Quark mass renormalization with non-exceptional momenta

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RBC/UKQCD collaborations

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This talk is based on the works:

- Non-perturbative renormalization of quark bilinear operators and B_K using domain wall fermions [arXiv:0712.1061]
  - RBC and UKQCD collaborations: Y.Aoki, P.A.Boyle, N.H.Christ, C.Dawson, M.A.Donnellan, T.Izubuchi, A.Juttner, S.Li, R.D.Mawhinney, J.Noaki, C.T.Sachrajda, A.Soni, R.J.Tweedie, A.Yamaguchi
  - idea of non-exceptional momenta and a demonstration
- Quark bilinear operators renormalized in MOM-scheme for the symmetric subtraction point [in preparation]
  - C. Sturm, Y. Aoki, N. H. Christ, T. Izubuchi, and A. Soni
  - construction of a non-exceptional RI-MOM scheme for mass renormalization and 1 loop matching

quark mass

- results from N_f=2+1 domain-wall fermions (β=2.13)
  - (talk by E. Scholz, RBC/UKQCD: arXiv:0804.0473)

$$m_{ud}^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 3.71(0.16)_{\mathrm{stat}}(0.18)_{\mathrm{syst}}(0.33)_{\mathrm{ren}}\mathrm{MeV},$$

$$m_{s}^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 107.3(4.4)_{\mathrm{stat}}(4.9)_{\mathrm{syst}}(9.7)_{\mathrm{ren}}\mathrm{MeV},$$

- error from the renormalization dominates
  - systematic error in our renormalization program
- how it is obtained ?
- how we can improve ?

RI-MOM scheme

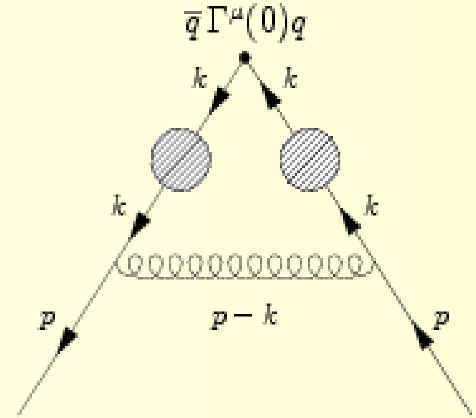
- impose renormalization condition on the vertex functions with off-shell quark states with momentum *p* at mass less limit: [Martinelli et al NPB445(95)81].
- renormalization condition on the vertex function Π of bilinear operator $O = \overline{u}\Gamma d$

$$\frac{Z_O}{Z_q}\frac{1}{12}\mathrm{Tr}(\Pi_O P_O) = 1 \qquad \text{at } p^2 = \mu^2,\ m \to 0$$

- matching to a continuum scheme ($\overline{\mathrm{MS}}$) must be done at high energy to reduce
  - truncation error of continuum perturbation theory
  - contamination of non-perturbative effect (NPE)

  $\longrightarrow$ These indeed are the main sources of the systematic error
- Window: $\Lambda_{QCD} \ll p \ll a^{-1}$

Typical NPE contamination in RI scheme



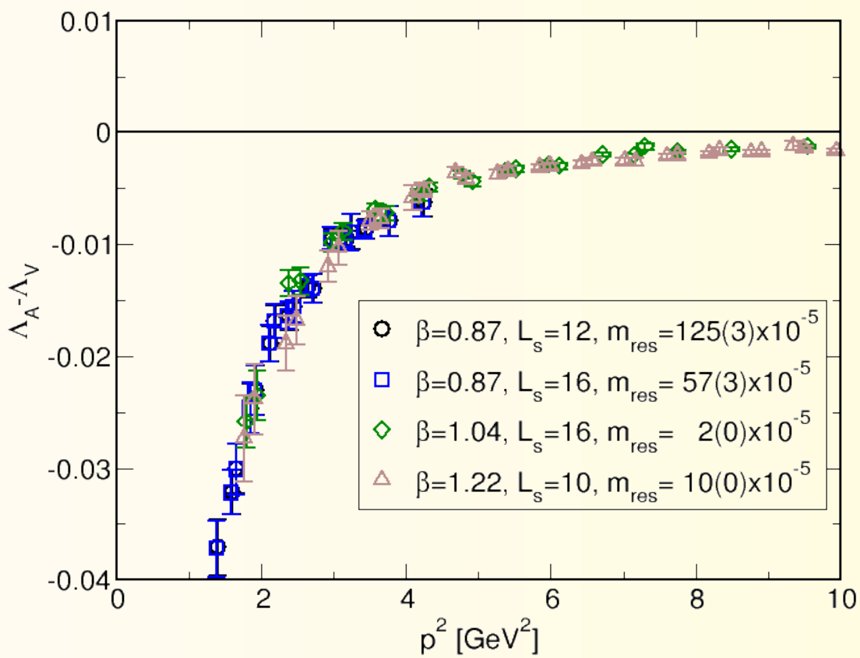
- $1/p^2$ through Weinberg's theorem

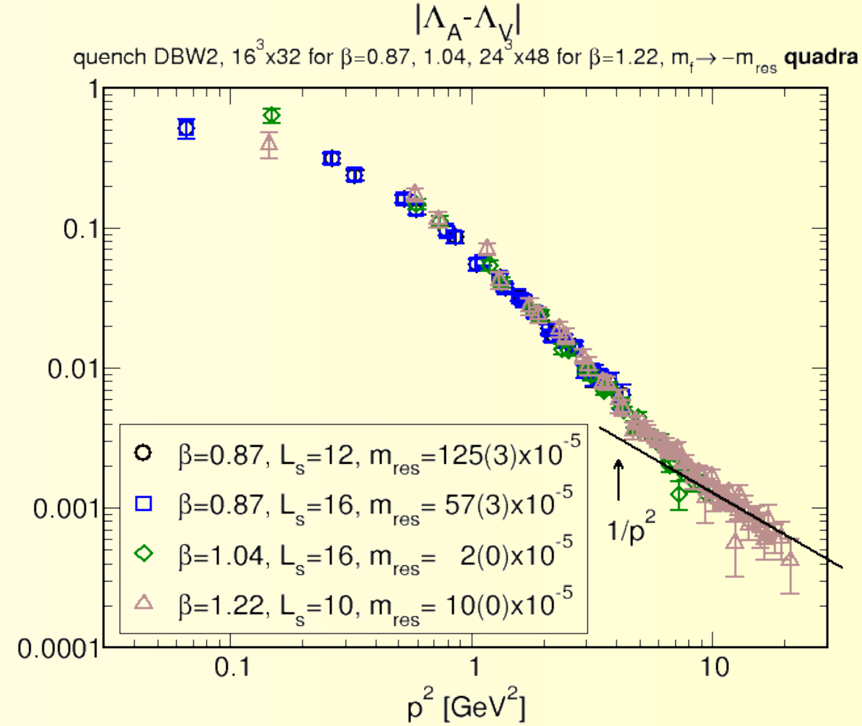
  for the exceptional momenta

  used in the conventional RI:
  - one gluon exchange: $1/p^2$
  - upper part affected by different NP depending on the opeator
  - P: with pion pole
  - S: double pole (quench) from topological near zero mode
  - A-V: $1/p^2$
- suppressed if you can make sure momenta in every part of the diagram scales as p: non-exceptional momenta
  - P: no pion pole
  - S: no double pole
  - A-V: $1/p^6$ perhaps invisible

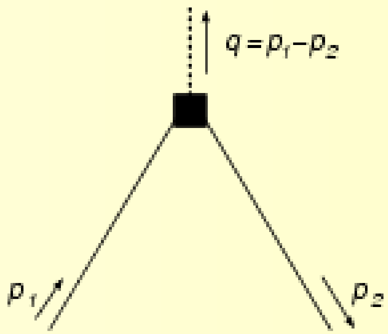
$\Lambda_A - \Lambda_V$

- quench DWF: $a^{-1} = 1.3,\ 2,\ 3$ GeV





sRI scheme



- uses symmetric point: $p_1^2 = p_2^2 = q^2$; $(q_\mu = 0 \text{ in RI})$
- renormalization conditions: $\dfrac{Z_O}{Z_q}\dfrac{1}{12}\mathrm{Tr}(\Pi_O P_O) = 1$

$$sRI: \begin{cases} P_S = 1 \\ P_P = \gamma_5 \\ P_V = \frac{1}{q^2}\not{q}q_\mu \\ P_A = \frac{1}{q^2}\gamma_5\not{q}q_\mu \end{cases} \qquad RI: \begin{cases} P_S = 1 \\ P_P = \gamma_5 \\ P_V = \frac{1}{4}\gamma_\mu \\ P_A = \frac{1}{4}\gamma_5\gamma_\mu \end{cases}$$

- through Ward identity the conditions on V, A are compatible with

sRI: $Z_q(sRI)=Z_q(RI')$

$$\frac{1}{p^2}\mathrm{Tr}[-i\not{p}S_R^{-1}(p)]\Big|_{p^2=\mu^2} = 1$$

RI: V, (A at large p^2)

$$\frac{1}{12}\mathrm{Tr}\left[-i\frac{\partial}{\partial\not{p}}S_R^{-1}(p)\right]\Big|_{p^2=\mu^2} = 1$$

- S, P → $Z_m=1/Z_S=1/Z_P$: condition on propagator

sRI: $$\lim_{m_R\to 0}\frac{1}{12m_R}\mathrm{Tr}[S_R^{-1}(p)]_{p^2=\mu^2} = 1 + \lim_{m_R\to 0}\frac{1}{24m_R}\mathrm{Tr}[q_\mu\Pi^\mu_{A,R}\gamma_5]$$

$sRI \to \overline{\mathrm{MS}}$ perturbative matching

- $sRI \to \overline{\mathrm{MS}}$ conversion factor

$$C_m = 1 + \frac{\alpha_s}{4\pi} C_F c_m^{(1)} \qquad c_m^{(1)} = \begin{cases} 0.484 - 0.172\xi & (sRI) \\ 4 - \xi & (RI) \end{cases}$$

  - sRI: smaller constant and gauge dependence

- size of 1-loop correction at $\mu = 2\ \mathrm{GeV}$ in Landau gauge

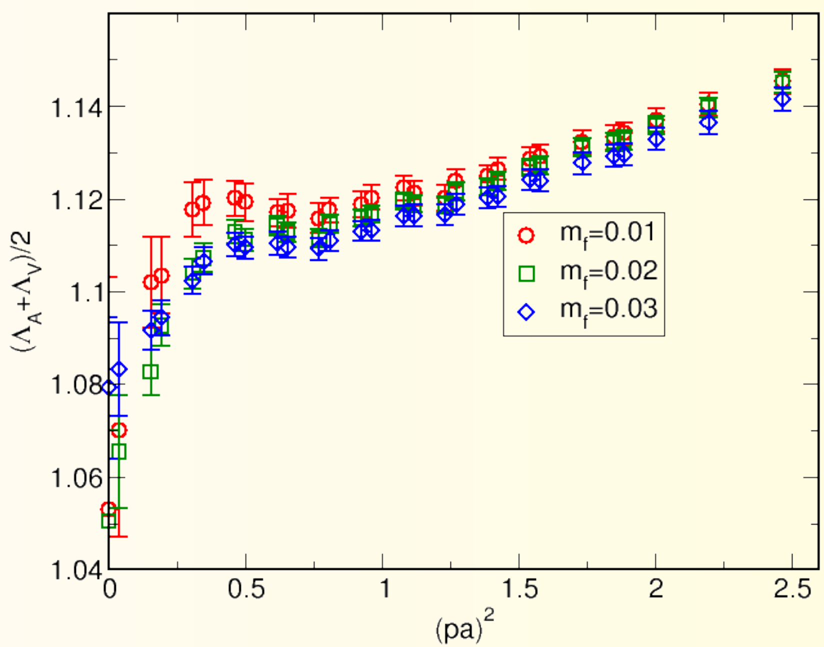
$$\begin{cases} 1.5\% & (sRI) \\ 12.3\% & (RI,\ 1\ \mathrm{loop}) \\ 6.2\% & (RI,\ 3\ \mathrm{loop}) \end{cases}$$

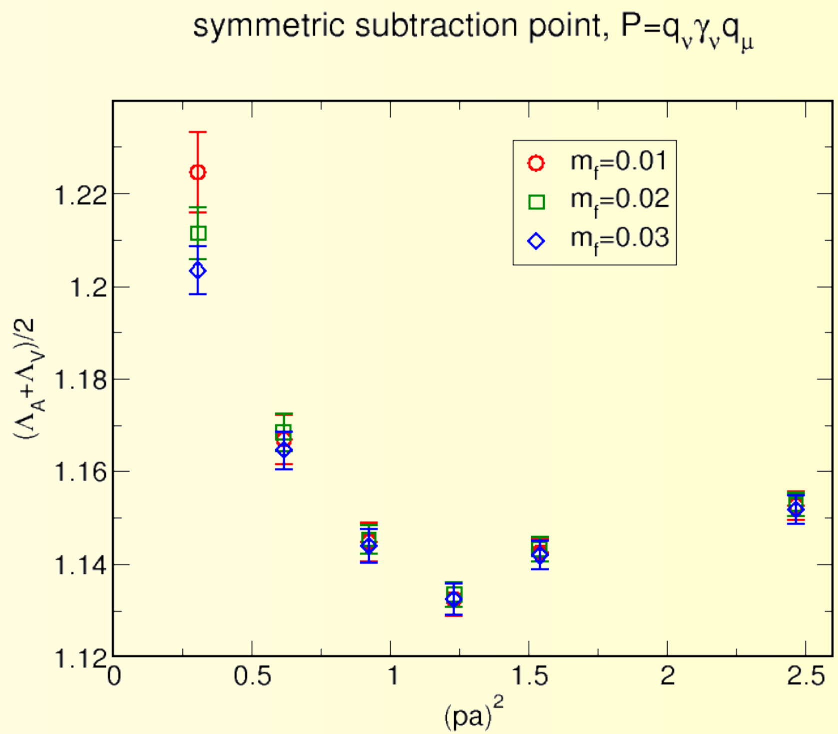
  - sRI: very small already at 1 loop

RI and sRI NPR: $(\Lambda_A + \Lambda_V)/2$ for Z_q

DWF $\quad n_f = 2+1,\ \beta = 2.13,\ 16^3 \times 32$

- RI: $\Lambda_A = \frac{1}{48}\mathrm{Tr}(\Pi^\mu_A \cdot \gamma_5\gamma_\mu)$
- sRI: $\Lambda_A = \frac{1}{12q^2}\mathrm{Tr}(\Pi^\mu_A \cdot \gamma_5 \not{q} q_\mu)$

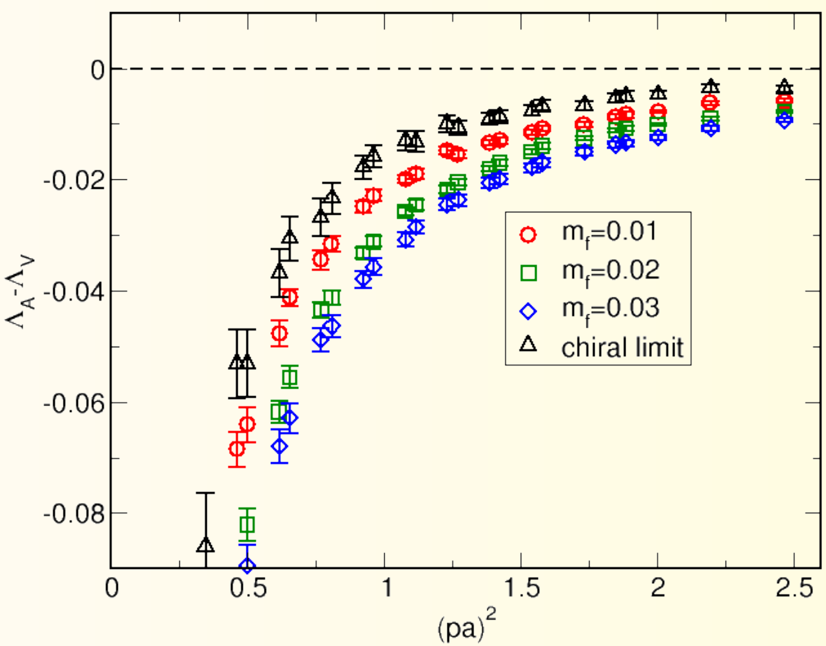




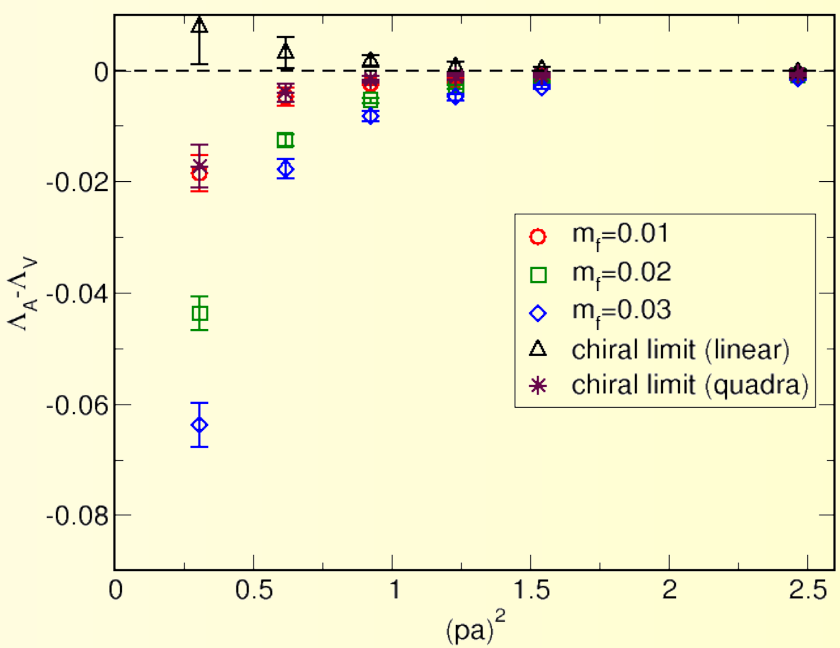
– tiny mass dependence

RI and sRI NPR: $(\Lambda_A - \Lambda_V)$

- RI

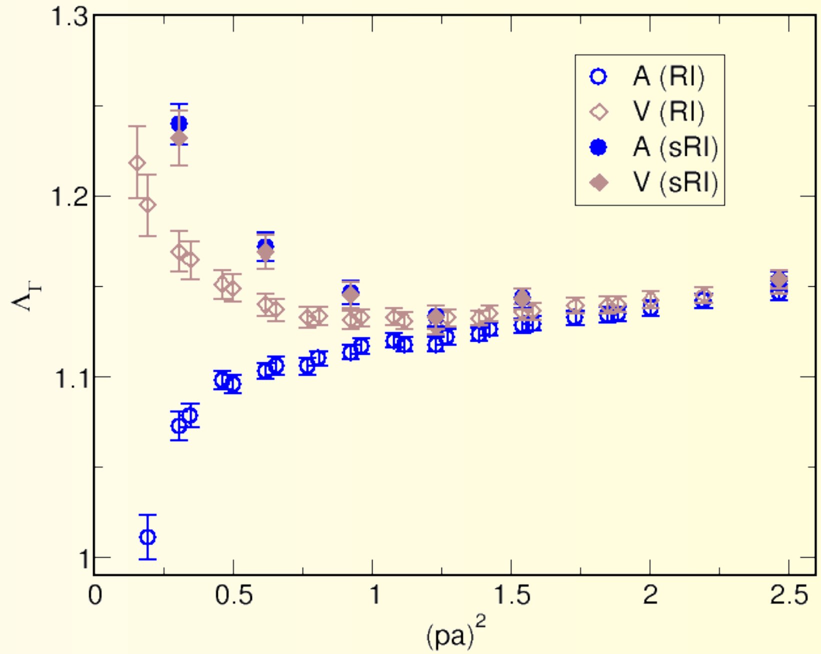


- sRI



  - invisible symmetry breaking effect at the chiral limit

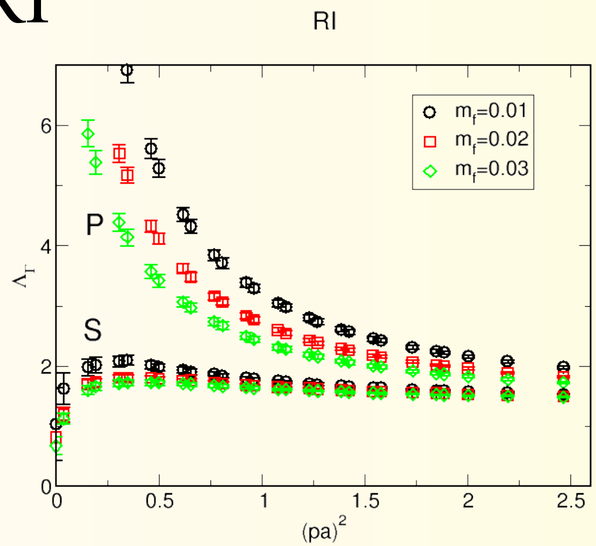
A and V for RI and sRI



- A-V difference for RI, no difference for sRI
  - compatible with Ward identities
- $\Lambda_V(RI) = Z_q/Z_A \to \Lambda_V(sRI) = Z'_q/Z_A$ for large p²
  - compatible with PT: these are same at $O(\alpha_s)$

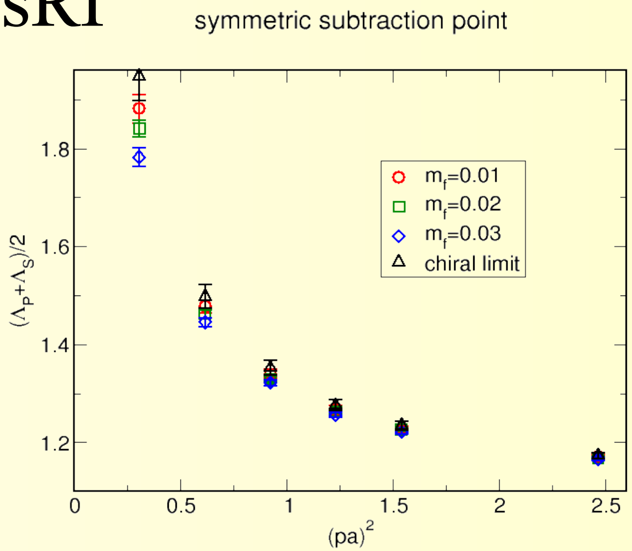
RI and sRI $\Lambda_{S,P} = Z_q Z_m$

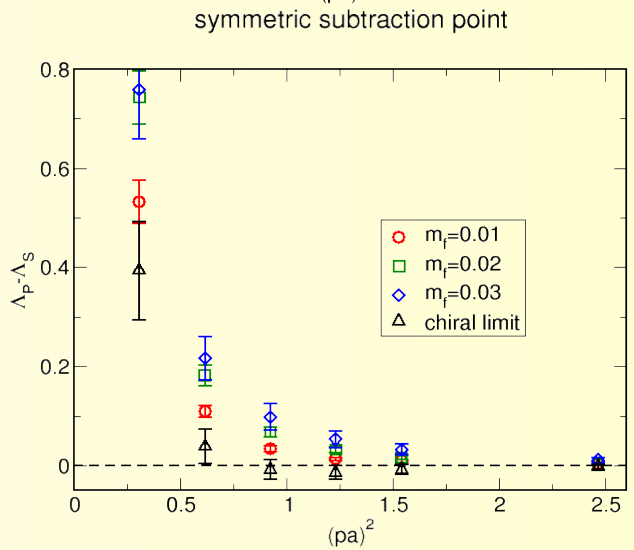
- RI



- mass dependence
  - large (RI)→large m_s error
  - small (sRI)
- S, P symmetry
  - broken (RI), intact (sRI)

- sRI





error budget on Z_m

RI

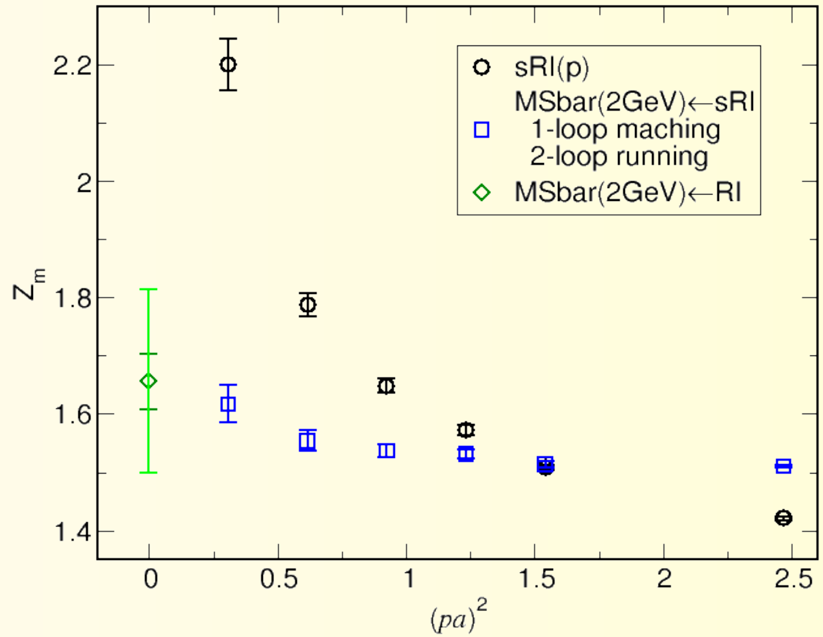
- 7% from $m_s \neq 0$
  - NP contamination
- 6% from $O(\alpha_s^3)$
- 3% statistical
- 9% in total

sRI

- much smaller
- 1.5% already at $O(\alpha_s)$
- a little smaller
  - drastic improvement possible

  $\rightarrow$ talks: C.Kelly, D.Brömmel
- total: hopefully very small

sRI Z_m



- 1 loop matching and 2 loop running makes flat $(pa)^2$ dep
- preliminary result, $(pa)^2 \to 0$ extrapolation not attempted
- consistent with $RI \to \overline{\mathrm{MS}}$ which has large systematic error

Summary

- conventional RI scheme uses exceptional momenta, thus has sizable non-perturbative contamination. Non-exceptional momenta are better.
- sRI scheme using non-exceptional momenta constructed.
- $\mathrm{sRI} \to \overline{\mathrm{MS}}$ matching calculated in 1 loop. The correction is already very small.
- Using NPR data with 2+1 f DWF, sRI scheme NPR plots are shown. Large reduction of the systematic is observed.
- Preliminary analysis on Z_m shown. It looks promising.