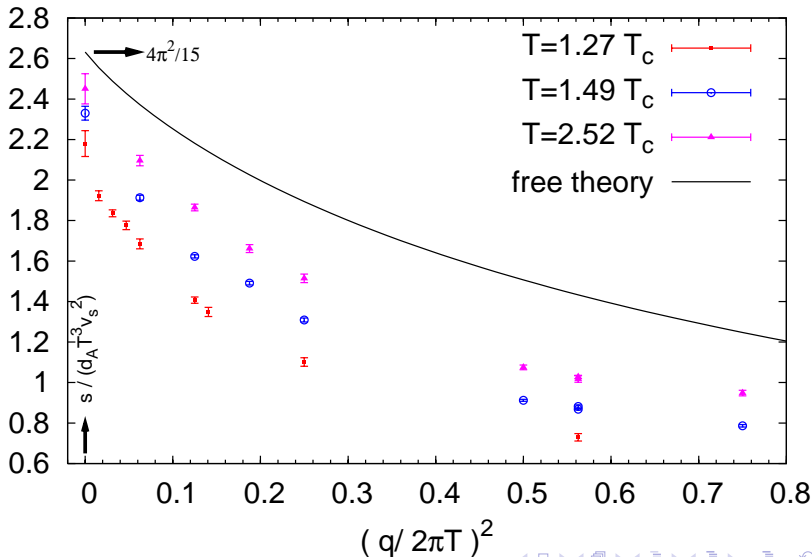
$$\begin{aligned}\widehat{\rho}(\omega, \mathbf{q}, T) = & \\ & \frac{\frac{2}{\pi} \tanh(\frac{\omega}{2T}) \Gamma_s (\epsilon + P) q^4}{(\omega^2 - v_s^2 q^2)^2 + (\Gamma_s \omega q^2)^2} \cdot (1 - \tanh^2 \omega/2T) \\ & + \frac{2d_A q^4}{15(4\pi)^2} \tanh \frac{\omega}{2T} \cdot \tanh^2 \omega/2T\end{aligned}$$

$$\begin{aligned}\widehat{\rho}(\omega, q, T) = & \\ & \frac{2}{\pi} \frac{\tanh(\frac{\omega}{2T}) \Gamma_s (\epsilon + P) q^4}{(\omega^2 - v_s^2 q^2)^2 + (\Gamma_s \omega q^2)^2} \cdot (1 - \tanh^2 \omega/2T) \\ & + \frac{2d_A q^4}{15(4\pi)^2} \tanh \frac{\omega}{2T} \cdot \tanh^2 \omega/2T\end{aligned}$$

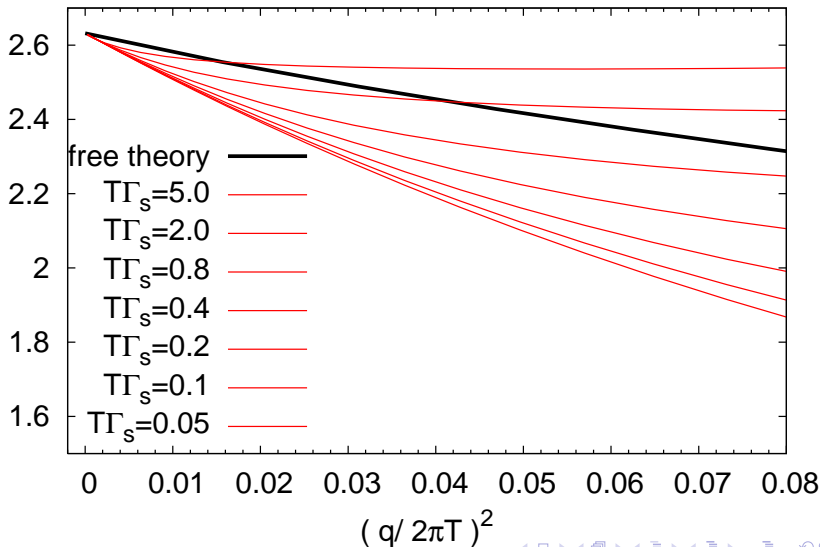
- thermodynamics known \Rightarrow only one free parameter, Γ_s (!)
- the \tanh^2 are arbitrary, but for small enough q , it will not matter
- caveat: the behavior for $\omega \approx \Lambda_{\overline{\text{MS}}} \approx T$ is not known
- we *know* that the sound peak dominates the Euclidean correlator $C(x_0 \approx \frac{1}{2T}, q)$ for sufficiently small q
- it is *assumed* that $\frac{d}{dq^2} C(\frac{1}{2T}, q = 0)$ is also dominated by the sound peak rather than the $\omega \approx T$ region.
- it is at least true at weak coupling and in strongly coupled SYM.

Energy density two-point function

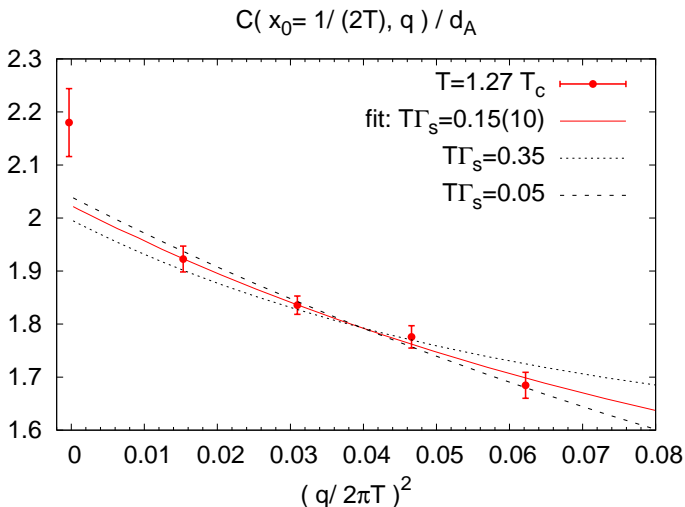
$$C(x_0=1/(2T), q) / d_A$$



$$C(x_0 = 1/(2T), q) / d_A$$



Fitting the small momenta ($T = 1.27 T_c, 12 \times 48^3, \xi = 2$)



- used $v_s^2 = 0.263(18)$ [Boyd. et al '96, CPPACS '01], $\frac{s}{T^3} = 4.54(14)$

Conclusions

- 1 the observed strong dynamics at RHIC calls for a theoretical understanding of transport coefficients beyond PQCD \Rightarrow lattice QCD
- 2 subpercent-level precision gluonic correlators have been achieved thanks to multi-level algorithm, variance-reduced discretizations of $T_{\mu\nu}$, treelevel improvement and anisotropic lattices
- 3 new method based on the energy density correlator at $0 < \mathbf{q} \ll 2\pi T$ (sound channel) \Rightarrow the functional form of ρ is largely known *a priori*
- 4 preliminary result: $\Gamma_s T = \frac{4}{3} \frac{\eta + \zeta}{s} = 0.15(10)$ at $1.3T_c$
- 5 hopefully soon: *prediction* for the LHC.

Backup slides

The “inverse problem”: solving for $\rho(\omega)$

Given $C_i = \int_0^\infty d\omega \rho(\omega) \frac{\cosh \omega L_0 \tau_i / 2}{\sinh \omega L_0 / 2}$ ($i = 1, \dots, N$), reconstruct $\rho(\omega)$.

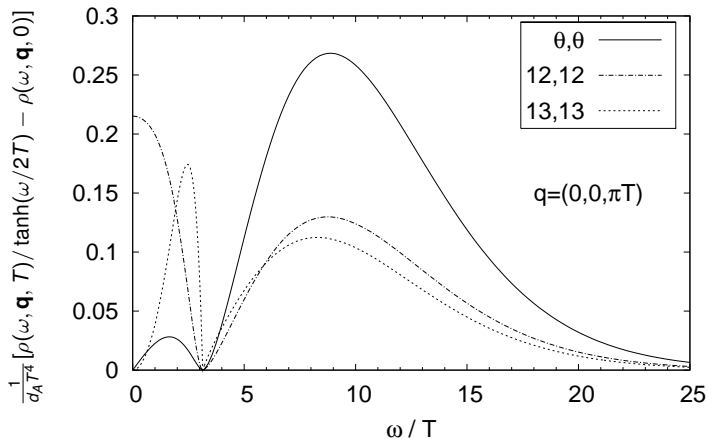
- our estimator for ρ : $\hat{\rho}(\omega) = m(\omega)[1 + \hat{a}(\omega)]$, $\hat{a}(\omega) = \sum_{\ell=1}^N c_\ell u_\ell(\omega)$
- $m(\omega) > 0$; $m(\omega \gg T)$ has the correct perturbative behavior
- basis u_ℓ determined by singular-value decomposition of $M(x_0, \omega) \stackrel{\text{def}}{=} K(x_0, \omega)m(\omega)$: $M^t = UV^t$, $u_\ell(\omega)$ = columns of U
- we are only able to reconstruct a ‘fudged’ version of the genuine $a(\omega)$:

$$\hat{a}(\omega) = \int \hat{\delta}(\omega, \omega') a(\omega') d\omega' \quad \hat{\delta}(\omega, \omega') = \sum_{\ell=1}^N u_\ell(\omega) u_\ell(\omega').$$

- $\hat{\delta}(\omega, \omega')$ is called the **resolution function** (how complete is the basis?)
- we would like the resolution function to resemble a delta-function.

Backus & Gilbert, (geophysics, 1968)

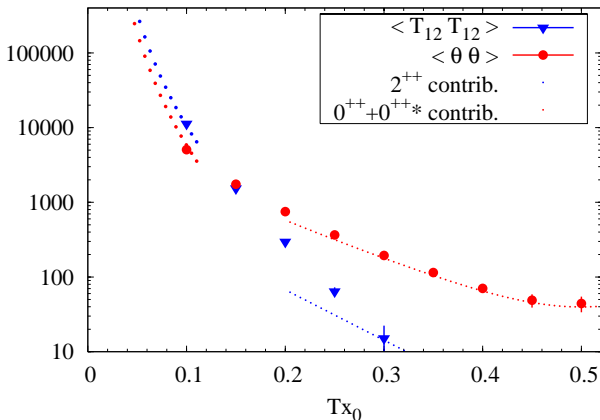
Perturbative thermal spectral functions



$$\rho_{\theta,\theta}(\omega, q, T=0) = \left(\frac{11\alpha_s N_c}{6\pi} \right)^2 \frac{d_A \theta(\omega-q)}{4(4\pi)^2} (\omega^2 - q^2)^2$$

Correlators in the confined phase

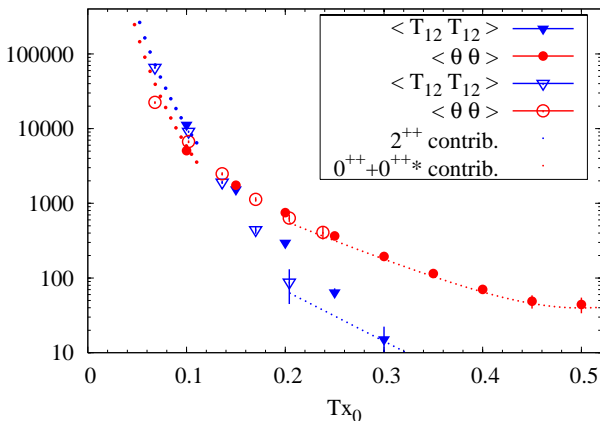
Treelevel Improved Correlators at $T=T_c/2$ ($\beta=6.2, 20^4$)



- in the scalar channel, the two stable glueballs almost saturate the correlator beyond 0.5fm
- calculation made possible by the **multi-level algorithm** (HM '03)

Correlators in the confined phase

Treelevel Improved Correlators at $T=T_c/2$ ($20^4, 28^4$)



- in the scalar channel, the two stable glueballs almost saturate the correlator beyond 0.5fm
- calculation made possible by the **multi-level algorithm** (HM '03)

Spectral functions in free theory: analytic expressions

For a polynomial P , define

$$\mathcal{I}([P], \omega, q, T) = \theta(\omega - q) \int_0^1 dz \frac{P(z) \sinh \frac{\omega}{2T}}{\cosh \frac{\omega}{2T} - \cosh \frac{qz}{2T}} + \theta(q - \omega) \int_1^\infty dz \frac{P(z) \sinh \frac{\omega}{2T}}{\cosh \frac{qz}{2T} - \cosh \frac{\omega}{2T}}.$$

Then, for instance,

$$\begin{aligned} \rho_{\theta, \theta}(\omega, q, T) &= \left(\frac{11\alpha_s N_c}{6\pi} \right)^2 \frac{d_A}{4(4\pi)^2} (\omega^2 - q^2)^2 \mathcal{I}([1], \omega, q, T), \\ \mathcal{I}([1], \omega, q, T) &= -\frac{\omega}{q} \theta(q - \omega) + \frac{2T}{q} \log \frac{\sinh(\omega + q)/4T}{\sinh |\omega - q|/4T} \end{aligned}$$

- in all other channels, spectral function is a linear combination of polylogarithms

HM, 2008

Lattice perturbation theory

The “clover” discretization of $T_{\mu\nu}$:

$$\theta_{00}(x) = \frac{1}{g_0^2} \text{Re Tr} \left\{ \widehat{Z}_{d\tau}(g_0, \xi_0) \sum_k \widehat{F}_{0k}(x)^2 - \widehat{Z}_{d\sigma}(g_0, \xi_0) \sum_{k<l} \widehat{F}_{kl}(x)^2 \right\}.$$

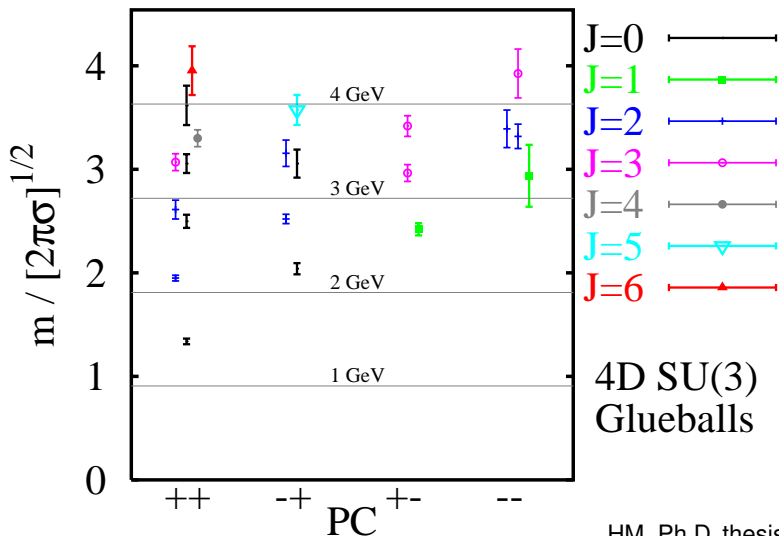
Perturbation theory on the anisotropic lattice:

$$\begin{aligned} & a_\sigma^3 \sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}} \langle \widehat{F}_{\mu\nu}^a(0) \widehat{F}_{\rho\sigma}^a(0) \widehat{F}_{\alpha\beta}^b(y) \widehat{F}_{\gamma\delta}^b(y) \rangle_0 \\ &= \frac{d_A}{a_\sigma^5} \int_{-\pi}^{\pi} \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\tilde{\phi}_{\mu\nu\alpha\beta}(\mathbf{p}, \tau) \tilde{\phi}_{\rho\sigma\gamma\delta}(\mathbf{p} + \mathbf{q}, \tau) + \tilde{\phi}_{\mu\nu\gamma\delta}(\mathbf{p}, \tau) \tilde{\phi}_{\rho\sigma\alpha\beta}(\mathbf{p} + \mathbf{q}, \tau) \right], \end{aligned}$$

where

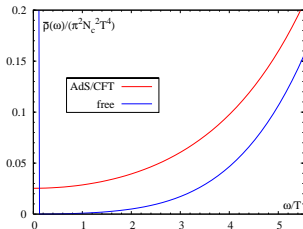
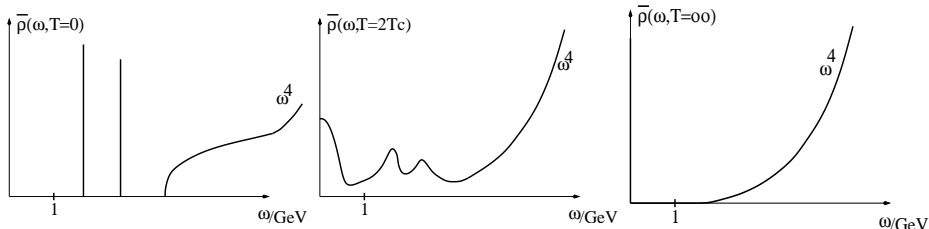
$$\begin{aligned} \tilde{\phi}_{\mu\nu\alpha\beta}(\mathbf{p}, \tau) &\equiv \xi_0 \int_{-\pi}^{\pi} \frac{dp_0}{2\pi} \frac{e^{ip_0\tau}}{\xi_0^2 \hat{p}_0^2 + \hat{\mathbf{p}}^2} \times \\ &\left[\delta_{\nu\beta} \xi_0^{\delta_\mu + \delta_\alpha} \cos^2(p_\nu/2) \sin p_\mu \sin p_\alpha + \delta_{\mu\alpha} \xi_0^{\delta_\nu + \delta_\beta} \cos^2(p_\mu/2) \sin p_\nu \sin p_\beta \right. \\ &\left. - \delta_{\mu\beta} \xi_0^{\delta_\nu + \delta_\alpha} \cos^2(p_\mu/2) \sin p_\nu \sin p_\alpha - \delta_{\nu\alpha} \xi_0^{\delta_\mu + \delta_\beta} \cos^2(p_\nu/2) \sin p_\mu \sin p_\beta \right]. \end{aligned}$$

Spectrum of the pure gauge theory



HM, Ph.D. thesis 04

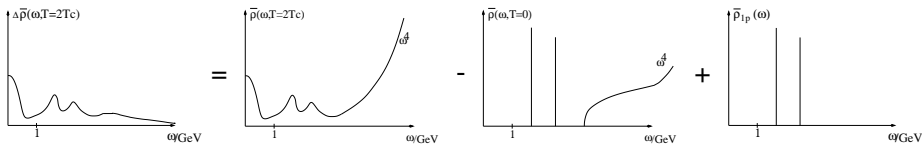
Expected form of the spectral function for η



- at $T \gg T_c$, perturbation theory predicts a peak at $\omega = 0$ of width $O(\alpha_s NT)$
- “strong coupling \leftrightarrow smooth spectral function”
- “weak coupling \leftrightarrow spiky spectral function”
- in strongly coupled $\mathcal{N} = 4$ SYM, $\rho(\omega)$ is very smooth
- $\rho(\omega)$ for ζ : no peak at $\omega = 0 \Rightarrow$ more favorable.

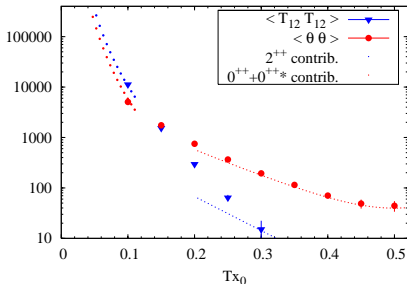
Strategy I, pictorially

$$\Delta\rho = \rho - \rho_{T=0} + \rho_{1p}$$



- 1 compute $\rho_{T=0}(\omega)$
- 2 compute the one-particle contributions to $\rho(\omega)$, ρ_{1p}
- 3 compute $\Delta C(x_0, T) \equiv C(x_0, T) - \int d\omega K(\omega x_0, \omega/T) [\rho_0(\omega) - \rho_{1p}(\omega)]$
- 4 $\Delta C = \int d\omega K(\omega x_0, \omega L_0) \Delta\rho(\omega)$
- 5 $\eta = \pi \Delta\rho(\omega)/\omega|_{\omega \rightarrow 0}$

Treelevel Improved Correlators at $T=T_c/2$ ($\beta=6.2, 20^4$)

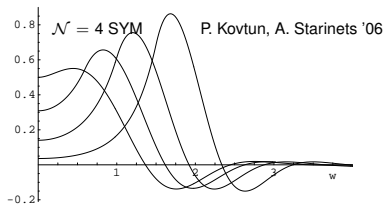


Subtracting $\rho(\omega, \mathbf{p} \neq \mathbf{0})$

- $\mathbf{q} = (0, 0, q)$: $\rho_{12,12}(\omega, \mathbf{q}, T = 0) = \frac{d_A \theta(\omega - q)}{10(4\pi)^2} (\omega^2 - q^2)^2$
- exploit the linearity of the problem to cancel the ω^4 contribution, e.g. via

- $C(x_0, \mathbf{q} = \mathbf{0}) - \frac{4}{3} C(x_0, \mathbf{q}) + \frac{1}{3} C(x_0, 2\mathbf{q})$

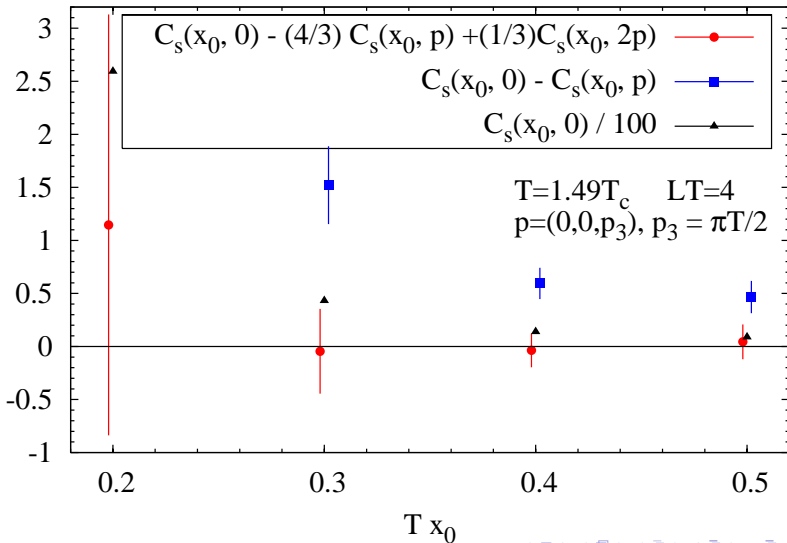
- this inverse problem is better conditioned
- if $\mathbf{q} = (0, 0, q)$ large enough, estimate $\rho(\omega, \mathbf{p})/\omega|_{\omega=0}$ in PT (or neglect it).
- if $\mathbf{q} = (q, 0, 0)$, hydro predicts $\rho(\omega, \mathbf{q})/\omega|_{\omega=0} = 0$



Finite-temperature part of the tensor spectral function $(\rho(\omega) - \rho_0(\omega))/\omega$, plotted in units of $\pi^2 N_C^2 T^4$ as a function of $\omega/2\pi T$. Different curves correspond to values of the momentum $\mathbf{p}/2\pi T$ equal to 0, 0.6, 1.0, and 1.5.

Differences of $\mathbf{p} \neq 0$ correlators

Tensor correlators (10×20^3 , $\xi=2$, clover discretization)

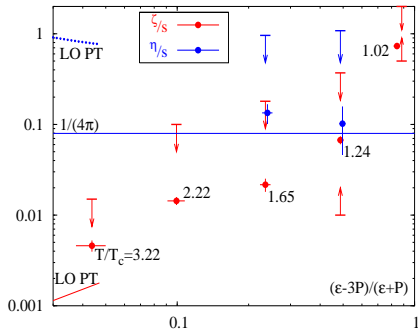
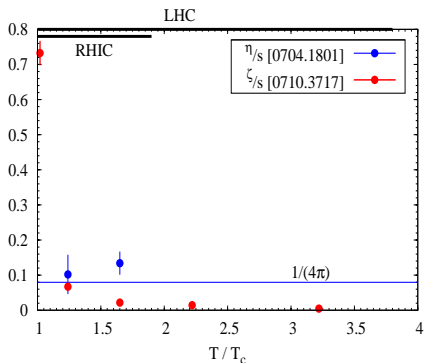


Non-zero momentum correlators (anisotropic lattices)

$$\langle T_{12} T_{12} \rangle, 20^3 \times 10, \xi = 2$$

$$\langle \theta \theta \rangle, 16^3 \times 12, \xi = 2$$

Summary



Perturbative and AdS/CFT calculations:

$$\eta/s, \zeta/s = \begin{cases} \frac{0.484}{\pi^2 \alpha_s^2 \log(0.608/\alpha_s)}, & \frac{1.25\alpha_s^2}{\pi^2 \log(4.06/\alpha_s)} \\ 1/(4\pi), & 0 \end{cases}, \quad \begin{matrix} N_f = 0 \text{ PT} \\ \mathcal{N} = 4 \text{ SYM.} \end{matrix}$$

Arnold, Moore, Yaffe '03; Arnold, Dogan, Moore '06;

Policastro, Son, Starinets '01; Kovtun, Son, Starinets '04

Universal properties of spectral functions

Ward Identities: $\mathbf{q} = (0, 0, q)$

$$\rho_{00,00}(\omega, \mathbf{q}) = \frac{q^4}{\omega^4} \rho_{33,33}(\omega, \mathbf{q})$$

Hydrodynamics prediction at small (ω, \mathbf{q}) :

$$\frac{\rho_{01,01}(\omega, \mathbf{q})}{\omega} \underset{\omega, q \rightarrow 0}{\sim} \frac{\eta}{\pi} \frac{q^2}{\omega^2 + (\eta q^2 / (\epsilon + P))^2},$$

$$\frac{\rho_{00,00}(\omega, \mathbf{q})}{\omega} \underset{\omega, q \rightarrow 0}{\sim} \frac{\Gamma_s (\epsilon + P)}{\pi} \frac{q^4}{(\omega^2 - v_s^2 q^2)^2 + (\Gamma_s \omega q^2)^2},$$

$$\Gamma_s = \frac{\frac{4}{3}\eta + \zeta}{\epsilon + P} = \frac{\frac{4}{3}\eta + \zeta}{Ts}$$

Teaney, 2006