Exploring Excited Hadrons

Colin Morningstar

(Carnegie Mellon University)

Lattice 2008: Williamsburg, VA

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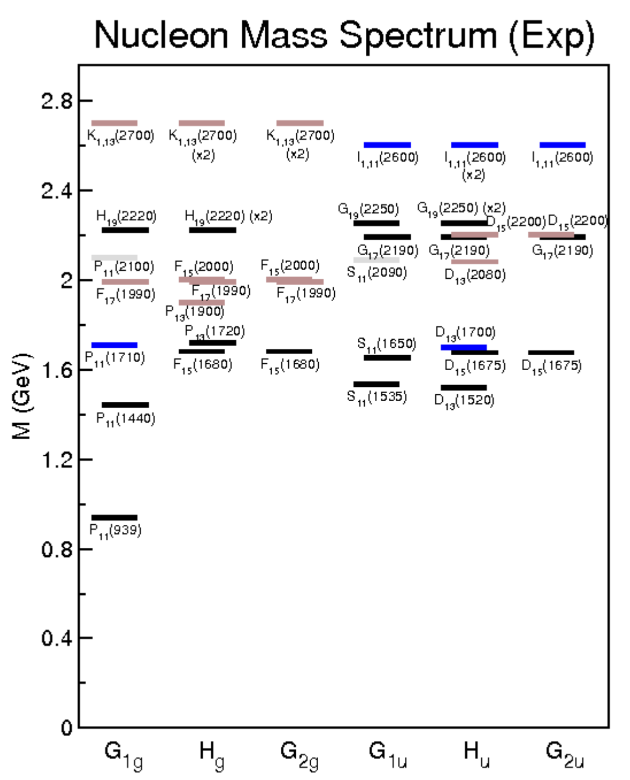
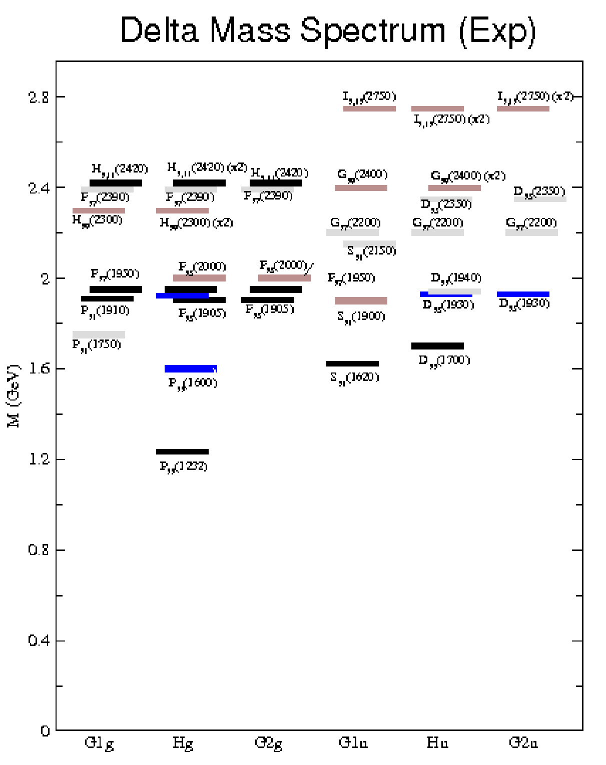
The frontier awaits

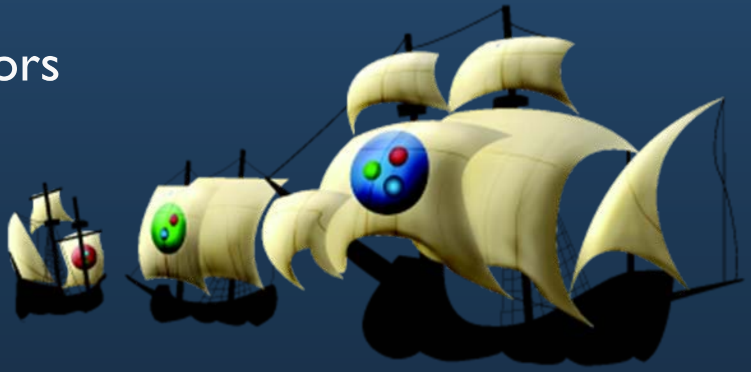
- experiments show many excited-state hadrons exist
- significant experimental efforts to map out QCD resonance spectrum → JLab Hall B, Hall D, ELSA, etc.
- great need for *ab initio* calculations → lattice QCD





The challenge of exploration!

- most excited hadrons are unstable (resonances)
- excited states more difficult to extract in Monte Carlo calculations
  - correlation matrices needed
  - operators with very good overlaps onto states of interest
- must extract all states lying below a state of interest
  - as pion get lighter, more and more multi-hadron states
- best multi-hadron operators made from constituent hadron operators with well-defined relative momenta
  - need for all-to-all quark propagators
- disconnected diagrams



Outline

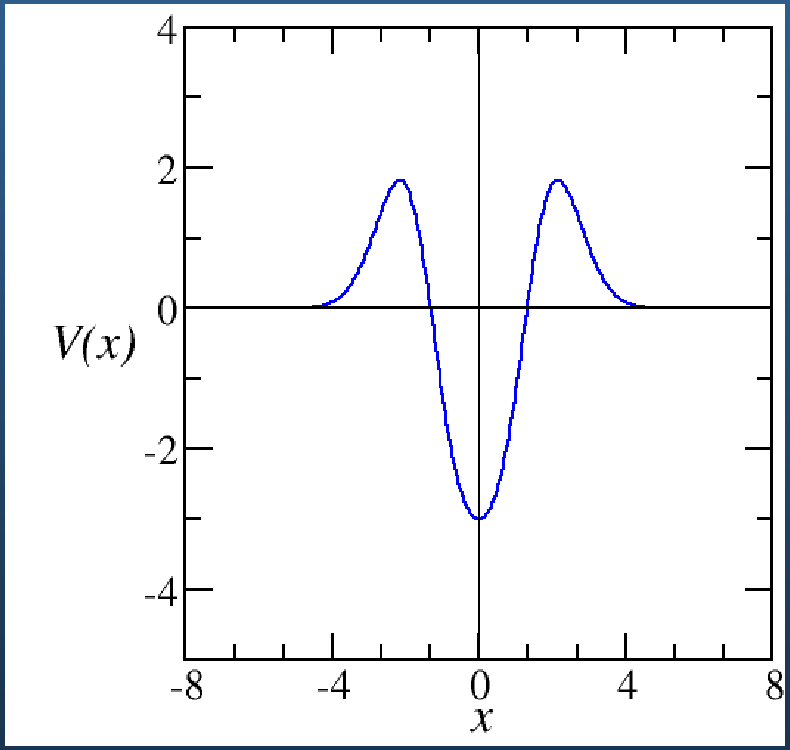
- Resonances in a box
- Extracting excited stationary state energies
  - recent results
  - operator technology
    - field smearing
    - symmetry
  - all-to-all quark propagators
    - variance reduction with dilutions
    - a new development
- Outlook

Resonances in a box

Resonances in a box: an example

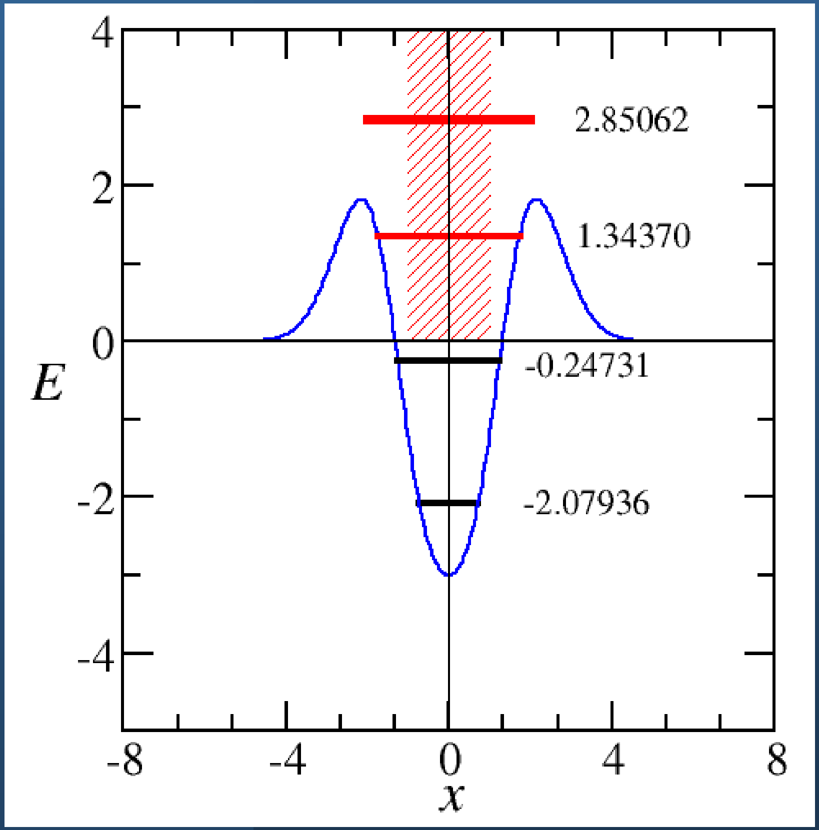
- Consider simple 1D quantum mechanics example
- Hamiltonian

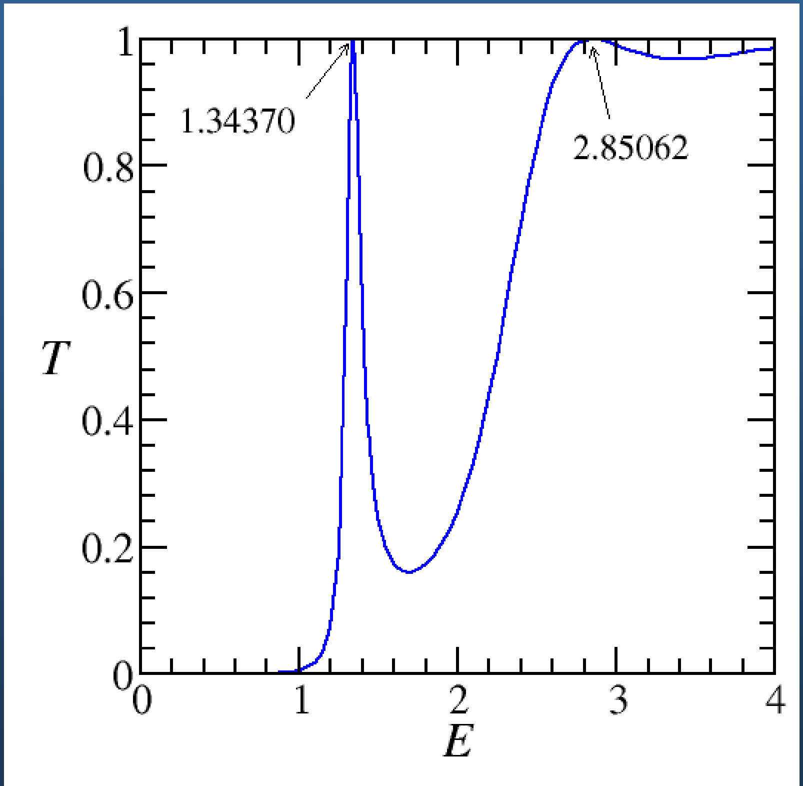
$$H = \frac{1}{2} p^2 + V(x) \qquad V(x) = (x^4 - 3)\, e^{-x^2/2}$$



1D example spectrum

- Spectrum has two bound states, two resonances for *E*<4

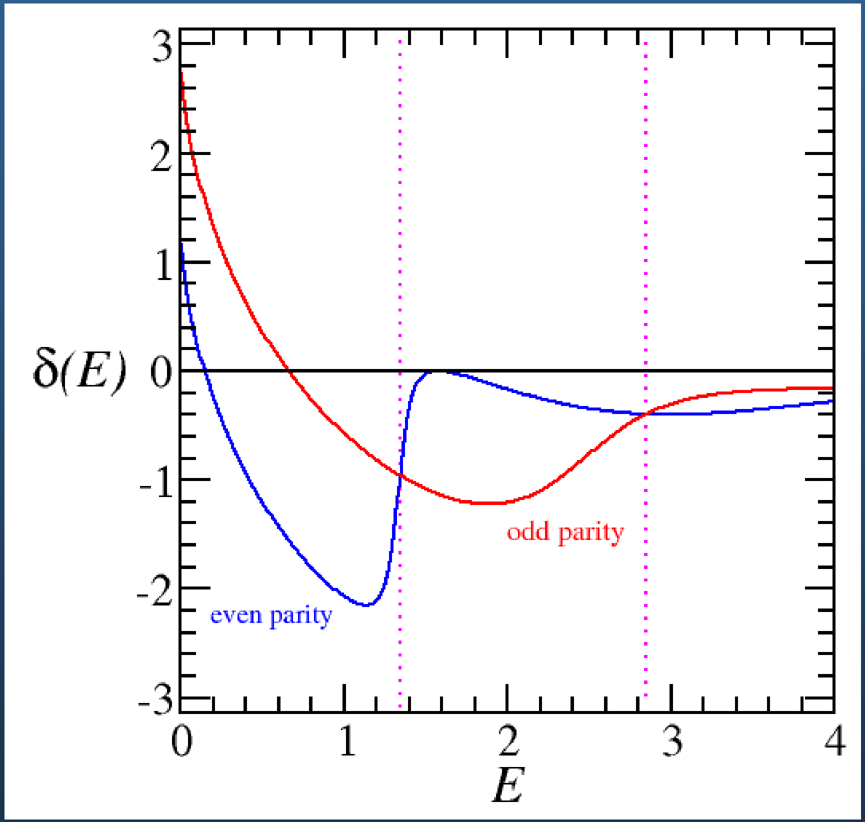




transmission coefficient

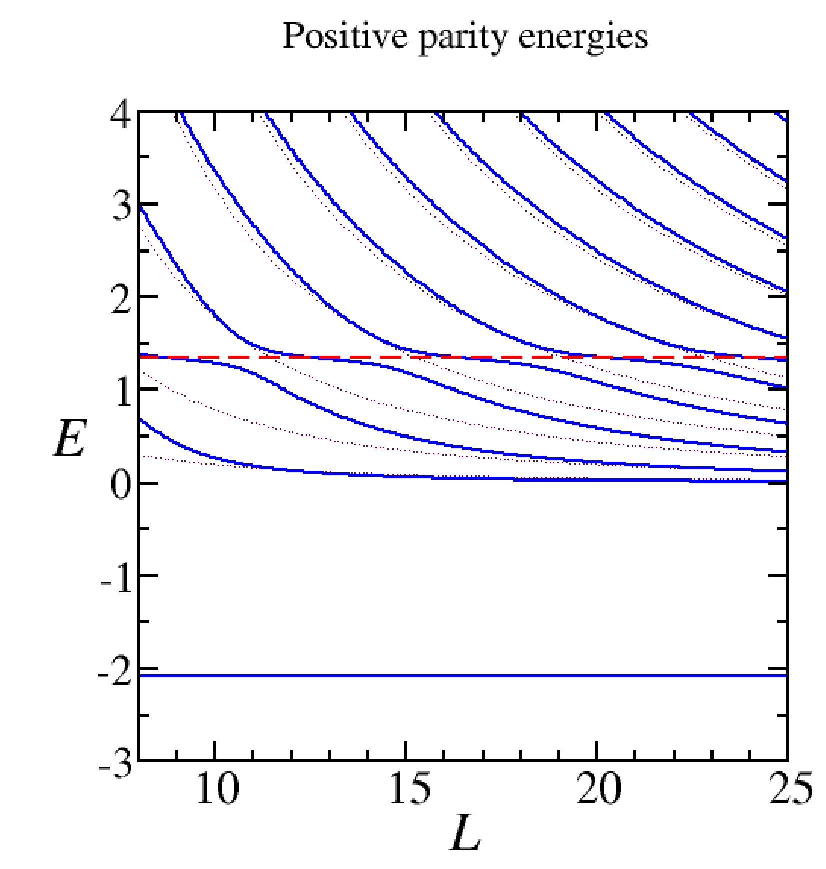
Scattering phase shifts

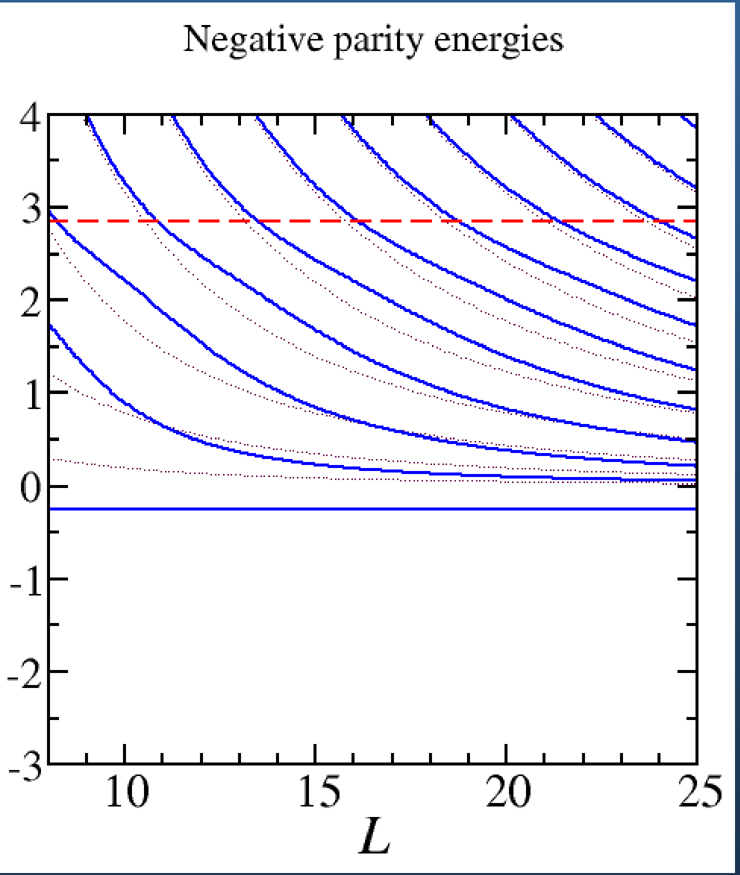
- define even- and odd-parity phase shifts $\delta_{\pm}$
  - phase between transmitted and incident wave



Spectrum in box (periodic b.c.)

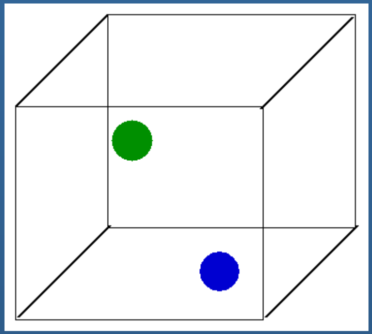
- spectrum is discrete in box (momentum quantized)
- narrow resonance is avoided level crossing, broad resonance?





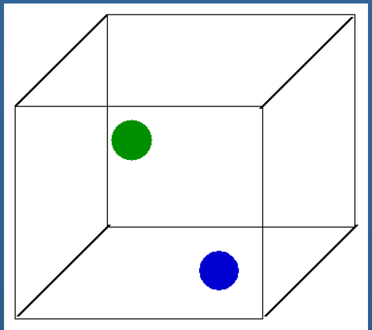
Dotted curves are V=0 spectrum

Unstable particles (resonances)



- our computations done in a periodic box
  - momenta quantized
  - discrete energy spectrum of stationary states → single hadron, 2 hadron, …
- how to extract resonance info from box info?
- approach 1: crude scan
  - if goal is exploration only → "ferret" out resonances
  - spectrum in a few volumes
  - placement, pattern of multi-particle states known
  - resonances → level distortion near energy with little volume dependence
  - short-cut tricks of McNeile/Michael, Phys Lett B556, 177 (2003)

Unstable particles (resonances)



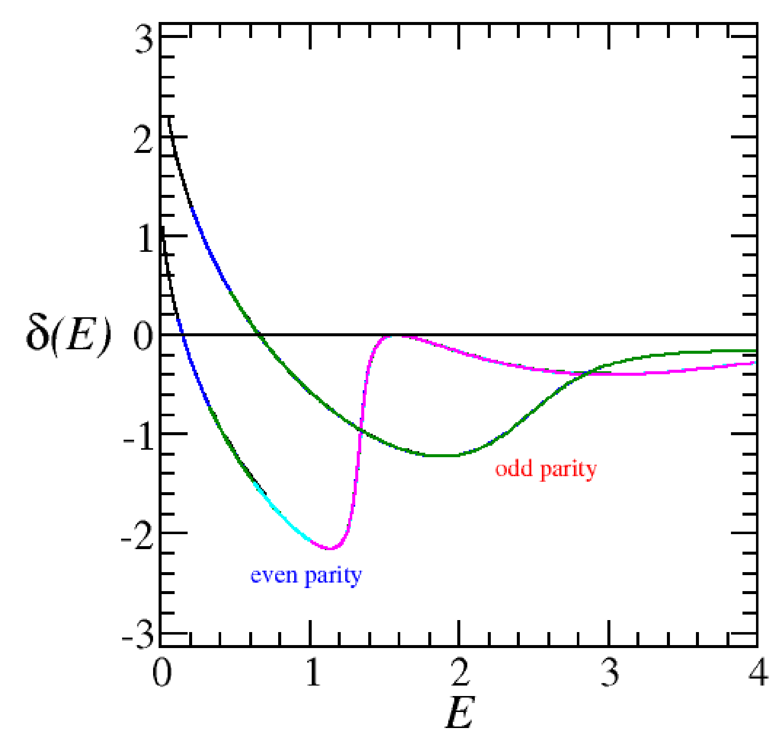
- approach 2: phase-shift method
  - if goal is high precision → work much harder!
  - relate finite-box energy of multi-particle *model* to infinite-volume phase shifts
  - evaluate energy spectrum in several volumes to compute phase shifts using formula from previous step
  - deduce resonance parameters from phase shifts
  - early references
    - B. DeWitt, PR **103**, 1565 (1956) (sphere)
    - M. Luscher, NPB**364**, 237 (1991) (ρ-$\pi\pi$ in cube)
- approach 3: histogram method
  - recent work for pion-nucleon system:
  - V. Bernard et al, arXiv:0806.4495 [hep-lat]

1D example: phase-shift method

- periodic boundary condition of box leads to condition

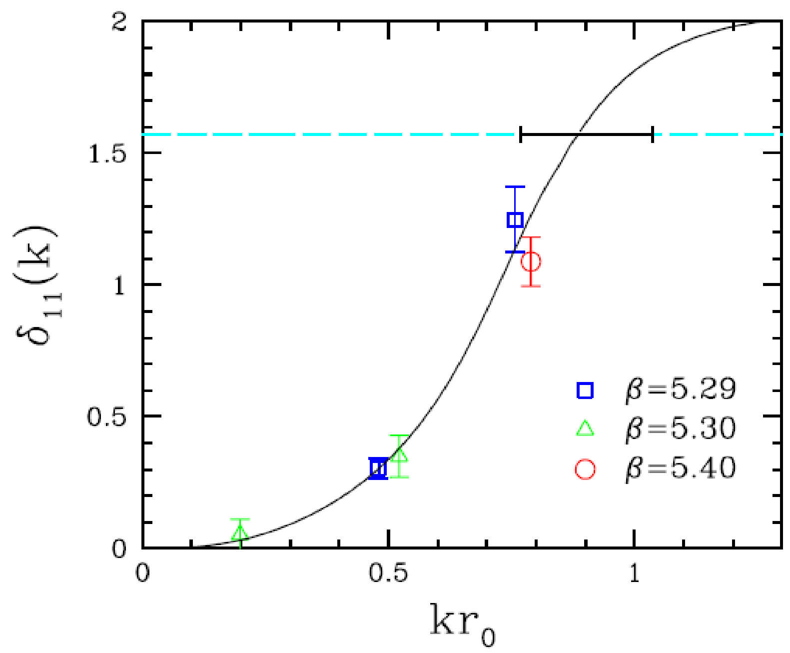
$$\exp\left(-iL\sqrt{2E}\right) = \exp\left(2i\delta_{\pm}(E)\right)$$

- calculate shifts $\delta_{\pm}(E)$ from finite-box energies



Mass and width of ρ

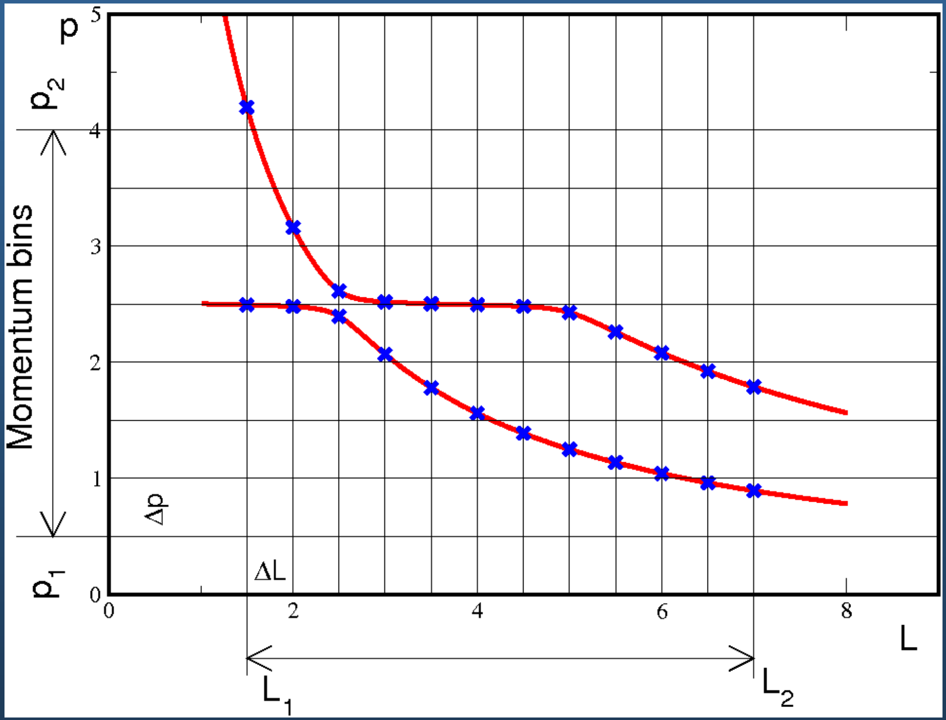
- Schierholz et al (this conference)
- Breit-Wigner fit: $\Gamma_\rho = 200^{+130}_{-100}\ \mathrm{MeV}$



$$\delta_{11}(k) - n\pi = \arctan\left(\frac{\pi^{3/2} q}{Z_{00}(1; q^2)}\right), \quad q = \frac{k L}{2\pi}$$

New histogram method

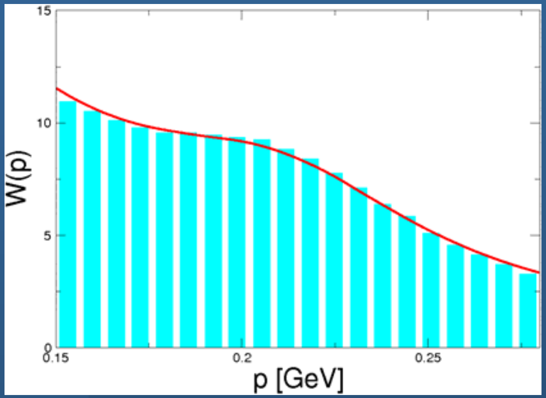
- use of probability distribution to study resonant structure
- reference: V. Bernard et al., arXiv:0806.4495 [hep-lat]



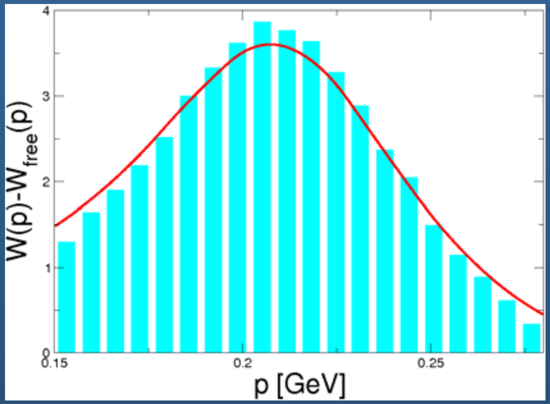
- count times eigenvalue occurs in particular momentum bin
- normalize to get probability distribution *W(p)*

Histogram method test

- tested on synthetic data to study Δ resonance



Before subtracting free result

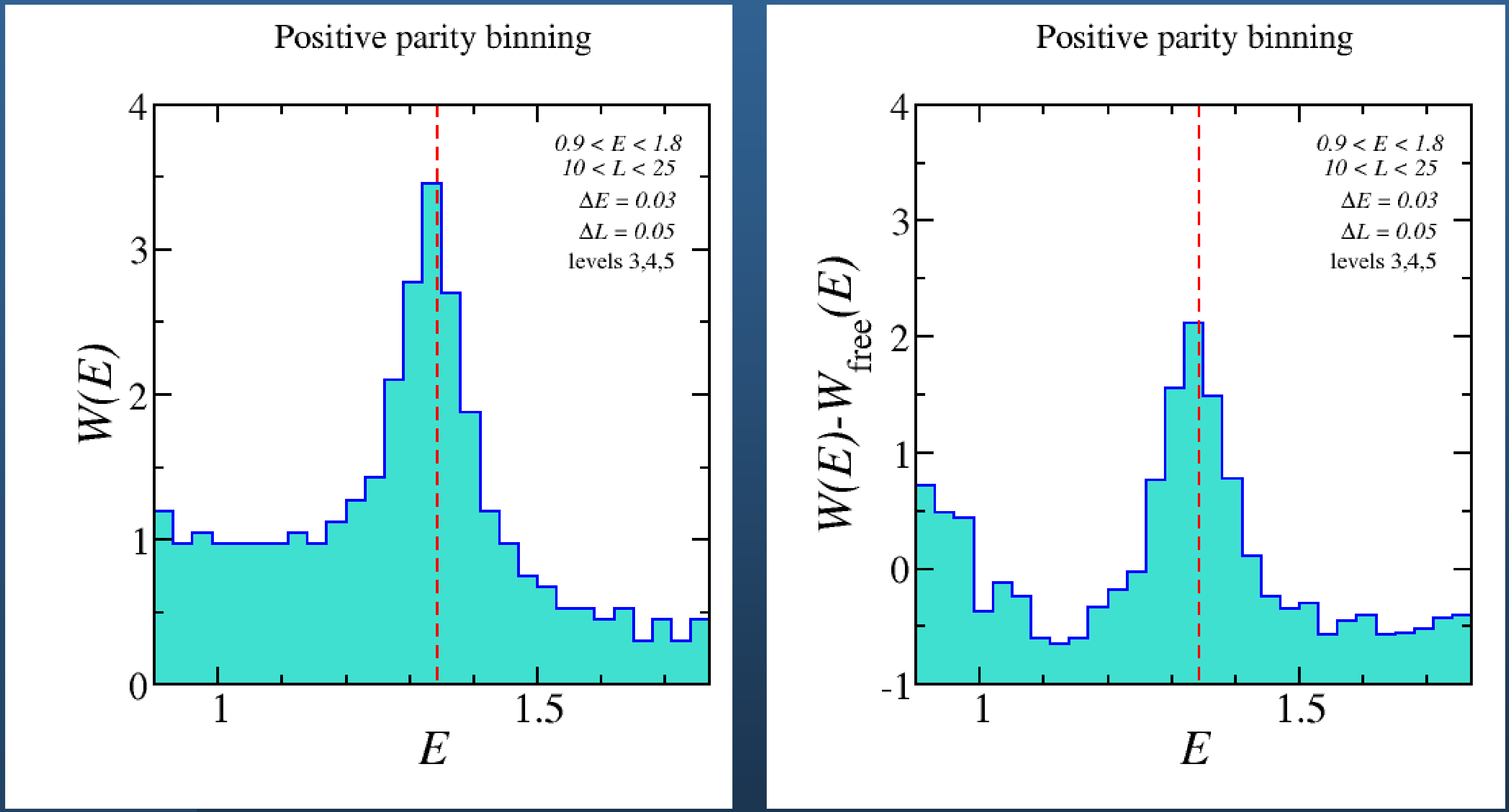


After subtracting free result

- no prior theoretical bias
- possibility of seeing resonant structure even when avoid level crossing washed out by broad resonance

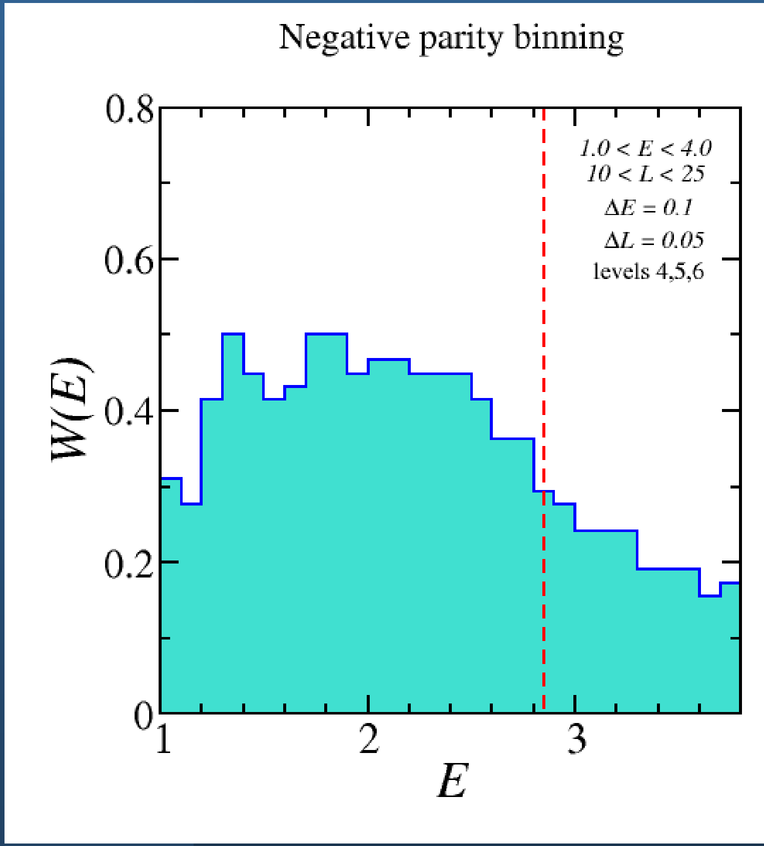
1D example: histogram method

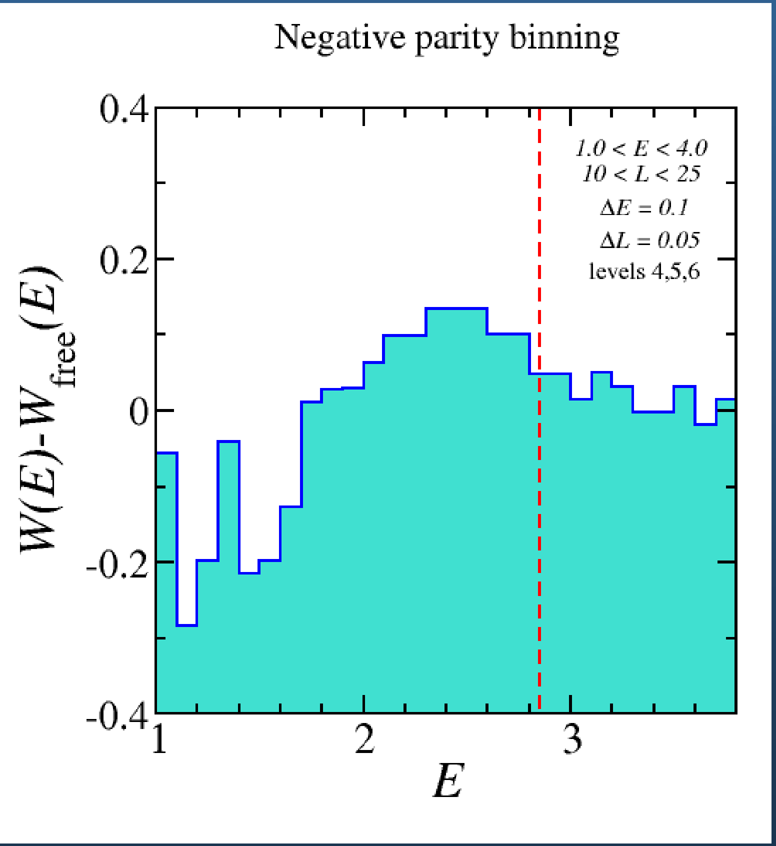
- even parity channel



1D example: histogram method

- odd parity channel





Excited stationary states

Excited-state energies from Monte Carlo

- extracting excited-state energies requires matrix of correlators
- for a given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle 0 | O_\alpha(t) O^+_\beta(0) | 0 \rangle$
  one defines the N *principal correlators* $\lambda_\alpha(t, t_0)$ as the eigenvalues of

  $$C(t_0)^{-1/2} C(t)\, C(t_0)^{-1/2}$$

  where t_0 (the time defining the "metric") is small
- can show that $\lim_{t\to\infty} \lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha}(1 + e^{-t\Delta E_\alpha})$
- N principal effective masses defined by $m^{\text{eff}}_\alpha(t) = \ln\left(\dfrac{\lambda_\alpha(t, t_0)}{\lambda_\alpha(t+1, t_0)}\right)$
  now tend (plateau) to the N lowest-lying stationary-state energies
- analysis:
  - fit each principal correlator to single exponential
  - optimize on earlier time slice, matrix fit to optimized matrix
  - both methods as consistency check

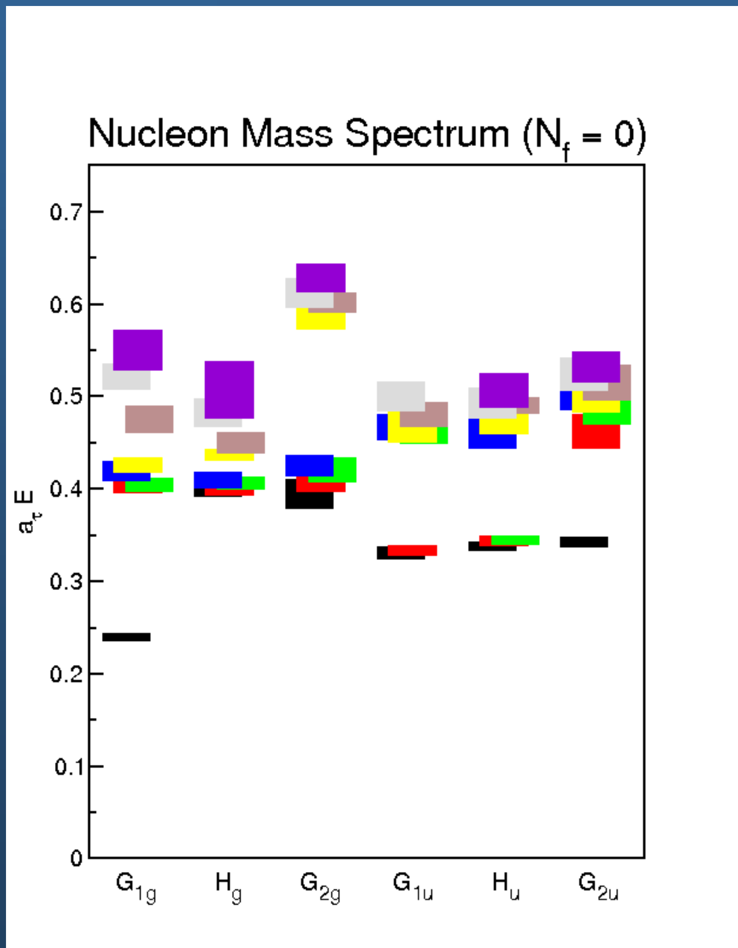
Recent excited-state results

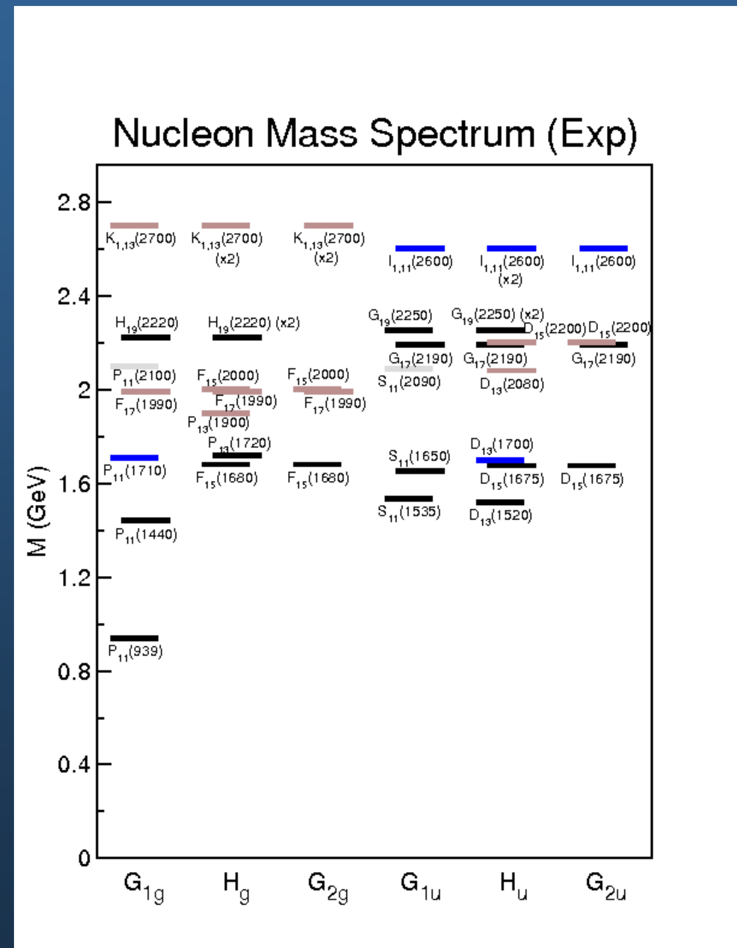
- *Lattice QCD determination of patterns of excited baryon states,* S. Basak, R. Edwards, G. Fleming, K. Juge, A. Lichtl, C. Morningstar, D. Richards, I. Sato, S. Wallace, Phys Rev D76, 074504 (2007)
  - quenched first results for nucleons/deltas
  - 239-16^3 64 and 167-24^3 64 quenched anisotropic Wilson, a_s~0.1 fm, a_s/a_t~3, m_π~490 MeV

- *Derivative sources in lattice spectroscopy of excited mesons,* C. Gattringer, L. Glozman, C. Lang, D. Mohler, S. Prelovsek, arXiv:0802.2020 [hep-lat]
  - 99-16^3 32 quenched chiral-improved fermion, LW gauge, a_s~0.15 fm, range of m_π

Nucleon spectrum: first results

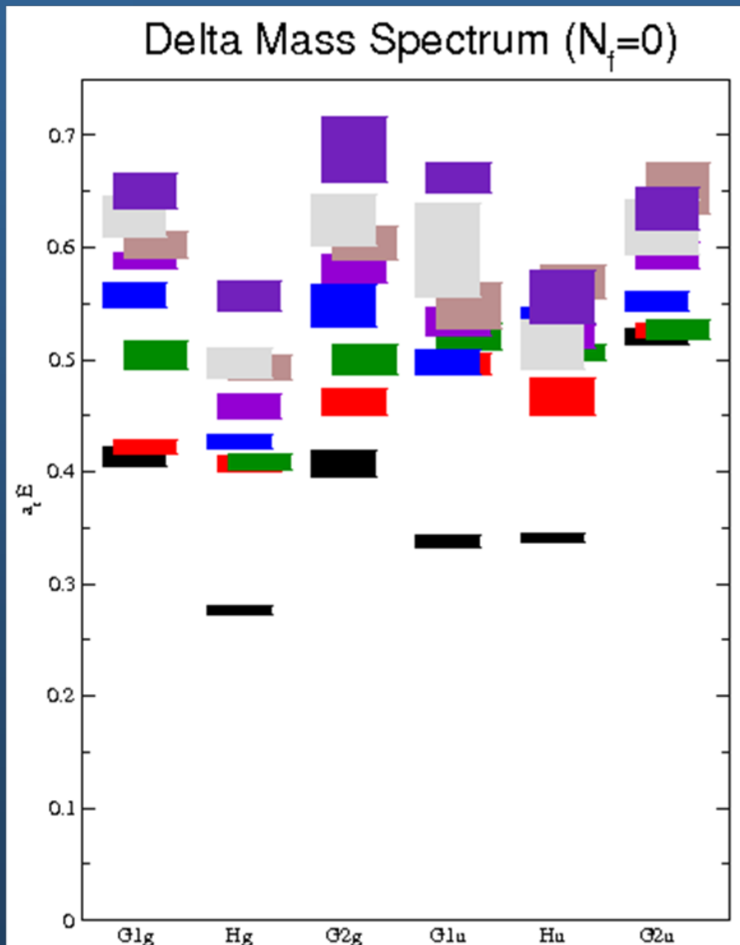
- 200 quenched configs, 12^3 48 anisotropic Wilson lattice, a_s~0.1 fm, a_s/a_t~3, m_π~700 MeV (A. Lichtl thesis)

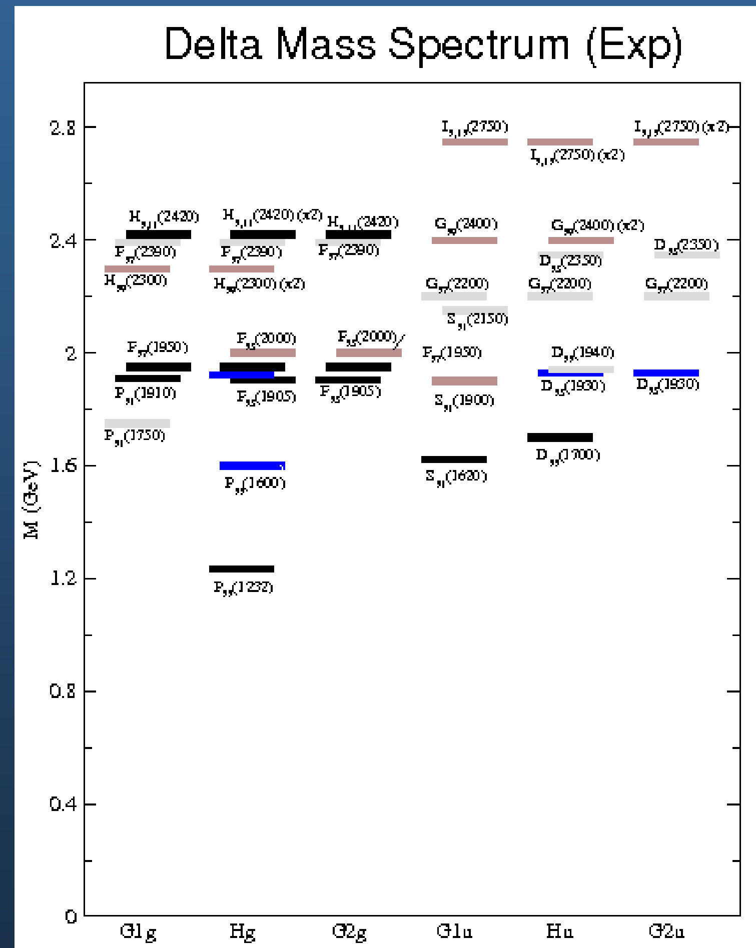




Delta spectrum: first results

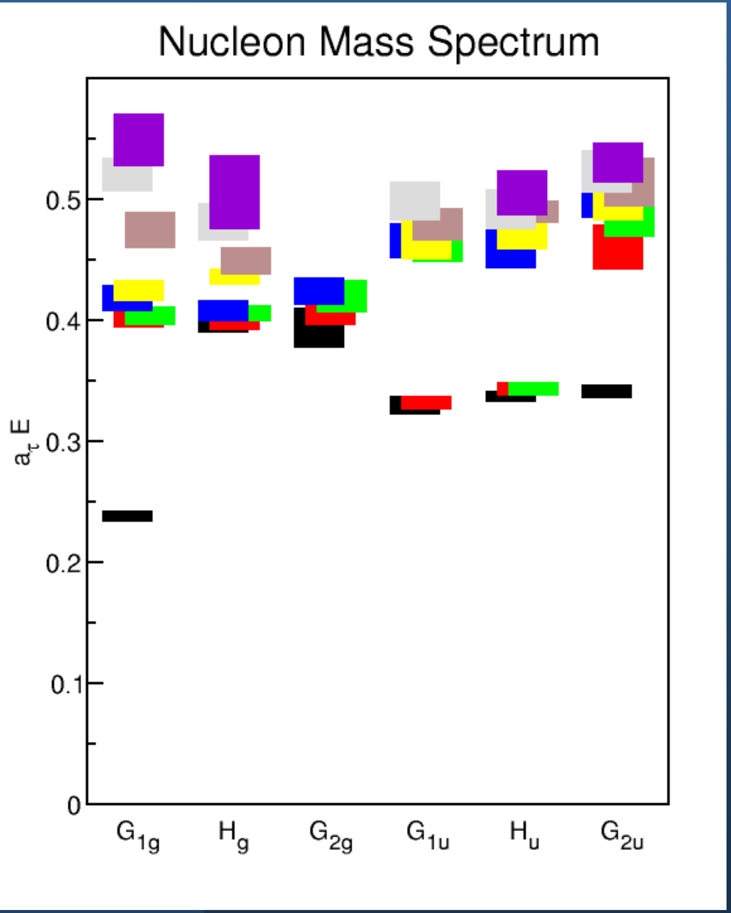
- 200 quenched configs, 12^3 48 anisotropic Wilson lattice, a_s~0.1 fm, a_s/a_t~3, m_π~700 MeV (J. Bulava)

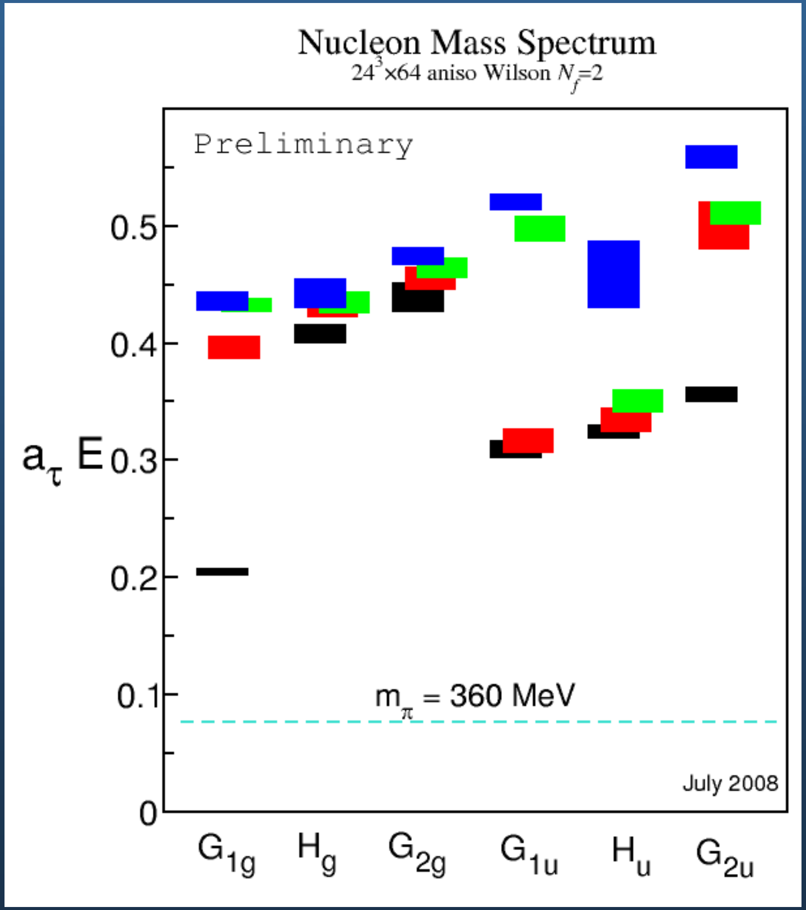




Inclusion of quark loops

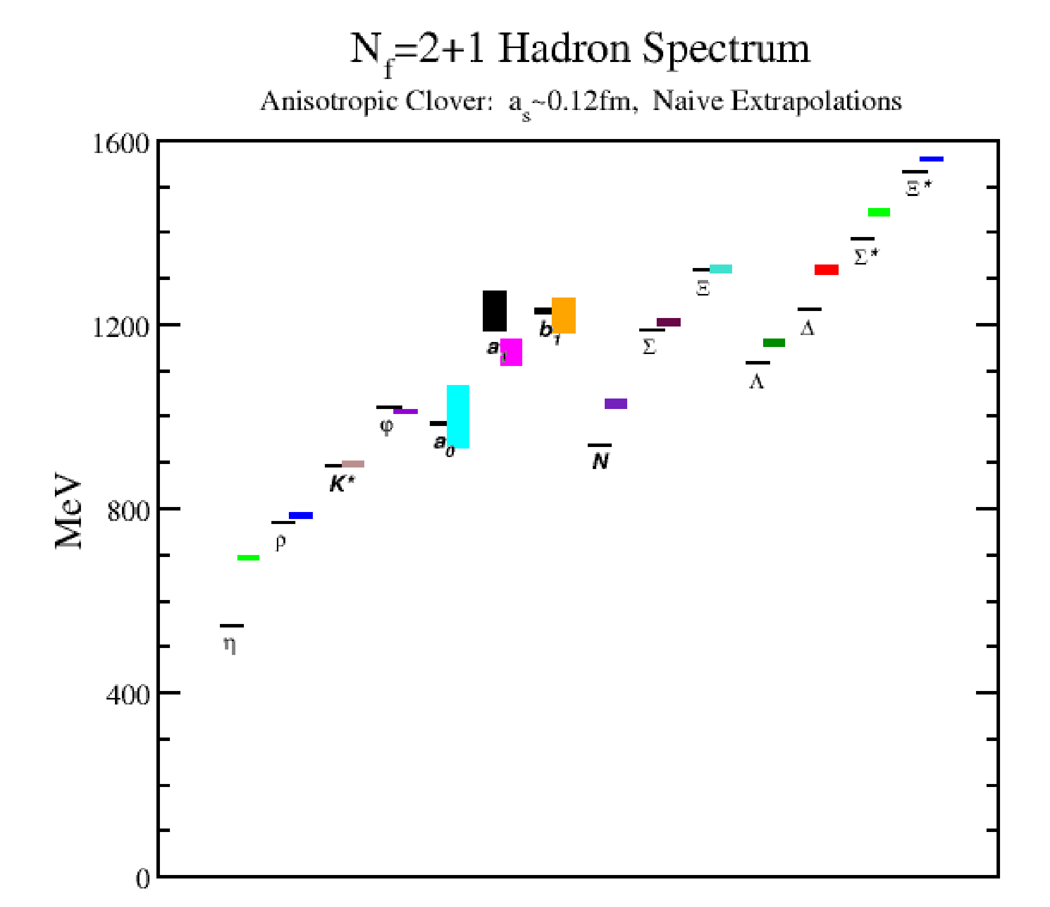
- Left: $N_f=0$ $m_\pi=700$ MeV
- Right: $N_f=2$ $m_\pi=360$ MeV (Spectrum collaboration, preliminary)





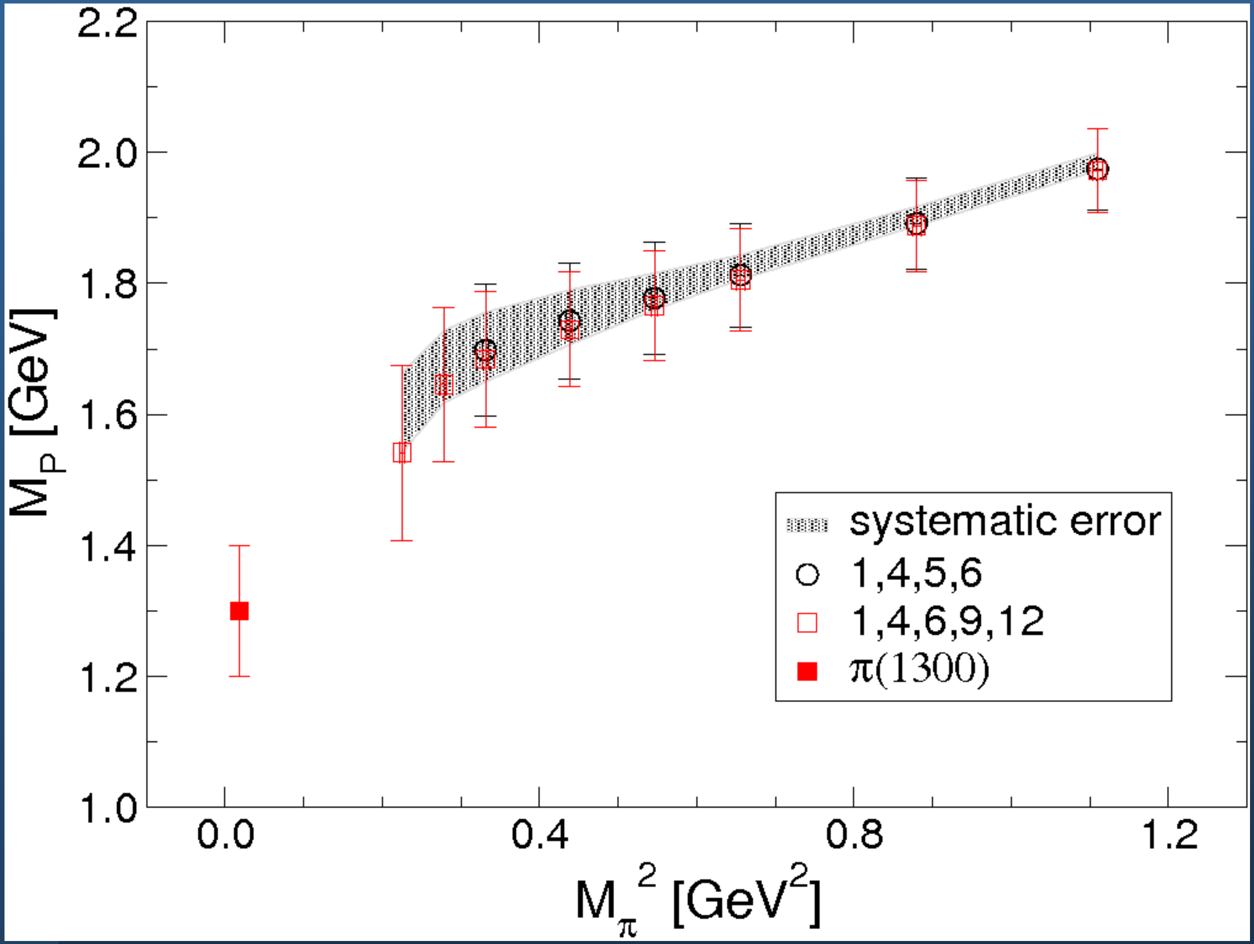
“Standard” hadrons

- Preliminary results from QCD Spectrum collaboration (Edwards talk)
- Scale setting using mass ratios (Ω, K: Peardon talk)



0^{-+} first-excited meson

- Gattringer et al.



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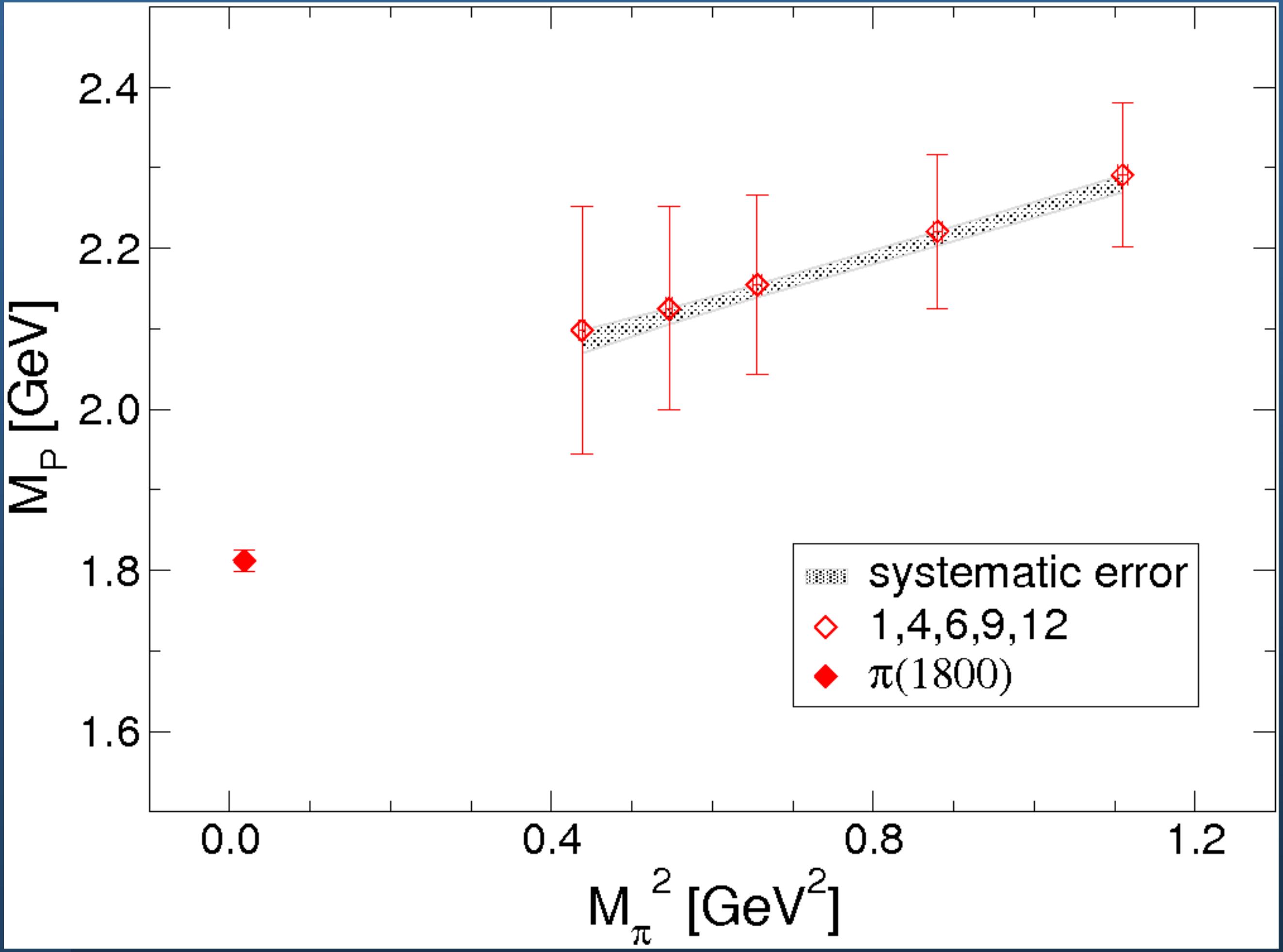
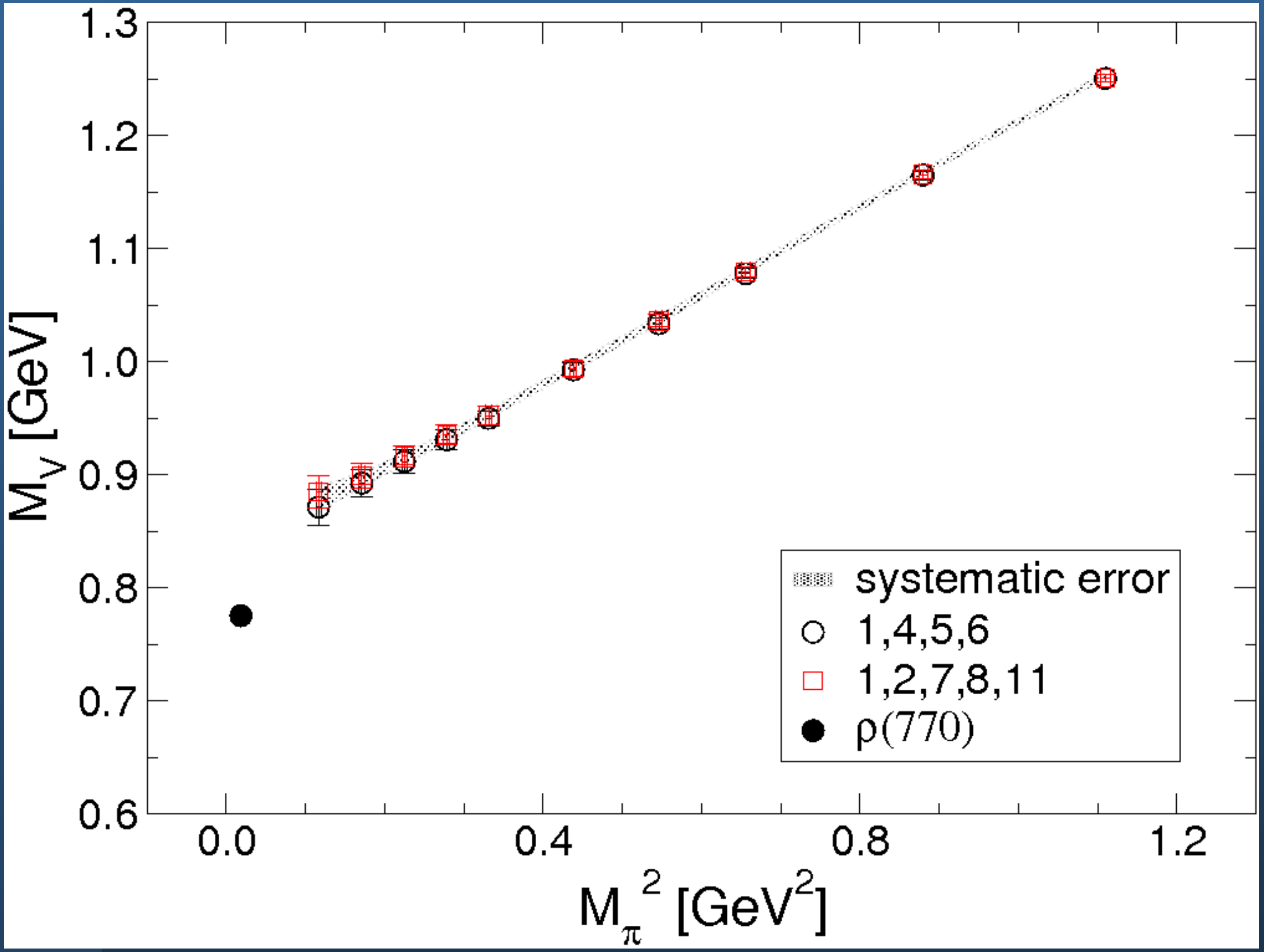


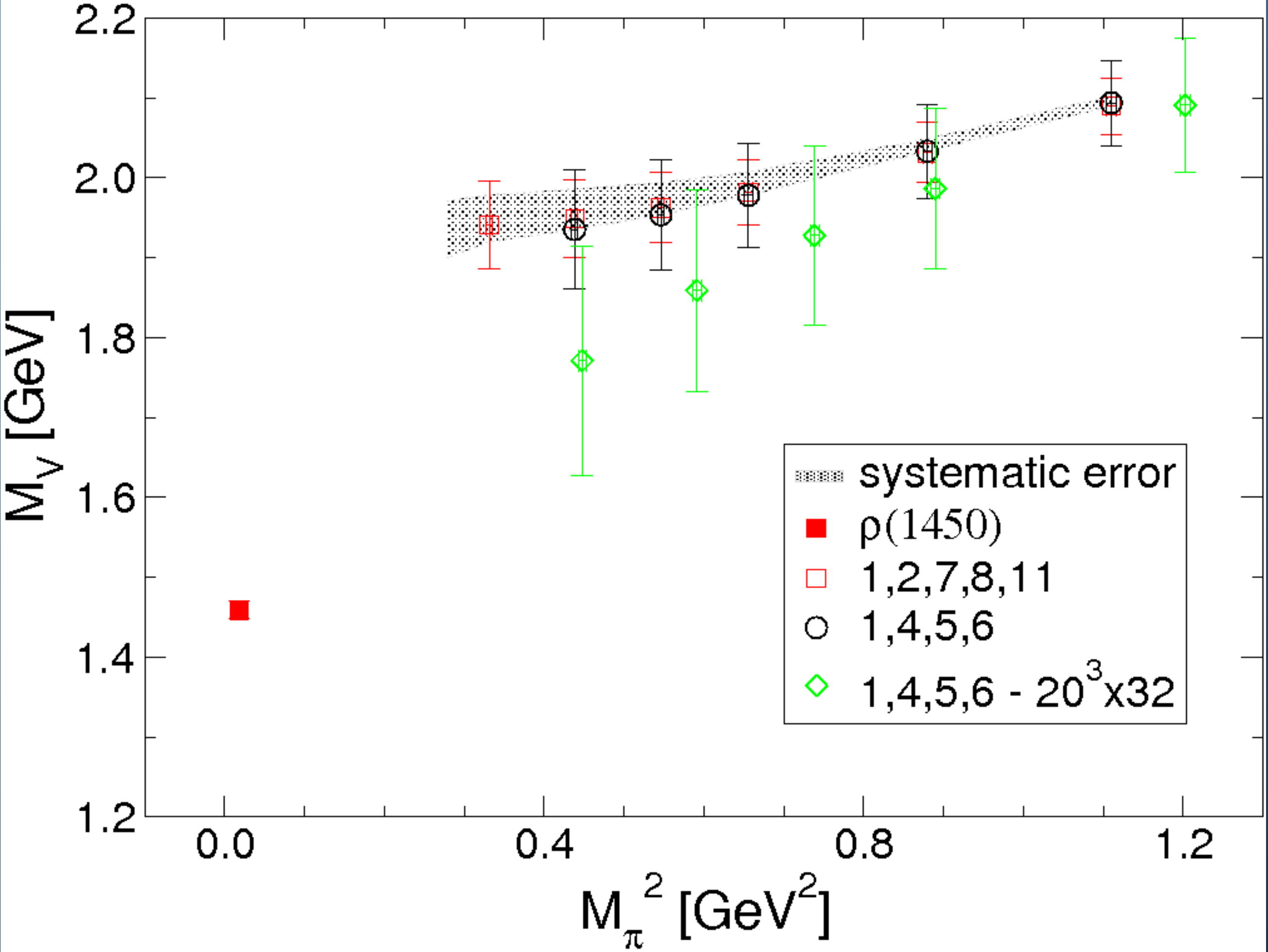
1^{--} ground state meson

- Gattringer et al.



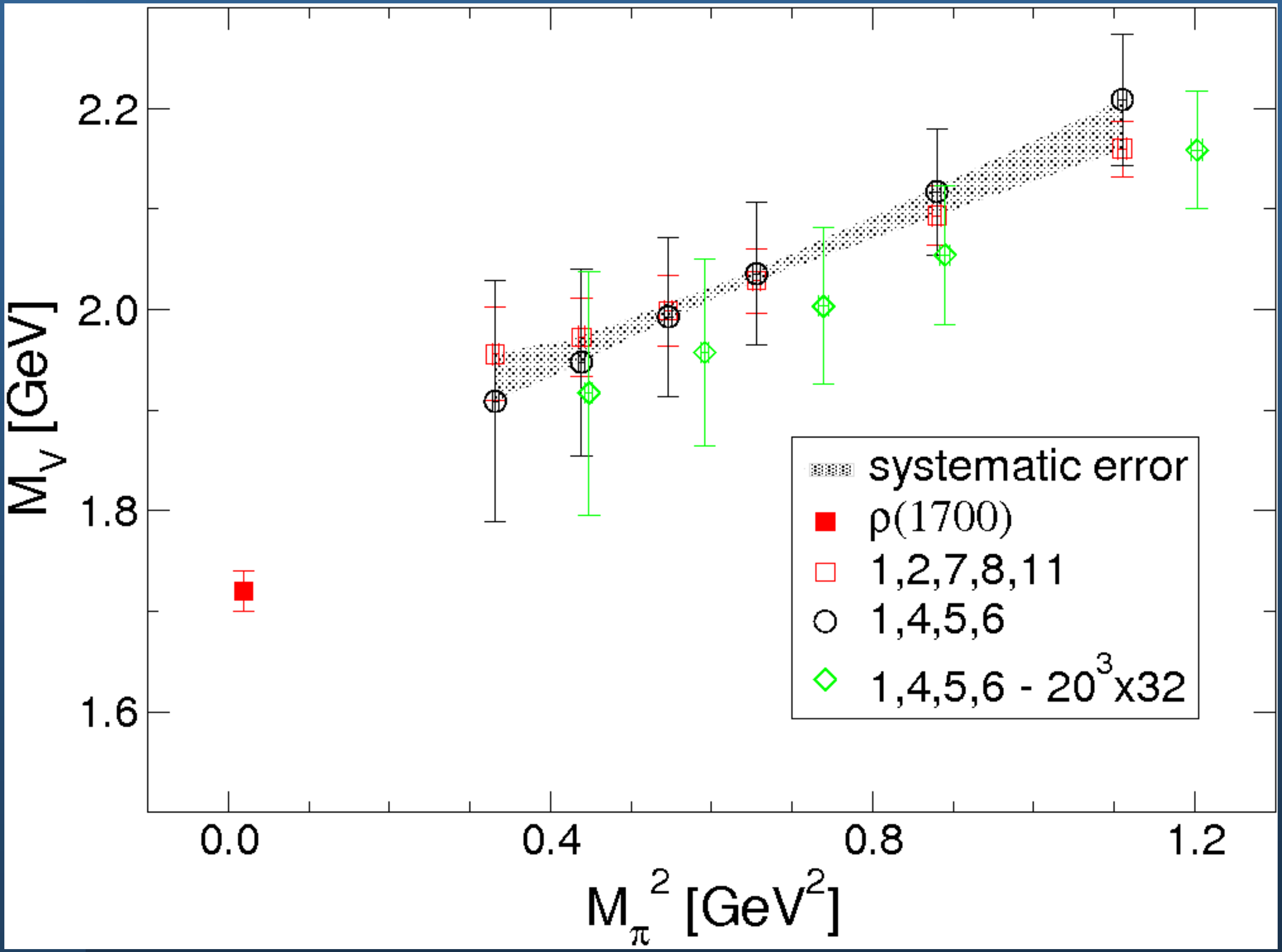
1^{--} first-excited state meson

- Gattringer et al.



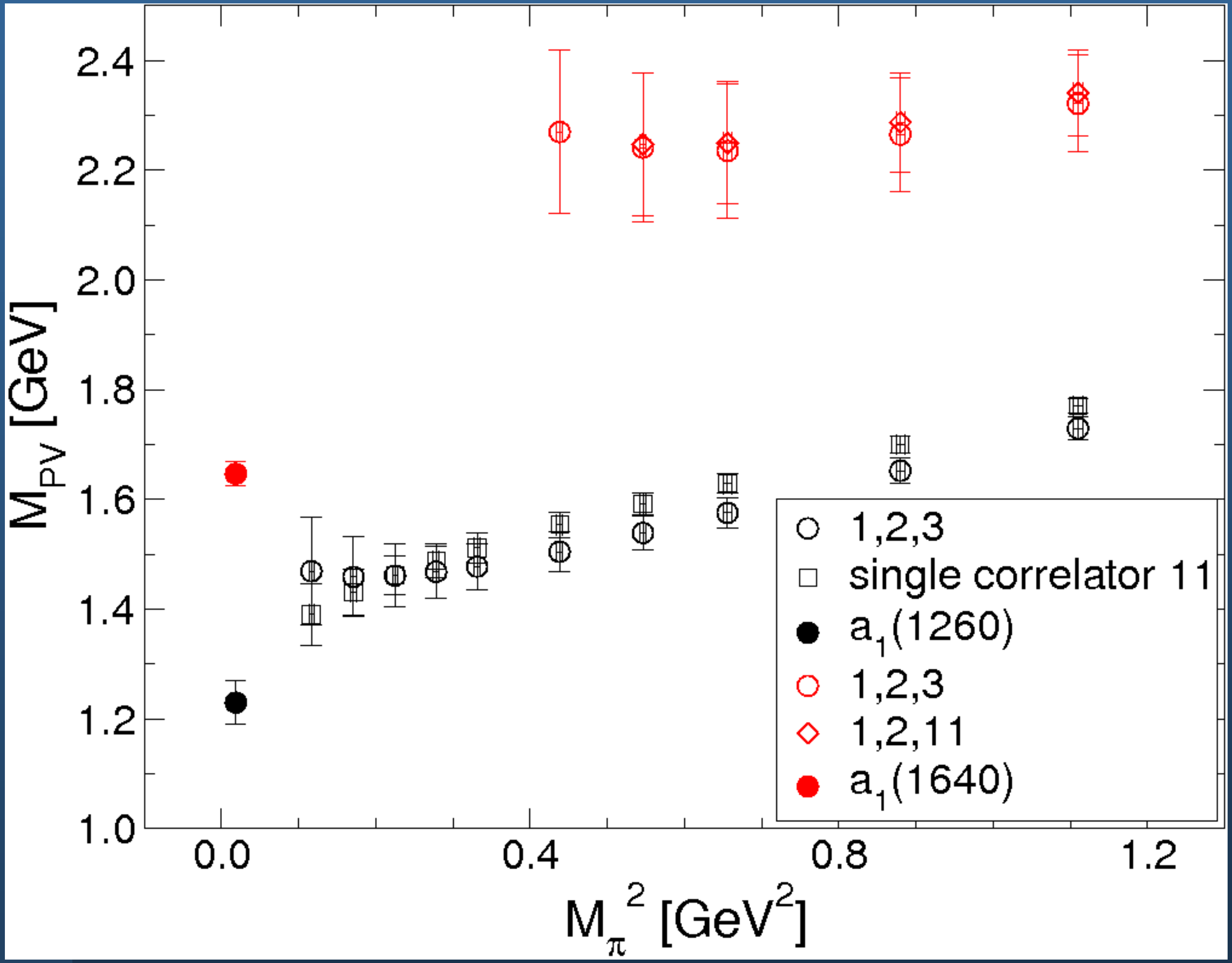
1^{--} second-excited state meson

- Gattringer et al.



1^{++} ground and first-excited state mesons

- Gattringer et al.



Search for light scalar tetraquarks with I=0, ½, 1

Sasa Prelovsek (Spectrum, Tuesday afternoon)

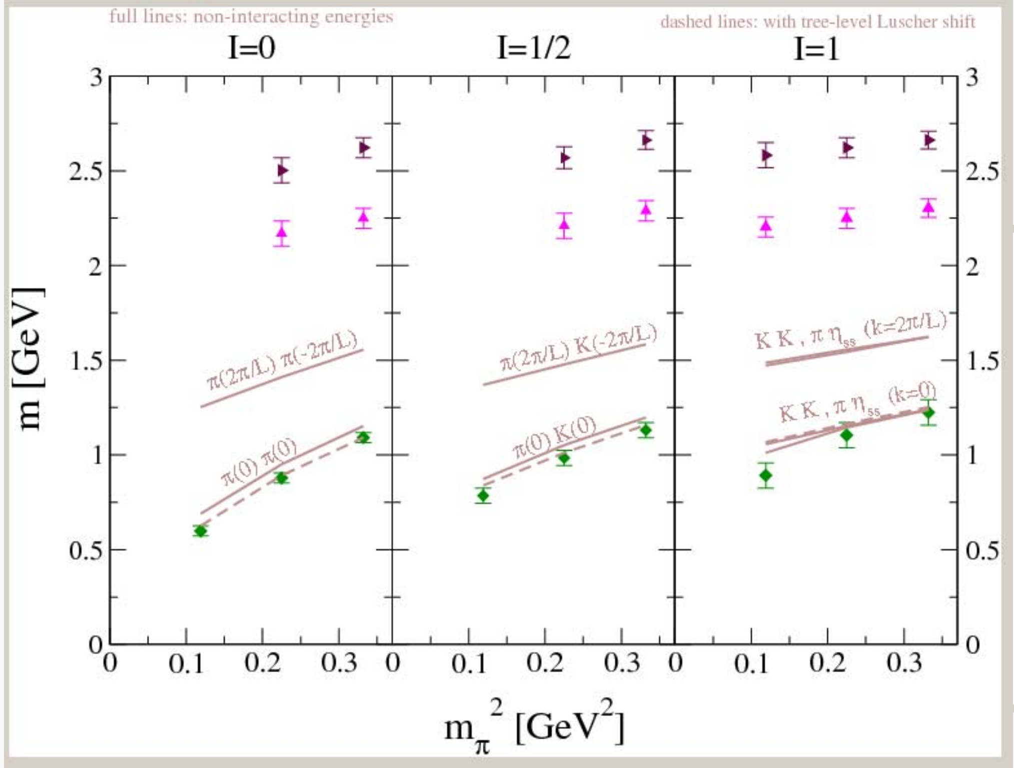
Simulation:

➢diquark anti-diquark interpolators

➢ 3 different smearings: 3x3 correlation matrix

➢ variational analysis: 3 states extracted

➢ Chiraly Improved f., mpi=300-600 MeV, L=12 & 16



Motivation:

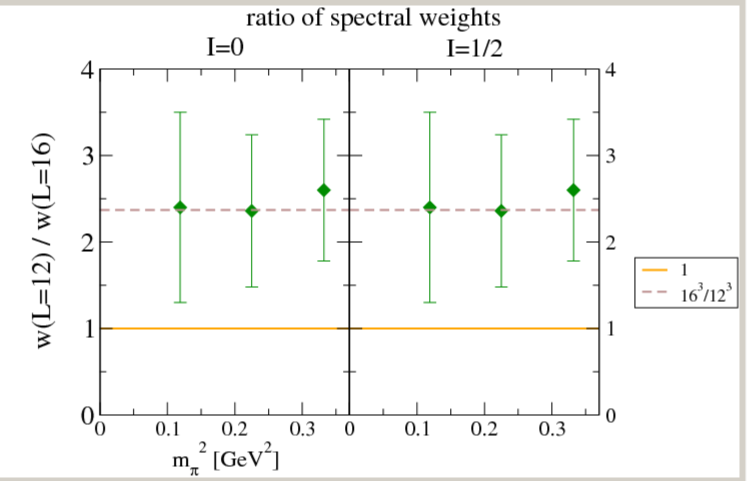
still not known whether some of the scalar resonances below 2 GeV correspond to qq or tetraquarks

Results:

➢ ground-state eigenvalue: tower of scattering states

➢ excited states: m>2 GeV

➢ **no indication for light tetraquarks found**



Operator design issues

- statistical noise increases with temporal separation t
- use of very good operators is crucial or noise swamps signal
- recipe for making better operators
  - crucial to construct operators using *smeared* fields
    - link variable smearing
    - quark field smearing
  - spatially extended operators
  - use large *set* of operators (variational coefficients)

Three stage approach (PRD72:094506,2005)

- concentrate on baryons at rest (zero momentum)
- operators classified according to the irreps of O_h

$$G_{1g}, G_{1u}, G_{2g}, G_{2u}, H_g, H_u$$

- (1) basic building blocks: smeared, covariant-displaced quark fields

$$(\widetilde{D}_j^{(p)}\widetilde{\psi}(x))_{Aa\alpha} \qquad p\text{ - link displacement } (j = 0, \pm 1, \pm 2, \pm 3)$$

- (2) construct elemental operators (translationally invariant)

$$B^F(x) = \phi^F_{ABC}\varepsilon_{abc}(\tilde{D}_i^{(p)}\tilde{\psi}(x))_{Aa\alpha}(\tilde{D}_j^{(p)}\tilde{\psi}(x))_{Bb\beta}(\tilde{D}_k^{(p)}\tilde{\psi}(x))_{Cc\gamma}$$

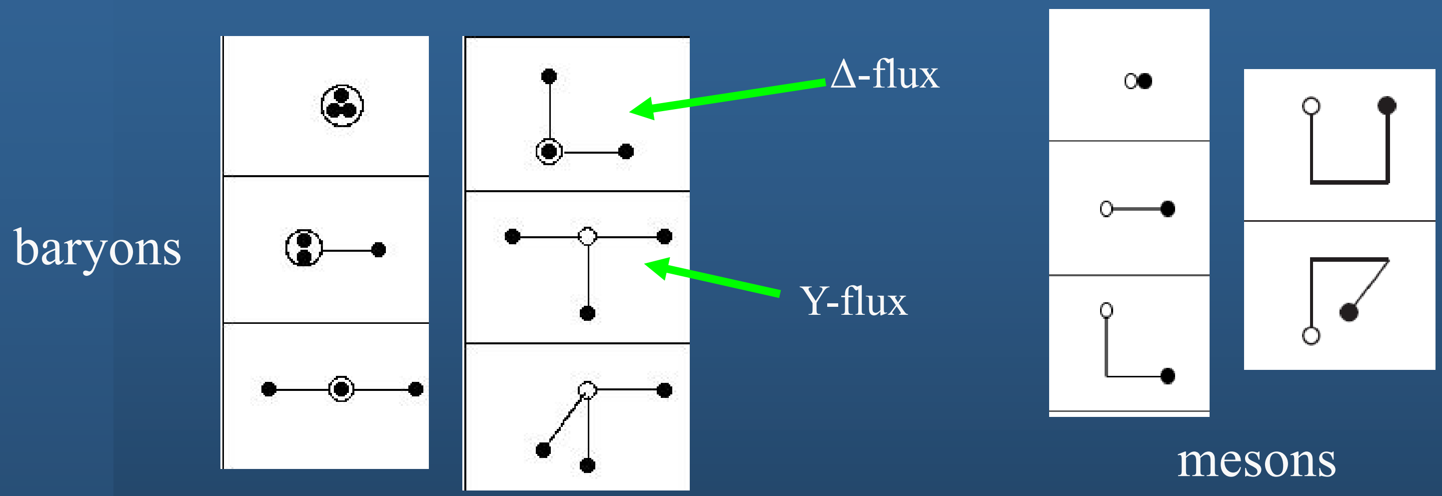
  - flavor structure from isospin
  - color structure from gauge invariance

- (3) group-theoretical projections onto irreps of O_h

$$B_i^{\Lambda\lambda F}(t) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R\in O_h^D} D_{\lambda\lambda}^{(\Lambda)}(R)^* U_R B_i^F(t) U_R^+$$

Incorporating orbital and radial structure

- displacements of different lengths build up radial structure
- displacements in different directions build up orbital structure



- operator design minimizes number of sources for quark propagators
- useful for mesons, tetraquarks, pentaquarks even!
- can even incorporate hybrid meson operators

Spin identification and other remarks

- spin identification possible by pattern matching

J	$n^J_{G_1}$	$n^J_{G_2}$	n^J_H
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\frac{11}{2}$	1	1	2
$\frac{13}{2}$	1	2	2
$\frac{15}{2}$	1	1	3
$\frac{17}{2}$	2	1	3

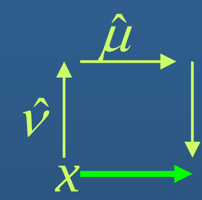
total numbers of operators assuming two different displacement lengths

Irrep	Δ, Ω	N	Σ, Ξ	Λ
G_{1g}	221	443	664	656
G_{1u}	221	443	664	656
G_{2g}	188	376	564	556
G_{2u}	188	376	564	556
H_g	418	809	1227	1209
H_u	418	809	1227	1209

- total numbers of operators is huge → uncharted territory
- ultimately must face two-hadron scattering states

Quark- and gauge-field smearing

- smeared quark and gluon fields fields → dramatically reduced coupling with short wavelength modes
- link-variable smearing (stout links PRD**69**, 054501 (2004))
  - define $C_\mu(x) = \sum_{\pm(\nu\neq\mu)} \rho_{\mu\nu} U_\nu(x) U_\mu(x+\hat{\nu}) U_\nu^+(x+\hat{\mu})$



  - spatially isotropic $\rho_{jk} = \rho, \quad \rho_{4k} = \rho_{k4} = 0$
  - exponentiate traceless Hermitian matrix

$$\Omega_\mu = C_\mu U_\mu^+ \qquad Q_\mu = \frac{i}{2}\left(\Omega_\mu^+ - \Omega_\mu\right) - \frac{i}{2N}\mathrm{Tr}\left(\Omega_\mu^+ - \Omega_\mu\right)$$

  - iterate $U_\mu^{(n+1)} = \exp\left(iQ_\mu^{(n)}\right) U_\mu^{(n)}$

$$U_\mu \to U_\mu^{(1)} \to \cdots \to U_\mu^{(n)} \equiv \widetilde{U}_\mu$$

- quark-field smearing (covariant Laplacian uses smeared gauge field)

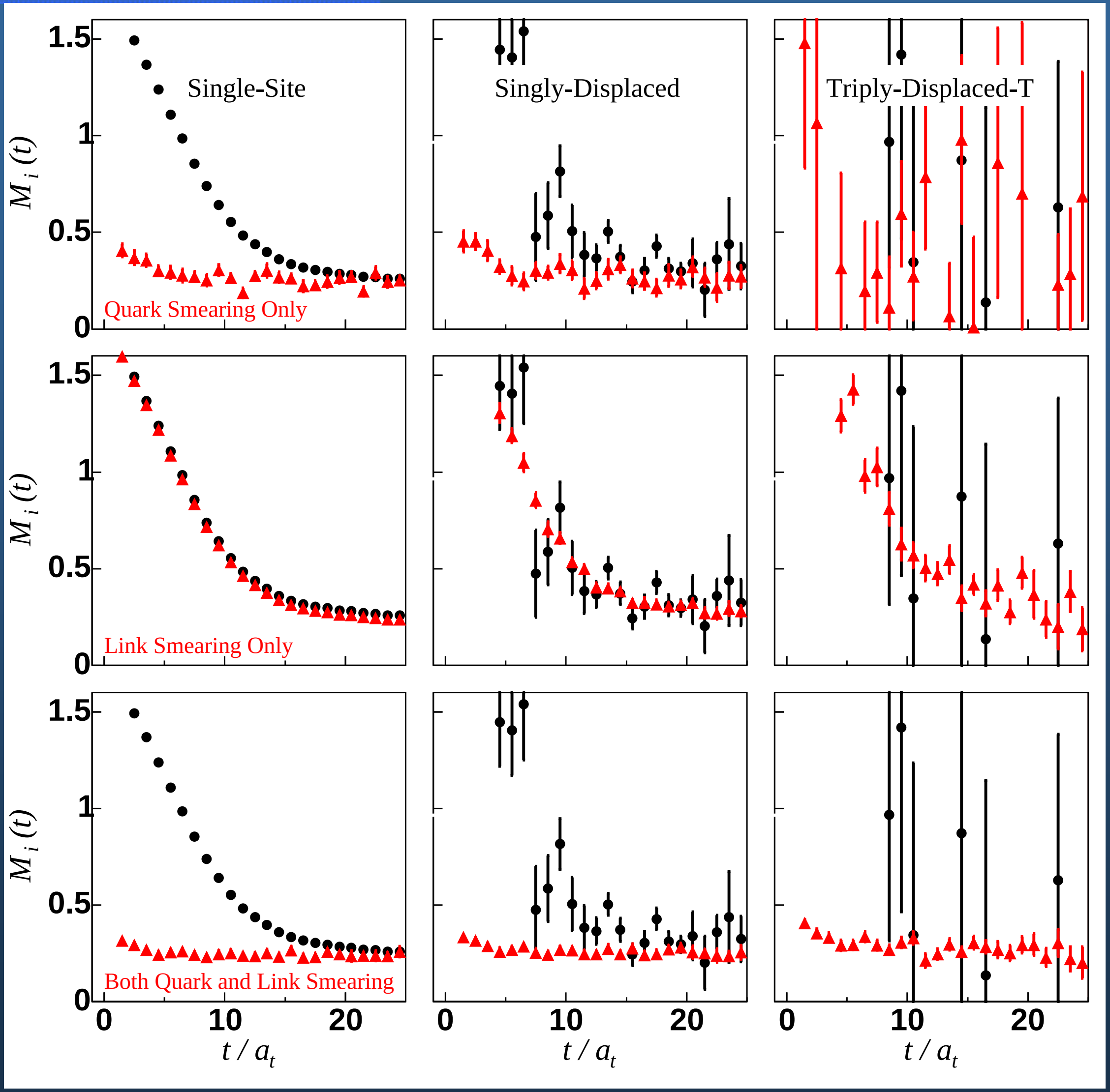
$$\tilde{\psi}(x) = \left(1 + \frac{\sigma_s}{4n_\sigma}\tilde{\Delta}^2\right)^{n_\sigma} \psi(x)$$

Importance of smearing

- Nucleon G_{1g} channel
- effective masses of 3 selected operators
- noise reduction from link variable smearing, especially for displaced operators
- quark-field smearing reduces couplings to high-lying states

$$\sigma_s = 4.0, \quad n_\sigma = 32$$
$$n_\rho \rho = 2.5, \quad n_\rho = 16$$

- less noise in excited states using $\sigma_s = 3.0$



Operator selection

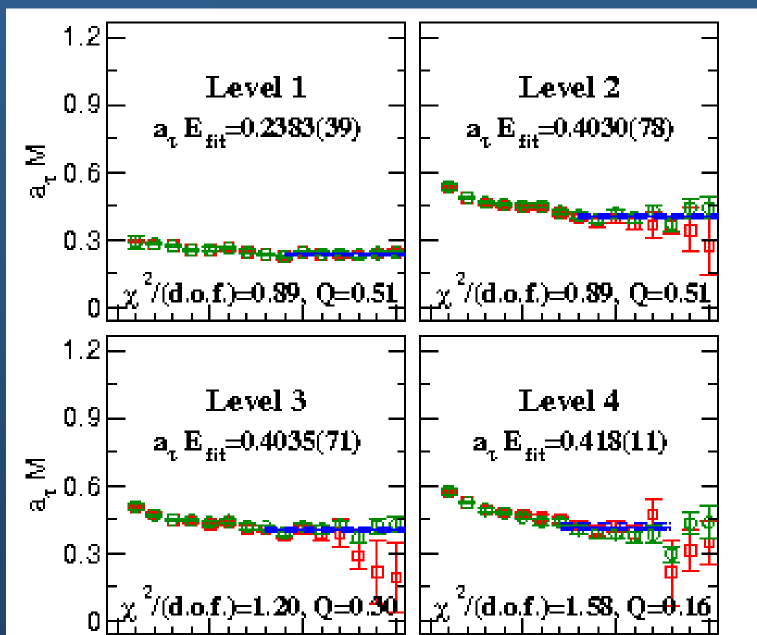
- rules of thumb for "pruning" operator sets
  - noise is the enemy!
  - prune first using intrinsic noise (diagonal correlators)
  - prune next within operator *types* (single-site, singly-displaced, *etc.*) based on condition number of
  - prune across all operators based on condition number
- best to keep a variety of different types of operators, as long as condition numbers maintained

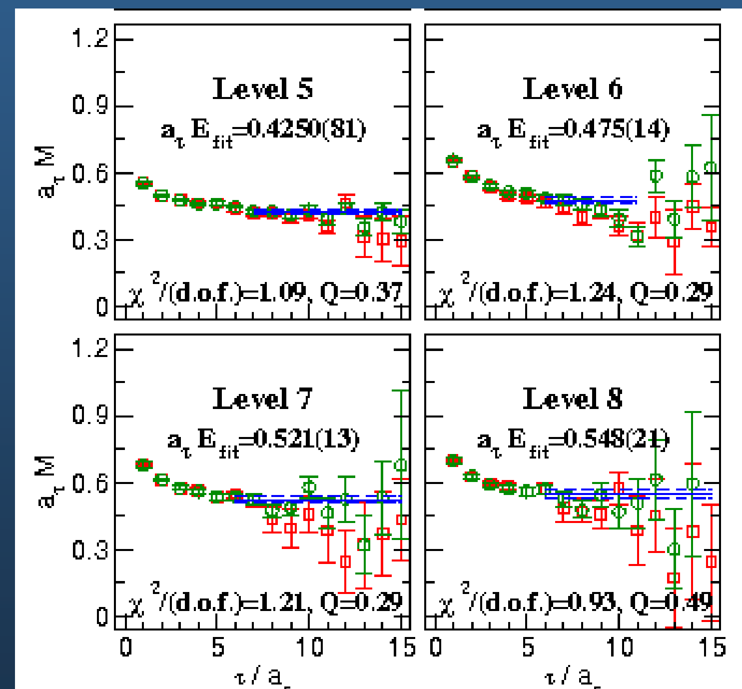
$$\hat{C}_{ij}(t) = \frac{C_{ij}(t)}{\sqrt{C_{ii}(t)C_{jj}(t)}}, \qquad t = 1$$

- typically use 16 operators to get 8 lowest lying levels

Nucleon G_{1g} effective masses

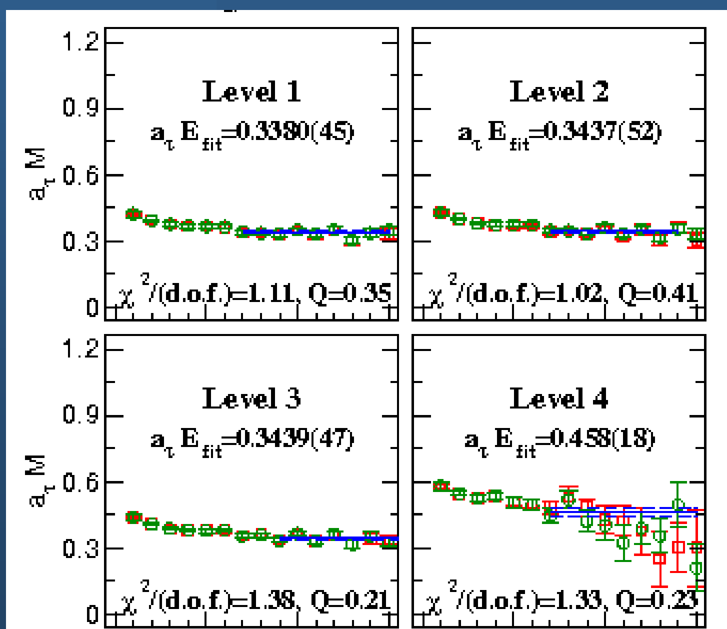
- 200 quenched configs, 12^3 48 anisotropic Wilson lattice, a_s~0.1 fm, a_s/a_t~3, m_π~700 MeV
- nucleon G_{1g} channel
- green=fixed coefficients, red=principal

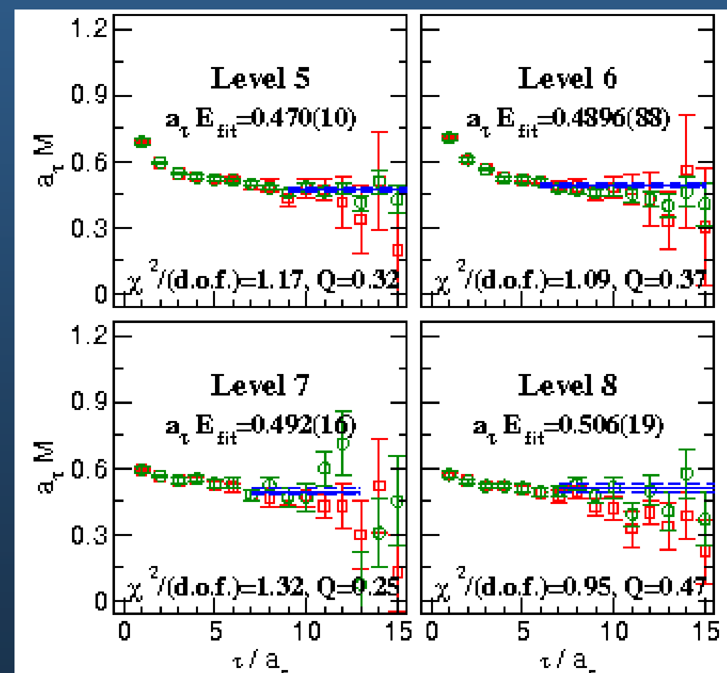




Nucleon H_u effective masses

- 200 quenched configs, 12^3 48 anisotropic Wilson lattice, a_s~0.1 fm, a_s/a_t~3, m_π~700 MeV
- nucleon H_u channel
- green=fixed coefficients, red=principal





Spatial summations

- baryon at rest is operator of form

$$B(\vec{p}=0,t)=\frac{1}{V}\sum_{\vec{x}}\varphi_B(\vec{x},t)$$

- baryon correlator has a double spatial sum

$$\left\langle 0\left|\overline{B}(\vec{p}=0,t)\ B(\vec{p}=0,0)\right|0\right\rangle=\frac{1}{V^2}\sum_{\vec{x},\vec{y}}\left\langle 0\left|\overline{\varphi}_B(\vec{x},t)\ \varphi_B(\vec{y},0)\right|0\right\rangle$$

- computing all elements of propagators exactly not feasible
- translational invariance can limit summation over source site to a single site for local operators

$$\left\langle 0\left|\overline{B}(\vec{p}=0,t)\ B(\vec{p}=0,0)\right|0\right\rangle=\frac{1}{V}\sum_{\vec{x}}\left\langle 0\left|\overline{\varphi}_B(\vec{x},t)\ \varphi_B(0,0)\right|0\right\rangle$$

All-to-all stochastic quark propagators

- good baryon-meson operator of total zero momentum has form

$$B(\vec{p},t)M(-\vec{p},t) = \frac{1}{V^2}\sum_{\vec{x},\vec{y}} \varphi_B(\vec{x},t)\varphi_M(\vec{y},t)e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

- cannot limit source to single site for multi-hadron operators
- disconnected diagrams (scalar mesons) will also need many-to-many quark propagators
- estimates of all quark propagator elements are needed!

Matrix inversion

- quark propagator is just inverse of Dirac matrix M
- noise vectors η satisfying $E(\eta_i)=0$ and $E(\eta_i\eta_j^*)=\delta_{ij}$ are useful for stochastic estimates of inverse of a matrix M
- Z_4 noise is used $\{1, i, -1, -i\}$
- define $X(\eta)=M^{-1}\eta$ then

$$E(X_i\eta_j^*) = E\left(\sum_k M_{ik}^{-1}\eta_k\eta_j^*\right) = \sum_k M_{ik}^{-1} E\left(\eta_k\eta_j^*\right) = \sum_k M_{ik}^{-1}\delta_{kj} = M_{ij}^{-1}$$

- if can solve $M X^{(r)}= \eta^{(r)}$ for each of N_R noise vectors $\eta^{(r)}$ then we have a Monte Carlo estimate of all elements of M^{-1}:

$$M_{ij}^{-1} \approx \frac{1}{N_R}\sum_{r=1}^{N_R} X_i^{(r)}\eta_j^{(r)*}$$

- variances in above estimates usually unacceptably large
- introduce variance reduction using source *dilution*

Source dilution for single matrix inverse

- dilution introduces a complete set of projections:

$$P^{(a)}P^{(b)} = \delta^{ab}P^{(a)}, \qquad \sum_a P^{(a)} = 1, \qquad P^{(a)\dagger} = P^{(a)}$$

- observe that

$$M^{-1}_{ij} = M^{-1}_{ik}\delta_{kj} = \sum_a M^{-1}_{ik}P^{(a)}_{kj} = \sum_a M^{-1}_{ik}P^{(a)}_{kk'}\delta_{k'j'}P^{(a)}_{j'j}$$

$$= \sum_a M^{-1}_{ik}P^{(a)}_{kk'}E\left(\eta_{k'}\eta^*_{j'}\right)P^{(a)}_{j'j} = \sum_a M^{-1}_{ik}E\left(P^{(a)}_{kk'}\eta_{k'}\eta^*_{j'}P^{(a)}_{j'j}\right)$$

- define $\eta^{[a]}_k = P^{(a)}_{kk'}\eta_{k'}, \qquad \eta^{[a]*}_j = \eta^*_{j'}P^{(a)}_{j'j}, \qquad X^{[a]}_k = M^{-1}_{kj}\eta^{[a]}_j$

  so that $M^{-1}_{ij} = \sum_a E\left(X^{[a]}_i \eta^{[a]*}_j\right)$

- Monte Carlo estimate is now

$$M^{-1}_{ij} \approx \frac{1}{N_R}\sum_{r=1}^{N_R}\sum_a X^{(r)[a]}_i \eta^{(r)[a]*}_j$$

- $\sum_a \eta^{[a]}_i \eta^{[a]*}_j$ has same expected value as $\eta_i\eta^*_j$, but reduced variance (statistical zeros → exact)

Dilution schemes for spectroscopy

- Time dilution (particularly effective)

$$P^{(B)}_{a\alpha;b\beta}\left(\vec{x},t;\vec{y},t'\right)=\delta_{ab}\delta_{\alpha\beta}\delta\left(\vec{x},\vec{y}\right)\delta_{Bt}\delta_{Bt'},\qquad B=0,1,\ldots,N_t-1$$

- Spin dilution

$$P^{(B)}_{a\alpha;b\beta}\left(\vec{x},t;\vec{y},t'\right)=\delta_{ab}\delta_{B\alpha}\delta_{B\beta}\delta\left(\vec{x},\vec{y}\right)\delta_{tt'},\qquad B=0,1,2,3$$

- Color dilution

$$P^{(B)}_{a\alpha;b\beta}\left(\vec{x},t;\vec{y},t'\right)=\delta_{Ba}\delta_{Bb}\delta_{\alpha\beta}\delta\left(\vec{x},\vec{y}\right)\delta_{tt'},\qquad B=0,1,2$$

- Spatial dilutions?
  - even-odd

Source-sink factorization

- baryon correlator has form

$$C_{l\bar{l}} = c_{ijk}^{(l)} c_{\bar{i}\bar{j}\bar{k}}^{(\bar{l})*} Q_{i\bar{i}}^{A} Q_{j\bar{j}}^{B} Q_{k\bar{k}}^{C}$$

- stochastic estimates with dilution

$$C_{l\bar{l}} \approx \frac{1}{N_R} \sum_{r} \sum_{d_A d_B d_C} c_{ijk}^{(l)} c_{\bar{i}\bar{j}\bar{k}}^{(\bar{l})*} \left(\varphi_i^{(Ar)[d_A]} \eta_{\bar{i}}^{(Ar)[d_A]*} \right) \times \left(\varphi_j^{(Br)[d_B]} \eta_{\bar{j}}^{(Br)[d_B]*} \right) \left(\varphi_k^{(Cr)[d_C]} \eta_{\bar{k}}^{(Cr)[d_C]*} \right)$$

- define

$$\Gamma_l^{(r)[d_A d_B d_C]} = c_{ijk}^{(l)} \varphi_i^{(Ar)[d_A]} \varphi_j^{(Br)[d_B]} \varphi_k^{(Cr)[d_C]}$$

$$\Omega_l^{(r)[d_A d_B d_C]} = c_{ijk}^{(l)} \eta_i^{(Ar)[d_A]} \eta_j^{(Br)[d_B]} \eta_k^{(Cr)[d_C]}$$

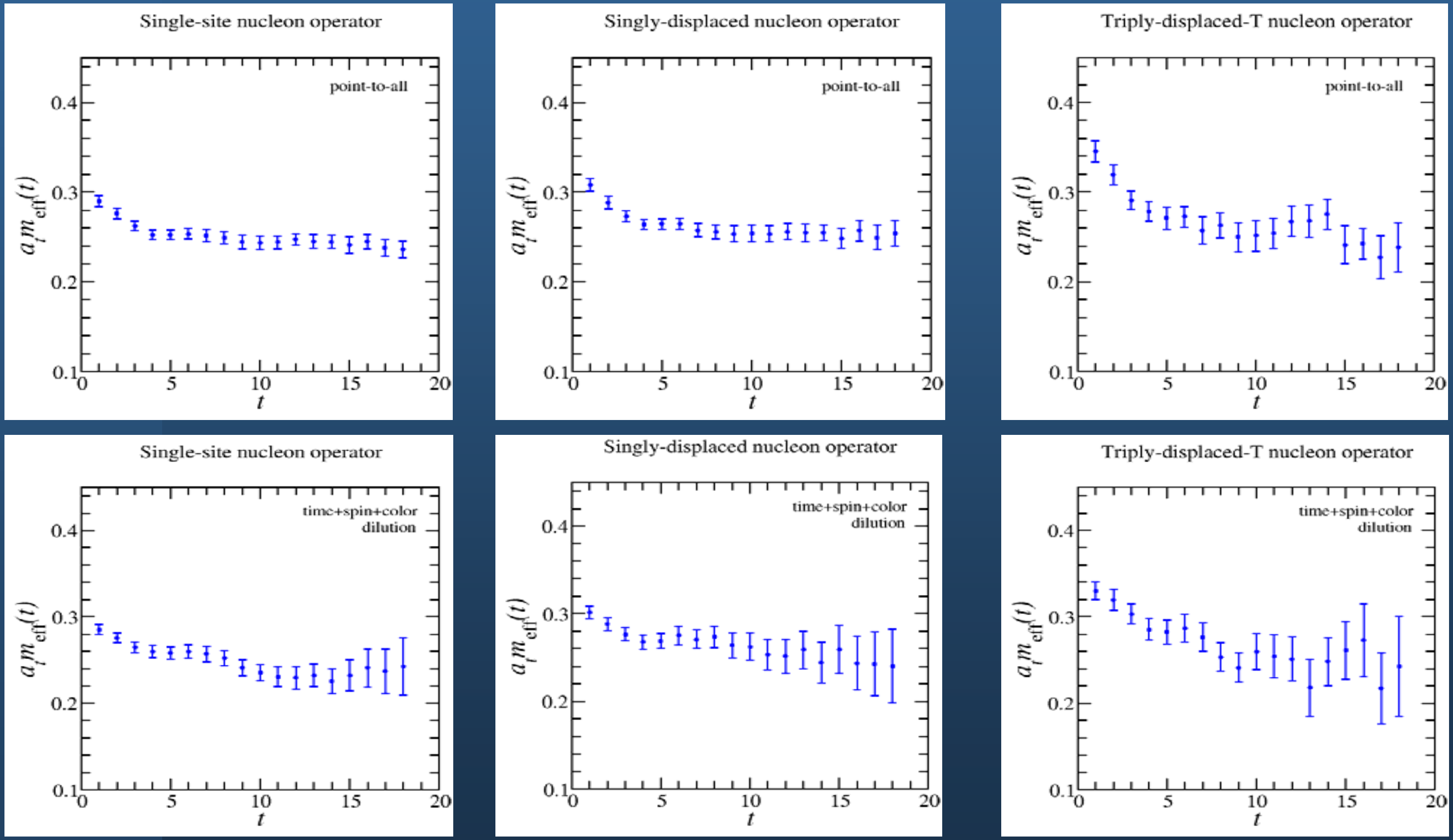
- correlator becomes dot product of source vector with sink vector

$$C_{l\bar{l}} \approx \frac{1}{N_R} \sum_{r} \sum_{d_A d_B d_C} \Gamma_l^{(r)[d_A d_B d_C]} \Omega_{\bar{l}}^{(r)[d_A d_B d_C]*}$$

- store *ABC* permutations to handle Wick orderings

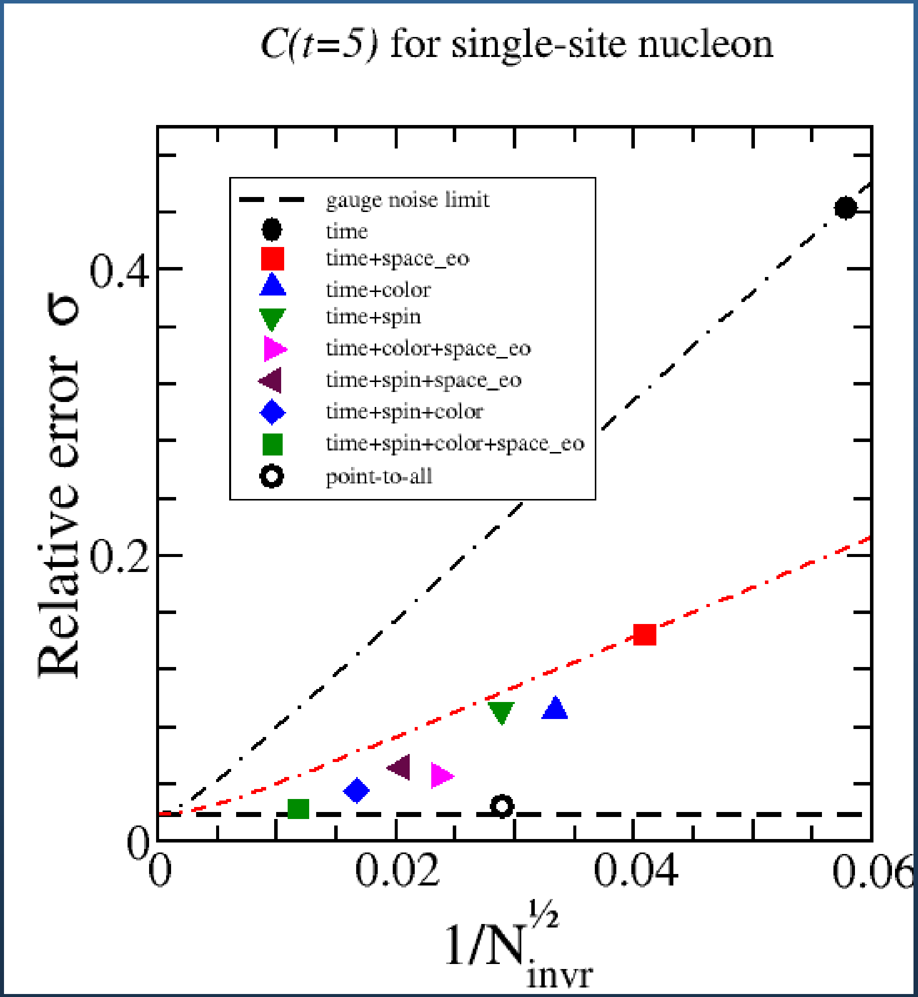
Dilution tests (see J. Bulava talk)

- 100 quenched configs, 12^3 48 anisotropic Wilson lattice, a_s~0.1 fm, a_s/a_t~3, m_π~700 MeV
- three representative operators: SS,SD,TDT nucleon operators



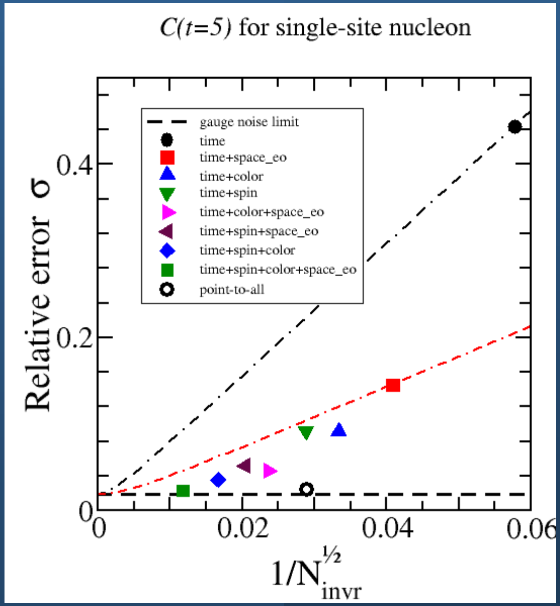
Dilution tests (continued)

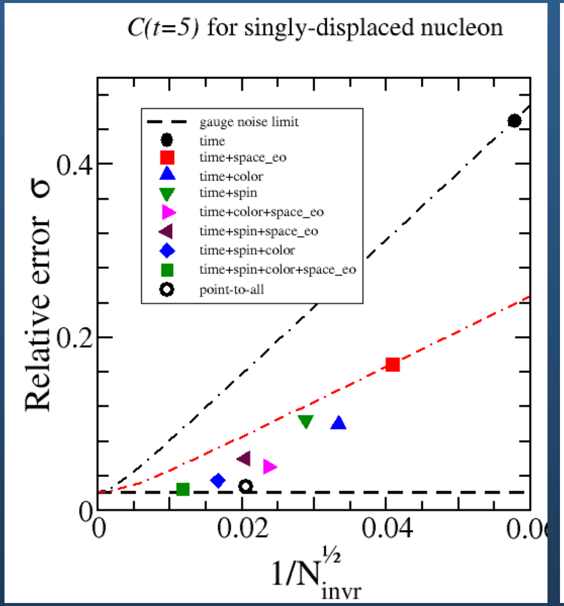
- 100 quenched configs, 12^3 48 anisotropic Wilson lattice

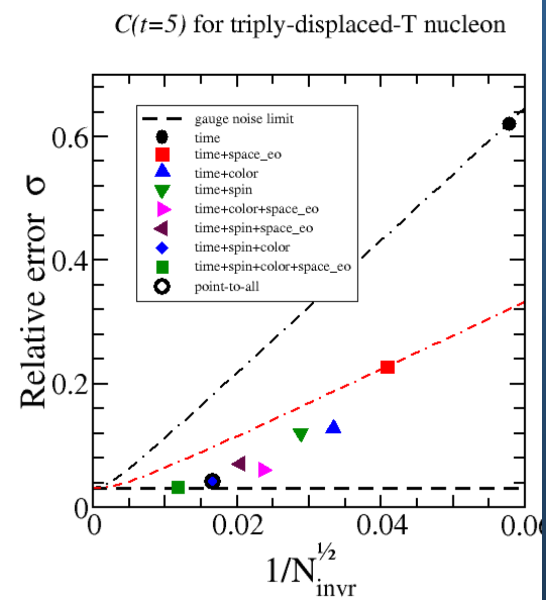


Dilution tests (continued)

- 100 quenched configs, 12^3 48 anisotropic Wilson lattice
- SS,SD,TDT nucleon operators
- same conclusions for other *t* values of *C(t)*, fitted mass







Future directions

- testing new method:
  - clever choice of quark-field smearing makes exact computations with all-to-all quark propagators possible!!
  - will work for disconnected diagrams
- will compare with stochastic with dilutions method

QCD Spectrum Collaboration

- A. Lichtl (Brookhaven Nat. Lab.)
- J. Bulava, C. Morningstar, J. Foley (Carnegie Mellon U.)
- R. Edwards, B. Joo, H.W. Lin, D. Richards (Jefferson Lab.)
- E. Engelson, S. Wallace (U. Maryland)
- K.J. Juge (U. of Pacific)
- N. Mathur (Tata Institute)
- M. Peardon, S. Ryan (Trinity Coll. Dublin)

Configuration generation

- significant time on USQCD (DOE) and NSF computing resources
- anisotropic clover fermion action (with stout links) and anisotropic improved gauge action
  - tunings of couplings, aspect ratio, lattice spacing in progress (Justin Foley, Robert Edwards talks)
- anisotropic Wilson configurations generated during clover tuning
- current goal:
  - three lattice spacings: a = 0.125 fm, 0.10 fm, 0.08 fm
  - three volumes: $V = (3.2\ \mathrm{fm})^4, (4.0\ \mathrm{fm})^4, (5.0\ \mathrm{fm})^4$
  - 2+1 flavors, $m_\pi \sim$ 350 MeV, 220 MeV, 180 MeV
- USQCD Chroma software suite

QCD Spectrum Collaboration talks

- J. Bulava, *Stochastic all-to-all propagators for baryon correlators*
- E. Engelson, *Lattice QCD determination of patterns of excited baryon masses*
- J. Foley, *Tuning improved anisotropic actions in lattice perturbation theory*
- R. Edwards, *Three flavor anisotropic clover fermions*
- K.J. Juge, *Multi-hadron operators with all-to-all quark propagators*
- N. Mathur , *Cascade baryon spectrum from lattice QCD*
- M. Peardon, *Determining bare quark masses for Nf = 2+1 dynamical simulations*

Summary

- discussed issues with unstable hadrons (resonances)
- discussed extraction of excited states in Monte Carlo calculations
  - correlation matrices needed
  - operators with very good overlaps onto states of interest
- must extract all states lying below a state of interest
  - as pion get lighter, more and more multi-hadron states
- multi-hadron operators → relative momenta
  - need for all-to-all quark propagators
- disconnected diagrams
- *exploration of excited hadrons*

  is well underway