Time: Tuesday, 5:00
Room: Chesapeake A

Eigenvalue Distributions of Quark Matrix at Finite Isospin Chemical Potential

Presenter:

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Co-authors:

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Abstract

We compare eigenvalue distributions of phase quenched Lattice QCD and Random Matrix Theory (RMT).

We calculated eigen-value distributions of quark matrix on $8^3 \times 4$ lattice by $N_f = 2$ KS fermions. We performed fittings between these lattice data and RMT at coupling $\beta = 5.30$ and iso-vector chemical potential μa = 0.0, 0.004773, 0.1 and 0.2 (weak non-hermiticity) and then find good agreement.

Our data indicates that F_π decreases as the iso-vector chemical potential increases.

1. Research situation at $\mu\neq0$

	RMM	LGT
SU(2) Full[1]	○	○
SU(3) Quench[2]	○	○
SU(3) Phase Quench	○	This talk
SU(3) Full	○	×

[1] Osborn, Splittorfff & Verbarrschot (2005), Akemann & Bittner (2006)
[2] Akemann & Wettig (2004)

Finite baryon-number density in SU(3) Finite density lattice QCD

→ introduces chemical potential μ

quark matrix determinant positive, real for $\mu=0$

complex for $\mu\neq0$

numerical study becomes difficult !

2. Formulation
Lattice calculation

Fermion : Kogut-Susskind (Staggered)

Quark matrix determinant is complex

⇒ one may perform Monte Carlo simulation

Quenching measure $$\langle O \rangle_q = \frac{\int DU\, O\, e^{-\beta S_g}}{\int DU e^{-\beta S_g}}$$

Phase quenching measure $$\langle O \rangle_0 = \frac{\int DU \quad \left|\det \Delta\right|^{N_f/4} O\, e^{-\beta S_g}}{\int DU \quad \left|\det \Delta\right|^{N_f/4}\, e^{-\beta S_g}}$$

N_f=2 Phase quench, SU(3), $8^3 \times 4$ lattice, β=5.3, ma=0.05

Calculated eigenvalues:

all eigenvalues ($N_C \times N_V$=6144) in 980 configurations

the smallest 100 eigenvalues in 15,000 /10,000 / 5,000 configurations

Random Matrix Model

- •G.Akemann and G.Vernizzi, 2003
- •G.Akemann, 2003
- •J.Osborne, 2004

N_f=2 Phase quenched spectral density

in weak non-Hermiticity limit

$$\rho^{(N_f=2)}(\xi) = \rho^{(N_f=0)}(\xi)\left(1 - \frac{\left|K_s(\xi,\eta^*)\right|^2}{K_s(\eta,\eta^*)K_s(\xi,\xi^*)}\right)$$

where quenched density is given by

$$\rho^{(N_f=0)}(\xi) = \frac{1}{4\pi\alpha^2}\left|\xi\right|^2 K_0\left(\frac{\left|\xi\right|^2}{4\alpha^2}\right) \mathrm{e}^{-\frac{1}{4\alpha^2}\mathrm{Re}(\xi^2)} K_s\left(\xi,\xi^*\right).$$

$$K_s\left(\xi,\xi^*\right) \equiv \int_0^1 dt\, e^{-2\alpha^2 t} I_0\left(\xi\sqrt{t}\right) I_0\left(\xi^*\sqrt{t}\right) \qquad I_0(z) = J_0(iz)$$

Bridge between LGT and RMM

$$\xi = za \cdot V\Sigma = za \cdot \pi / d$$

rescaled eigenvalue which is used in RMM

measured eigenvalue on the lattice

$$\eta = ma \cdot V\Sigma = ma \cdot \pi / d$$

rescaled mass

given mass on the lattice

$$\alpha^2 = (\mu a)^2 F_\pi^2 V$$

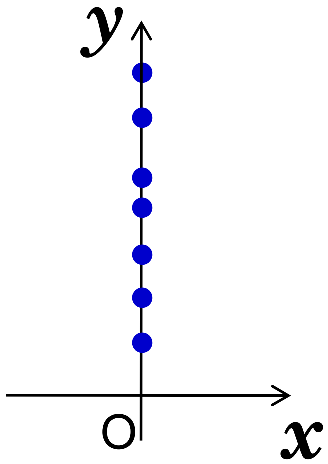
given chemical potential on the lattice

a: lattice spacing
d: mean level spacing
V: lattice volume
Σ: chiral condensate
F_π: pion decay const.

Mean level spacing *d* is very very important !

Is *d* 1-dimensional or 2-dimensional spacing?
It seems that we should think of *d* as 1-dimensional spacing.

μ=0 Banks –Casher formula

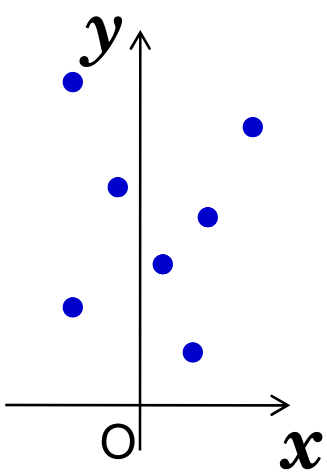


$$\Sigma = \langle \bar{\psi}\psi \rangle = -\frac{\pi\rho(0)}{V} = -\frac{\pi}{Vd} \propto \frac{1}{d}$$

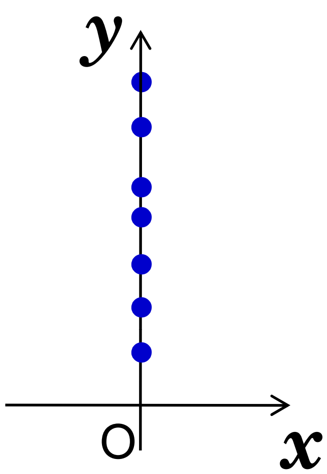
Measure the mean level spacing *d* between neighbor eigenvalues.

μ≠0

for the smallest 7eigenvalues



Project eigenvalues on *y*-axis



Calculate the mean level spacing *d* on *y*-axis

3. Comparison of RMM result and lattice data

Our purpose again

Eigen-value distribution function of Lattice

$$\rho(x,y)$$

$$\int \rho(x,y)\,dxdy = N$$

$$= 3\times 8^3 \times 4 = 6144$$

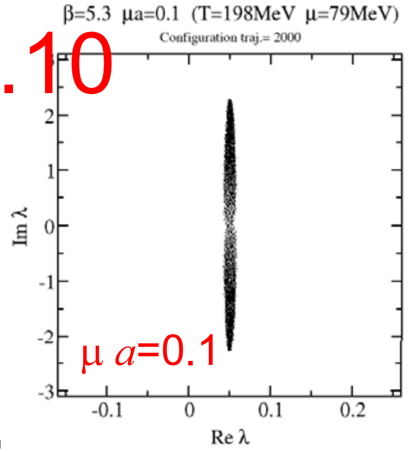
⟺

Spectral density of RMM

$$\rho^{(N_f=2)}(\xi) = \rho^{(N_f=0)}(\xi)\left(1-\frac{\left|K_s(\xi,\eta^*)\right|^2}{K_s(\eta,\eta^*)K_s(\xi,\xi^*)}\right)$$

We want to determine parameters in which the lattice data reappear.

$8^3 \times 4$ lattice, N_f=2, β=5.3, ma=0.05, μa=0.10



① Calculate mean level-spacing *d*, and rescale lattice data by it.

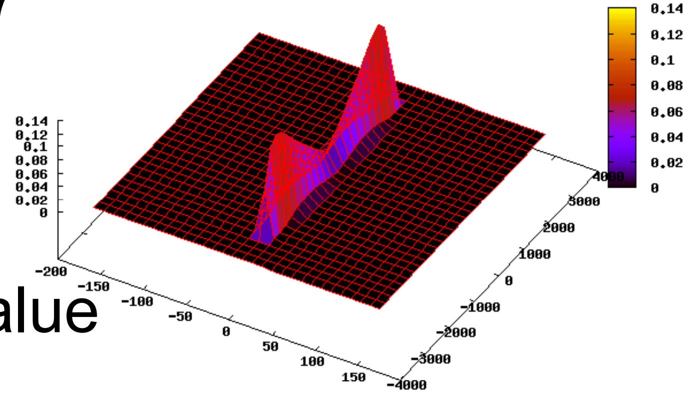
(15,000 configurations, the smallest 7 eigenvalues)

$$d = 2.775 \times 10^{-3}$$

$$\xi = za \cdot \pi / d$$

rescaled eigenvalue which is used in RMM

measured eigenvalue on the lattice



The aerial view is obtained from 980 configurations.

② Obtain the rescaled mass *η* by *d*

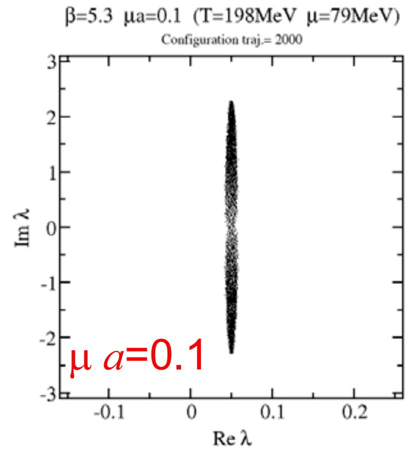
$$\eta = ma \cdot \pi / d = 57.6$$

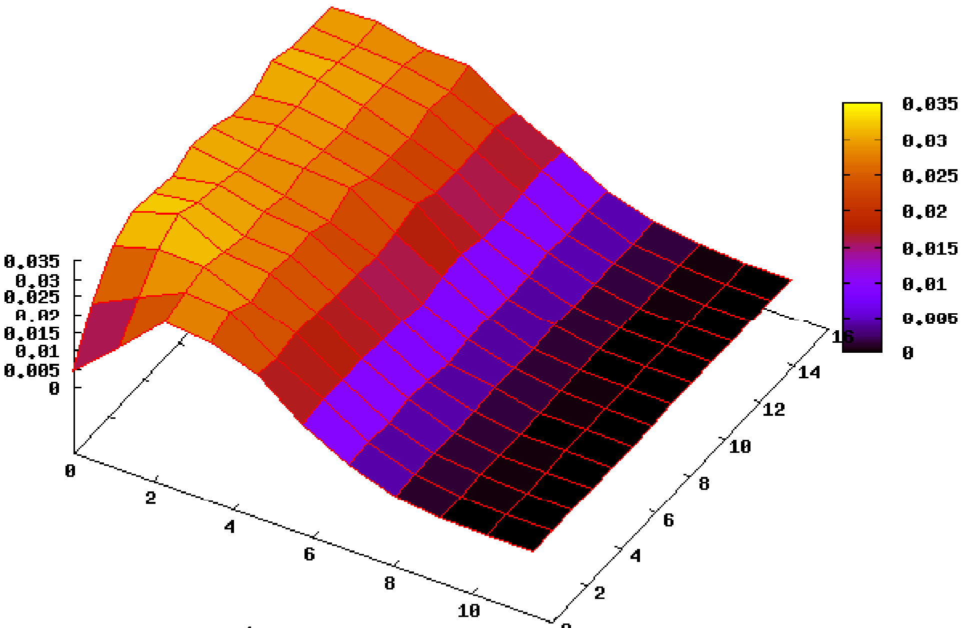
These values are determined uniquely.

③ Put η and choose α suitably in RMM

☆ Choose α in order to match those distribution latitudes, peaks and plateaus on the real and imaginary axies.

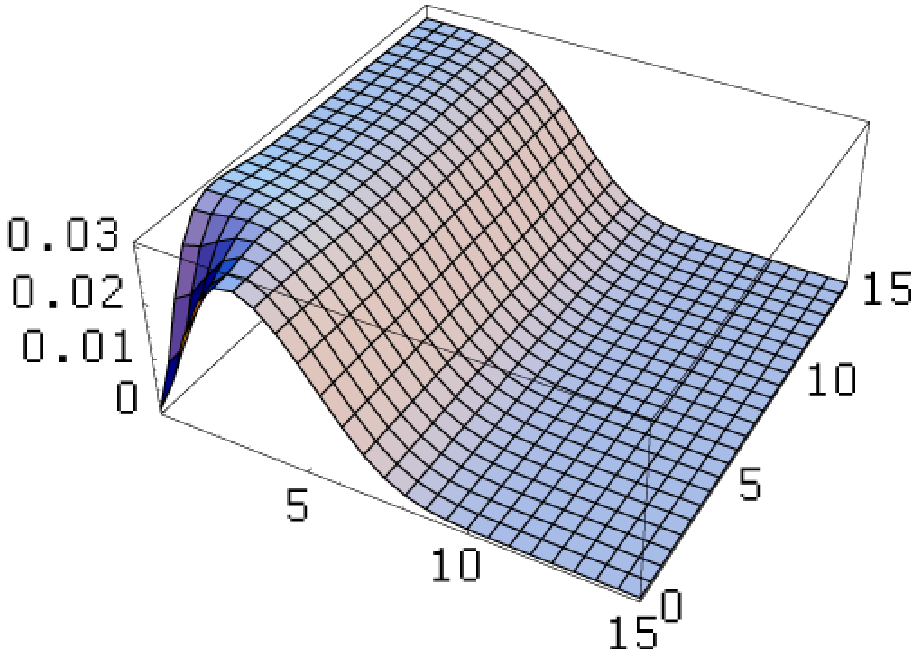
☆ Then α =1.68 is obtained.





LGT $8^4 \times 4$ lattice, N_f=2, β=5.3, ma=0.05, μa=0.10

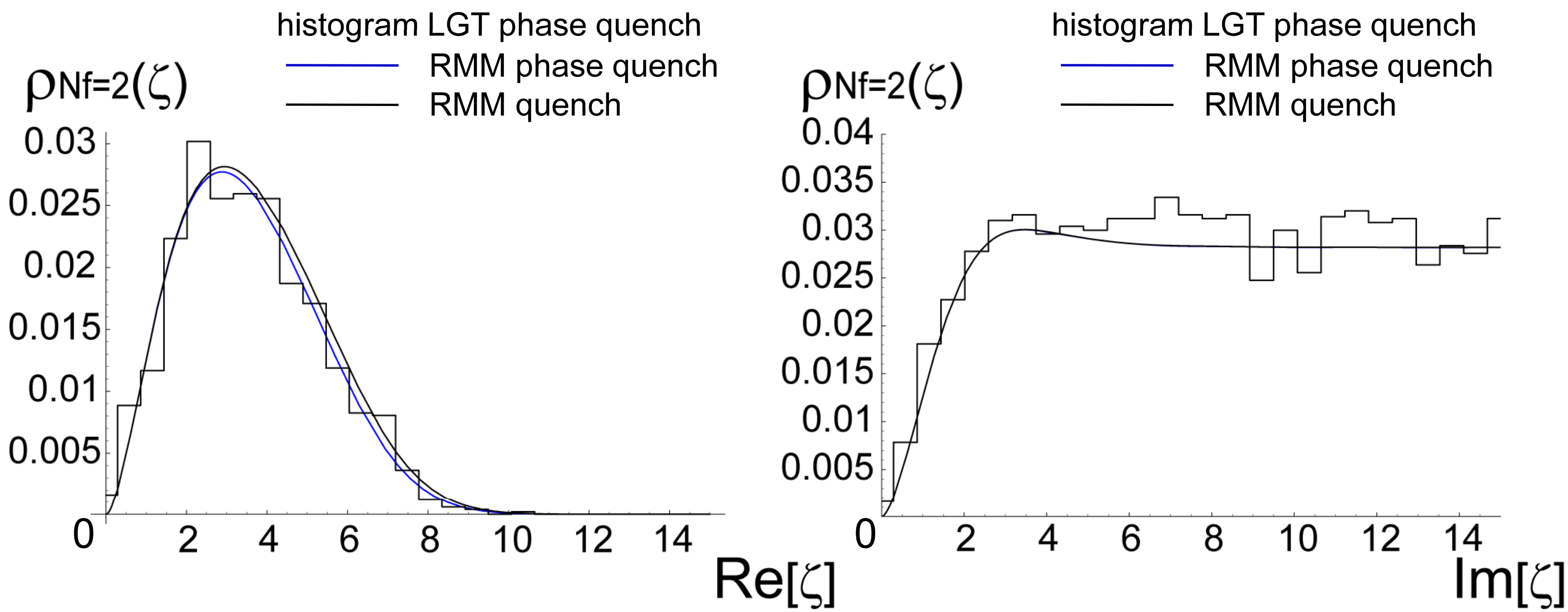
$$\xi = za \cdot \pi / d$$



RMM N_f=2, α=1.68, η=57.6

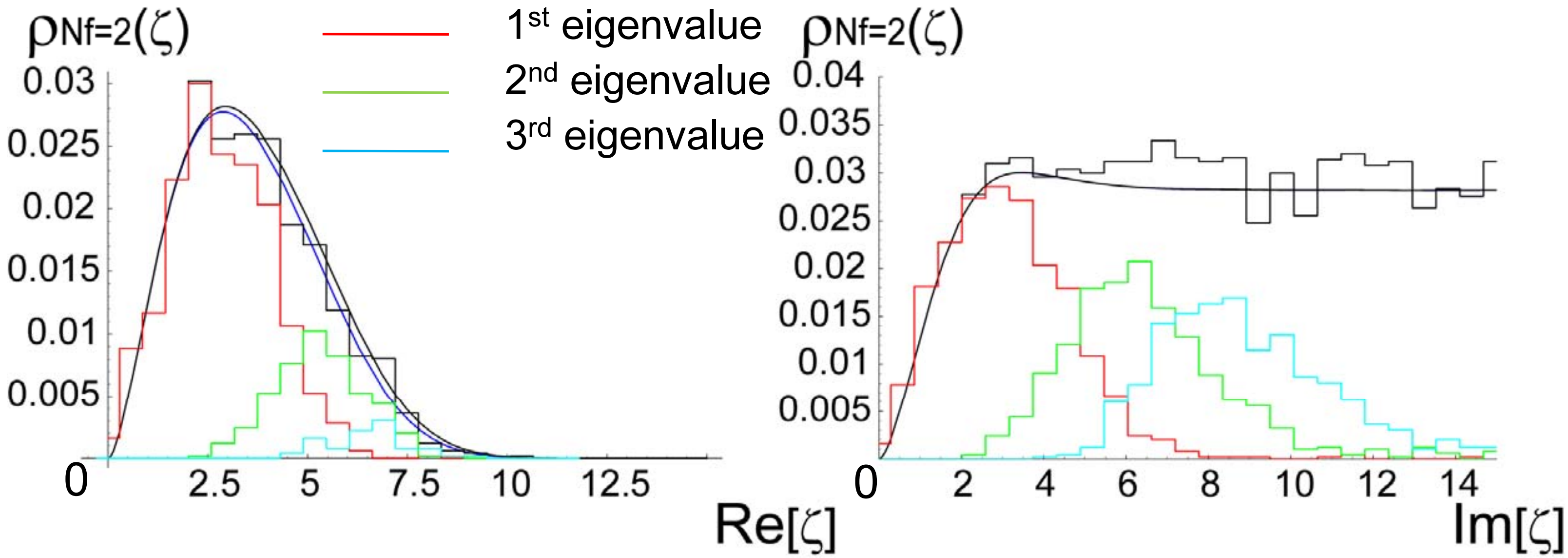
μa=0.10

15000 configurations

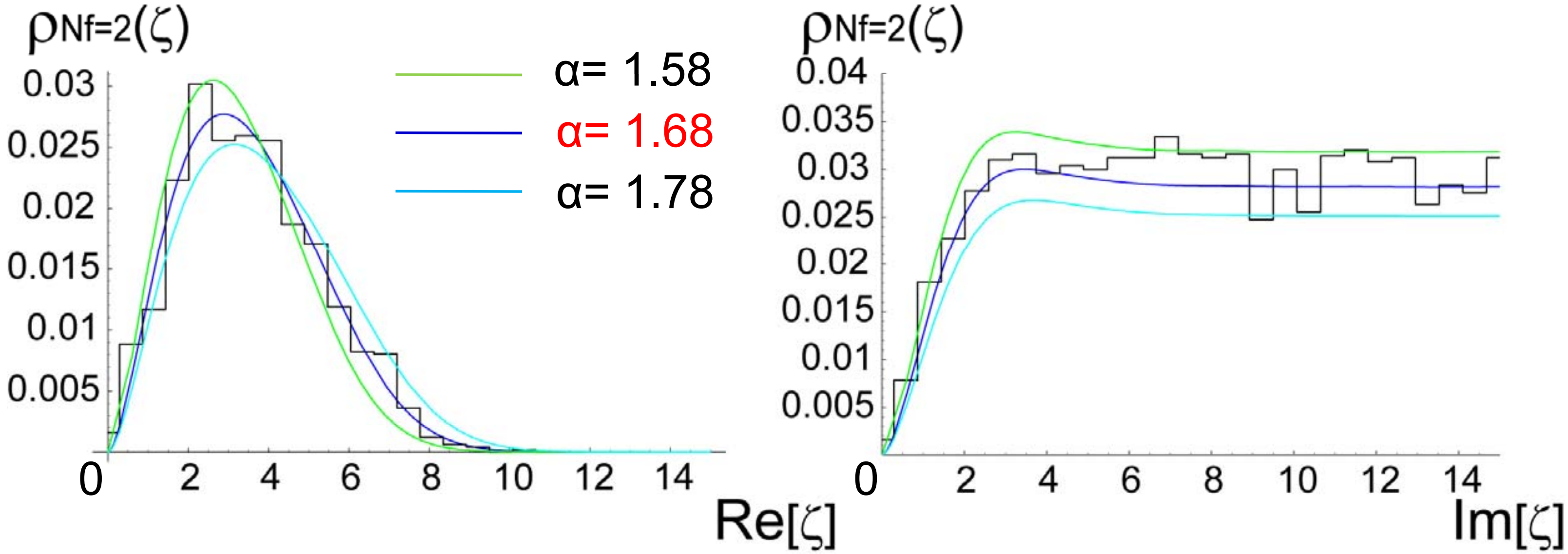


- Charts coincide without tuning of those normalizations.
- Because the phase effect is small, it is difficult to know which of RMM graphs corresponds to LGT graph.
- Free parameter is α only.

Distribution of the first 3 eigenvalues in LGT



Tuning of parameter *α*

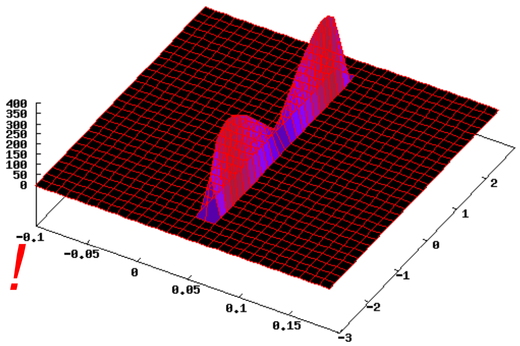


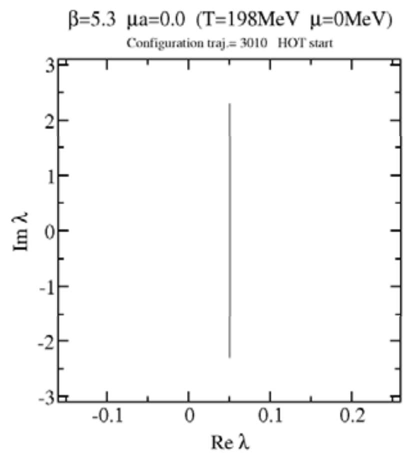
μa=0.00

$$d = 2.284 \times 10^{-3}$$

$$\alpha^2 = (\mu a)^2 F_\pi^2 V = 0.0$$

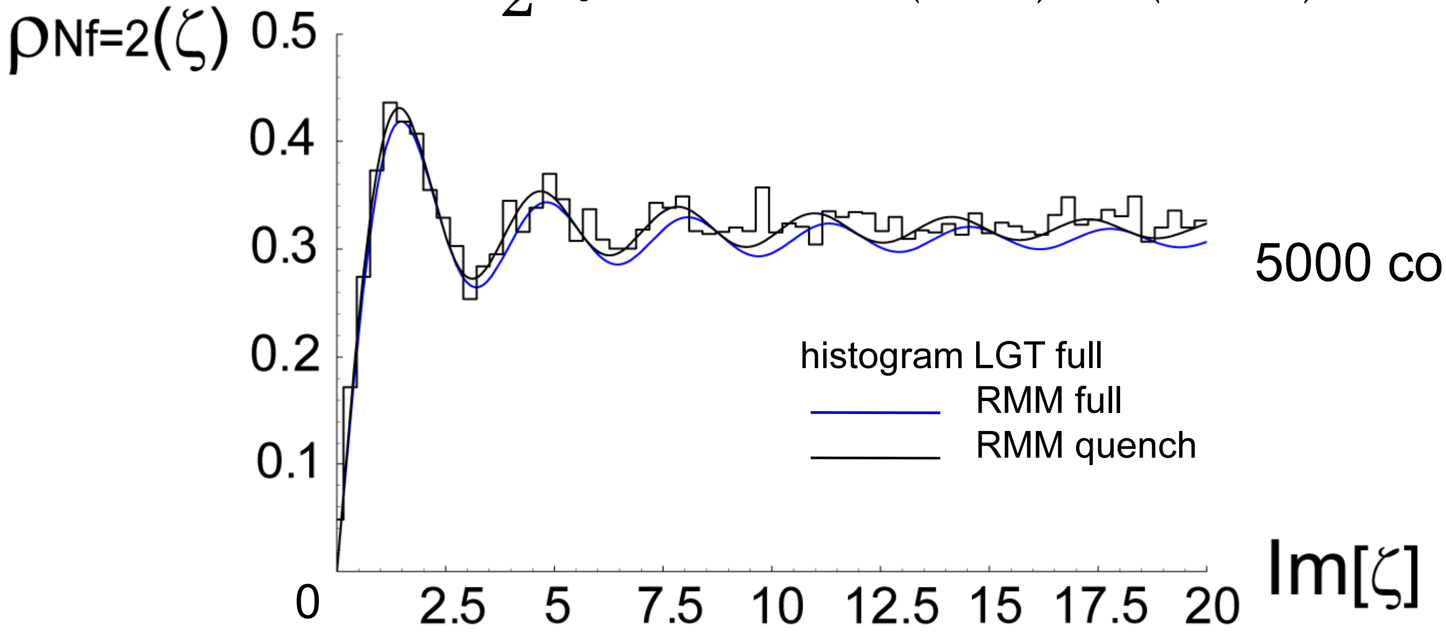
No free parameter !





Spectral density of RMM

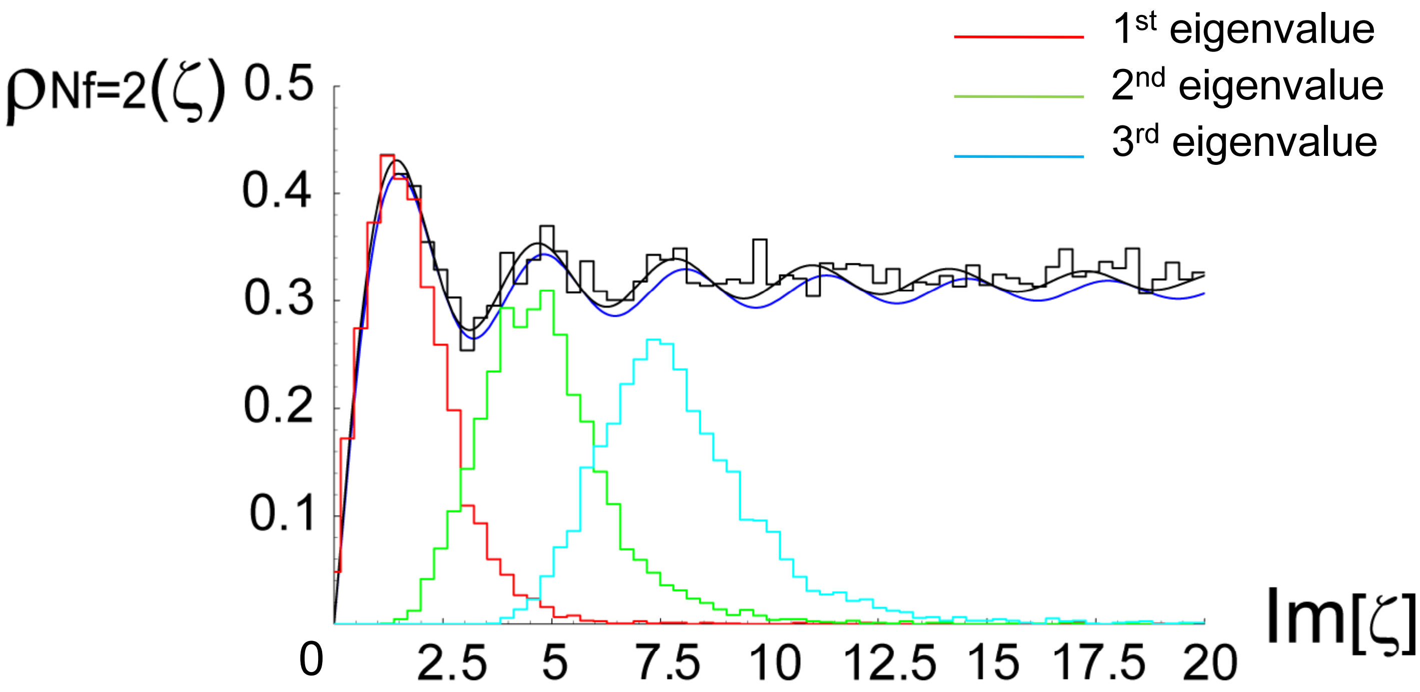
$$\rho^{(N_f=0)}(\xi) = \frac{y}{2}\int_0^1 dt\, e^{-2\alpha'^2 t} I_0\left(\xi\sqrt{t}\right) I_0\left(\xi^*\sqrt{t}\right) \qquad \xi = x + iy$$



5000 configurations

- This statistics are not so rich. The first three peaks of LGT full are very well in agreement with the one of RMM full.

Distribution of the first 3 eigenvalues in LGT

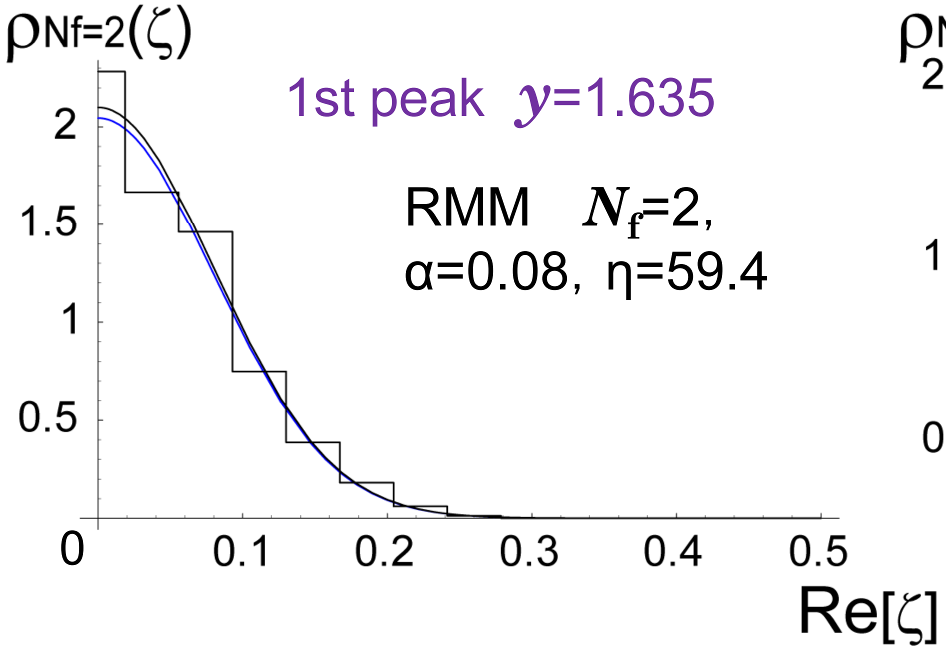


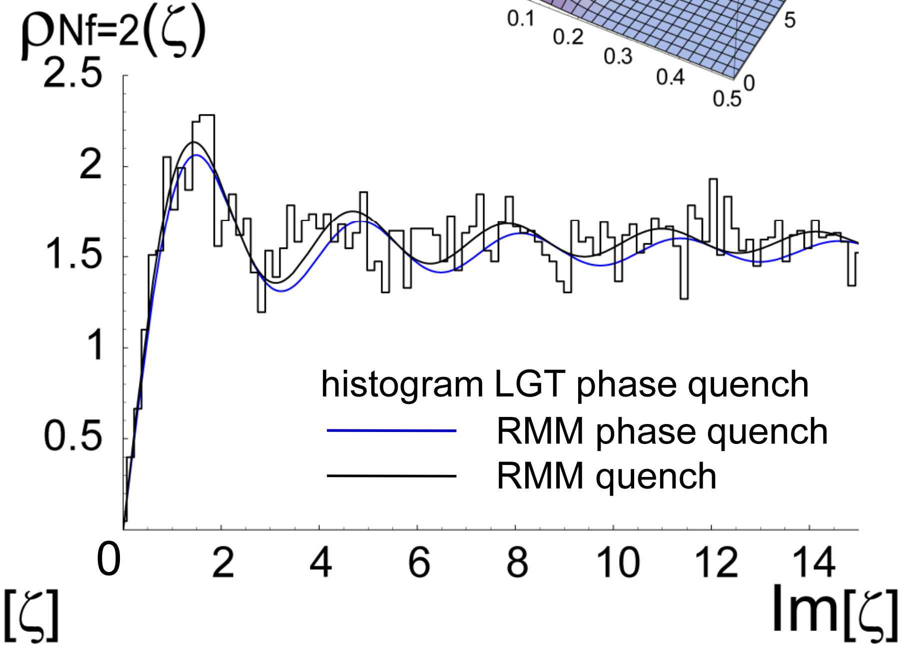
μa=0.004773

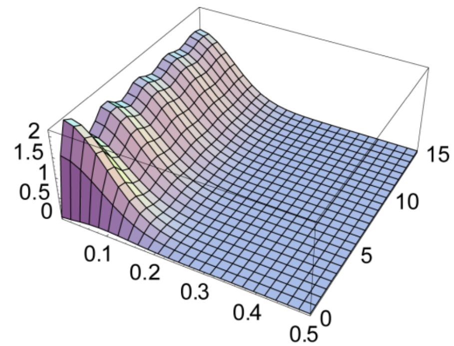
$$d = 2.661 \times 10^{-3}$$

This aerial view is the almost same one at μa=0.0. The close-up near the origin has very narrow distribution width.

15,000 configurations

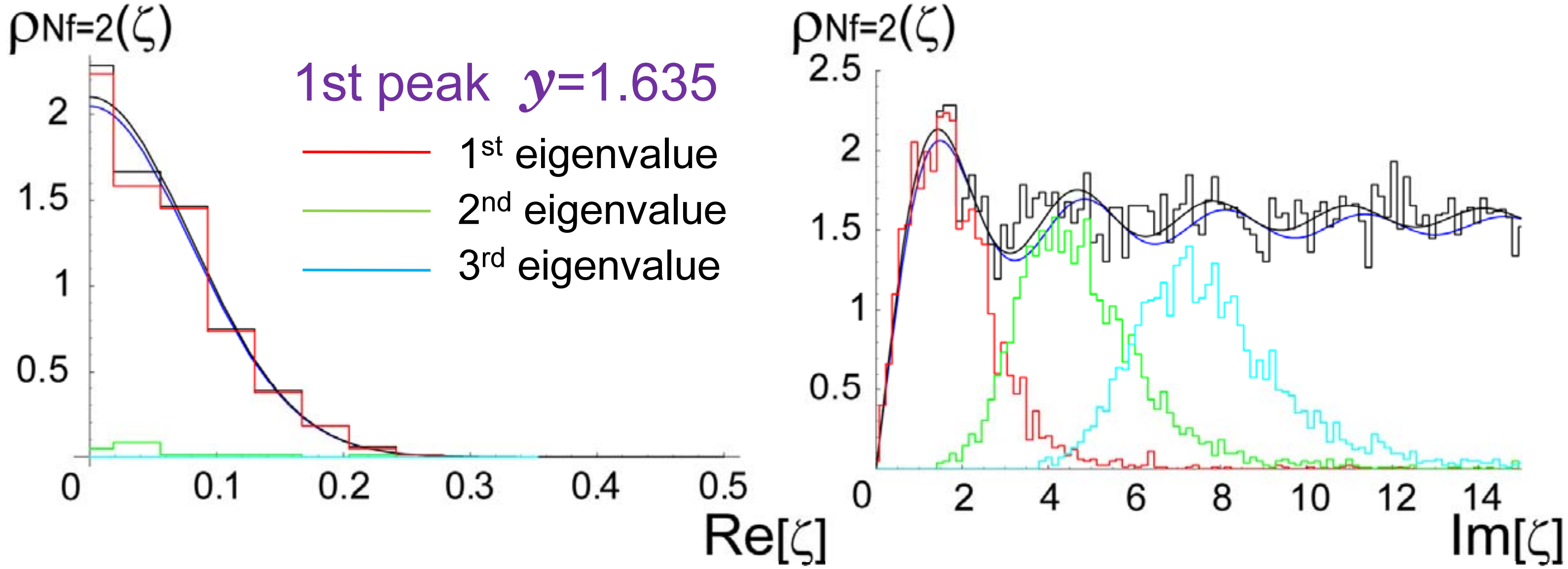




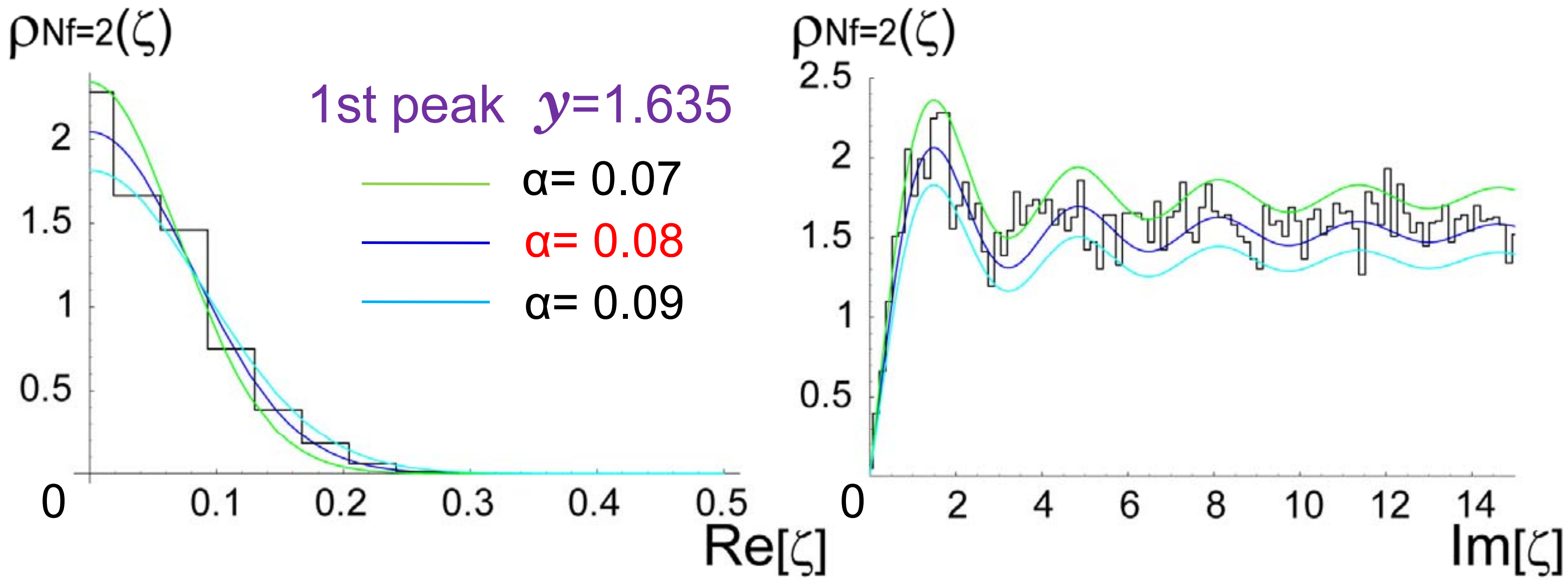


- This statistics are not so poor. It seems that only the first peak of LGT is in agreement with RMM.

Distribution of the first 3 eigenvalues in LGT

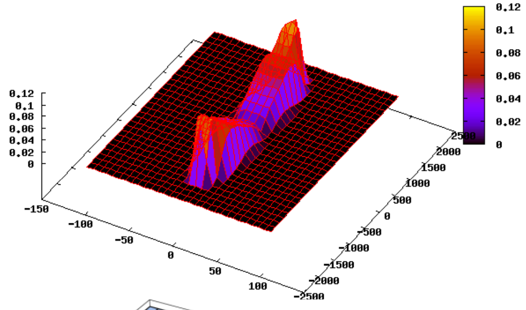


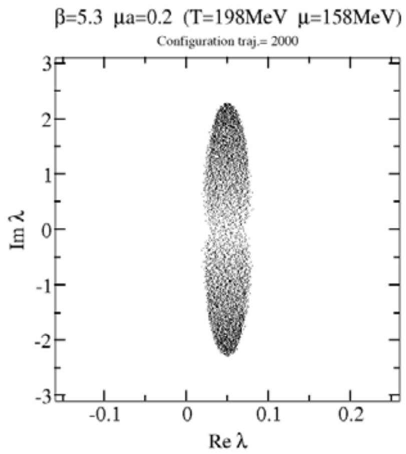
Tuning of parameter α

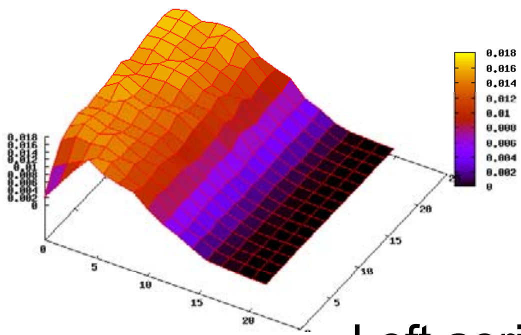


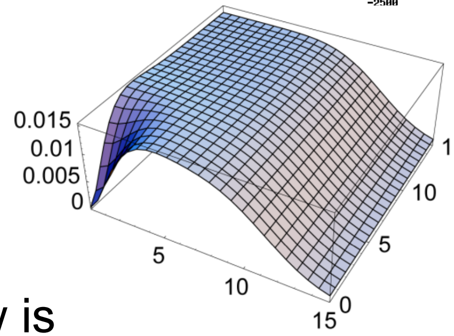
μa=0.20 10,000 configurations

$$d = 4.341 \times 10^{-3}$$



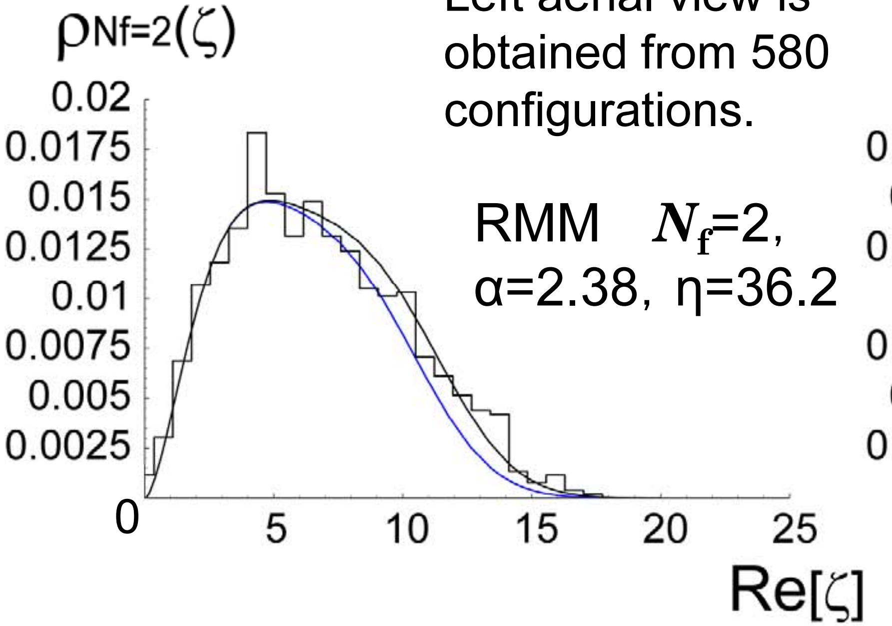


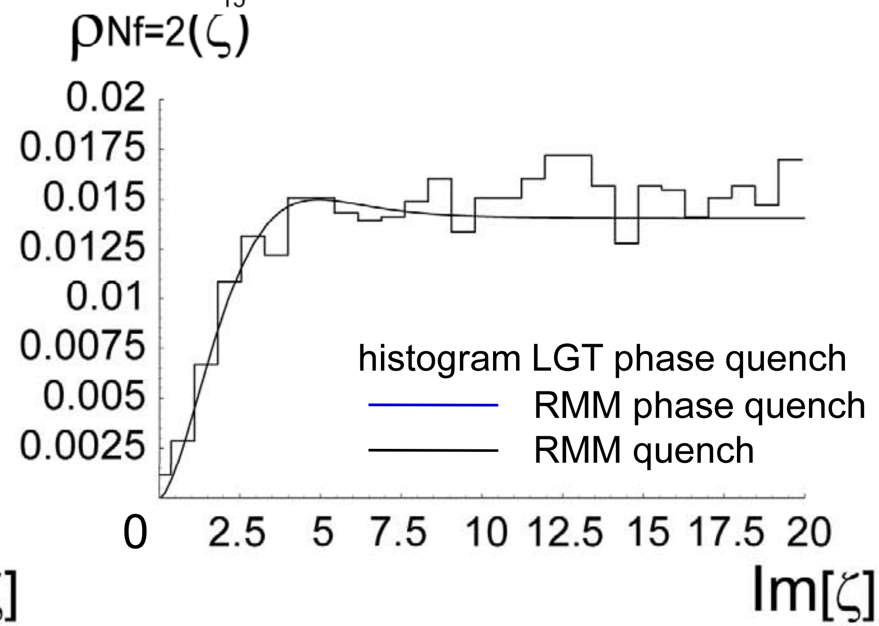




Left aerial view is obtained from 580 configurations.

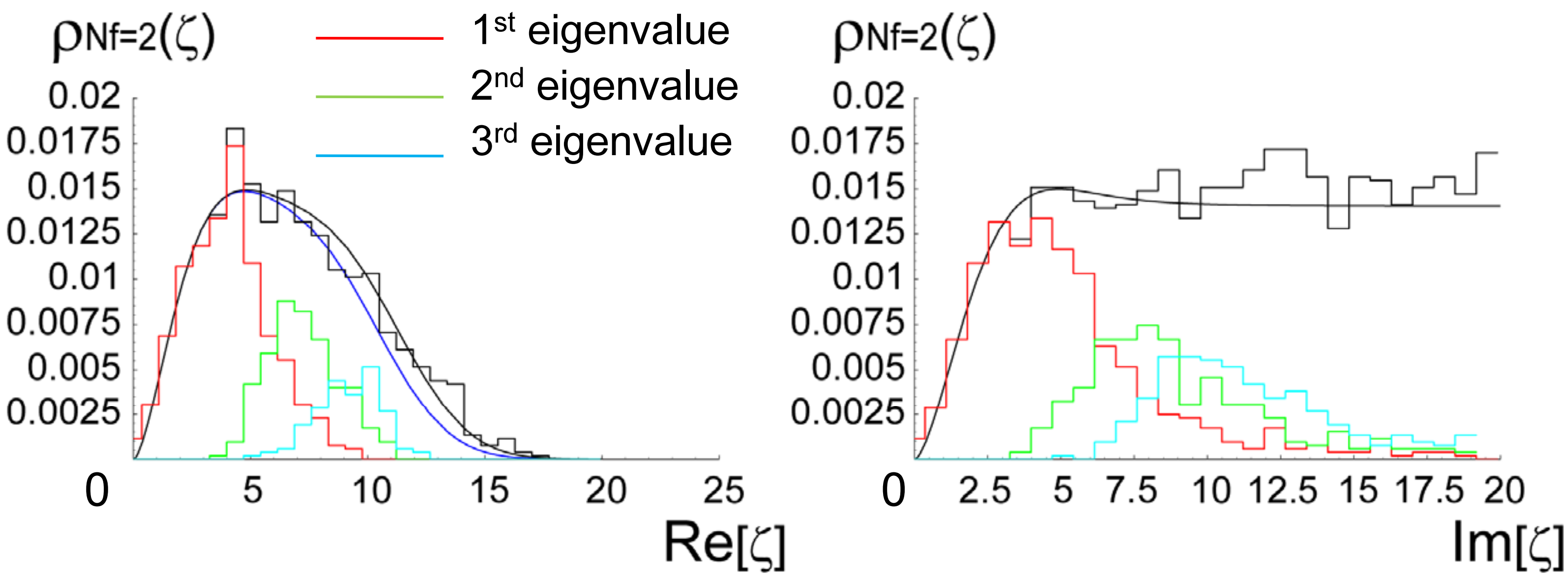
RMM N_f=2, α=2.38, η=36.2



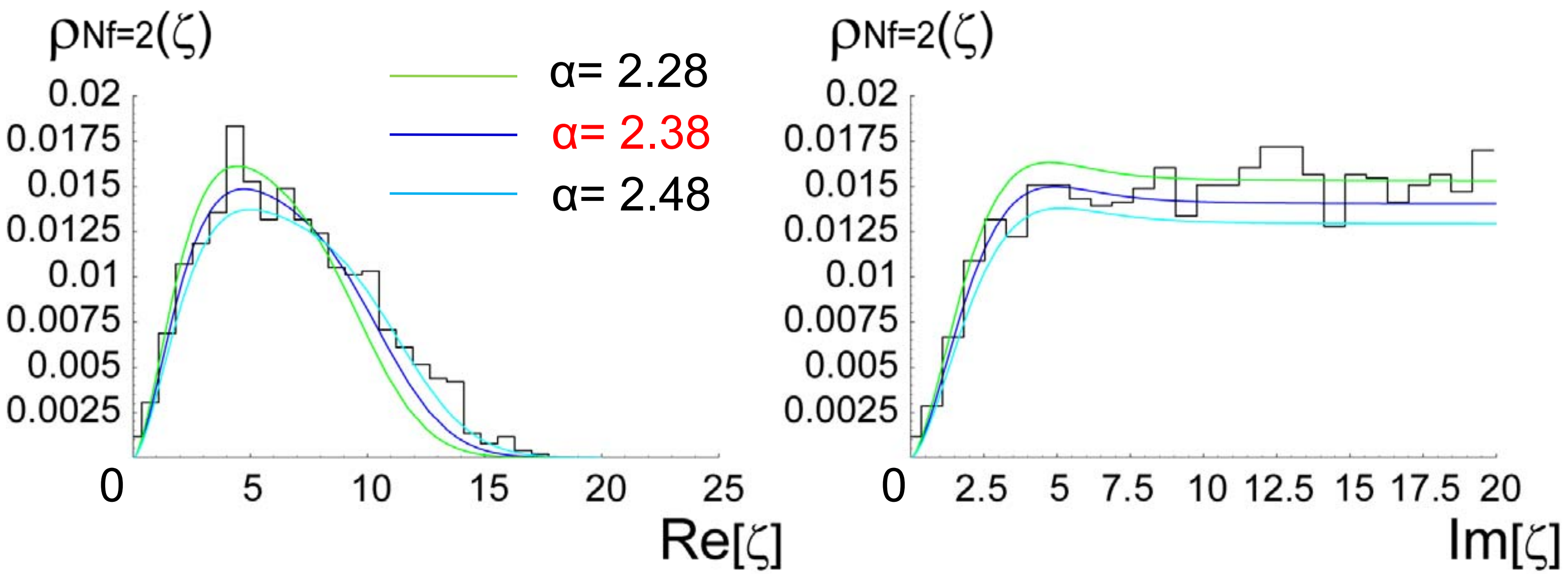


- There is phase effect at μa=0.2. It seems that statistics are still insufficient in order to know whether the phase quenched graph of LGT corresponds to the same graph of RMM

Distribution of the first 3 eigenvalues in LGT



Tuning of parameter α



4. Pion decay constant F_π

$$\alpha / \mu a = F_\pi / \sqrt{V}$$

$\beta = 5.30$

μa	α_{fit}	$\alpha_{fit} / \mu a$
0.0 confinement	none	none
0.004773 ($\beta < \beta_C$= 5.3197(9)) confinement	0.08	16.8
0.1 ($\beta < \beta_C$=5.314(1)) confinement	1.68	16.8
0.2 ($\beta > \beta_C$=5.298(2)) deconfinement	2.38	11.9

- β_C is from Kogut and Sinclear (2004).
- It seems that F_π on β_C or in deconfinement phase is smaller than F_π in confinement phase.

5. Summary

A) We have the phase quenched configurations that calculated on $8^3 \times 4$ lattice. To analyze the distributions of the eigenvalues, we compared the distributions with RMM calculations.

B) In case of μa=0.00, we have the full QCD configurations that are N_f=2, ma=0.05. There is no free parameter. The first three peaks of LGT quench are very well in agreement with the one of RMM quench.

C) In case of μa=0.004773, 0.1, 0.2, it is possible to fit the RMM graph to the LGT one by tuning only α parameter.

E) We estimated the variations of F_π at $\mu \boldsymbol{a}$=0.004773, 0.1, 0.2, it seems that F_π at $\mu \boldsymbol{a}$=0.004773, 0.1(confinement phase) is larger than F_π at $\mu \boldsymbol{a}$=0.2 (β > ~ β_C, almost on β_C or deconfinement phase).

F) In future work, we try to estimate of the variations of F_π at $\mu \boldsymbol{a}$=0.17 at which β is a little smaller than β_C.

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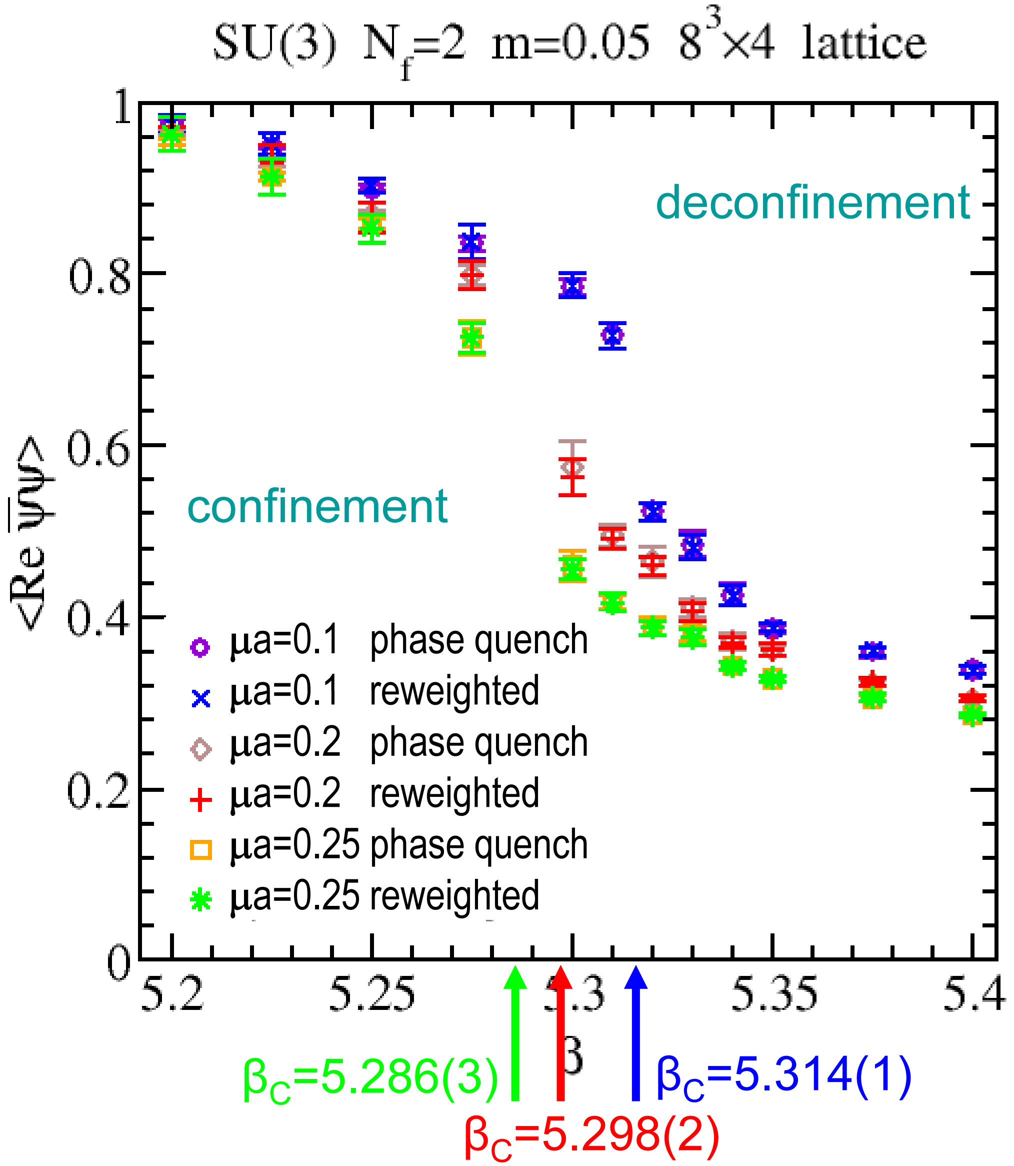
Neutron star

Backup slides

Chiral condensate $<\bar{\psi}\psi> = \frac{1}{V}\frac{\partial}{\partial(2ma)}\ln Z$



The bellow graph exhibits both of no phase case and re-weighted case.

No phase : $\langle\bar{\psi}\psi\rangle$ are the averages over 4000 trajectories each trajectories.

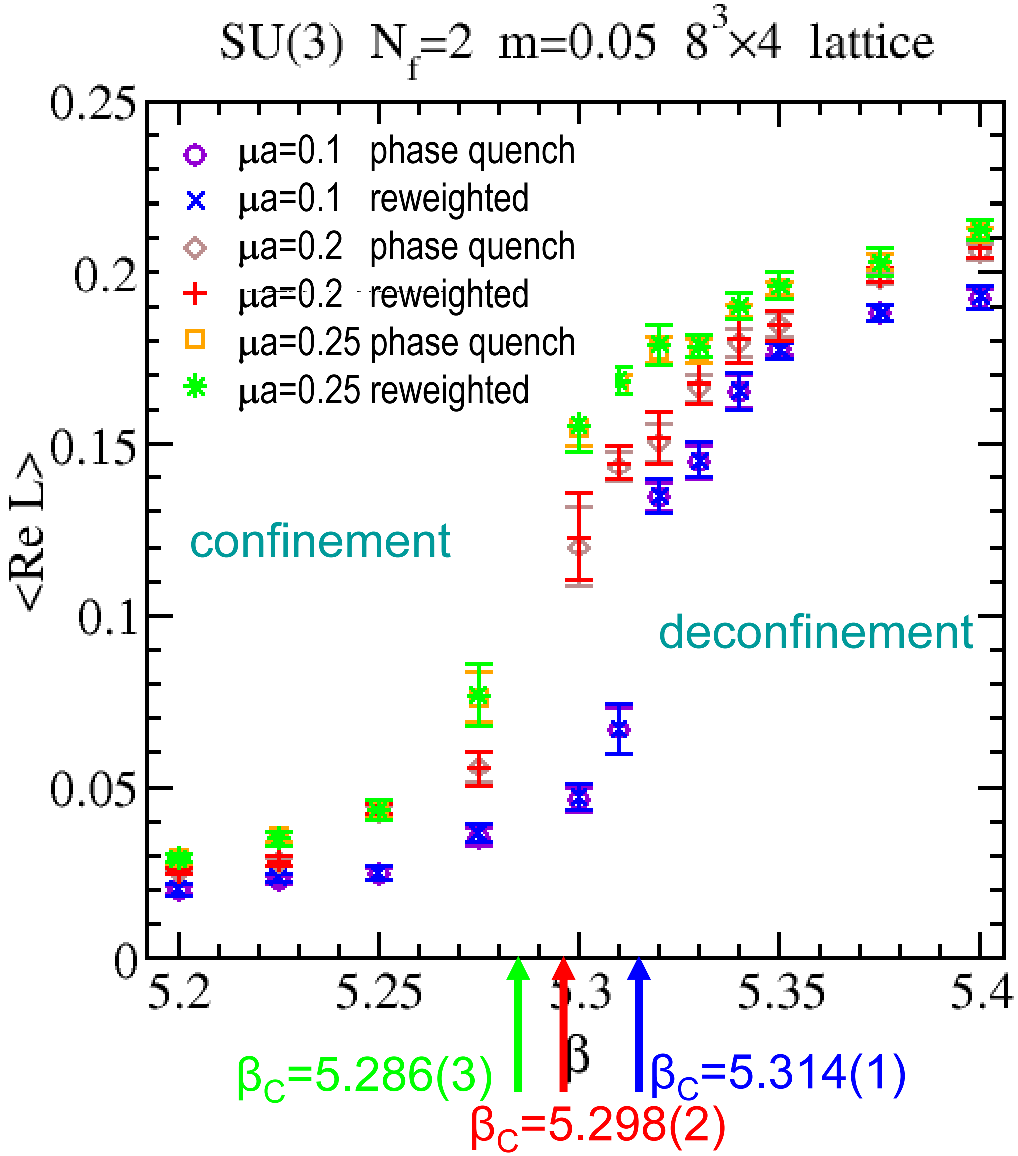
Re-weighted : det*Δ* is calculated each 10 trajectories. $\langle\bar{\psi}\psi\rangle$ are the averages over 4000 trajectories

These signs overlap mutually.

Phases of $\langle\bar{\psi}\psi\rangle$ are factorized. We can't confirm the phase effect.

Polyakov line $< L >= \frac{1}{3} Tr(U_{t1\,t2} U_{t2\,t3} \ldots U_{tn-1\,tn})$

We attempt the similar consideration to Polyakov line.



The effect of re-weighting was not seen as well as the case of Chiral condensate.

⇩

We want to examine the effect of re-weighting with more bigger μa.

At β=5.2, CG doesn't converge in the density region beyond μa=2.8.

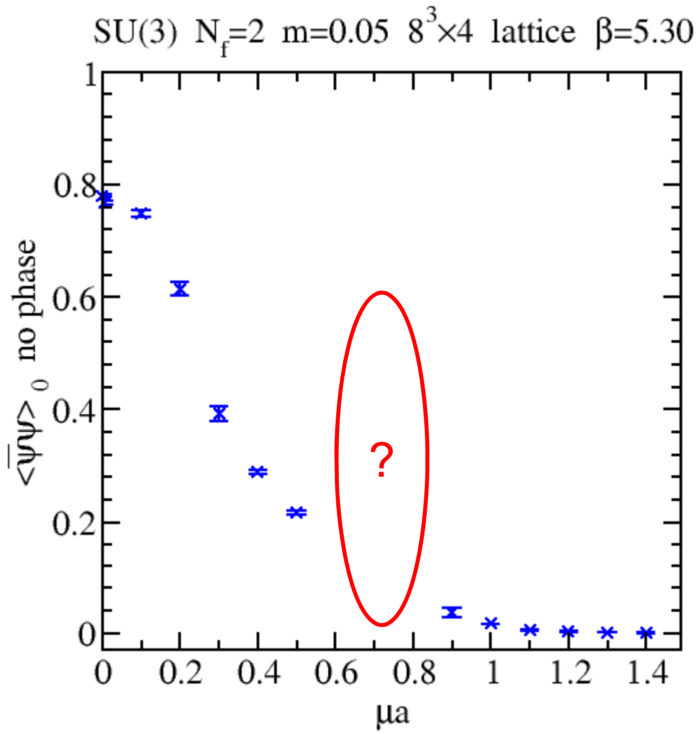
⇩

Does CG work well in the high density region (almost μa=1.2) ?

Phase Quenched

Chiral condensate

$$<\bar{\psi}\psi>=\frac{1}{V}\frac{\partial}{\partial(2ma)}\ln Z$$

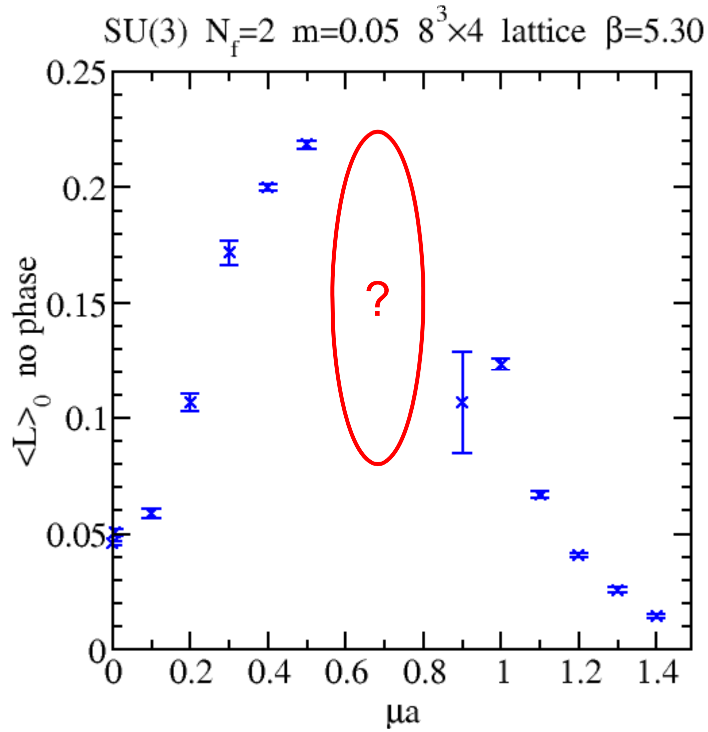


As μa increases,
chiral symmetry is restore.

$$<L>=\exp(-\frac{1}{T}\varepsilon)$$

Polyakov line

$$<L>=\frac{1}{3}Tr(U_{t1\,t2}U_{t2\,t3}\ldots U_{tn-1\;tn})$$



As μa increases,

confinement phase

⟹ deconfinement phase

⟹ confinement phase (Why?)

Chiral condensate

$$\langle \bar{\psi}\psi \rangle = -\frac{\pi \rho(0)}{V} = -\frac{\pi}{Vd} \propto \frac{1}{d}$$

μa	***d*** **measured**	$\langle \bar{\psi}\psi \rangle$ **measured**	$\langle \bar{\psi}\psi \rangle \cdot d$
0.0	2.569×10^{-3}	0.7803	2.005×10^{-3}
0.004773	2.661×10^{-3}	0.7681	2.044×10^{-3}
0.1	2.775×10^{-3}	0.7484	2.077×10^{-3}
0.2	4.341×10^{-3}	0.6146	2.668×10^{-3}

Lattice calculation

Formulation

QCD Lagrangian

$$L = \bar{\psi}\left(i\gamma_\mu D^\mu - m_f\right)\psi + \frac{1}{2}F^a_{\mu\nu}F^{\mu\nu}_a$$

N_f : flavors

Baryon number operator

$$\hat{N} = \int d^3x\,\bar{\psi}\gamma_4\psi$$

Partition function

$$Z = \int DU\,D\bar{\psi}\,D\psi\,\exp[-\int_0^{1/T} d\tau \int d^3x(L + \mu\bar{\psi}\gamma_4\psi)]$$

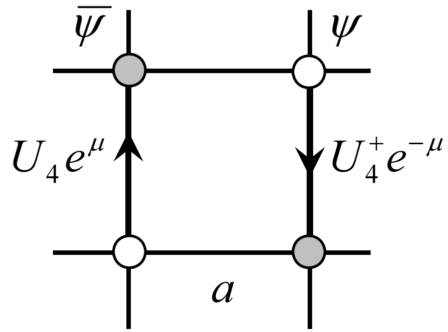
$$= \int DU\,(\det\Delta)^{N_f/4} e^{-S_g}$$

S_g: gauge action

Fermion matrix (Kogut-Susskind (Staggered))

$$\Delta(x,y) = m\delta_{x,y} + \frac{1}{2}\sum_{i=1}^{3}(-1)^{x_1+\cdots+x_{i-1}}\{U_i(x)\delta_{x+\hat{i},y} - U_i^+(y)\delta_{x,y+\hat{i}}\}$$

$$+\frac{1}{2}(-1)^{x_1+x_2+x_3}\{\underline{e^{\mu a}}U_4(x)\delta_{x+\hat{4},y} - \underline{e^{-\mu a}}U_4^+(y)\delta_{x,y+\hat{4}}\}$$



a : lattice spacing

Re-weighting method

$$\langle O \rangle = \frac{1}{Z}\int DU\,(\det \Delta)^{1/2}\,Oe^{-\beta S_g} = \frac{\int DU\,|\det \Delta|^{1/2} e^{i\theta/2} O e^{-\beta S_g}}{\int DU\,|\det \Delta|^{1/2} e^{i\theta/2} e^{-\beta S_g}}$$

$$= \frac{\int DU\,|\det \Delta|^{1/2} e^{i\theta/2} O e^{-\beta S_g}}{\int DU\,|\det \Delta|^{1/2} e^{-\beta S_g}} \Big/ \frac{\int DU\,|\det \Delta|^{1/2} e^{i\theta/2} e^{-\beta S_g}}{\int DU\,|\det \Delta|^{1/2} e^{-\beta S_g}}$$

$$= \frac{\langle O\, e^{i\theta/2} \rangle_0}{\langle e^{i\theta/2} \rangle_0}$$

μα=0.00

Spectral density of RMM

$$\rho^{(N_f=2)}(\xi) = \rho^{(N_f=0)}(\xi)\left(1 - \frac{\left|K_s(\xi,\eta^*)\right|^2}{K_s(\eta,\eta^*)K_s(\xi,\xi^*)}\right)$$

$$\alpha^2 = \mu^2 F_\pi^2 V$$

quench density For α<<1.0 $K_\nu(x) \approx \sqrt{\pi/2x}\,\exp(-x)$

$$\rho^{(N_f=0)}(\xi) = \frac{1}{4\pi\alpha^2}|\xi|^2 K_0\left(\frac{|\xi|^2}{4\alpha^2}\right) \mathrm{e}^{-\frac{1}{4\alpha^2}\mathrm{Re}(\xi^2)} K_s\left(\xi,\xi^*\right) \qquad \xi = x + iy$$

$$\approx \frac{1}{4\pi\alpha^2}|\xi|^2 \sqrt{\frac{\pi}{2|\xi|^2/4\alpha^2}}\, \mathrm{e}^{-\frac{1}{4\alpha^2}|\xi|^2}\, \mathrm{e}^{-\frac{1}{4\alpha^2}\mathrm{Re}(\xi^2)} \int_0^1 dt\, e^{-2\alpha^2 t} I_0\left(\xi\sqrt{t}\right) I_0\left(\xi^*\sqrt{t}\right)$$

$$\underset{x \to 0.0}{=} \frac{1}{\sqrt{2\pi}\alpha}\, \mathrm{e}^{-\frac{1}{2\alpha^2}x^2} \times \frac{y}{2}\int_0^1 dt\, e^{-2\alpha'^2 t} I_0\left(\xi\sqrt{t}\right) I_0\left(\xi^*\sqrt{t}\right)$$

$$\underset{\alpha \to 0.0}{\to} \delta(x) \times \frac{y}{2}\int_0^1 dt\, e^{-2\alpha'^2 t} I_0\left(\xi\sqrt{t}\right) I_0\left(\xi^*\sqrt{t}\right)$$