The electric dipole moment of the nucleon from lattice QCD with imaginary vacuum angle theta

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for the theta collaboration

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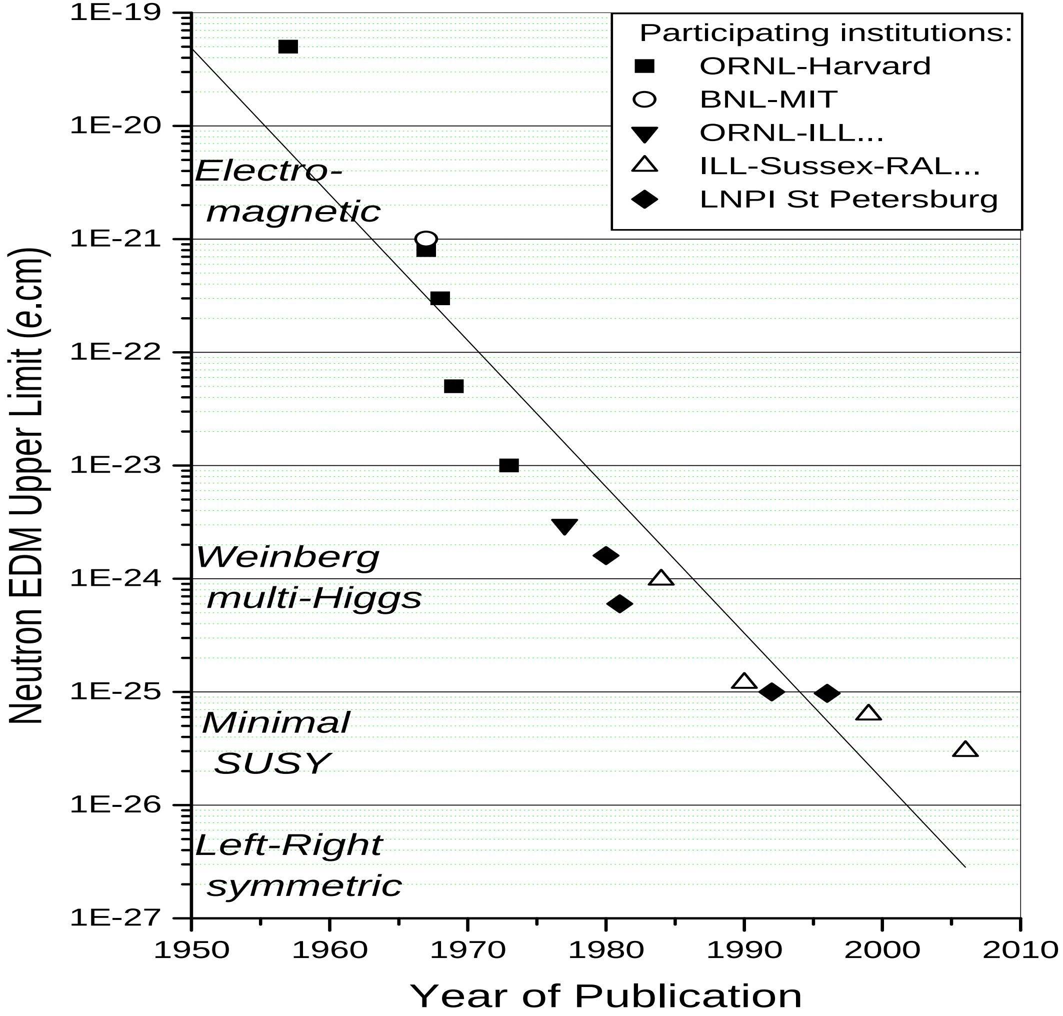
Electric Dipole Moment(EDM)

- Permanent EDM is a signature of T(Time reversal symmetry, $=$CP) violation
- Various candidates of CP violations
  - Electro Weak: CKM phase in quark mass matrix
    very small
  - New Physics: SUSY, left-right, multi Higgs
  - Vacuum angle θ
    In QCD gauge invariant CP odd terms are allowed

$$S_\theta = i\theta \frac{1}{32\pi^2} \int d^4x F_{\mu\nu}(x) \tilde{F}_{\mu\nu}(x) = i\theta Q$$

Neutron EDM

Since 50 years ago



$$|d_n| < 2.9 \times 10^{-13} e \times \text{fm}$$

Baker et al. (2007)

Harris (2007)

NEDM on lattice

89 Aoki-Gocksch	N_f=0 Wilson	Electric Field
04 Berruto et al.	N_f=0 DWF	F_3
05 Berruto et al.	N_f=2 DWF	F_3
06 CP-PACS	N_f=0 DWF	F_3
06 CP-PACS	N_f=0 DWF	Electric Field
06 QCDSF	N_f=0 overlap	F_3
06 Blum, Izubuchi, Doi	N_f=2 DWF	Electric Field
08 CP-PACS	N_f=2 Clover	Electric Field

- 2 methods(so far)
  - External Electric Field
    Measure a spin splitting of energy
  - Electric form factor F_3 $\Longleftarrow$ in this talk
    Measure 3pt function with momenta p, and $p \to 0$
- Dynamical simulations are important
  NEDM is very sensitive to sea quark mass, $d_n = 0$ in the chiral limit
- Reweighting method have been used $\Longleftarrow$ noisy (needs large statistics)
  In usual QCD simulations, θ=0
  In the real world θ is real
  But one can not do Full QCD HMC simulations with real θ

LQCD with imaginary θ

Motivation

- To calculate d_N/θ

  In lattice QCD, θ is one of the input parameters

  d_N/θ from lattice
  $|d_n|$ from ex.
  $\rightarrow |\theta| <?$

- To check feasibility of lattice QCD simulations at imaginary θ

Simulations with imaginary θ

There are 2 choices

- gluonic: $-\theta F\tilde{F}$

  needed smearing/cooling

- fermionic: $m\theta\bar{\psi}\gamma_5\psi$

  by using anomalous chiral WT relation

rotation by θ

$$m \longrightarrow m\mathrm{e}^{i\frac{\theta}{N_f}\gamma_5}$$

Action with θ

We choose fermionic way

$$S_F + S_\theta = \bar{\psi}\,\{D + [\cos(\theta/N_f) + i\,\sin(\theta/N_f)\,\gamma_5]\,m\}\,\psi$$

$$\begin{aligned}\bar{m} &= \cos(\theta/N_f)\,m \\ \bar{\theta} &= \tan(\theta/N_f)\,N_f\end{aligned}$$

$$S_F = \bar{\psi}\,\left\{D + \bar{m} + i\,(\bar{\theta}/N_f)\,\gamma_5\,\bar{m}\right\}\psi$$

Lattice action

$$S_F = \bar{\psi}\,\{D + \bar{m} + i\,(\theta_R/N_f)\,Z_m Z_P\,\gamma_5\,\bar{m}\}\,\psi$$

Z_m: the renormalization constant of the quark mass
Z_P : the renormalization constant the pseudoscalar density
θ_R: renormalized vacuum angle

$$\theta_R = (Z_m Z_P)^{-1}\;\bar{\theta}$$

<u>chiral fermion:</u> nicer, definitely, extremely expensive
$Z_m Z_P = 1$ and $\theta_R = \bar{\theta}$
<u>clover fermion:</u> relatively cheap
$Z_m Z_P = 1$ and $\theta_R = \bar{\theta}$ in the continuum limit

We employ N_f=2 flavors of dynamical clover fermions
(chiral symmetry is violated)

$$a(D+\bar{m}) \longrightarrow D^{lat} = D^{Wilson} + T^{clover}$$

$$a\bar{m} \longrightarrow \frac{1}{2\kappa} - \frac{1}{2\kappa_c} \qquad \text{VWI quark}$$

$$D^{Wilson}_{x,y} = \delta_{x,y} - \kappa \sum_{\mu} \left\{ (1-\gamma_\mu) U_\mu(x) \delta_{x+\mu,y} + (1+\gamma_\mu) U^\dagger_\mu(x-\mu) \delta_{x-\mu,y} \right\}$$

$$T^{clover}_{x,y} = \left\{ \frac{i}{2} c_{SW} \kappa \sigma_{\mu\nu} F_{\mu\nu}(x) + i(1 - \frac{\kappa}{\kappa_c}) \frac{\theta_R}{2} Z_m Z_P \gamma_5 \right\} \delta_{x,y}$$

In simulations with dynamical quarks, $\gamma_5 D \gamma_5 = D^\dagger$ is required
The vacuum angle $\bar{\theta}$ is taken to be purely imaginary

$$\bar{\theta} = -i\,|\bar{\theta}| \equiv -i\,\bar{\theta}^{\,I}$$

$$\Downarrow$$

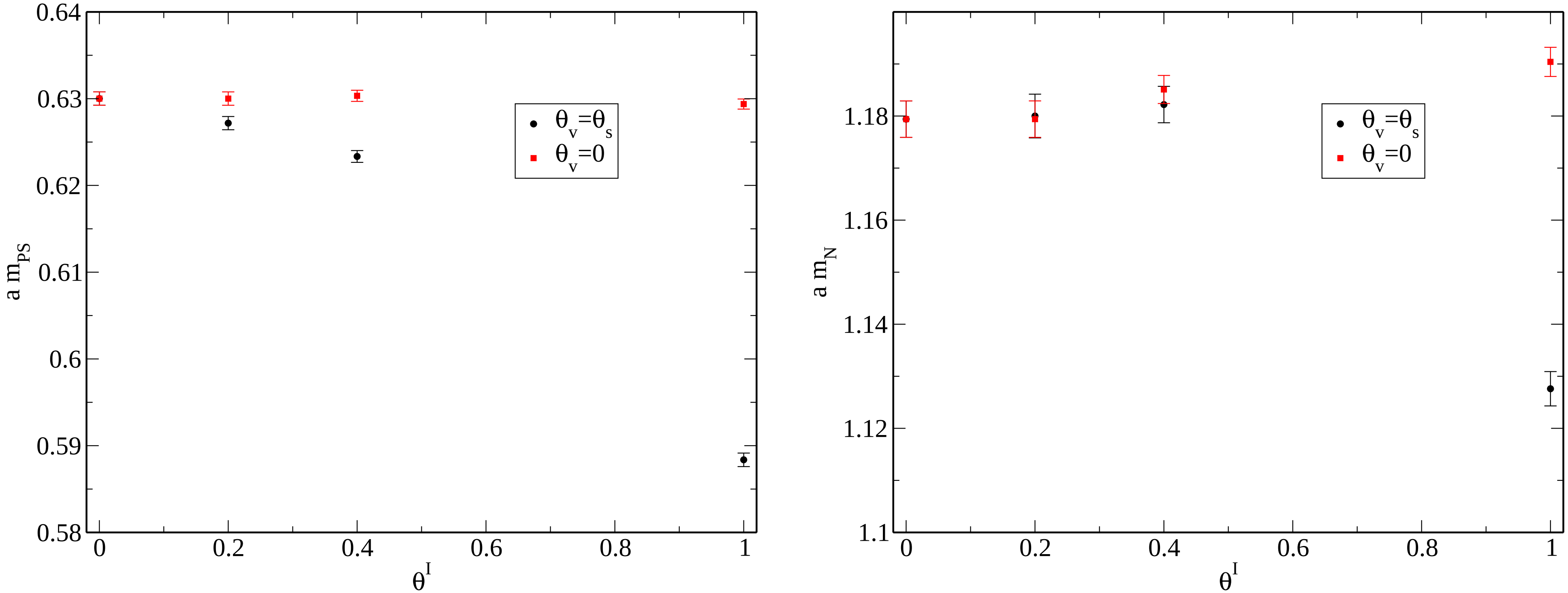
The Boltzmann weight positive definite

Simulation

Lattice Size: $16^3 \times 32$
β: 2.1(Iwasaki)
κ: 0.1357
κ_c: 0.138984
Lattice Spacing: a=0.1076(13) fm
am_π: 0.6309(8)

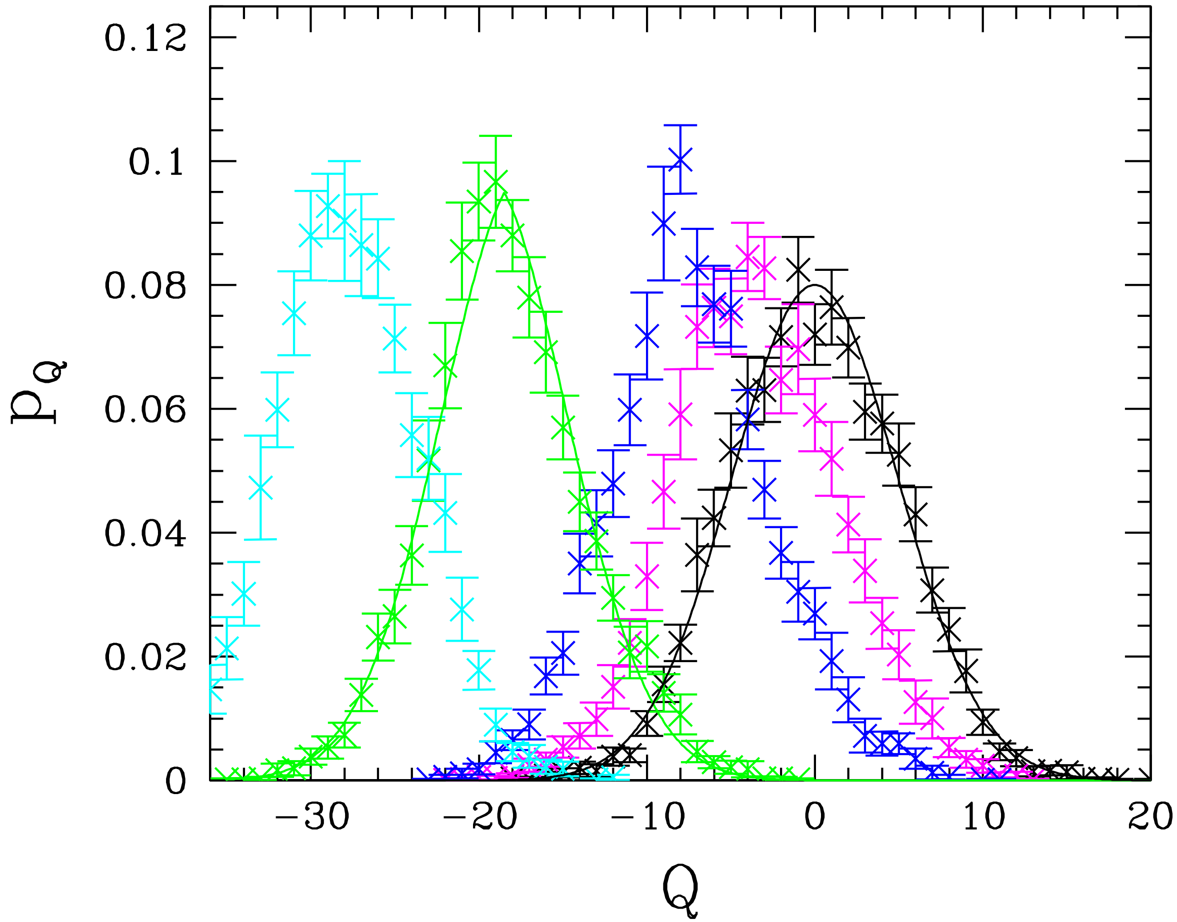
same action as CP-PACS

$\bar{\theta}^I$	# of traj.	$\langle P \rangle$	τ_{int}^P	$\langle Q \rangle$	$\langle Q^2 \rangle - \langle Q \rangle^2$	$\mathrm{e}^{-\Delta H}$
0.0	9000	0.598059(13)	3.24(68)	-0.06(31)	24.9(14)	1.0008(25)
0.2	9000	0.598045(11)	3.30(73)	-3.52(46)	24.1(15)	1.0029(35)
0.4	7000	0.598045(17)	3.02(41)	-7.35(36)	22.7(17)	1.0001(37)
1.0	6000	0.598078(16)	4.1(19)	-18.38(30)	21.7(15)	1.0001(14)
1.5	6000	0.598081(16)	2.56(61)	-27.84(37)	18.1(13)	0.9950(28)



a m_PS
0.58
0.59
0.6
0.61
0.62
0.63
0.64
0
0.2
0.4
0.6
0.8
1
θ^I
θ_v=θ_s
θ_v=0
a m_N
1.1
1.12
1.14
1.16
1.18
0
0.2
0.4
0.6
0.8
1
θ^I
θ_v=θ_s
θ_v=0

Topological charge distribution



$P(Q)$ is changed by $\bar{\theta}^I$

Effective value of θ

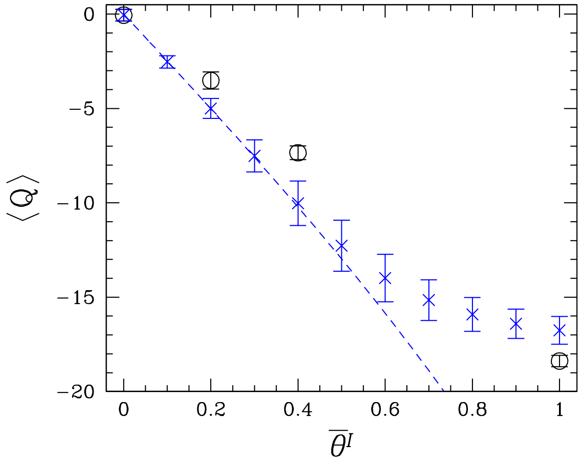
$$\theta^{\text{input}} = (1 - \frac{\kappa}{\kappa_c^\theta})\bar{\theta}_R^I Z_m^\theta Z_P^\theta$$

ex.

Input parameter $\theta^{\text{input}} = 0.00472572$

If $\kappa_c^\theta = 0.138984$ and $Z_m^\theta Z_P^\theta = 1 \quad \rightarrow \quad \bar{\theta}_R^I = 0.4$

- We could define renormalized θ by $Q(\theta)$



$$\theta_R^I \sim \theta^I \times 0.75$$

- One could also check the effect of θ in other hadronic observable

$$m_\pi(\theta) = m_\pi(0) \cos(i\theta/N_f)?$$

Brower et al. (2003), Aoki et al. (2007)

Nucleon form factors

The electromagnetic current between nucleon states

$$\langle p', s' | J_\mu | p, s \rangle = \bar{u}_\theta(\vec{p}\,', s')\, \mathcal{J}_\mu\, u_\theta(\vec{p}, s)$$

$$\begin{aligned}\mathcal{J}_\mu &= \gamma_\mu F_1^\theta(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2^\theta(q^2)}{2m_N^\theta} \\ &+ i\,\theta \left[(\gamma q\, q_\mu - \gamma_\mu\, q^2)\, \gamma_5\, F_A^\theta(q^2) + \sigma_{\mu\nu} q_\nu\, \gamma_5 \frac{F_3^\theta(q^2)}{2m_N^\theta} \right]\end{aligned}$$

$q = p' - p$, $q^2 = -(E' - E)^2 + (\vec{p}\,' - \vec{p})^2$, $\gamma p = i E \gamma_4 + \vec{\gamma}\vec{p}$

Electric dipole moment

$$d_N^\theta = \lim_{q^2 \to 0} \frac{e F_3^\theta(q^2)}{2m_N^\theta}$$

The Dirac spinors are modified by a phase in the θ vacuum

$$\begin{aligned}u_\theta(\vec{p}, s) &= \mathrm{e}^{i\alpha(\theta)\gamma_5}\, u(\vec{p}, s) \\ \bar{u}_\theta(\vec{p}, s) &= \bar{u}(\vec{p}, s)\, \mathrm{e}^{i\alpha(\theta)\gamma_5}\end{aligned}$$

Spinor relation is modified to

$$\sum_{s',s} u_\theta(\vec{p},s')\bar{u}_\theta(\vec{p},s) = \mathrm{e}^{i\alpha(\theta)\gamma_5}\left(\frac{-i\gamma p + m_N^\theta}{2E_N^\theta}\right)\mathrm{e}^{i\alpha(\theta)\gamma_5}$$

The lowest order in θ

$$\sum_{s',s} u_\theta(\vec{p},s')\bar{u}_\theta(\vec{p},s) = \frac{-i\gamma p + m_N(1-2\alpha'\bar{\theta}^I\gamma_5)}{2E_N}.$$

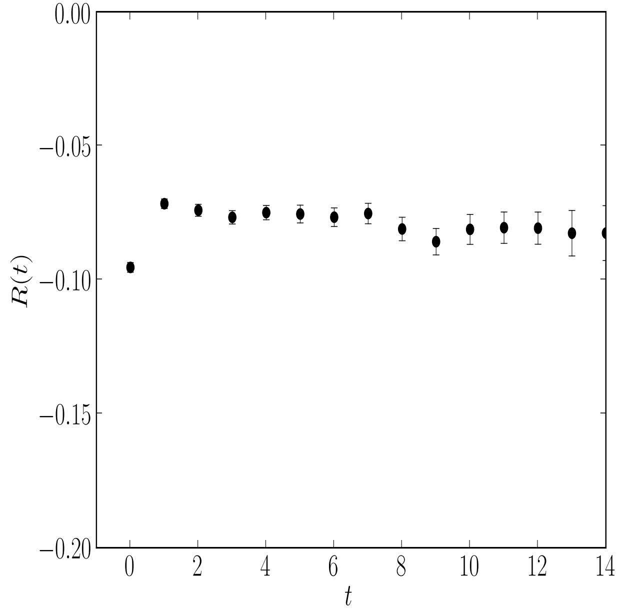
We are primarily interested in the electric dipole moment in the limit $\theta \longrightarrow 0$, it is sufficient to consider the lowest order expansion only

d_n for $\bar{\theta}^I \leq 0.4$ in this work

$$\mathrm{Tr}\,[G^{\theta}_{NN}(t;0)\Gamma_4] \simeq \frac{1}{2}|Z_N|^2\,\mathrm{e}^{-m_N t},$$
$$\mathrm{Tr}\,[G^{\theta}_{NN}(t;0)\Gamma_4\gamma_5] \simeq -\alpha'\bar{\theta}^{\,I}\,\frac{1}{2}|Z_N|^2\,\mathrm{e}^{-m_N t},$$

$$\Gamma_4 = (1+\gamma_4)/2$$

taking ratio $R(t) = \frac{\mathrm{Tr}\,[G^{\theta}_{NN}(t,0)\Gamma_4\gamma_5]}{\mathrm{Tr}\,[G^{\theta}_{NN}(t;0)\Gamma_4]} \simeq -\alpha'\bar{\theta}^{\,I}$



$\bar{\theta}^{\,I} = 0.4$

Nucleon form factors

$$R_\mu(t',t;\vec{p}',\vec{p}) = \frac{G^{\theta\,\Gamma}_{NJ_\mu N}(t',t;\vec{p}',\vec{p})}{\mathrm{Tr}\,[G^\theta_{NN}(t';\vec{p}')\Gamma_4]}$$

$$\times \left\{ \frac{\mathrm{Tr}\,[G^\theta_{NN}(t;\vec{p}')\Gamma_4]\,\mathrm{Tr}\,[G^\theta_{NN}(t';\vec{p}')\Gamma_4]\,\mathrm{Tr}\,[G^\theta_{NN}(t'-t;\vec{p})\Gamma_4]}{\mathrm{Tr}\,[G^\theta_{NN}(t;\vec{p})\Gamma_4]\,\mathrm{Tr}\,[G^\theta_{NN}(t';\vec{p})\Gamma_4]\,\mathrm{Tr}\,[G^\theta_{NN}(t'-t;\vec{p}')\Gamma_4]} \right\}^{1/2}$$

$$= \sqrt{\frac{E^{\theta\,\prime}\,E^\theta}{(E^{\theta\,\prime}+m^\theta_N)\,(E^\theta+m^\theta_N)}}\,F(\Gamma,\mathcal{J}_\mu),$$

$$F(\Gamma,\mathcal{J}_\mu) = \frac{1}{4}\,\mathrm{Tr}\,\Gamma\left[\mathrm{e}^{i\alpha(\theta)\gamma_5}\frac{E^{\theta\,\prime}\gamma_4 - i\vec{\gamma}\vec{p}\,' + m^\theta_N}{E^{\theta\,\prime}}\,\mathrm{e}^{i\alpha(\theta)\gamma_5}\right]$$

$$\times\,\mathcal{J}_\mu\left[\mathrm{e}^{i\alpha(\theta)\gamma_5}\frac{E^\theta\gamma_4 - i\vec{\gamma}\vec{p} + m^\theta_N}{E^\theta}\,\mathrm{e}^{i\alpha(\theta)\gamma_5}\right]$$

$$\mathcal{J}_\mu = \gamma_\mu F^\theta_1(q^2) + \sigma_{\mu\nu}q_\nu\frac{F^\theta_2(q^2)}{2m^\theta_N}$$

$$+\,i\,\theta\left[(\gamma q\,q_\mu - \gamma_\mu\,q^2)\,\gamma_5\,F^\theta_A(q^2) + \sigma_{\mu\nu}q_\nu\,\gamma_5\frac{F^\theta_3(q^2)}{2m^\theta_N}\right]$$

momenta: quantized in units of $2\pi/L$

- conventional (periodic) boundary condition(BC)
  $p = \frac{2\pi}{16} \sim 0.4$ is not small $\longrightarrow$ noisier & large extrap.
- twisted boundary condision(TBC) Bedaque (2004)

$$\psi(x_k + L) = \mathrm{e}^{i\,\alpha_k}\,\psi(x_k), \quad k = 1, 2, 3.$$

the dispersion relation for the nucleon

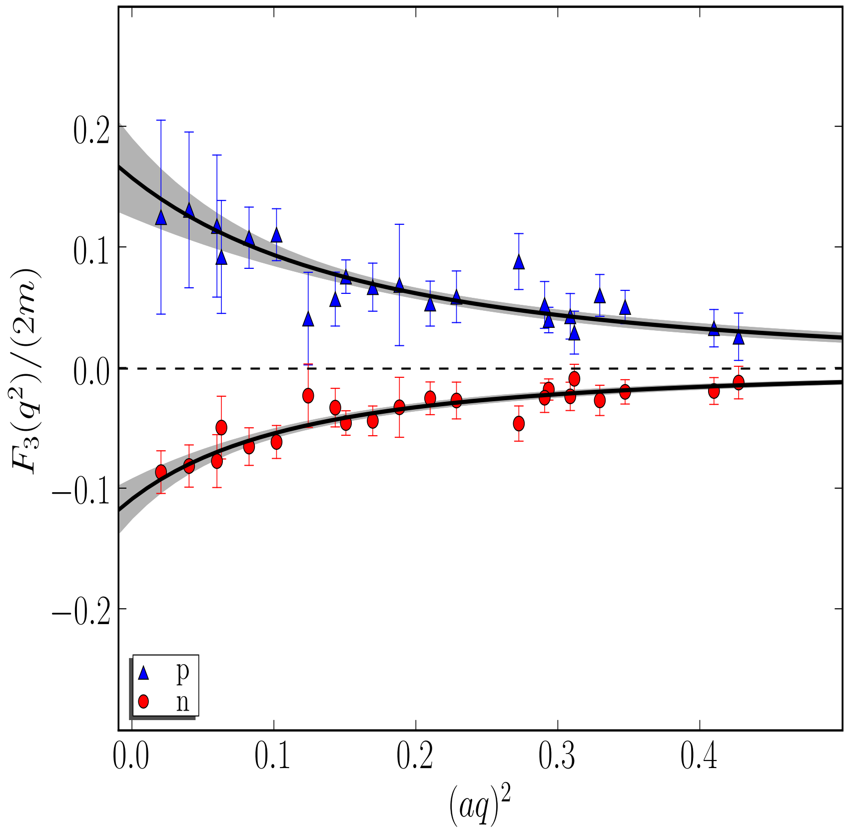
$$E = \sqrt{m_N^2 + (\vec{p} + \vec{\alpha})^2}$$

choice of twist angles

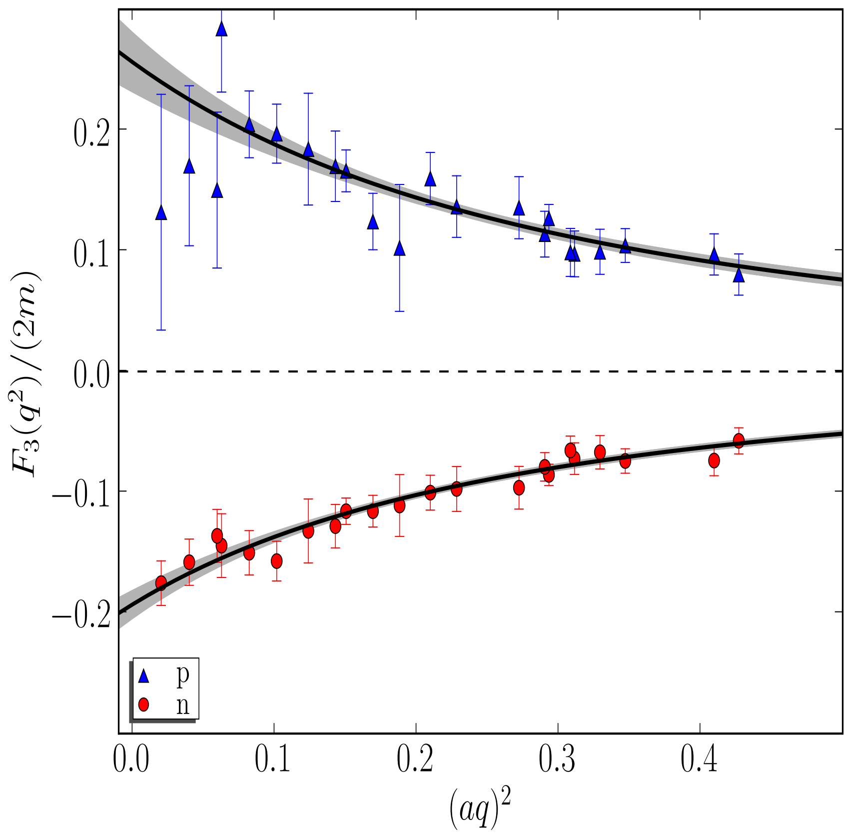
$$\vec{\alpha} = \frac{2\pi}{L}\,(0, 0, 0)$$
$$\vec{\alpha} = \frac{2\pi}{L}\,(0.36, 0, 0)$$
$$\vec{\alpha} = \frac{2\pi}{L}\,(0.36, 0.36, 0)$$
$$\vec{\alpha} = \frac{2\pi}{L}\,(0.36, 0.36, 0.36)$$

Preliminary results

$\bar{\theta}^{\,I} = 0.2$



$\bar{\theta}^{\,I} = 0.4$

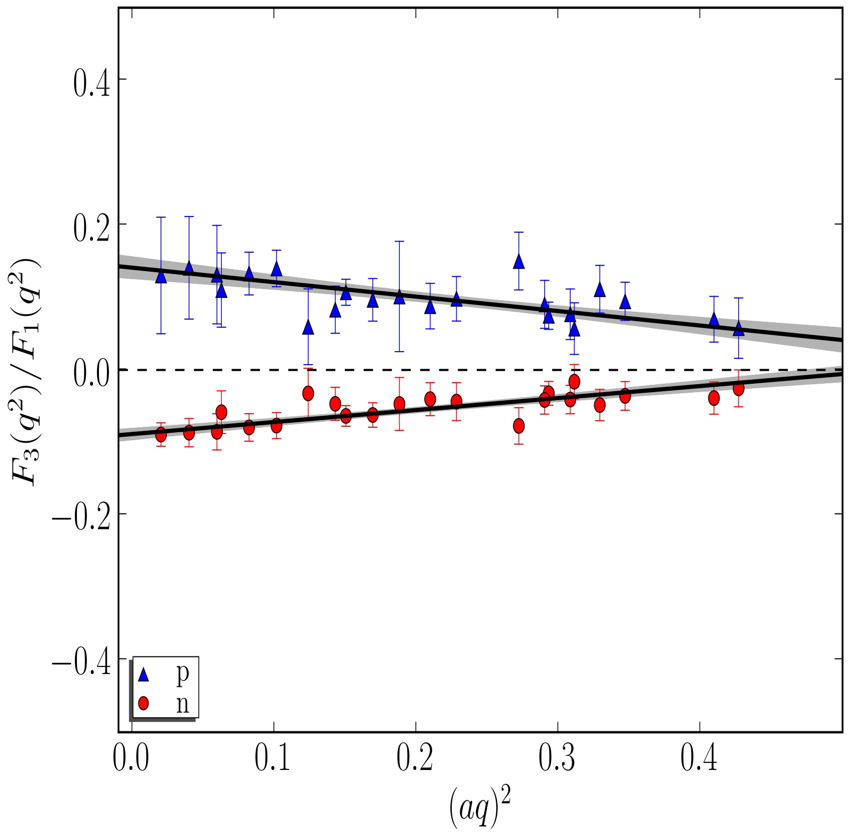


Fit using a dipole ansatz

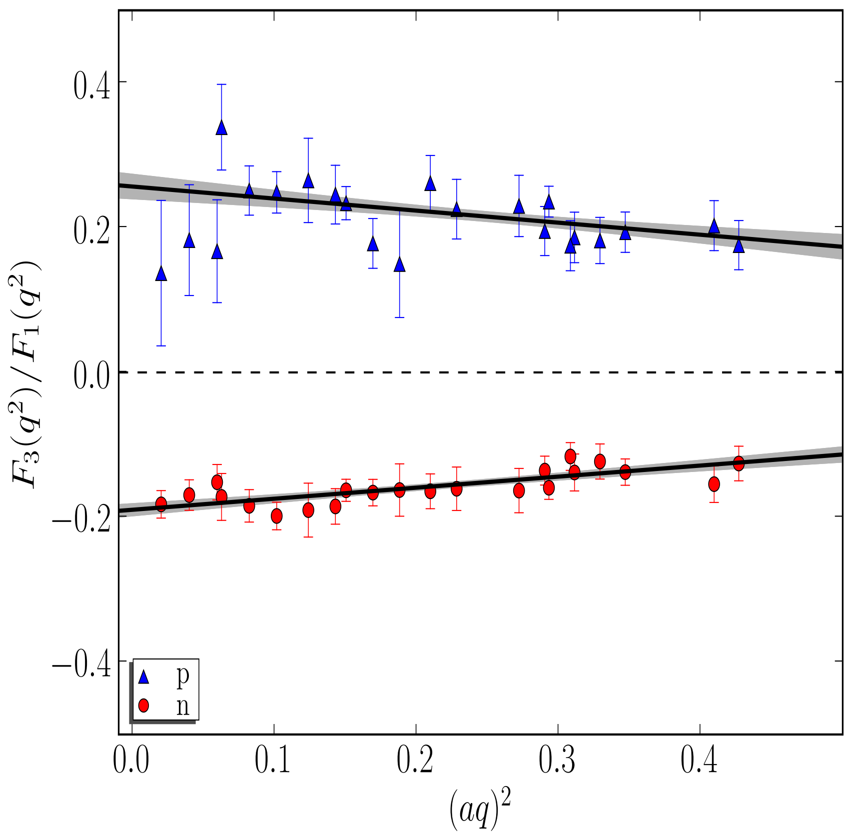
$$F_3^{\theta}(q^2) = \frac{F_3^{\theta}(0)}{(1 + q^2/M^2)^2}$$

The renormalization constant Z_V is needed

$\bar{\theta}^{\,I} = 0.2$



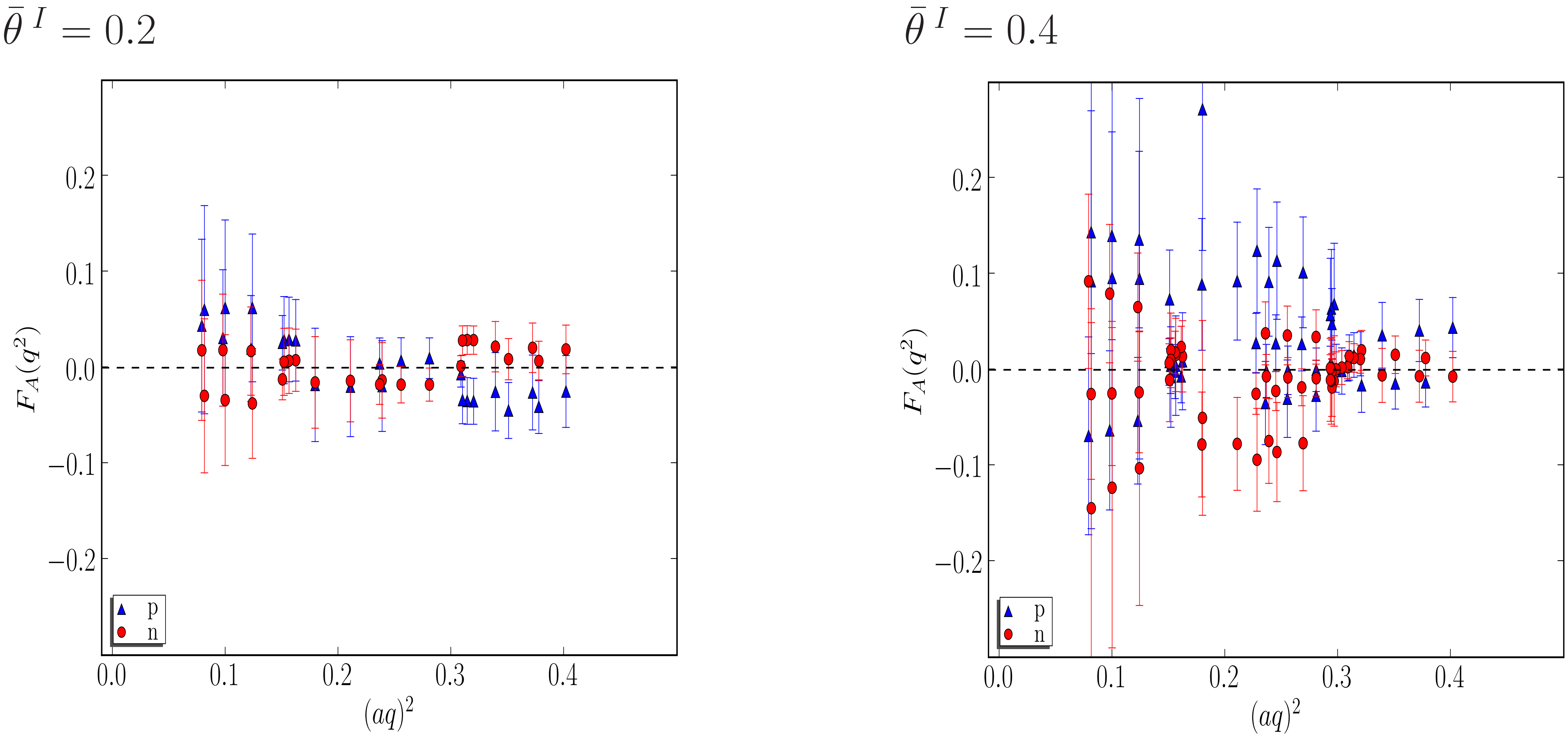
$\bar{\theta}^{\,I} = 0.4$

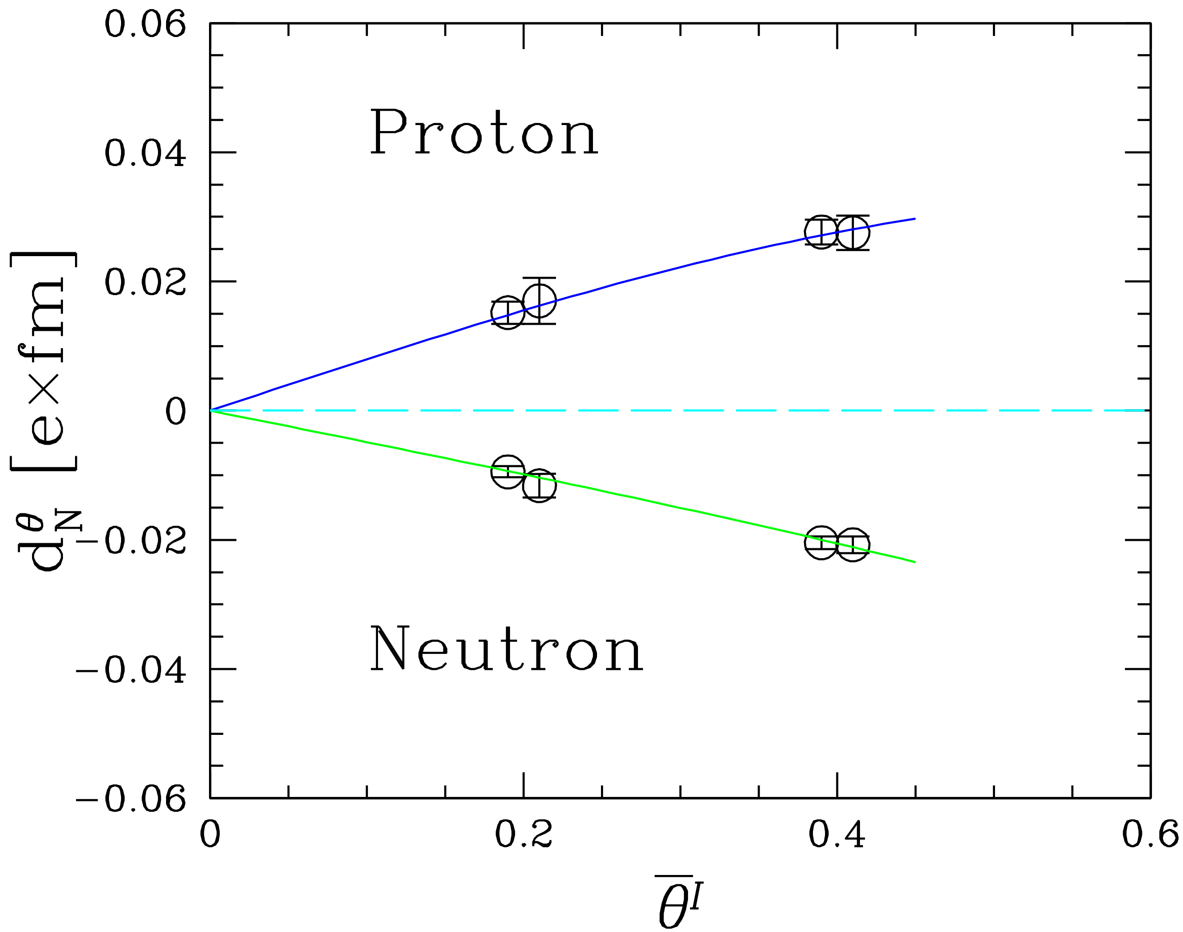


$$F_3^\theta(0) = \lim_{q^2 \to 0} \frac{F_3^\theta(q^2)}{F_1^p(q^2)}$$

Z_V cancels

Anapole form factor





fit

$$d_N^{\theta} = \frac{\partial d_N^{\theta}}{\partial \bar{\theta}^I}\,\bar{\theta}^I + c\,\bar{\theta}^{I\;3}$$

gives at $\bar{\theta}^I = 0$

$$\frac{\partial d_N^\theta}{\partial \bar{\theta}^I} = \quad 0.080(10)_{\textsf{stat+fit}}(?)_{\textsf{sys}} \quad [e \times \textsf{fm}] \quad \textsf{Proton}$$

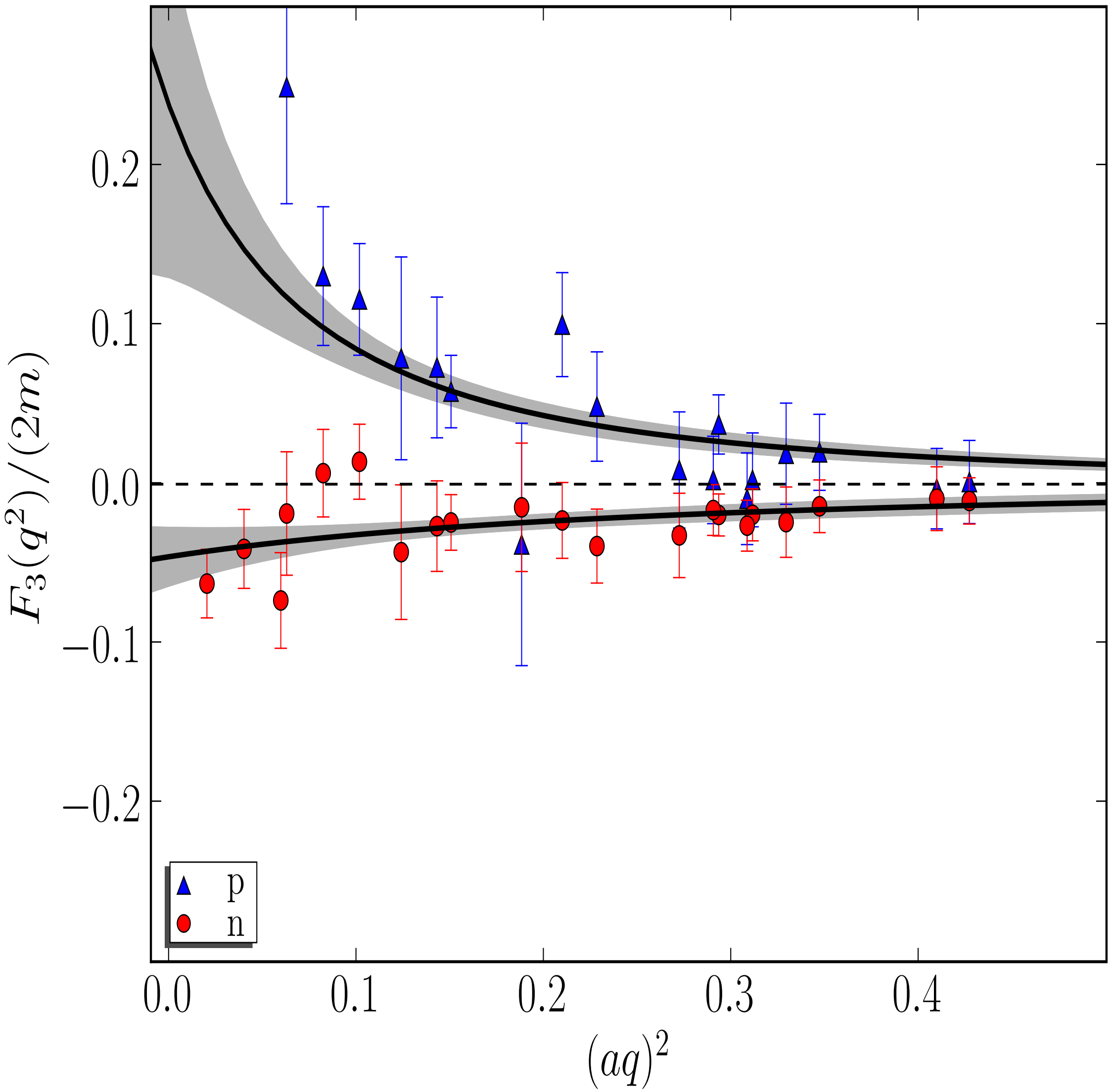
$$\frac{\partial d_N^\theta}{\partial \bar{\theta}^I} = -0.049(5)_{\textsf{stat+fit}}(?)_{\textsf{sys}} \quad [e \times \textsf{fm}] \quad \textsf{Neutron}$$

$$\Downarrow$$

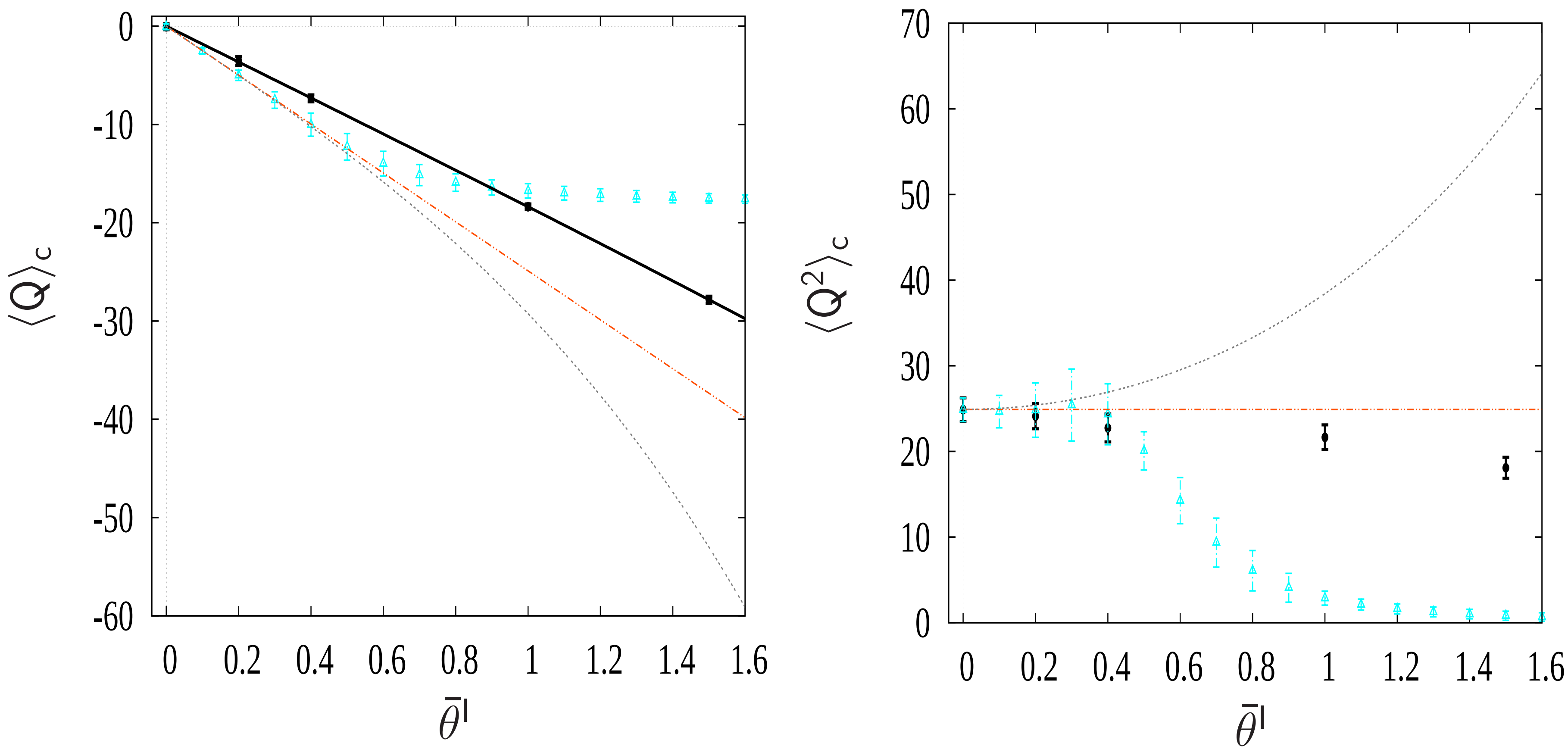
$$|\theta| < 6 \times 10^{-12}$$

preliminary

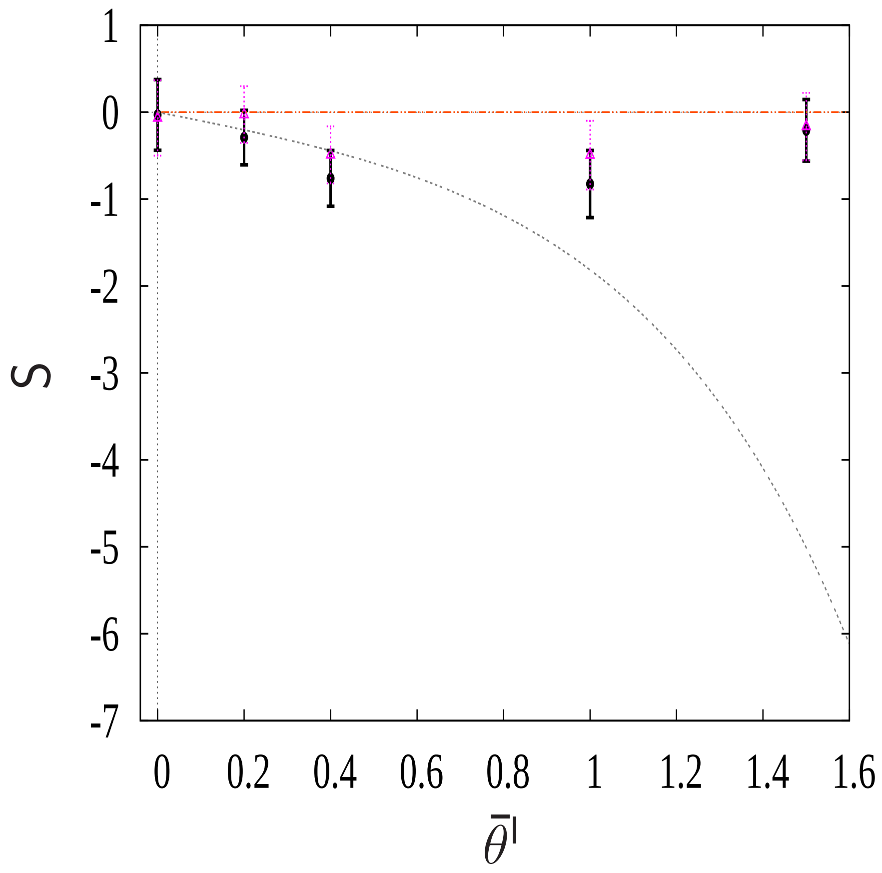
$\bar{\theta}_s^{\,I} = 0.4,\ \bar{\theta}_v^{\,I} = 0.0$

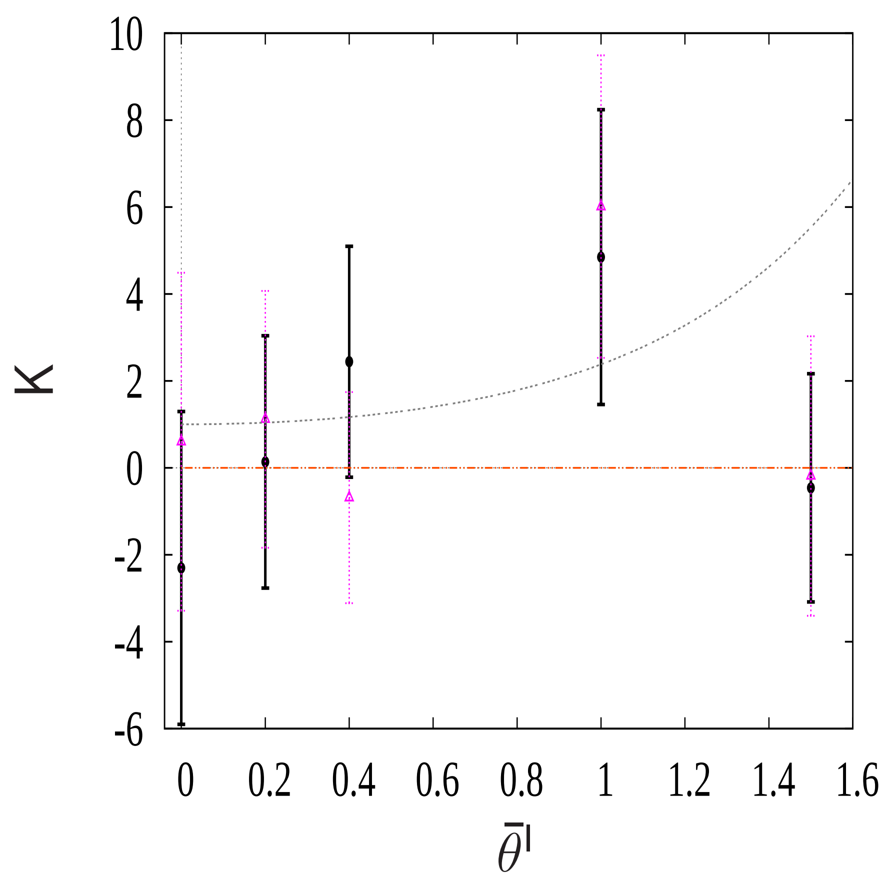


Charge distribution and θ vacuum



$$\langle Q^n \rangle_c \equiv i^n \frac{\partial^n}{\partial \theta^n} \ln Z(\theta)$$





$$S = \frac{\langle Q^3 \rangle_c}{\langle Q^2 \rangle_c}$$

$$K = \frac{\langle Q^4 \rangle_c}{\langle Q^2 \rangle_c}$$

Conclusions and future plans

- Have performed simulations of QCD with $N_f = 2$ flavors of dynamical quarks at imaginary vacuum angle θ
- The use of partially twisted boundary conditions has allowed us to compute the relevant neutron form factor $F_3(q^2)$ with high precision over the entire range of momenta down to $(aq)^2 \approx 0.02$

  $\longrightarrow$ greatly facilitated the extrapolation to $q^2 = 0$
- Successfully obtain signal disentangled from statistical noise
- Plan
  - Sea quark mass dependence (χPT)

    EDM is zero in the chiral limit

    In this work m_π is very heavy ~ 1 GeV
  - Volume dependence

    In this work $V \sim (1.7\ \text{fm})^3 \longrightarrow$ small for baryon
  - Dependence of the boundary conditions

    We used periodic BC for sea quark TBC for valence quark
  - Gluonic operator instead of the pseudoscalar operator