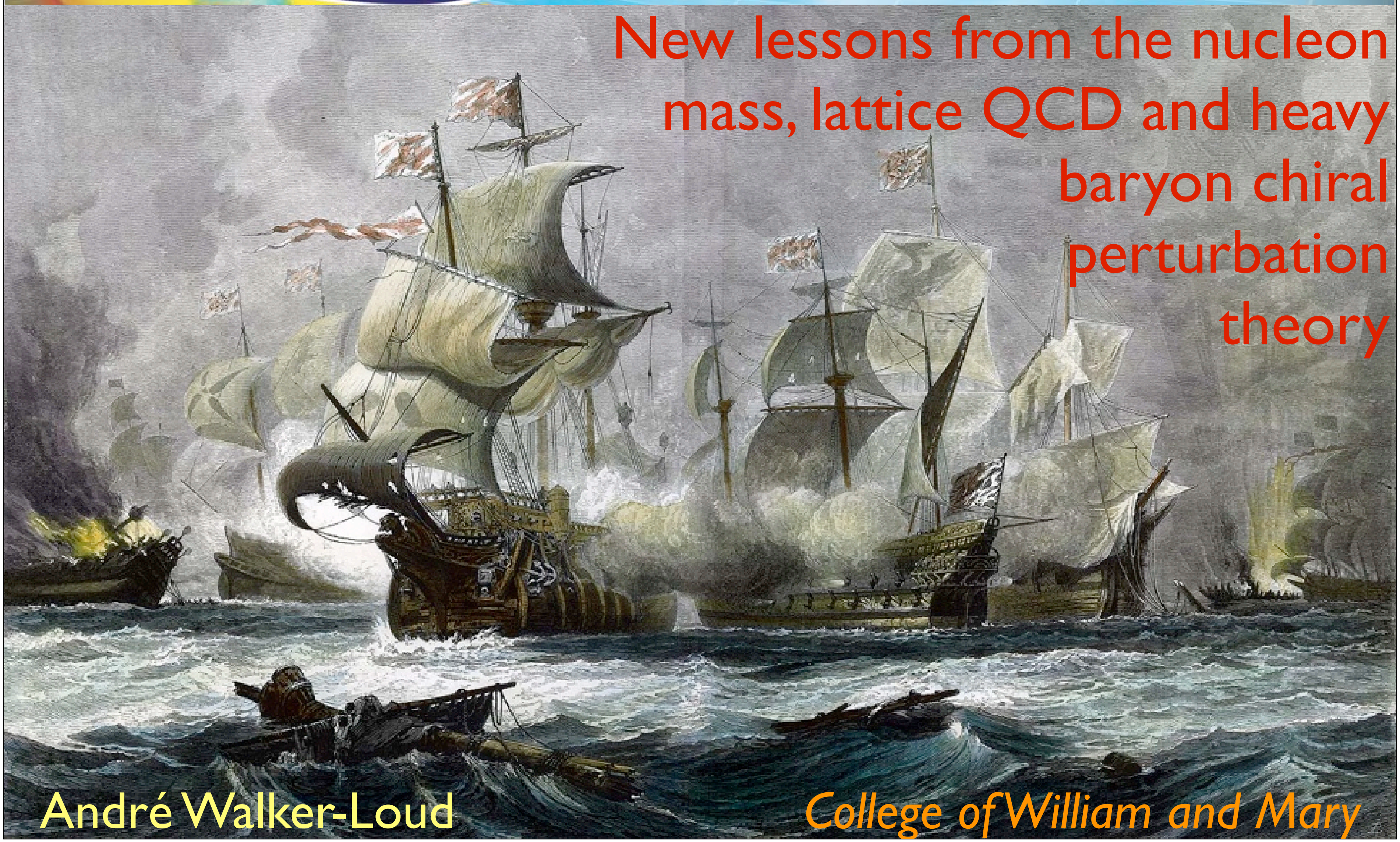
Lattice 2008

# New lessons from the nucleon mass, lattice QCD and heavy baryon chiral perturbation theory



André Walker-Loud

*College of William and Mary*

# PREVIEW

- Heavy baryon chiral perturbation theory
    - Comparison with recent lattice results
        - Unexpected findings and new lessons

# Hadron Masses and the QCD Trace Anomaly

The QCD trace anomaly, arising from the breaking of scale invariance in massless QCD, provides

$$\mathcal{T}_\mu^{\ \mu} = \frac{\beta_{QCD}}{2g_s} G^a_{\mu\nu} G^{\mu\nu}_a \qquad \text{where} \qquad \mathcal{T}^{\mu\nu} = \frac{i}{2} \bar{\psi}\, \gamma^\mu \overleftrightarrow{D}^\nu\, \psi$$

is the energy momentum tensor.

Adding the quark masses, we have

$$\mathcal{T}_\mu^{\ \mu} = \frac{\beta_{QCD}}{2g_s} G^a_{\mu\nu} G^{\mu\nu}_a + m_u \bar{q}_u q_u + m_d \bar{q}_d q_d + m_s \bar{q}_s q_s + \ldots$$

The first term provides the dominant mass for all light hadrons (no c, b, t), except for the pseudo-Goldstone modes. For example,

$$\langle N(\mathbf{p})|\mathcal{T}_\mu^{\ \mu}|N(\mathbf{p})\rangle = M_N\, \bar{u}(\mathbf{p}) u(\mathbf{p})$$

$$= \langle N(\mathbf{p})| \frac{\beta_{QCD}}{2g_s} G^a_{\mu\nu} G^{\mu\nu}_a + m_u \bar{q}_u q_u + m_d \bar{q}_d q_d + m_s \bar{q}_s q_s |N(\mathbf{p})\rangle$$

The leading chiral correction to all light hadron masses then scales as $m_q$ , aside from the pseudo-Goldstone modes for which $m^2_{\pi,K,\eta} \simeq B(m_{q_1} + m_{q_2})$

# (Heavy) Baryon Chiral Perturbation Theory

Heavy baryon chiral perturbation theory is an effective field theory describing the interactions of pions with non-relativistic nucleons. Based upon heavy quark effective theory. E. Jenkins and A. Manohar PLB 255 (1991)

Writing down a relativistic effective nucleon Lagrangian, one immediately runs into trouble

$$\mathcal{L} = \bar{\psi}_N \, (i\partial\!\!\!/ - M_0) \, \psi_N + g_A \bar{\psi}_N \, \gamma_\mu \gamma_5 A^\mu \, \psi_N + \ldots$$

$$A_\mu = \frac{\partial_\mu \phi}{f} + \ldots \qquad \phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}$$

Leading order equation of motion

$$i\partial\!\!\!/ \psi_N = M_0 \, \psi_N \qquad M_0 \sim \Lambda_\chi$$

How do you count powers of $\dfrac{i\partial\!\!\!/}{\Lambda_\chi} = \dfrac{M_0}{\Lambda_\chi}$ ?

# (Heavy) Baryon Chiral Perturbation Theory

E. Jenkins and A. Manohar

Similar to the phase one introduces to convert the Klein-Gordon equation to the Schroedinger equation

$$N_v(x) = \frac{1+\not{v}}{2} e^{iM_0 v\cdot x} \psi_N(x)$$

in the rest frame $\quad v_\mu = (1, \mathbf{0})$

$$\longrightarrow \frac{1+\gamma_0}{2}$$ non-relativistic projector

$$\longrightarrow \mathcal{L} = \bar{N}_v iv\cdot\partial N_v + 2g_A \bar{N}_v S\cdot A N_v + 2\alpha_M \bar{N}_v N_v \mathrm{tr}(\mathcal{M}_+) + \mathcal{O}\left(\frac{1}{\Lambda_\chi}\right) + \mathcal{O}\left(\frac{1}{M_0}\right)$$

$$\mathcal{M}_+ = m_Q + \ldots$$

the nucleon momentum can be parameterized as $\; P_\mu = M_0 v_\mu + k_\mu$

$$\partial_\mu N_v = ik_\mu N_v$$

consistent power counting with $\quad k^2 \sim m_\pi^2 \ll \Lambda_\chi^2$

# (Heavy) Baryon Chiral Perturbation Theory

E. Jenkins and A. Manohar

Including the delta degrees of freedom explicitly adds a slight complication. One can not simultaneously phase away the nucleon mass in the chiral limit along with the delta mass in the chiral limit. This introduces a new parameter in the theory

$$\Delta_0 = M_\Delta - M_N \Big|_{m_q=0}$$

phenomenologically $\Delta \simeq 290 \text{ MeV}$

$$\mathcal{L} = \bar{N} i v \cdot \partial N + 2\alpha_M \bar{N} N \mathrm{tr}(\mathcal{M}_+) - \bar{T}^\mu \left[ i v \cdot \partial - \Delta_0 \right] T_\mu - 2\bar{\gamma}_M \bar{T}^\mu T_\mu \mathrm{tr}(\mathcal{M}_+)$$

$$+2g_A \bar{N} S \cdot A N + 2g_{\Delta\Delta} \bar{T}^\mu S \cdot A T_\mu + g_{\Delta N} \left( \bar{T}^\mu A_\mu N + \bar{N} A^\mu T_\mu \right)$$

leading axial charge of nucleon

leading axial charge of delta

leading axial transition charge

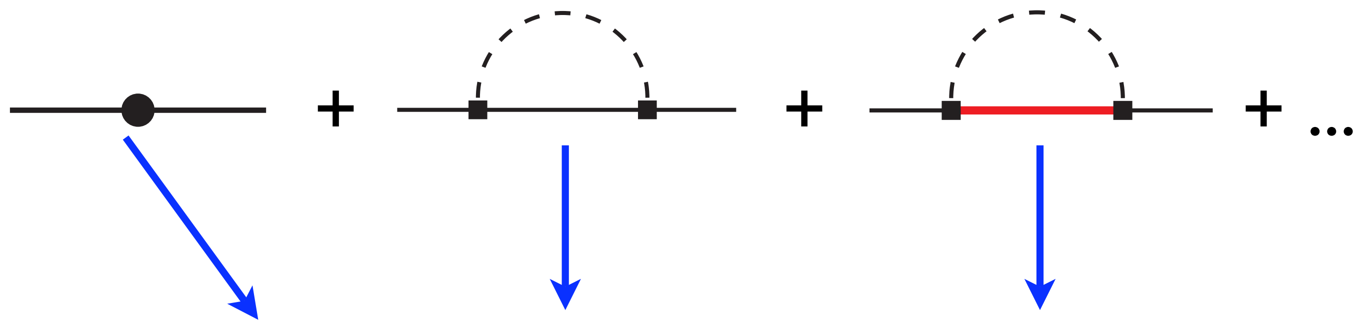
these charges are phenomenologically very important and they also give rise to the leading non-analytic pion mass dependence of the baryon masses.

# (Heavy) Baryon Chiral Perturbation Theory

E. Jenkins and A. Manohar

## nucleon mass to nlo

$-i\Sigma =$




$$M_N = M_0 - 2\alpha_M(\mu)\, m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\Delta N}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu)\,,$$

$$\mathcal{F}(m, \Delta, \mu) = (\Delta^2 - m^2 + i\epsilon)^{3/2} \ln\left(\frac{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}\right) - \frac{3}{2}\Delta\, m^2 \ln\left(\frac{m^2}{\mu^2}\right) - \Delta^3 \ln\left(\frac{4\Delta^2}{m^2}\right)$$

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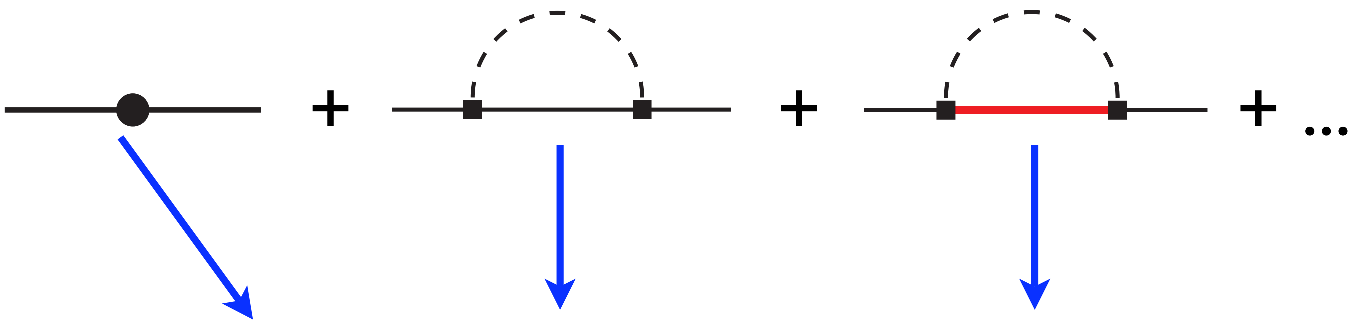

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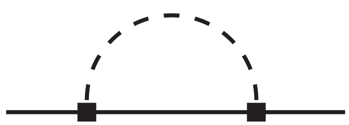

$$\sim \frac{g_A^2}{f_\pi^2} \int \frac{d^d k}{(2\pi)^d} \frac{S\cdot k\, S\cdot k}{(v\cdot k + i\epsilon)(k^2 - m_\pi^2 + i\epsilon)} \qquad \frac{k^{d+2}}{k^3}$$

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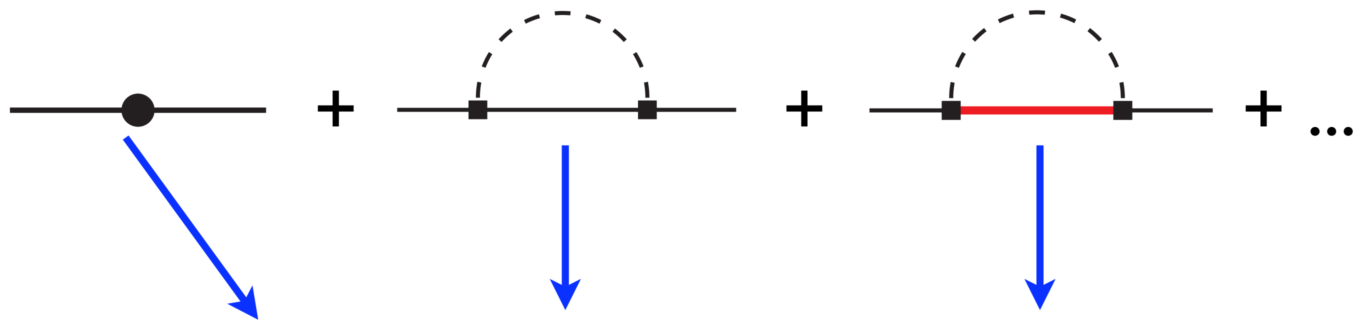


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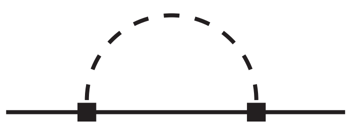

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$$\sim \frac{g_A^2}{f_\pi^2} m_\pi^3$$

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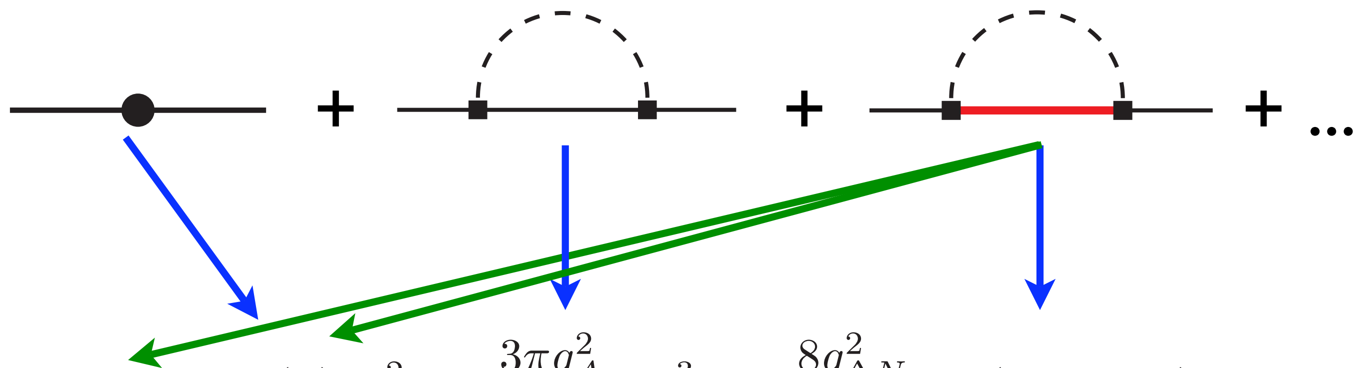


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$$M_0 \to M_0 + \frac{16 g_{\Delta N}^2}{9} \frac{\Delta^3}{\Lambda_\chi^2} + d_3^M \frac{\Delta^3}{\Lambda_\chi^2} \qquad \alpha_M(\mu) = \alpha_M + \frac{2 g_{\Delta N}^2}{3} \frac{\Delta}{\Lambda_\chi^2} - 2 g_{\Delta N}^2 \frac{\Delta}{\Lambda_\chi^2} \ln\left(\frac{\mu^2}{\mu_0^2}\right) + d_1^\alpha \frac{\Delta}{\Lambda_\chi^2}$$

$$g_A \to g_A \left(1 + d_1^g \frac{\Delta}{\Lambda_\chi} + d_2^g \frac{\Delta^2}{\Lambda_\chi^2} + \ldots\right)$$

# (Heavy) Baryon Chiral Perturbation Theory

E. Jenkins and A. Manohar

The price: Lorentz invariance is lost...

However, it can be recovered order-by-order in $\frac{1}{M_0}$

This is know as Reparameterization Invariance (RPI)

M. Luke and A. Manohar PLB 286 (1992)

Consider a small shift in the parameterization of the nucleon momentum

$$P_\mu = M_0 v_\mu + k_\mu \qquad\qquad v \to v + \frac{\epsilon}{M_0} \quad k \to k - \epsilon$$

- Requiring the theory to be invariant under this reparameterization recovers Lorentz invariance order-by-order in inverse powers of the nucleon mass.

- This happens through a constraint of the coefficients of certain operators in the effective Lagrangian. Implementing RPI, one finds the coefficients of higher dimensional operators are exactly constrained by coefficients of lower dimensional operators

# (Heavy) Baryon Chiral Perturbation Theory

E. Jenkins and A. Manohar

## Reparameterization Invariance

$$\bar{N} i v \cdot \partial N \longrightarrow \bar{N} i v \cdot \partial N - \bar{N} \frac{\partial^2}{2M_0} N + \mathcal{O}\left(\frac{1}{M_0^2}\right)$$

The coefficient of the operator $\bar{N} \frac{(v \cdot \partial)^2}{2M_0} N$ is unconstrained, but one can use the equations of motion to make a field redefinition to convert the Lagrangian to the form

A. Manohar PRD 56 (1997)

$$\mathcal{L} = \bar{N} i v \cdot \partial N - \bar{N} \frac{\partial_\perp^2}{2M_0} N \qquad \text{with} \qquad \partial_\perp^2 = \partial^2 - (v \cdot \partial)^2$$

this provides the familiar form of the non-relativistic propagator if this leading kinetic operator is re-summed to all orders

$$\frac{i}{v \cdot k + i\epsilon} \longrightarrow \frac{i}{v \cdot k - \frac{\vec{k}^2}{2M_0} + i\epsilon}$$

# (Heavy) Baryon Chiral Perturbation Theory

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## Nucleon mass to NNLO

B. Tiburzi and A. Walker-Loud    Nucl.Phys. A 764 (2006)

$$M_N = M_0 - 2\alpha_M(\mu) m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\Delta N}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu)$$
$$+ m_\pi^4 \left[ b_M(\mu) + \frac{8 g_{\Delta N}^2 \alpha_M(\mu)}{(4\pi f_\pi)^2} - \frac{9 g_{\Delta N}^2}{4 M_0 (4\pi f_\pi)^2} - \frac{45 g_A^2}{32 M_0 (4\pi f_\pi)^2} \right] + \frac{8 g_{\Delta N}^2 \alpha_M}{(4\pi f_\pi)^2} m_\pi^2 \mathcal{J}(m_\pi, \Delta, \mu)$$
$$+ \frac{m_\pi^4}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{\mu^2}\right) \left[ 6\alpha_M(\mu) - \frac{3 b_A(\mu)}{4\pi f_\pi} - \frac{27 g_A^2}{16 M_0} - \frac{5 g_{\Delta N}^2}{2 M_0} \right], \tag{29}$$

## known to NNNLO without explicit deltas

J. McGovern and M. Birse    PRD 74 (2006)

## Lattice spacing corrections to this formula are known for

- Wilson fermions — S. Beane and M. Savage    PRD 68 (2003)
- twisted mass fermions — A. Walker-Loud and J.M.S. Wu    PRD 75 (2005)
- domain-wall on staggered — B. Tiburzi    PRD 72 (2005)
- staggered on staggered — J. Bailey    PRD 77 (2008)

# (Heavy) Baryon Chiral Perturbation Theory

## Analyticity and kinematic thresholds

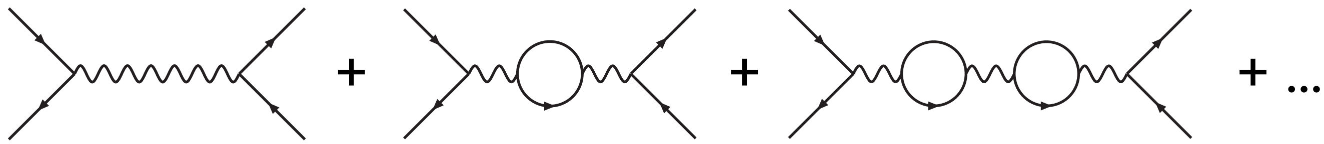
Because we have approximated the nucleon as an infinitely heavy static field, a naive use of the theory will lead to incorrect analytic structure.

Static quantities such as the nucleon mass will be insensitive to these problems in the range of convergence of the theory.

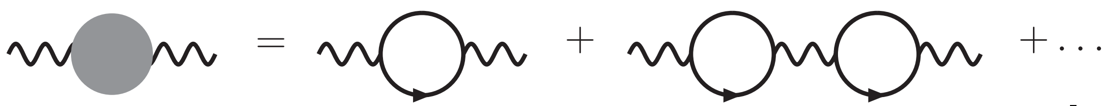
Dynamic quantities with external momentum insertions, such as pion-nucleon scattering, the scalar form factor, etc need to be analyzed carefully, to see if for kinematic reasons, the power counting must be rearranged.

# (Heavy) Baryon Chiral Perturbation Theory

Consider electron-positron scattering near the Z-pole

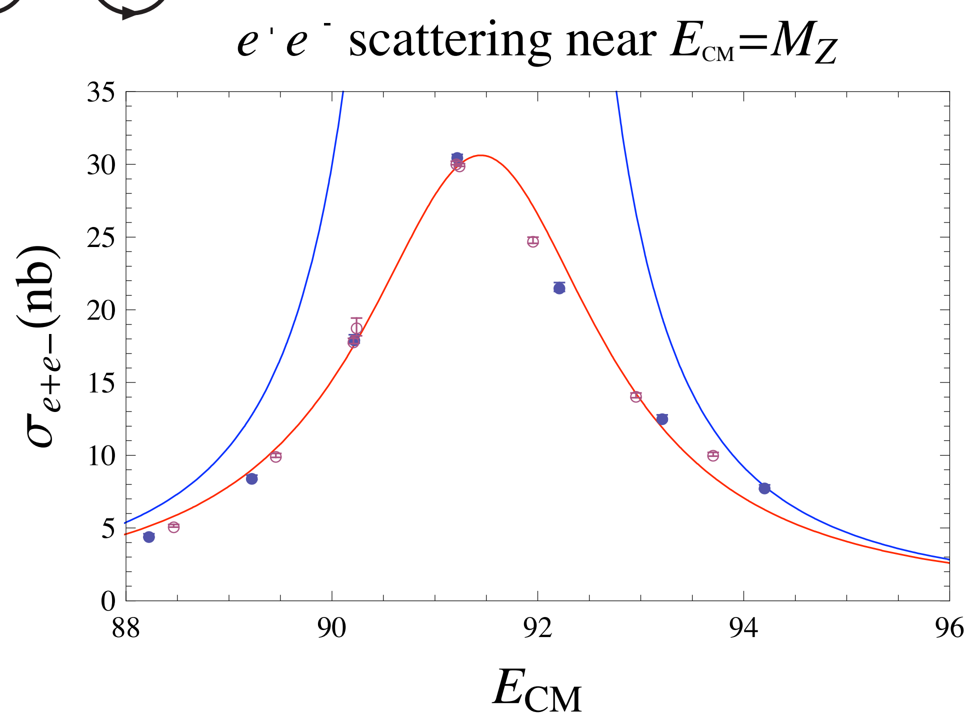


at any finite order, the sum fails to reproduce the observed cross section



$$\sigma_{e^+e^-}(s) \sim \frac{1}{(s-M_Z^2)^2}$$

$$\longrightarrow \frac{1}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$



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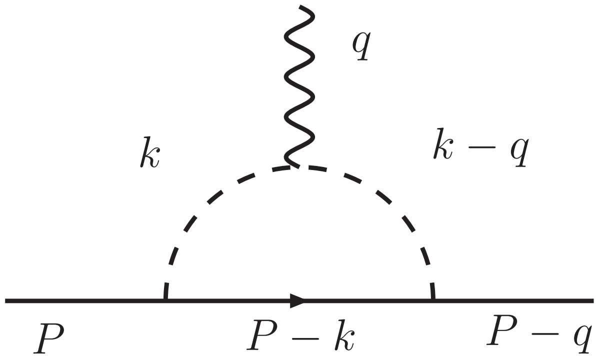
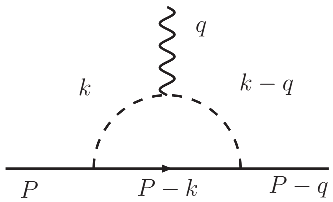



$$\langle N(P)|m_q \bar{q} q|N(P-q)\rangle$$

Evaluation of this process using the static nucleon propagator leads to an unphysical singularity

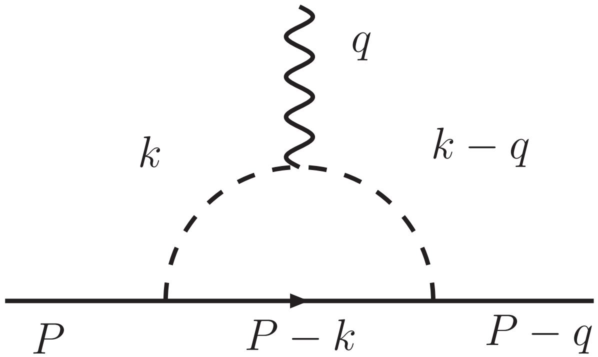
To begin, lets re-sum the kinetic correction to the nucleon propagator

$$G_N = \frac{i(1+\not{v})/2}{v\cdot k - \frac{\vec{k}^2}{2M_0} + i\epsilon} \qquad\qquad \frac{1+\not{v}}{2} N_v = N_v$$




$$\sim \frac{g_A^2}{f_\pi^2} \int \frac{d^d k}{(2\pi)^d} \frac{\frac{1+\not{v}}{2} h(S,k)}{(v\cdot k - \frac{\vec{k}^2}{2M_0})((k-q)^2 - m_\pi^2)(k^2 - m_\pi^2)}$$

# (Heavy) Baryon Chiral Perturbation Theory

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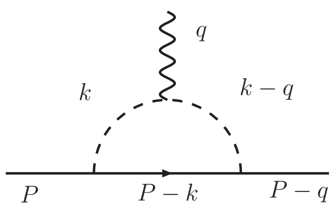


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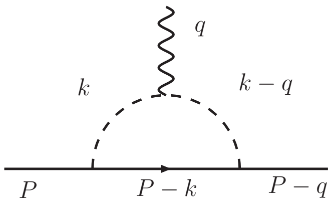


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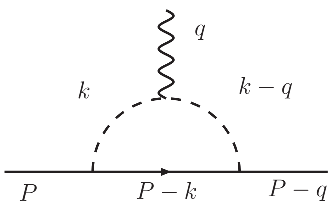
recall, the coefficient of the operator $\bar{N}\frac{(v\cdot\partial)^2}{2M_0}N$ is not physical, since we are free to make a field redefinition to absorb it. Therefore we are free to change

$$\vec{k}^2 \longrightarrow k^2$$

Also, let me pull the $2M_0$ out of the nucleon propagator

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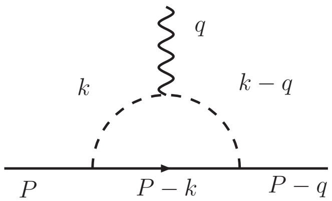


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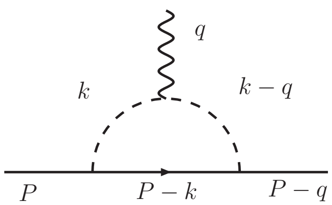


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For on-shell nucleons, $M_0\not{v}\to\not{P}$

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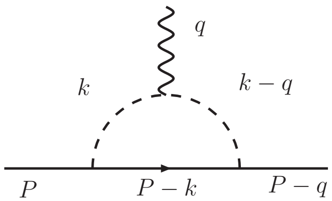


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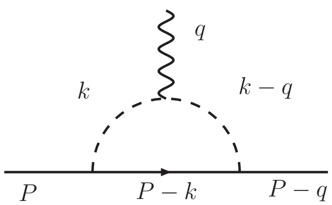


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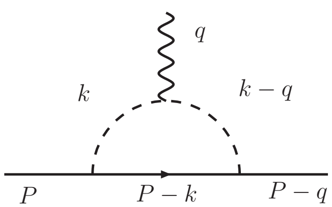


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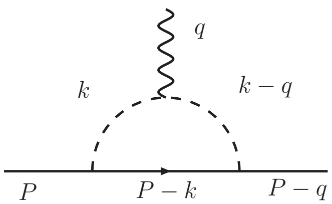
# (Heavy) Baryon Chiral Perturbation Theory




$$\sim \frac{g_A^2}{f_\pi^2} \int \frac{d^d k}{(2\pi)^d} \frac{[\not{P} - \not{k} + M_0]\, h'(S,k)}{(M_0^2 - (P-k)^2)((k-q)^2 - m_\pi^2)(k^2 - m_\pi^2)}$$

up to numerator structure, this is what we would get if we naively started from the relativistic Lagrangian,

$$\mathcal{L} = \bar{\psi}_N\ (i\not{\partial} - M_0)\ \psi_N + g_A \bar{\psi}_N\, \gamma_\mu \gamma_5 A^\mu\, \psi_N + \ldots$$

Becher and Leutwyler, in 1999, wrote a nice paper showing how to regulate this integral in a manner which preserves chiral symmetry and can be expanded to reproduce the heavy baryon expansion.

*covariant baryon* $\chi PT$ T. Becher and H. Leutwyler Eur.Phys.J.C9 (1999)

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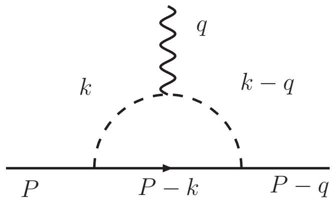


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$$\sigma(t) = \frac{3\pi g_A^2 \, m_\pi}{4(4\pi f_\pi)^2}(t - 2m_\pi^2)\left[\sqrt{\frac{m_\pi^2}{t}} \ln \frac{2 + \sqrt{t/m_\pi^2}}{2 - \sqrt{t/m_\pi^2}} - \ln\left(1 + \frac{m_\pi/(2M_0)}{\sqrt{1 - t/(4m_\pi^2)}}\right)\right]$$

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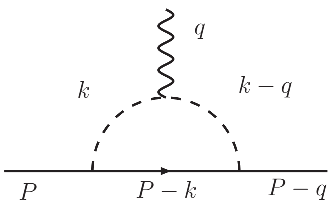


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near $t \sim 4m_\pi^2$ each term diverges

# (Heavy) Baryon Chiral Perturbation Theory

Is covariant baryon chiral perturbation theory a new effective field theory?

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Near kinematic thresholds, the power counting of heavy baryon $\chi\mathrm{PT}$ may change, as we have seen.

Keeping the full expressions determined in the covariant formalism, away from these thresholds is equivalent to the power-counting

$$m_\pi \sim M_0$$

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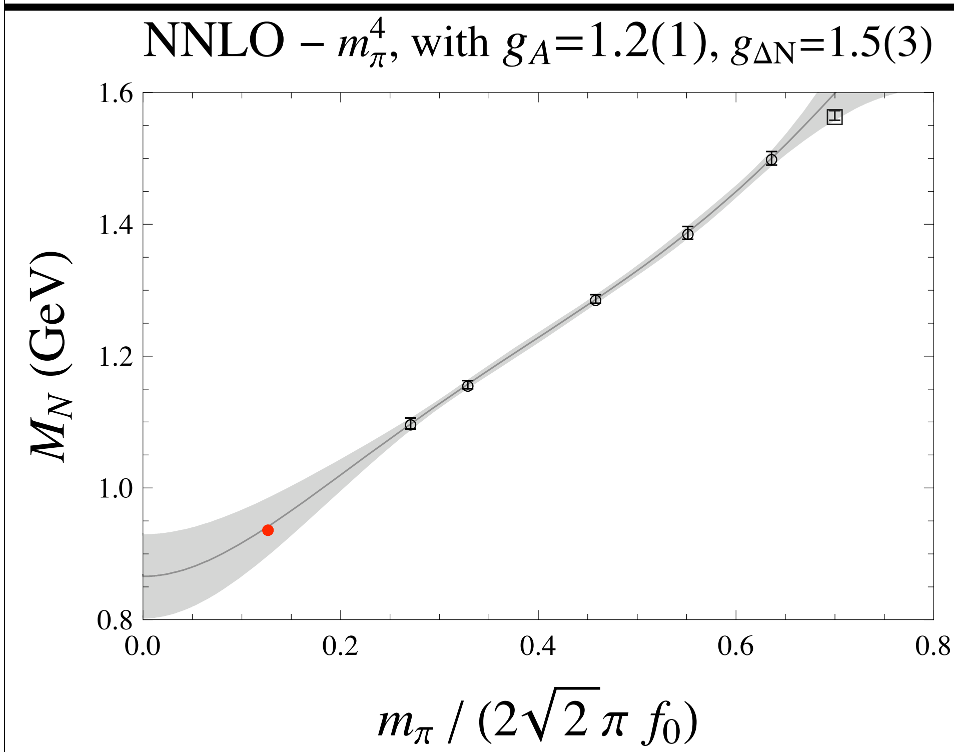
working away from the kinematic thresholds and not including the delta degrees of freedom is equivalent to the power-counting

$$m_\pi \sim M_0 \ll M_\Delta - M_N$$

# Comparing to Lattice Results

# Comparing to LHP results

arXiv:0806.4549




Mixed Action with Domain-Wall valence fermions on Asqtad improved rooted-staggered action

props computed by NPLQCD and LHP

Mixed Action EFT for baryons is known B. Tiburzi PRD 72 (2005) but can't use it

NNLO Heavy Baryon Fit

$$M_N = 954 \pm 42 \pm 20 \text{ MeV}$$

statistical

varying inputs

| NNLO, Eq. (29) | range | $M_0$[GeV] | $\alpha_M$[GeV$^{-1}$] | $b_M$[GeV$^{-3}$] | $b_A$ | $\chi^2$ | d.o.f. | $M_N$[MeV] |
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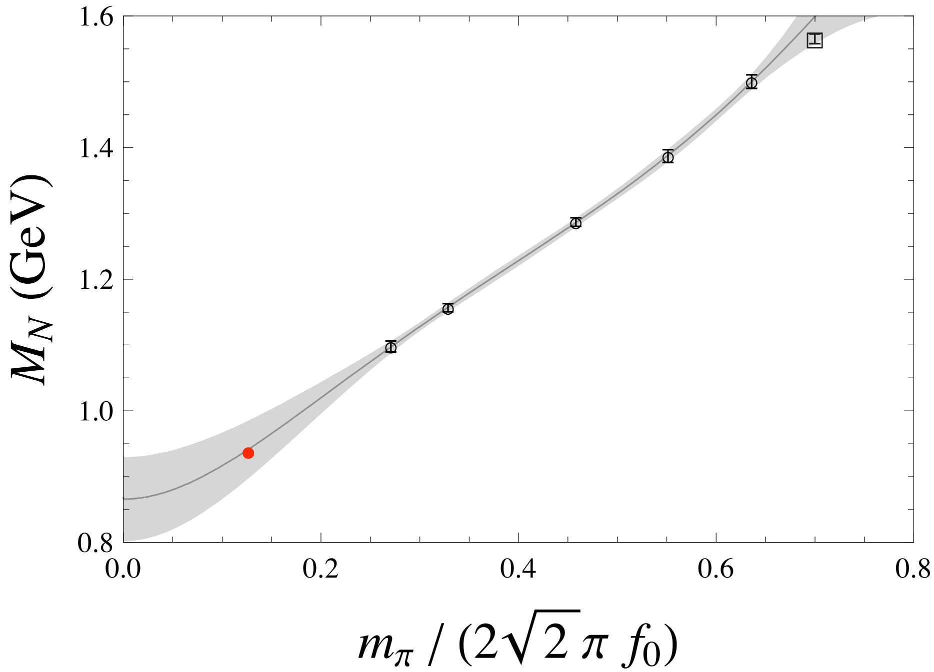


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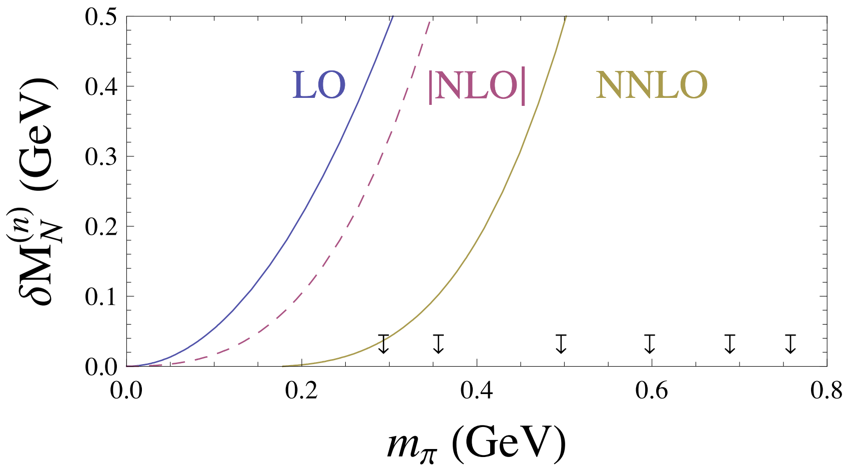


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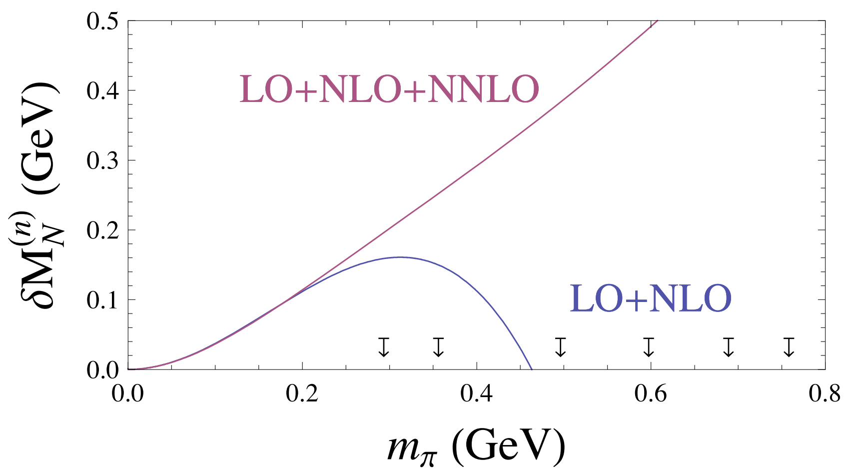


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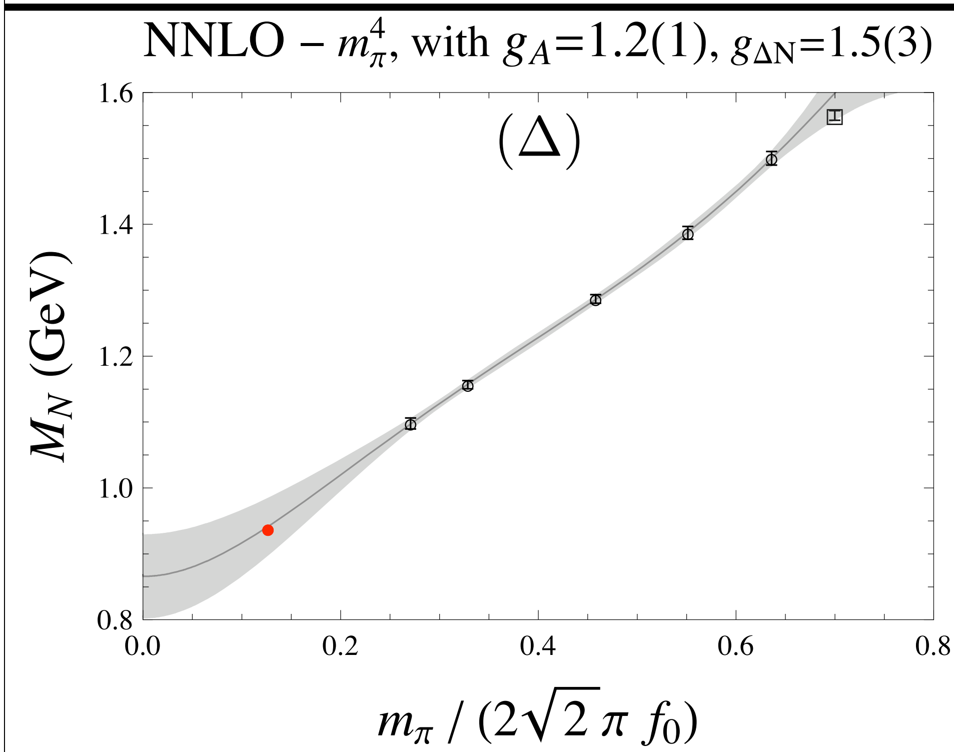


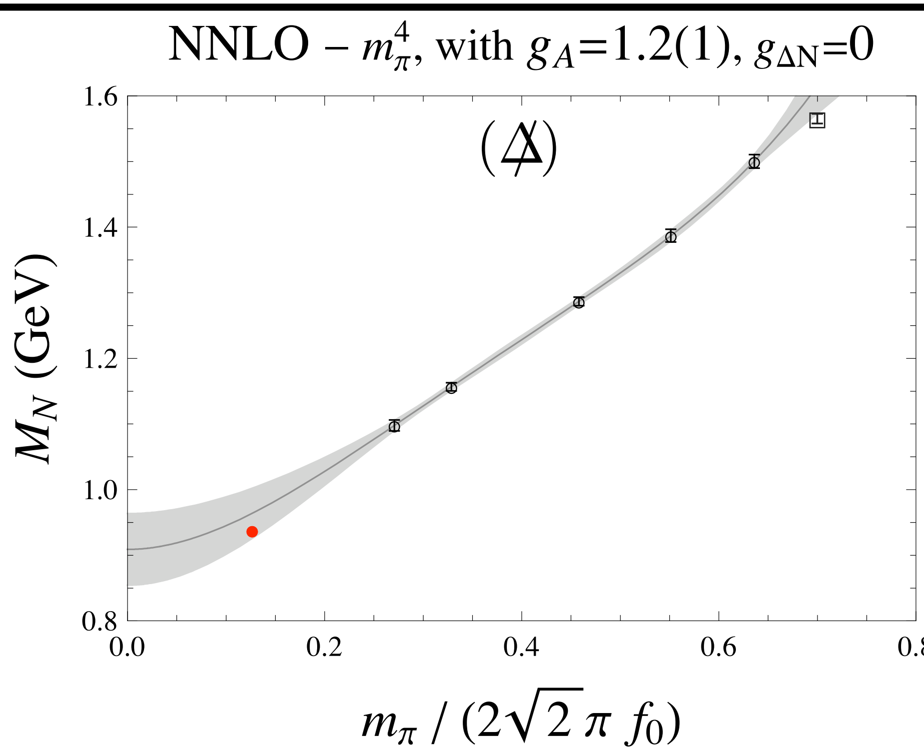


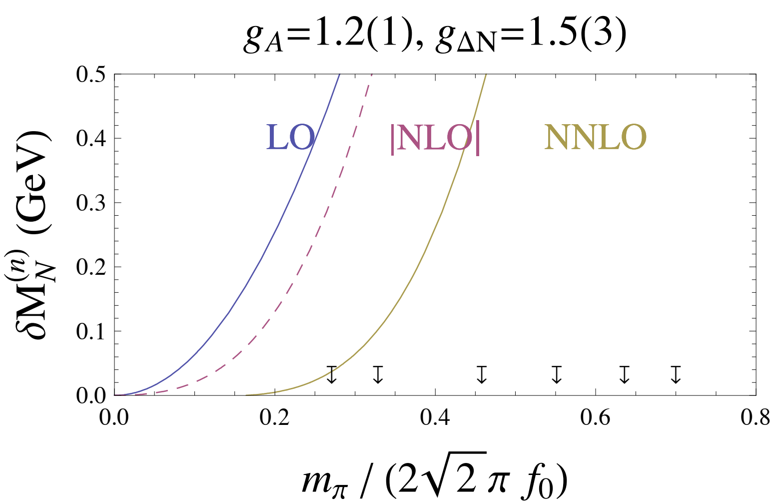


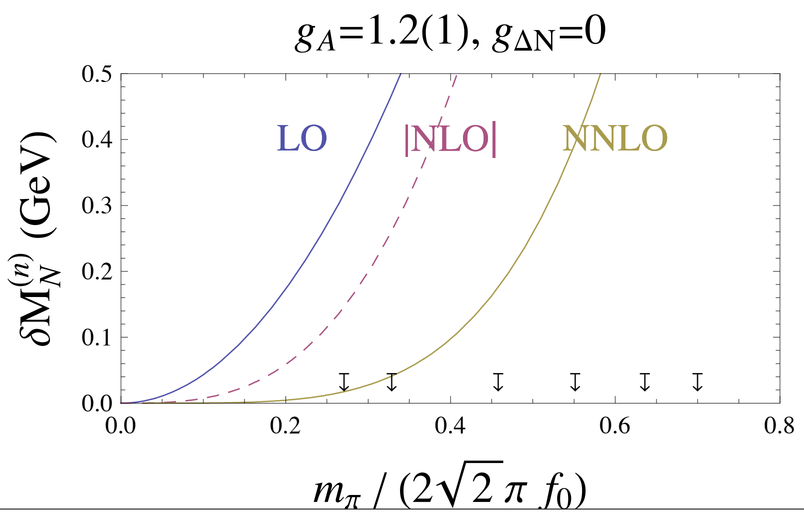

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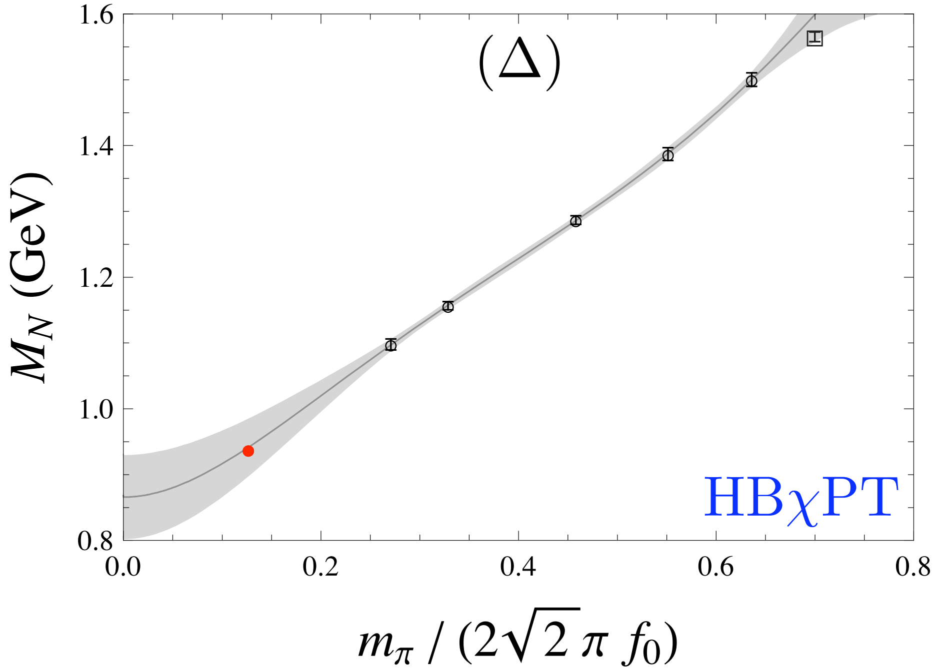
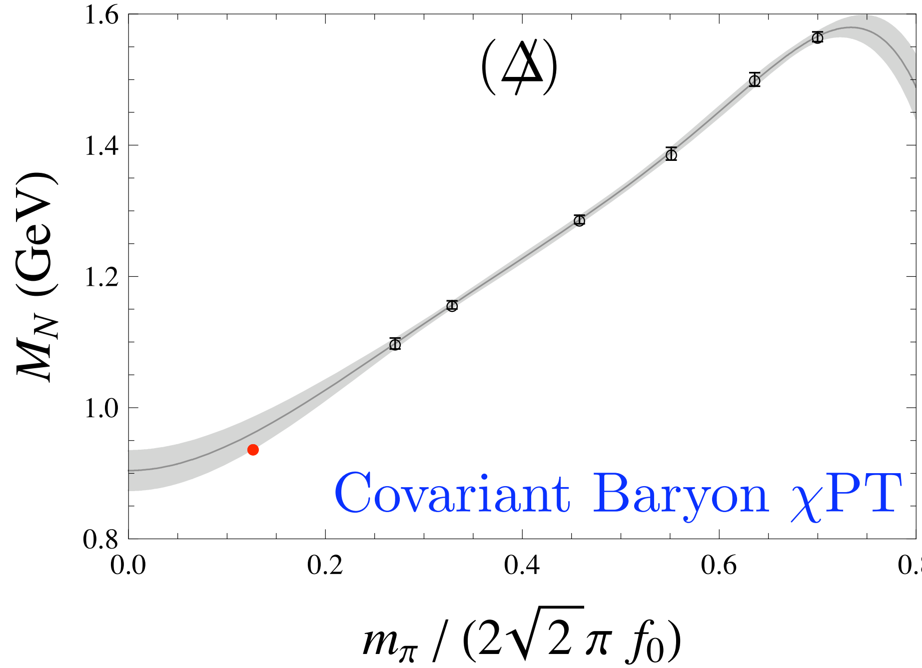



NNLO Covariant, with $g_A$=1.2(1)

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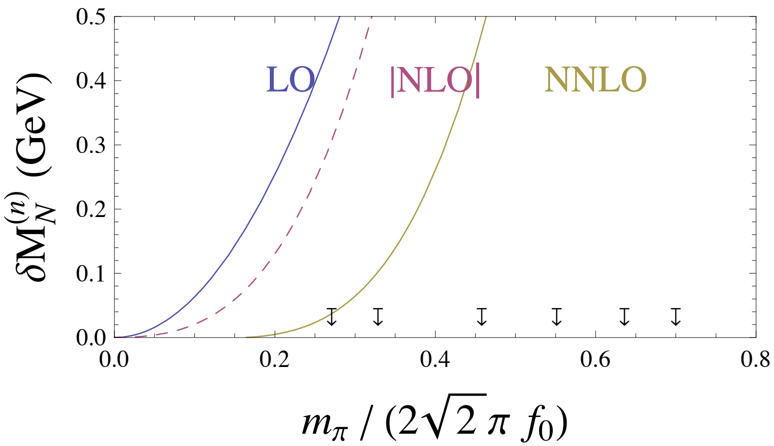


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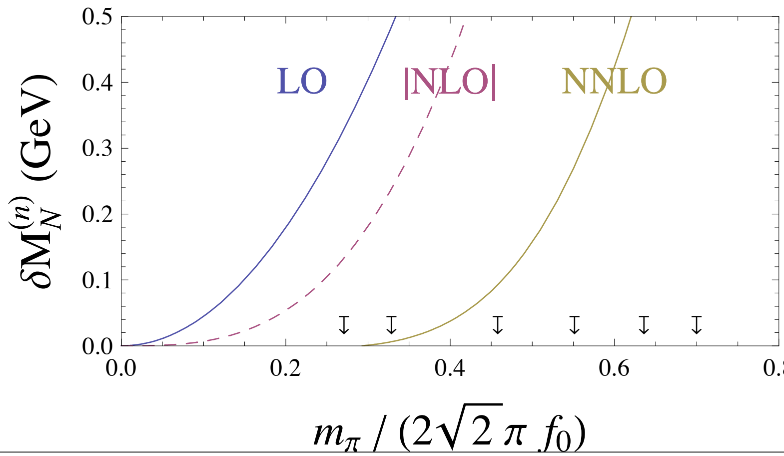

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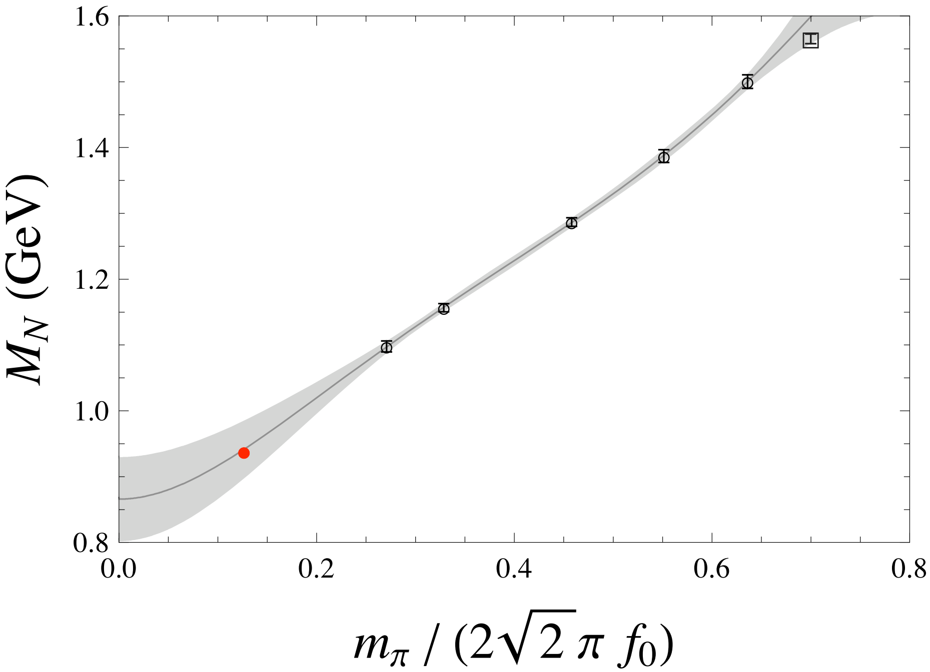
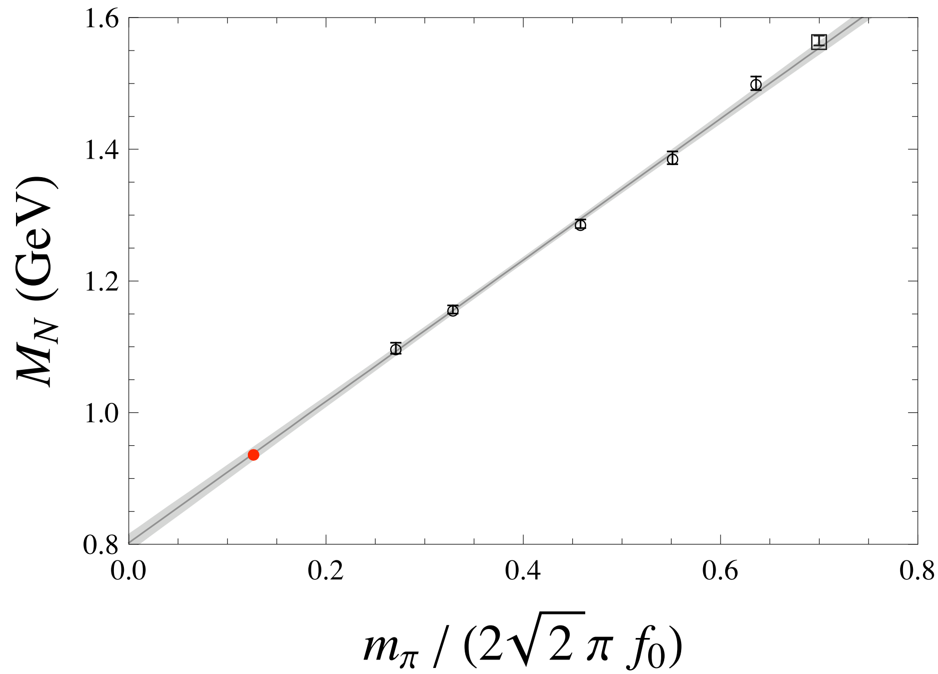



$M_N = \alpha_0^N + \alpha_1^N\, m_\pi$




## NNLO Heavy Baryon Fit

$$M_N = 954 \pm 42 \pm 20 \text{ MeV}$$

$$M_N = \alpha_0^N + \alpha_1^N m_\pi = 938 \pm 9 \text{ MeV}$$

$\chi^2/\mathrm{dof}$ 1.46

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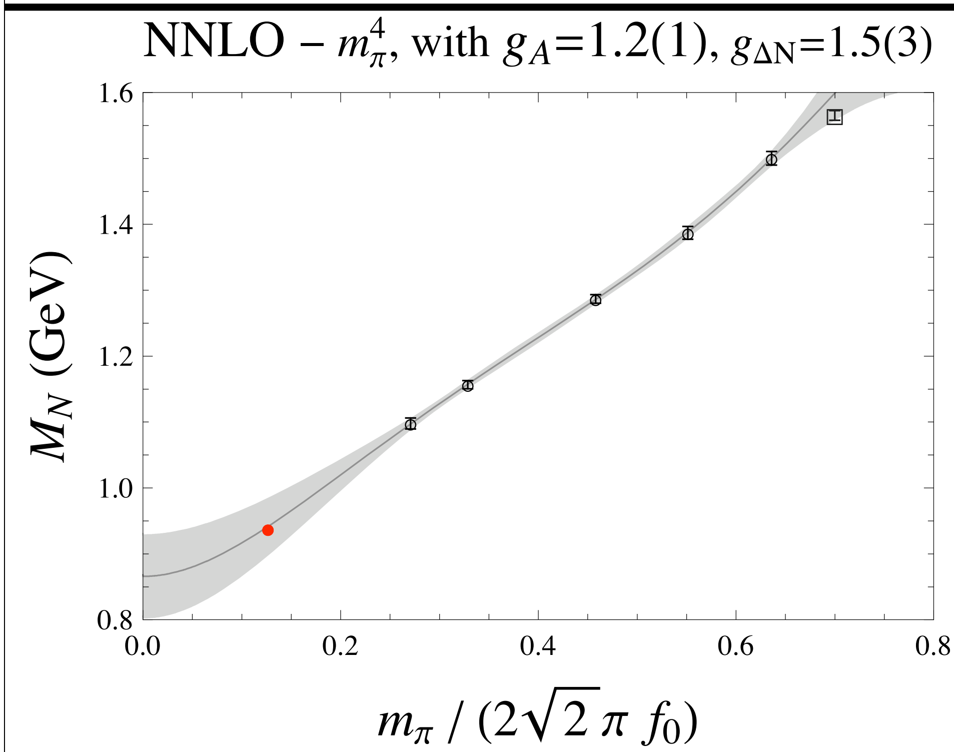


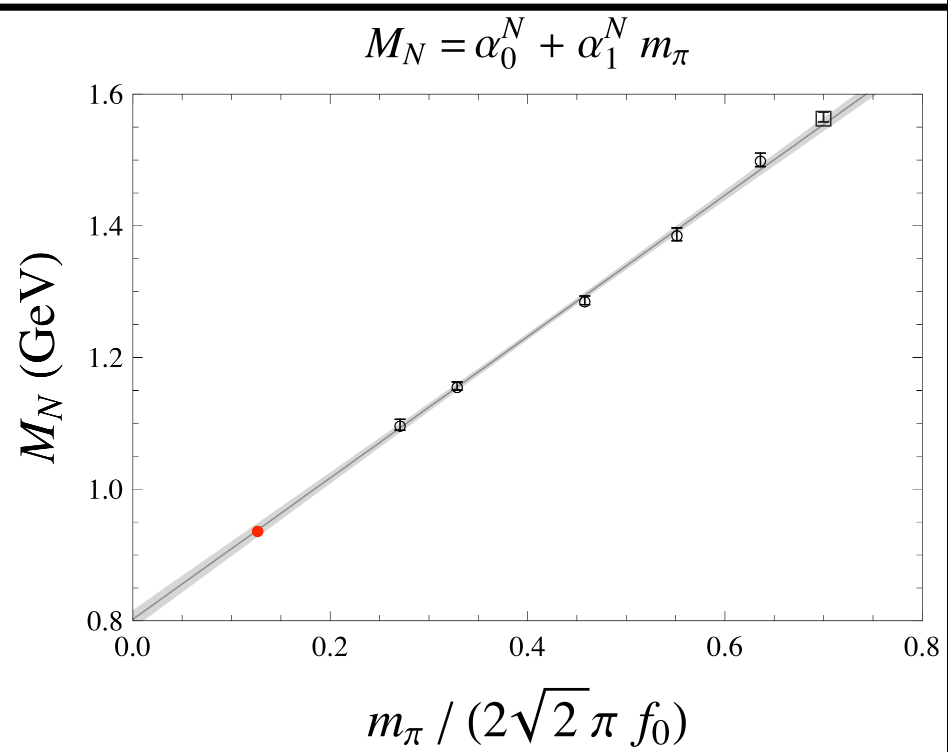


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$$= 938 \pm 9 \text{ MeV}$$

$\chi^2/\text{dof}$

1.46

$$\sigma_{\pi N} = \langle N(p)|m_u \bar{u}u + m_d \bar{d}d|N(p)\rangle$$
$$= m_q \frac{\partial}{\partial m_q} M_N$$

$$\frac{\sigma_{\pi N}}{m_q} \xrightarrow[m_q \to 0]{} \infty\,!$$

$M_N$ : BMW, ETM, JLQCD, LHP, MILC, NPLQCD, PACS-CS, QCDSF/UKQCD, RBC/UKQCD

- Budapest-Marseille-Wupertal (BMW) Collaboration $m_\pi \geq 193$ MeV 2+1 clover improved
  see talk by Chirstian Hoelbling Monday 3:50 3 lattice spacings, multiple volumes
- European Twisted Mass (ETM) Collaboration $m_\pi \geq 311$ MeV 2 flavor twisted mass
  arXiv:0803.3190 3 lattice spacings
- Japanese Lattice QCD (JLQCD) Collaboration $m_\pi \geq 288$ MeV 2 flavor overlap
  arXiv:0806.4744
- Lattice Hadron Physics (LHP) Collaboration $m_\pi \geq 293$ MeV 2+1 DWF on MILC
  arXiv:0806.4549
- MIMD Lattice Computation (MILC) Collaboration $m_\pi \geq 217$ MeV 2+1 staggered
  arXiv:0711.0021 3 lattice spacings, multiple volumes
- PACS-CS Collaboration arXiv:0807.1661 $m_\pi \geq 156$ MeV 2+1 clover improved
  see talk by Daisuke Kadoh Monday 4:10
- QCDSF/UKQCD Collaboration $m_\pi \geq 240$ MeV 2 flavor clover improved
- RBC/UKQCD Collaborations $m_\pi \geq 331$ MeV 2+1 DWF
  see talk by Tom Blum Monday 6:00

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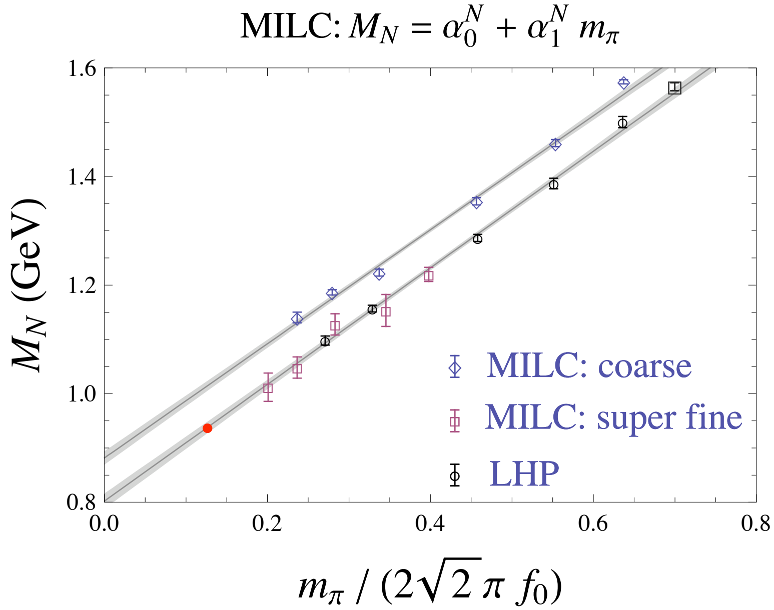

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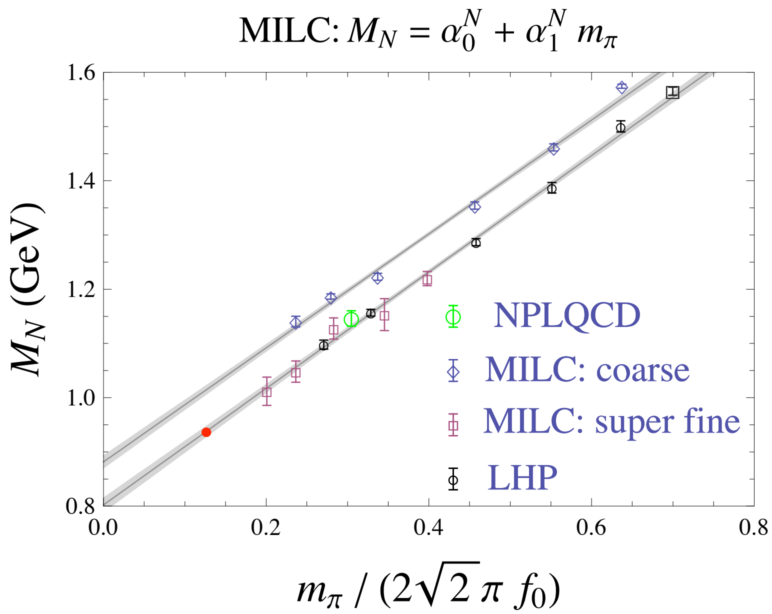

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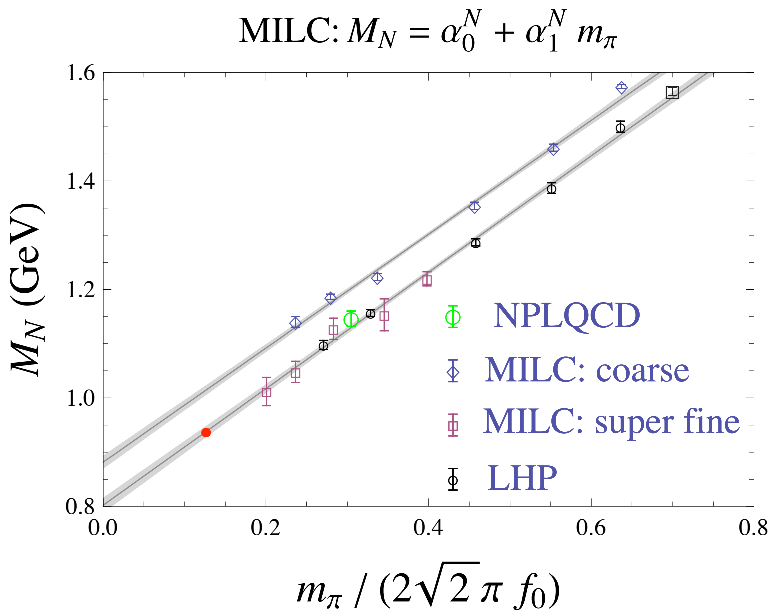


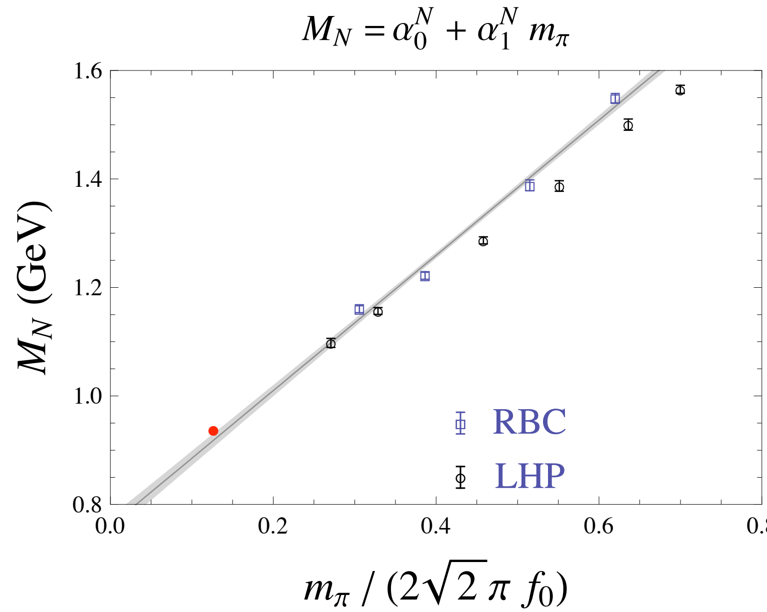

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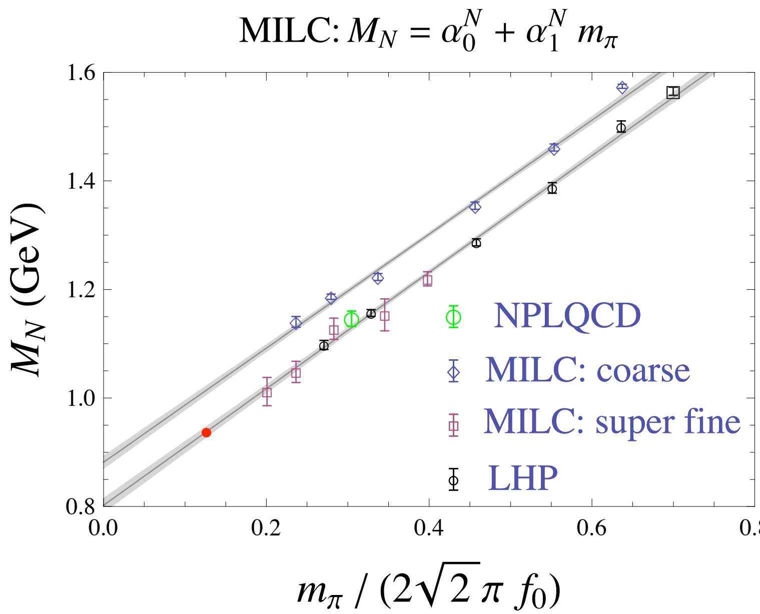


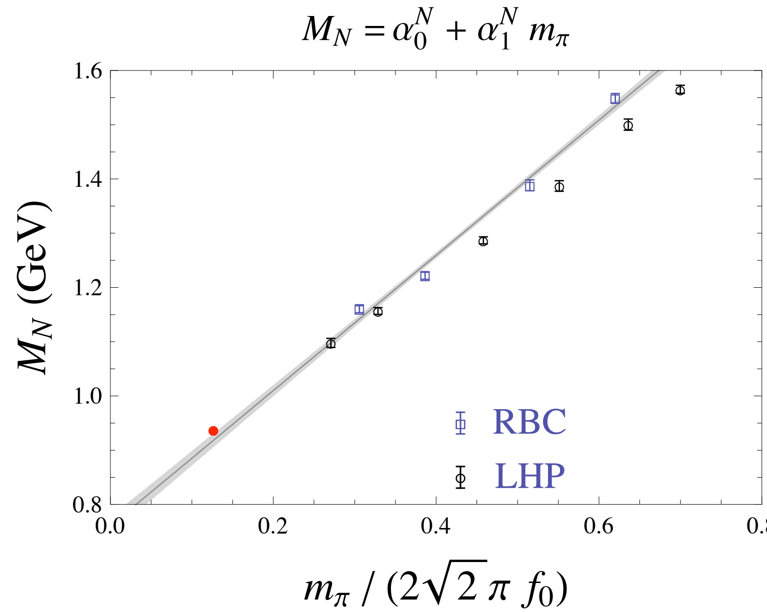


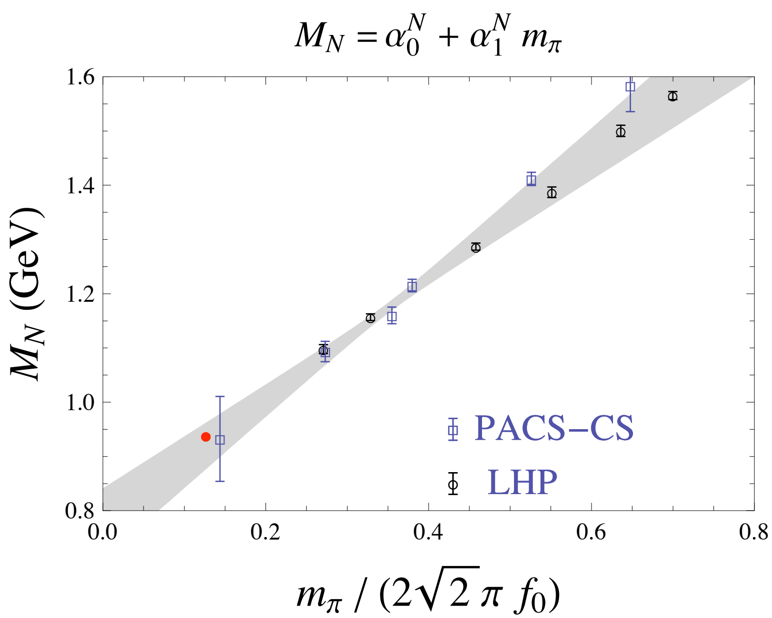

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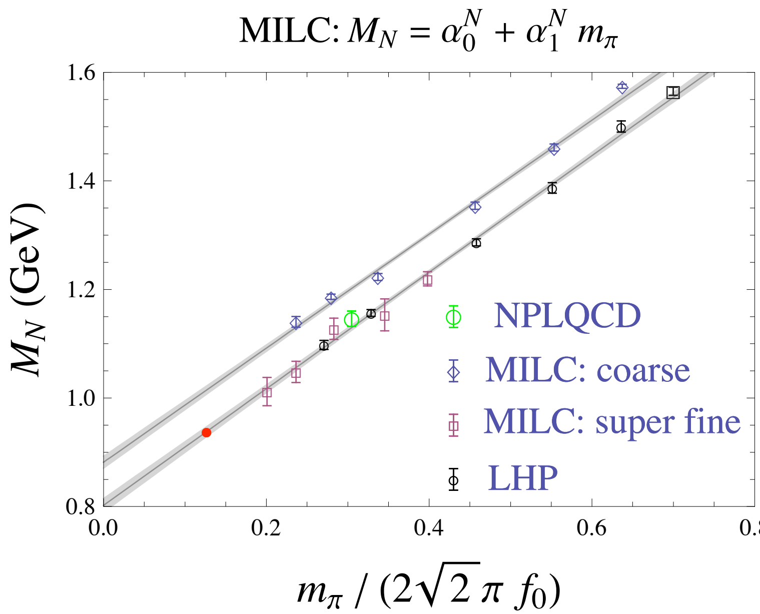


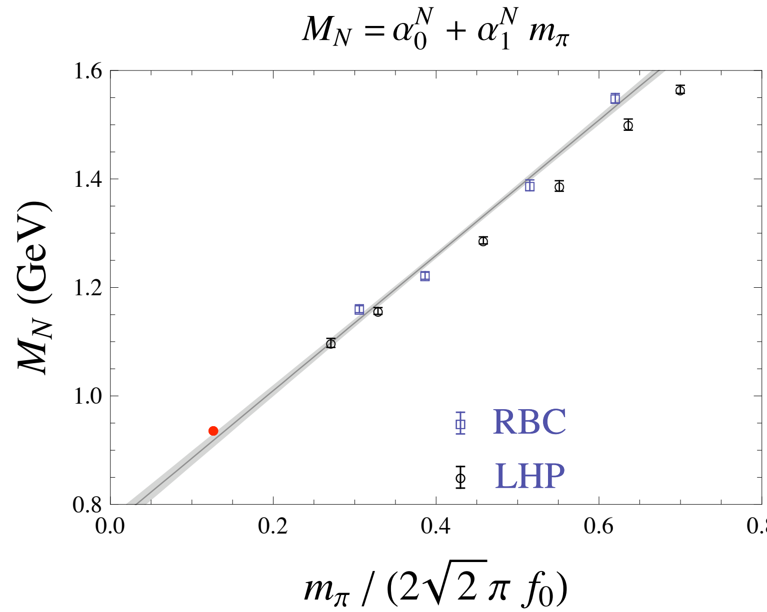


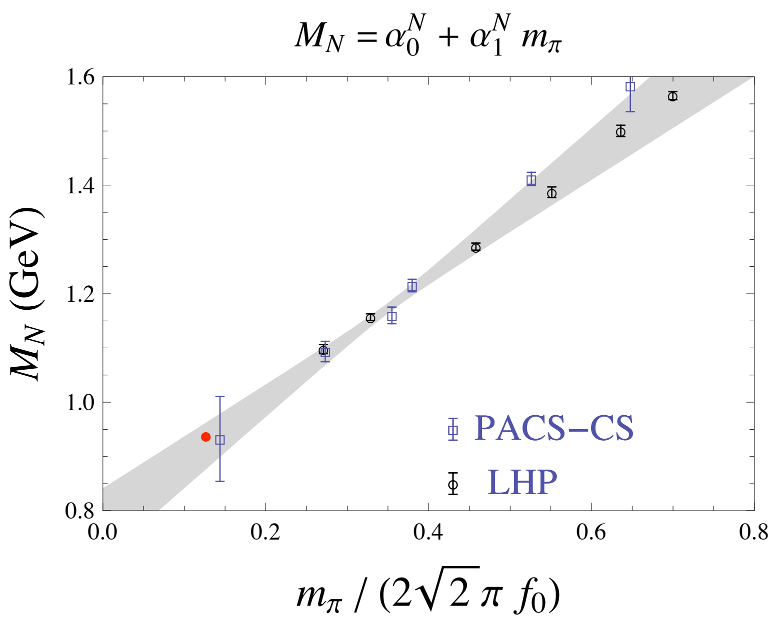


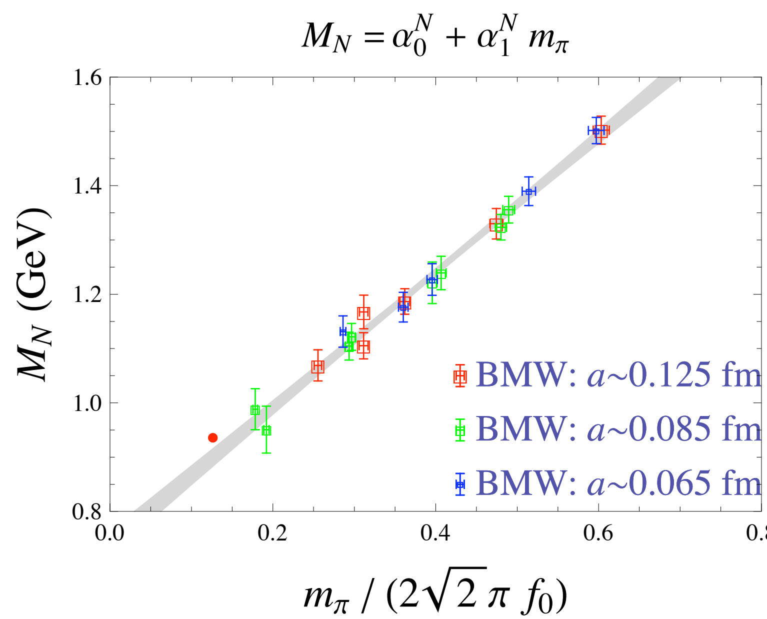

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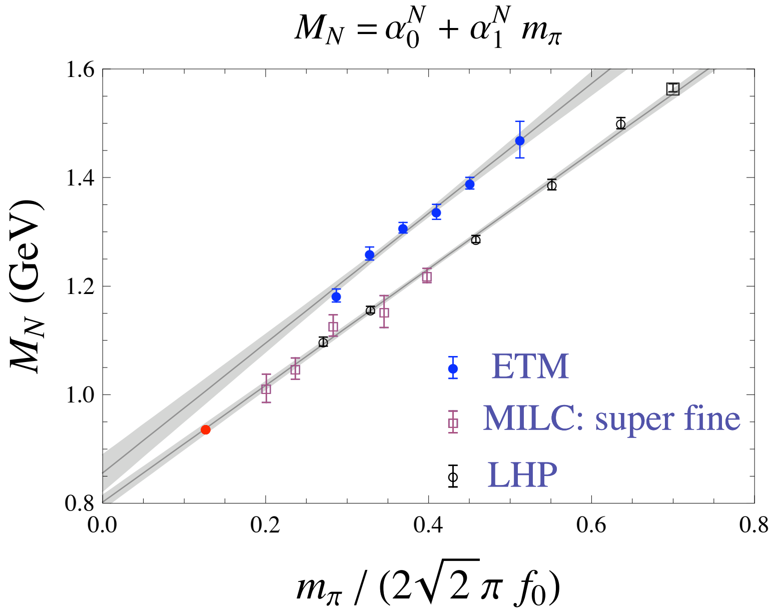


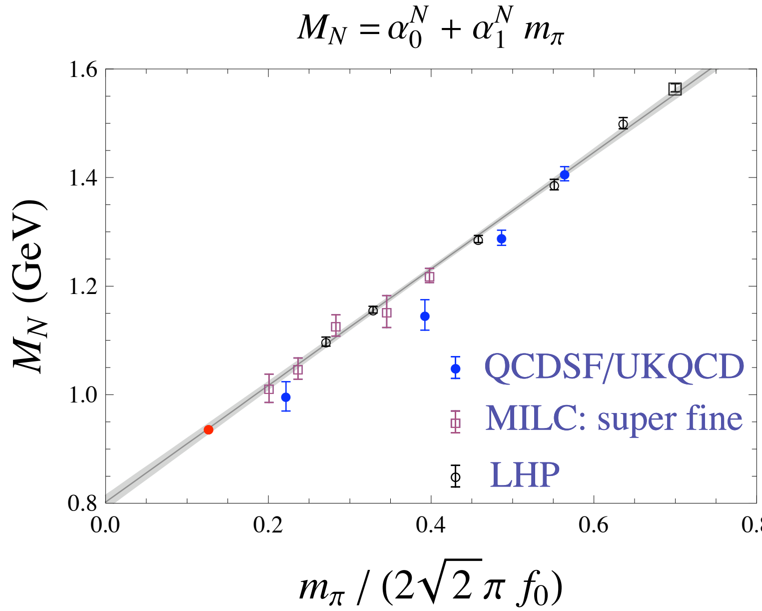


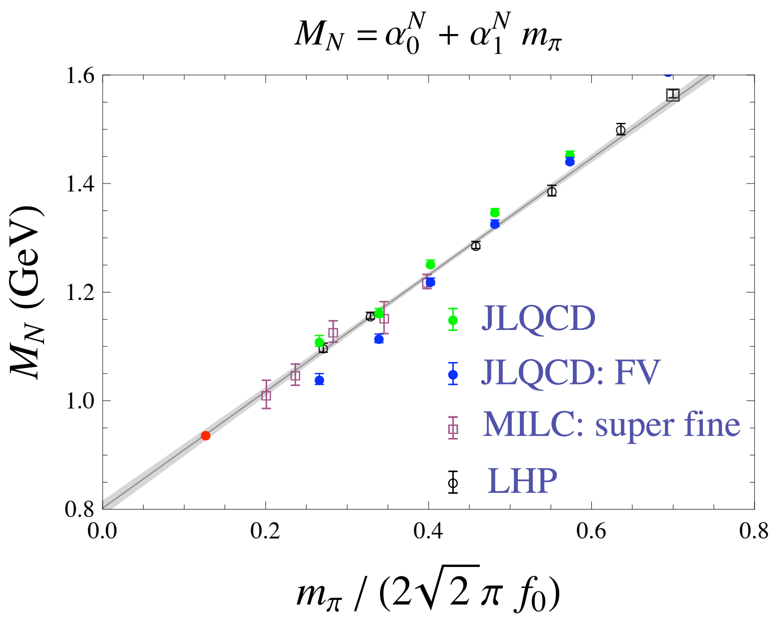

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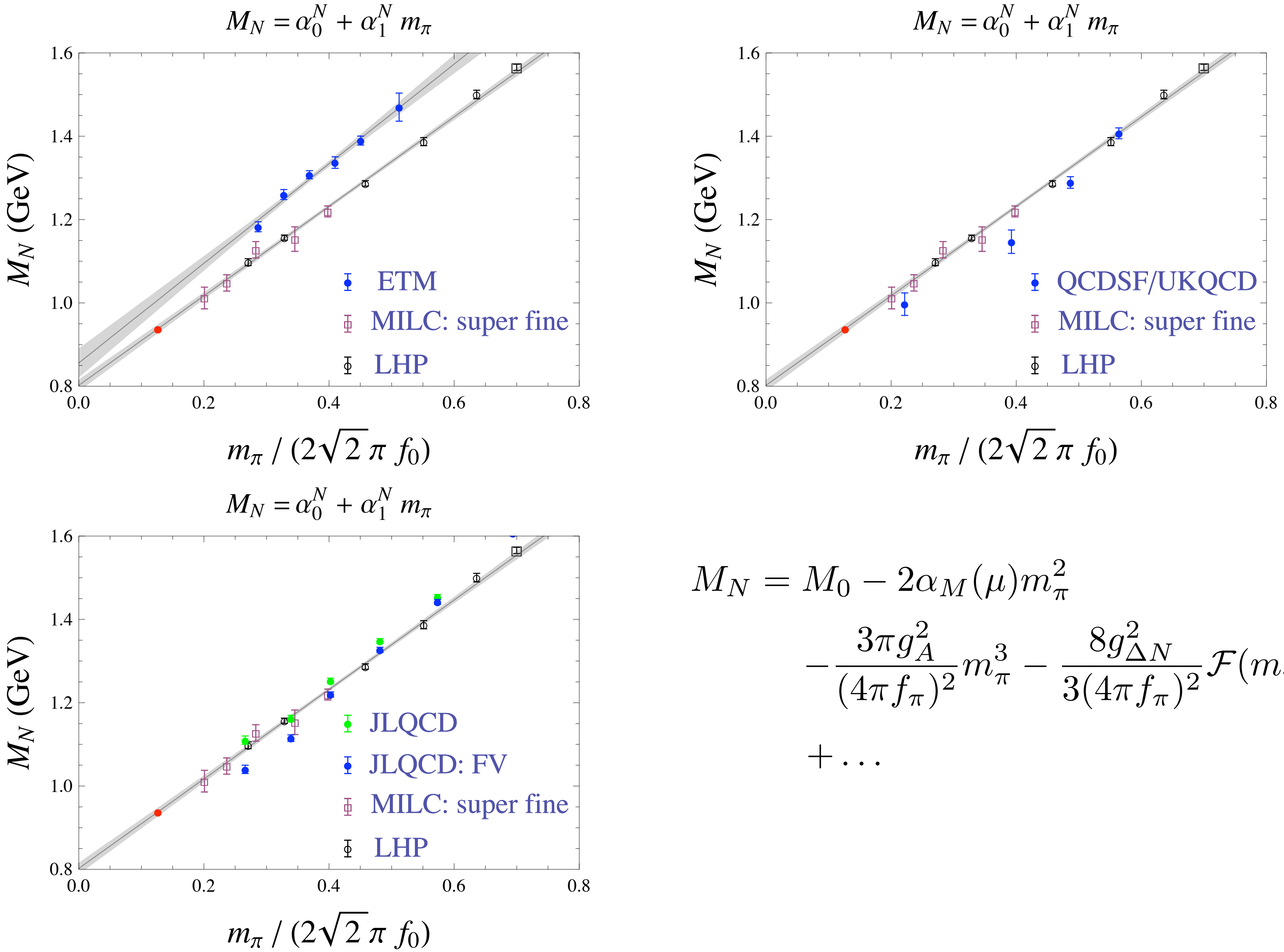



$$M_N = M_0 - 2\alpha_M(\mu) m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\Delta N}^2}{3(4\pi f_\pi)^2}\mathcal{F}(m_\pi, \Delta, \mu) + \ldots$$

# New Lessons

What is needed to have confidence in chiral extrapolation?

- Ideally, $g_A, g_{\Delta N}$ would come out of analysis and not be inputs

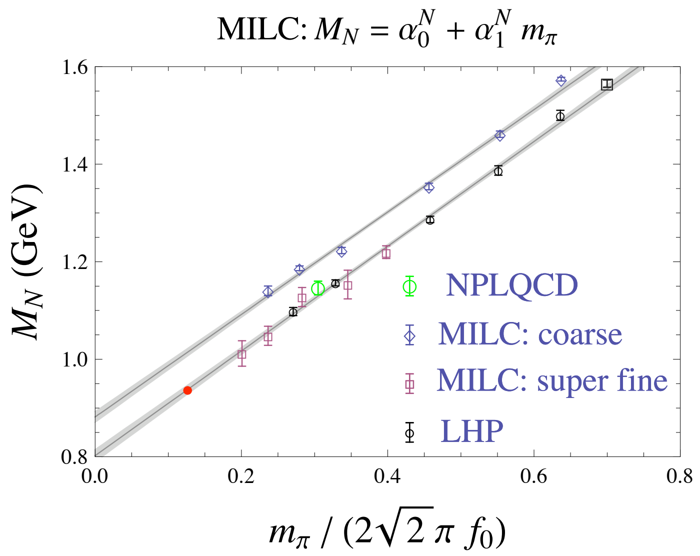
  This is unlikely, but possible

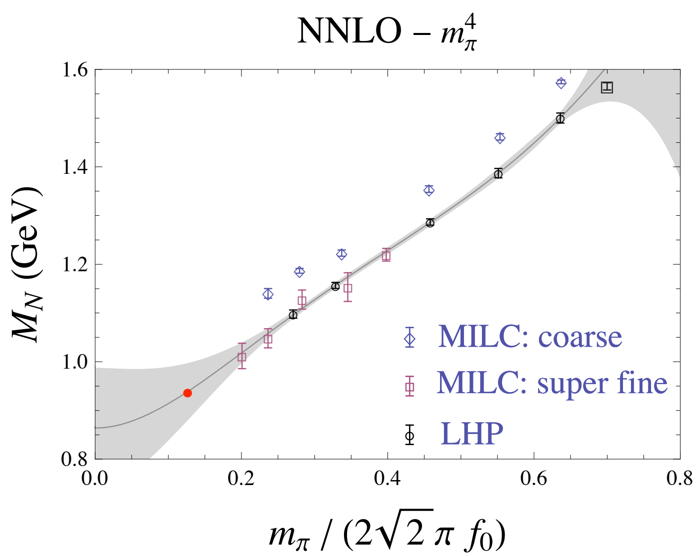
  $g_A = 1.1(3.5)\,,\; g_{\Delta N} = 1.5(11.3)$

  global fit to $g_A, g_{\Delta N}$ and $M_N$

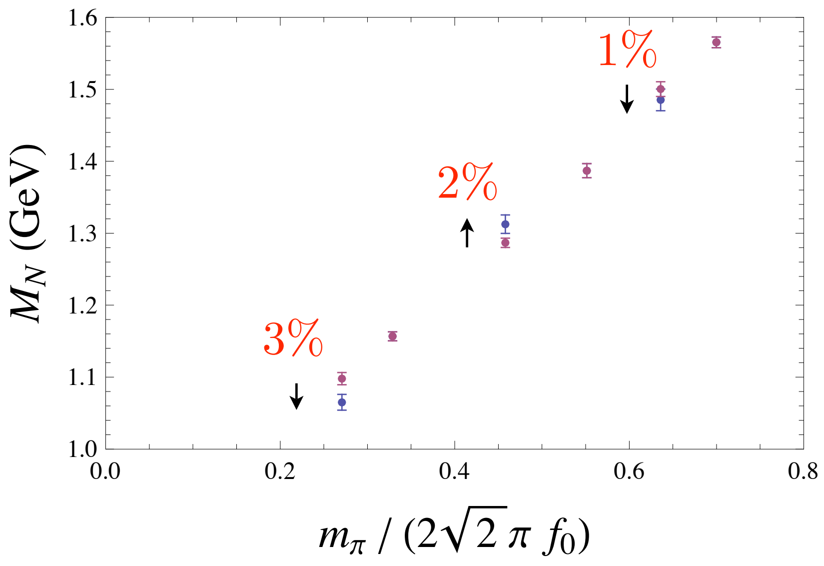
- Also, unphysical extrapolation functions should be ruled out by analysis

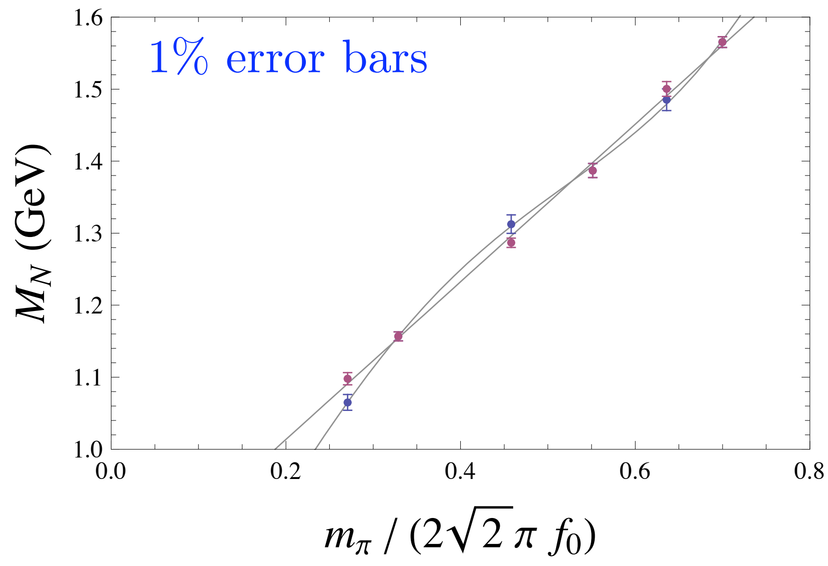
  Assume the "straight line" behavior is accidental: what precision is needed to rule out this extrapolation function?

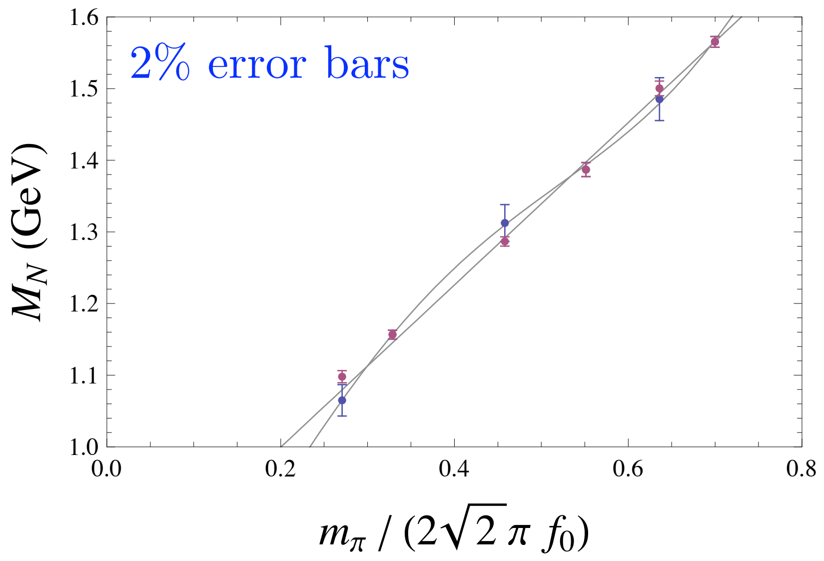


# What is needed?




| $M_N = \alpha_0^N + \alpha_1^N m_\pi$ , | $\chi^2/\text{dof}$ |
|---|---|
| 2% error bars | 0.52 |
| 1% error bars | 2.08 |

- Need 1% error bar to rule out the straight line fit!!!

# CONCLUSIONS

- Heavy baryon extrapolation formulae is in good statistical agreement with the lattice results
  - although resulting convergence is not great
- All 2+1 flavor results show an unexpected trend with the theoretically unmotivated fit ansatz $M_N = \alpha_0^N + \alpha_1^N m_\pi$ describing the results remarkably well.
- To distinguish between this straight line and the expected behavior from heavy baryon chiral perturbation theory, each nucleon mass point needs to be known at the ~1% level, including lattice spacing and finite volume systematics
- The 2-flavor results are systematically different from the 2+1 flavor results, and they are not consistent with each other.
- Larger systematic effects in the nucleon mass than previously thought
- Determining the nucleon axial charge from the mass extrapolation is likely not possible

$$M_N = M_0 - 2\alpha_M(\mu) m_\pi^2 - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\Delta N}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) \; + \ldots$$

- Move towards global analysis, in which the axial charges are fit simultaneously with the masses

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