QCD Thermodynamics

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Why Study QCD thermodynamics?

- Early universe
- Heavy ion collisions
  - RHIC, LHC, FAIR
  - Plasma signatures, structure
  - Modeling: hydrodynamics
- Dense stars
  - Stellar structure, observables
  - Strange matter
- Theory
  - Chiral symmetry restoration
  - Color/flavor locking
  - AdS/CFT predictions

What lattice QCD can contribute

- At RHIC temperatures we need nonperturbative calculations.
- We can do equilibrium thermodynamics at zero baryon density
  - Phase diagram
  - Transition temperature
  - Equation of state
  - Transport coefficients
- Nonequilibrium difficult
  - Extrapolate from lattice results using models - e.g. hydrodynamics
- Nonzero baryon density is very challenging
  - See next talk by Shinji Ejiri

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Progress in the Past Year

- Large HotQCD study. (R. Gupta, M. Cheng talks).
- Insights that affect the determination of T_c. (Karsch talk).
- New ideas for computing the equation of state. (Umeda talk).
- New ideas and methods for computing transport coefficients. (Meyer talk).
- New result for spatial string tension. (RBC/Bielefeld).
- No time to cover QCD phase structure at high N_f. (Deuzeman talk)
- My apologies to those whose work I am not covering.

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Outline

- Introduction
- Lattice methodology
- Cutoff issues with various actions
- Determination of T_c at zero baryon number density
  - A variety of observables and their problems
  - T_c confusion diminished
- Equation of state
- Plasma structure
  - Transport coefficients
  - Spatial string tension
- Conclusions

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Lattice methodology: General approach

- Set finite N_τ
- Simulate quantum partition function $Z = \mathrm{Tr}\exp(-H/T)$
- $T = 1/(aN_\tau)$
- Usually fix N_τ and vary g^2 to vary a, so $T \to \infty$ as $g^2, a \to 0$.
- Set quark masses (use the same H but $T \to 0$)
  e.g. Lines of constant physics are standard now

$$m_\pi/m_\rho = \mathrm{const} \quad m_K/m_\phi = \mathrm{const}$$

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Lattice methodology: continuum limit

- Approach the continuum by increasing N_τ and repeating.
- Warning! Risk of lattice cutoff effects at low N_τ.

At $T_c \approx 180$ MeV we have

N_τ	4	6	8	10	12
a (fm)	0.27	0.18	0.14	0.11	0.09

So by today's standards $N_\tau = 4$ looks crude, even for improved actions.

Lattice methodology: Action choices

- Pure Yang-Mills quite well studied.
- QCD with 2 or 2+1 flavors.
  - Staggered: most thoroughly studied
  - Wilson: has been limited to rather heavy quark masses. (JLQCD; Bornyakov et al.)
  - Domain wall and overlap: in infancy
- Improvement is essential. Calculational cost

$$\begin{aligned} \text{standard cost} &\sim a^{-7} \\ \text{EOS cost} &\sim a^{-11} \end{aligned}$$

  - Improvement tends to fatten the action operators
  - Locality could become an issue for improved-action thermodynamics
  - Want localization length $\ell \ll aN_\tau = 1/T$

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Cutoff issues: free fermions

- Cutoff effects for various actions [Hegde et al (arXiv:0801.4883)]

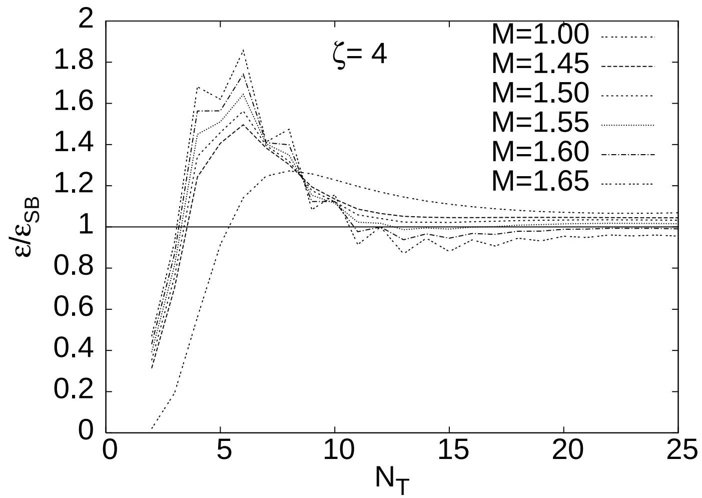
$$\frac{P}{T^4} = \sum_{k=0}^{\infty} A_{2k} P_{2k}(\mu/\pi T) \left(\frac{\pi}{N_\tau}\right)^{2k}$$

action	A_2/A_0	A_4/A_0	A_6/A_0
standard staggered	$248/147$	$635/147$	$3796/189$
Naik	0	$-1143/980$	$-365/77$
p4	0	$-1143/980$	$73/2079$
standard Wilson	$248/147$	$635/147$	$13351/8316$
hypercube	-0.242381	0.114366	-0.0436614
overlap/ domain wall	$248/147$	$635/147$	$3796/189$

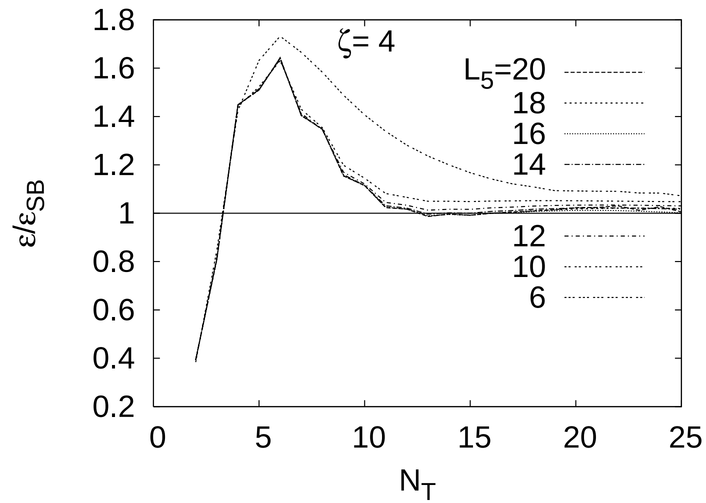
- Quarks do become free at high T, so why design for bad scaling?
- DWF, overlap, Wilson: dispersion relation should be improved!

Cutoff issues: free chiral fermions

- Gavai and Sharma (arXiv:0805.2866)
  Free chiral fermions.



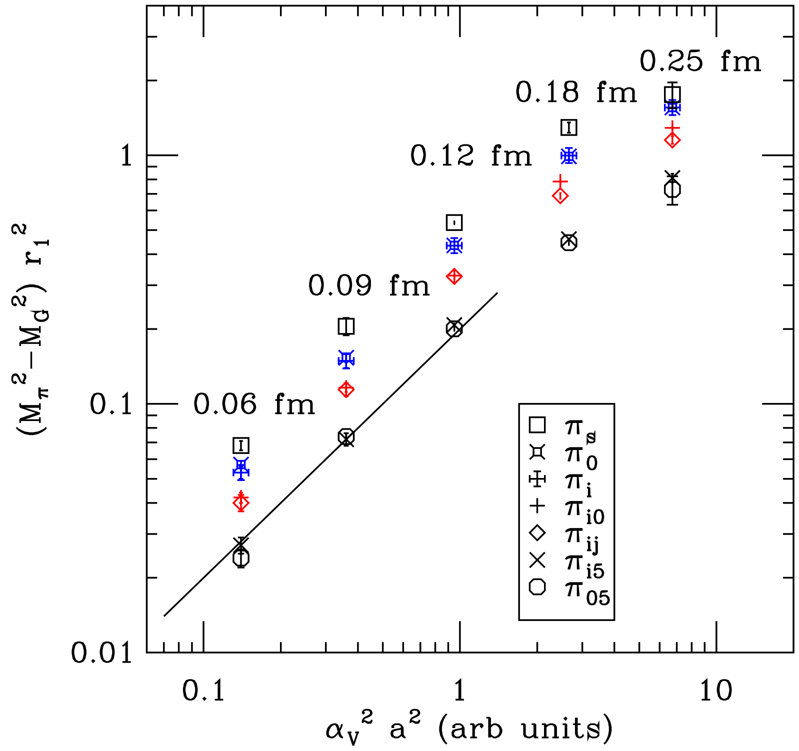
Free overlap



Free domain wall $M = 1.55$

- Signs of negative transfer-matrix eigenvalues and slow scaling.

Cutoff issues: staggered fermions



MILC (2001-8)

Splitting of the pion taste multiplet *vs* $\alpha^2 a^2$.

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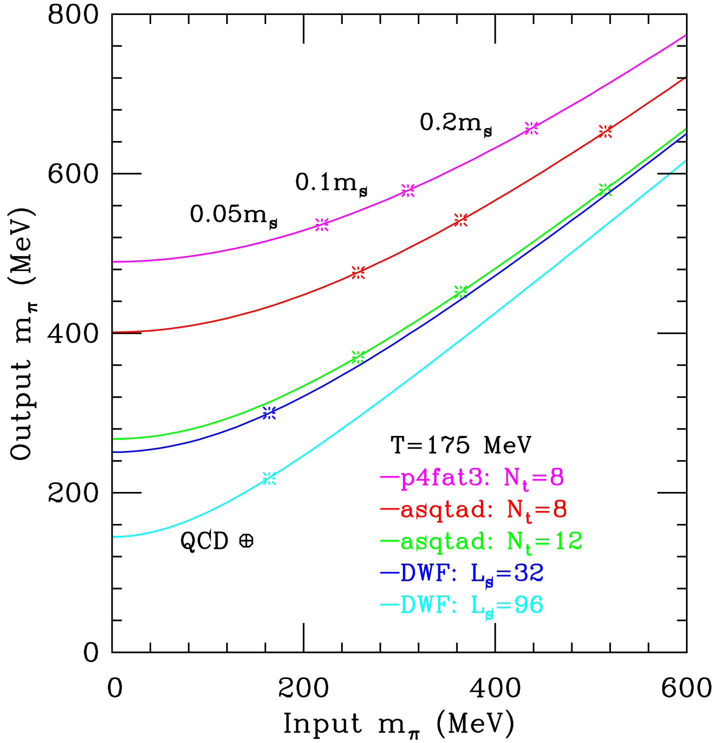
Cutoff issues: staggered and DWF

- Input pion mass
  - Staggered: Goldstone pion.
  - DWF: pion with $m_{\text{res}} = 0$.
- Output pion mass
  - Staggered: RMS mass.
  - DWF: propagator pion.



- Truth in labeling! If the staggered Goldstone pion is at it's physical mass, but the RMS pion mass is large, one can't claim to have reached the physical point.
- Important for deconfining phenomena? Maybe not. Hagedorn pileup of states.
- Important for chiral critical behavior? Maybe so. Wrong universality class.
- Important for $T < 100$ MeV? Certainly. Wrong pion masses.

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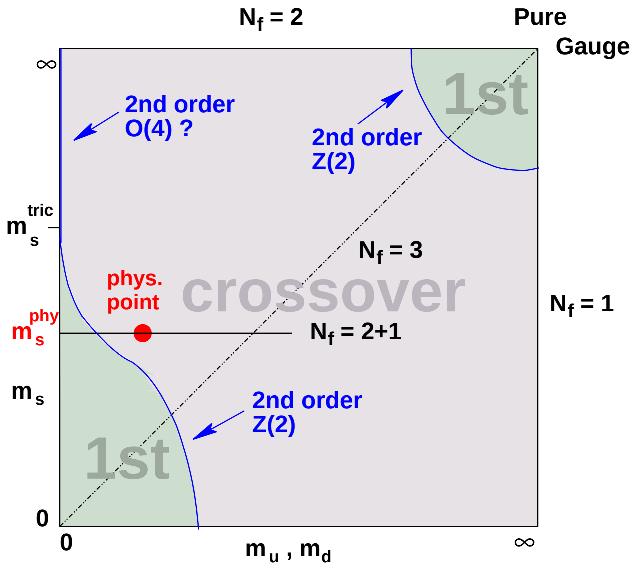
Phase Diagram at Zero Baryon Density



- Long-standing consensus: Crossover at the physical point
  Recent strong case: Y. Aoki, G. Endrödi, Z. Fodor, S.D. Katz and K.K. Szabó [Nature 443, 675 (2006)].
- Dissent: first order for $N_f = 2$? Bielefeld(1996), JLQCD (1996-8). Pisa/Genoa/BNL group (2005-2008). G. Cossu (this conference).
  But! so far only $N_\tau = 4$ with conventional staggered fermions.

How to determine T_c

- For a crossover it is not uniquely defined.
  - Why do we need T_c?
  - How precisely do we need it?
  - What is the relevant observable?
- Phenomenology at the physical point. Here it is good enough to determine the temperature range over which a quantity, such as the energy density or entropy density changes rapidly. Each observable may give a different answer.
- Field theory at a critical point. T_c is unambigous, precision is achievable, and it may even be useful. The observable must have a sensible continuum limit and, to be effective, it should expose the critical behavior.
- Related issue: how to set the lattice scale?

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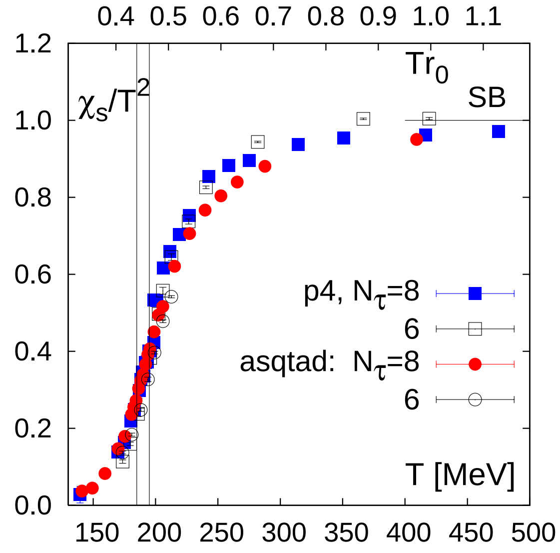
Transition signatures

- Deconfinement-type observables
  - Quark number susceptibility
  - Energy density or entropy
  - Polyakov loop -> heavy quark screening free energy
- Chiral-type observables
  - Light quark chiral condensate
  - Light quark chiral susceptibilities

Deconfinement-type observables

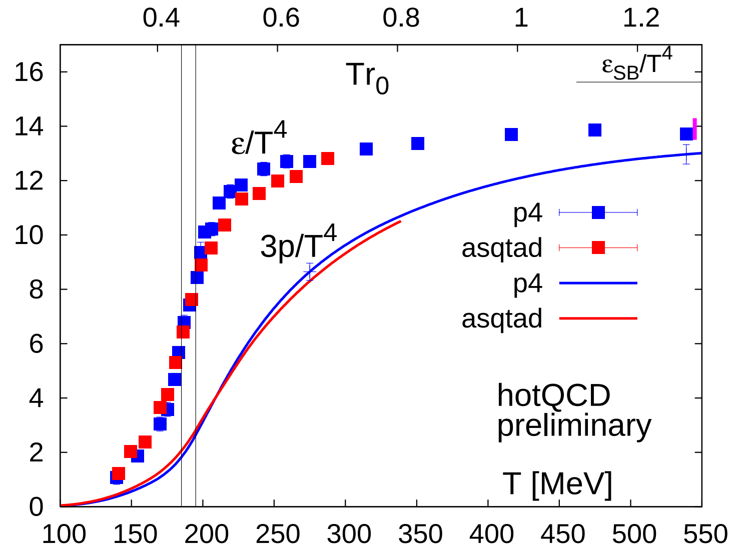


Strange quark number susceptibility

$$\chi_s = \langle N_s^2 \rangle / (VT)$$

Measures strangeness fluctuations

LCP $m_\ell / m_s = 0.1$



Equation of state

$$\begin{aligned} \varepsilon &= \text{energy density} \\ 3p &= \text{three times pressure} \end{aligned}$$

$N_\tau = 8$, LCP $m_\ell / m_s = 0.1$

[HotQCD preliminary (2008)]

Synthesis of order parameters

- Singular part of the free energy $f_s = -T \log Z$ in the chiral limit. [Hatta and Ikeda (2002), Karsch(2007)]

$$\begin{aligned} f_s(T,\mu_q) &= b^{-1} f_s(tb^{1/(2-\alpha)}) \sim t^{2-\alpha} \qquad [\alpha \approx -0.25 \text{ for } O(4)] \\ t &= \left| \frac{T - T_c}{T_c} \right| + c \left(\frac{\mu_q}{T_c} \right)^2 \end{aligned}$$

- Slope in T of quark number susceptibility at $\mu_q = 0$ (Inflection point at max slope)

$$\frac{\partial(\chi_l/T^2)}{\partial T} \sim \frac{\partial^3 f_s}{\partial \mu^2 \partial T} \sim t^{1-\alpha}$$

  Weak. Masked by analytic contributions.

- Specific heat and quartic quark number fluctuations at $\mu_q = 0$.

$$C_V \sim \frac{\partial^2 f_s}{\partial T^2} \sim t^{-\alpha} \qquad\qquad c_4^q \sim \frac{\partial^4 f_s}{\partial \mu_q^4} \sim t^{-\alpha}$$

- $c_4^q = (\langle N_q^4 \rangle - 3 \langle N_q^2 \rangle)$ is a good observable. Karsch (2007).

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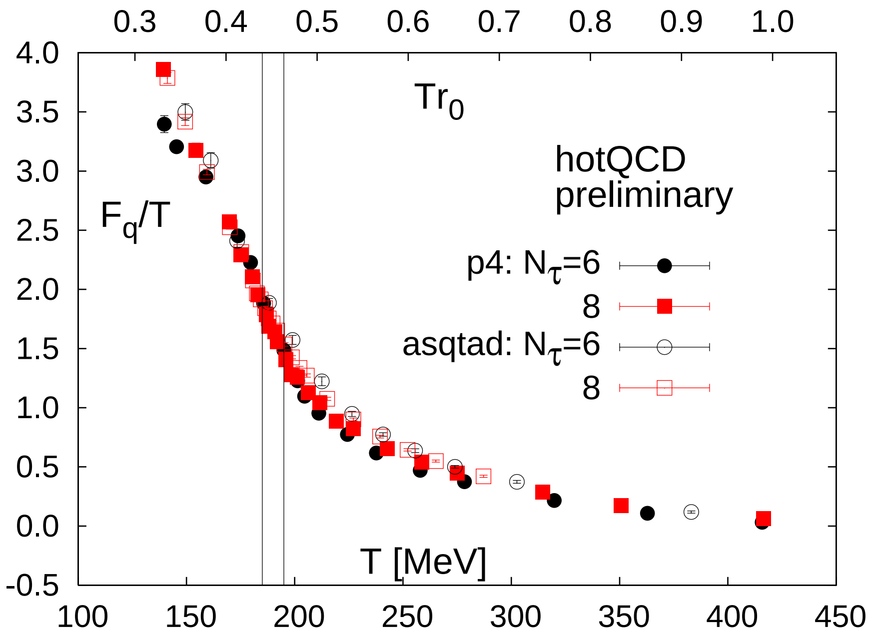
Deconfinement-type observables

Heavy quark free energy



$N_\tau = 8$, LCP $m_{ud}/m_s = 0.1$

- Related to Polyakov loop, the deconfinement order parameter.

$$F_q(T) = -T \log[P_{\rm renorm}(T)].$$

- Useful for phenomenology, but will it show critical behavior?
- Polyakov loop susceptibility? Peak weakens with increasing N_τ.

Chiral-type: chiral condensate

- Chiral condensate for light quarks at small m and a: Chiral and UV singularities:

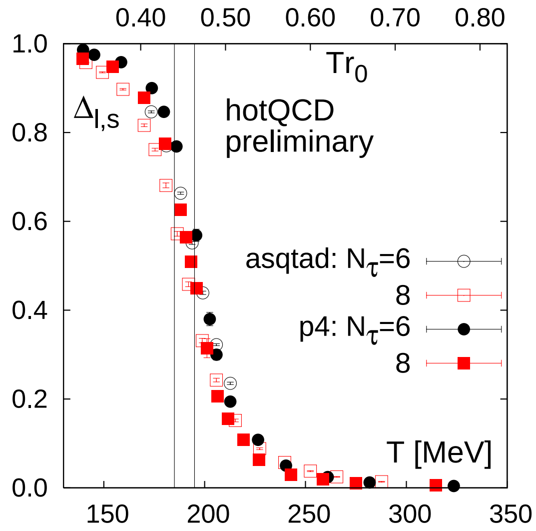
$$\langle\bar\psi\psi\rangle(a,m,T) \sim \begin{cases} c_{1/2}(a,T)\sqrt{m} + c_1 m/a^2 + \text{analytic} & T < T_c \\ c_1 m/a^2 + c_\delta m^{1/\delta} + \text{analytic} & T = T_c \\ c_1 m/a^2 + \text{analytic} & T > T_c \end{cases}$$

- Bielefeld/RBC difference eliminates the m/a^2 term.

$$\begin{aligned} D_{\ell,s}(T) &= \langle\bar\psi\psi\rangle|_\ell - \frac{m_\ell}{m_s}\langle\psi\psi\rangle|_s \\ \Delta_{\ell,s}(T) &= D_{\ell,s}(T)/D_{\ell,s}(T=0) \end{aligned}$$

$N_\tau = 8$, LCP $m_{ud}/m_s = 0.1$

[HotQCD preliminary (2008)]



- It is not a primary phenomenological variable like entropy density, but it has a clear chiral limit.

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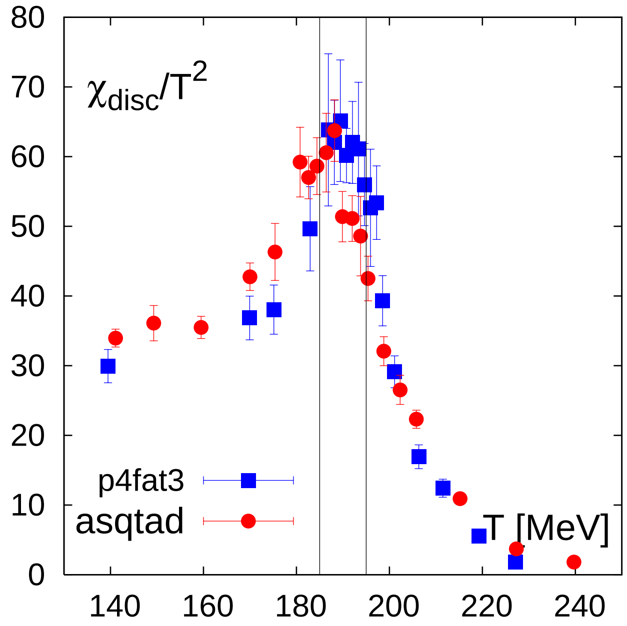
Chiral-type: chiral susceptibility

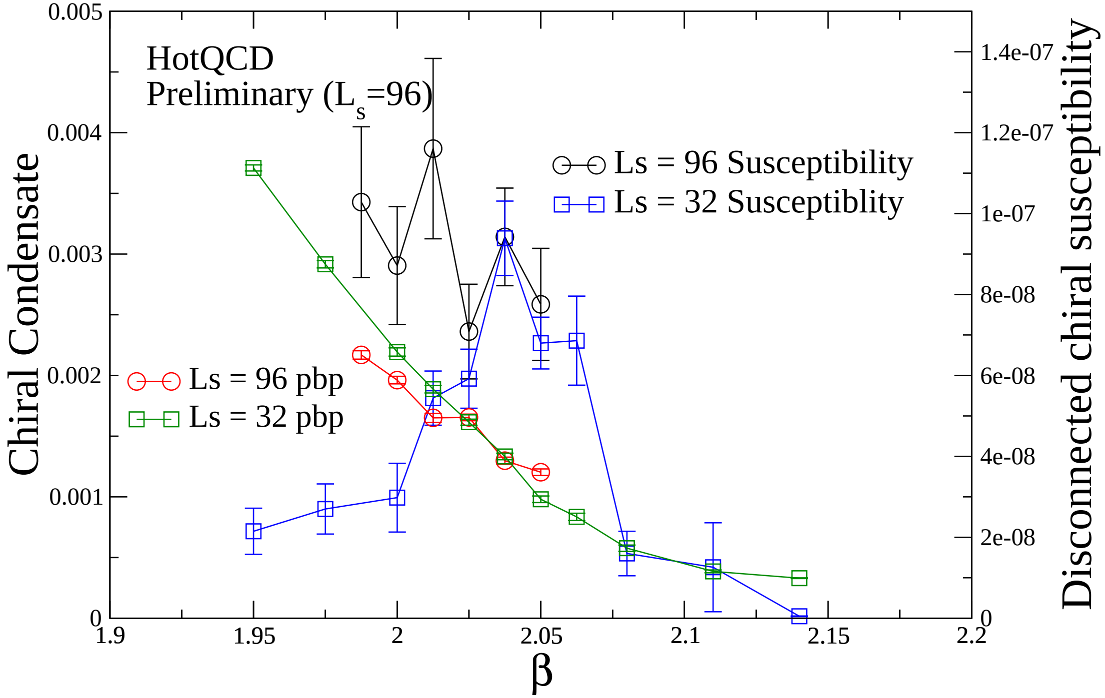


[HotQCD preliminary (2008)] light quark, $N_\tau = 8$, LCP $m_\ell/m_s = 0.1$

$$\chi_{\rm disc} = \frac{T}{V}\left[\left\langle(\mathrm{Tr}\, M^{-1})^2\right\rangle - \left\langle \mathrm{Tr}\, M^{-1}\right\rangle^2\right]$$

Chiral susceptibility: Domain Wall

New exploratory DWF study



$$16^3 \times 8 \times L_s,\, N_\tau = 8,\, m_\pi/m_\rho \approx 0.37$$

M Cheng (this conference)

Chiral susceptibility

- The chiral susceptibility is an integrated correlator

$$\begin{aligned} C(x,T) &= \langle \bar\psi\psi(x)\bar\psi\psi(0)\rangle \\ \chi &= C(p=0,T) = \int d^4x\, C(x,T) \end{aligned}$$

- Ultraviolet singularity (continuum)

$$C(x,T) \to 1/x^6 \quad \text{(small } x\text{)}$$

- Bad continuum limit. Will get noisy. With lattice regularization

$$\chi \to 1/a^2$$

- Budapest/Wuppertal renormalization

$$m_q^2[\chi(m_q,T) - \chi(m_q,0)]/T^4$$

- Shifts peak to lower T, because m_q^2/T^2 decreases with increasing T. Zero in chiral limit.

Susceptibility and correlators

- In the chiral limit for $T < T_c$, there is also an infrared chiral singularity (3D analog of chiral log). Karsch (this conference);

$$\chi_{\rm isosinglet} \sim \begin{cases} c_1/a^2 + c_{1/2}(a,T)/(2\sqrt{m}) + \text{analytic} & T < T_c \\ c_1/a^2 + c_\delta m^{1/\delta - 1} + \text{analytic} & T = T_c \\ c_1/a^2 + \text{analytic} & T > T_c \end{cases}$$

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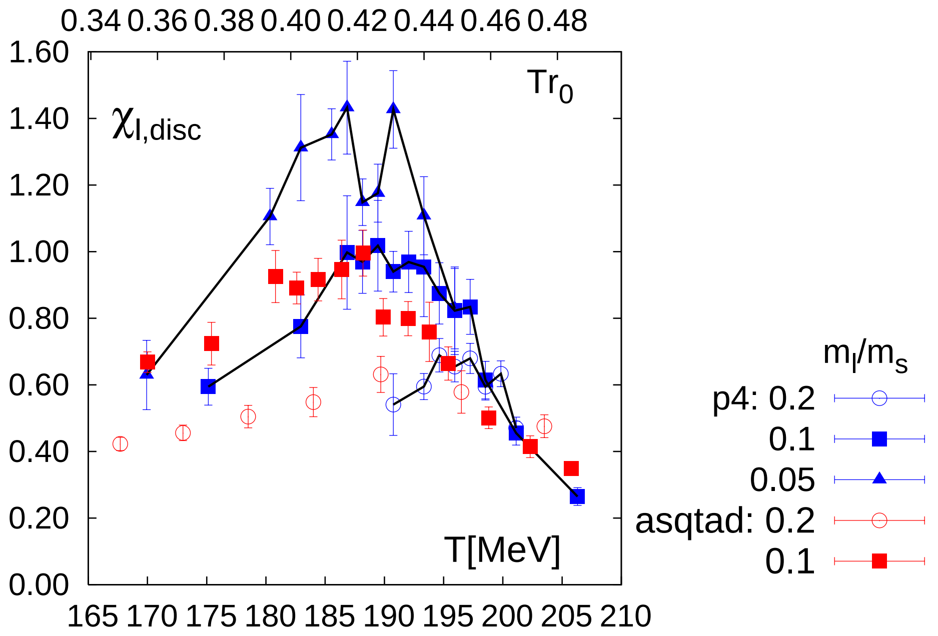
Chiral susceptibility



[Bielefeld/RBC + HotQCD preliminary (2008)]

$N_\tau = 8$

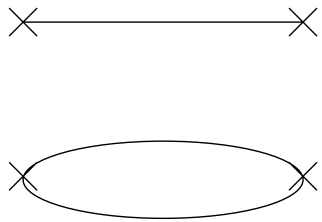
- Peak region is broad and noisy.
- Not good in the chiral limit. Probably OK at a nonchiral critical point.
- Let's analyze the screening spectrum of the correlator...

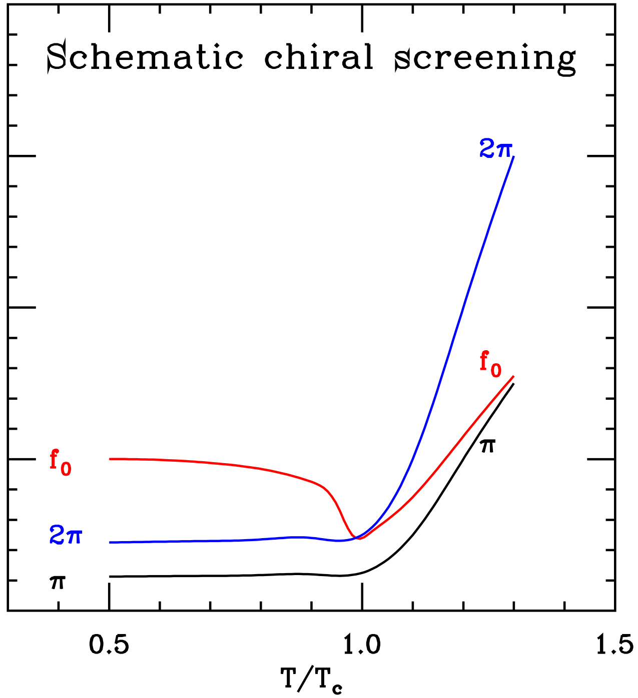
Susceptibility and screening masses

- $C(p=0,T) \sim \int d\mu^2 \, \rho(\mu^2,T)/\mu^2$ peaks when screening masses get small.
- Consider contributions to the isosinglet chiral susceptibility.

f_0

2π





In chiral limit π and 2π shift to zero for $T < T_c$:

- Produces the chiral singularity for $T < T_c$.
- Causes a steep drop in the chiral susceptibility for $T > T_c$
- T_c is near the edge of the cliff.

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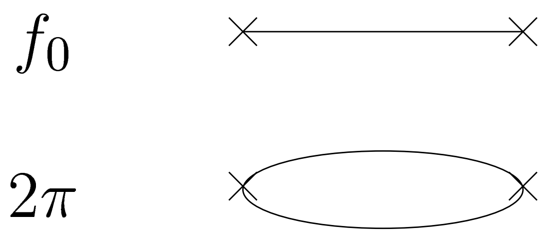

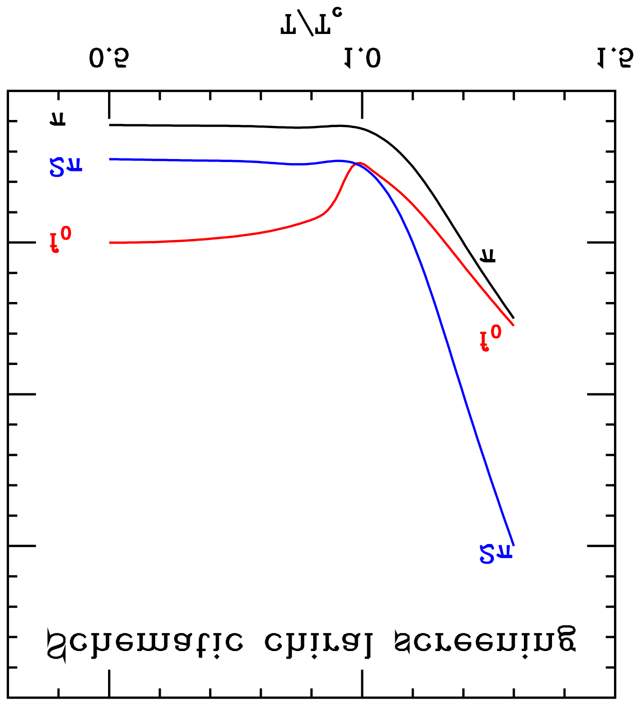

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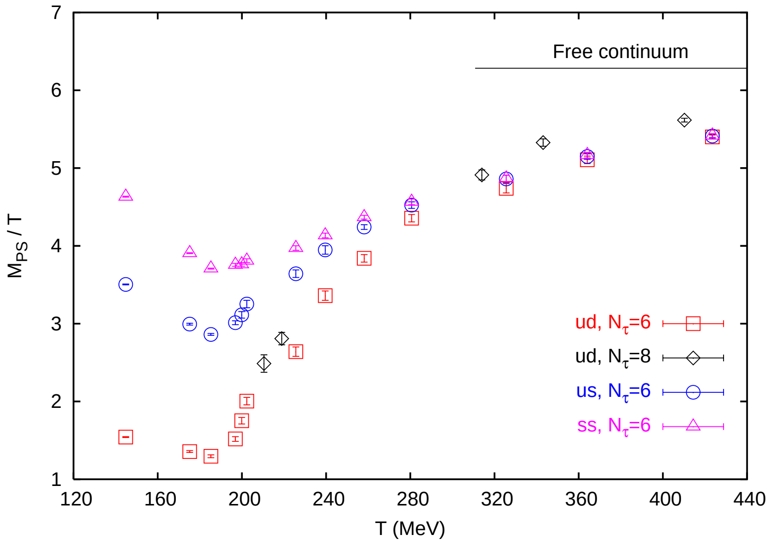
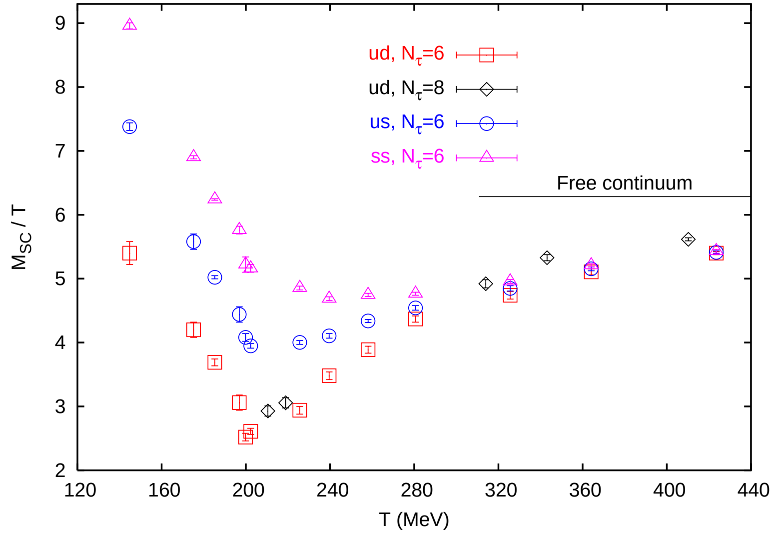
Screening masses



Pseudoscalars: (π, K, etc.)

[Datta *et al.* (new, this conference)]



Scalars: (a_0, etc.)

$24^3 \times 6$ and $32^3 \times 8$; LCP $m_{\pi,G} \approx 220$ MeV, $m_K \approx 500$ MeV.
E. Laermann talk.

- Screening masses may be more robust than the chiral susceptibility.
  - No UV or chiral divergences.
  - No renormalization.

Scale setting

- Use Sommer parameter r_0 (or r_1)? (Favored by many groups)
- Use f_K? (Budapest/Wuppertal)
- The f_K scale can give a 10 - 20% lower T than r_0 for current m_q and a.
- Best choice: the one that gives the best scaling of thermodynamic observables.
- Y. Aoki, Z. Fodor, S.D. Katz and K.K. Szabo, (2006) found that r_0 gives better scaling for T_c determined from the quark number susceptibility or Polyakov loop.
- No reason so far to abandon Sommer parameter scale.

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Confusion about T_c

- Budapest/Wuppertal (Y. Aoki, Z. Fodor, S.D. Katz and K.K. Szabo, 2006) reported that at the physical point
  - $T_c = 151(3)(3)$ MeV from a peak in the chiral susceptibility
  - $T_c = 175(2)(4)$ from the inflection point in the quark number susceptibility. Also from Polyakov loop.
- *cf.* MILC (2004) $169(12)(4)$, RBC (2006) $192(7)(4)$ based on r_0 scale.
- The current conventional wisdom is that there is only one critical temperature.
- As BW has carefully explained, the lower chiral number can be attributed to
  - Nonuniqueness of the definition of the crossover temperature.
  - BW renormalization of the susceptibility. Shifts a peak to lower T.
  - BW use of f_K scale: Tends to lower all inferred temperatures.
- New reason this year:
  - The chiral susceptibility is problematic. We should probably be locating the edge of the "cliff" instead of fitting a parabola to a peak.

Equation of State

- Standard integral method

$$
\begin{aligned}
I &= \varepsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a} \\
\frac{p}{T} &= \left.\frac{\partial \ln Z}{\partial V}\right|_T
\end{aligned}
$$

- Next, *assume* that V is large enough that $\ln Z \propto V$.
  - Gliozzi(hep-lat/0701020) and Panero (this conference) examine the deviations from the Stefan-Boltzmann law caused by finite size effects. Important at high T where we want to compare with perturbation theory.
- Proceed to integrate.

$$
\begin{aligned}
\frac{p}{T} &= \frac{\ln Z}{V} \\
\left.\frac{Vp}{T}\right|_a - \left.\frac{Vp}{T}\right|_{a_0} &= -\int_{a_0}^{a} \frac{V'}{T'}(\varepsilon' - 3p')\, d\ln a'
\end{aligned}
$$

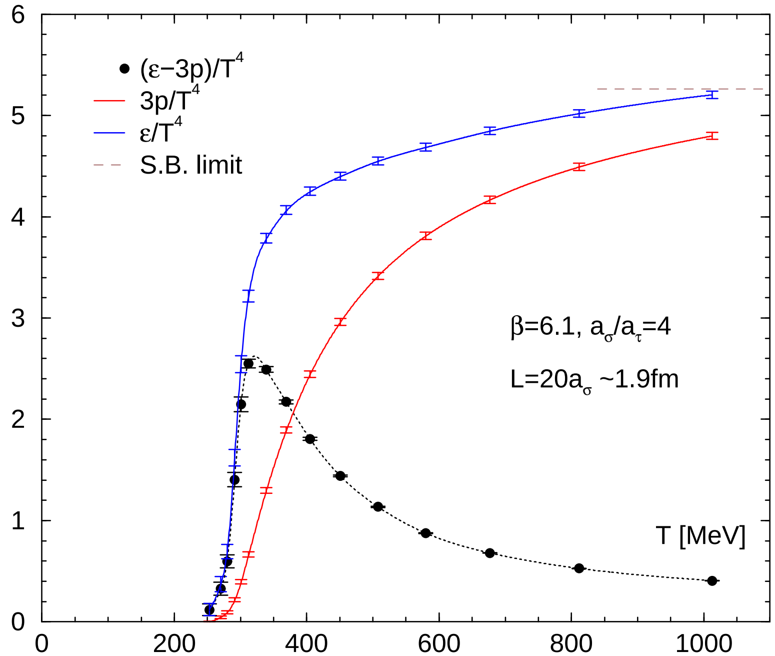
- We must subtract out the vacuum pressure and energy density.

EoS: New T integral method

- WHOT-QCD collaboration (pronounced "hot"): S. Aoki, S. Ejiri, T. Hatsuda, N. Ishii, K. Kanaya, H. Ohno and T. Umeda. See Umeda (this conference).

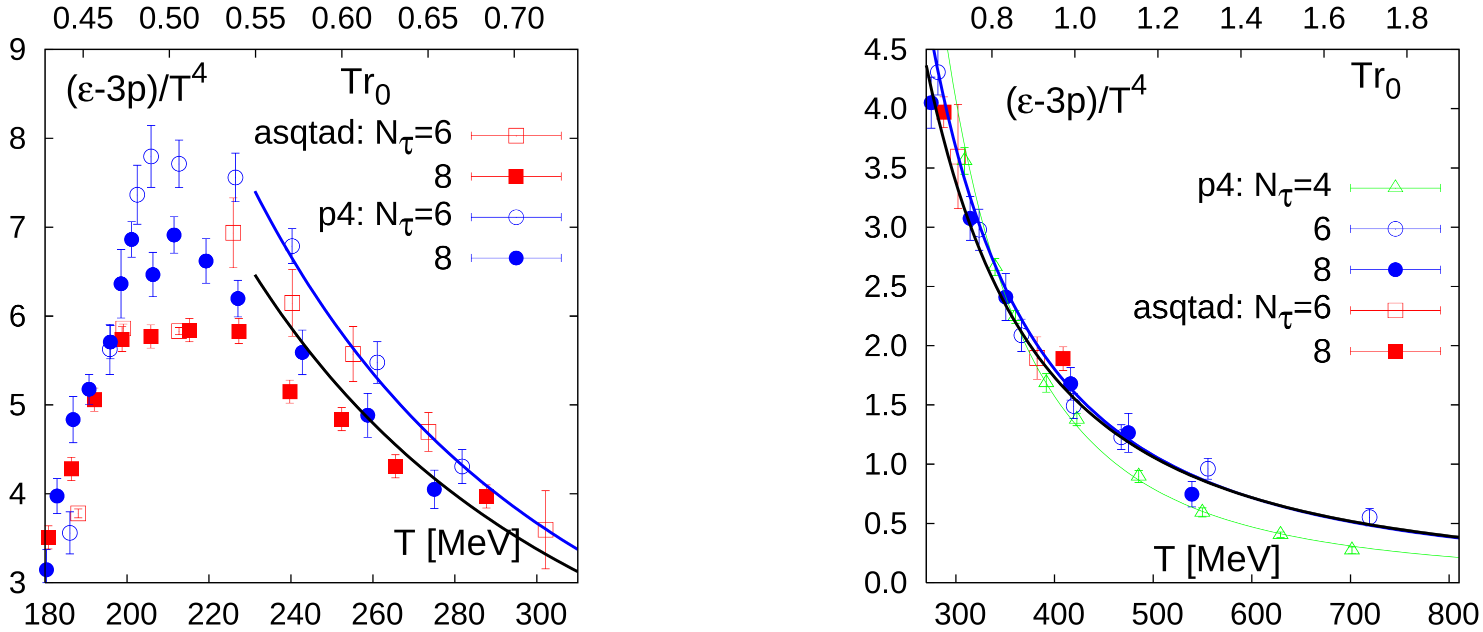
$$(\varepsilon - 3p)/T^4 = \frac{d(p/T^4)}{d \ln T}$$

- Vary N_τ at fixed g^2, quark masses. Do $\ln T$ integral.
- Anisotropic $a_t \ll a_s$ helps overcome the discrete T resolution.



EOS for SU(3) Yang-Mills. Anisotropic lattice.

Interaction measure



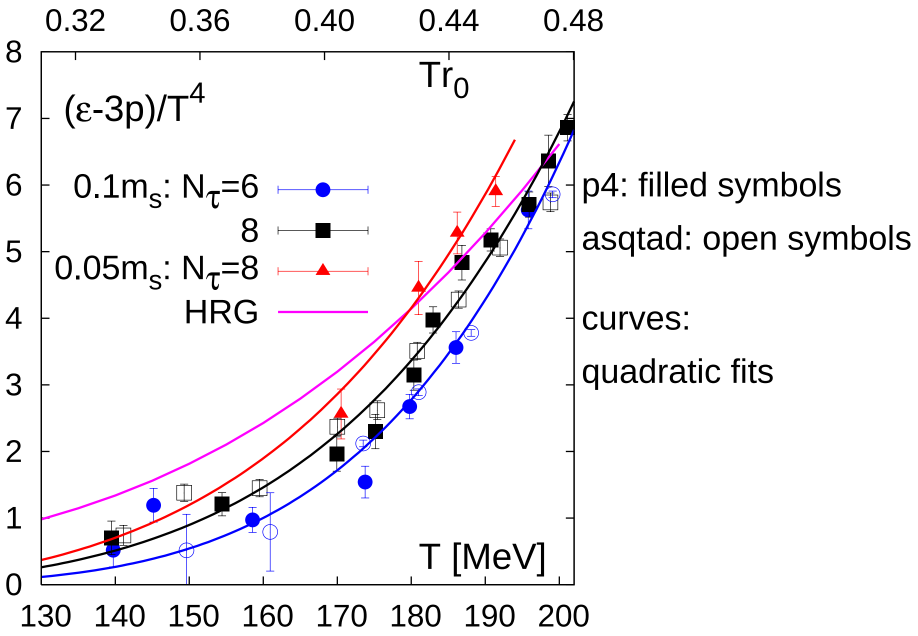
LCP $m_\ell/m_s = 0.1$ [HotQCD preliminary (2008)]

- For $T \in [250, 700]$ MeV, can be fit to

$$(\varepsilon - 3p)/T^4 = b/T^2 + c/T^4$$

- No indication of perturbative $1/\log T$ terms.

Interaction measure



[RBC/Bielefeld/HotQCD preliminary (2008)]

Magenta line: hadron resonance gas (HRG) with masses < 2.5 Gev.

- We may soon be able to test models such as HRG.

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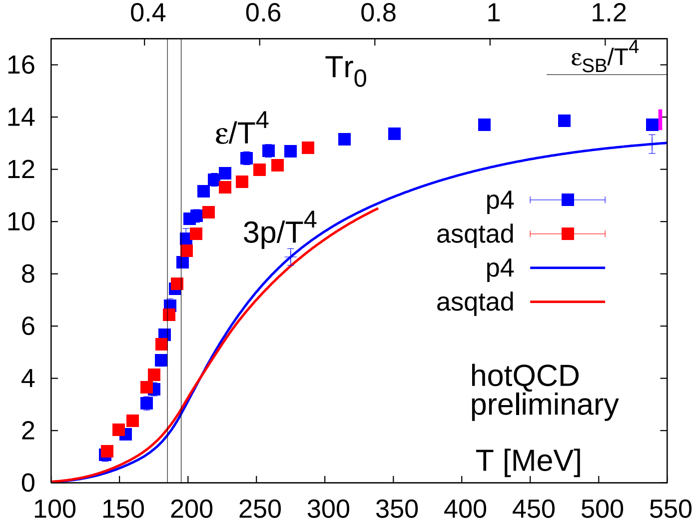
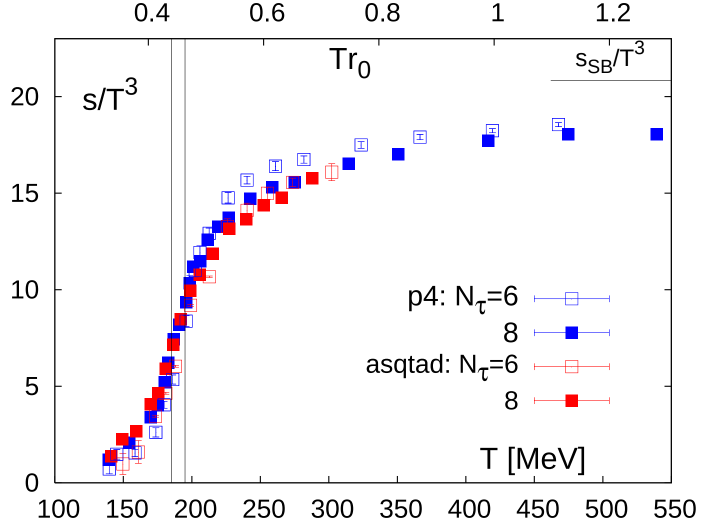
EOS at zero baryon number density

Energy density and 3 times pressure



Entropy density



LCP $m_\ell/m_s = 0.1$ [HotQCD(2008)] .

Plasma structure: Transport coefficients

- Analysis of RHIC heavy ion collisions suggests that high T matter is a good fluid
- Hydrodynamics modeling needs shear (η) and bulk (ζ) viscosities
- These are obtained from correlators of the energy-momentum tensor at temperature T

$$C(x_0, \mathbf{x}, T) = \langle T_{\mu\nu}(x_0, \mathbf{x}) T_{\rho\sigma}(0) \rangle$$

- We need the spectral function ρ from the Kubo formula

$$C(x_0, \mathbf{q}, T) = \int_0^\infty d\omega\, \rho(\omega, \mathbf{q}, T) \frac{\cosh \omega(x_0 - 1/2T)}{\sinh(\omega/2T)}.$$

$$\eta(T) = \pi \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, 0, T)}{\omega} \qquad \zeta(T) = \frac{\pi}{9} \lim_{\omega \to 0} \frac{\rho_{ii,jj}(\omega, 0, T)}{\omega}$$

- Going from Euclidean $C(x_0)$ to $\rho(\omega)$ is a really difficult inverse problem.
- At $N_\tau = 8$ we have only 5 x_0's after symmetrization!
- Like inferring a dinosaur from a toe bone. It helps to know the dinosaur.
- Meyer paleontology talk on Friday.

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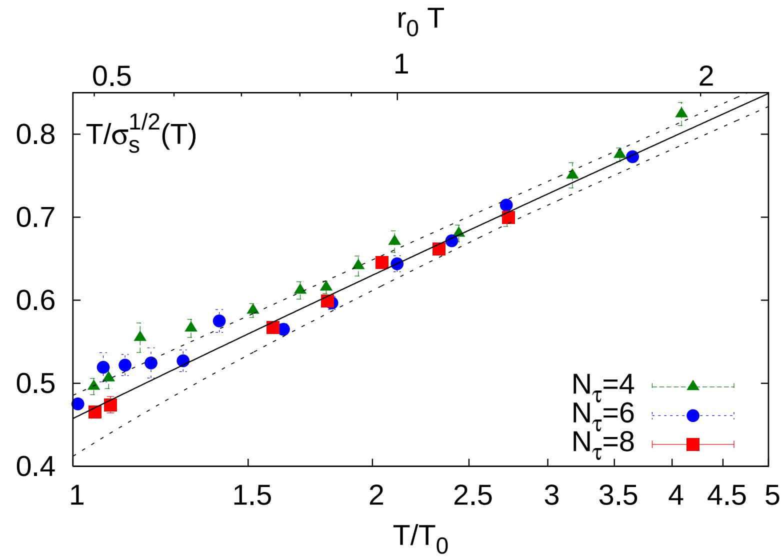
Plasma structure: spatial string tension

- For $T \gg T_c$ QCD can be described by an effective 3D confining theory.
  - Quarks acquire a large 3D mass $\sqrt{(\pi T)^2 + m_q^2}$
  - A_0 becomes a scalar field. We get a gauge-Higgs theory.
- The spatial Wilson loop gives the potential and 3D string tension.



Points: measured 4D values of spatial string tension

Curves: predictions of 3D-reduced theory.

[Cheng *et al* (RBC/Bielefeld) arXiv:0806.3264]

- Surprise: should dimensional reduction really work as low as $1.5T_c$?

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Conclusions

- New high statistics results from HotQCD show good agreement between two different staggered fermion methods (p4fat3, asqtad).
- New results will help hydrodynamic modeling of heavy-ion collisions.
- First exploratory (expensive) DWF results with small residual mass look promising.
- We are learning more about the chiral behavior of the chiral susceptibility.
- More work is needed to confirm scaling and to approach the physical point and the critical point.
- New methods
  - WHOT-QCD collaboration: equation of state.
  - Meyer: transport coefficients.
  - We are making progress.
- New result
  - Dimensional reduction and spatial string tension.