Heavy **flavour** phenomenology **from** lattice **QCD**

Elvira Gámiz

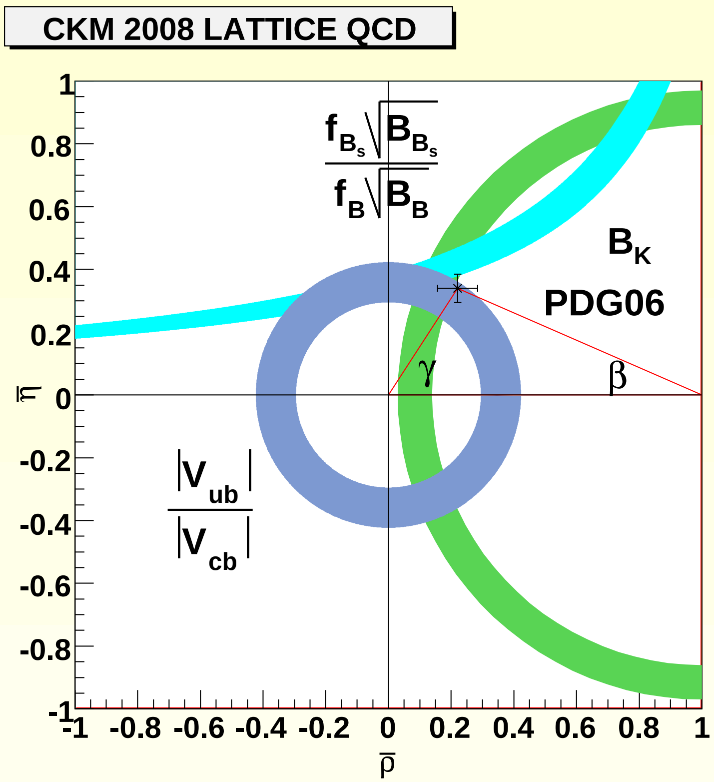
Lattice 2008

· Williamsburg (Virginia), 18 July 2008 ·

Outline

1. Impact of lattice QCD calculations on heavy flavour physics
2. Decay constants: $P \rightarrow l\nu$
   - f_D and f_{D_s}: test of lattice (QCD)
   - f_B and f_{B_s}
3. Semileptonic decays
   - $B \rightarrow D^*(D)l\nu$: determination of $|V_{cb}|$
   - $B \rightarrow \pi l\nu$: determination of $|V_{ub}|$
   - $D \rightarrow K(\pi)l\nu$: determination of $|V_{cd(s)}|$
4. $B^0 - \overline{B}^0$ mixing
5. Heavy quark masses: m_c and m_b
6. Conclusions and outlook

1. Impact of lattice QCD calculations on heavy flavour physics



- \# Determination of fundamental parameters of the SM
  - * CKM matrix elements: $|V_{ub}|$, $|V_{cs}|$, $|V_{cd}|$, $|V_{cb}|$
  - * heavy quark masses: m_b, m_c

C. Davies

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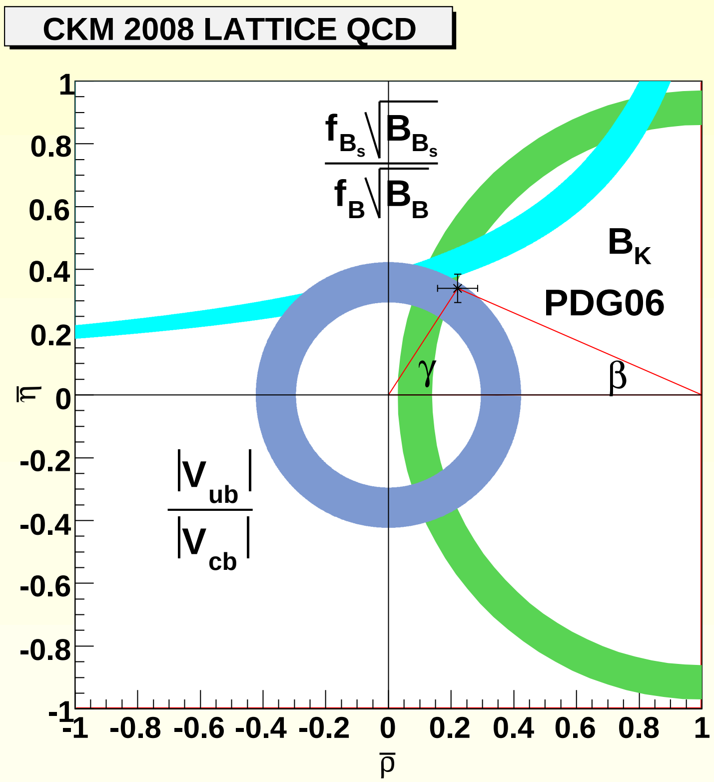

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Constraining NP models.

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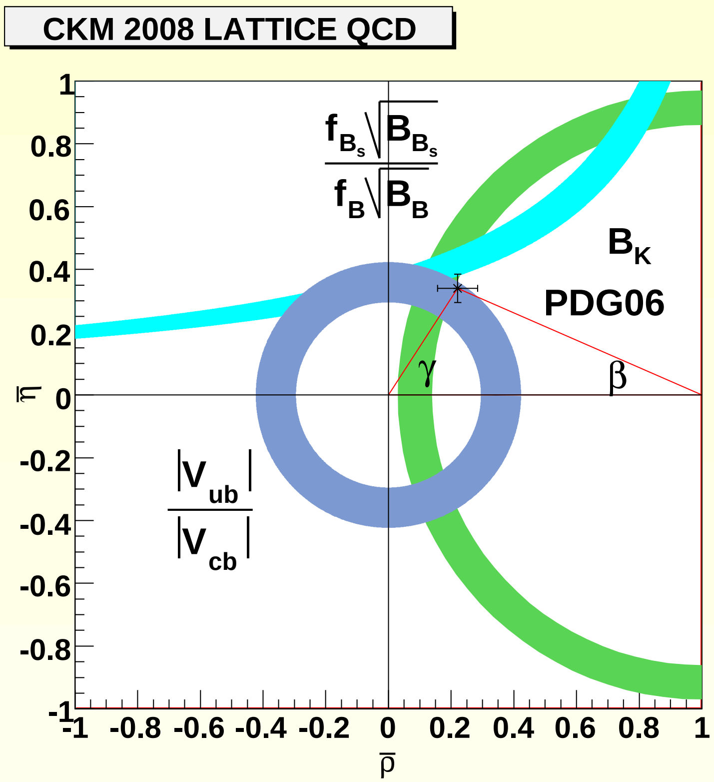

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C. Davies

\# In conjunction with experimental measurements ...

* CDF and DØ tagged angular analysis of $B_0^s \to J/\Psi\phi$

In conjunction with experimental measurements ...

* $B^0 - \bar{B}^0$ mixing observables

Observable	source	% error
ΔM_s	CDF	<1
ΔM_d	PDG07	<1

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* Leptonic decays branching fractions **CLEO-c**, 0806.2112

Observable	% error in corresponding decay constant
$Br(D_s \to \mu\nu)/\ Br(D_s \to \tau\nu)$	3/6.5
$Br(D \to \mu\nu)$	4

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* Semileptonic decays branching ratios **BaBar, Belle, CLEO-c**

Observable	% error in corresponding CKM element
$Br(D \to K(\pi)e\nu)$	1.5/4.5
$Br(B \to \pi l\nu)$	6
$Br(B \to D^* l\nu)$	1.5

Non-perturbative theory inputs still main source of error

$\rightarrow$ Need to reduce lattice errors to $\leq 5\%$

$N_f = 2+1$ calculations + all the sources of systematic errors analyzed: `chiral extrapolation, discretization (continuum limit), renormalization, finite volume, ...`

* Results relevant for phenomenology rely on χPT to go to physical masses $\rightarrow$ validity of χPT techniques to have accurate results

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Hints of discrepancies between SM expectations and some flavour observables (see, for example, **E. Lunghi**, talk at BEACH08)

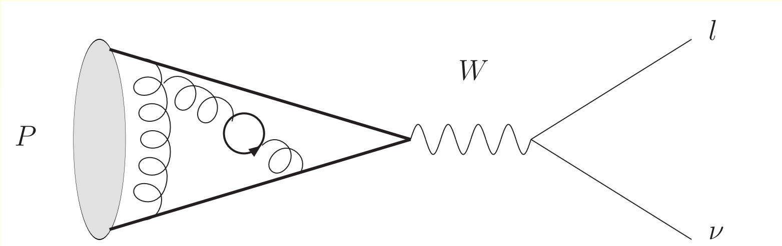
* B_s^0 mixing phase **UTfit coll.**, arXiv:0803.0659

* f_{D_s} **B. Dobrescu and A. Kronfeld**, arXiv:0803.4340 (**talk by A. Kronfeld**)

* $sin(2\beta)$ **E. Lunghi and A. Soni**, arXiv:0803.0512 (**talk by A. Soni**)

Improvement in calculation of `decay constants`, ξ and `form factors` needed for the extraction of V_{cb} and V_{ub} is crucial.

2. Decay constants: $P \to l\nu$



2.1. f_D and f_{D_s}: test of lattice QCD

2.2. f_B and f_{B_s}

f_D and f_{D_s}: test of lattice QCD

Charm quark is in between the heavy and light mass regimes

* Heavy quark effective theories do not give accurate results
* Relativistic descriptions: Maintain cut-off effects under control requires

  ** Improved actions and currents.

  ** Fine enough lattices

Heavy quark formalisms for D mesons

Fermilab action: `Relativistic` clover action with Fermilab (HQET) interpretation

* Smooth interpolation between static limit and light quarks

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HISQ (Highly improved staggered action): No tree level a^2 errors (Asqtad) + reduction of $\mathcal{O}(a^2\alpha_s)$ and $\mathcal{O}((am_Q)^4)$ errors (by a factor of ~ 3)

→ Very precise results for charm physics: charmonium and D

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Twisted mass QCD at maximal twist (tuning a single parameter)

* Meson masses and decay constants $\mathcal{O}(a)$ improved.
* No need for renormalization for decay constants (PCAC)
* Mass renormalization multiplicative and calculated NP

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$\mathcal{O}(a)$ **improved Wilson**: improvement in action and currents.

f_D and f_{D_s}: test of lattice QCD

FNAL/MILC $\boxed{N_f = 2 + 1}$ (talk by P. Mackenzie) **Preliminary**

Reanalysis of existing data completed with all systematic errors analyzed.

`Heavy valence quarks`: Fermilab action

`Light quarks`: improved staggered (Asqtad)

MILC ensembles: 3 values of $a = 0.15, 0.12, 0.09\ fm$ with 3-5 light sea quark masses (down to $m_s/10$).

* For each sea quark mass: 8-12 valence quark masses (including full QCD points).

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Renormalization partially NP: $Z_{A4}^{Qq} = \rho_{A4}^{Qq} \sqrt{Z_{V_4}^{QQ} Z_{V_4}^{qq}}$

* $Z_{V_4}^{QQ(qq)}$ calculated NP (1.4% error)
* ρ_{A4}^{Qq} very close to 1 ($\leq$ 0.3% error)

$\rightarrow$ **small error** $\sim 1.4\%$

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Simultaneous chiral and continuum extrapolation: `Staggered` χ`PT`

* `NLO` + analytic `NNLO` + explicit $\mathcal{O}(a^2)$
* Remove the dominant light discretization errors

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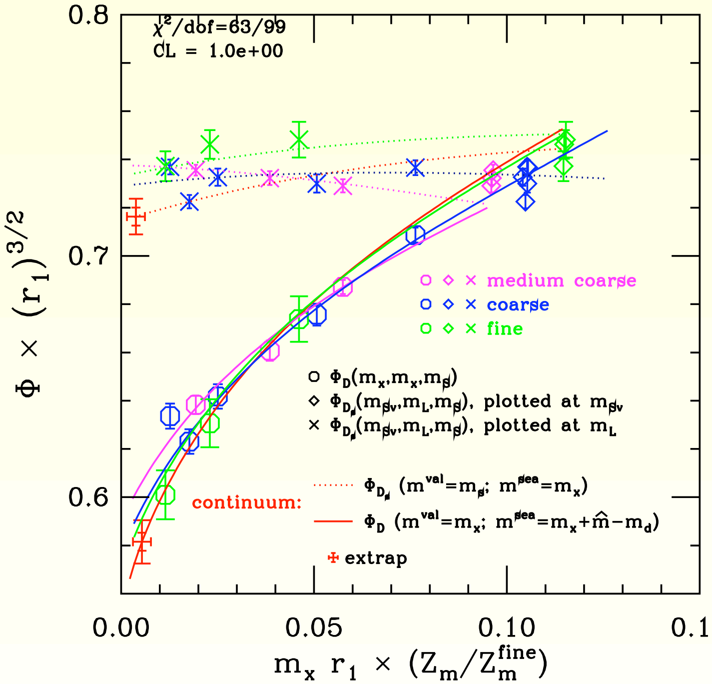
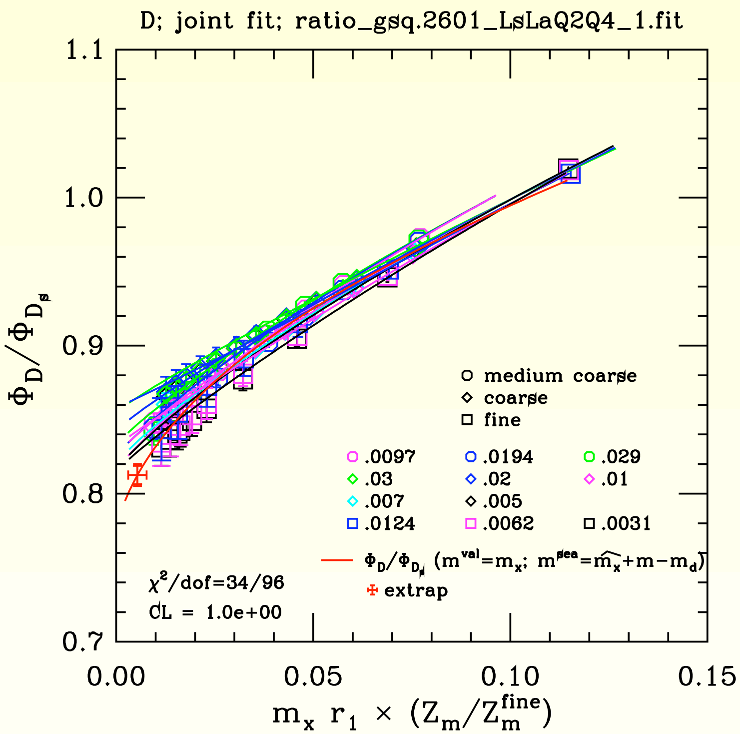
Simultaneous fit to all the data: $\rightarrow$ f_D and f_{D_s}





Fits sensitive to logarithms

f_D and f_{D_s}: test of lattice QCD

FNAL/MILC $N_f = 2+1$ (talk by P. Mackenzie) **Preliminary**

Error budget (in %)

source	f_D	f_{D_s}	f_{D_s}/f_D
statistics	1.5	1.0	1.0
inputs (r_1, $m_{s,d,u}$)	2.1	1.4	0.6
inputs (m_c)	2.7	2.7	$<$0.1
renorm.	1.4	1.4	$<$0.1
HQ disc.	2.7	2.7	0.3
LQ disc.	2.6	1.2	1.6
FV	0.6	0.2	0.6
total syst.	5.3	4.5	1.8

$$f_D = 207(11)\mathrm{MeV}$$

$$f_{D_s} = 249(11)\mathrm{MeV}$$

$$f_{D_s}/f_D = 1.200(27)$$

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Future improvements:
- * Smaller lattice spacings (existing: $a = 0.06\ fm$, generating: $a = 0.04\ fm$)
- * Quadruple number of configurations
- * Technical improvements to reduce statistical errors.
- * Improved determination of inputs: r_1, m_c

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HPQCD, PRL 100(2008)062002 $\boxed{N_f = 2+1}$

`Charm and light valence quarks`: Highly improved staggered (HISQ)

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No renormalization needed (PCAC): $f_P m_P^2 = (m_a + m_b)\langle 0|\bar{a}\gamma_5 b|P\rangle$

Bayesian fit of the masses and decay constants to the chiral and continuum limits: `continuum NLO ChPT` + $\mathcal{O}(a^2)$

$\mathcal{O}(a^2) \propto \alpha_s a^2, \alpha_s^3 a^2, \alpha_s^3 a^2 log(x_{u,d}), \alpha_s^3 a^3 x_{u,d}$ with $x_q \propto m_q$

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$$f_{D_s} = (241 \pm 3)\mathrm{MeV} \qquad f_D = (208 \pm 4)\mathrm{MeV} \qquad f_{D_s}/f_D = 1.162(9)$$

Very good agreement with **FNAL/MILC**.

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ETMC $\boxed{N_f = 2}$ (talk by C. Tarantino) **Preliminary**

Twisted mass QCD at maximal twist.

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Lattice spacing: $a = 0.1\ fm, 0.0855\ fm, 0.0667\ fm$ with light quark masses (`full QCD`) $m_s/5 - m_s/2$, several m_s and m_c around the physical ones (interpolation)

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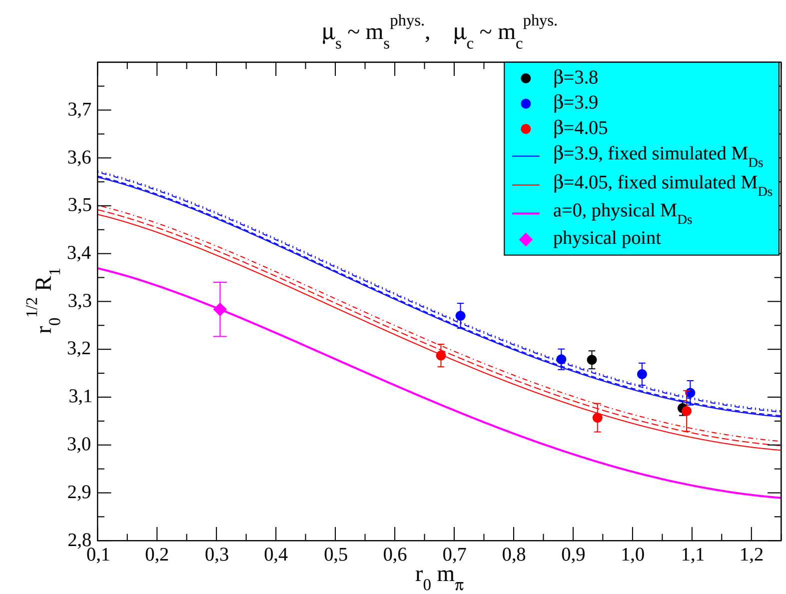
Combined fit: meson mass dependence (`NLO`) + $\mathcal{O}(a^2)$ terms

* Use $SU(2)$ HMχPT
* Decay constants extracted from the ratios:

$$R_1 = f_{D_s}\sqrt{M_{D_s}}/f_K$$

$$R_2 = \left[f_{D_s}\sqrt{M_{D_s}}/f_K\right] / \left[f_D\sqrt{M_D}/f_\pi\right]$$

(smooth chiral behaviour)



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$$f_D = (197 \pm 7 \pm 12)^* \mathrm{MeV} \qquad f_{D_s} = (244 \pm 4 \pm 11)^* \mathrm{MeV}$$

$$f_{D_s}/f_D = (1.24 \pm 0.04 \pm 0.02)^*$$

* Estimate of the errors is preliminary: statistics $\pm$ systematics (continuum extrapolation and chiral extrapolation).

Systematic errors dominated by cut-off effects.

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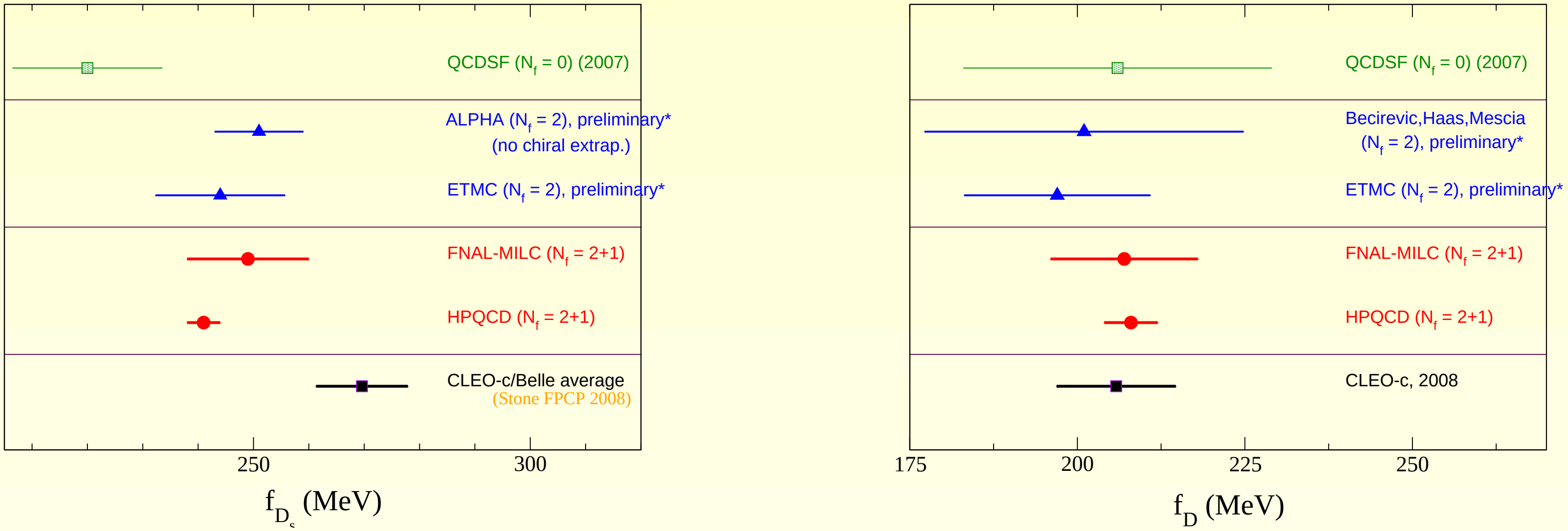
Good agreement with complete $N_f = 2 + 1$ calculations.

* But still missing part of the vacuum polarization effects.

Disagreement for f_{D_s} between lattice and experiment



\# $> 3\sigma$ discrepancy between experiment and **HPQCD** f_{D_s}
(1.6σ with **FNAL/MILC**
and all lattice numbers smaller than experiment).

* Experiment - **HPQCD** agree in f_K, f_π, f_D, m_D, m_{D_s}, $\frac{2m_{D_s} - m_{\eta_c}}{2m_D - m_{\eta_c}}$
(with errors $\leq 2\%$).

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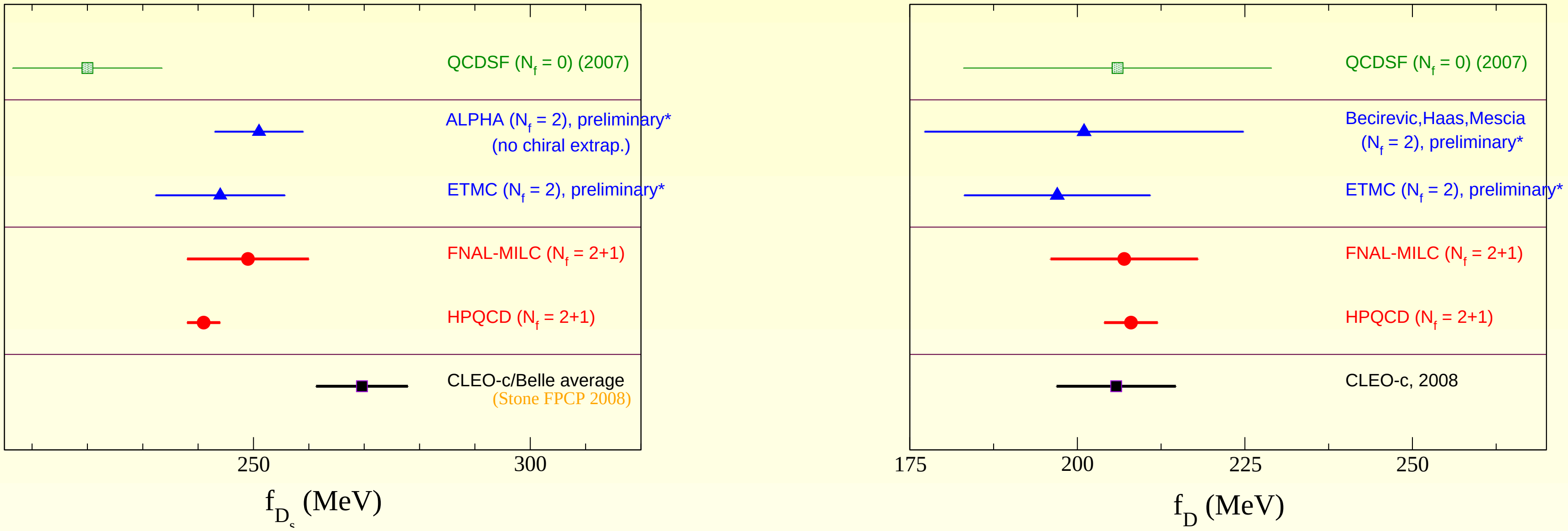

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(with errors $\leq 2\%$).

Good check: other $N_f = 2 + 1$ calculations with $< 5\%$ accuracy or better.

Disagreement for f_{D_s} between lattice and experiment

Experimental issues to be addressed:

* Experiment uses $V_{cs} = V_{ud}$/PDG's global CKM fit.
* Radiative corrections $D_s \to D_s^* \gamma \to \mu\nu\gamma$ estimated to be 1%.

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Sensitive to **BSM** physics: Starting to see evidence of nonstandard leptonic decays of D_s mesons? (**talk by A. Kronfeld**)

f_B and f_{B_s}

Extraction of CKM matrix elements: $\underbrace{B(B^- \to \tau^- \bar{\nu}_\tau)}_{\textbf{experiment}} \propto |V_{ub}|^2 \underbrace{f_B^2}_{\textbf{lattice}}$

Decay constants needed in the SM prediction for processes potentially very sensitive to BSM effects: for example, f_{B_S} for $B_s \to \mu^+\mu^-$

$B^- \to \tau^- \bar{\nu}_\tau$ is a sensitive probe of effects from charged Higgs bosons.

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Extrapolation method:

Relativistic simulations at masses $\sim m_c$ $\longrightarrow$ **fit functions determined by HQET** bottom

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Step Scaling Method (HQET):

* Simulate b in a small volume: calculate an observable $O(L_0, m_b)$.
* Eliminate finite size effects through SS functions:
  ** $\sigma(L, s, m_h) = \frac{O(sL, m_h)}{O(L, m_h)}$ for $s > 1$ and $m_h < m_b$
  ** Assume mild dependence of finite size effects on high energy scale
* Extrapolate SS functions in $1/m_h$ to m_b

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HQET: static + $1/M$

f_B and f_{B_s}

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`Light quarks`: improved staggered (Asqtad)

Same set-up as for the f_D, f_{D_s} determination.

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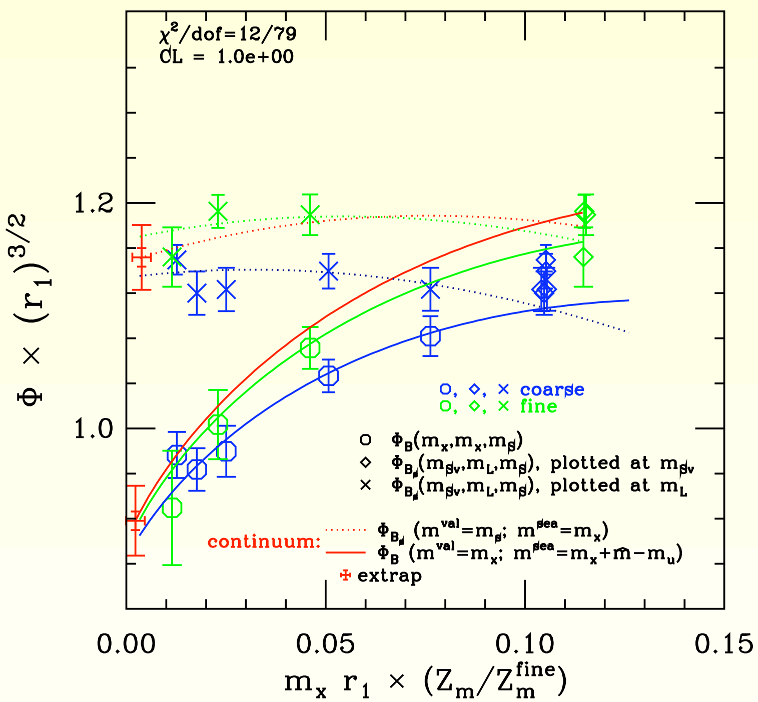

* After chiral and continuum extrapolations with SχPT

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$f_{B_s}/f_B = 1.25 \pm 0.04$

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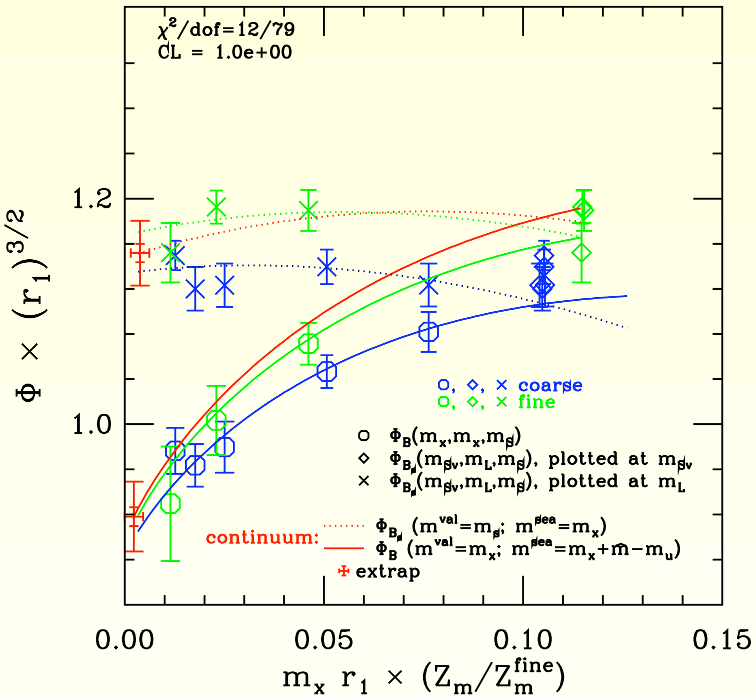

* After chiral and continuum extrapolations with SχPT
* Error dominated by statist. and light quark discretization errors + chiral extrap.
* Error in the ratio f_{B_s}/f_{B_d} is 3%

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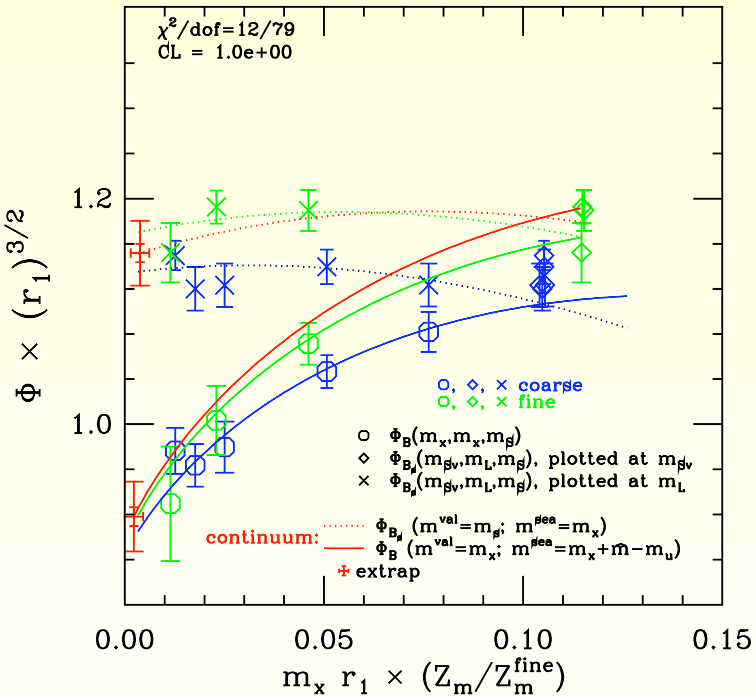

* After chiral and continuum extrapolations with SχPT
* Error dominated by statist. and light quark discretization errors + chiral extrap.
* Error in the ratio f_{B_s}/f_{B_d} is 3%

$f_B = (195 \pm 11)\mathrm{MeV}$ $\quad f_{B_s} = (243 \pm 11)\mathrm{MeV}$

$f_{B_s}/f_B = 1.25 \pm 0.04$

agree with HPQCD, PRL 95(2005)212001

$$f_B = (216 \pm 22)\mathrm{MeV} \quad f_{B_s} = (260 \pm 26)\mathrm{MeV} \quad f_{B_s}/f_B = 1.20 \pm 0.03$$

HPQCD errors dominated by higher-order perturbative renormalization

f_B and f_{B_s}

ALPHA $\boxed{N_f = 0}$

Action and currents: $\mathcal{O}(a)$ improved Wilson action.

f_B and f_{B_s}

ALPHA $\boxed{N_f = 0}$

Action and currents: $\mathcal{O}(a)$ improved Wilson action.

extrapolation+static

M. Della Morte et al, JHEP 0802(2008)078

* Continuum static approximation
 + relativistic QCD with masses
 around $m_c \rightarrow$ interpolation to
 the physical point of B_s

$$r_0^{3/2} \frac{F_{PS}\sqrt{m_{PS}}}{C_{PS}(M/\Lambda_{\overline{MS}})} = A\left(1 + \frac{B}{r_0 m_{PS}}\right)$$

* 10% correction of the slope at
 the physical b `quark` mass.

f_B and f_{B_s}

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SSM+static

D. Guazzini et al, JHEP 0802(2008)078

* Combine HQET and Step Scaling Method (SSM)
  ** SS functions calculated for several masses around m_c and static limit $\rightarrow$ interpolation to B_s
* Extrapolation in $1/(Lm_h)$ linear $\sim$ quadratic
* Corrections to the static limit very small at the b quark mass.

f_B and f_{B_s} ALPHA $\boxed{N_f = 0}$

extrapolation+static **SSM+static**

$$\boxed{f_{B_s} = 191(6)\mathrm{MeV}} \qquad \boxed{f_{B_s} = 193(7)\mathrm{MeV}}$$

Both results are in very good agreement

  * Also interesting to compare with HQET including $1/m$ corrections.

Inclusion of static point improves control over heavy quark mass dependence

f_B and f_{B_s} ALPHA $\boxed{N_f = 0}$

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Inclusion of static point improves control over heavy quark mass dependence

Quenched effects very large in f_{B_s}:

$$\sim 250 \quad \underbrace{\Rightarrow}_{quenching} \quad \sim 190$$

 * Promising methods to extend to unquenched simulations

Static-Light studies in progress

RBC/UKQCD $N_f = 2+1$ (**talk by T. Ishikawa**)

Light quarks formulation: domain wall.

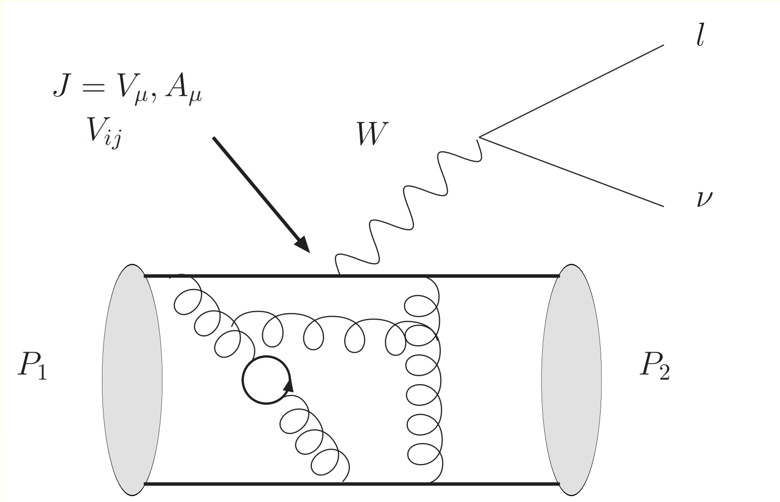
f_B and $B^0 - \bar{B}^0$ mixing analyses in progress.

ETMC $N_f = 2$ (**talk by M. Wagner**)

Light quarks formulation: tmQCD.

Spectrum results presented for B_s mesons.

3. Semileptonic decays



New lattice techniques

* Use of **double ratios** with cancellation of statistical and systematic errors and simpler χPT expressions

* Choose an adequate (model independent) parametrization of the shape to describe the form factor in the allowed q^2 region.

* Twisted boundary conditions allowed to go to smaller values of q^2.

Exclusive $B \to D^* l\nu$: determination of $|V_{cb}|$

$B \to D^* l\nu$ rate depend on four form factors:

$$F^{B\to D^*}(\omega) = h_{A_1}(\omega)\sqrt{\frac{H_0^2(\omega)+H_+^2(\omega)+H_-^2(\omega)}{\lambda(\omega)}}$$

  * ...but at zero recoil $\propto |V_{cb} h_A(1)|$.

Experimental errors at zero recoil for $B \to D^* l\nu$ smaller than for $B \to Dl\nu$.

$|V_{cb}|$ needed as an input in ϵ_K and rare kaon decays ($Br(K \to \pi\nu\bar{\nu})$).

  * More relevant after progress in B_K.

Exclusive $B \to D^* l\nu$: determination of $|V_{cb}|$

FNAL/MILC (J. Laiho 2008) $N_f = 2+1$

MILC configurations, Asqtad for light quarks and Fermilab action for heavy quarks.

Exclusive $B \to D^* l\nu$: determination of $|V_{cb}|$

FNAL/MILC (J. Laiho 2008) $N_f = 2+1$

MILC configurations, Asqtad for light quarks and Fermilab action for heavy quarks.

New double ratio method: $|h_A(1)|^2 = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma_j\gamma_5 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\bar{B}|\bar{b}\gamma_4 b|\bar{B}\rangle}$

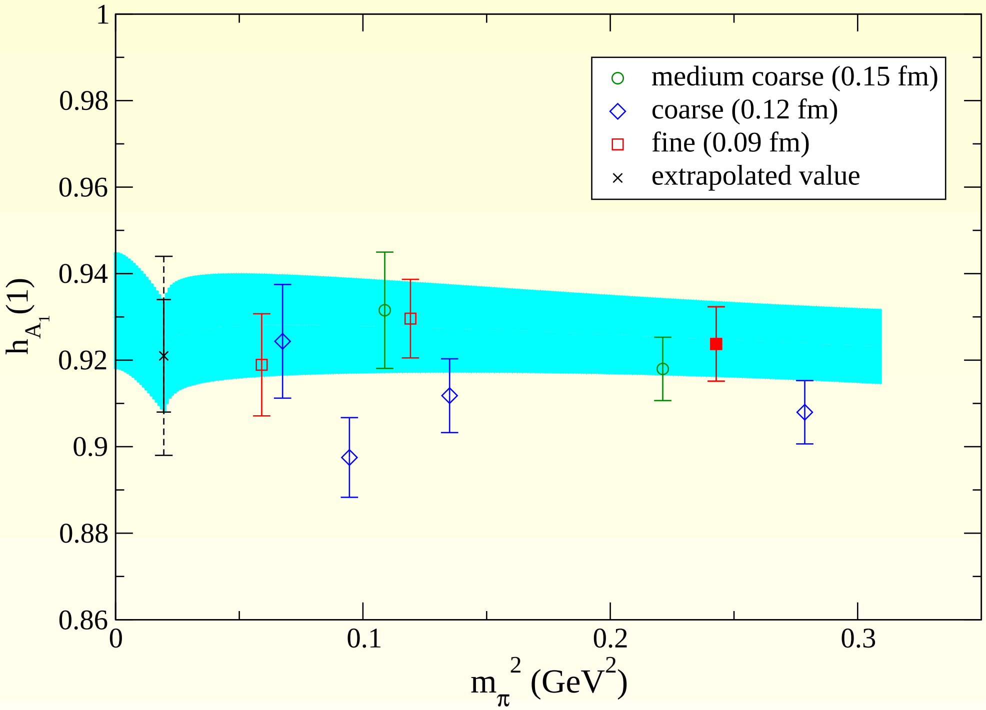
* Cancellation of statistical and systematic errors (particularly, renormalization mostly cancel).

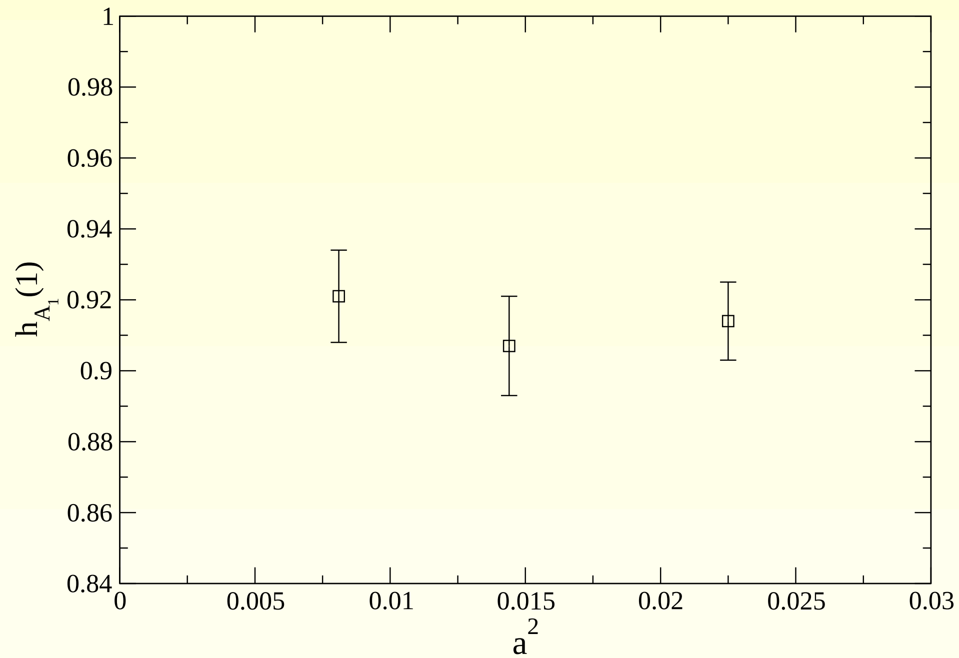
* h_{A_1} given directly to all orders in HQET

* Ratio can be calculated at tuned $m_{b,c}$ $\to$ computationally more efficient than previous FNAL/MILC method

Exclusive $B \to D^* l\nu$: determination of $|V_{cb}|$

FNAL-MILC (J. Laiho 2008) $N_f = 2 + 1$





full QCD points

\# Use NLO + analytic NNLO SχPT.

\# Very mild chiral and continuum extrapolations

Exclusive $B \to D^* l\nu$: determination of $|V_{cb}|$

FNAL-MILC (J. Laiho 2008) $N_f = 2 + 1$

uncertainty	$h_{A_1}(1)$
statistical	1.4%
g_π	0.9%
NLO vs partial NNLO ChPT fits	0.9%
discretization errors	1.5%
kappa tuning	1.0%
perturbation theory	0.3%
u_0 tuning	0.4%
Total	2.7%

$$h_{A_1}(1) = 0.921(13)_{stat.}(21)_{syst.}$$

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$$h_{A_1}(1) = 0.921(13)_{stat.}(21)_{syst.}$$

HFAG average: $h_A(1)|V_{cb}| = (36.0 \pm 0.6) \times 10^{-3}$

$$\Longrightarrow \qquad |V_{cb}| = (38.8 \pm 0.6_{exp} \pm 1.0_{theo}) \times 10^{-3}$$

Inclusive determination is $|V_{cb}| = 41.7(0.7)_{total} \times 10^{-3}$ (2σ difference)

Exclusive $B \to D^* l\nu$: determination of $|V_{cb}|$

G.M. Divitiis, R. Petronzio and N. Tantalo $N_f = 0$ **Preliminary**

\# **Twisted flavour boundary condit.**: Calculate $F^{B\to D^*}(\omega)$ for $\omega \geq 1$.

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\# **SS method**: SS functions almost insensitive to initial heavy quark mass m_h for $m_h > m_c$.

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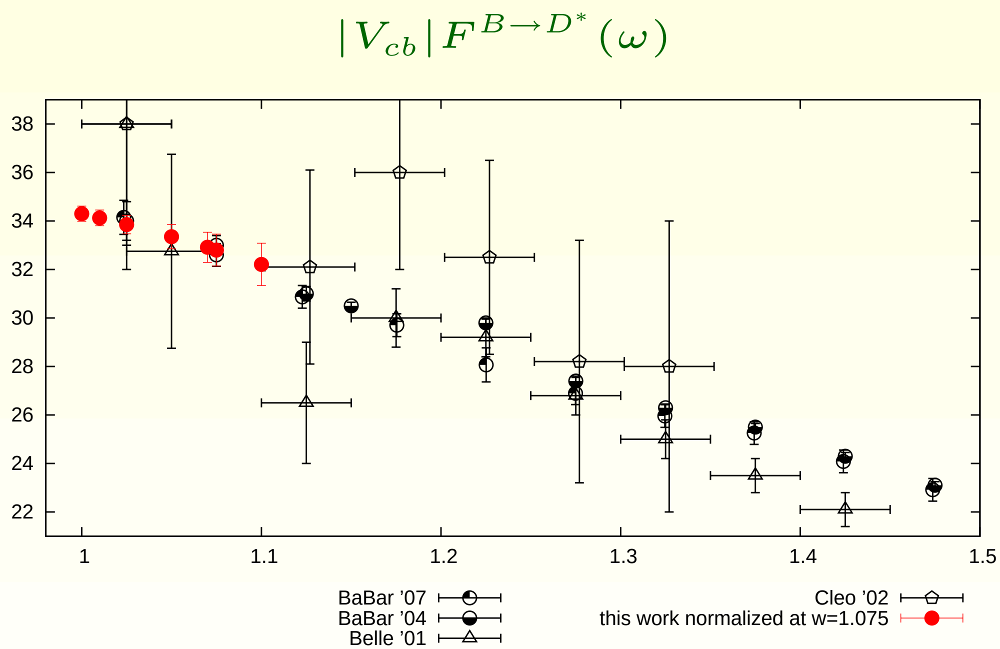

← **Matching experimental and lattice data at $\omega = 1.075$**

$$F^{B\to D^*}(1) = 0.917 \pm 0.008 \pm 0.005$$

* First error is statist. (including extrap.) and second error is renorm. factors in the small V

Exclusive $B \to Dl\nu$: determination of $|V_{cb}|$

G.M. Divitiis, E. Molinaro, R. Petronzio and N. Tantalo, JHEP 0710(2007)062; PLB 655(2007)45 $N_f = 0$

Form factors for $H_i \to H_f l\nu$ with $H_{i,f} = B, D$ and $l = e, \mu, \tau$

Same methodology as for $B \to D^* l\nu$

Exclusive $B \to Dl\nu$: determination of $|V_{cb}|$

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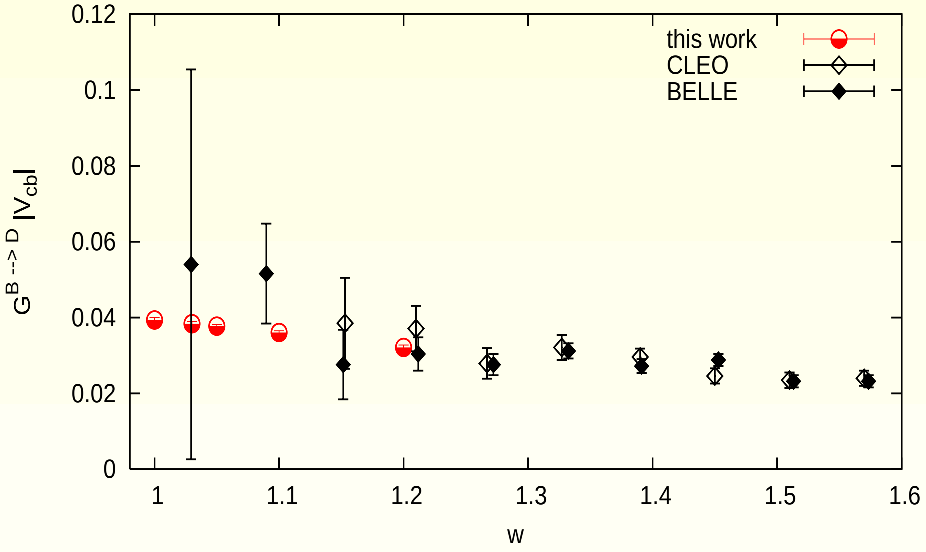

Twisted flavour boundary condit.

$\longrightarrow$ **Form factors for** $1 \leq \omega = v_i \cdot v_f \leq 2$

where experimental data are available

$\longrightarrow$ **no need for extrapolation**

Exclusive $B \to Dl\nu$: determination of $|V_{cb}|$

G.M. Divitiis, E. Molinaro, R. Petronzio and N. Tantalo, JHEP 0710(2007)062; PLB 655(2007)45 $N_f = 0$

Calculation of $\Delta^{D\to B}(\omega)$ (linear combination of the 2 form factors), which parametrizes the difference between $B \to De, \mu\nu_{e,\mu}$ and $B \to D\tau\nu_\tau$.

* $\Delta^{D\to B}$ can be extracted from $\frac{d\Gamma(B\to D\tau\nu_\tau)}{d\Gamma(B\to De,\mu\nu_{e,\mu})}$

$\Longrightarrow$ to be checked by experiment.

** Independent of CKM inputs.

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  $\Longrightarrow$ to be checked by experiment.

  ** Independent of CKM inputs.

* Lepton-flavour universality checks on the extraction of V_{cb} are possible.

* The ratio of partially integrated rates $Br(B \to D\tau\nu_\tau)/Br(B \to De\nu_e)$ is a good place to look for charged Higgs contributions to low energy observables (**J.F. Kamenik and F. Mescia, arXiv:0802.3790**)

$B \to \pi l \nu$: determination of $|V_{ub}|$

$$Br(B \to \pi l \nu) = |V_{ub}|^2 \int_0^{q^2_{max}} dq^2 f_+^{B\to\pi}(q^2)^2 \times (\text{known factors})$$

Problem: Poor overlap in q^2 between lattice and experiment
$\to$ increases the total error

Work in progress to reduce total error.

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* **Moving NRQCD**: Generate data at low q^2 + keeping statical errors under control K. Wong Lattice2007.

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* **z-fit**: combine lattice and experimental data over full q^2 region using model-independent expression based on analyticity and unitarity to describe the shape of the form factor

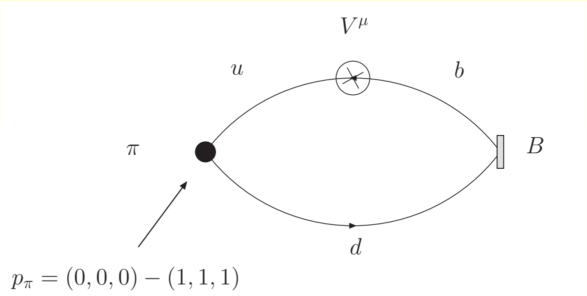
Arnesen et al.; Becher & Hill; P. Ball; P. Mackenzie and R. Van de Water

FNAL/MILC $N_f = 2+1$ (talk by R. Van de Water) **Preliminary**

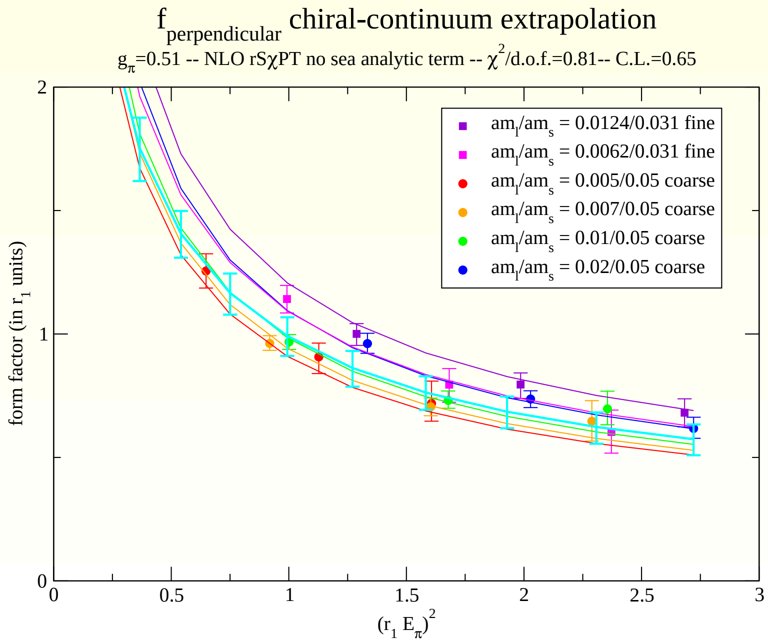
$B \to \pi l \nu$: determination of $|V_{ub}|$

FNAL/MILC $N_f = 2+1$ (talk by R. Van de Water) **Preliminary**

MILC configurations: $a = 0.15\ fm, 0.12\ fm, 0.09\ fm$, full QCD for nine light quark masses.



$$C_{3,\mu}(t_x, t_y, \vec{p}) = \sum_{\vec{x}\vec{y}} e^{i\vec{p}\cdot\vec{y}} \langle O_\pi(0) V_\mu O_B^\dagger \rangle$$



SχPT

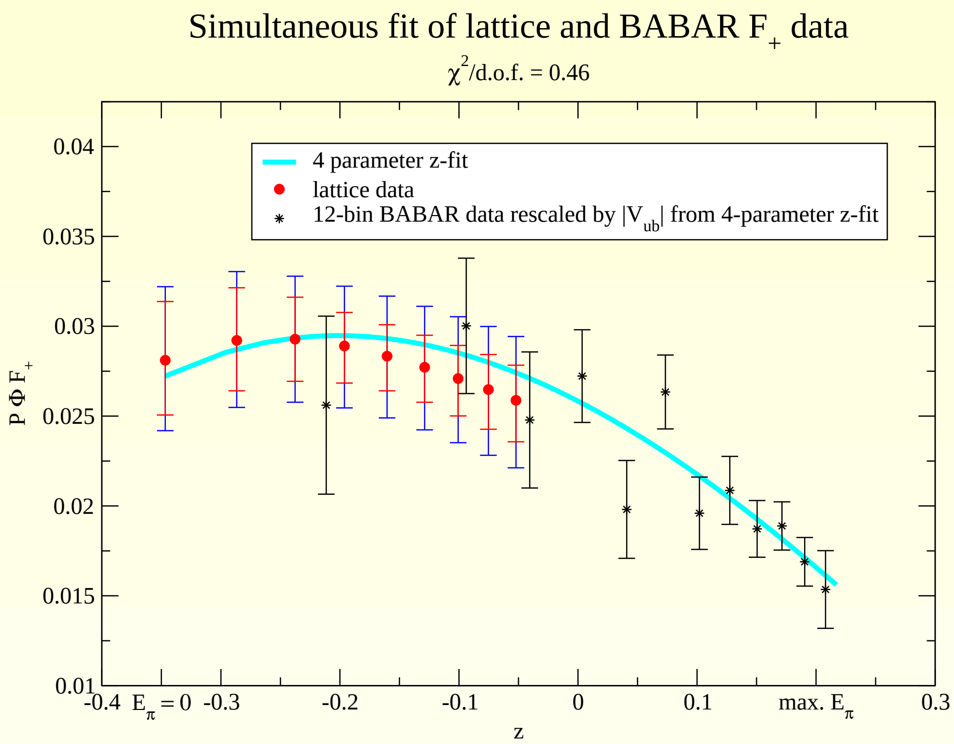
continuum + chiral extrapolation

(separate fit for $f_\perp$ and $f_\parallel$)

* NLO for $f_\perp$ (dominated by B^* pole)
* NLO + $m_q E_\pi$ + E_π^3 + $m_q E_\pi^2$ for $f_\parallel$.

$B \to \pi l \nu$: determination of $|V_{ub}|$

FNAL/MILC $N_f = 2+1$ (talk by R. Van de Water) **Preliminary**



Simultaneous **z-fit** to lattice and BaBar data gives a **model independent** determination of $|V_{ub}|$

* Lattice error dominated by statistics and chiral+continuum extrapolation errors.

(8% and 7%)

$$|V_{ub}| = (2.94 \pm 0.35) \times 10^{-3}$$ (12% error)

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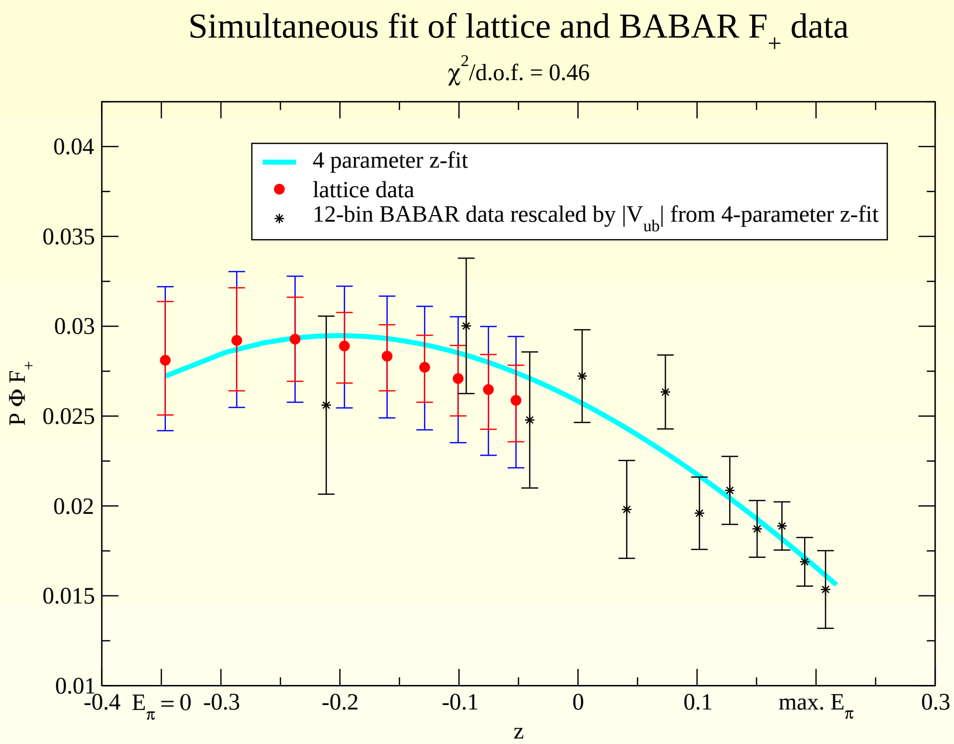

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\# **Improvements**: Double number of configurations, randomize spatial origin, finer lattice spacing, partial quenched points ...

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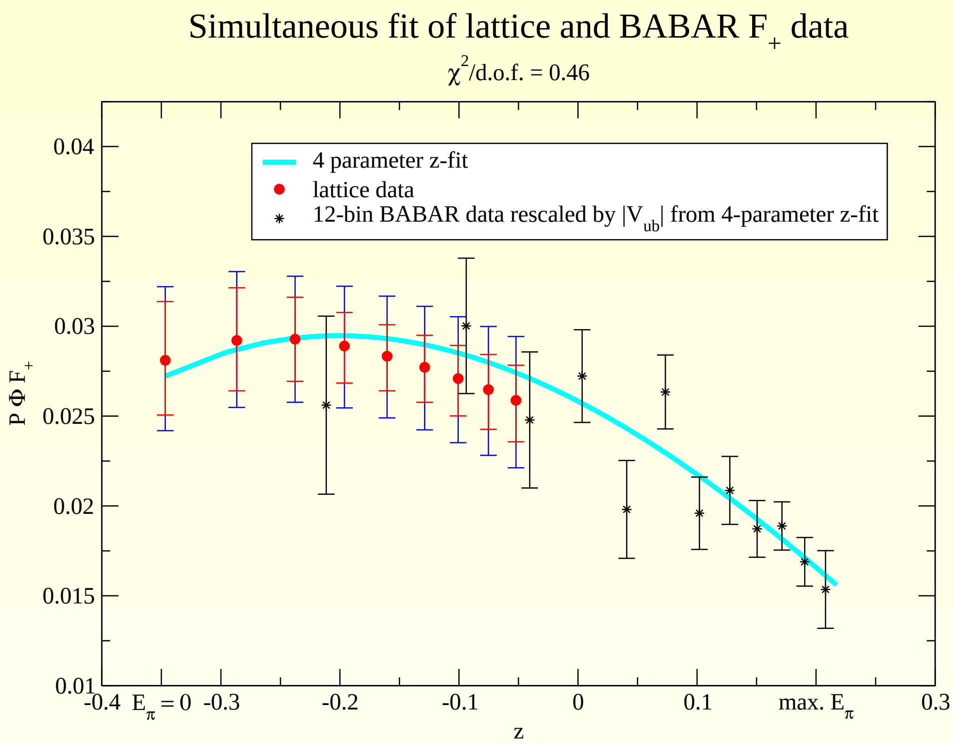

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\# **Improvements**: Double number of configurations, randomize spatial origin, finer lattice spacing, partial quenched points ...

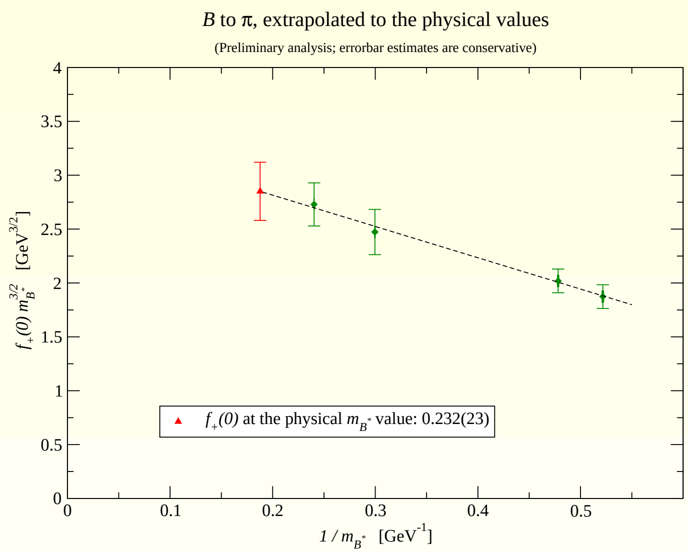
\# 2σ lower than inclusive determinations.

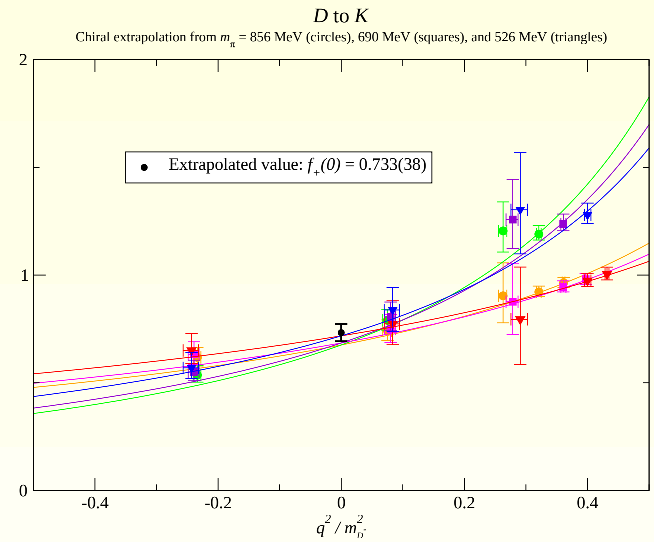
$B \to \pi l\nu$: determination of $|V_{ub}|$

QCDSF $N_f = 0$ **Preliminary**

\# Calculate form factors for $B \to \pi(K)l\nu$, $D \to \pi(K)l\nu$ and $D_s \to Kl\nu$, with $\mathcal{O}(a)$ improved Wilson at a single $a = 0.04\ fm$.

\# Physical c, quite heavy b (no need of an important $1/m_H$ extrapolation) and 3 light masses $m_\pi^{min} = 526 MeV$ (very heavy).





Becirevic-Kaidalov parametrization

$f_+^{B\to\pi}(0) = 0.232(23)^*$ $\quad$ $f_+^{B\to K}(0) = 0.29(3)^*$

$f_+^{D\to\pi}(0) = 0.668(38)^*$ $\quad$ $f_+^{D\to K}(0) = 0.733(38)^*$ $\quad$ $f_+^{D_s\to K}(0) = 0.598(20)^*$

* Systematic error analysis still in progress.

D **meson decays:** $V_{cd(s)}$ **from** $D \to \pi(K) l\nu$

$Br(D \to K e\nu)$ + lattice form factors $\to$ best determination of V_{cs}

FNAL/MILC, PRL95(2005)122002

\+ CLEO-c

$$|V_{cs}| = 1.015 \pm 0.015 \pm 0.106$$

$Br(D \to \pi e\nu)$ + (improved) lattice form factors $\to$ potentially best determination of V_{cd}

D meson decays: $V_{cd(s)}$ from $D \to \pi(K) l\nu$

$Br(D \to Ke\nu)$ + lattice form factors → best determination of V_{cs}

FNAL/MILC, PRL95(2005)122002

+ CLEO-c

$$|V_{cs}| = 1.015 \pm 0.015 \pm 0.106$$

$Br(D \to \pi e\nu)$ + (improved) lattice form factors → potentially best determination of V_{cd}

Semileptonic-leptonic decays **ratios**

$$\frac{1}{\Gamma(D^+ \to l\nu)} \frac{d\Gamma(D \to \pi l\nu)(q^2)}{dq^2} \quad \text{and} \quad \frac{1}{\Gamma(D_s \to l\nu)} \frac{d\Gamma(D \to K l\nu)(q^2)}{dq^2}$$

* independent of $|V_{cq}|$ → consistency check

* Smoother chiral extrapolation to the physical pion mass.

D meson decays: V_{cd} from $D \to \pi l \nu$

ETMC $N_f = 2$ **Preliminary**

Twisted mass QCD at maximal twist.

* $a \simeq 0.086\ fm$ and $V * T = 24^3 * 48$.

* Four values of $am_h = 0.25(\sim am_c) - 0.46$ and six values of $am_l^{sea} = am_l^{val.}$ $(0.3 \leq m_\pi(GeV) \leq 0.6)$.

* Use all-to-all propagators computed with a stochastic method and twisted boundary conditions.

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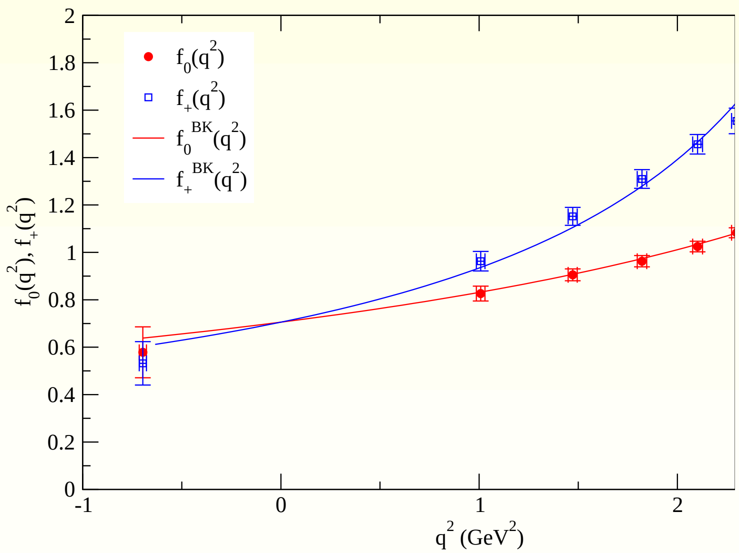

BK parametrization:

$$f_+(q^2) = \frac{f(0)}{(1-q^2/M_{D^*}^2)}\left(1 - \alpha q^2/M_{D^*}^2\right)$$

$$f_0(q^2) = \frac{f(0)}{(1-q^2/(\beta M_{D^*}^2))}$$

`Extrapolation` (linear) to physical π / `interpolation` to physical D.

D **meson decays:** V_{cd} **from** $D \to \pi l \nu$

Becirevic, Haas and Mescia $N_f = 2$ **Preliminary**

Improved $\mathcal{O}(a)$ Wilson

* Configurations from QCDSF
* $a \simeq 0.08\ fm$, $m_\pi^{sea} = 770, 585, 380\ MeV$ and m_h close to m_c.

Double ratio

* $D - meson$ at rest and inject momenta to the pion.
* Twisted boundary conditions
* NP renormalization mostly cancelled

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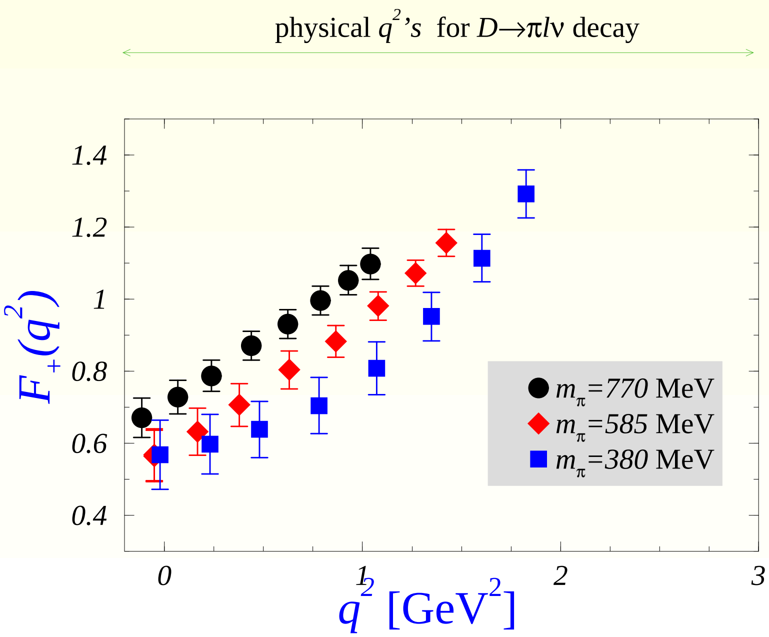

Qualitative change in the shape of f_+ and f_0 when m_π goes to physical value.

(polar behaviour more visible)

Extrapolation (linear and HMχPT) to physical π.

* No sensitive to logarithms

D **meson decays:** V_{cd} **from** $D \to \pi l \nu$

source	$[f_+^{D\to\pi}(q^2 = 1 GeV^2)/f_{D+}] GeV^{-1}$
ETMC (linear fit) $N_f = 2$	4.39(31)$_{stat.}$ **Preliminary**
Becirevic et al. (linear fit) $N_f = 2$	3.76(54) **Preliminary**
Becirevic et al. (HMχPT fit) $N_f = 2$	4.32(56) **Preliminary**
reconstructed from CLEO	4.51(53)

Direct experimental determination of $f_+^{D\to\pi}/f_{D+}$.

D meson decays: V_{cd} from $D \to \pi l \nu$

source	$[f_+^{D\to\pi}(q^2 = 1GeV^2)/f_{D+}]GeV^{-1}$
ETMC (linear fit) $N_f = 2$	4.39(31)$_{stat.}$ **Preliminary**
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reconstructed from CLEO	4.51(53)

Direct experimental determination of $f_+^{D\to\pi}/f_{D+}$.

Need study of discretization errors.

D meson decays: V_{cd} from $D \to \pi l \nu$

source	$[f_+^{D\to\pi}(q^2 = 1GeV^2)/f_{D+}]GeV^{-1}$
ETMC (linear fit) $N_f = 2$	4.39(31)$_{stat.}$ **Preliminary**
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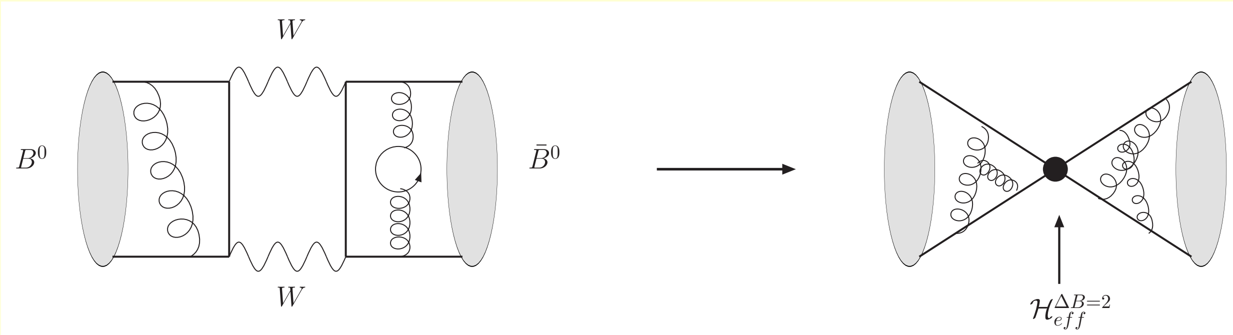
♣ ETMC result for $D \to K l \nu$:

$$f_+^{D\to\pi}(0)/f_+^{D\to K}(0) = 0.90 \pm 0.05_{stat.}$$

FNAL/MILC (2005), $N_f = 2 + 1$ $f_+^{D\to\pi}(0)/f_+^{D\to K}(0) = 0.87 \pm 0.03 \pm 0.09$

4. $B^0 - \bar{B}^0$ mixing: $\Delta M_{d,s}$, $\Delta\Gamma_{d,s}$ and ξ

theoretically: In the Standard Model



$\Delta M_q|_{theor.} \propto$

$|V_{tq}^* V_{tb}|^2 \quad f_{B_q}^2 \hat{B}_{B_q}$

;

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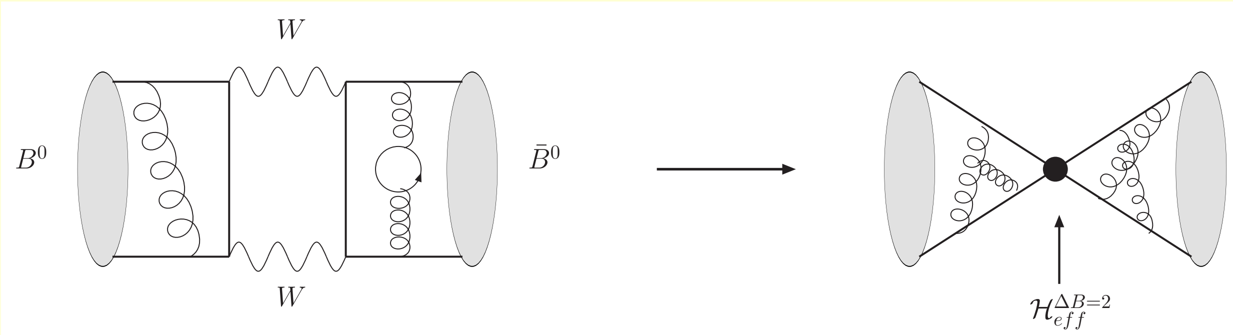

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$|V_{tq}^* V_{tb}|^2$ $\quad f_{B_q}^2 \hat{B}_{B_q}$

Non-perturbative input in $B^0 - \bar{B}^0$ analysis

$\frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_L | B_q^0 \rangle(\mu)$ with $O_L \equiv [\overline{b^i}\, q^i]_{V-A} [\overline{b^j}\, q^j]_{V-A}$

For $\Delta\Gamma_s$ one needs either O_S and O_L, or O_3 and O_L

$$O_S \equiv [\overline{b^i}\, s^i]_{S-P} [\overline{b^j}\, s^j]_{S-P} \qquad O_3 \equiv [\overline{b^i}\, s^j]_{S-P} [\overline{b^j}\, s^i]_{S-P}$$

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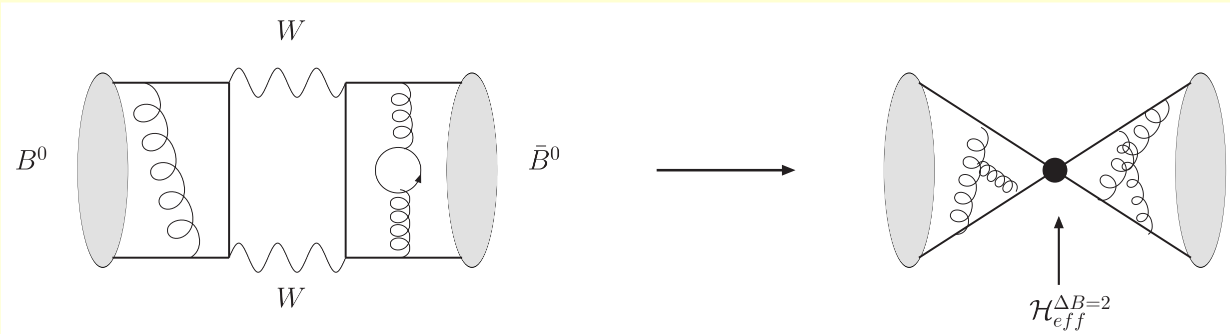

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$B^0 - \bar{B}^0$ system very sensitive to NP effects. Recent suggestions of NP effects in $B_s^0 - \bar{B}_s^0$ and $B_d^0 - \bar{B}_d^0$ mixing:

Bona et al. (UTfit Col.), arXiv:0803.0659; **E. Lunghi and A. Soni**, arXiv:0803.0512

E. Buras and A. Guadagnoli, arXiv:0805.3887

* **talk by A.Soni**: SM prediction for $\sin(2\beta)$ using $\Delta F = 2$ inputs (ξ and $\hat{B}_K$) disagrees by $\sim 2\sigma$ with direct experimental measurements via tree-level $B_d \to \psi K_s$ and penguin-loop $b \to s$ decays

** Independent of (controversial) $|V_{ub}|$

** It would imply the existence of a BSM CP-odd phase

$$\underbrace{\frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}}_{\xi}$$

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** Independent of (controversial) $|V_{ub}|$

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Important input for SM tests: ξ

$$\left|\frac{V_{td}}{V_{ts}}\right| = \underbrace{\frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}}_{\xi} \sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}$$

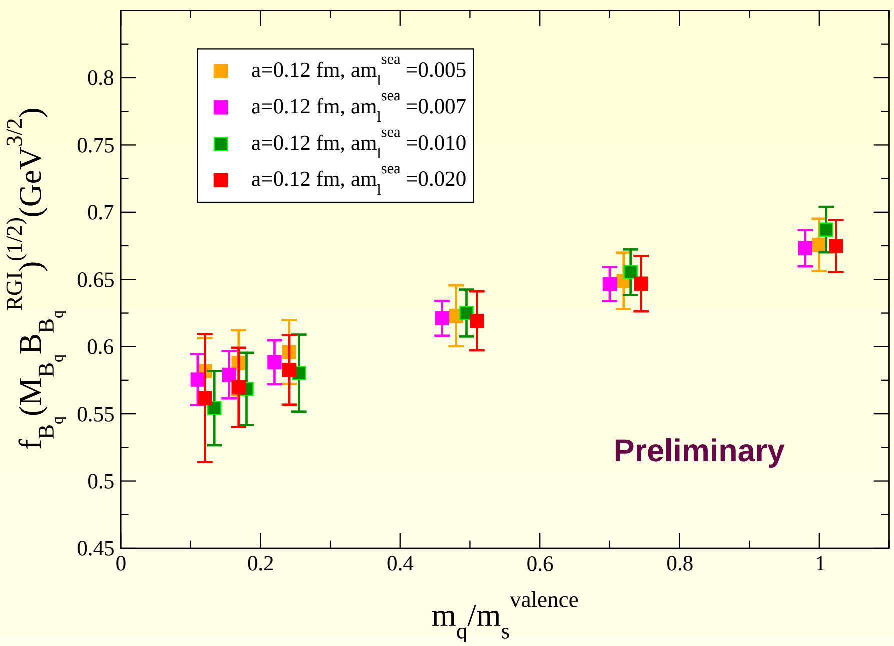
* Many uncertainties in the theoretical (lattice) determination cancel totally or partially in the ratio

$B^0 - \bar{B}^0$ **mixing:** $N_f = 2+1$ **Preliminary**

	FNAL/MILC (talk by R. Todd Evans)	HPQCD
# Calculation of all the matrix elements needed to determine $\Delta M_{d,s}$, $\Delta\Gamma_{d,s}$ and ξ		
# MILC configurations: Asqtad for `light sea` (and `valence`) quarks ($m_\pi^{min.} \simeq 230\mathrm{MeV}$)		
b `quarks`	Fermilab	NRQCD
a(fm)	0.15, 0.12, 0.09	0.12, 0.09
`light sea masses`	3 + 4 + 2	4 + 2
`light valence masses`	6 for each sea mass	full QCD
# Simultaneous fits of the `2-pt` and `3-pt` correlators for any four-fermion operator		
# Perturbative renormalization: one loop.		

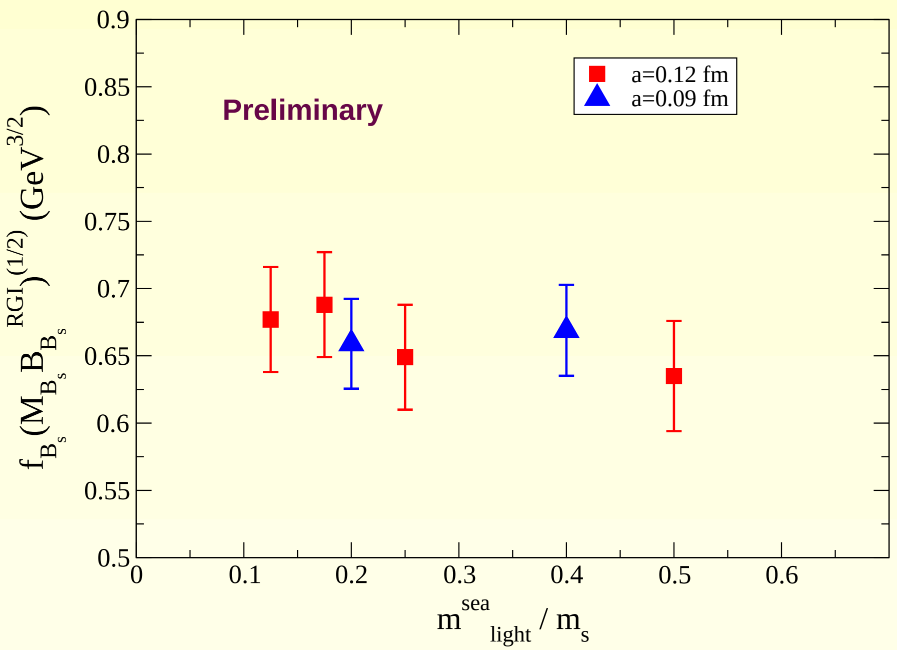
$B^0 - \bar{B}^0$ mixing: $N_f = 2+1$ Preliminary results for $f_{B_q}\sqrt{M_{B_q}\hat{B}_{B_q}}$

FNAL/MILC



Renormalization not applied yet.

HPQCD



All systematic sources included in error bars.

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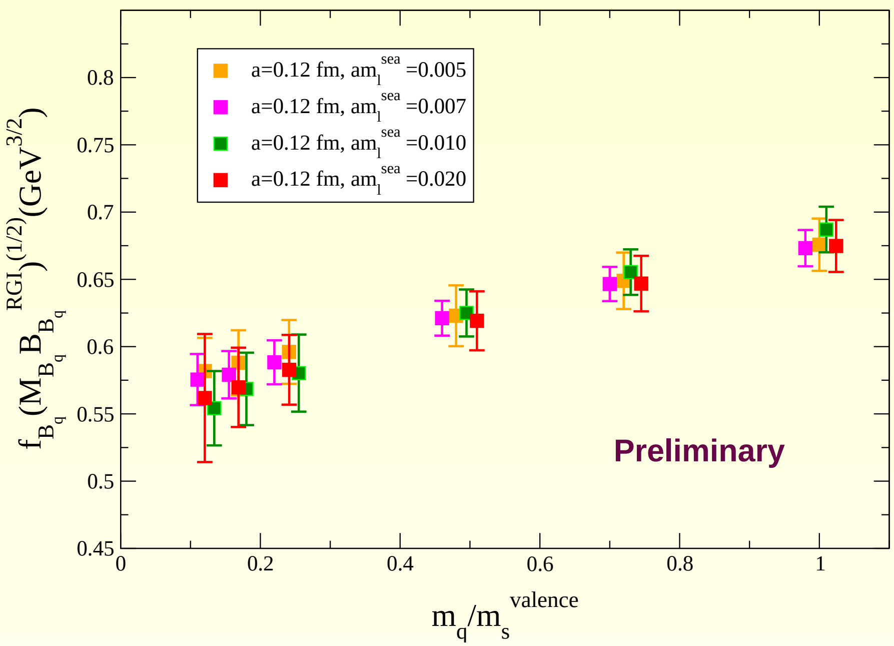

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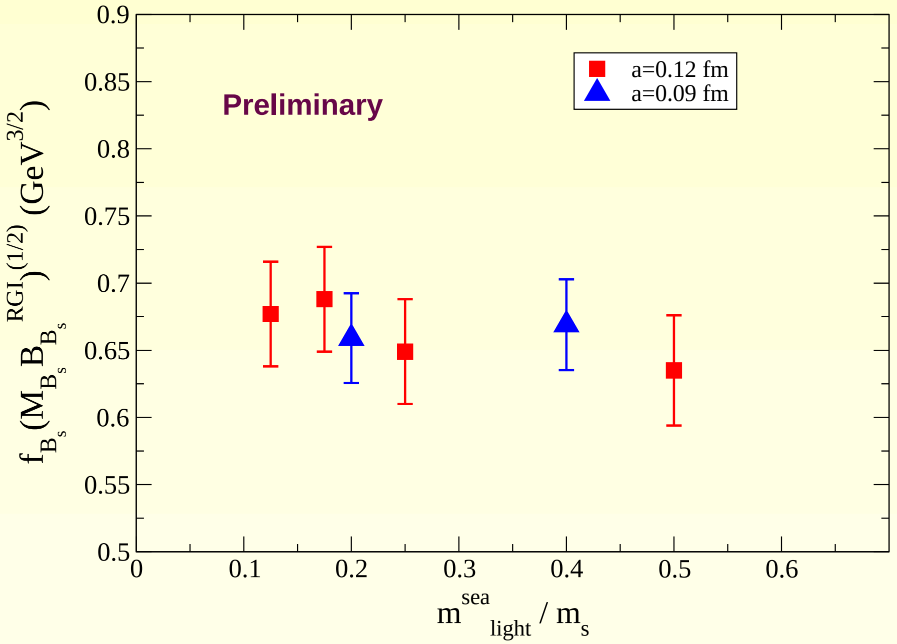

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Statistics+fitting errors: $1-4\%$ (B^0_s-B^0_d)

Very mild dependence on light sea quark masses.

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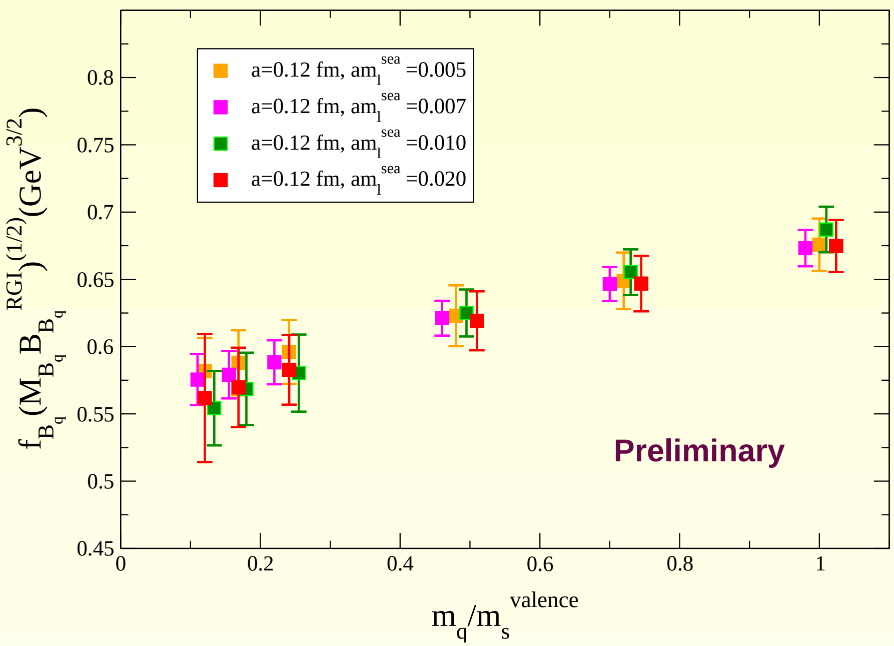

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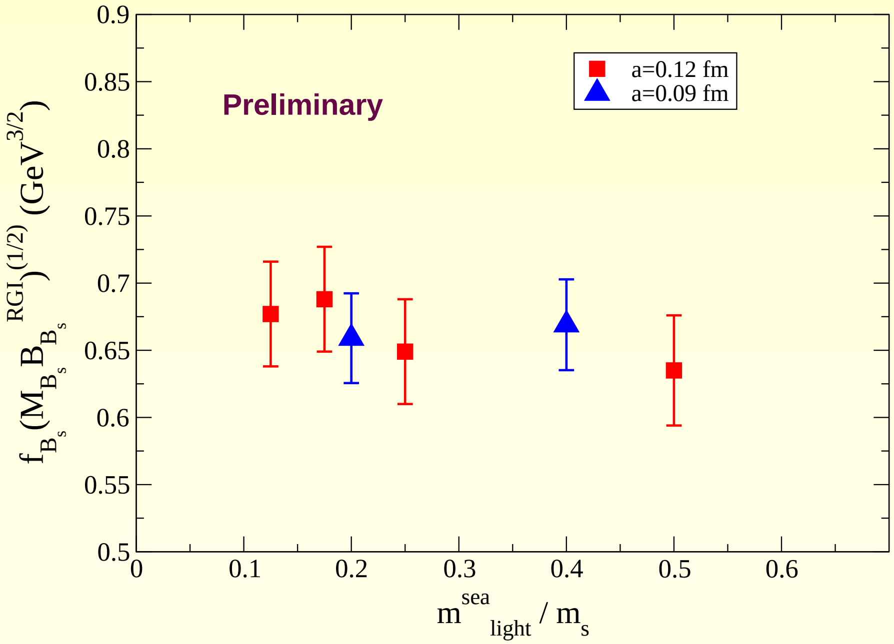

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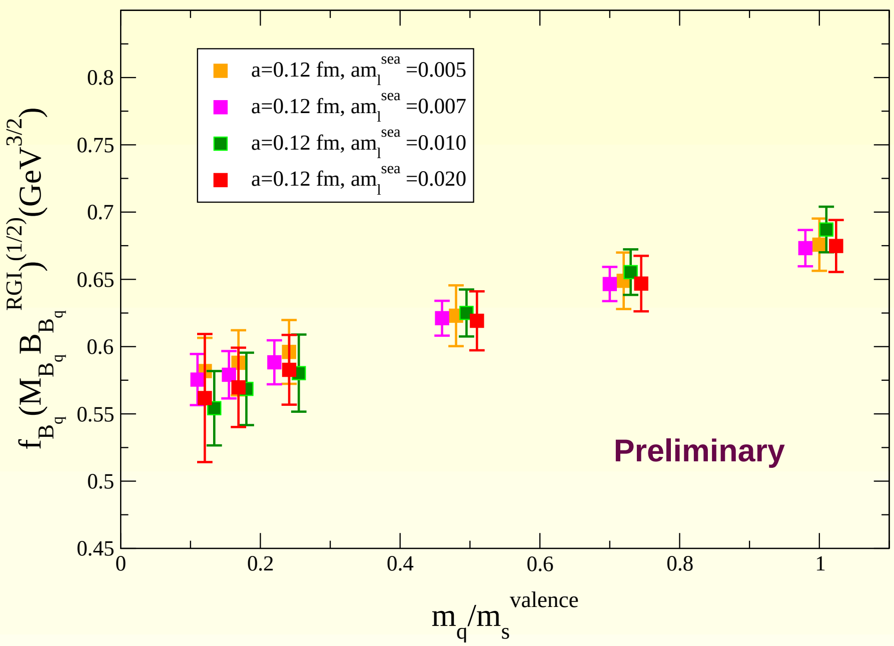

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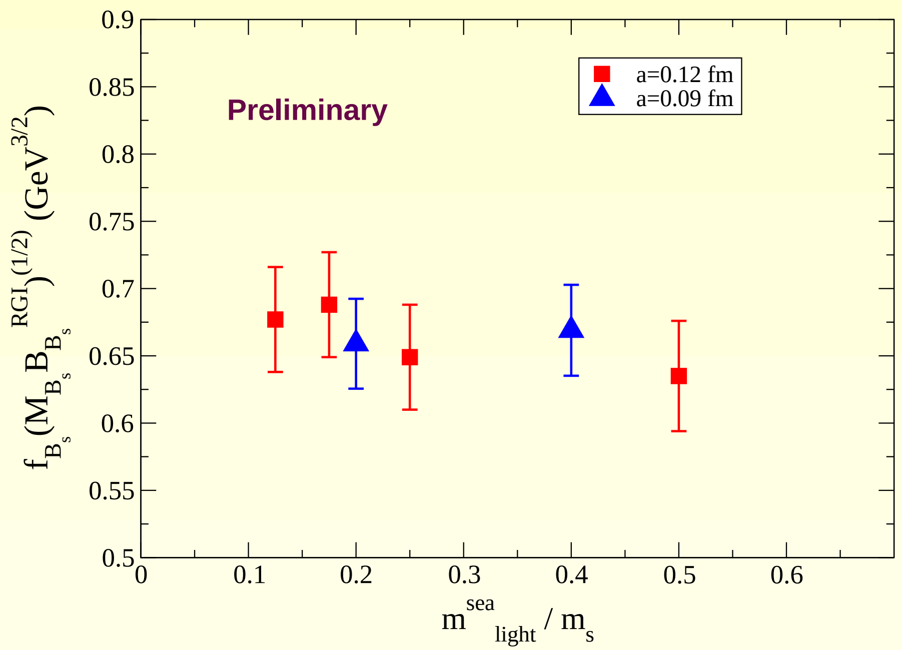

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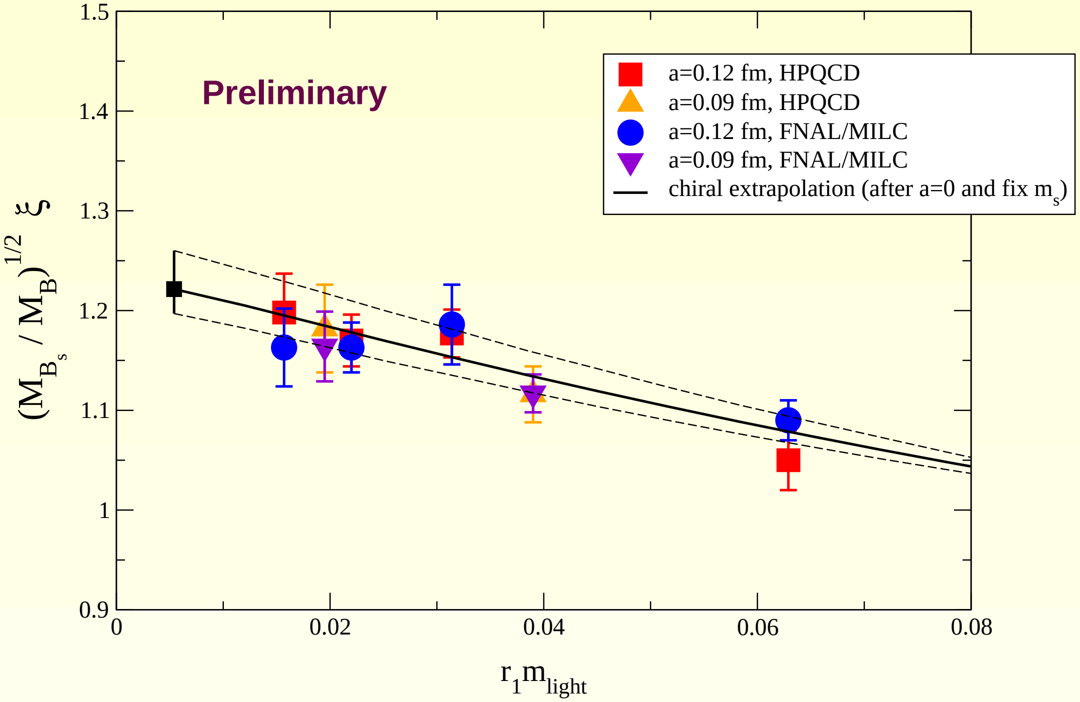
* HPQCD Relativistic corrections (after power law subtraction) are $\sim 5-6\%$ for coarse and $\sim 3-4\%$ for fine.

$B^0 - \bar{B}^0$ mixing: $N_f = 2+1$ Preliminary results for ξ

(FNAL/MILC & HPQCD)



* Only full QCD for FNAL/MILC shown.

Statistical errors: $1 - 3\%$

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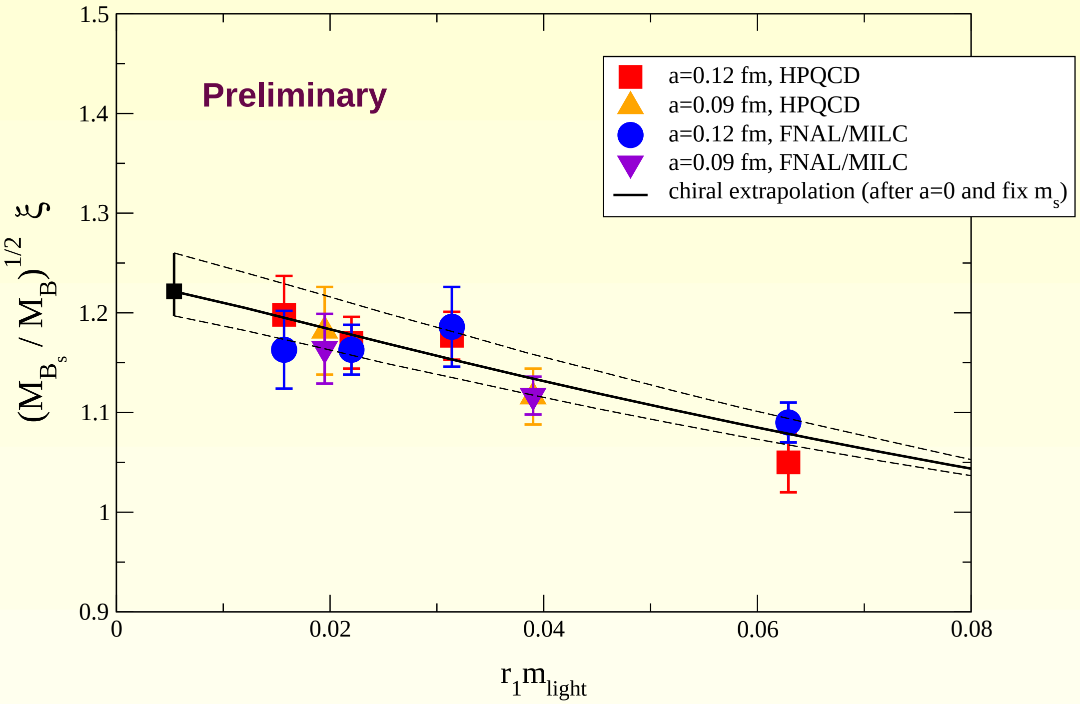

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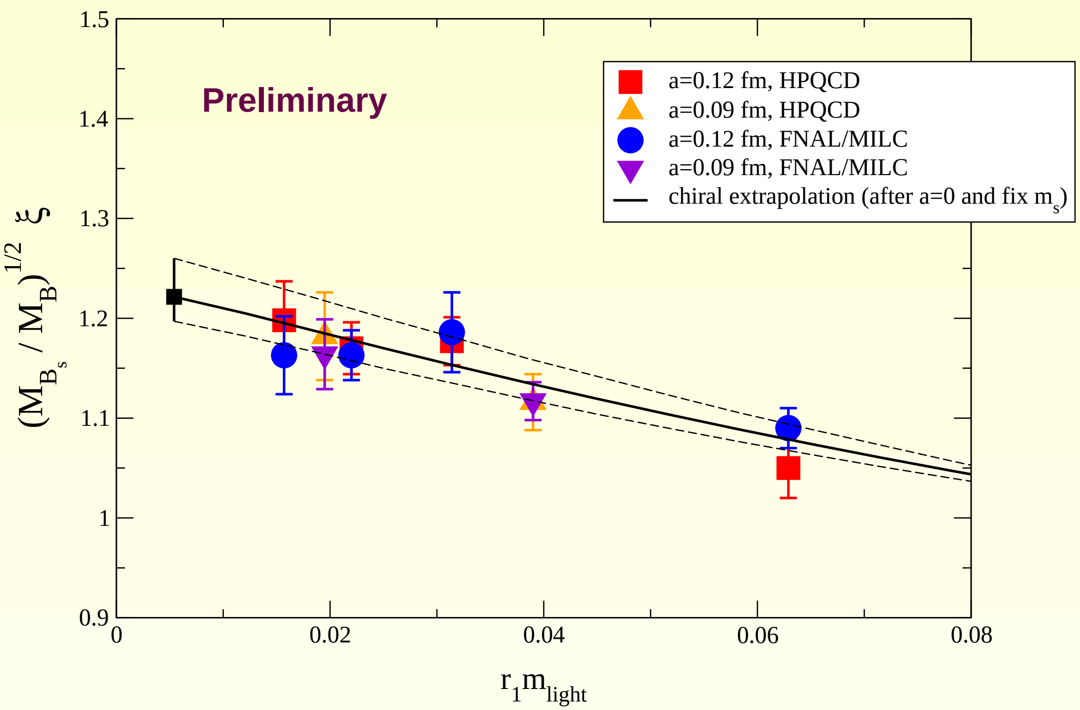

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FNAL/MILC: Simultaneous chiral and continuum extrapolation with SχPT at NLO + NNLO analytic terms:

$$\xi = 1.211 \pm 0.038 \pm 0.024_{estimate}$$

5. Heavy quark masses

Charm quark mass m_c

HPQCD, Chetyrkin, Kühn, Steinhauser & Sturm, arXiv:0805.2999

$N_f = 2 + 1$ (**talk by P. Lepage**)

Method analogous to the extraction of m_c from dispersion relations using perturbative **determination of zero-momentum** moments of current-current correlators **and** experimental **data from $e^+e^- \to hadrons$.**

m_c extracted from

* moments of charm-quark P, V and A correlators
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* **HISQ** action used to determine moments G_n ($j_5 \equiv \bar{\psi}_c \gamma_5 \psi_c$)

$$G_n \equiv \sum_t (t/a)^n G(t) \quad \text{with} \quad G(t) \equiv a^6 \sum_{\vec{x}} (am_{0c})^2 \langle 0 | j_5(\vec{x}, t) j_5(0, 0) | 0 \rangle$$

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- * moments of charm-quark P, V and A correlators
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$$G_n = \frac{g_n(\alpha_{\overline{MS}}(\mu), \mu/m_c)}{\left(am_c^{\overline{MS}}(\mu)\right)^{n-4}}$$

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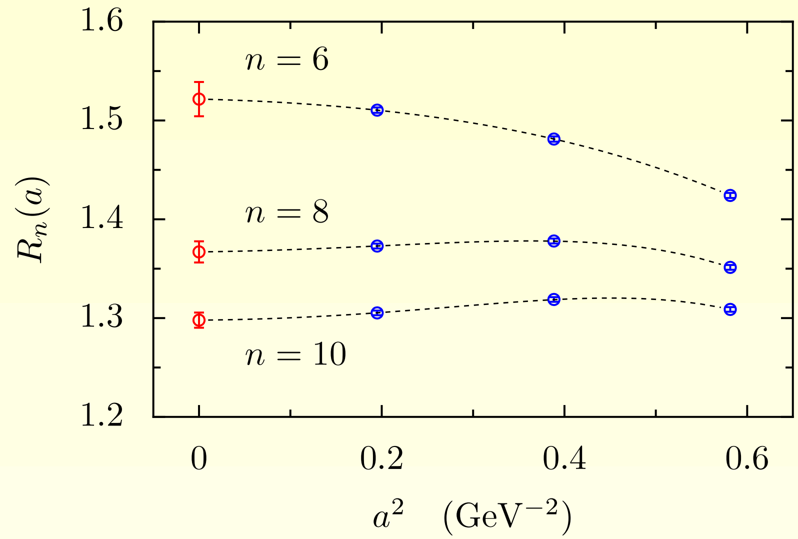

* Cut-off effects decrease with n, but n should be small enough so perturbation theory is applicable.

* Averaging over $n = 6, 8, 10$:

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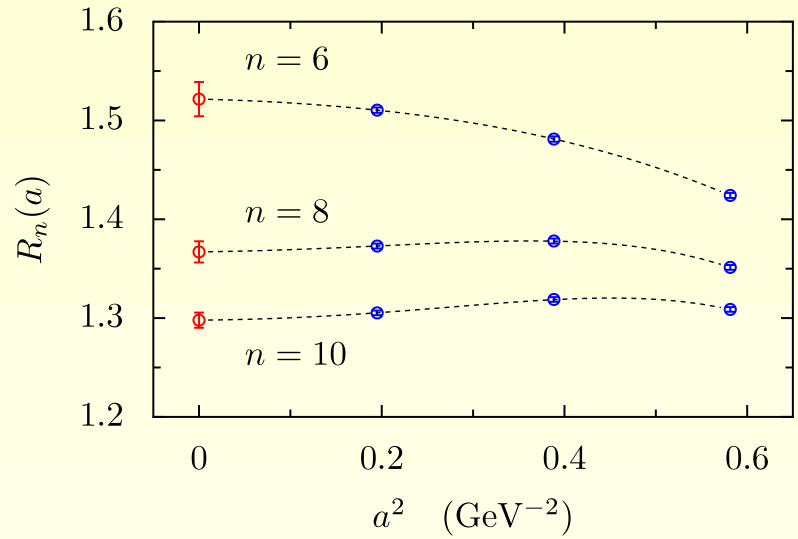

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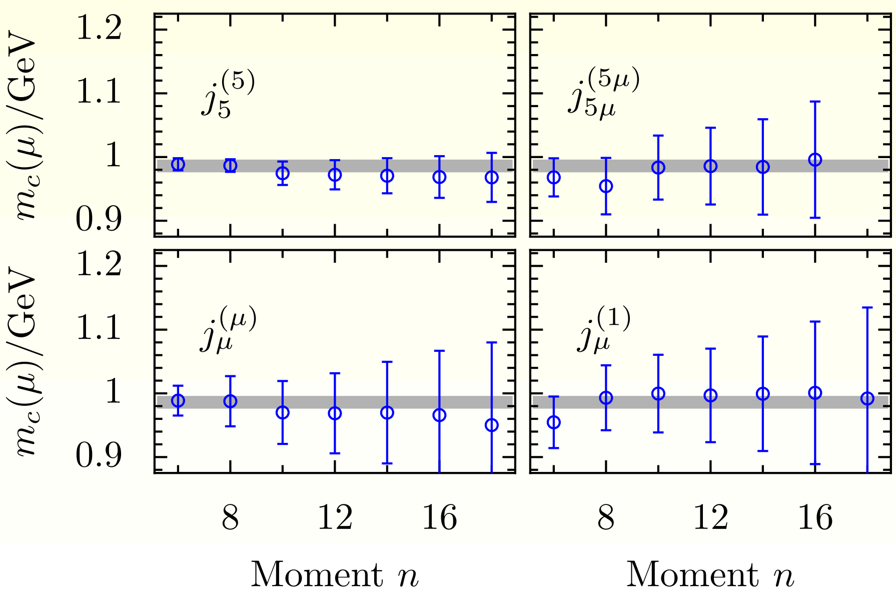

$j_\mu^{(1)} \equiv \bar{\psi}_c(x + a\hat{\mu})\gamma_\mu \psi_c(x)$;

$j_\mu^{(\mu)} \equiv \bar{\psi}_c(x)\gamma_\mu \psi_c(x)$; $j_{5\mu}^{(5\mu)} \equiv \bar{\psi}_c(x)\gamma_5\gamma_\mu \psi_c(x)$

Agreement for different momenta/correlators

→ check of systematic errors and negligible taste-changing effects

Charm quark mass m_c

HPQCD, Chetyrkin, Kühn, Steinhauser & Sturm, arXiv:0805.2999

$N_f = 2 + 1$ (**talk by P. Lepage**)

Updated value: including superfine ($a = 0.06\ fm$) lattice data and new $\mathcal{O}(\alpha_s^3)$ contributions in the perturbation theory for $n = 8$ **Preliminary**

$$m_c^{\overline{MS}}(m_c) = 1.269(9) GeV \qquad m_c^{\overline{MS}}(3GeV) = 0.988(10) GeV$$

continuum analysis: $m_c^{\overline{MS}}(3GeV) = 0.986(13) GeV$ $\surd$

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The method can be applied to determination of m_b with

* Υ currents.
* NRQCD for the $b-$quarks
* (Multiplicative) renormalization factors cancelled by taking ratios

$$R_n^{(j)} \equiv \frac{am^j}{2am_{0b}} \left(\frac{G_n^{(j)}}{G_{n-2}^{(j)}} \frac{G_n^{(j0)}}{G_{n-2}^{(j0)}} \right)^{1/2}$$

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Interesting application: Non perturbative calculation of renormalization coefficients for $H-H$ and $H-L$ currents.

Charm quark mass m_c

HPQCD $N_f = 2+1$ (talk by I. Allison) **in progress**

Different approach using also HISQ formulation.

* Use 2-loop perturbation theory (traditional and `high`-β techniques).

$$m_c^{\overline{MS}}(3GeV) = 0.983(25)GeV$$

* Determination of $m_c/m_s \rightarrow$ extraction of m_s.

TWQCD, PLB 651(2007)171 $N_f = 0$

Exploratory study: DW quarks in a small volume/fine lattice.

$m_c^{\overline{MS}}(m_c) = 1.16 \pm 0.04^{*}\mathrm{GeV}$ from η_c $m_b^{\overline{MS}}(m_b) = 4.65 \pm 0.05^{*}\mathrm{GeV}$ from $\Upsilon(9460)$

* error is an estimate not including all systematics.

Prediction for $m_{\eta_b} = 9383(4)(2)\mathrm{MeV}$ agrees with recent experimental measurement by **BaBar**, arXiv:0807.1086: $m_{\eta_b(1S)} = 9388.9^{+3.1}_{-2.3} \pm 2.7\mathrm{MeV}$

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Guazzini, Sommer and Tantalo, JHEP 0801(2008)076 $N_f = 0$

Same set-up as for the f_{B_s} calculation: **HQET** and **SS** method.

* RGI mass fixed using physical m_{B_s}
* Non-perturbative renormalization

$$M_b^{RGI} = 6.88(10)\mathrm{GeV} \rightarrow \boxed{\overline{m}_b^{\overline{MS}}(\overline{m}_b) = 4.42(6)\mathrm{GeV}}$$

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ALPHA, JHEP 0701 (2007) 007 $N_f = 0$

NP **HQET**: static + $1/m$

$$M_b^{RGI} = 6.758(86)\mathrm{GeV} \rightarrow \boxed{\overline{m}_b^{\overline{MS}}(\overline{m}_b) = 4.347(48)\mathrm{GeV}}$$

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Need accurate unquenched determination

6. Conclusions and outlook

Precise lattice calculations of hadronic matrix elements in the heavy sector are needed for extracting Standard Model parameters and are crucial for testing the SM.

* Complementary to direct searches in studying and constraining possible NP.

* Possible indications of NP already in `leptonic decays` and $B^0 - \bar{B}^0$ mixing: f_{D_s}, ξ, $|V_{cb}|$, $|V_{ub}|$, m_b , ...

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Important progress made in calculations of decay constants, form factors for B semileptonic decays and m_c ...

... more results in progress for D form factors, B^0 mixing and m_b.

Current and future calculation benefiting from

* Improved of actions and operators.
* Improved statistics: `all-to-all propagators`, `RW sources`, `smearing techniques` ...
* Improved methods: Twisted boundary conditions, model-independent parametrization of form factors, double ratios.

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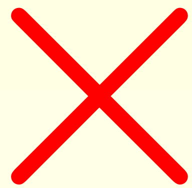
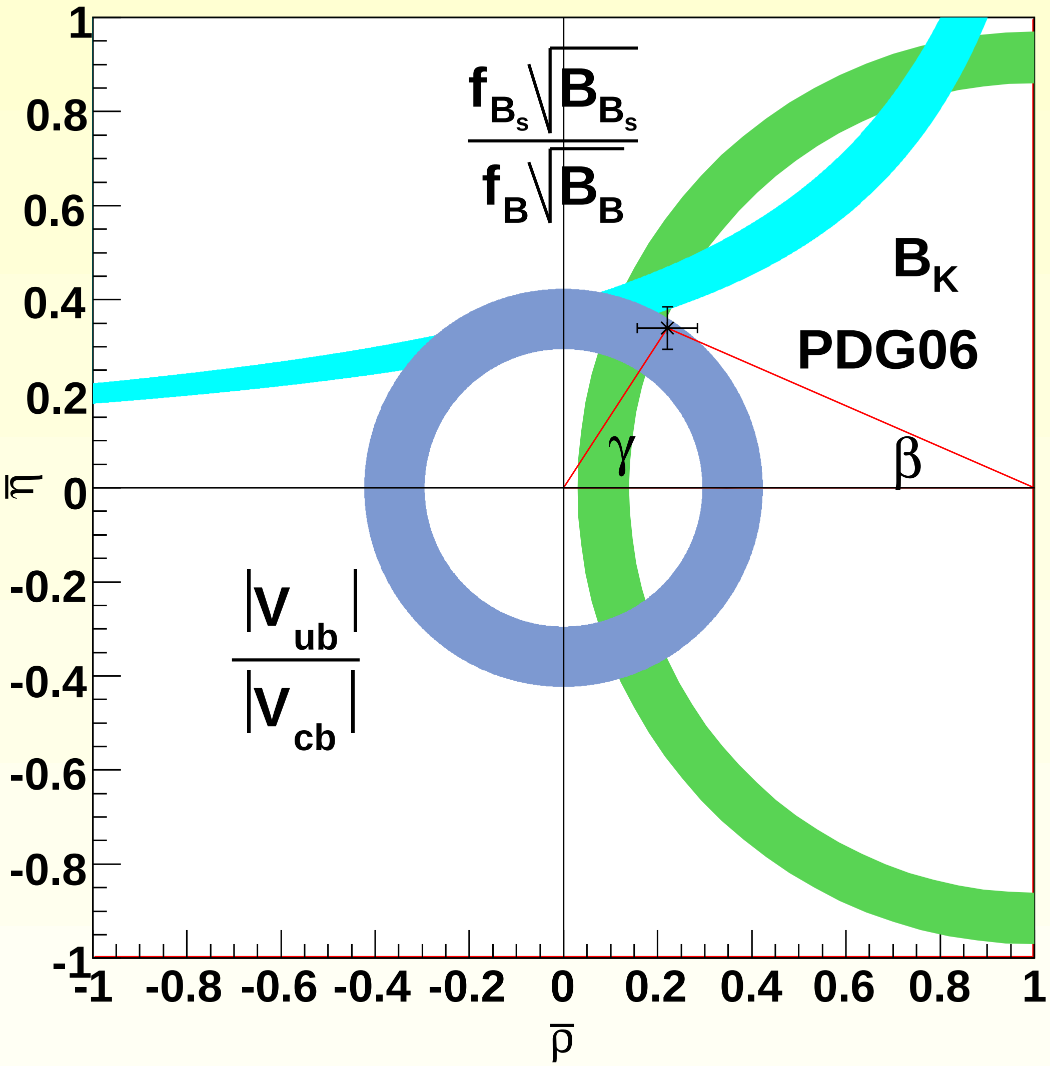
B^0 mixing results can be extended to matrix elements of four-fermion which only contribute BSM. Same for short-distance contributions to D^0 mixing.

Thanks to: Benoit Blossier, Christine Davies, Aida El-Khadra, Todd Evans, Eduardo Follana, Benjamin Haas, Jack Laiho, Peter Lepage, Paul Mackenzie, Marco Panero, Silvano Simula, Junko Shigemitsu, Amarjit Soni, Nazario Tantalo, Cecilia Tarantino, Ruth Van de Water, Georg von Hippel.

For sending material and useful discussions



CKM 2008 LATTICE QCD



* $\hat{B}_K$ from RBC/UKQCD

* $\frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$ preliminary result from FNAL/MILC

* $|V_{ub}|$ from Flynn and Nieves, 0705.3553

* $|V_{cb}|$ from Jack Laiho, LAT2007

* $|V_{us}|$ from $K_{l2}^{exp.} + \underbrace{\frac{f_K}{f_\pi}}_{\text{HPQCD}}$

C. Davies & C. McNeile

B^0 and D^0 mixing beyond the SM

Effects of heavy new particles seen in the form of effective operators built with **SM** degrees of freedom

$$\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i$$

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$$\tilde{Q}_{1,2,3}^q = Q_{1,2,3}^q \text{ with the replacement } (\mathrm{I} \pm \gamma_5) \rightarrow (\mathrm{I} \mp \gamma_5)$$

where ψ_q is a heavy fermion field (b or c) and ψ_f a light fermion field.

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- $C_i, \widetilde{C}_i$ Wilson coeff. calculated for a particular BSM theory
- $\langle \bar{F^0} | Q_i | F^0 \rangle$ calculated on the `lattice`

SM predictions + BSM contributions + experiment

→ constraints on BSM physics

F. Gabbiani et al, Nucl.Phys.B477 (1996) general SUSY extensions

D. Bećirević et al, Nucl.Phys.B634 (2002) general SUSY models

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HPQCD: Same code can be used for the B^0 mixing analysis.

* One-loop renormalization coefficients already calculated

E. G. ,J. Shigemitsu and H. Trottier, arXiv:0804.1557

Charm quark mass m_c

HPQCD, Chetyrkin, Kühn, Steinhauser & Sturm, arXiv:0805.2999

$N_f = 2+1$ (**talk by P. Lepage**)

In producing the moments one must control $\mathcal{O}((am_c)^n)$ and parameters' tuning errors (a, am_c).

Define reduced moments

lattice | continuum PT

$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & g_4/g_4^{(0)} \quad \text{for } n=4, \\ \dfrac{am_{\eta_c}}{2am_{0c}}\left(G_n/G_n^{(0)}\right)^{1/(n-4)} & \dfrac{r_n(\alpha_{\overline{MS}}, \mu/m_c)}{2m_c(\mu)/m_{\eta_c}} \quad \text{for } n \geq 6, \end{cases}$$

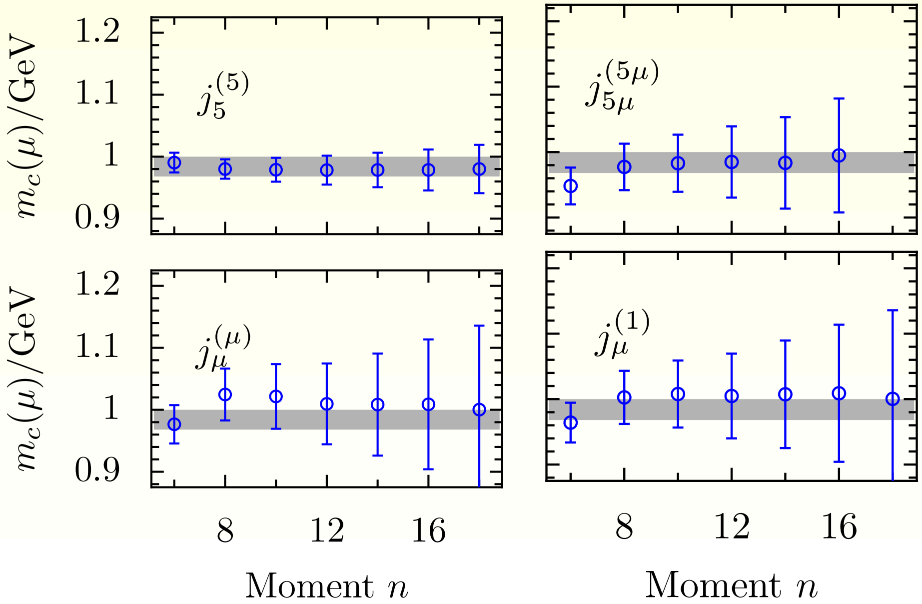
with $r_n = \left(g_n/g_n^{(0)}\right)^{1/(n-4)}$ and $G_n^{(0)}/g_n^{(0)}$ moments at first order in lattice/continuum PT $\rightarrow$ reduce discretization effects.

a^{-1}	$am_{0u/d}$	am_{0s}	am_{0c}	L/a	T/a	$N_{\rm cfg}$
1.31	0.013	0.066	0.850	16	48	631
1.60	0.014	0.055	0.660	20	64	595
2.26	0.007	0.037	0.430	28	96	566

Extra simulation performed to test sea quark dependence (also PT estimate) and finite volume effects ($am_{0,u/d} = 0.007,\ L/a = 24$) and ($am_{0,u/d} = 0.028,\ L/a = 20$) → errors $< 0.2\%$.

Analysis repeated with different correlators:

$$j_\mu^{(1)} \equiv \bar\psi_c(x + a\hat\mu)\gamma_\mu\psi_c(x);\ j_\mu^{(\mu)} \equiv \bar\psi_c(x)\gamma_\mu\psi_c(x);\ j_{5\mu}^{(5\mu)} \equiv \bar\psi_c(x)\gamma_5\gamma_\mu\psi_c(x)$$



Agreement for different momenta/correlators

→ check of systematic errors and negligible taste-changing effects

$B^0 - \bar{B}^0$ mixing

ETMC, JHEP 0805(2008)065 $N_f = 2$

Non-perturbative renormalization and Renormalization Group running of relevant four-fermion operators with $\mathcal{O}(a)$ improved Wilson fermions completed.

* Static (HYP2) heavy quarks
* Parity odd operators:
  ** Protected from non-continuum like operator mixing
  ** Can be mapped to parity even operators via addition of a chirally **tm** term.
* Schrödinger Functional methods.

Limitations current calculation

* **Increased statistical fluctuations at the three strongest couplings**
* **Need control over continuum extrapolation**

* **Improving statistics at the three strongest couplings**
* **Simulations closer to the continuum**
* **Removing $\mathcal{O}(a)$ discretization effects (improving operators)**